SIMULTANEOUS EVOLUTIONARY FITS TO JUPITER AND SATURN WITH FUZZY CORES

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Introduction

Early models of Jupiter and Saturn assumed homogeneous, adiabatic interiors, successfully explaining Jupiter's luminosity but not Saturn's temperature. Stevenson (1997) proposed that helium rain from hydrogen-helium immiscibility in Saturn's interior could slow cooling by releasing gravitational energy. This idea gained support after Galileo measured Jupiter's subsolar atmospheric helium, motivating non-homogeneous models with internal helium gradients.

Evolutionary studies with composition gradients show that a planet's initial structure strongly affects its evolution. However, many past models either neglected helium rain or failed to match the extended fuzzy cores suggested by recent data.

This work aims to build a unified evolutionary model for Jupiter and Saturn. Our approach incorporates helium rain, updated equations of state, modern atmospheric boundary conditions, and realistic heavy-element masses to match each planet's current effective temperature, atmospheric composition, radius, and gravitational moments. We also generalize our models by varying the density ratio parameter:

$$R_\rho \equiv \frac{\delta}{\varphi}\frac{(\nabla_T - \nabla_{\rm ad})}{\nabla_\mu}, \quad (1)$$

where $R_\rho = 0$ is Schwarzschild convection, $R_\rho = 1$ is Ledoux convection, while any value in between models semi-convection.

New Jupiter and Saturn Models with the APPLE Code

We use APPLE (Sur et al. 2024) [1] for computing all evolutionary models. We use the Chabrier-Debras 2021 EOS for the H-He mixture, while the heavy elements in the envelope are modeled using the AQUA EOS. We use the Lorenzen (2009-2011, see figure 1) H-He miscibility curve to model the helium rain and use the Chen et al. 2023 [2] atmospheric boundary condition to calculate the heat loss from the surface.

