Stellar Properties: what are they and what can we measure?
What are some examples of fundamental stellar properties?
<table>
<thead>
<tr>
<th>Fundamental</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Temperature ((T_{\text{eff}}))</td>
<td>Color</td>
</tr>
<tr>
<td>Radius ((R))</td>
<td>Spectral Type</td>
</tr>
<tr>
<td>Mass ((M))</td>
<td>Luminosity Class</td>
</tr>
<tr>
<td>Chemical Composition</td>
<td>Rotation Period</td>
</tr>
<tr>
<td>Surface Gravity ((g))</td>
<td>Activity Level / Cycle</td>
</tr>
<tr>
<td>Luminosity ((L))</td>
<td>Multiplicity</td>
</tr>
<tr>
<td>Density ((\rho))</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td></td>
</tr>
</tbody>
</table>
Fundamental Properties of Stars

Effective Temperature ($T_{\text{eff}}$)
Radius (R)
Mass (M)
Chemical Composition
Surface Gravity (g)
Luminosity (L)
Density ($\rho$)
Age
Fundamental Properties of Stars

Most fundamental properties are not independent

Effective Temperature ($T_{\text{eff}}$)
Radius (R)
Mass (M)
Chemical Composition
Surface Gravity (g)
Luminosity (L)
Density ($\rho$)
Age

Most fundamental properties are not independent

\[
\rho \propto \frac{M}{R^3}
\]
\[
g \propto \frac{M}{R^2}
\]
\[
L \propto R^2 \, T_{\text{eff}}^4
\]

Assumes: atmospheres are thin & stars ~ blackbodies
Mass, composition & age uniquely define other fundamental stellar properties

(Vogt-Russell “Theorem”)

Rich Townsend
(http://www.astro.wisc.edu/~townsend)
Mass, composition & age uniquely define other fundamental stellar properties
(Vogt-Russell “Theorem”)

Unfortunately, mass & age (+ some elements such as He) are hard to measure!

Rich Townsend
(http://www.astro.wisc.edu/~townsend)
Fundamental Properties of Stars

Effective Temperature \( (T_{\text{eff}}) \)
Radius \( (R) \)
Mass \( (M) \)
Chemical Composition
Surface Gravity \( (g) \)
Luminosity \( (L) \)
Density \( (\rho) \)
Age

“Easy”
Possible
Hard

Astrometry is critical for inferring \(~\) all fundamental parameters of stars
Fundamental Properties of Stars

Effective Temperature ($T_{\text{eff}}$)
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Astrometry is critical for inferring ~ all fundamental parameters of stars
Stellar Luminosity: Distance Modulus

\[ m_b - M_b = 5 \log_{10}(d) - 5 + A_b \]

- \( m_b \): observed (apparent) magnitude in band b
- \( M_b \): distance in pc
- \( A_b \): interstellar extinction in band b

\[ M_{\text{bol}} = M_b + BC_b \]

- \( BC = \text{bolometric correction (from model atmospheres)} \)

\[ M_{\text{bol}} - M_{\text{bol,\odot}} = -2.5 \log_{10}(L/L\odot) \]

Advantage: Observables (\( m_b \) & \( d \)) are available for lots of stars
Disadvantage: Need to be confident in your BC and extinction model!
Bolometric Fluxes: SED Fitting

\[ L = 4\pi f_{\text{bol}} d^2 \]

**Advantage:** Less model dependent than bolometric corrections

**Disadvantage:** Zeropoint offsets between photometric surveys

**Challenge (also true for BCs):** Depends on \( T_{\text{eff}} \) and extinction, which are highly degenerate

Gaia Bp/Rp spectra are critical for this! See Orlagh’s talk tomorrow

Mann+ 2016
Stellar Radii from Parallaxes

\[ R = \frac{d \alpha}{2} \]

**Angular size + Distance** gives a direct measurement of the star’s **Radius**

Problem: stellar diameters are small and require interferometry to be resolved.

*More on this by Roxanne tomorrow!*
Stellar Radii from Parallaxes

\[ R = d \alpha/2 \]

Angular size + Distance gives a direct measurement of the star’s Radius

Problem: stellar diameters are small and require interferometry to be resolved.

More on this by Roxanne tomorrow!

Alternative: Stefan-Boltzmann Law

\[ R_\star = \sqrt{\frac{F_{\text{bol}} d^2}{\sigma T_{\text{eff}}^4}} \]

But what is \( T_{\text{eff}} \) and how well do we know it?
Effective Temperatures

T\textsubscript{eff} characterizes the total radiative flux transported through the atmosphere. It can be regarded as an average of the temperature over depth in the atmosphere.

