

# Post-formation dynamical evolution

Smadar Naoz  
UCLA

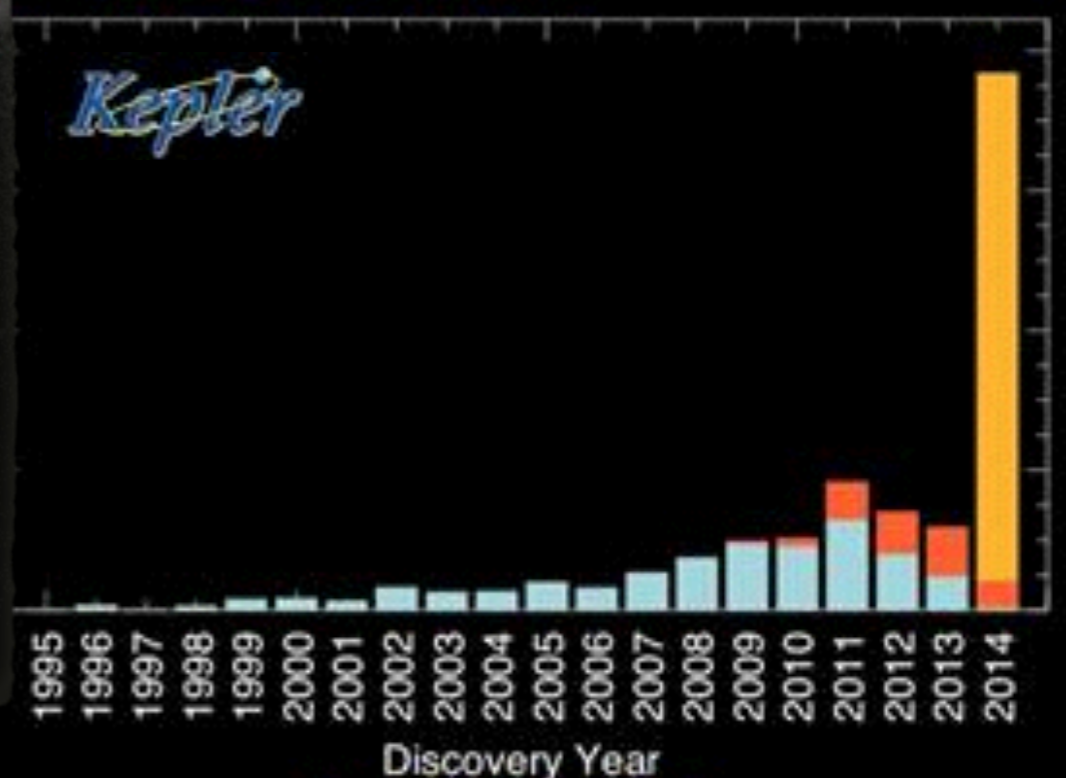
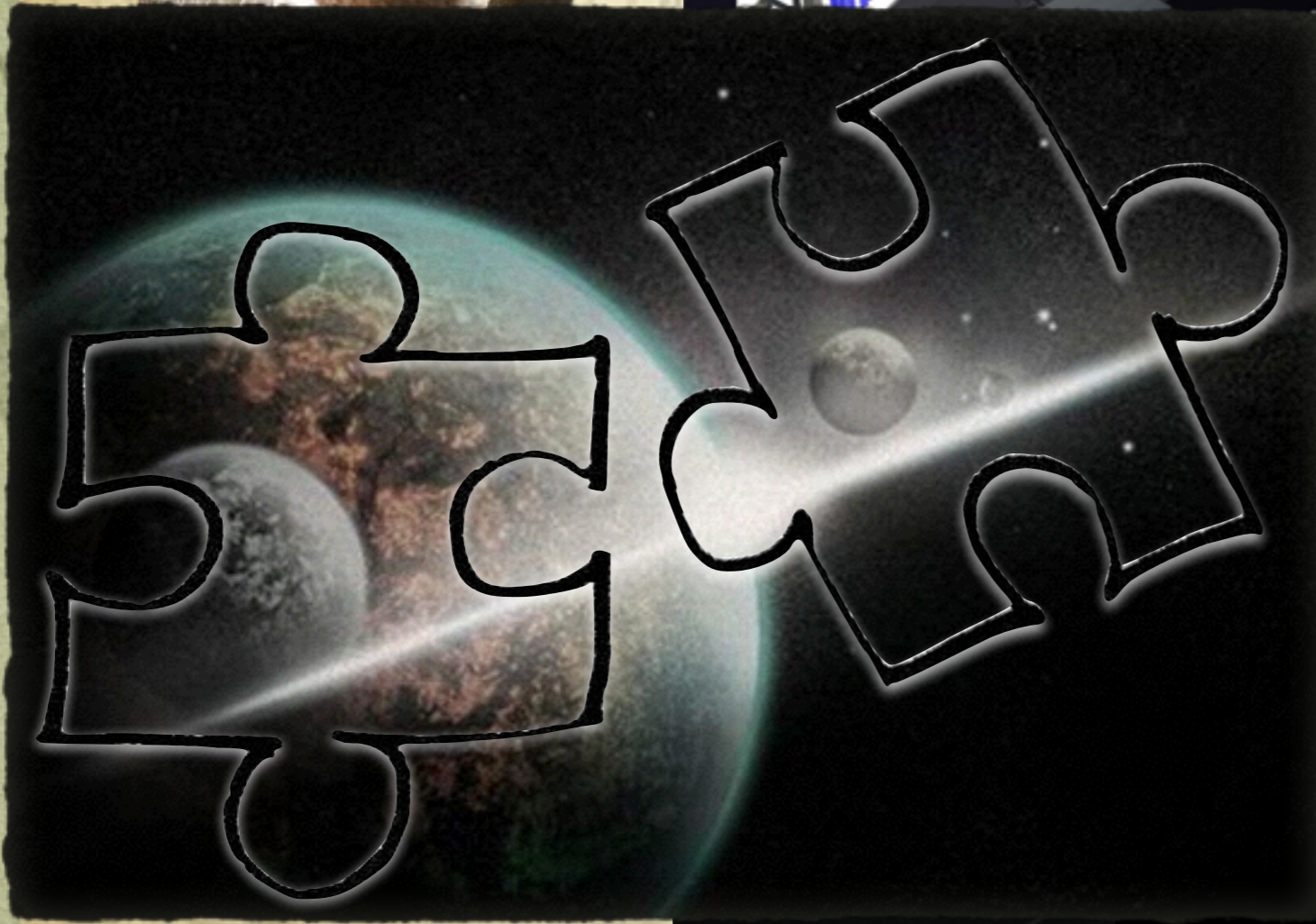
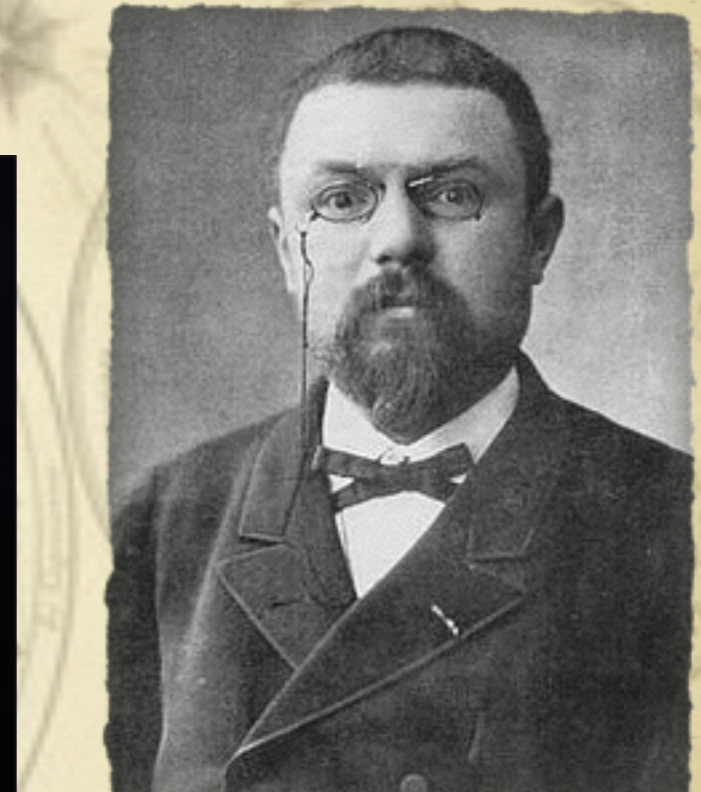


