Hands-on Session II

Fitting JWST Data: From Light Curves to Planet Spectra

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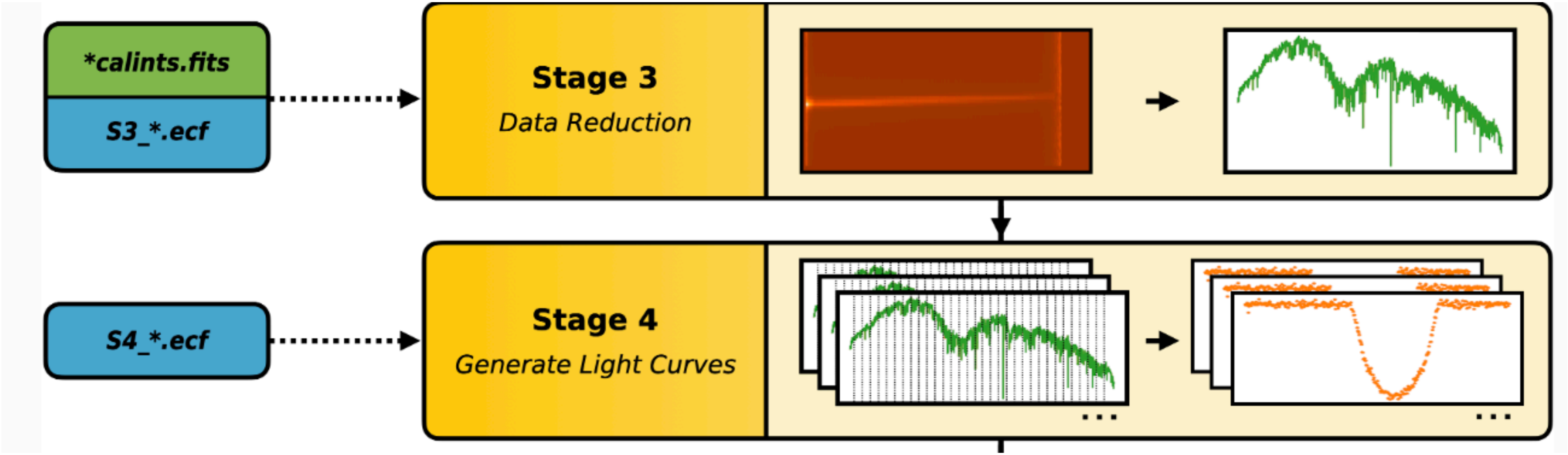
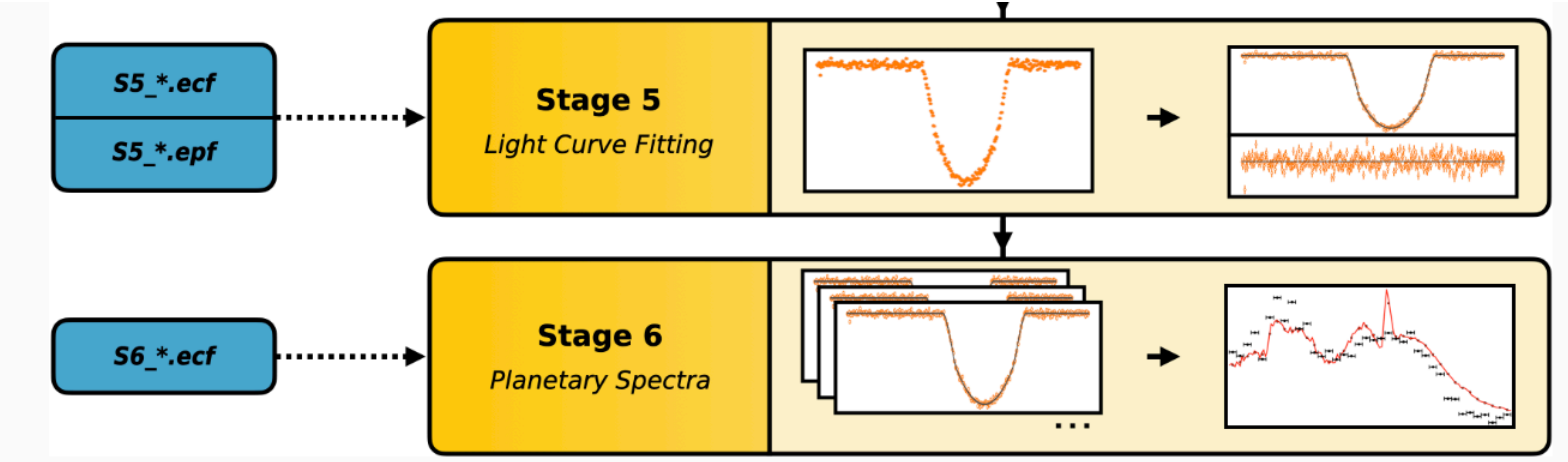
Recap from yesterday



2D images —> 1D spectra —> spectroscopic light curves

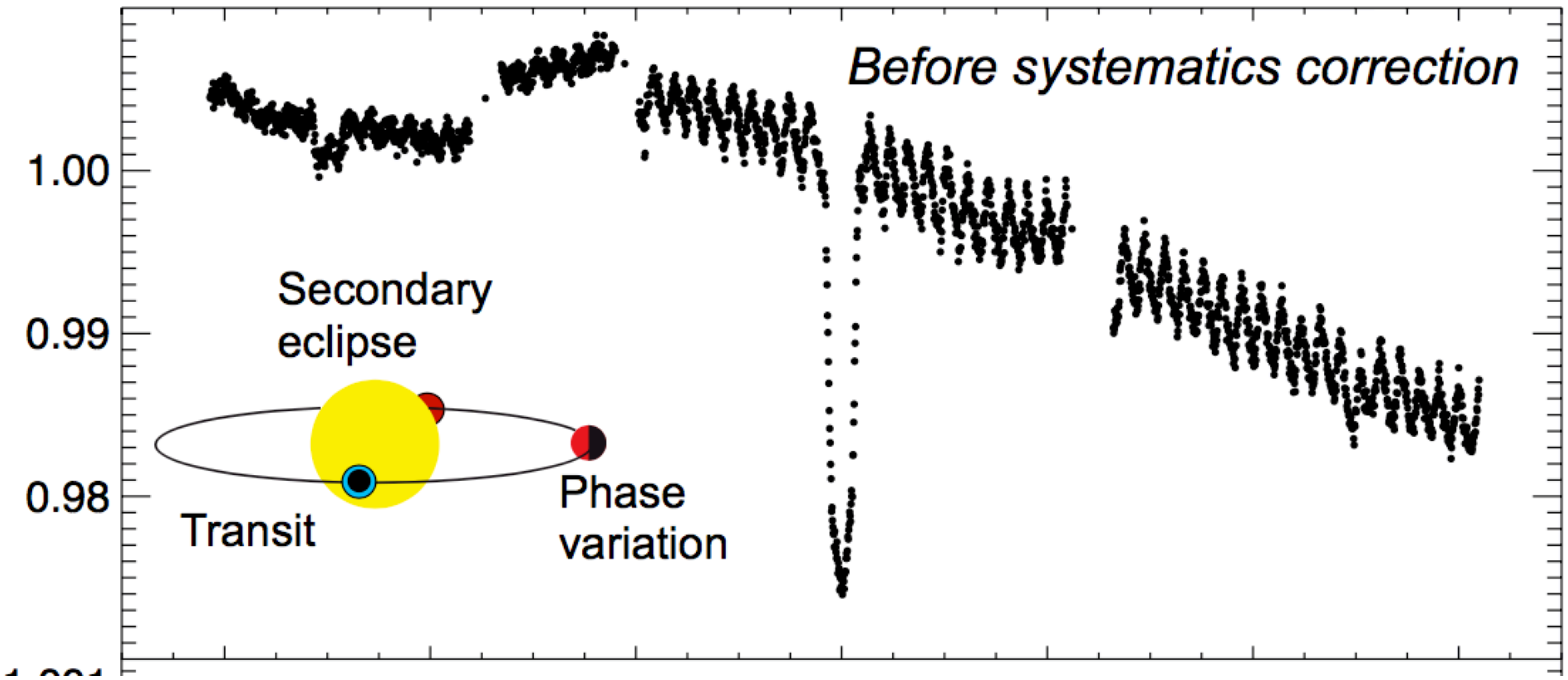
Today: goals for light curve fitting



We want to know:

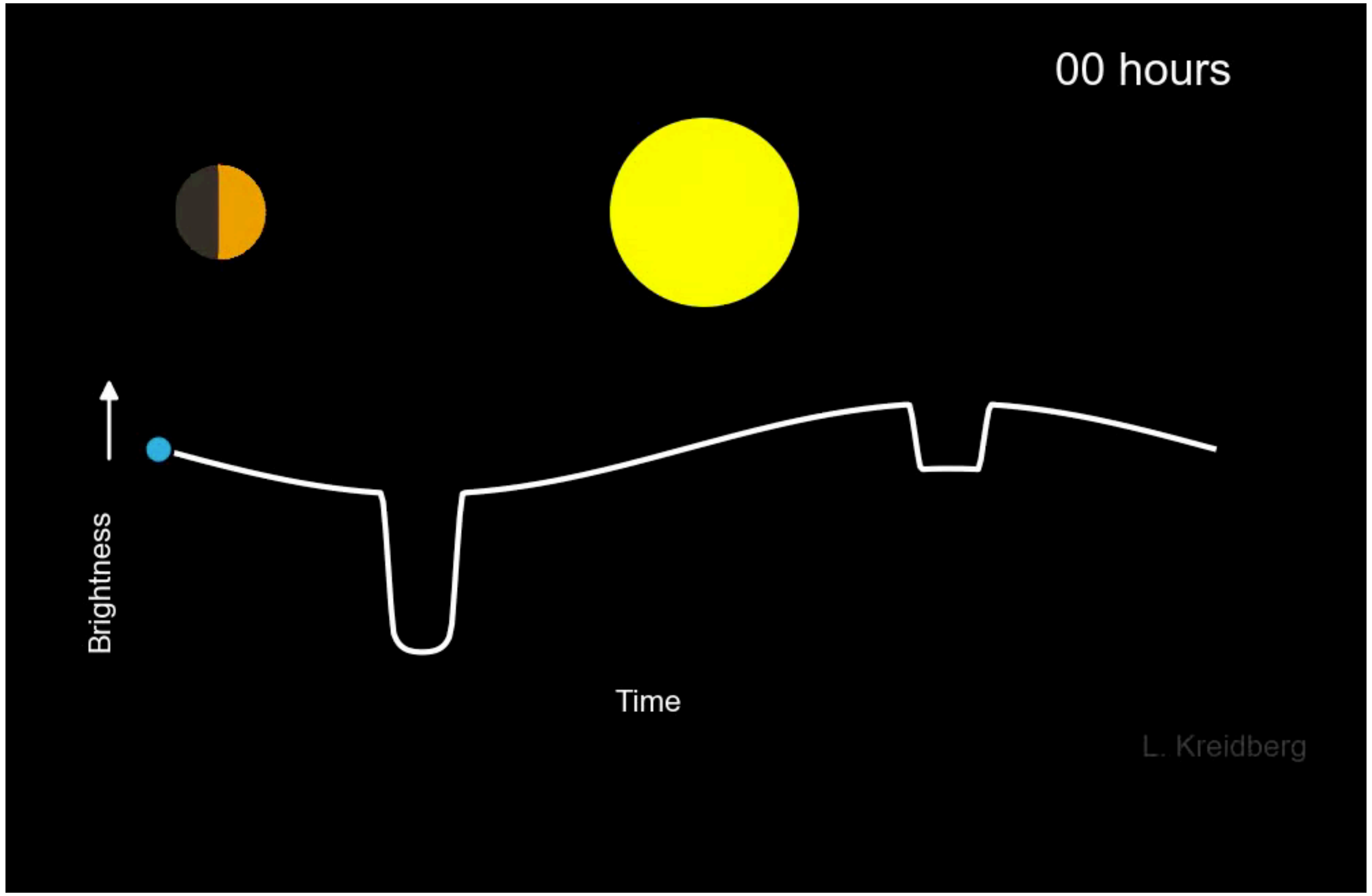
planet-to-star radius or planet-to-star flux versus wavelength

Light curve fitting: model the astrophysical signal and instrument systematics — simultaneously!



Raw light curve - Spitzer observations of HD 189733b
Knutson et al. 2008

Modeling the astrophysical signal



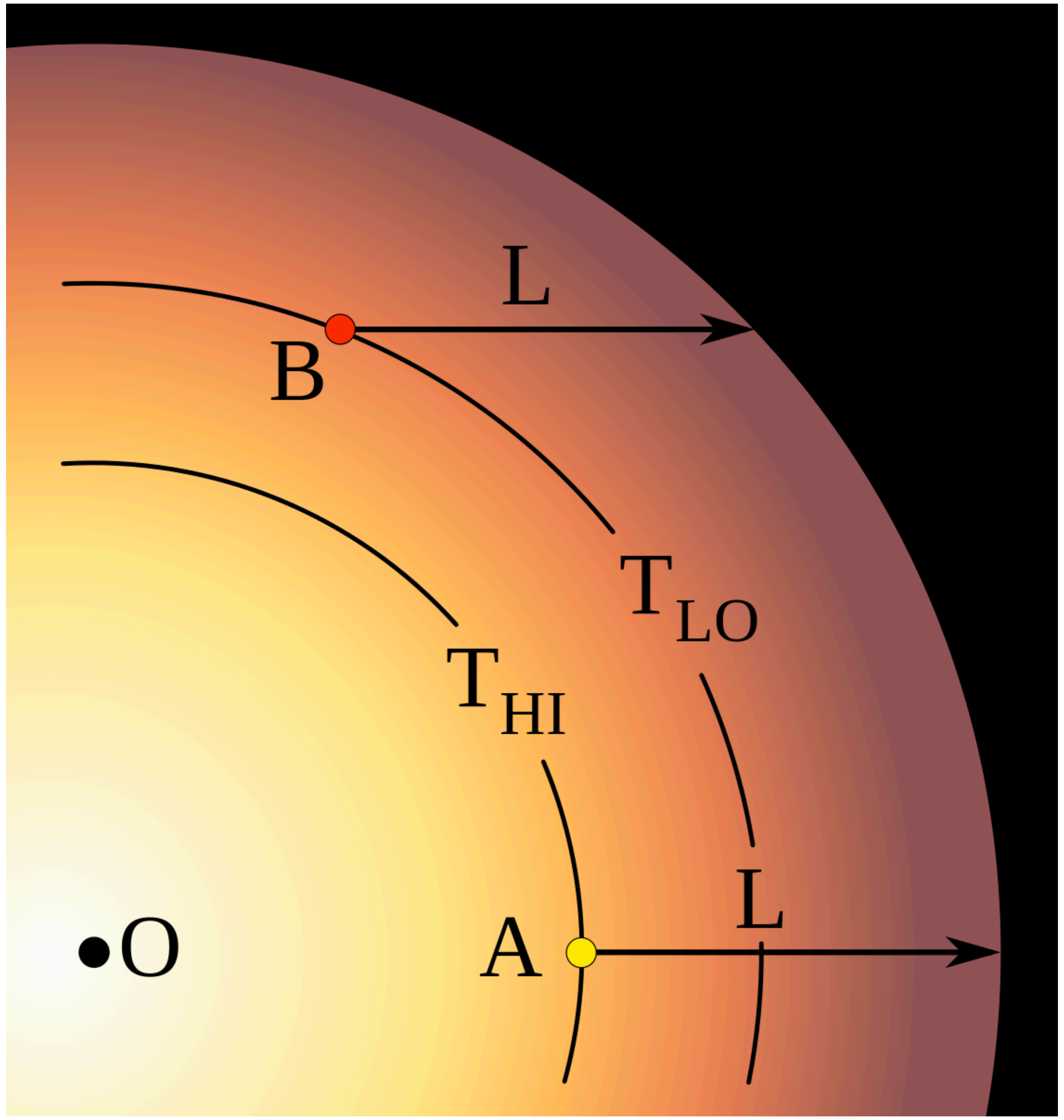
- Transits and eclipses
- Phase variation