A blackbody radiating the same amount of total energy would have a temperature T = T\textsubscript{eff}.

\[ F = \sigma T_{\text{eff}}^4 \]

\[ F = f_{\text{bol}} d^2/R^2 \]

\[ R = d \alpha/2 \]

\[ T_{\text{eff}} = (4 f_{\text{bol}} / \sigma \alpha^2)^{1/4} \]

Effective Temperature is defined through angular diameter & bolometric flux.
Effective Temperatures

High-Resolution Spectroscopy

\[ R_\star = \sqrt{\frac{F_{\text{bol}} d^2}{\sigma T^4_{\text{eff}}}} \]

\( T_{\text{eff}} \) from different methods (and absolute scale) can vary by up to ~2%. Sets a floor of ~4% on stellar radii!

Petigura 2015

Color-\( T_{\text{eff}} \) Relations, SED, IRFM

\[ \Delta T_{\text{eff}} = \text{Pinsonneault+ 2012} \]

Casagrande+ 2010

\[ \text{KIC IRFM } T_{\text{eff}}(J-K_s) - T_{\text{eff}}(\text{YREC}) \]

Spec. \( T_{\text{eff}} \) - \( T_{\text{eff}}(\text{YREC}) \)

YREC calib. range

\[ \text{Cluster IRFM } T_{\text{eff}}(J-K_s) - T_{\text{eff}}(\text{YREC}) \]

\[ \text{Cluster Mean IRFM } T_{\text{eff}} - T_{\text{eff}}(\text{YREC}) \]
Gaia parallaxes decreased radius uncertainties for Kepler stars by a factor of ~5-6!
Challenge (opportunity?): *unresolved binaries*. In general, causes overestimation of L and underestimation of $T_{\text{eff}}$. 

*Mathur+ 2017* 

*Berger+ 2018*
Fundamental Properties of Stars

Effective Temperature ($T_{\text{eff}}$)
Radius (R)
Mass (M)
Chemical Composition
Surface Gravity ($g$)
Luminosity (L)
Density ($\rho$)
Age

“Aeasy”
Possible
Hard

Astrometry is critical for inferring ~ all fundamental parameters of stars
Dynamical Masses from Astrometry

A hidden treasure: \(~10^5\) astrometric solutions from Gaia

<table>
<thead>
<tr>
<th>Quantity</th>
<th>This paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass, (M_A + M_B)</td>
<td>0.9168 ± 0.0148(M_{\odot})</td>
</tr>
<tr>
<td>Mass of (\mu) Cas A, (M_A)</td>
<td>0.7440 ± 0.0122(M_{\odot})</td>
</tr>
<tr>
<td>Mass of (\mu) Cas B, (M_B)</td>
<td>0.1728 ± 0.0035(M_{\odot})</td>
</tr>
</tbody>
</table>

~1-2% Masses!

Gaia Collaboration+ 2022
Asteroseismology: Densities, log(g) & Ages

Gaia luminosity is an important external constraint for testing model physics (e.g. convection)

Garcia & Ballot 2019

\[ \Delta \nu \propto \rho^{1/2} \]

\[ \nu_{\text{max}} \propto g T_{\text{eff}}^{-0.5} \]

\[ \delta \nu \propto \int_0^R \frac{dc}{dr} \frac{dr}{r} \]
Masses & Ages of Single Stars: “Isochrone” Fitting

Models (Mass, Age, Composition) predict quantities to be compared to observations ($T_{\text{eff}}$, $L$, [M/H], absolute mag, colors)

Available software tools (incomplete list!): 

- *isochrones*: https://github.com/timothydmorton/isochrones
- *isoclassify*: https://github.com/danxhuber/isoclassify
- *BASTA*: https://github.com/BASTAcode/BASTA
- *Param*: http://stev.oapd.inaf.it/cgi-bin/param


Age diagnostics not discussed here: rotation, chemical abundances, kinematics & clusters (more on this from Melissa & Marina tomorrow!)
Caveat: Stellar models have systematic errors

**Source of systematics:**
uncertain input physics such as convection, atmospheric boundary conditions, rotation, opacities and overshoot

Sets error floor of $\sim 5\%$ in mass and $\sim 20\%$ in age for solar-type stars, with variation across HRD

Always a good idea to establish systematic errors by using different model grids!
Empirical Relations for M Dwarfs

$T_{\text{eff}}$-L-R-M relations calibrated using interferometric angular diameters, bolometric fluxes and dynamical masses. Possible because M dwarfs hardly evolve!
A Stellar Properties “Cook Book” (for single, solar-type field stars)

**High-resolution spectrum**
- $T_{\text{eff}}, \log(g), [\text{M/H}]$

  - **Are $T_{\text{eff}}$ values consistent?**
    - **No**
      - Is $A_v$ negligible?
        - **Yes**
          - Repeat spectr. analysis with $T_{\text{eff,phot}}$ prior
        - **No**
          - Repeat phot. analysis with $T_{\text{eff,spec}}$ prior
    - **Yes**
      - Use different models to establish systematic errors

**SED/IRFM/Colors**
- $T_{\text{eff}}, A_v, f_{\text{bol}}$

  - **Luminosity**
    - **Radius**
      - $T_{\text{eff}}, [\text{M/H}], L + \text{Stellar Evolution Model}$
        - **Density, log(g), Mass, Age**
          - **Is log(g) consistent with spectroscopy?**
            - **No**
              - Repeat spectroscopic analysis
            - **Yes**
              - Use different models to establish systematic errors
How do Stellar Properties Impact Exoplanet Properties?
Fundamental Properties of Stars

Effective Temperature ($T_{\text{eff}}$)  
Radius (R)  
Mass (M)  
Chemical Composition  
Surface Gravity (g)  
Luminosity (L)  
Density ($\rho$)  
Age

Which stellar parameters are important for understanding exoplanets?
Fundamental Properties of Stars

Effective Temperature ($T_{\text{eff}}$)
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Which stellar parameters are important for understanding exoplanets?