Sagan Workshop  
July 2015

Copernicus 1661



# AstroDynamics

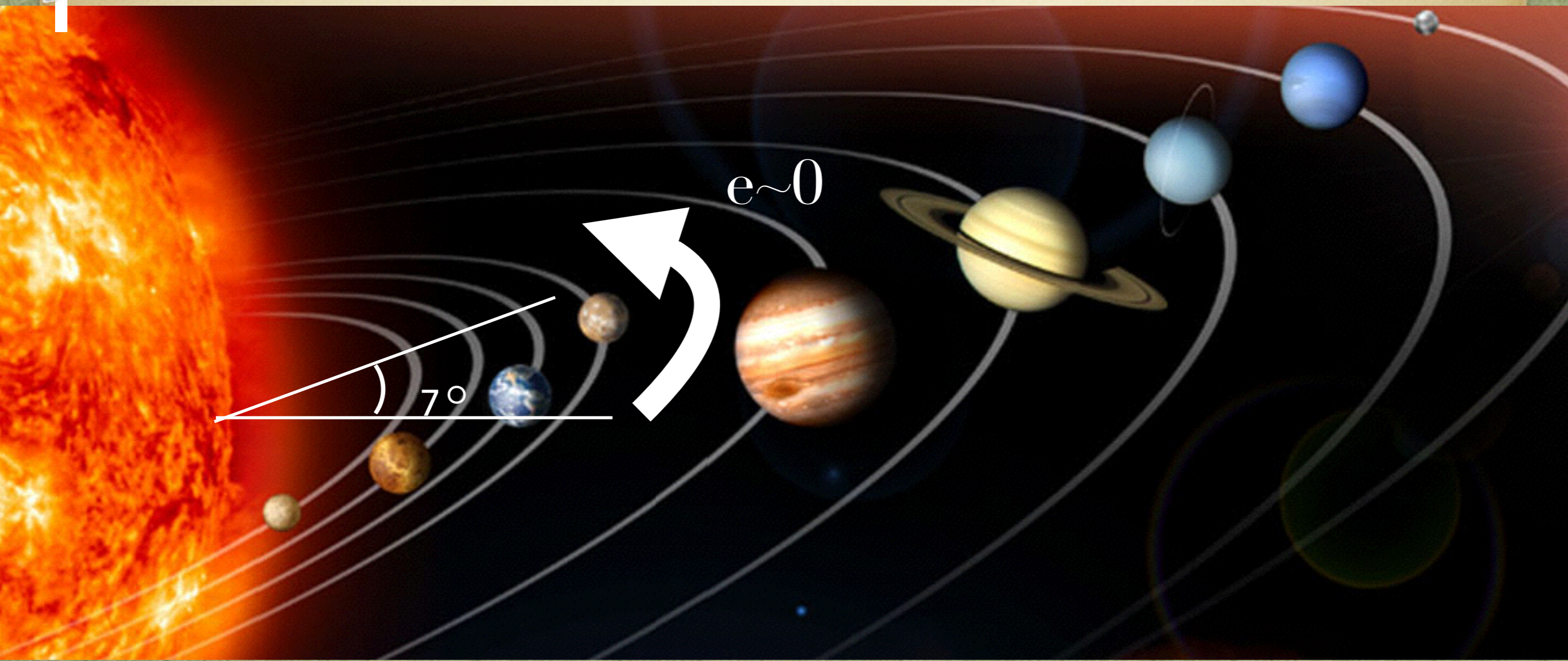


credit:Natalie Batalha

26 Feb 2014

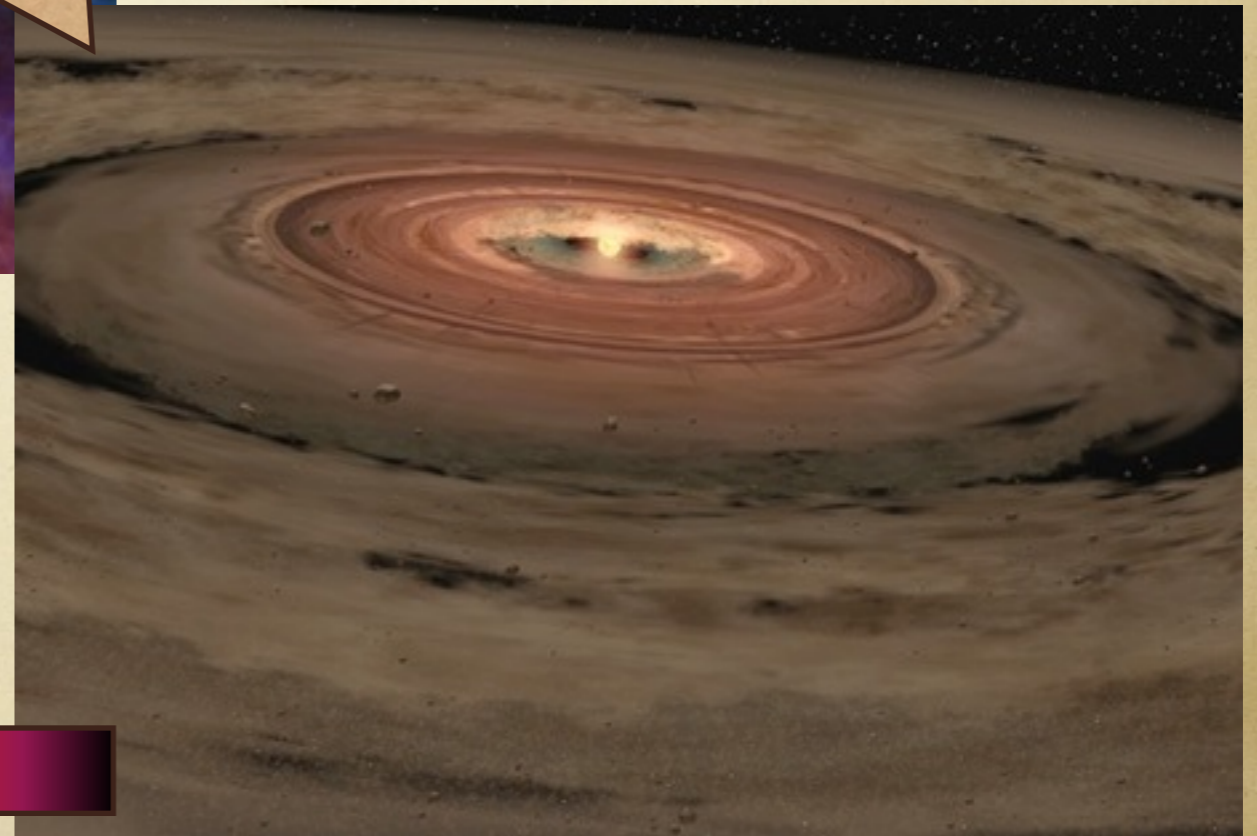
# Our solar System

the rotation of  
the sun

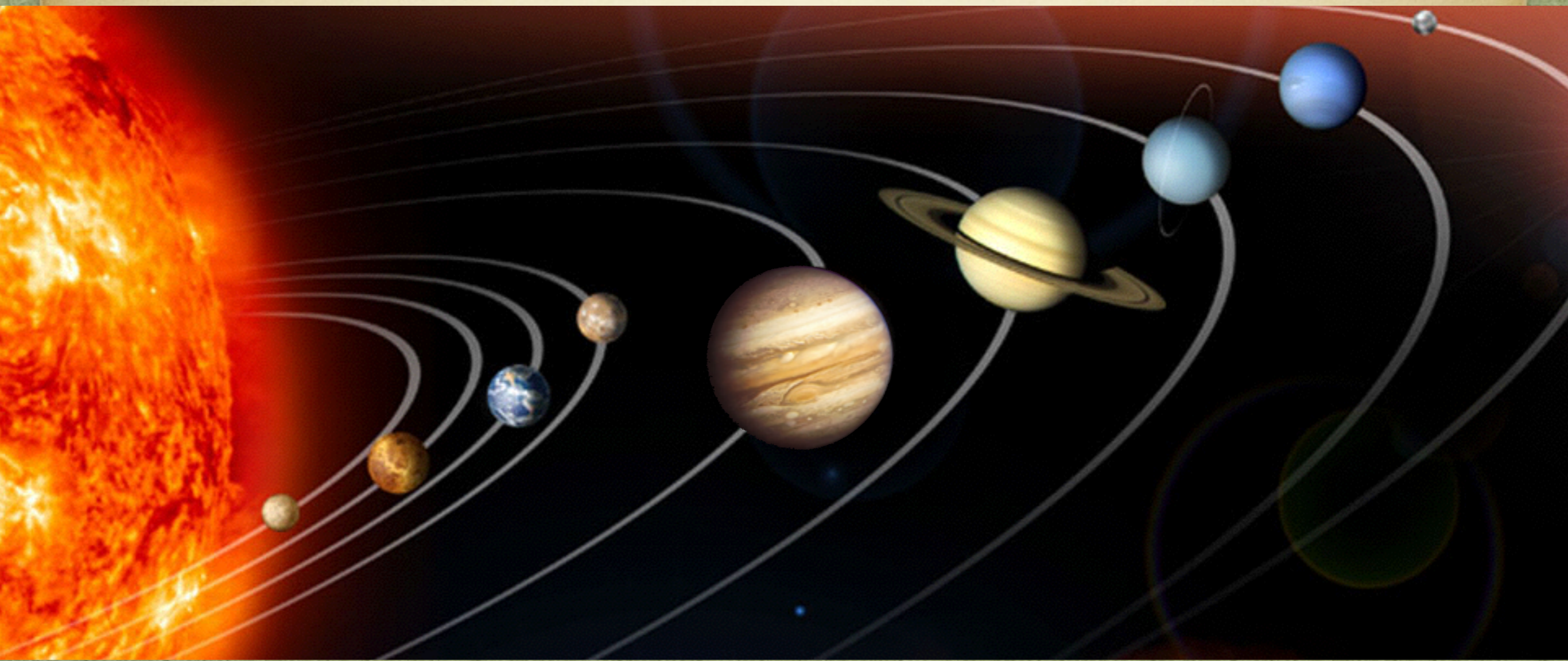


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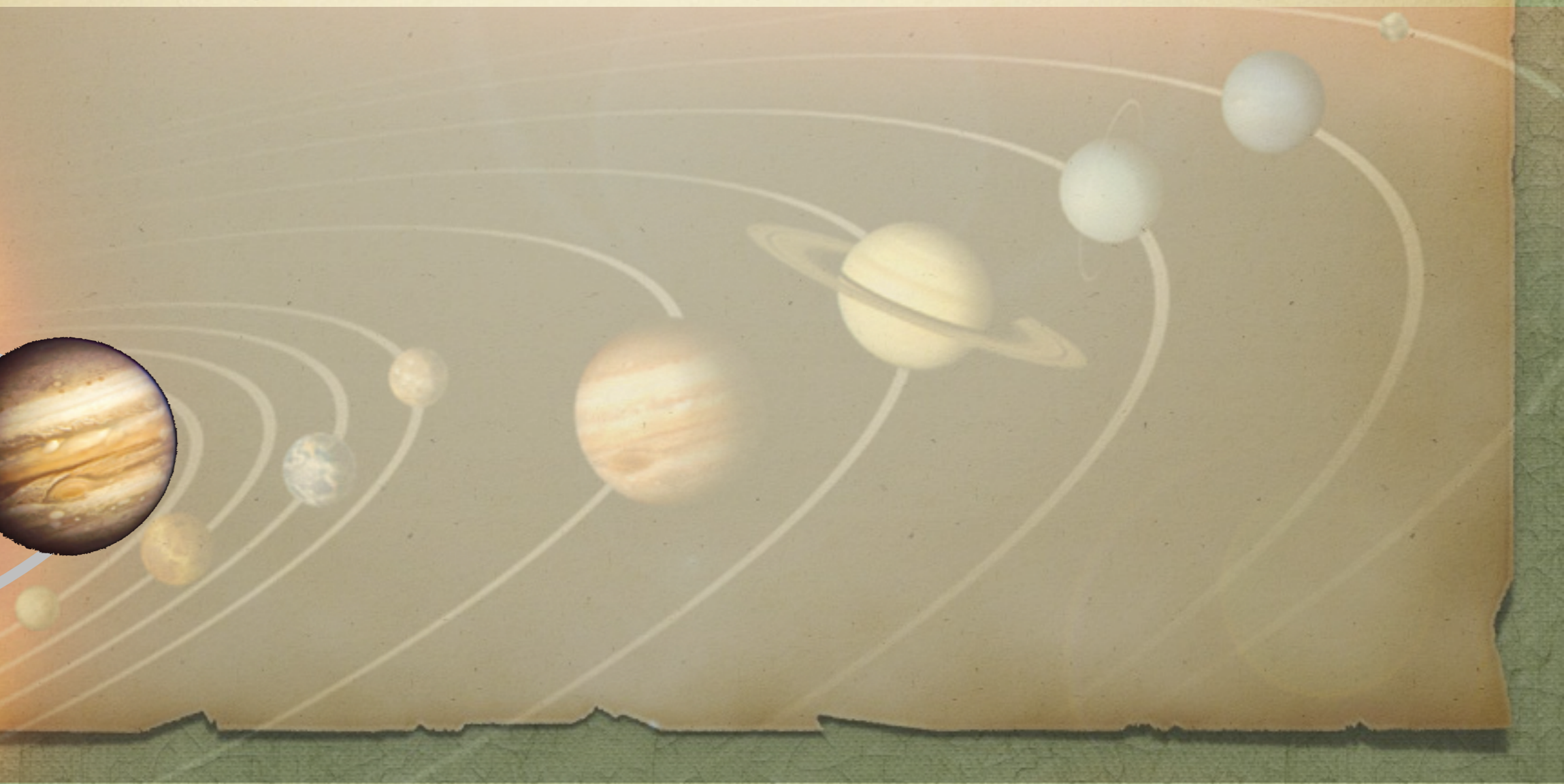
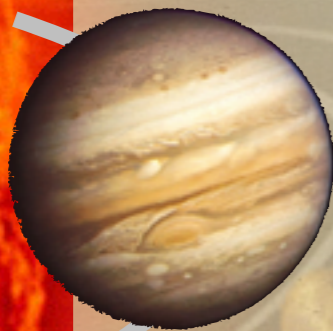
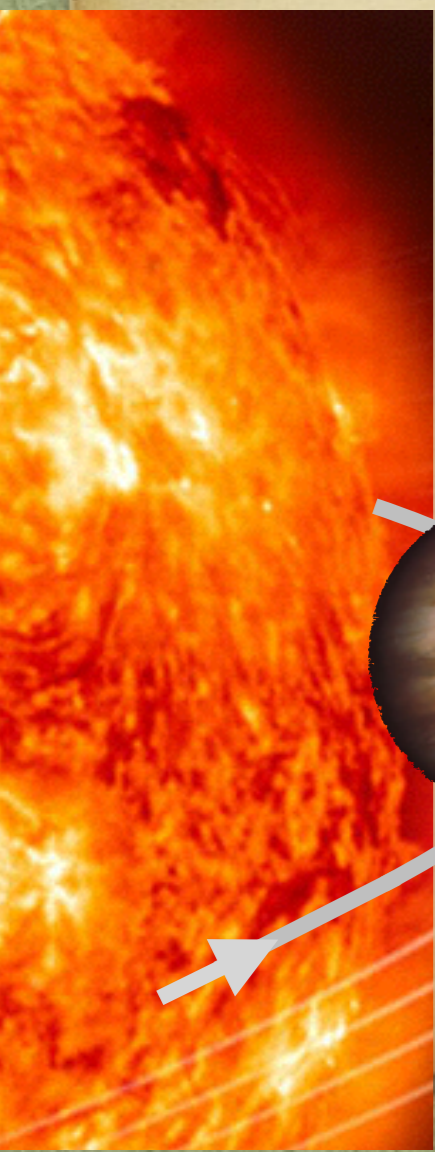
Formation story