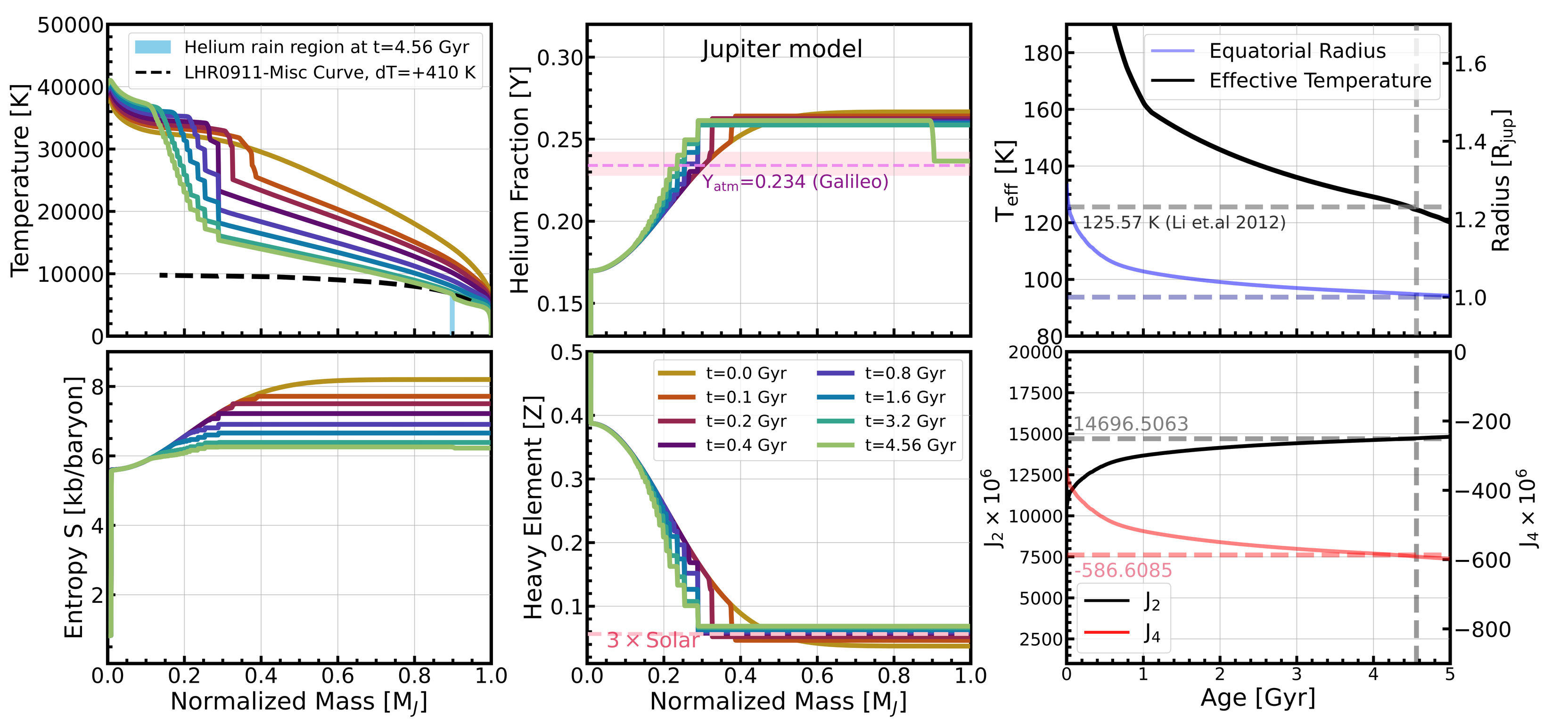

Figure 1: A very slightly improved evolutionary model for Jupiter [3] with an initial fuzzy core that matches within observational uncertainties the current values of the effective temperature, equatorial radius, atmospheric helium abundance, outer envelope metallicity, and J_2 and J_4 gravitational moments. This Jupiter model contains 42.5 M$_\oplus$ of heavy elements, including a compact core of 3 M$_\oplus$($Z = 1$). Helium rain begins at 4 Gyrs, based on the Lorenzen miscibility curves with a +410 K temperature shift, resulting in outer helium depletion to $Y = 0.236$, consistent with *Galileo* entry probe measurements.