All of them!
"Know Thy Star, Know Thy Planet"

\[ \delta = \left( \frac{r_p}{R_*} \right)^2 \]

Ikwut-Ukwa+ 2020

\[ K \propto \frac{m_p \sin i}{M_*^{2/3}} \]

Nielsen+ 2020

Han+ 2020

\[ q = \frac{m_p}{M_*} \]

Bohn+ 2020
"Know Thy Star, Know Thy Planet"

\[ \delta = \left( \frac{r_p}{R_*} \right)^2 \]

\[ K \propto \frac{m_p \sin i}{M_*^{2/3}} \]

Ikwut-Ukwa+ 2020

Han+ 2020

Bohn+ 2020

Nielsen+ 2020
\[ \delta = \left( \frac{r_p}{R_*} \right)^2 \]

\( T \) = Transit Duration  \( \tau \) = In/Egress Duration  \( \delta \) = Transit Depth
Limb
Darkening
Limb Darkening

\[ b=0 \quad \delta_1 \]
Limb Darkening

\[ \text{fixed} \quad \frac{R_p}{R_s} \]

\[ b=0 \quad \delta_1 \]

\[ b=0.8 \]
Limb Darkening

\[ \frac{R_p}{R_s} \]

- \( b=0 \) \( \delta_1 \)
- \( b=0.8 \) \( \delta_2 < \delta_1! \)
for circular orbits and $R_p \ll R_\star \ll a$:

(see Saeger & Mallen-Ornelas 2003 for a rigorous derivation)

$$T \approx T_0 \sqrt{1 - b^2},$$

$$T_0 \equiv \frac{R_\star P}{\pi a}$$

Can rewrite $a/R_s$ using Kepler’s 3rd law …
Big Result 1: Independent stellar parameters can be used to improve transit parameters!

Big Result 2: Transit observables can be used to measure stellar parameters

\[ T_0 \equiv \frac{R_\star P}{\pi a} \]

\[ \rho_{\star, \text{transit}} = \frac{3\pi}{GP^2} \left( \frac{a}{R_\star} \right)^3 \]
Eccentricities from asteroseismic densities + transit durations. Gaia parallaxes ($\rho \propto M/R^3$) should now allow this for many more systems (see also Xie+ 2016)
"Know Thy Star, Know Thy Planet"

\[ \delta = \left( \frac{r_p}{R_*} \right)^2 \]

\[ K \propto \frac{m_p \sin i}{M_*^{2/3}} \]

\[ q = \frac{m_p}{M_*} \]

Ikwut-Ukwa+ 2020

Nielsen+ 2020
Ages are important for masses of young planets & to test formation models.
Host Stars of Directly Imaged Earth-like Planets

Roman Coronagraph Instrument

6-m Class Astro2020 Flagship

John Krist (NASA JPL)

Juanola Parramon, Zimmerman, Roberge (NASA GSFC)
Ages of nearby bright stars may be critical for interpreting biosignatures on (future) directly imaged exoplanets.

Krissansen-Totton, Huber, MacGregor & O’Rourke (see also Bixel & Apai 2021)
Summary

• Astrometry from Gaia has revolutionized our ability to derive fundamental properties of stars: Luminosities and radii to $<\sim5\%(!)$ are now routinely possible. $F_{bol}$ and $T_{eff}$ are the new bottlenecks!

• Masses and ages of stars remain challenging: Binaries from Gaia will be critical to calibrate models that are required for most field stars

• Stellar properties are important for virtually all fields of exoplanet science, ranging from improving transit fits to interpreting biosignatures in (future) directly imaged planets. Much more on this tomorrow!

... and a big thank you to the Gaia team for these amazing astrometric datasets!
Extra Slides
Bolometric Fluxes: Systematic Errors

~2-4% offsets not uncommon. Can dominate the error budget on $L \propto f_{\text{bol}} d^2!$
Effects of Unresolved Binaries

Berger+ 2018
Asteroseismic rotation period constrains line-of-sight inclination to $i\sim60^\circ$ ($i\sim120^\circ$). Consistent with planetary orbit alignment?
Caveat: Stellar models have systematic errors

Source of systematics: uncertain input physics such as convection, atmospheric boundary conditions, rotation, opacities and overshoot

Sets error floor of ~5% in mass and ~20% in age for solar-type stars, with variation across HRD

Always a good idea to establish systematic errors by using different model grids!

Tayar et al. 2022