# Other solar systems

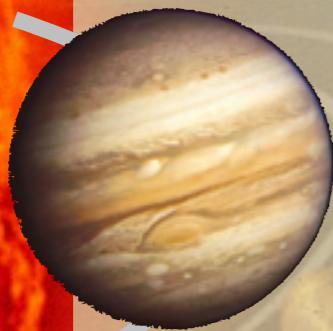
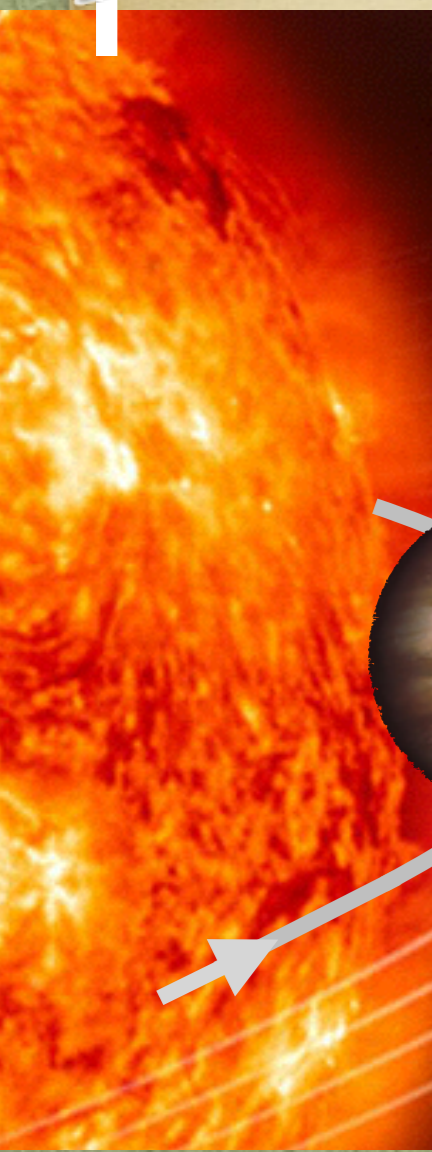