Transit and eclipse models

- Size of the planet relative to the star (Rp/Rs)
- Eclipse depth (fp/fs)
- Distance from the planet to the star (a/Rs)
- Inclination of the planet along the line of sight (i)
- Orbital period (P)
- Transit time (t0)
- Eccentricity (e)
- Limb darkening! [u1, u2, …, un]

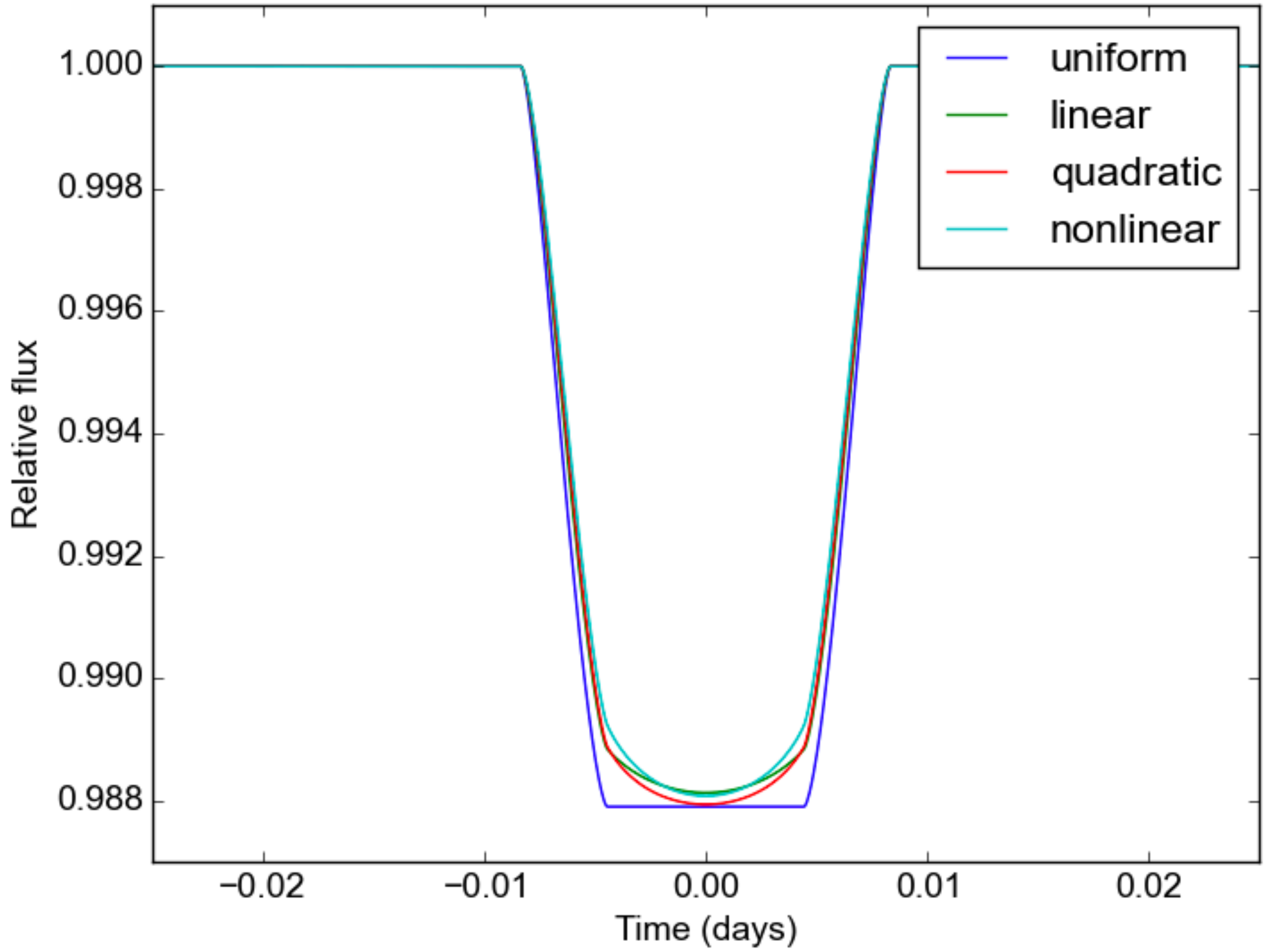
http://lkreidberg.github.io/batman

Limb darkening



- Can predict limb darkening from stellar models (e.g. PHOENIX, Kurucz), or fit for it in the light curve
- The transit depth and limb darkening are somewhat degenerate — be careful to get it right!

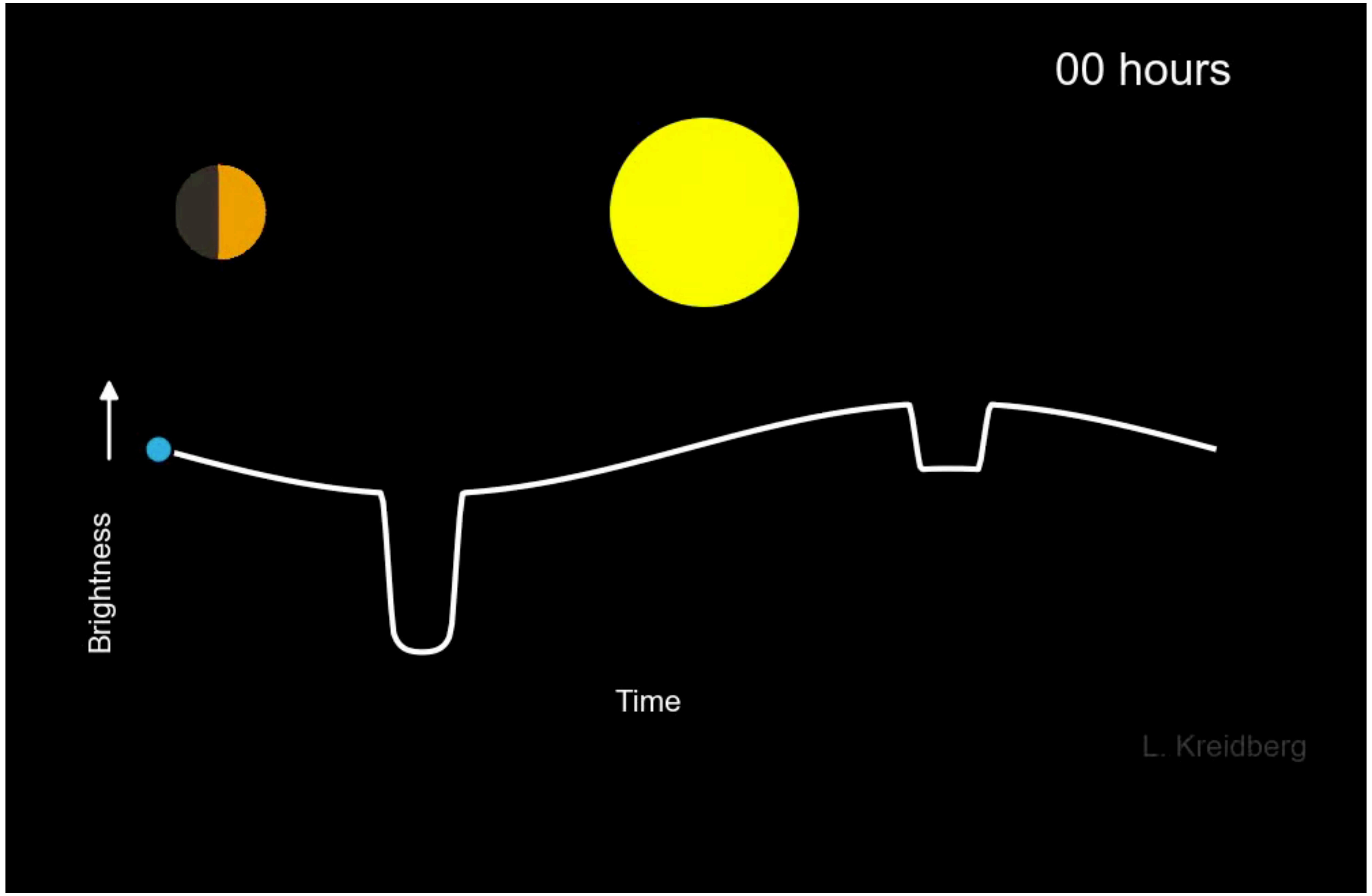
Limb darkening



Several popular parameterizations:

$$
\begin{aligned}
&(\text{uniform}) && I(\mu) = I_0 \\
&(\text{linear}) && I(\mu) = I_0[1 - c_1(1-\mu)] \\
&(\text{quadratic}) && I(\mu) = I_0[1 - c_1(1-\mu) - c_2(1-\mu)^2] \\
&(\text{nonlinear}) && I(\mu) = I_0[1 - c_1(1-\mu^{1/2}) - c_2(1-\mu) - c_3(1-\mu^{3/2}) - c_4(1-\mu^2)]
\end{aligned}
$$

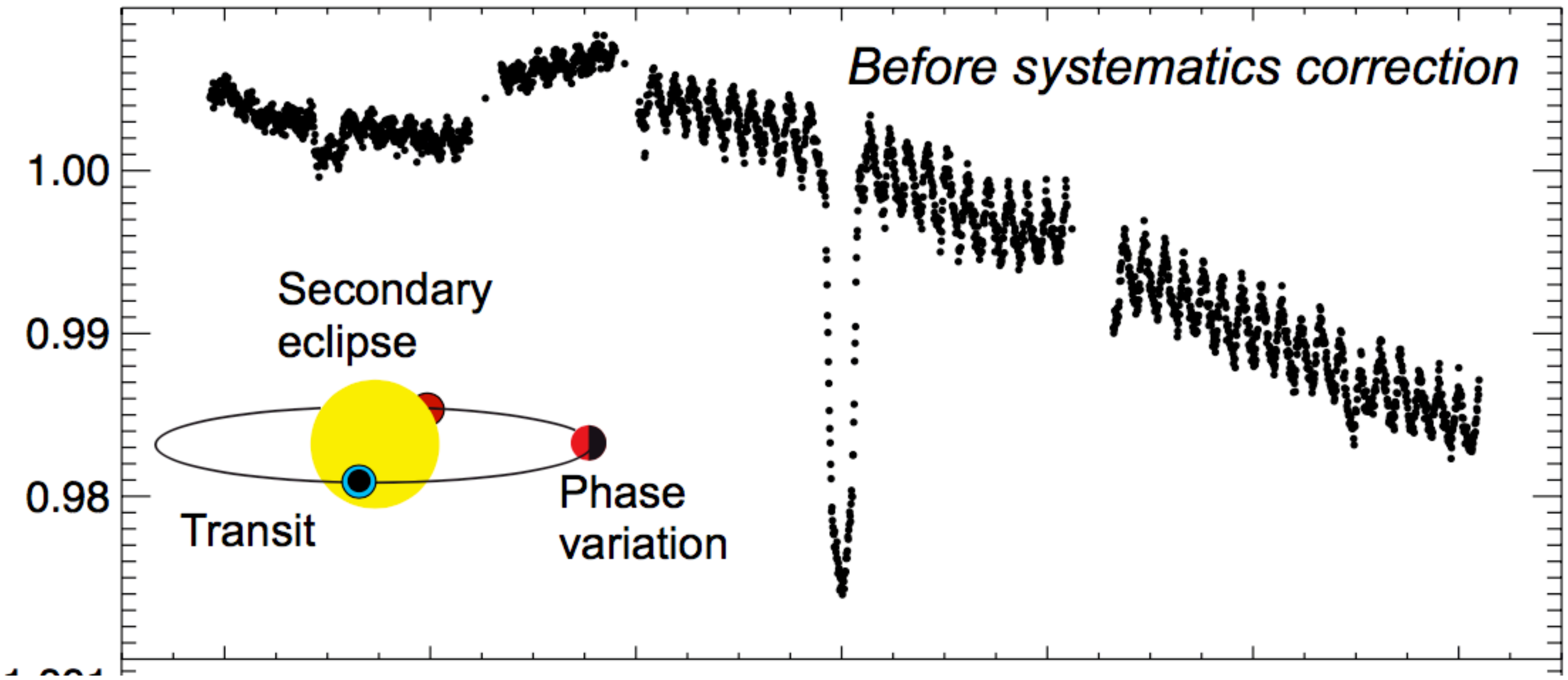
Phase variation



- Sinusoid(s)
- Spherical harmonics

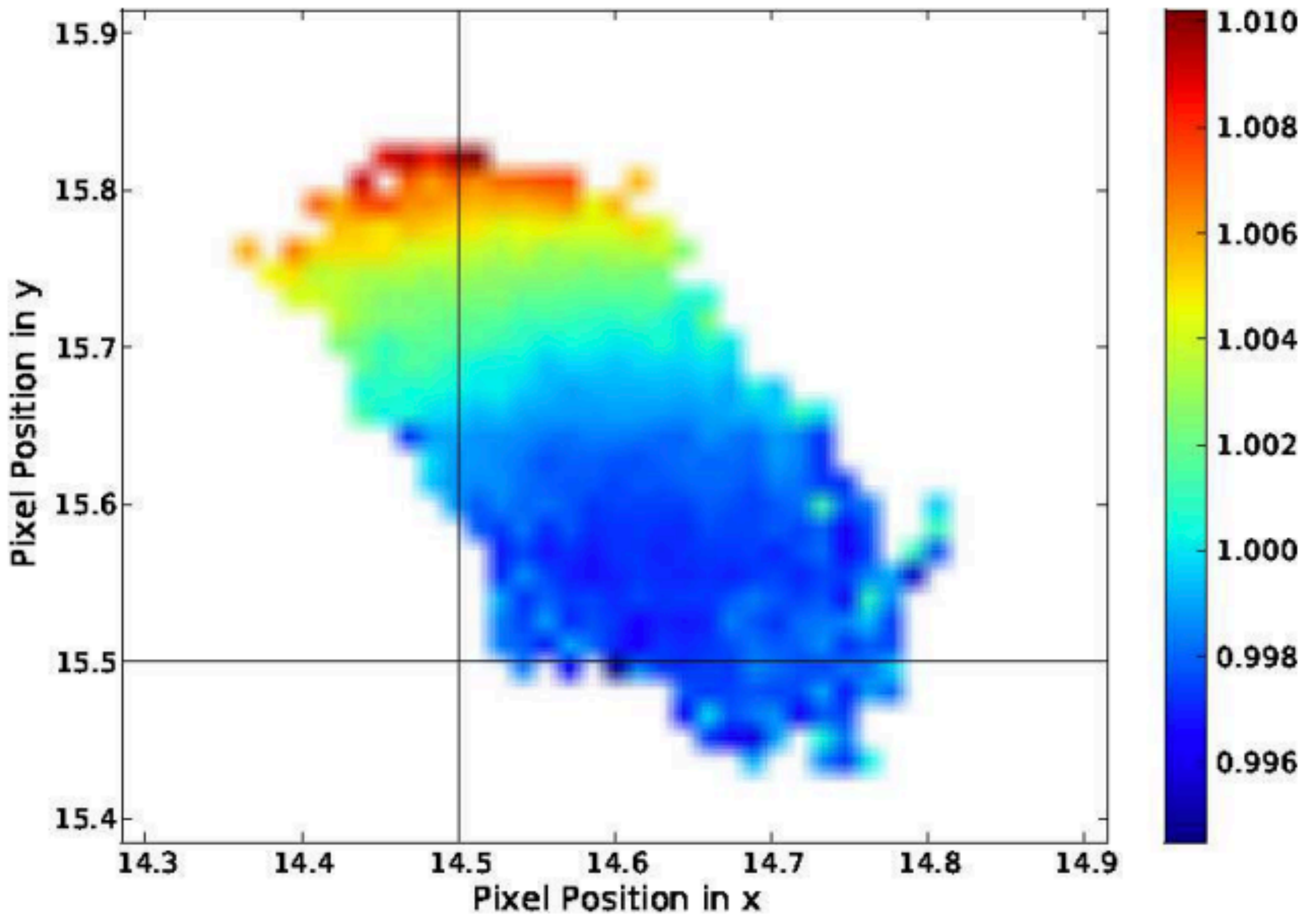
https://starry.readthedocs.io/en/latest/

