Planetary population synthesis - comparing theory and observation

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Christoph Mordasini
Division of Space Research and Planetary Sciences
Physics Institute
University of Bern, Switzerland
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1. Observational motivation
Observational motivation

- Enormous increase in observational data on exoplanets since 1995. Detections from ground and space (HARPS, HIRES, Kepler, TESS, NGTS, SPHERE, GPI, CARMENES, CHEOPS, ESPRESSO, WASP…)

- More to come soon (JWST, Gaia, PLATO, Roman ST, ARIEL, ELT, …)

We would like to use all these observations to better understand planet formation and evolution. But the field remains observationally driven, theory struggles to keep up. Why?

Diversity in exoplanet properties

- Radial velocities
- Transit photometry
- Direct imaging
- Microlensing

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Challenges in planet formation and evolution theory

Planet formation is a complex process

- Huge range in spatial scales: dust grains to giant planets
- Millions of dynamical timescales
- Multiple input physics: gravity, hydrodynamics, thermodynamics, radiative transport, magnetic fields, high-pressure physics,…
- Strong non-linear mechanisms and feedbacks
- Laboratory experiments only for special regimes
- Complete 3D radiation-magnetohydrodynamic numerical simulations too expensive

Cannot build theory based on first principles of physics only.

→ Theory needs observational guidance via comparison of observations and theoretical predictions
Comparing theory and observations

Compelling comparisons not so easy in practice:
• models for specific processes: difficult to test directly with observations. Each physical mechanism intermingles with many others. Only result of non-linearly combined action of all mechanisms observable.
• Often only limited knowledge about an individual exoplanet system (like period and radius / minimum mass).

But: very high number of exoplanets: they can be treated as a population.
• statistical constraints
• data from many different techniques: much more stringent constraints on theoretical models by combining M, a, e, R, L, spectra, …

We need a tool to use this wealth of constraints.
Population synthesis is a tool to:

• use all known exoplanets to constrain planet formation and evolution models
• test the implications of theoretical concepts
• predict the yield of future instruments
• provide a link between theory and observations

Statistical approach rather than comparing individual systems

• need to compute the formation of many planetary systems
• the approach and the physics must be simplified (typically low-dimensional)
• but it must capture the key effects

→ builds on all detailed studies of specific physical mechanism, combining them into a global end-to-end formation & evolution model

• depends on / reflects the general progress of the field
Combine constraints from all major detection methods plus Solar System

Today, formation theory cannot explain all these observations in one coherent picture. But at least for some, it can give us clues about possible mechanisms responsible for them.
2. Population synthesis principle
**The sequential planet formation paradigm**

**Star formation (t=0)**  
With protoplanetary gas disk (Class I - II)  
Without gas (Class III)  
MS End (10 Gyr)

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**STAR & PROTOPLANETARY DISK**

- "Gravitational instability"
- Dust & pebbles
- Planetesimals
- Protoplanets
- Giant planets
- Giant impacts
- Terrestrial planets

- "Core accretion"

**Dynamical evolution**

**Inward drift**

**Orbital migration**

**Thermodynamic & compositional evolution**

Global end-to-end models should - in principle - include all these effects... a formidable task
The essence of the method

- you need specialised models to know what is important

- while you get the essence, you have lost the subtlety of the original

- but what is left is a concentrate of many effects

- and lets you see the big picture (hopefully)
Distill how strongly?

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + (\rho \mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \left( \frac{P + \frac{B^2}{8\pi}}{\rho} \right) - \rho \nabla \Phi + \left( \frac{B}{4\pi} \cdot \nabla \right) \mathbf{B} + \eta \nabla \left( \nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \right) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \]

\[ \frac{d \Sigma}{dt} = \frac{3}{r} \frac{\partial}{\partial r} \left[ r^{1/2} \frac{\partial}{\partial r} \tilde{\nu} \Sigma r^{1/2} \right] + \dot{\Sigma}_w(r) \]

\[ \Sigma(r) = \Sigma_0 \left( \frac{r}{r_0} \right)^{-\alpha} e^{-t/\tau} \]
The population synthesis method

Models of individual processes
Accretion, migration, ...