# Other solar systems

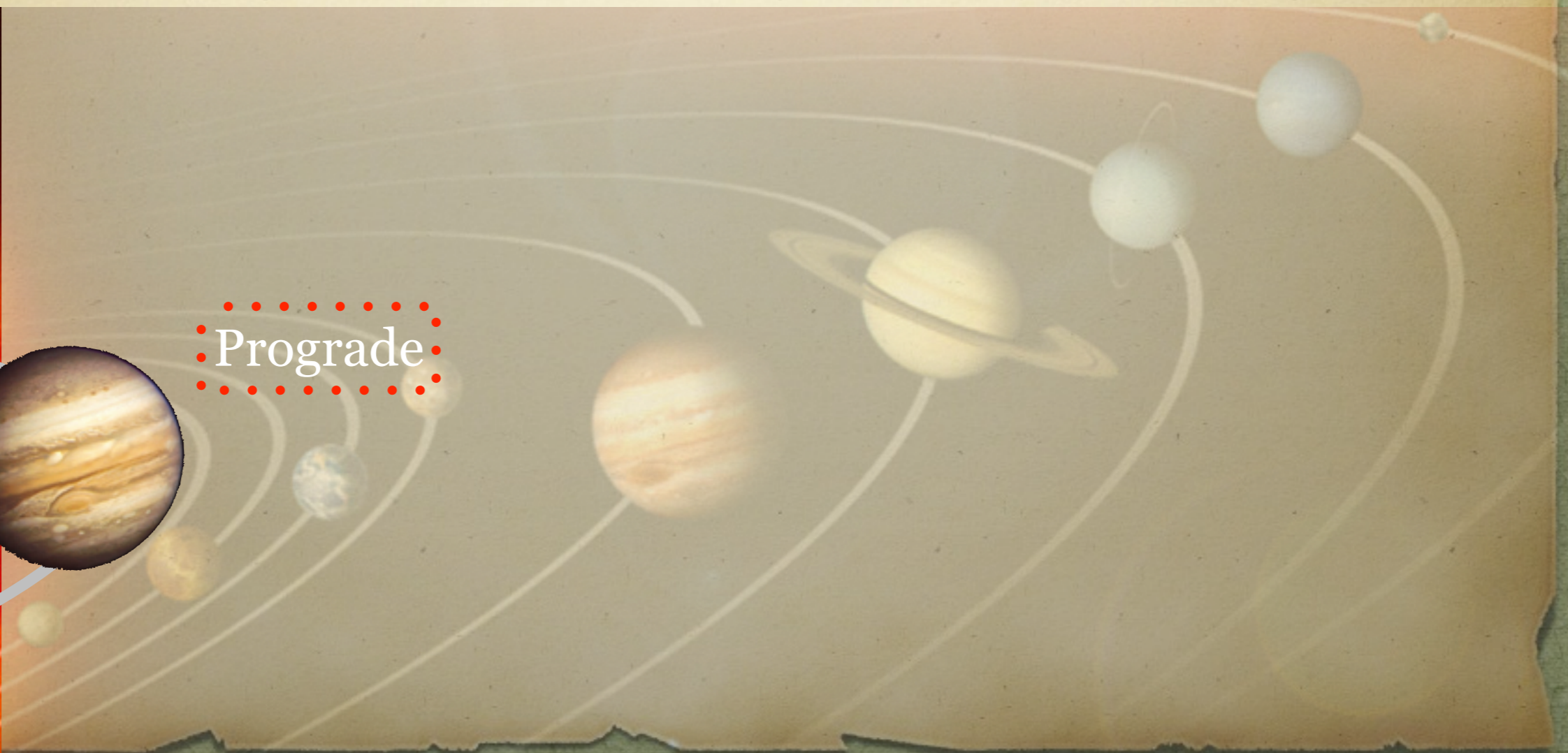


# Other solar systems

the rotation of  
the star

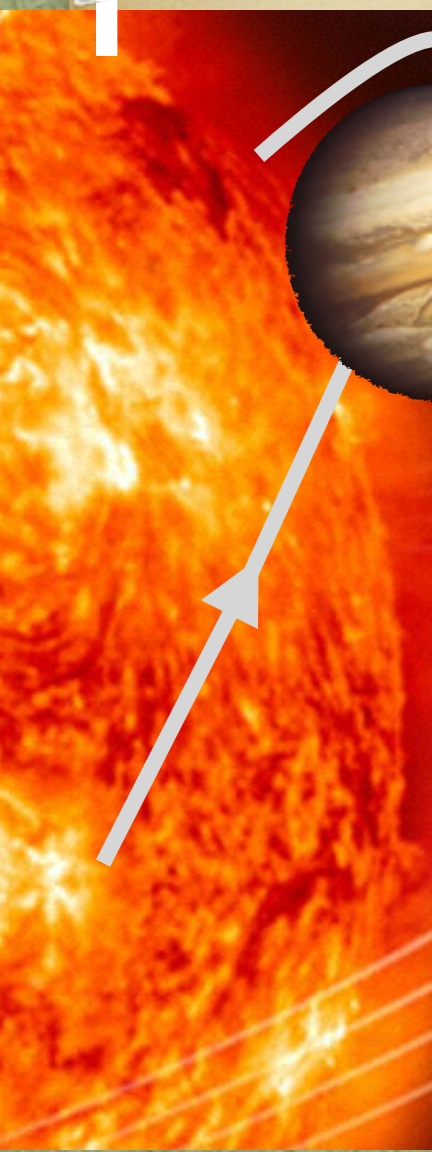


Prograde

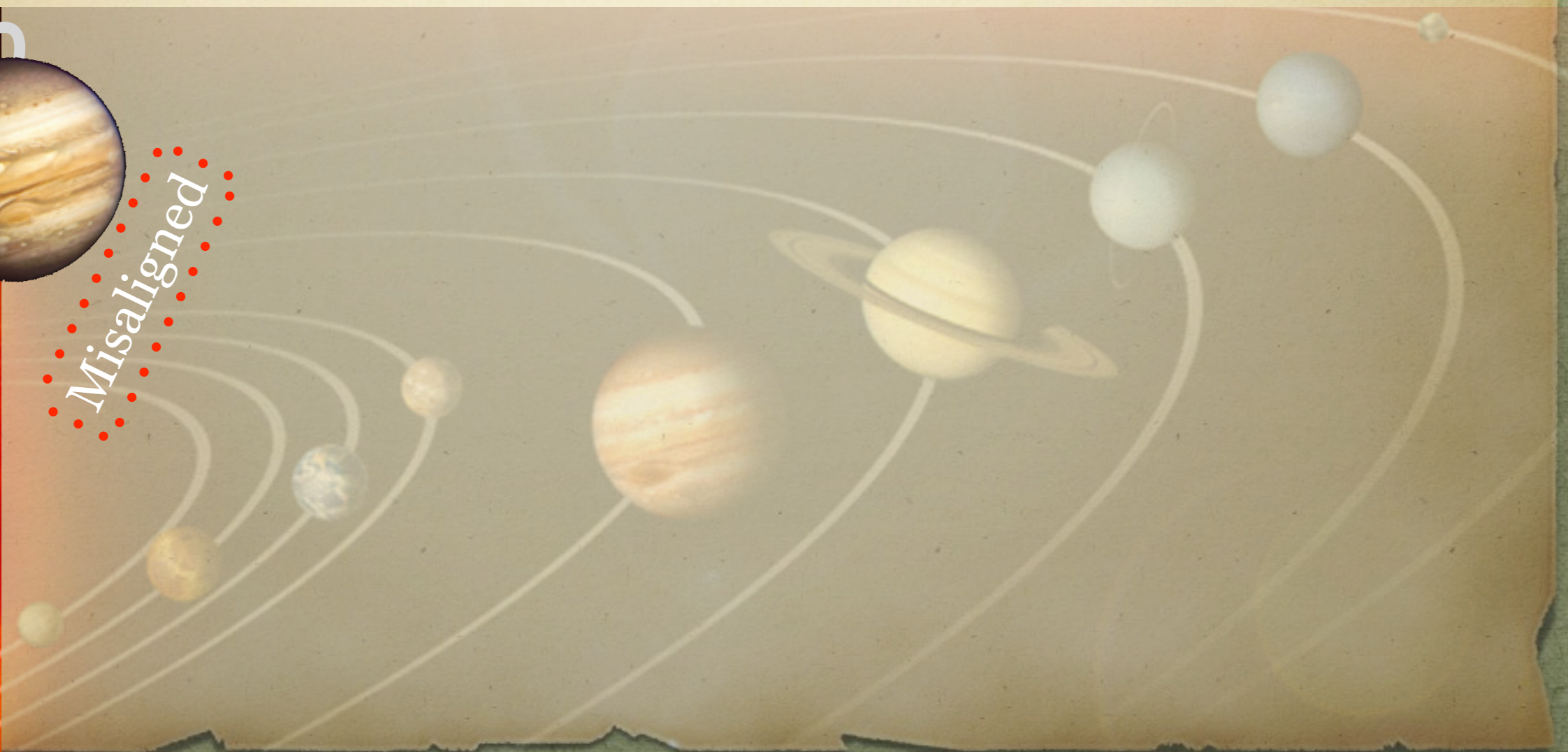


# Other solar systems

the rotation of  
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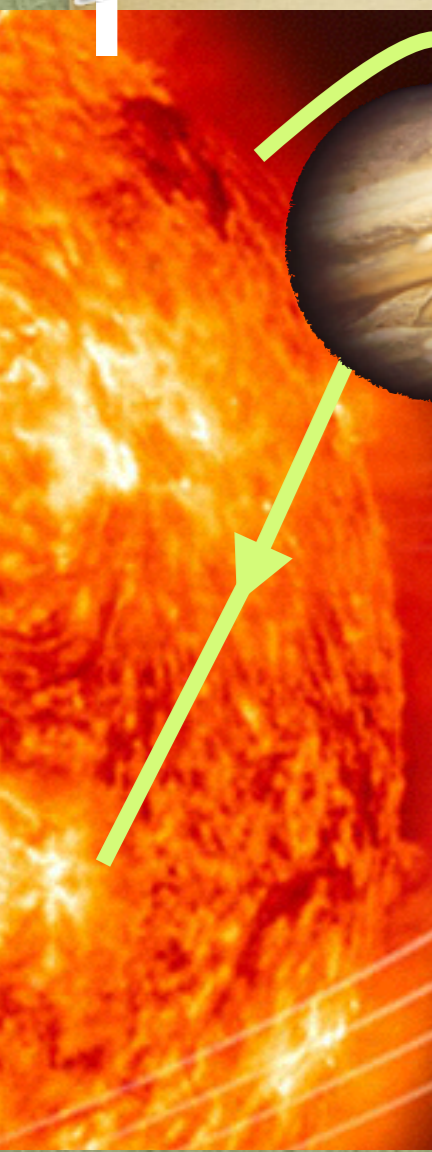
*Misaligned*