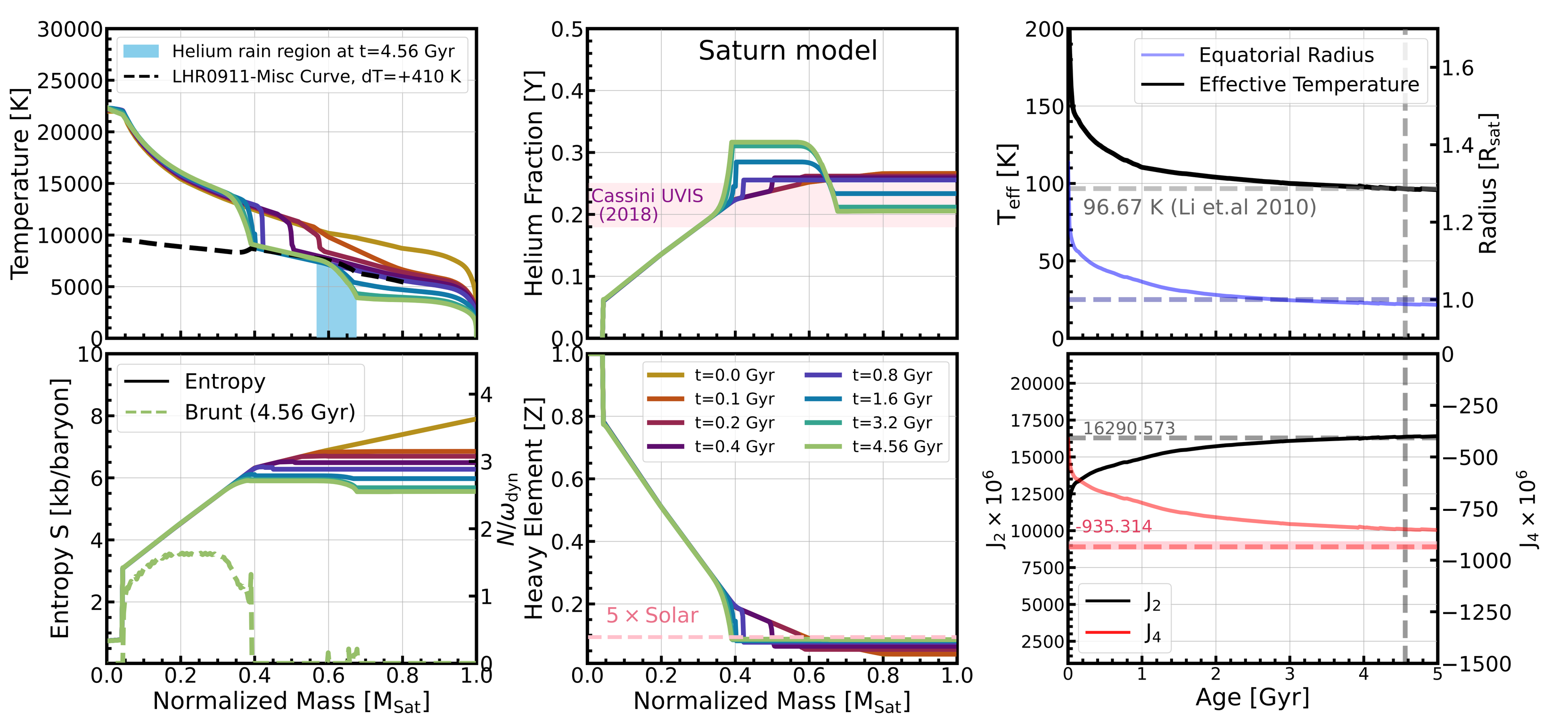

Figure 2: The best fit evolutionary model for Saturn with an initial fuzzy core that matches its current values of the effective temperature, equatorial radius, atmospheric helium abundance, outer envelope metallicity, Brunt-Väisälä frequency, and gravitational moments within observational uncertainties. This model contains 25 M$_\oplus$ of heavy elements, including a compact core of 4 M$_\oplus$. The Brunt-Väisälä ratio of approximately 1.7 at 4.56 Gyr, closely aligning with the value of 2 predicted by [4], is shown in the bottom left panel.

Quantity	Jupiter		Saturn	
	Measured	Our Model (4.56 Gyr)	Measured	Our Model (4.56 Gyr)
$T_{\rm eff}$ [K]	125.57 ± 0.07	124.6	96.67	96.54
Radius [km]	71492 ±4	72019.5	60268 ±4	59,551.8
$Y_{\rm atm}$	0.234 ±0.005	0.236	0.075-0.22	0.205
$(Z_{\rm atm}/Z_\odot)$	1.5–5	3.6	5.0–10.0	4.6
$J_2 \times 10^6$	14696.572	14731.6	16290.573	16365.7
$J_4 \times 10^6$	-586.609	-591.46	-935.314	-850.11