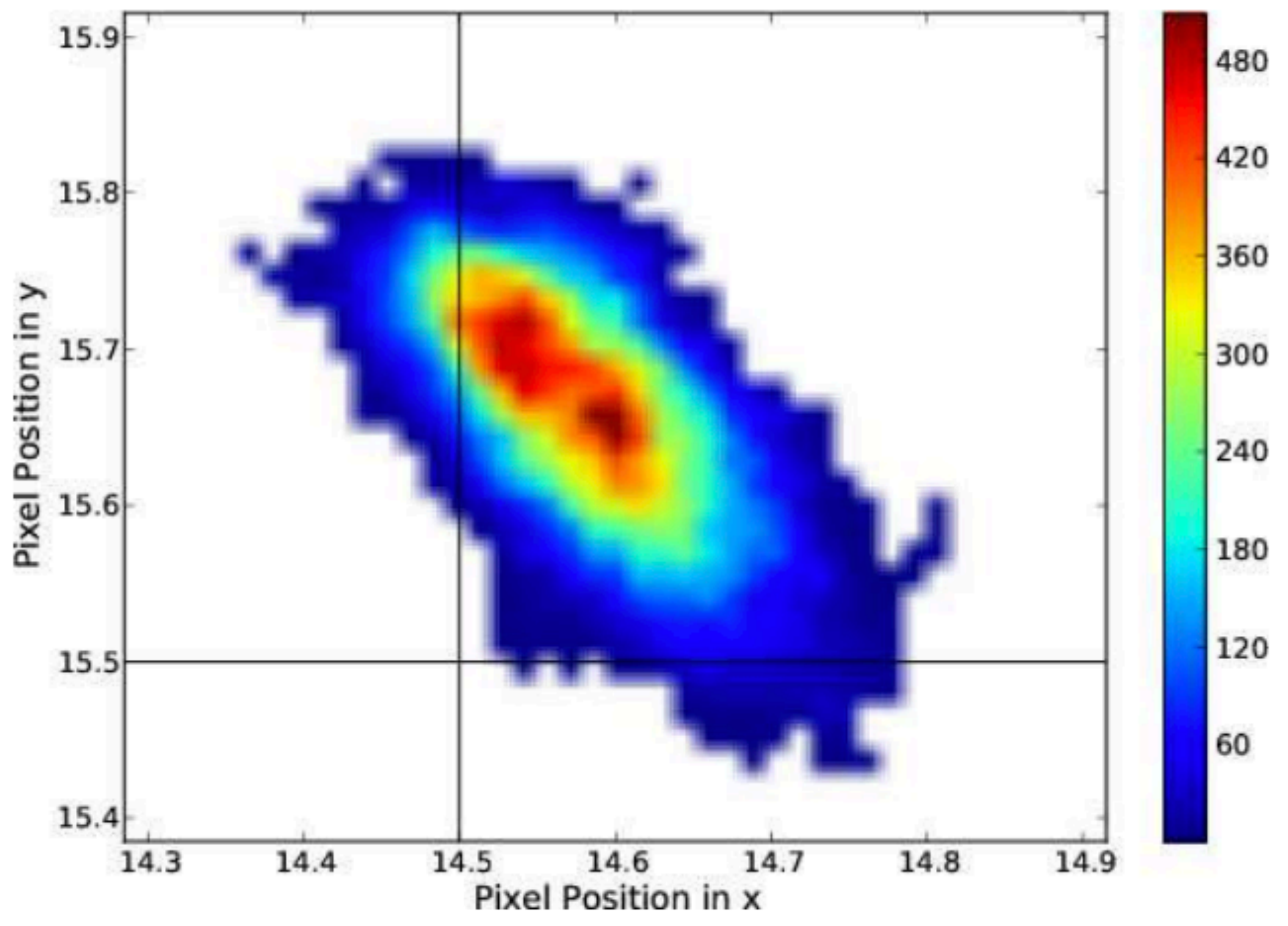
Instrument systematics 😭😭😭

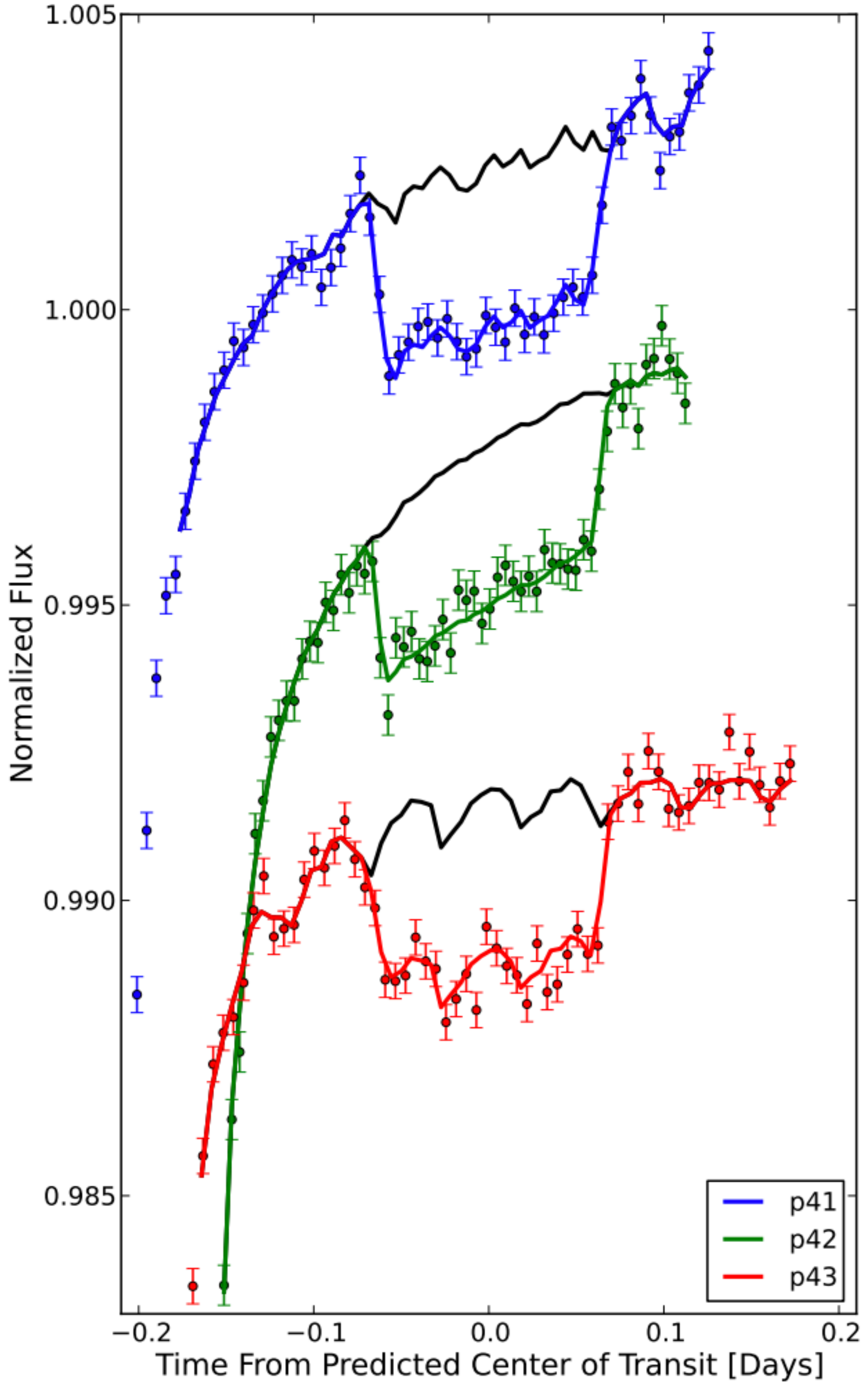


Raw light curve - Spitzer observations of HD 189733b
Knutson et al. 2008

Example instrument systematics

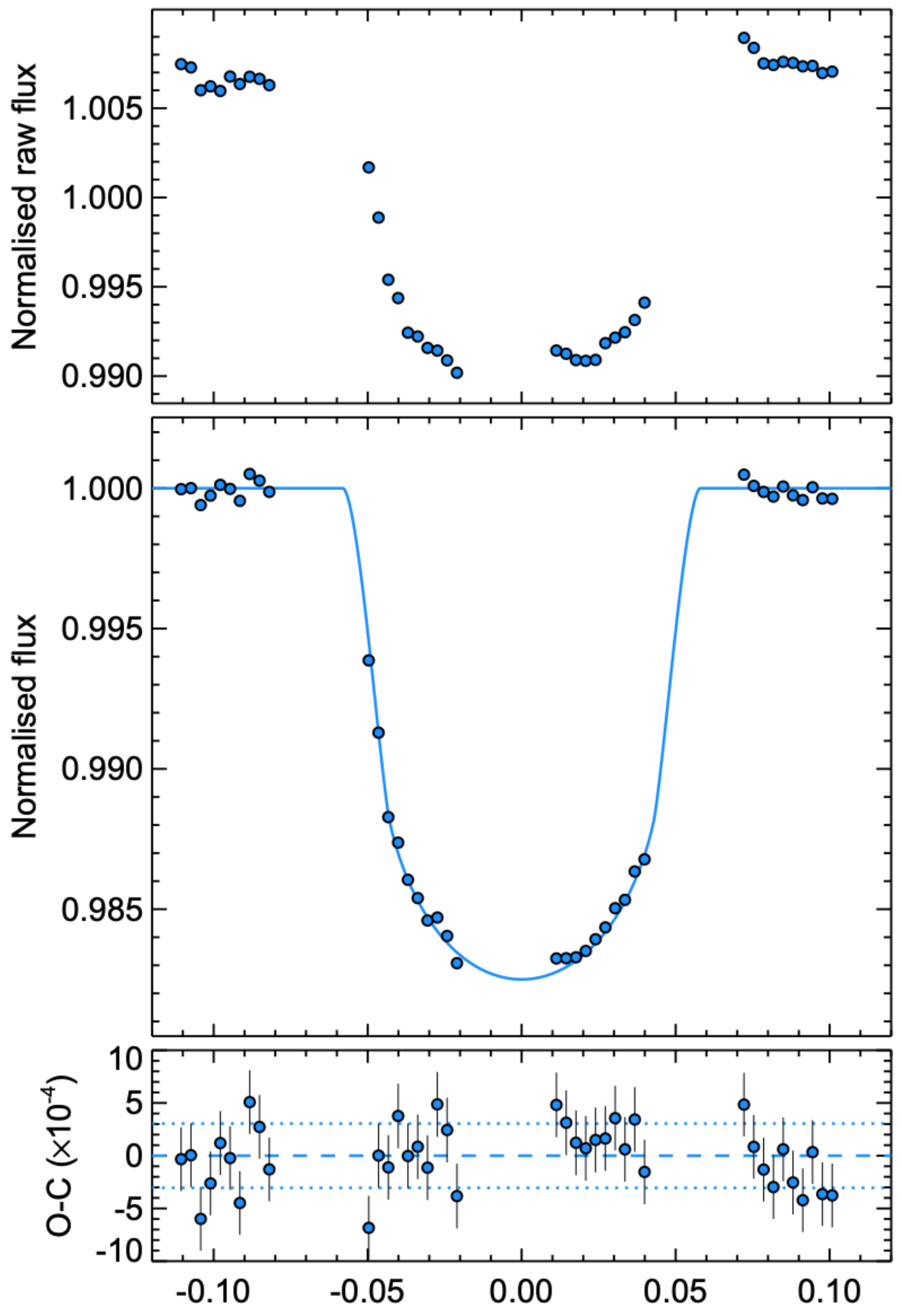






Spitzer
intrapixel sensitivity variation
Stevenson et al. 2012

Example instrument systematics

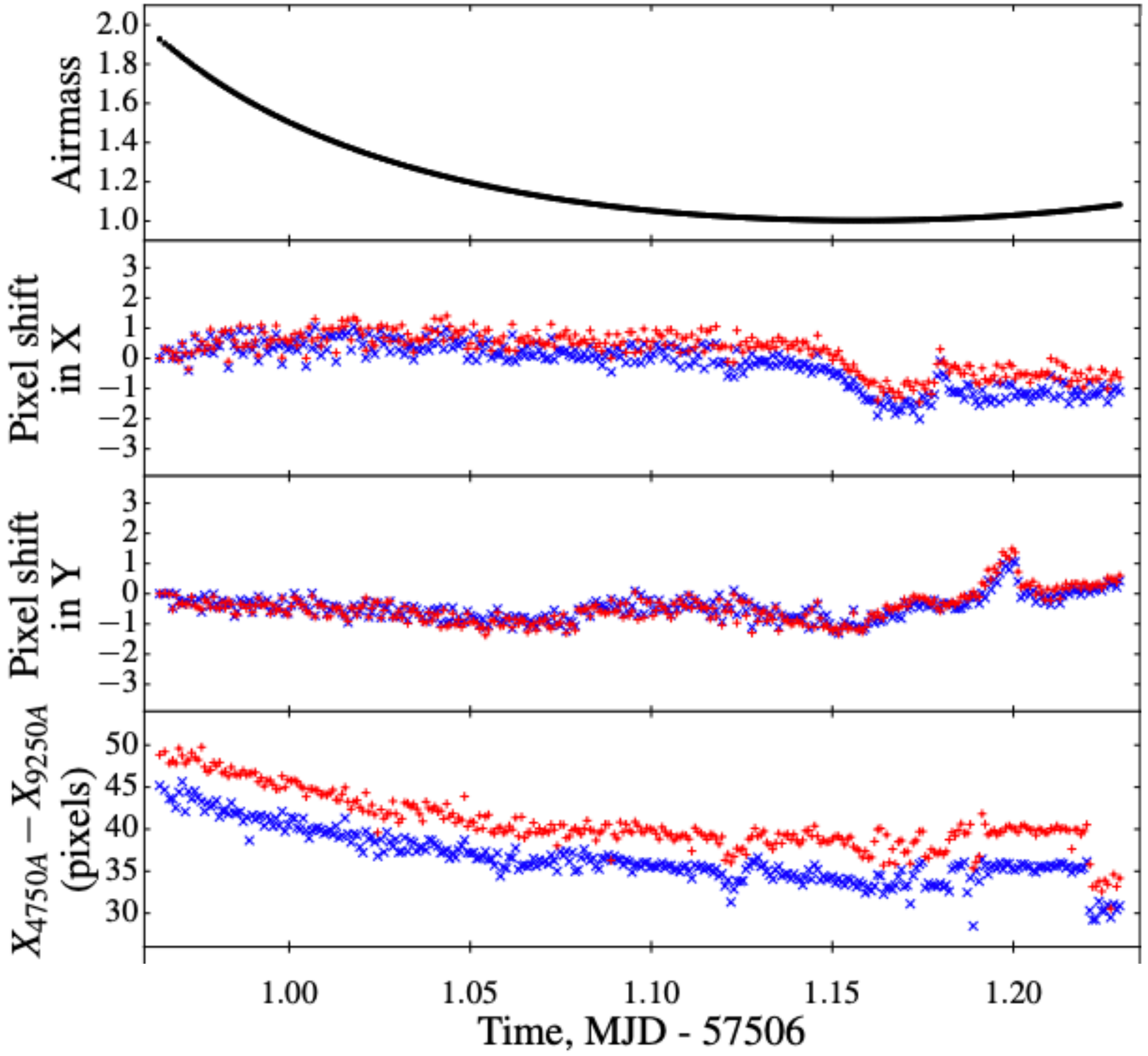


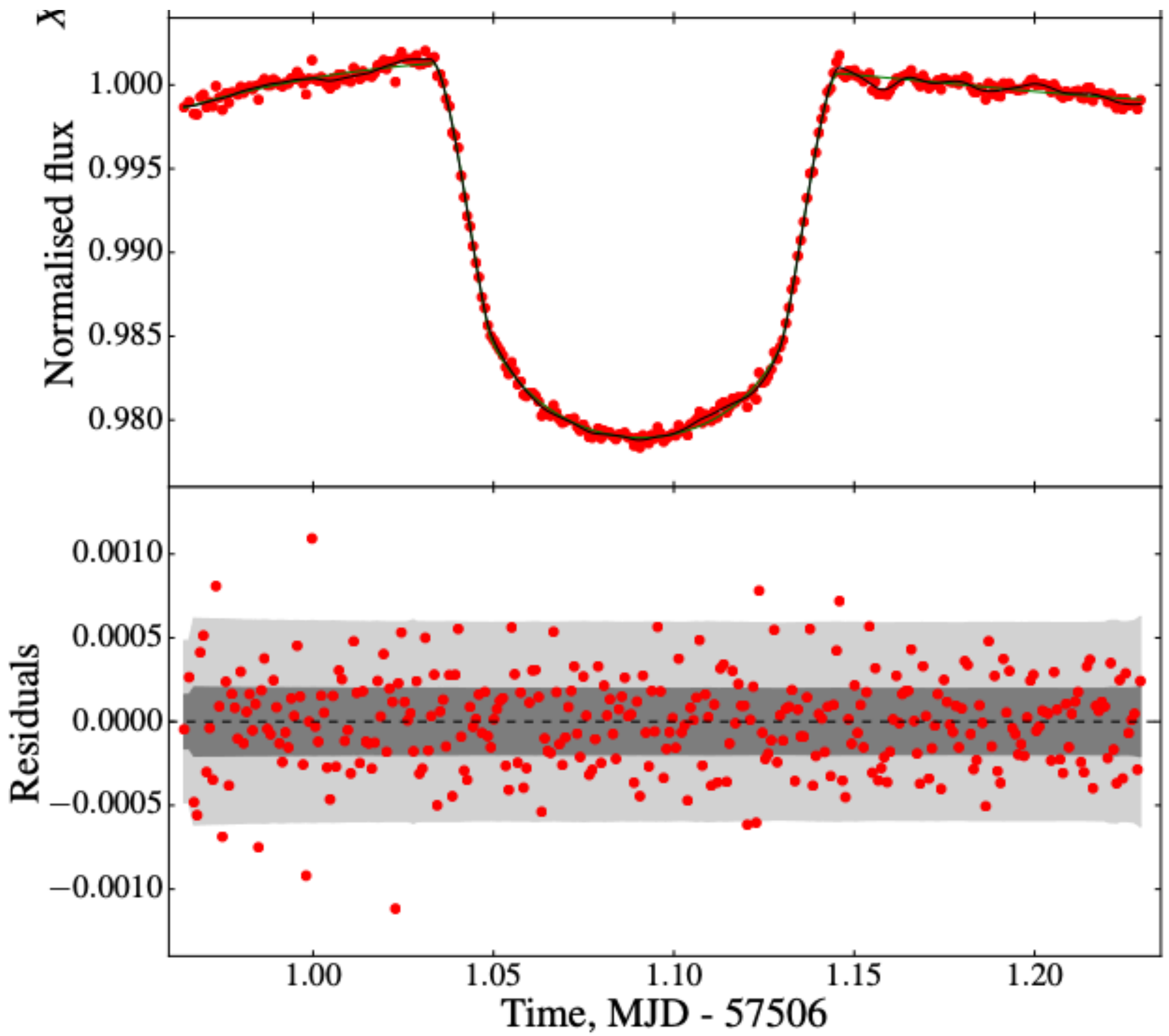
HST/STIS -

Warming and cooling of the telescope through its orbit causes changes in focus over time (thermal breathing)

Sing et al. 2013

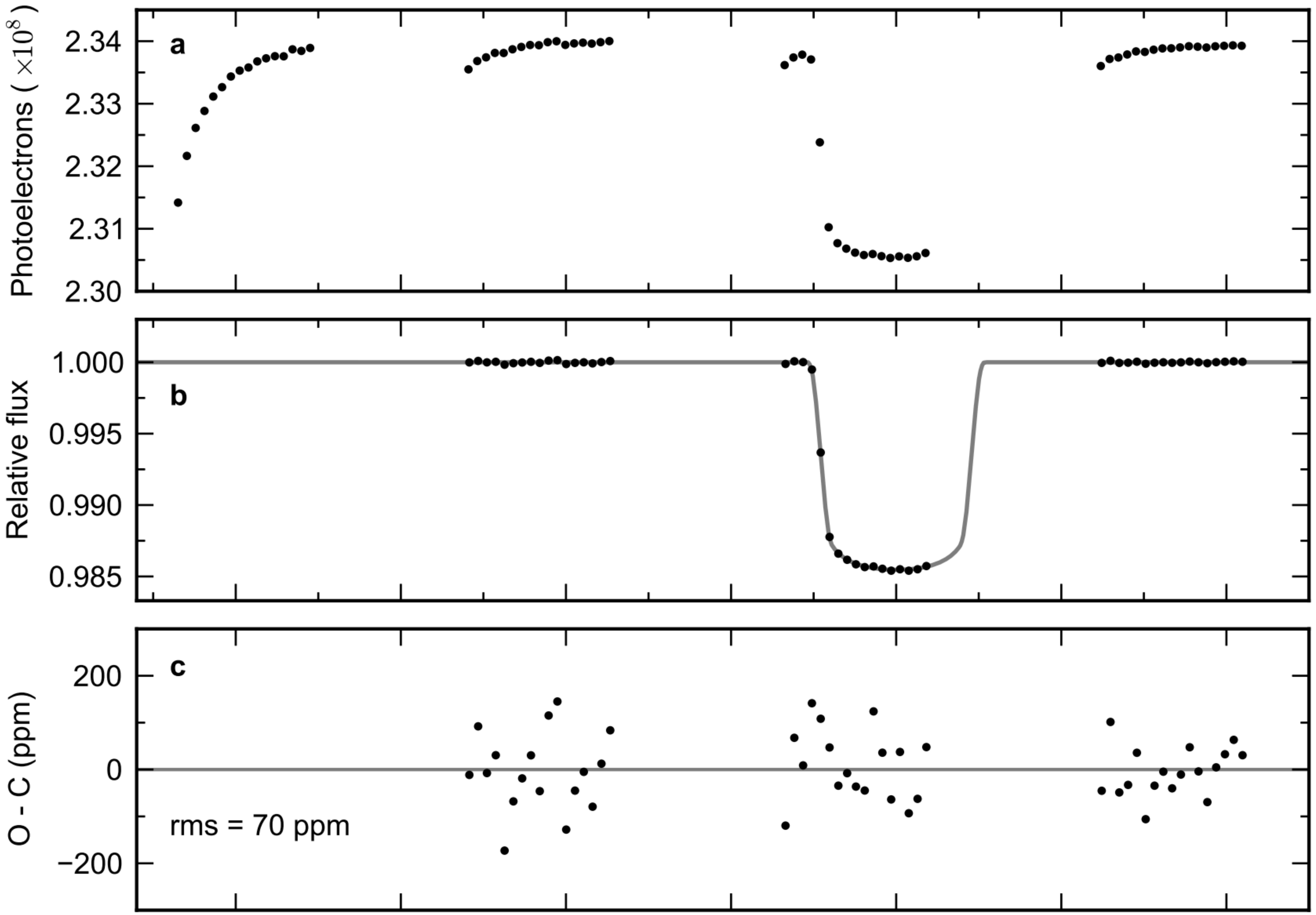