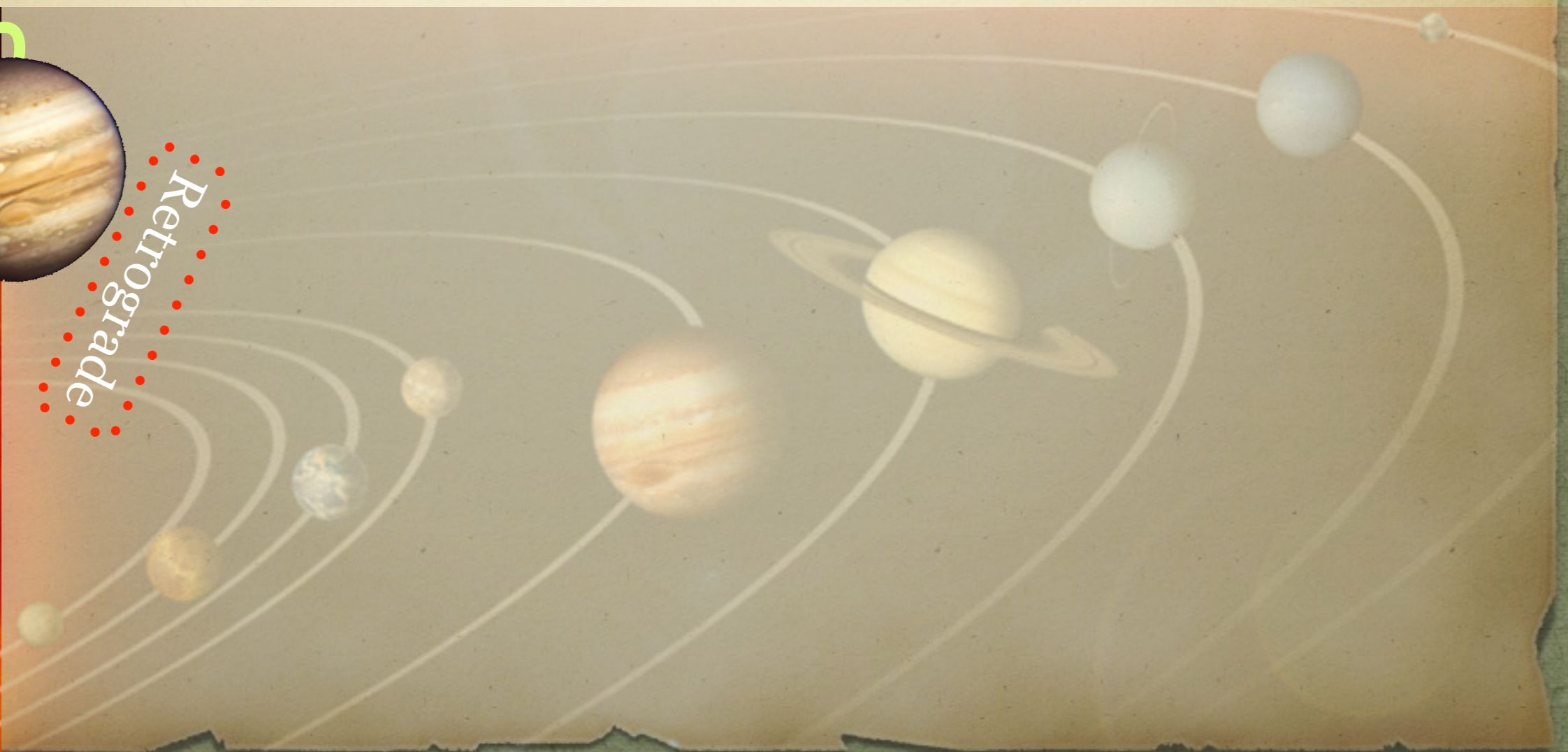


# Other solar systems

the rotation of  
the star



Retrograde

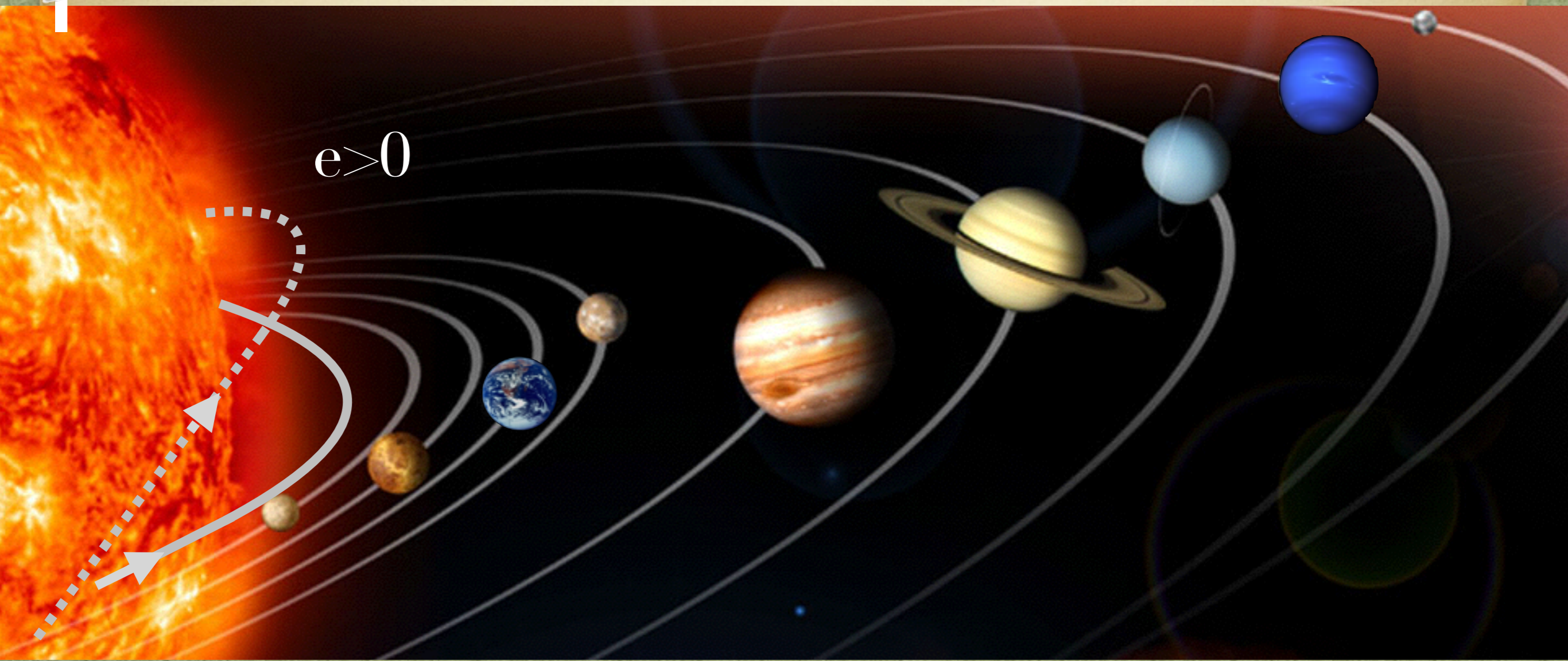


# Other solar systems

the rotation of  
the star



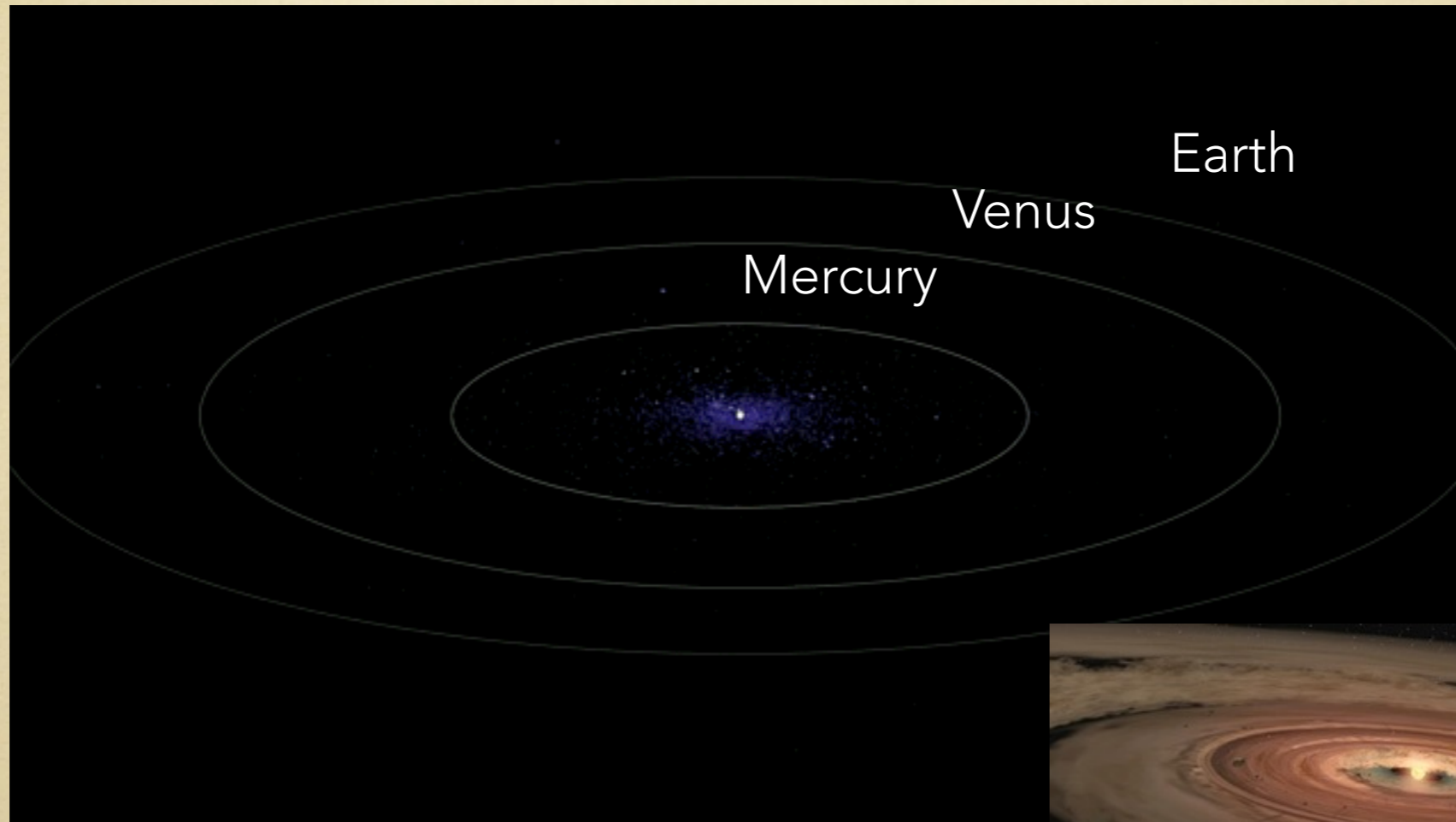
$e > 0$



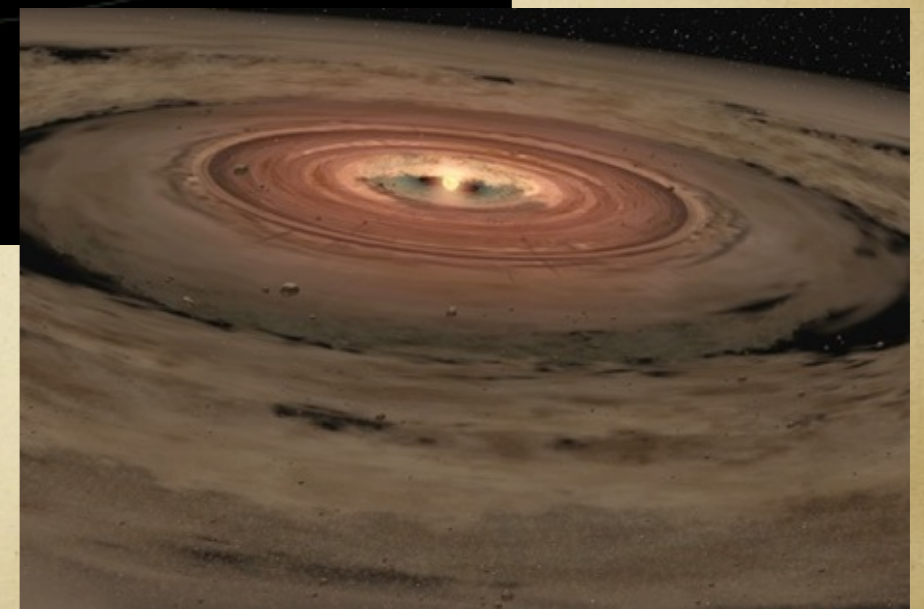
Kepler planet candidates are in a very close orbit

90% of all planets and planet candidates are in a very close orbit

Illustration of the planets and planet candidates as if they orbit a single star



movie by Alex Parker



Astrodynamics is alive!



# Selected dynamical processes

- Planet-Planet scattering
- Mean motions resonances
- “Classical” secular evolution
- The eccentric Kozai-Lidov (EKL) mechanism

# Selected dynamical processes

- **Planet-Planet scattering**
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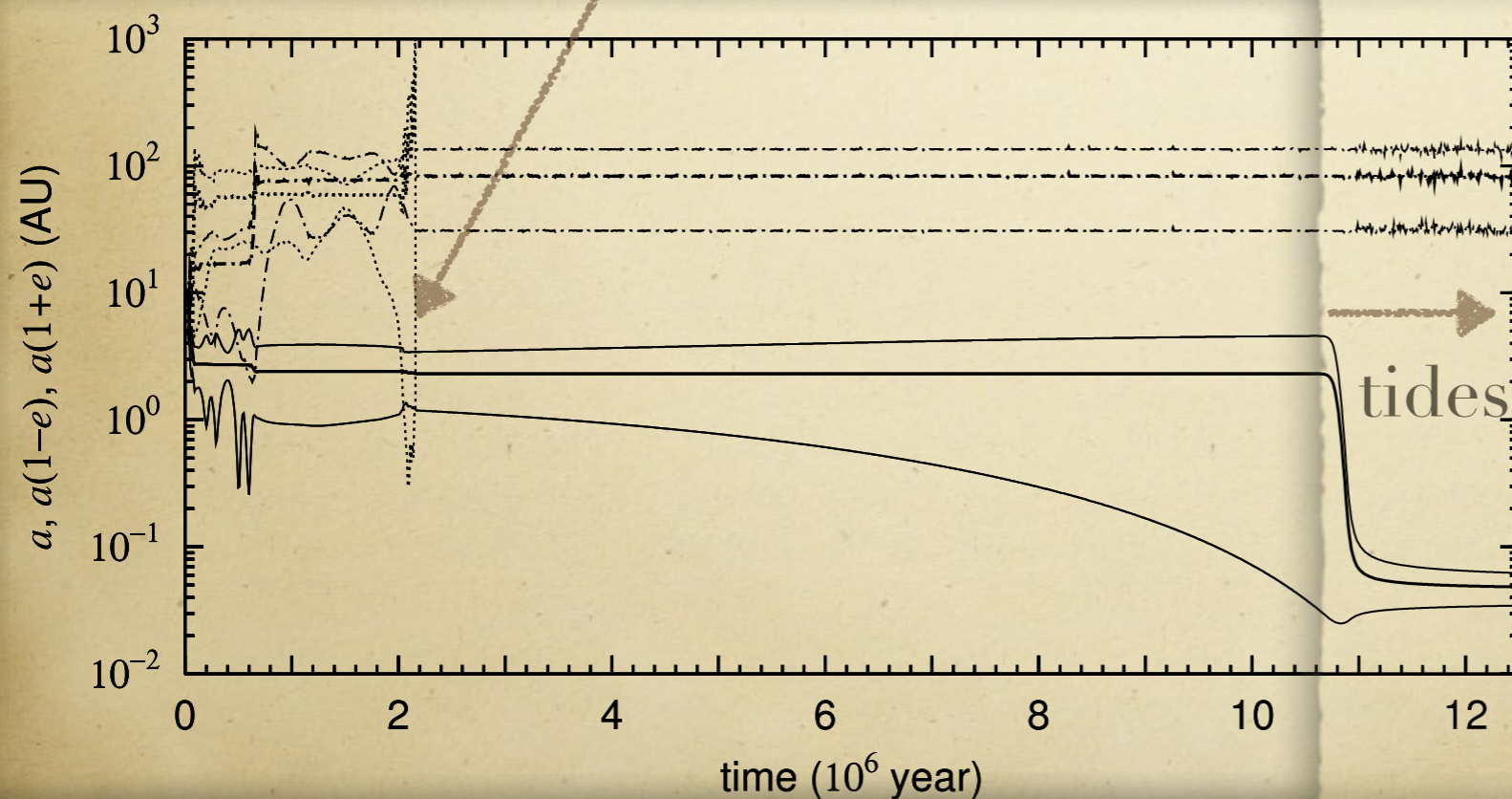
# Post evolution dynamics in planetary systems

1. Scattering events Rasio & Ford 1996

*Short Time Scale*  
larger than orbital

Packed systems, outcome: inclined, eccentric, ejection

ejection



# Selected dynamical processes

- Planet-Planet scattering
- **Mean motions resonances**
- “Classical” secular evolution
- The eccentric Kozai-Lidov (EKL) mechanism