Variation with Semi-convective Parameter R_ρ

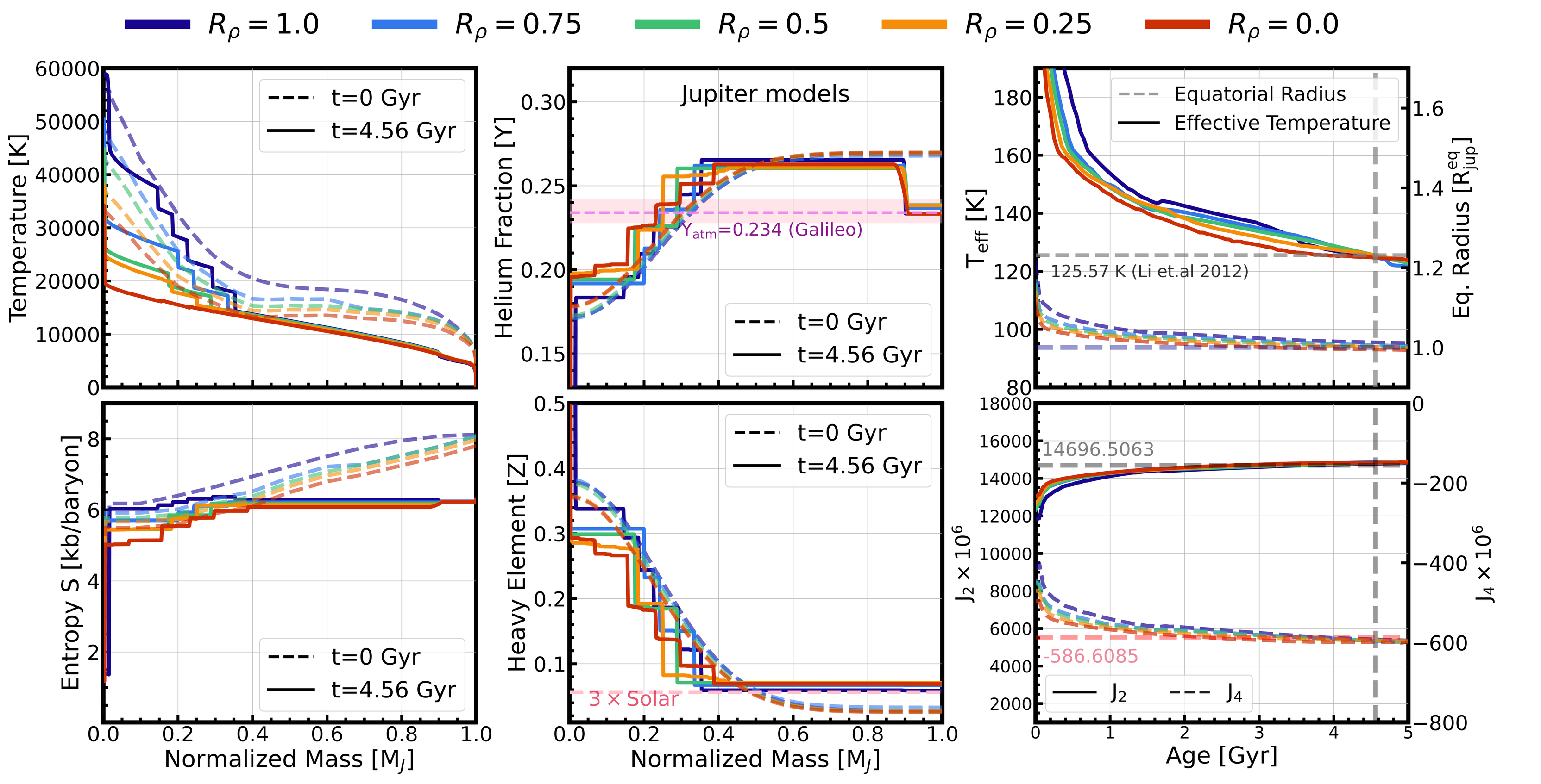

Figure 3: Same as Figure 1 but showing the variation of various quantities as a function of R_ρ at two different times.

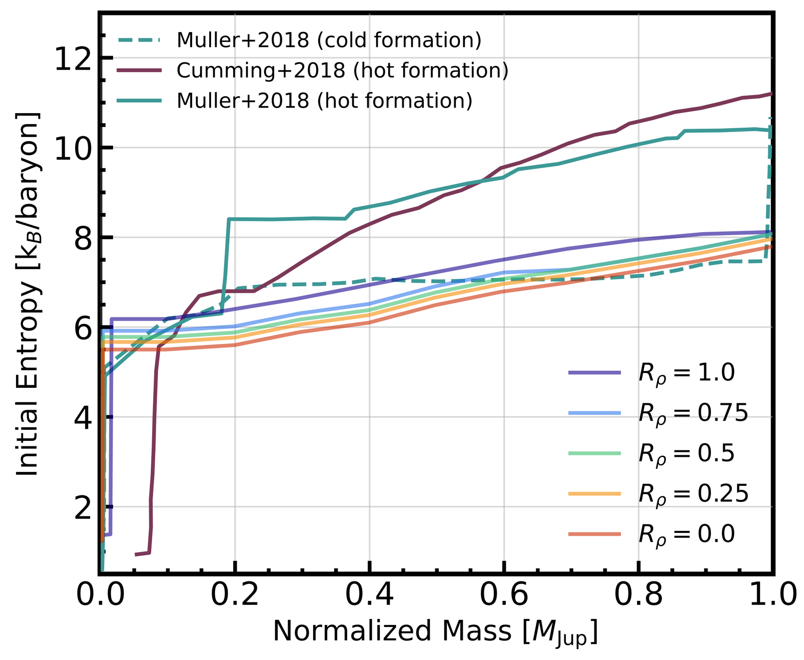

Figure 4: Initial entropy profiles for our Jupiter models with varying density ratio R_ρ, compared against hot-start (solid lines) and cold-start (dashed lines) initial conditions.