Global end-to-end formation & evolution model
Disk properties \(\rightarrow\) planet properties

Initial Conditions: Probability distributions of disk properties

- Disk gas mass
- Disk dust mass
- Disk lifetime

From observations

Draw and compute synthetic population

Apply observational detection bias

New instrumentation
better observational constraints

Predictions
(going back to the full synthetic population)

Stat.
Comparison:
Observable sub-population
- Frequencies
- Orbits, masses, radii, luminosities
- Architecture, multiplicity
- Correlations

Match
Model solution found

Observed population

No match: change parameters, improve model, reject model

Andrews+2018

One learns a lot even if a synthetic population does not match the observed one!
3. Input physics: global models

[Graph showing distribution of planet mass $M_p$ vs. semi-major axis $a$ in AU, with data points labeled 'Ida & Lin 2004']
Initial Conditions: Probability distributions of disk properties
- Disk gas mass
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From observations

Global end-to-end formation & evolution model

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An early (earliest?) population synthesis

Computer Simulation of the Formation of Planetary Systems

STEPHEN H. DOLE
The Rand Corporation
Santa Monica, California 90406

Received January 25, 1970; revised July 30, 1970

One of the many hypotheses about the formation of the solar system postulates that the planets were formed by the aggregation of particulate matter within a cloud of dust and gas surrounding the newly-formed sun. A test of the validity of one version of this hypothesis was obtained in a computerized Monte Carlo simulation of the process.

In the model used, nuclei are “injected” into the cloud one at a time, on elliptical orbits. The dimensions of the semimajor axis and the eccentricity of the orbit of each nucleus are determined by using random numbers. As the nuclei orbit within the cloud they grow by aggregation and gradually sweep out dust-free annular lanes. If they grow larger than a specified critical mass they can begin to accumulate gas from the cloud as well. If the orbit of a planet comes inside a certain interaction distance from a planet that was formed earlier, or if the orbits cross one another, the two bodies coalesce to form a single, more massive planet which may then continue to grow by aggregation. The process of injecting nuclei is continued until all the dust has been swept from the system. At this point the run is terminated and the machine output displays the masses and orbital parameters of the planets remaining in the final configuration.

Each planetary system produced by using a different random number sequence is unique. However, all the systems so produced share the major regular features of our solar system. The orbital spacings have patterns of regularity suggestive of “Bode’s law.” The innermost planets are small rocky bodies; the midrange planets are large gaseous bodies; the outermost planets are generally small. The general pattern of planetary mass distribution is similar to that in our solar system with masses ranging from less than that of Mercury to greater than Jupiter’s.
An early approach

- Disk model: static in time, exponential profile
- Accretion of solids: limited by feeding zone (restricted 3 body)
- Accretion of gas: if $M_{\text{core}} > k_{\text{crit}} \times M_{\text{crit}}$ found from $v_{\text{therm}} < v_{\text{esc}}$
- Termination of gas accretion: ~arbitrary parametrization
- Coalescence of embryos: if feeding zones overlap,
- Orbits: fixed (in situ formation, no migration, no N-body)
- Parameters: stellar mass, disk profile, seed mass, $k_{\text{crit}}$, max M.
- Monte Carlo variables: position and eccentricity of seed, disk mass, disk dust-to-gas ratio
“Monte carlo computer synthesis”

Pre-viscous-accretion disk theory (Lynden-Bell & Pringle 1974)

Pre-planetesimal accretion theory (Safronov 1972)

Pre-1D planetary structure theory (Mizuno 1978)

Pre-orbital migration theory (Goldreich & Tremaine 1979)

Reliance of global models on models for specific processes … and on observations

-Solar System-like with ~uniform spacing in log
-no close-in planets, no distant giant planets

~isolation mass
~critical mass

Dole 1970

Fig. 1. Examples of planetary systems generated in computer synthesis. $X_0$ is initiating number for random number series used in program. Positions of circles along line indicate mean orbital radius; numbers above circles and sizes of circles indicate planetary masses (radius of circle $\sim$ planetary mass$^{1/3}$); solid circles are terrestrial bodies; horizontal shading designates gas giants.
First modern model: Ida & Lin 2004