Example instrument systematics





Ground-based light curves — GPs are a common solution — Kirk et al. 2017

Example instrument systematics



HST/Wide Field Camera 3 charge trapping

Kreidberg et al. 2014

Putting it all together

Example model for HST/WFC3 data:

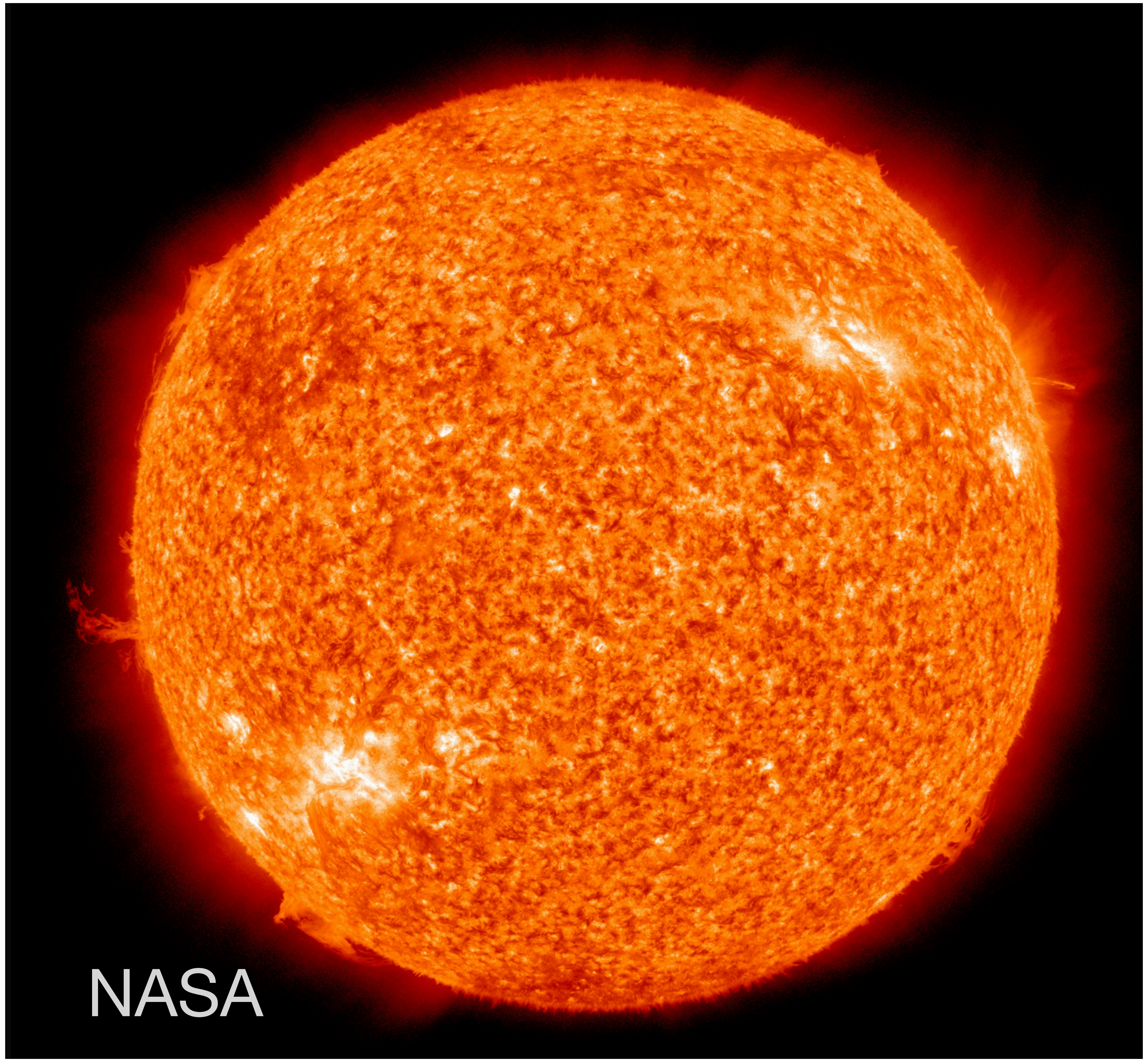
$$F_{\rm physical}(\lambda, t) = T(\lambda, t)$$

$$F_{\rm sys}(t) = (c\,S(t) + v_1\,t_{\rm v} + v_2\,t_{\rm v}^2)(1 - \exp(-a\,t_{\rm orb} - b))$$

Calculate best fit model parameters with a least squares fit, then estimate parameter uncertainties with e.g. MCMC

How do I know if my fit is good?