# Post evolution dynamics in planetary systems

## Resonances

### 2. Mean Motion Resonances

$$\frac{P_1}{P_2} \sim \frac{n}{m}$$

*Orbital Time Scale*

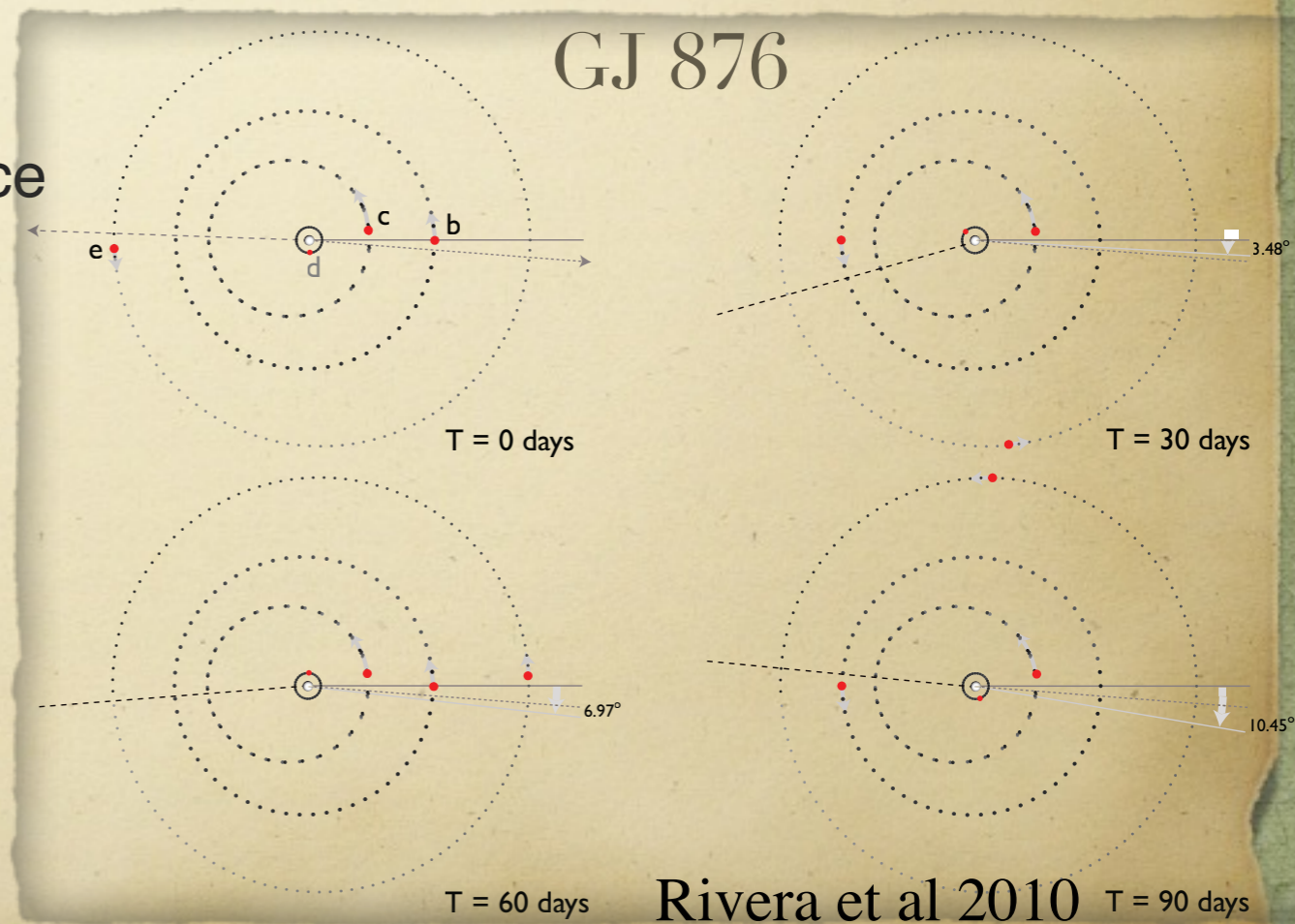
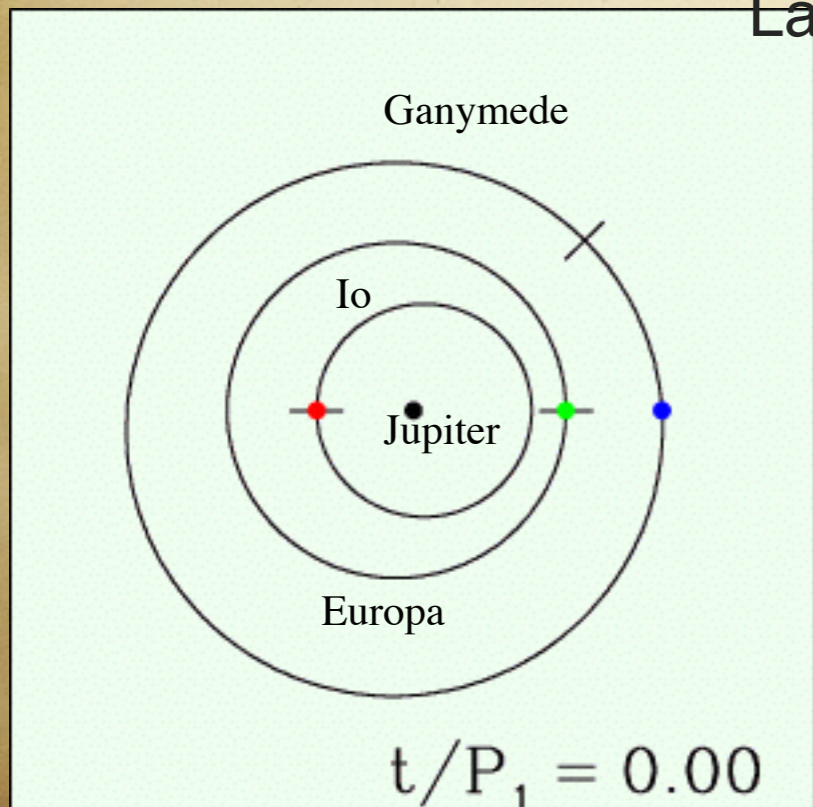
outcome: in most cases lead to unstable config.

sometimes we stable config.

### Galilean Satellites

Io, Europa, and Ganymede 1:2:4

Laplace resonance



# Post evolution dynamics

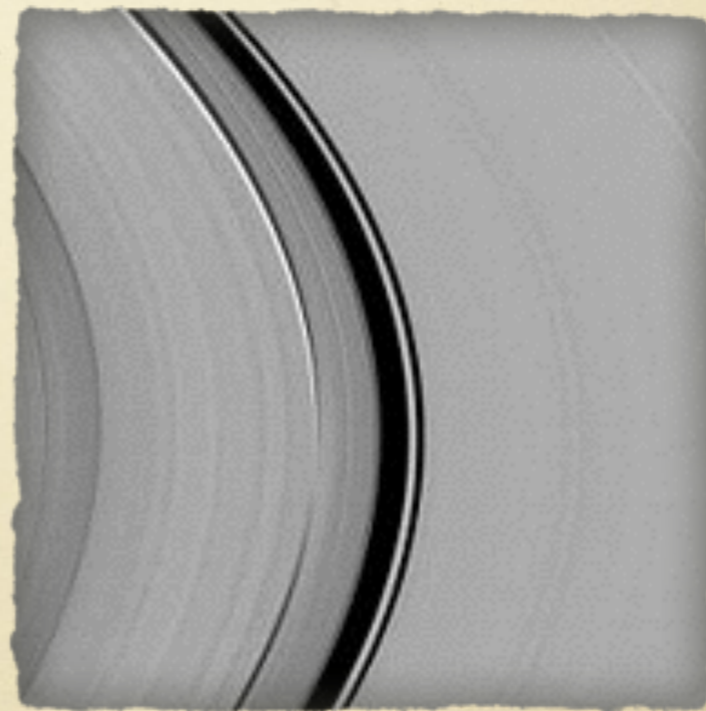
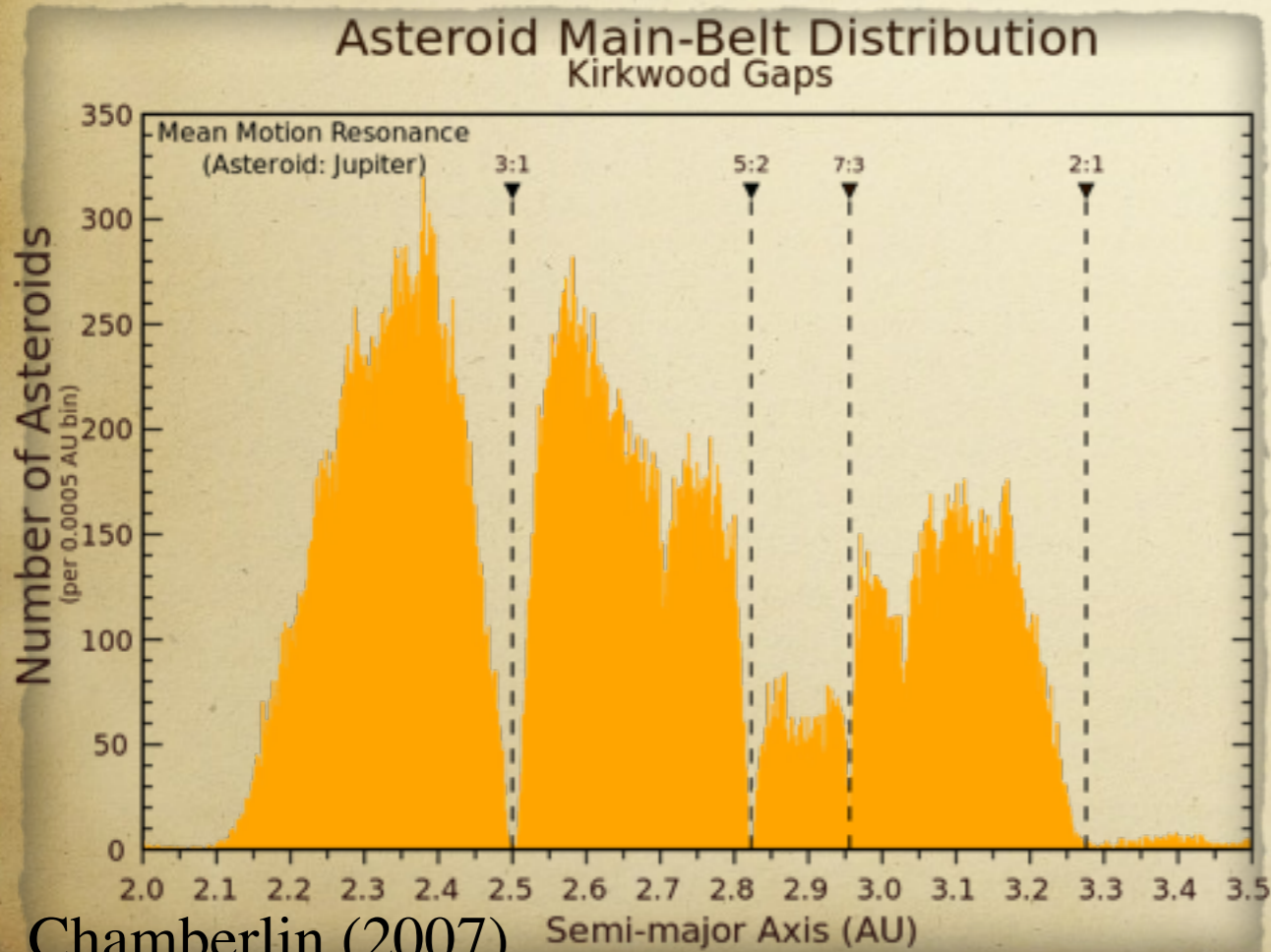
## in planetary systems