- Disk model: powerlaw, exponential decrease

- Accretion of solids: Safonov rate equation, isolation mass

- Accretion of gas: Parameterized KH-contraction, fitted $M_{\text{crit}}$

- Termination of gas accretion: Gap formation, disk dissipation
  Lubow 1999, Kley & Dirksen 2006

- Coalescence of embryos: 1 embryo per disk, later semi-analytical prescription (orbit crossing)

- Orbits: type I and II disk migration
  Goldreich & Tremaine 1979, Lin & Papaloizou 1986, Paardekooper et al. 2010, ...

- Monte Carlo variables: position of embryo, disk mass, dust-to-gas ratio, disk lifetime

Later several improvements: dead zones, local enhancement of solids, new type II mig., ...
First modern pop. synthesis

- aM: diversity
- Planetary desert
- Metallicity effect (correlation between metallicity and giant planet detection probability)
- termination of gas accretion
- effects of type II migration

Ida & Lin 2004
Overview of some population synthesis models

Core accretion

- **Ida & Lin Model**: with planetesimals. Fast, customised for pop. synthesis. First one-embryo per disk, then w. statistical N-body interactions.
  - Similar open source model available online at https://nexsci.caltech.edu/workshop/2015/


- **Lund Model** (Bitsch, Johansen, Ndugu, Liu and collaborators): with pebbles. Single embryo per disk. 2D-disk model.

- **Mc Master Model** (Pudritz, Alessi, Cridland, Hasegawa et al.): with planetesimals. Disk traps, astrochemistry, interior structures.

Gravitational instability

- **Forgan, Rice at al.; Nayakshin, Humphries et al.; Müller, Helled & Mayer**
- Fragmentation criteria, tidal downsizing, migration, clump contraction, ...
Bern Generation 3 formation & evolution model

Core accretion paradigm
Benz et al. 2014, Mordasini 2018, Emsenhuber et al. 2021a,b Schlecker et al. 2021a,b, Burn et al. 2021, Mishra et al. 2021

Emphasis of Generation III
• direct prediction of all important observable quantities
• ability to simulate planets ranging in mass from of Mars to super-Jupiters, at all orbital distances

Sub-models
Form. & evolution phase (0-10 Gyr)
Gas disk (0-τ_{disk} Myr)
Formation phase (0-20 Myr)
Evolution phase (20 Myr - 10 Gyr)

Stages: All Gas disc Formation Evolution

- 1D axisym. cst. α-model w. photoevap. & irradiation (Lynden-Bell & Pringle, Hollenbach, Chiang & Goldreich, …)
- Planetesimals as a surface density with dynamical state: eccentricity, inclination (Adachi, Ohtsuki, Chambers, …)
- Rate equation à la Safronv for planetesimal accretion rate (Safronv, Greenzweig & Lissauer, Ida & Makino, Inaba, …)
- Solution of 1D radially symmetric planetary structure equations to calculate H/He envelope internal structure and thus gas accretion rate, radius and luminosity (Bodenheimer & Pollack), w. D burning & XUV driven atm. escape
- Outer boundary conditions for envelope structure: attached, detached, isolated (Eddington gray)
- Internal structure and radius of the solid core (modified polytropic EOS, Seager)
- Type I & type II gas disk-driven orbital migration (Lin & Papaloizou, Tremaine, Paardekooper, …)
- N-body interaction among protoplanets: scattering, collisions, capture in MMR (Newton, Chambers, …)
**The Gen III Bern Model of planet formation and evolution**

**Simplification: rich in (micro)physics, but low dimensionality:**
- Planets: spherically symmetric (internal structure resolved radially in 1D)
- Disks: rotationally symmetric (resolved 1+1D, radial and vertical direction)

Still many effects neglected: early phases for solids (e.g., Voelkel+2020), disk winds (e.g., Suzuki et al. 2010), …