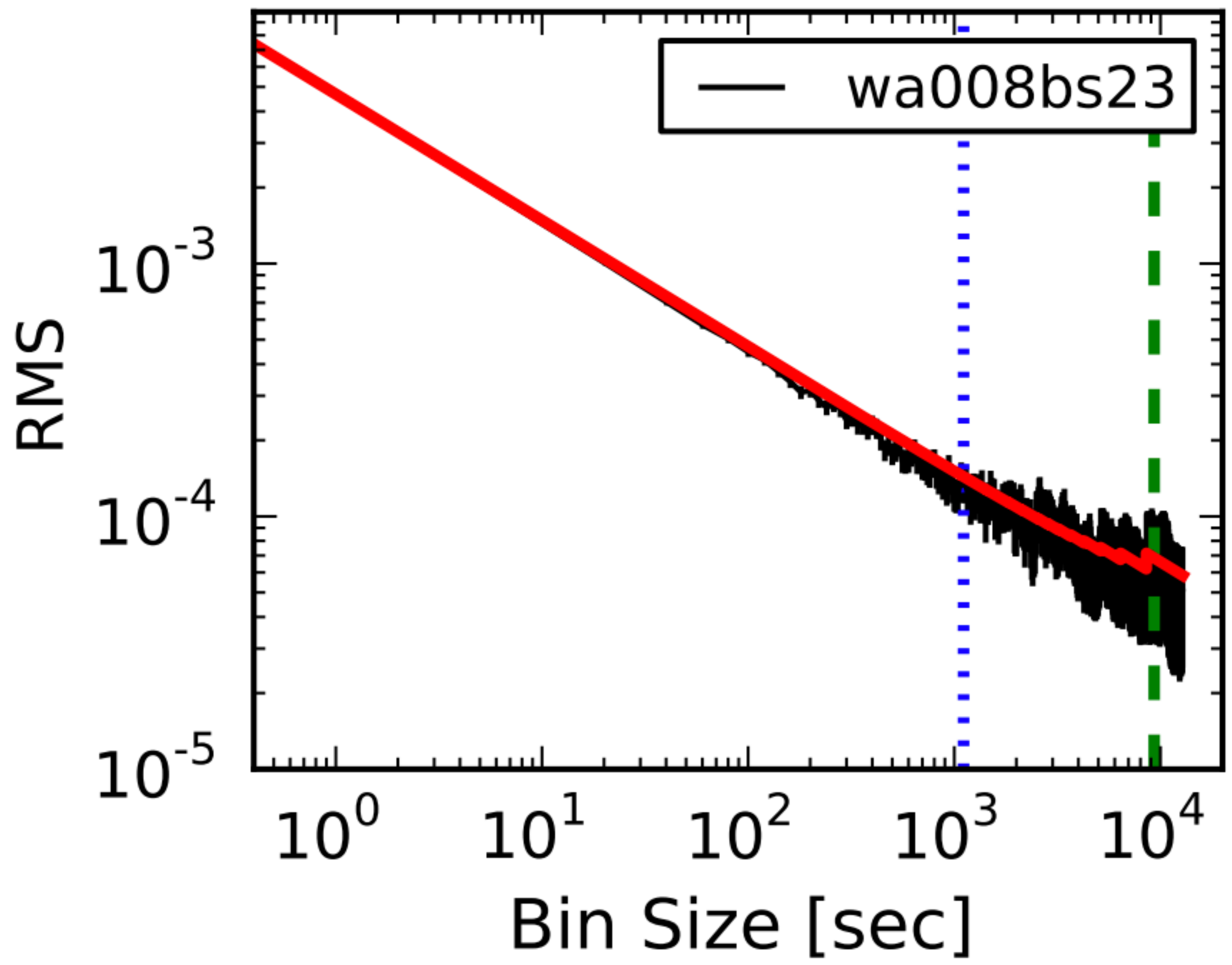
Stellar photon noise is the fundamental limit on precision



- In any time interval *t*, there is a low probability that any hydrogen atom emits a photon
- The sum of many low probability events is a Poisson process
- The expected number of photons in any time interval is thus: **N +/- sqrt(N)**
- Other noise sources: detector, stellar activity, Earth's atmosphere …

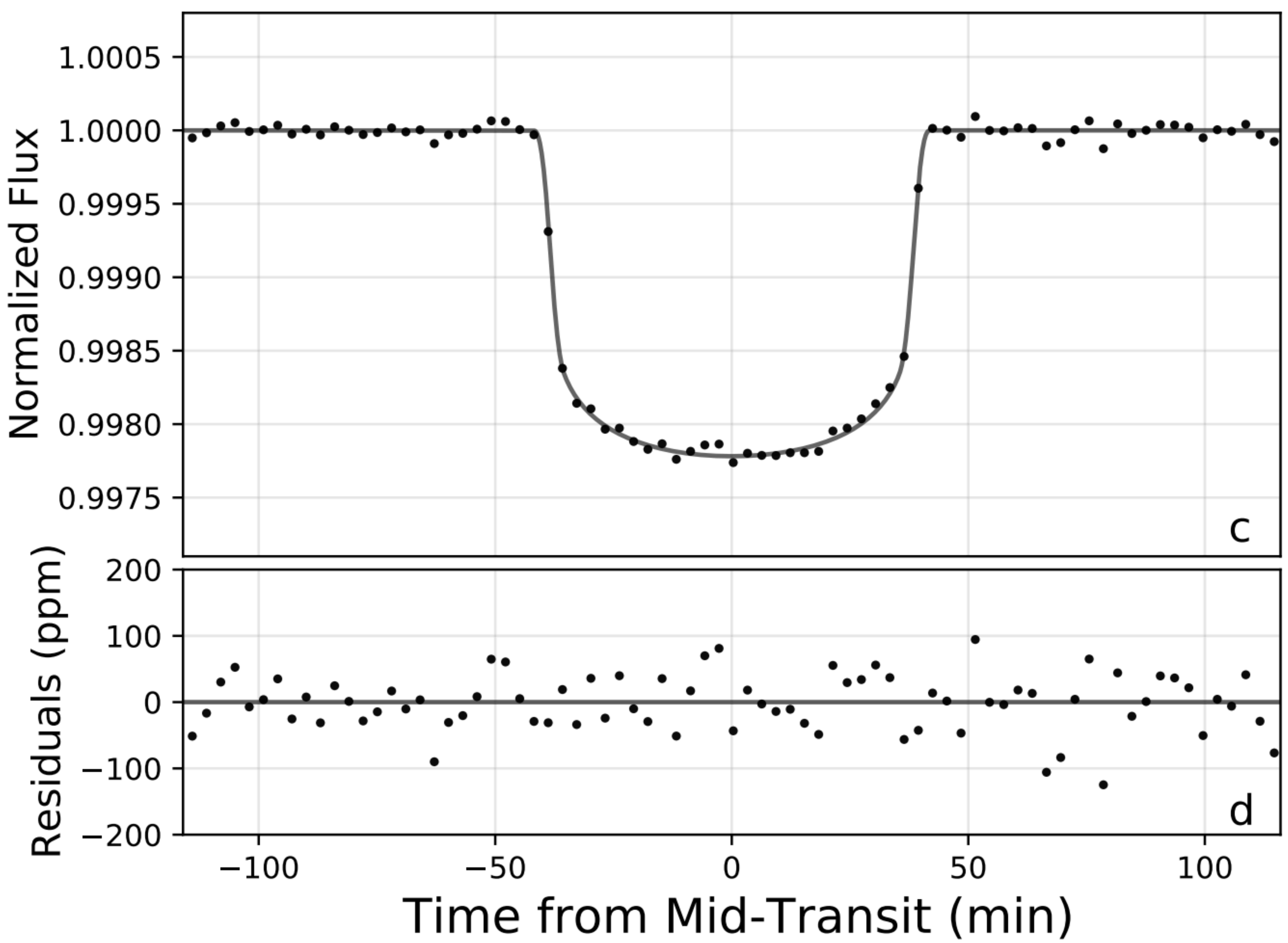
How do you know if your fit is good?

1. You reach the theoretical photon + read noise limit
2. Residuals are normally distributed
3. The light curve residuals bin down as 1/sqrt(N) (expected for Poisson noise)



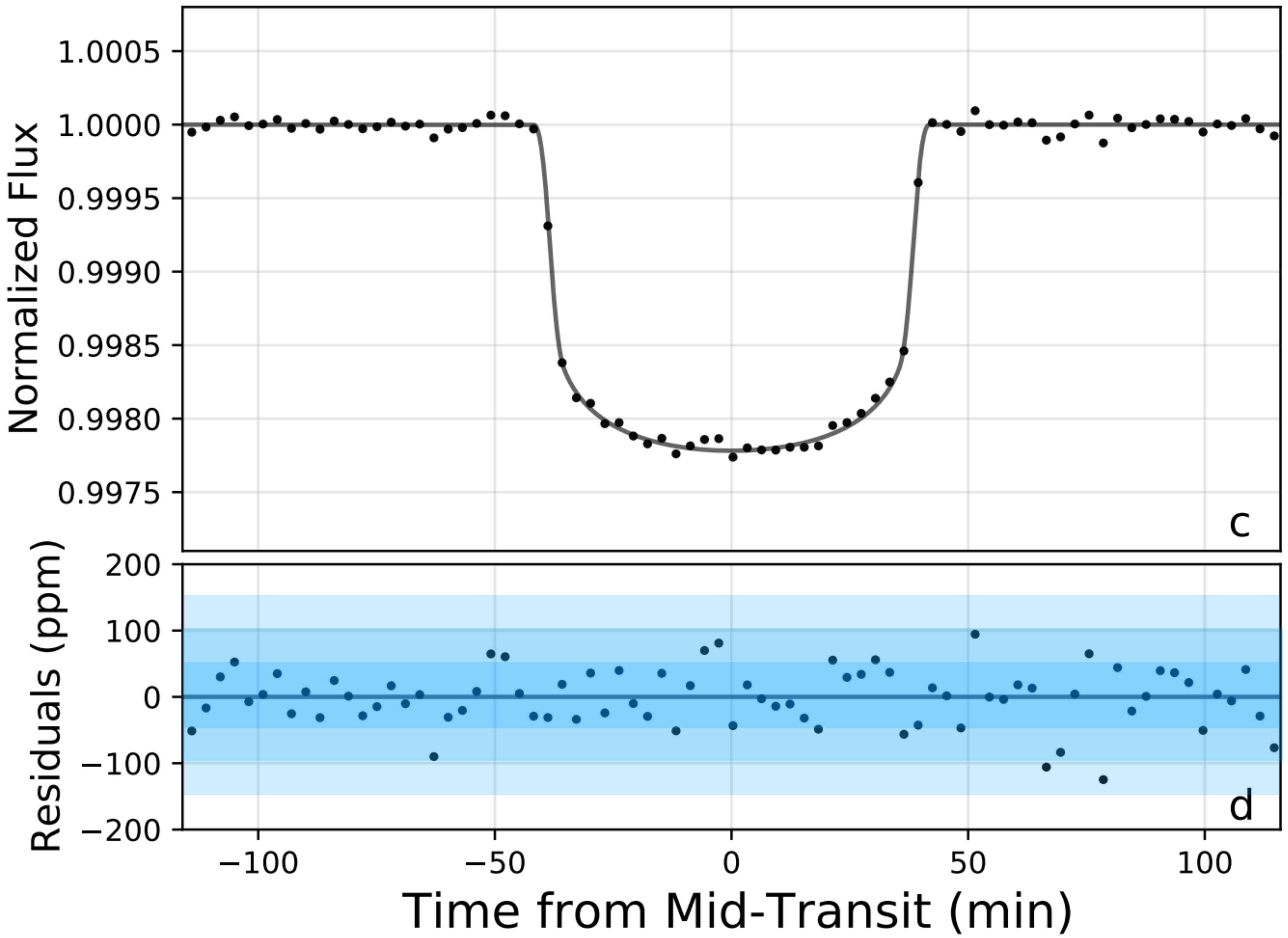
Allan deviation plot
Cubillos et al. 2013

How do you know if your best fit is good?

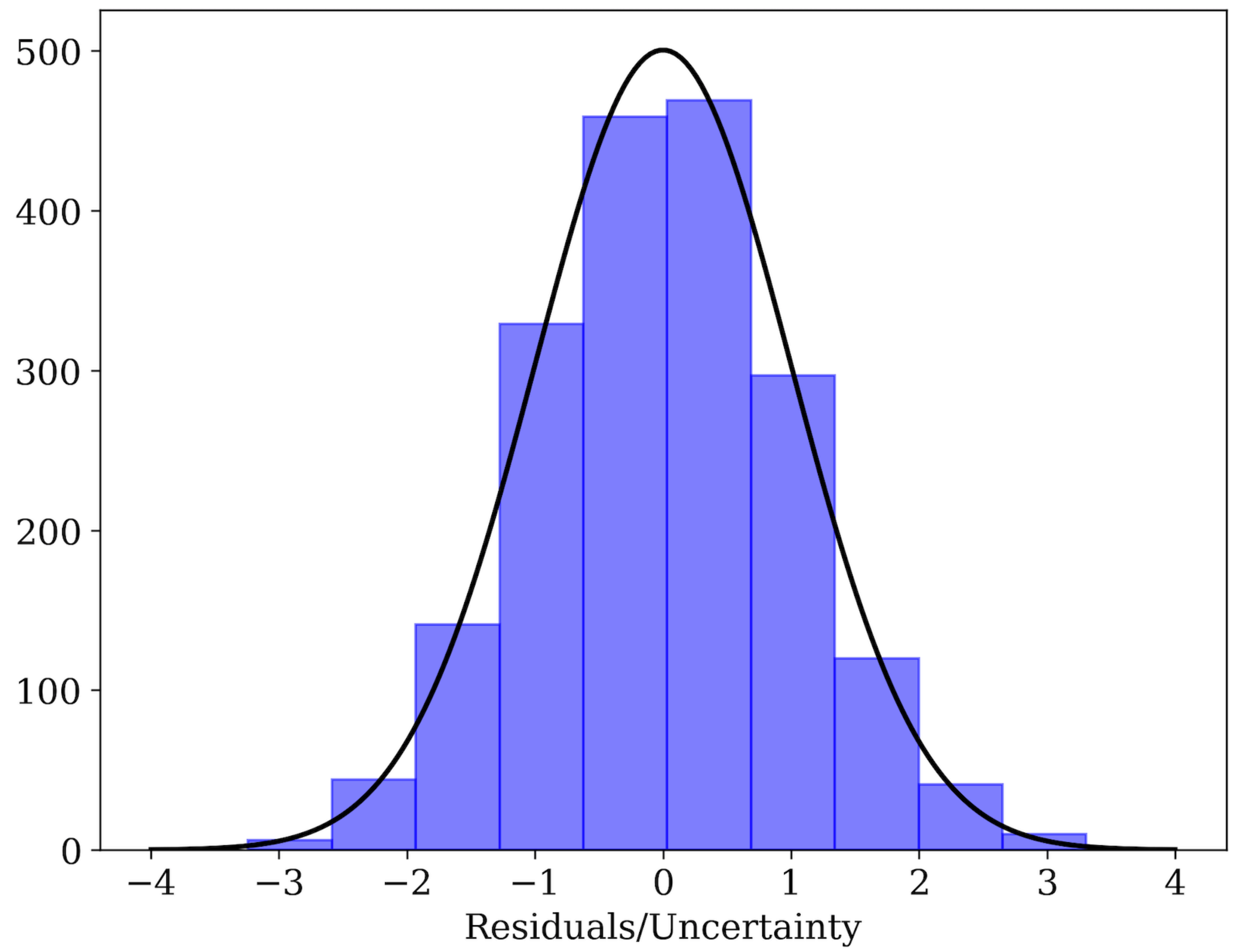


Diamond-Lowe et al. 2022

How do you know if your best fit is good?