### Resonances

#### 2. Mean Motion Resonances

$$\frac{P_1}{P_2} \sim \frac{n}{m}$$

*unstable config:*



# Post evolution dynamics

## in planetary systems

### Resonances

#### 2. Mean Motion Resonances

$$\frac{P_1}{P_2} \sim \frac{n}{m}$$

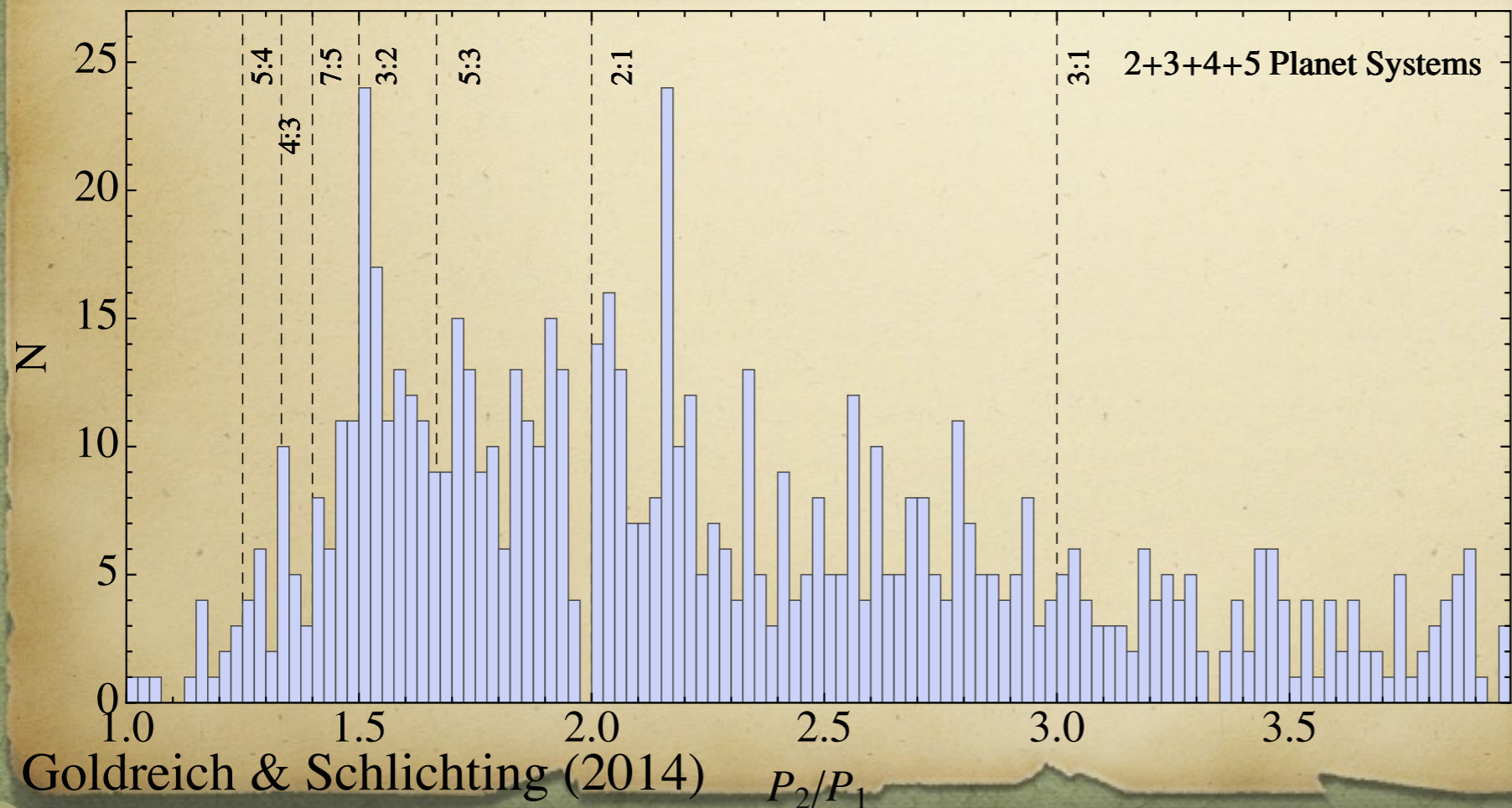
*Extrasolar...*

Problem: predicted migration rates: most planets in resonances.

But reality ...

Few possible explanations:

- Accretion of mass e.g., Petrovich et al (2013)
- Dissipation (maybe tides?) Lithwick & Wu (2012), Batygin & Morbidelli (2013)
- Resonance capture is temporary Goldreich & Schlichting (2014)
- and more ....



# Selected dynamical processes

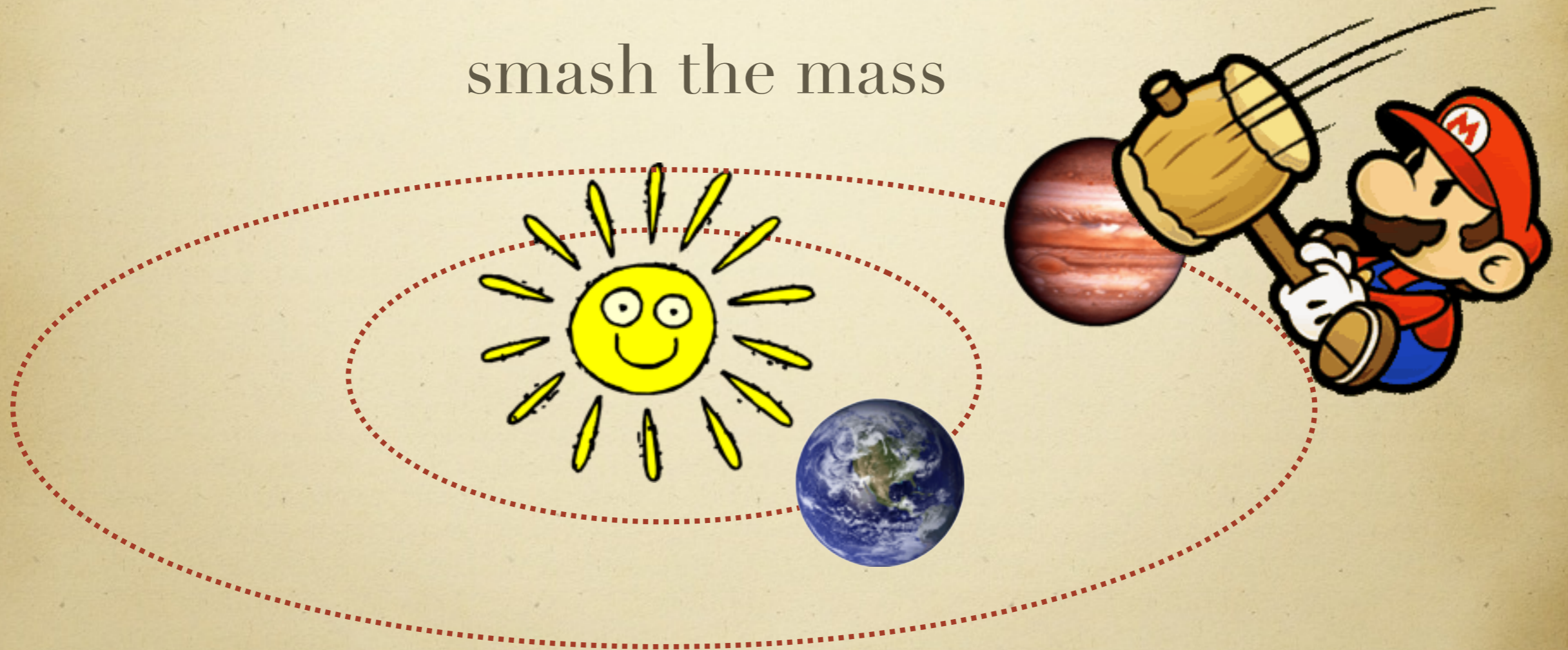
- Planet-Planet scattering
- Mean motions resonances
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- The eccentric Kozai-Lidov (EKL) mechanism

# Post evolution dynamics in planetary systems

## 3. The secular interactions

*Long Time Scale*

smash the mass



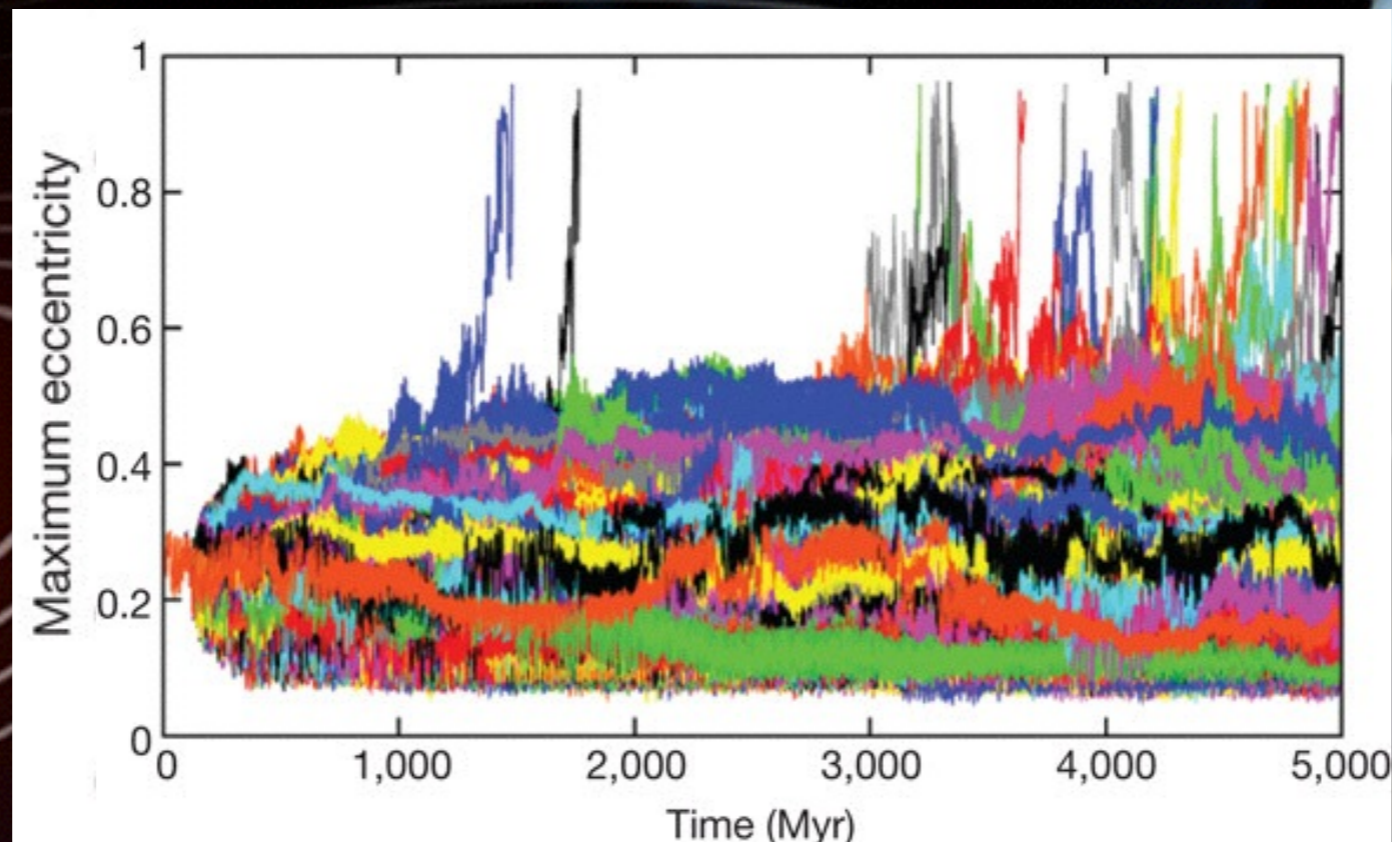
# Post evolution dynamics in planetary systems