Numerical simulation of 1 planetary system starting from 100 lunar mass embryos: about 3 months (mostly N-body and planetary internal structure calculation). Long calculation time makes parameter optimisation difficult (Chambers 2018).
4. Initial conditions
**Initial conditions**

Models of individual processes
- Accretion, migration, …

Global end-to-end formation & evolution model
- Disk properties ⇒ planet properties

Draw and compute synthetic population

Apply observational detection bias

Observed population

- No match: change parameters, improve model, reject model

Initial Conditions: Probability distributions of disk properties
- Disk gas mass
- Disk dust mass
- Disk lifetime
- From observations

Stat. Comparison:
- Observable sub-population
  - Frequencies
  - Orbits, masses, radii, luminosities
  - Architecture, multiplicity
  - Correlations
  - …..

New instrumentation
- better observational constraints

Predictions
- (going back to the full synthetic population)

Model solution found

One learns a lot even if a synthetic population does not match the observed one!
The imprint of disk properties

Planet-forming disks: large diversity too. Observational determination of distributions of
- Disk lifetimes (stellar cluster environ.)
- Disk gas masses
- Disk dust masses
- Disk sizes

Diversity of disks (Initial conditions)

Diversity of planets (End products)

Statistically reproducible with a population synthesis model

The ALMA revolution
Monte Carlo initial conditions

1 Metallicity
assume same in star and disk

Stellar [Fe/H] from spectroscopy. Gaussian distribution for [Fe/H] with \( \mu \sim 0.0, \sigma \sim 0.2 \) (e.g. Santos et al. 2003)

\[ f_{\text{dg}} = 0.0149 \times 10^{[\text{Fe/H}]} \]

2 Disk (gas) masses
From VANDAM survey of Perseus Class I disks (Tychoniec et al. 2018)

3 Disk lifetime
IR excess

vary lifetime via photoevaporation rate
alpha=2x10^{-3}

4 Inner disk edge

At corotation radius. Venuti+2017 rotation periods in NGC 2264 (~3 Myr):
log-normal distribution with mean of 4.74 days and \( \sigma \) of 0.3 dex.

Initial solid mass

Initial mass of solids in the disks.
Gray: sum of metals in Solar System planets today

It is not trivial to derive these distributions
5. Detection biases
**Detection bias**

- **Initial Conditions:** Probability distributions of disk properties
  - Disk gas mass
  - Disk dust mass
  - Disk lifetime
  - From observations

- **Draw and compute synthetic population**

- **Apply observational detection bias**

- **Observed population**

- **No match:** change parameters, improve model, reject model

- **Models of individual processes**
  - Accretion, migration, ...

- **Global end-to-end formation & evolution model**
  - Disk properties \(\Rightarrow\) planet properties

- **New instrumentation**
  - Better observational constraints

- **Predictions**
  - (going back to the full synthetic population)

- **Stat. Comparison:**
  - Observable sub-population
    - Frequencies
    - Orbits, masses, radii, luminosities
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- **Match**
  - Model solution found

**One learns a lot even if a synthetic population does not match the observed one!**
Radial velocity detection bias

Get sub-population of observable synthetic planets

Includes effects of
- Orbital eccentricity
- Stellar metallicity, rotation rate, and jitter
- Actual measurement schedule

Elodie \sim 10 \text{ m/s} \quad \text{Instrumental precision} \quad \text{HARPS} \sim 1 \text{ m/s}

Naef et al. 2004

Mayor et al. 2011

822 stars

Naef et al. 2004

Mayor et al. 2011
Transits, direct imaging, microlensing

Accounting properly for biases is important. Otherwise, the picture might be distorted (e.g. Hot Jupiters)
- Models need to predict not only masses and orbits but also radii and magnitudes
- Each technique probes different aspects of the theory: helps to beat the parameter dependency of global models, a weakness of this approach.
- Once we have the detectable sub-population, we can compare it with the actual observed one and learn if the model disagrees/agrees with the observations

Large surveys with a well defined bias are suited best for statistical comparisons
End of part A