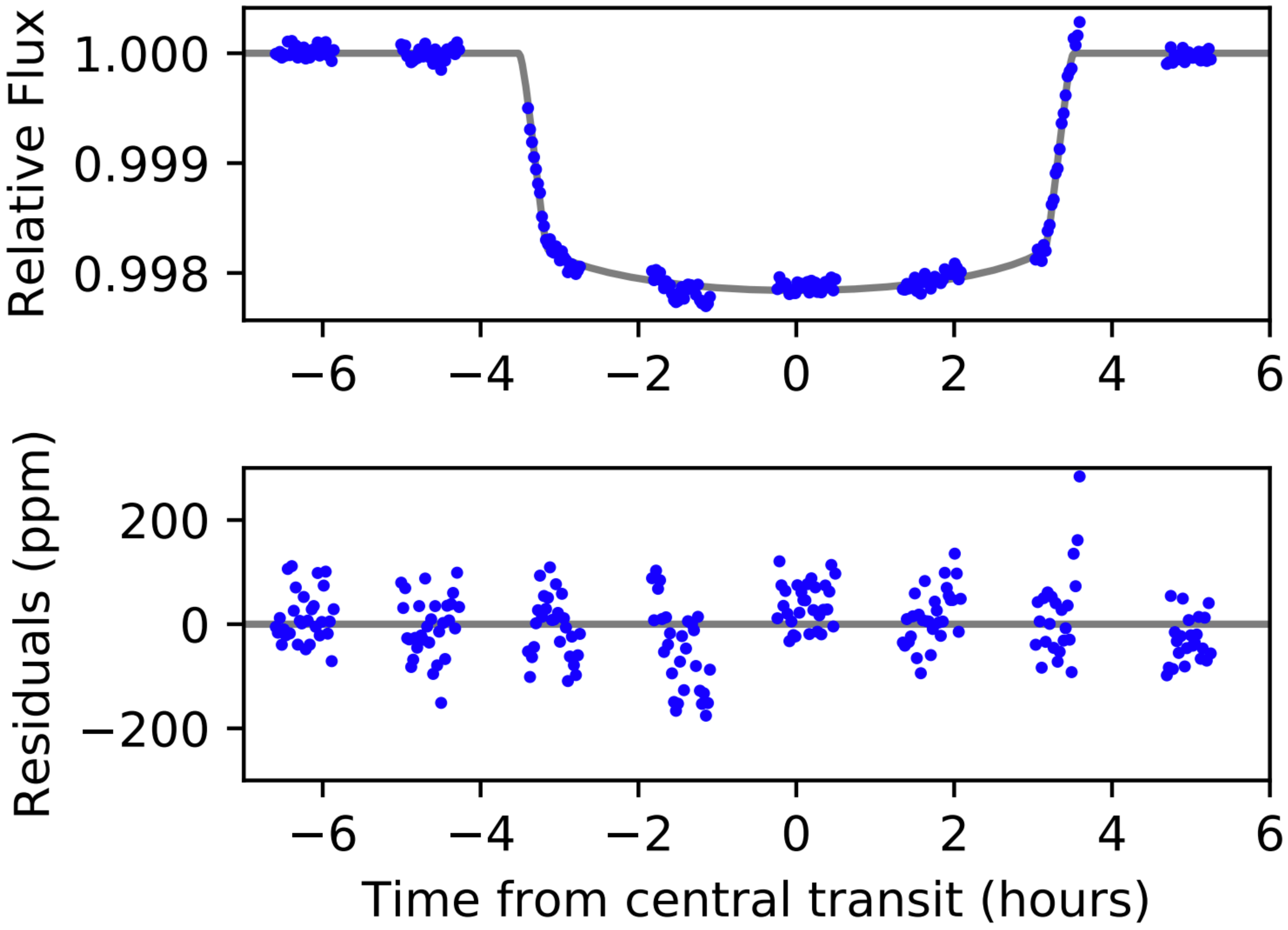
Residuals should be Gaussian and follow the 65-95-99.7 rule:



$$\Pr(\mu - 1\sigma \leq X \leq \mu + 1\sigma) \approx 68.27\%$$
$$\Pr(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95.45\%$$
$$\Pr(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 99.73\%$$

Diamond-Lowe et al. 2022

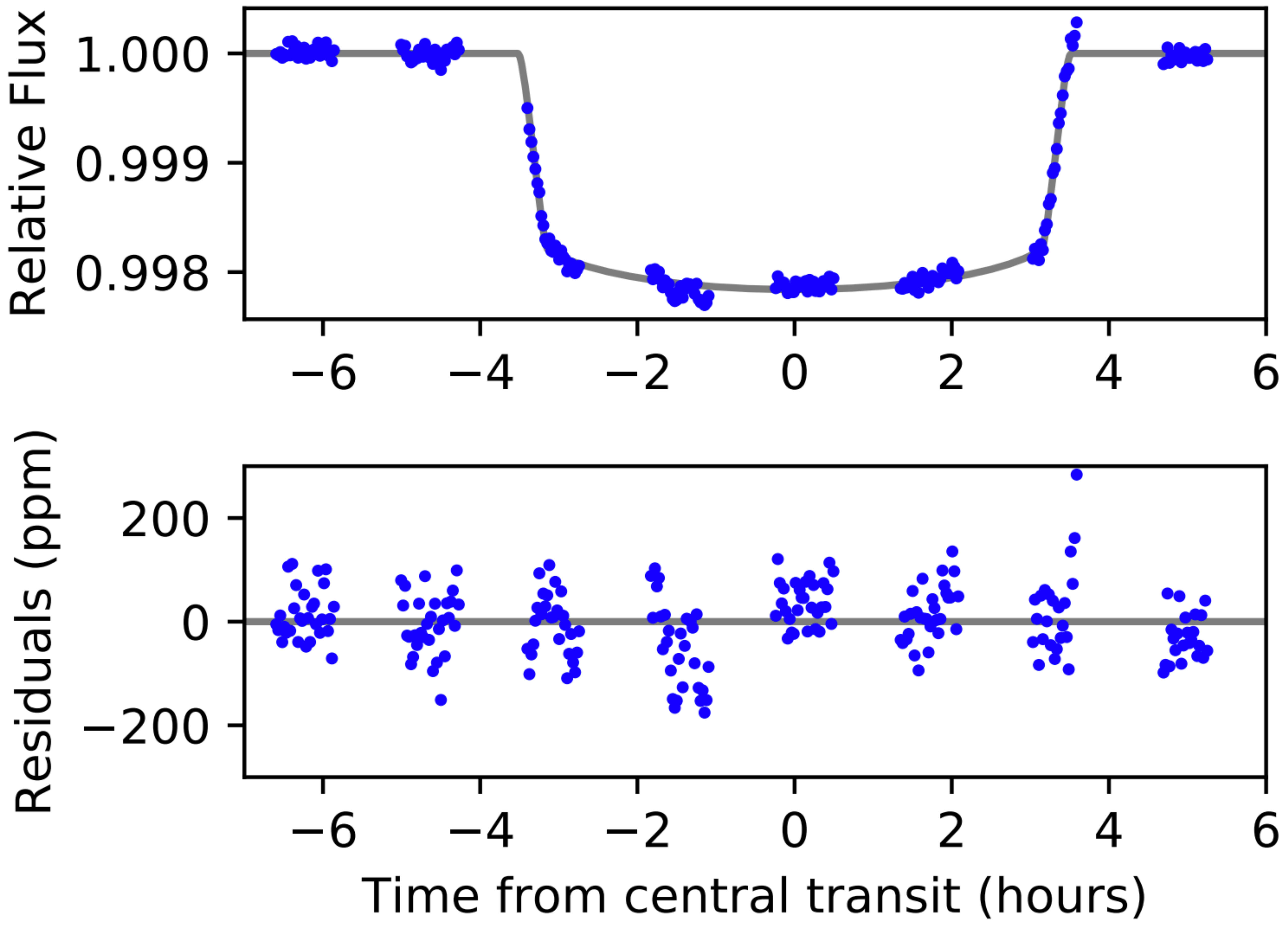
How do you know if your best fit is good?



Colón et al. 2020

**disclaimer: I reduced these data

How do you know if your best fit is good?



Noticeable correlated noise

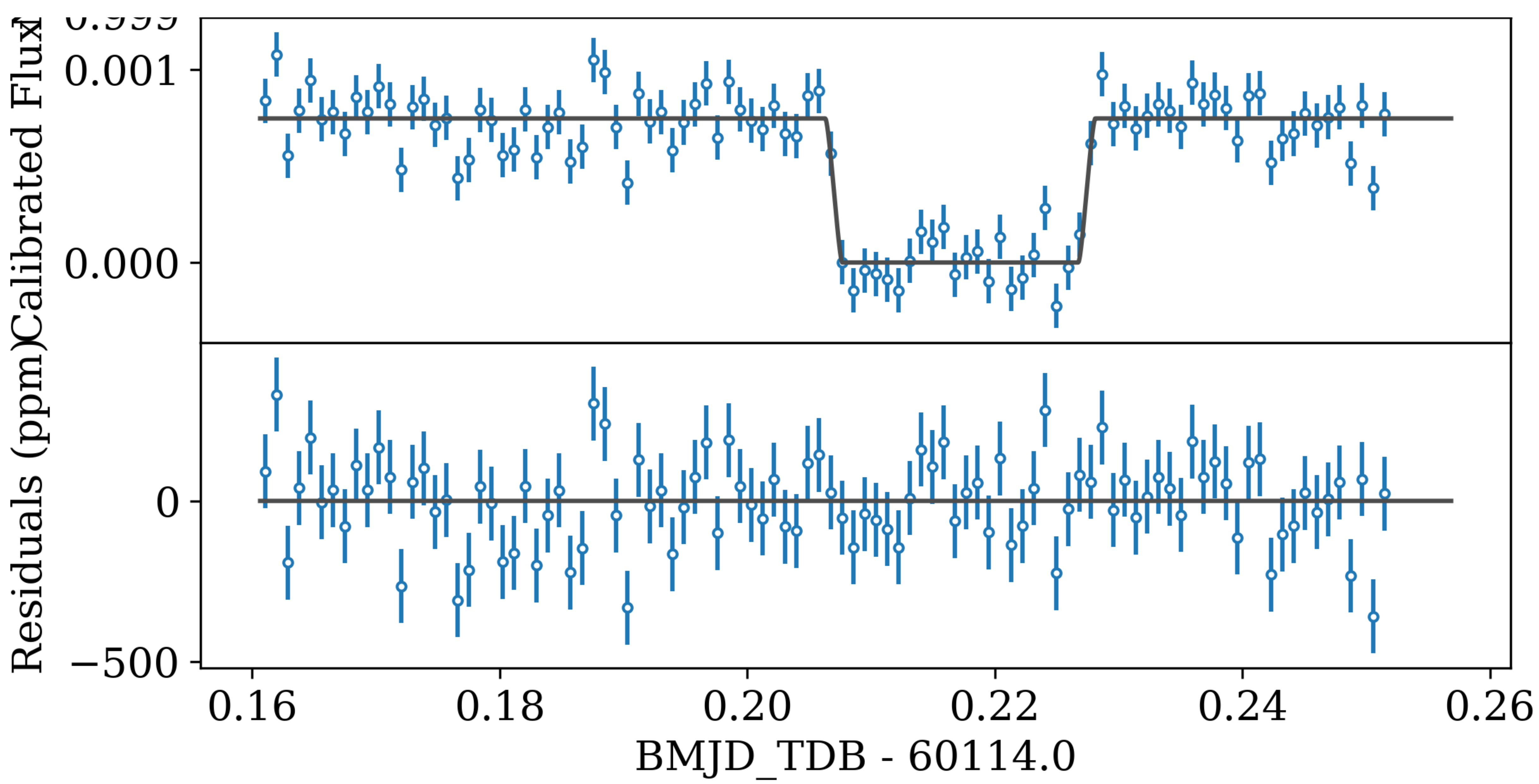
More outliers than expected for Gaussian noise

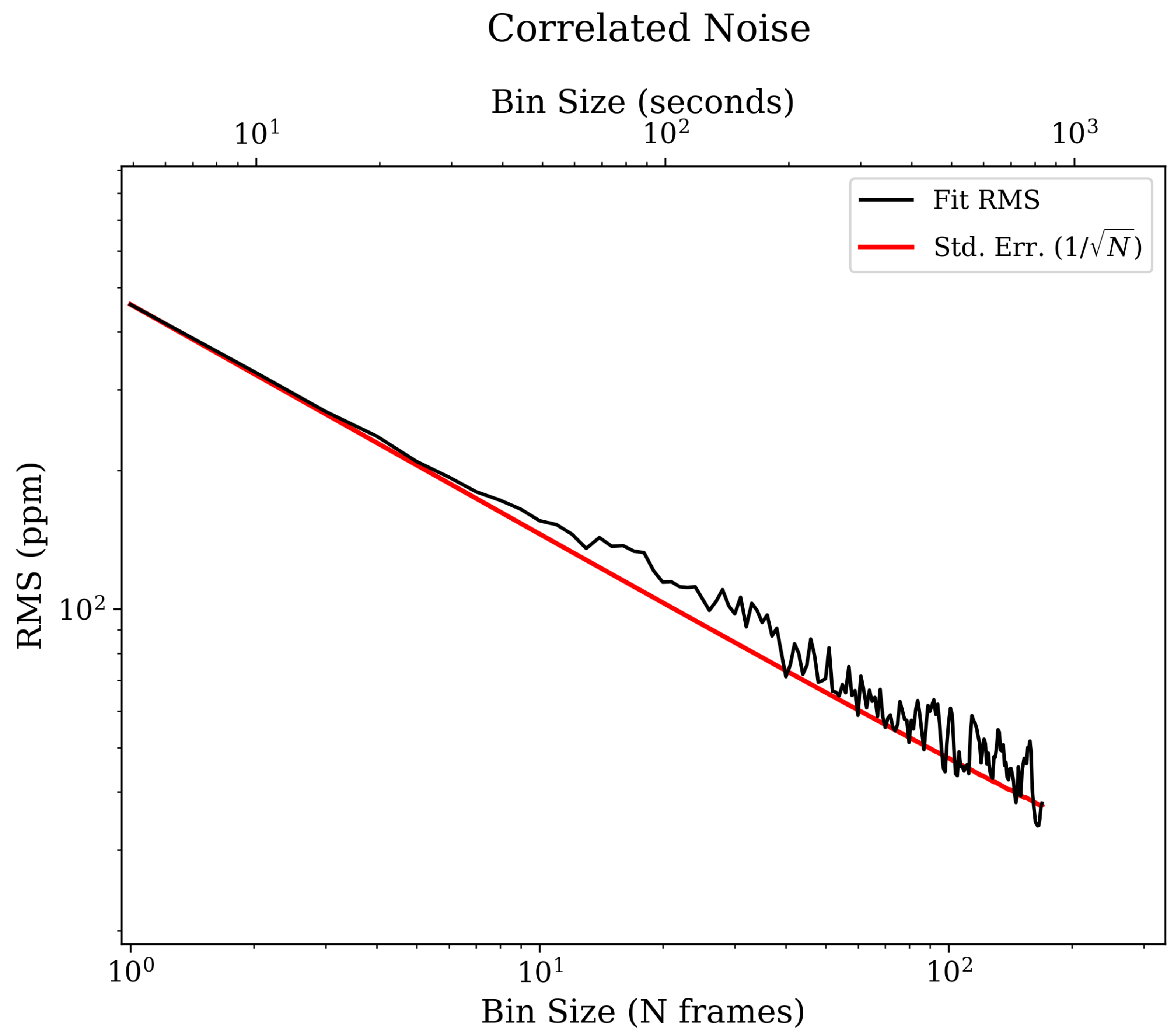
Colón et al. 2020

What do you do if you do *not* reach the photon noise or have correlated noise?! 😬