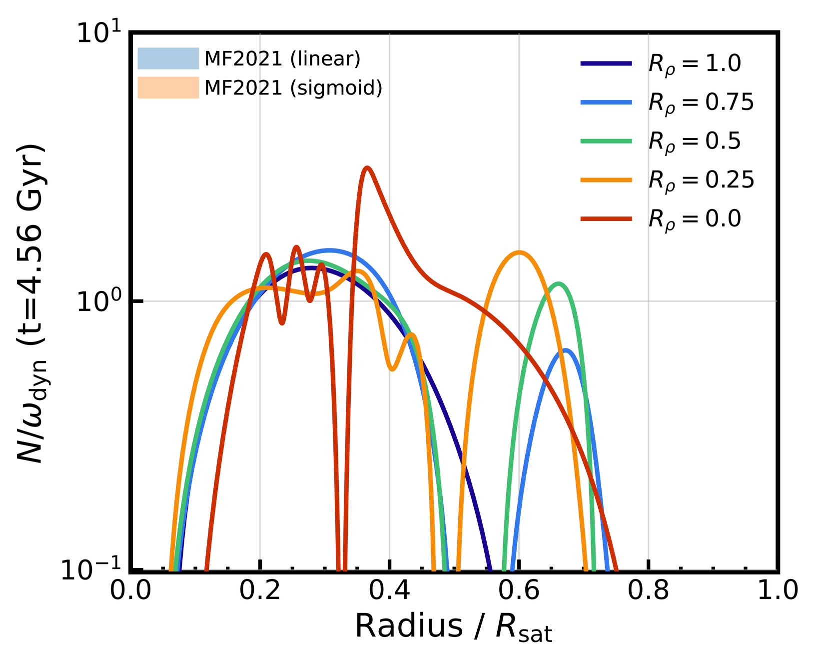

Figure 5: Brunt-Väisälä frequency for Saturn compared with the predicted seismologically inferred stratified zone from Mankovich Fuller (2021, MF2021), with a characteristic ratio of 2 extending to 60% of Saturn's radius.

Conclusions

- We find that planets start "cold" as opposed to "hot" initial conditions.
- Decreasing R_ρ increases the degree of mixing between the deep interior and the outer envelope.
- If a fuzzy heavy-element core survives from birth, then there is no helium ocean formed in Saturn. Helium piles up not in the interior but in an intermediate region bounded from above by the helium rain region.
- The predicted atmospheric helium abundance is ∼0.205, close to the measurements of Koskinen et al [5]. This is ∼3 times the prediction of Mankovich & Fortney 2020 [6] and is, in part, a consequence of the possible existence of an inner stable fuzzy core at the current epoch that has survived since formation.
- Models with lower R_ρ exhibit cooler interiors due to more efficient convective heat transport, which flattens temperature gradients and facilitates thermal equilibration throughout the planet.
- The radial extent of the fuzzy core remains nearly identical across all models, as it is constrained by the need to reproduce the measured gravitational moments.
- We simultaneously achieve a current Brunt-Väisälä ratio of ∼2 in the interior ∼50% of Saturn's radius, which in that region is stably stratified. The inner ∼40% by mass, as it is not convective, barely cools on solar-system timescales.
- As R_ρ decreases, two distinct regions of enhanced Brunt frequencies emerge: one within the dilute core and another in the helium rain layer. The mean Brunt frequency ratio increases as we move from $R_\rho = 1$ (Ledoux) to $R_\rho = 0$ (Schwarzschild), indicating stronger stable stratification in key interior regions.

References

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