credit: the internet

Try adding more free parameters

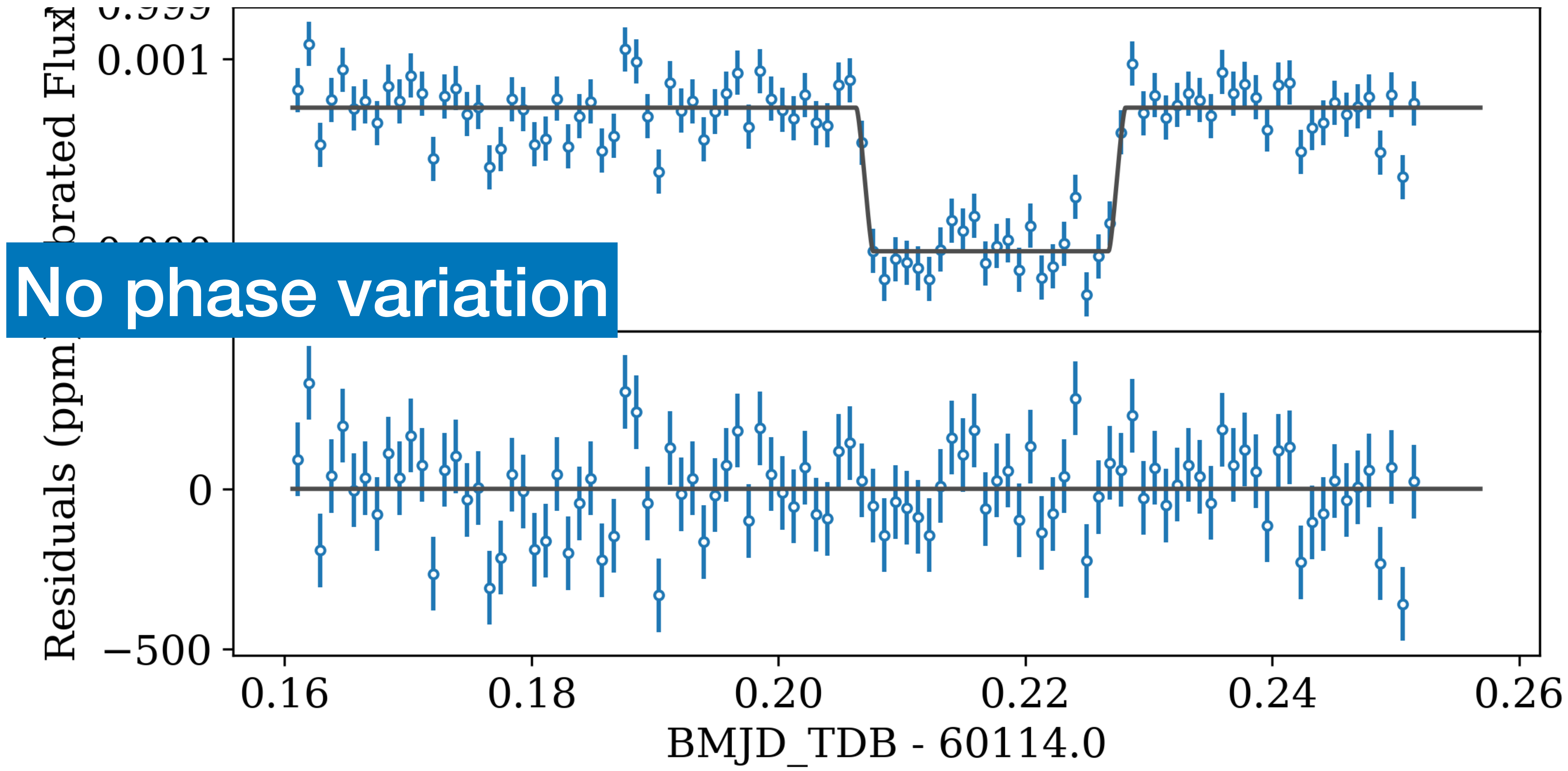


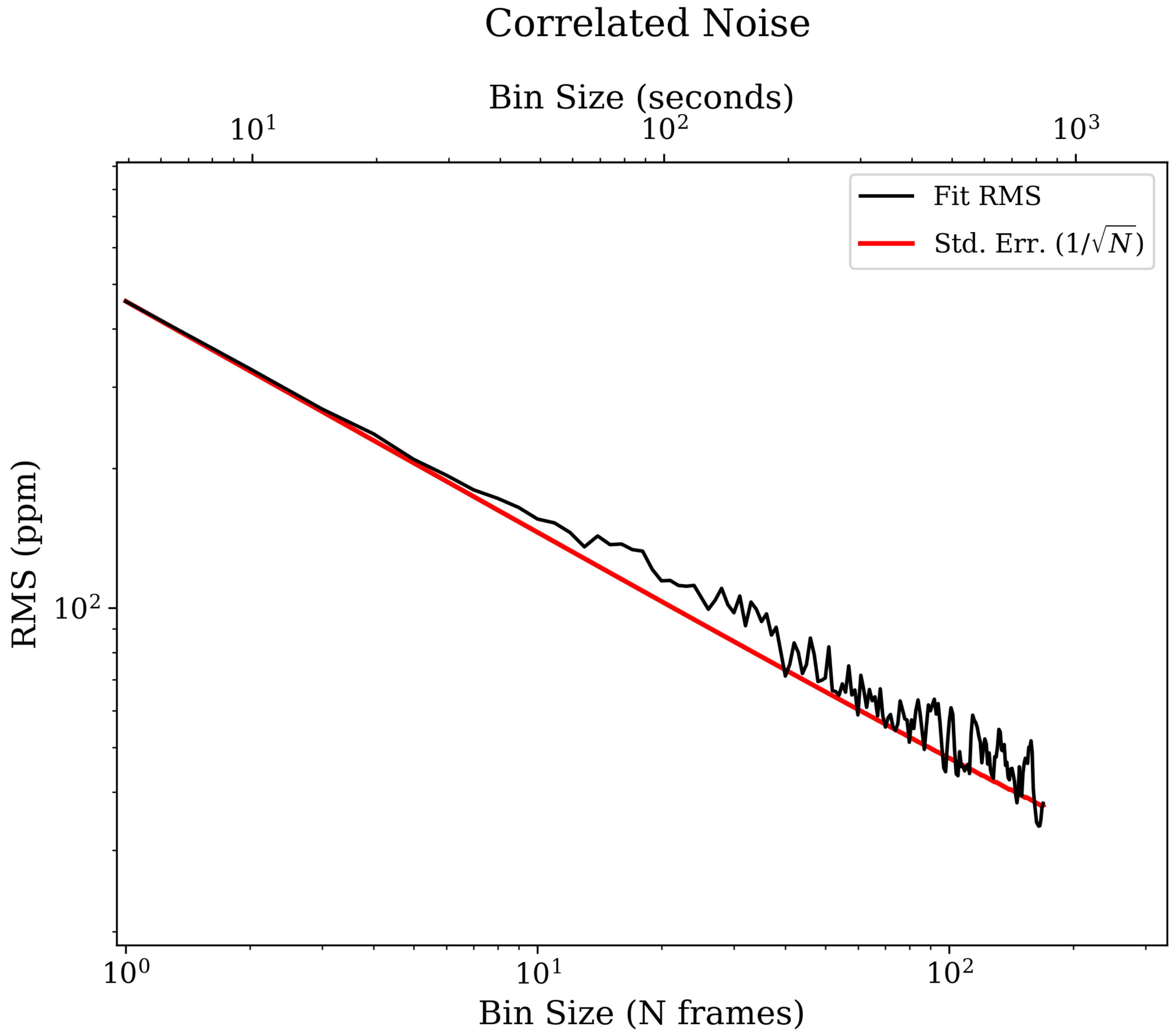


Look at the residuals to the light curve fit to see if any obvious trends are present

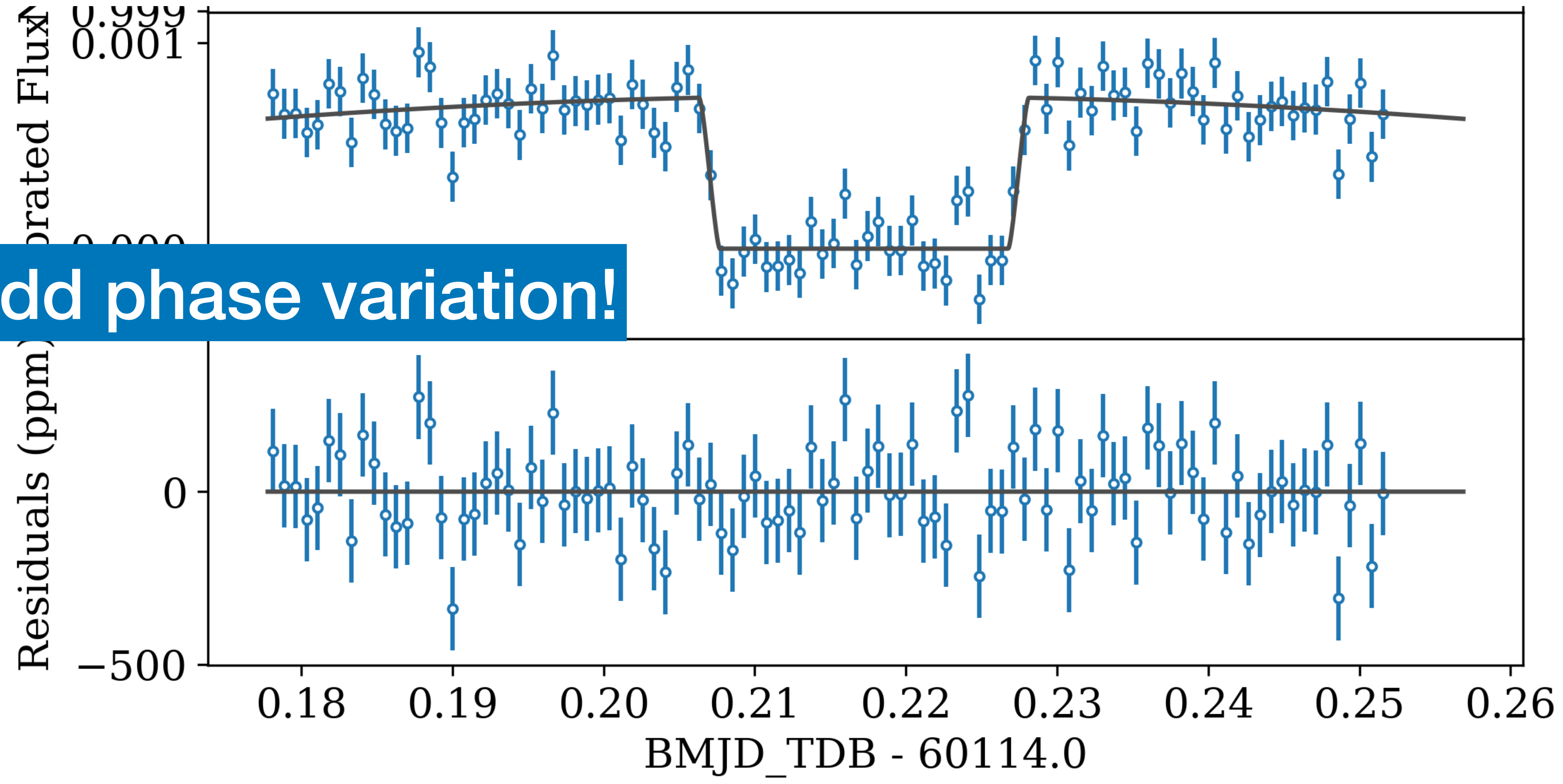
Add more free parameters to the fit until they are no longer statistically justified (by e.g. Bayesian evidence, Bayesian information criterion, Aikake information criterion)

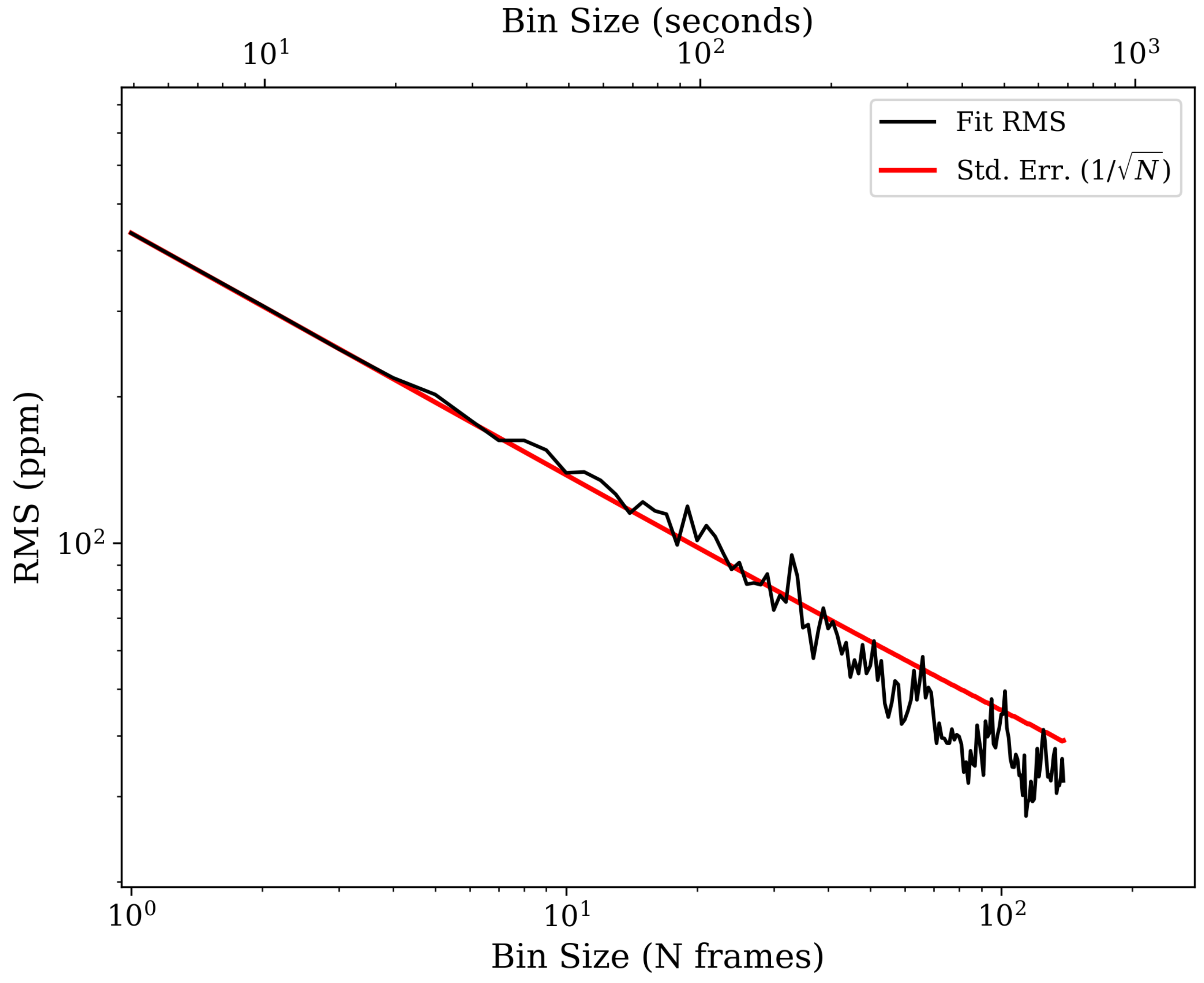
No phase variation



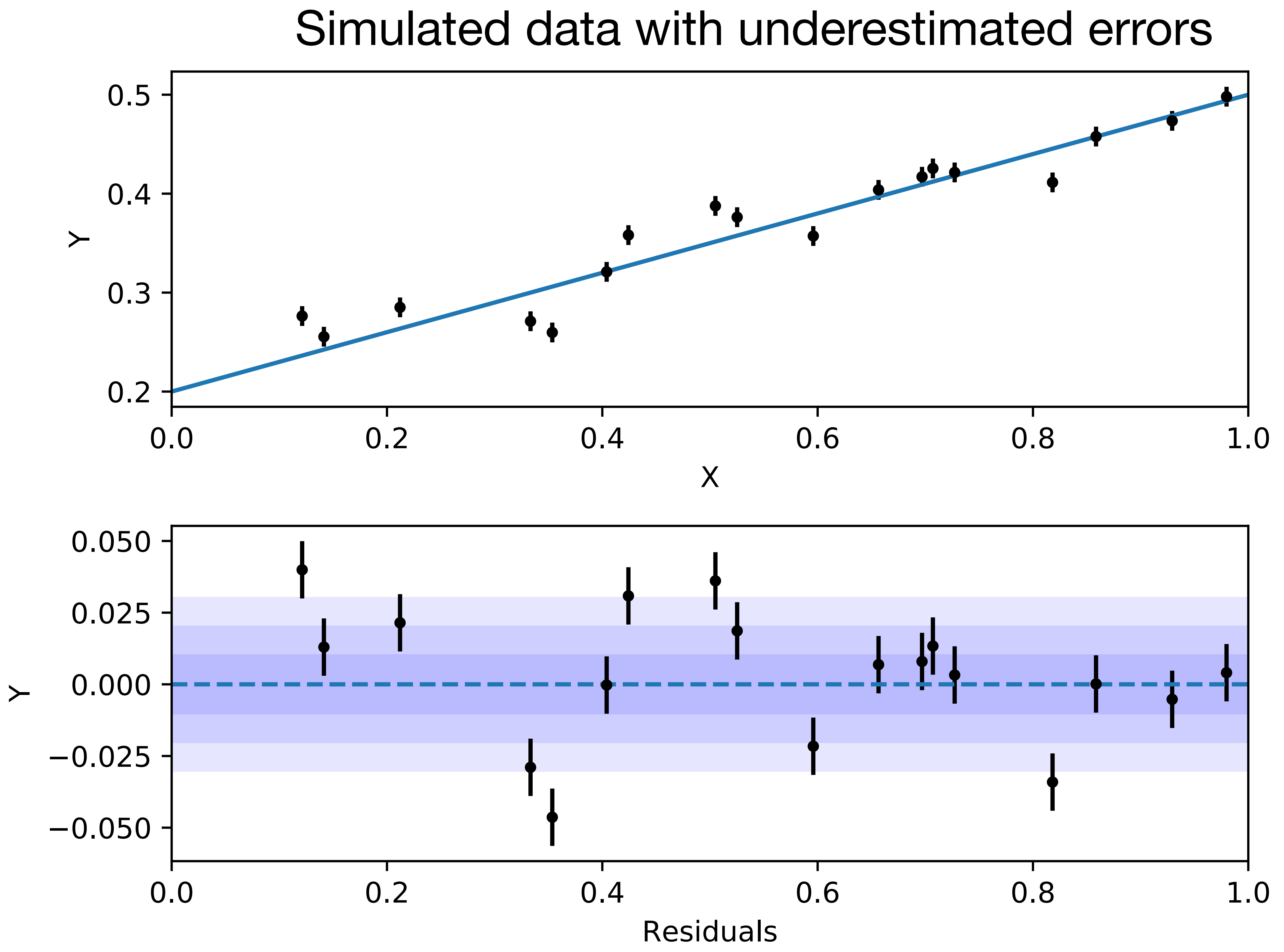


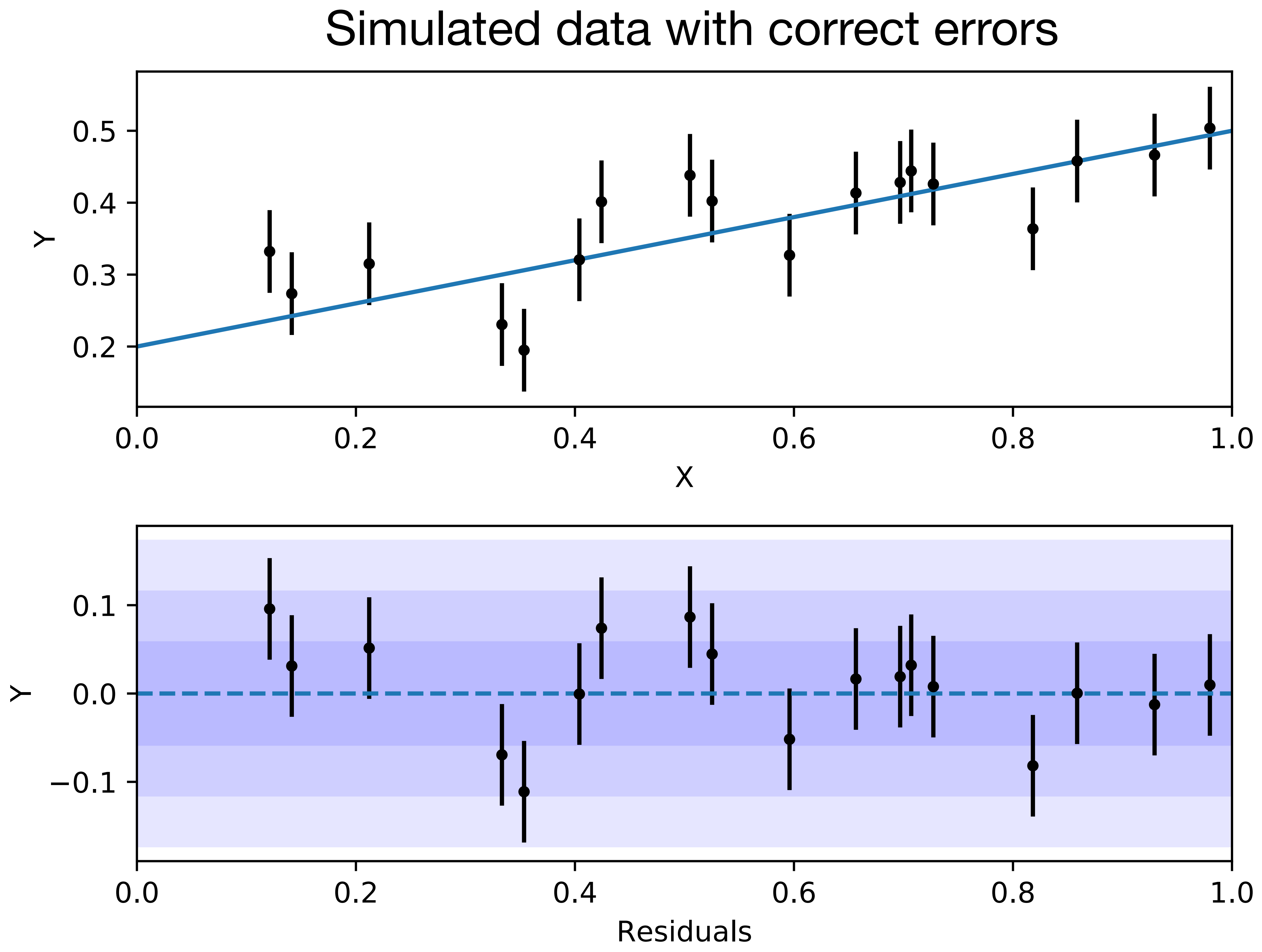
Add phase variation!





Uncorrelated noise, but still greater than photon limit? rescale error bars





$\chi^2_\nu = 5.4$

$\chi^2_\nu = 1.0$

Common approaches for red noise

State of the art — fit a Gaussian Process (e.g. Gibson et al. 2012)

Old school — Pont et al. 2006

- Estimate red noise from the ratio of actual to expected residuals
- Add it in quadrature to the white noise term

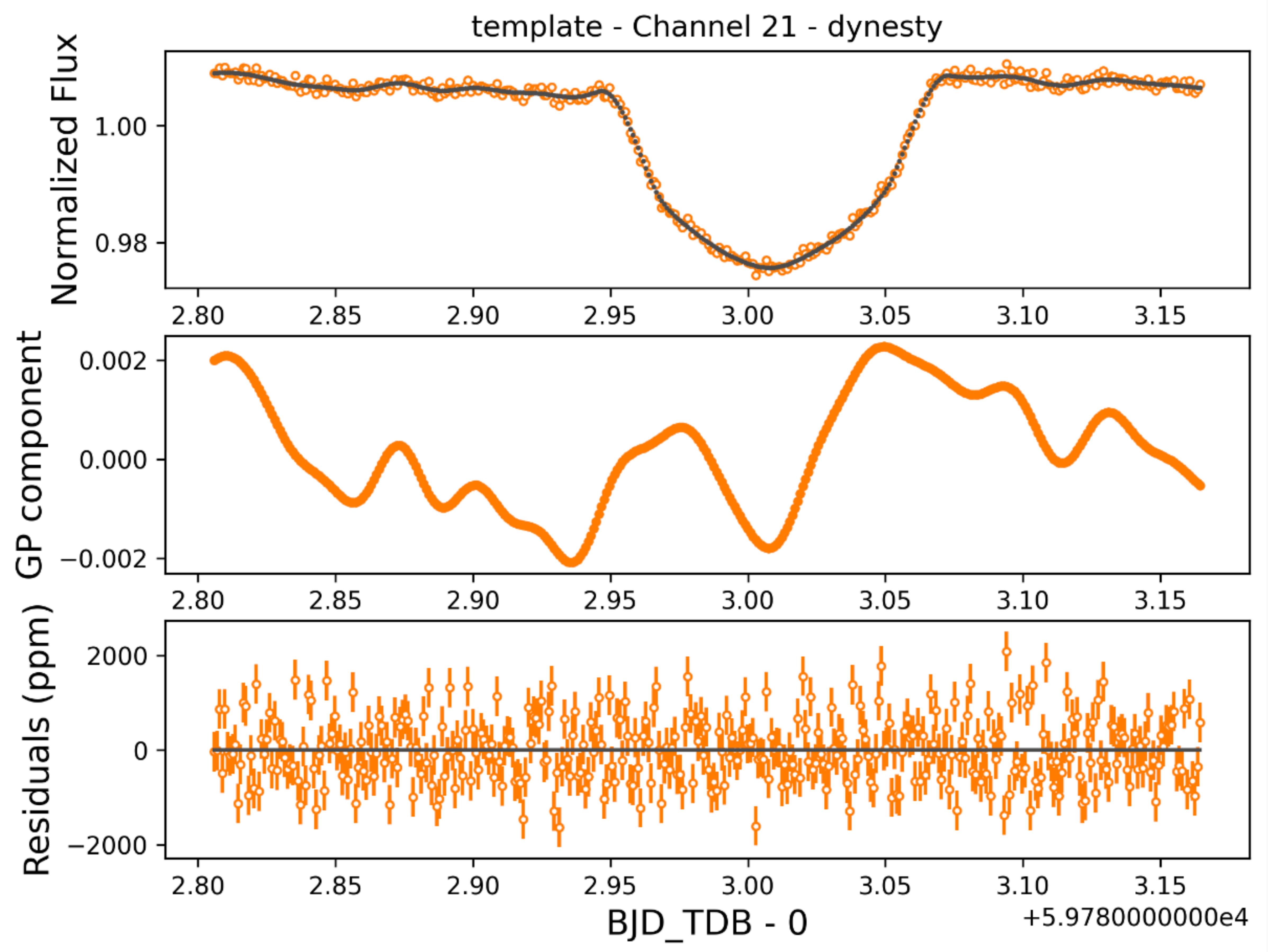
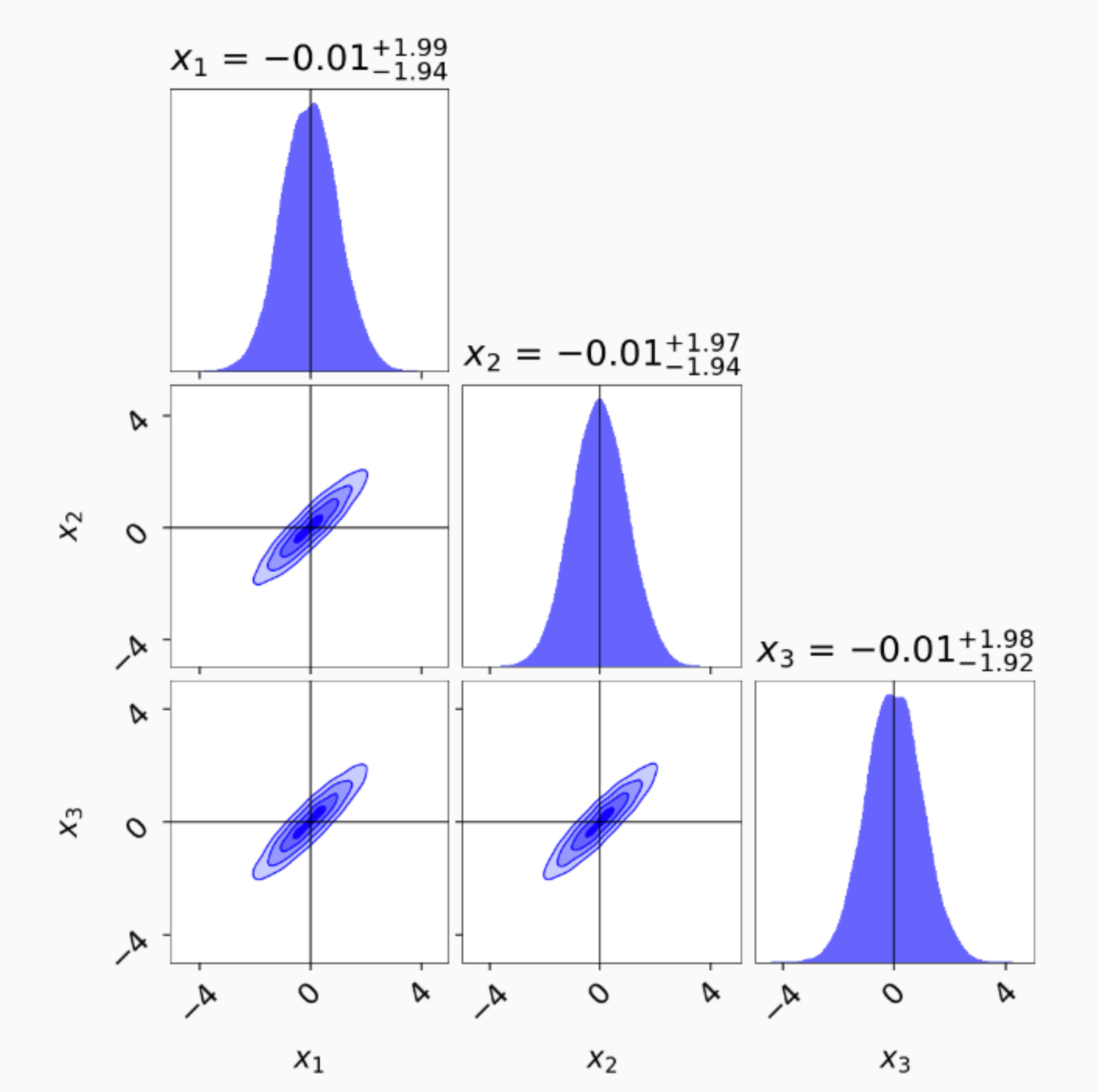


Fig 5102: Fitted Lightcurve, GP Model, and Residual Plot

How do you estimate uncertainties on the fit parameters?

1. First, make sure the error bars on the data are NOT underestimated!
2. Make initial guess with least squares fit
3. Estimate uncertainties with MCMC or nested sampling
4. Make sure your sampler is converged!



Example posterior distribution from dynesty.readthedocs.io

Raw data to transmission spectrum

When you are satisfied that:

- Your light curve fits reach the photon noise limit, accounting for both the astrophysical signal and systematic effects
- Your parameter estimation is converged

Then you may look at the transmission spectrum :)

https://eurekadocs.readthedocs.io/

Checklist for light curve fits

- Am I using the right number of free parameters?
- Does the rms of my fit reach the photon noise limit?
- Are the residuals from the light curve fit normally distributed?
- Is there any evidence for correlated (“red”) noise?
- Is the parameter estimation converged?