## 3. The secular interactions

~circular orbits, concentric, coplanar

*Long Time Scale*

Laskar & Gastineau (2009)



# Selected dynamical processes

- Planet-Planet scattering
- Mean motions resonances
- “Classical” secular evolution
- **The eccentric Kozai-Lidov (EKL) mechanism**

# Post evolution dynamics in planetary systems

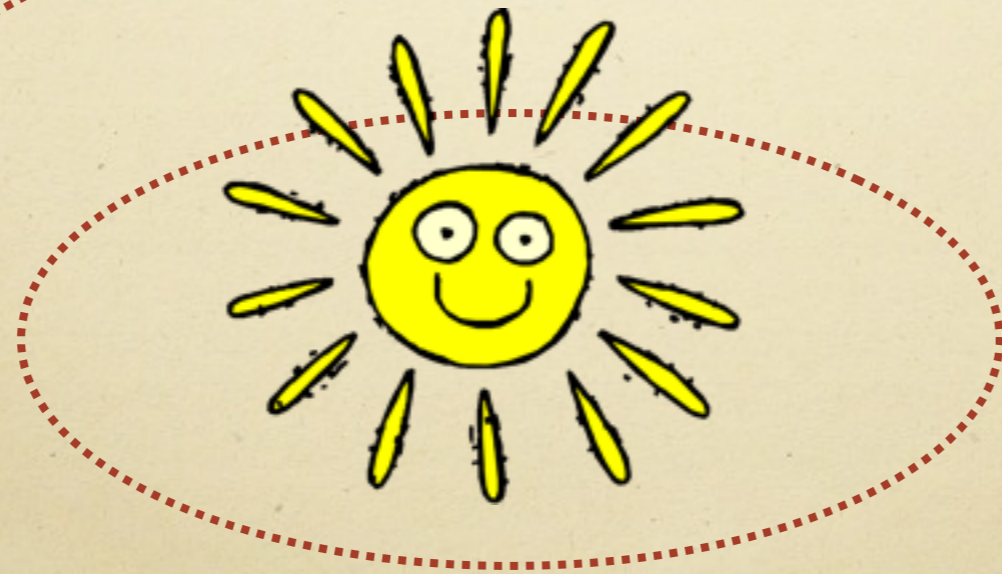
## 4. The secular interactions

~can be eccentric, hierarchical, inclined

*Long Time Scale*

Analytical treatment 3  
body config.

Perturbations from a far  
away perturber

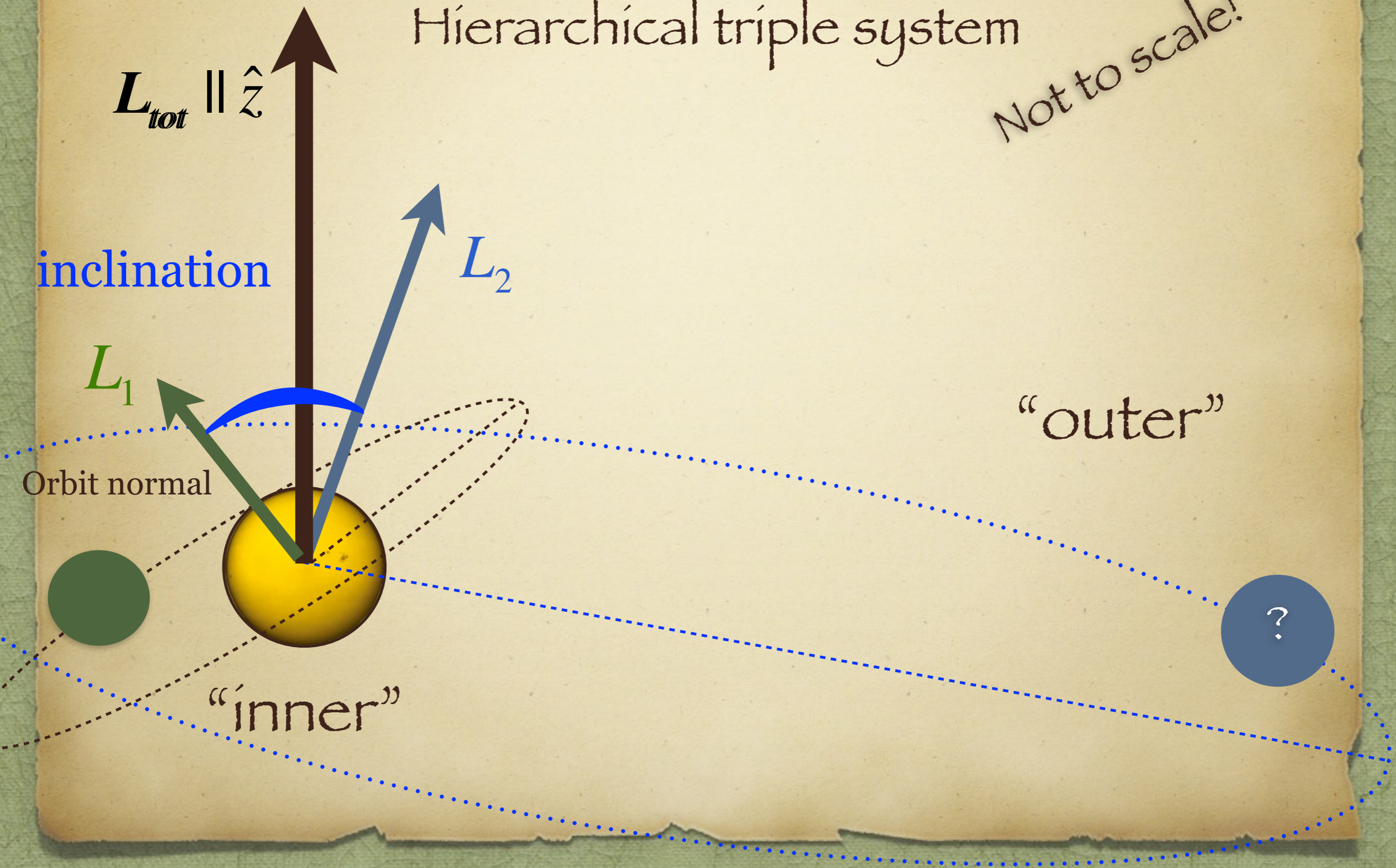




# The Kozai-Lidov Formalism

Hierarchical triple system

Not to scale!



inclination

$L_1$

$L_2$

"outer"

Orbit normal

"inner"

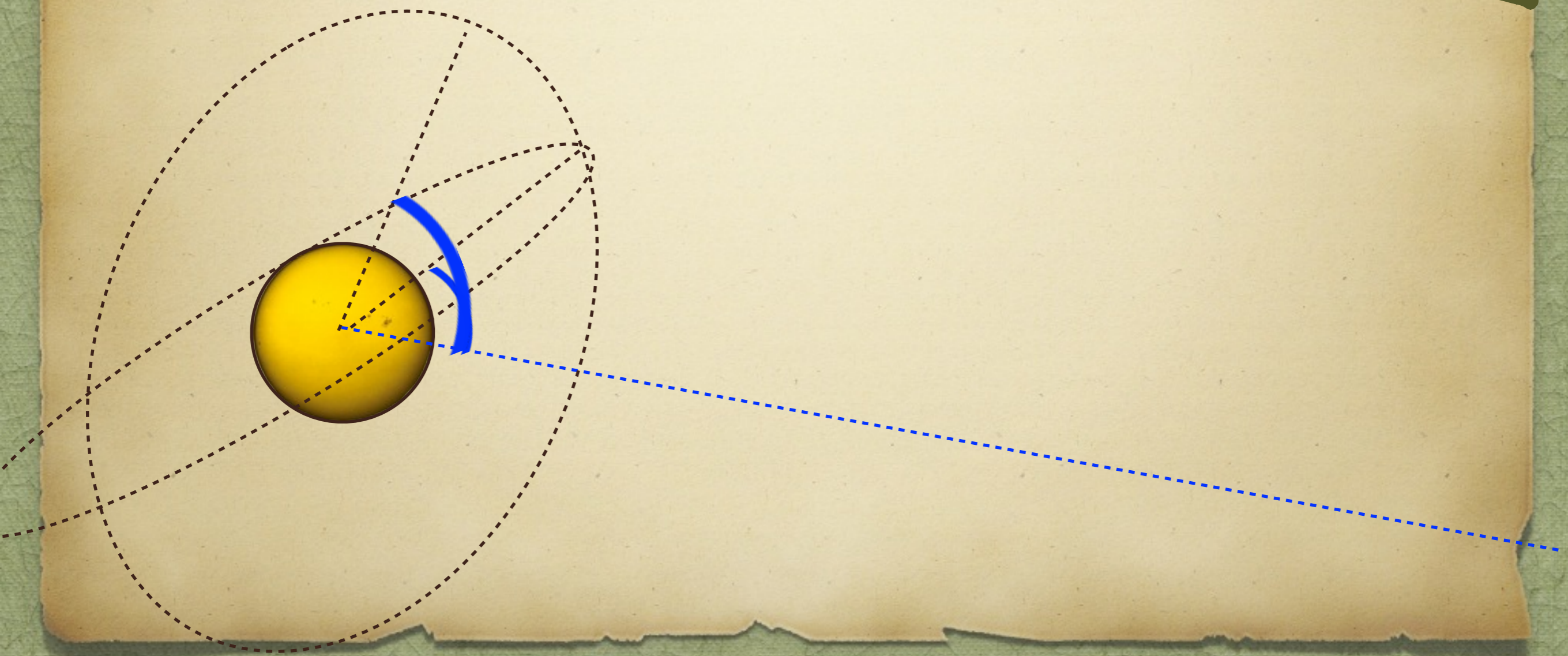
?

# The Kozai-Lidov Formalism

The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

For initially inclined system  $\gtrsim 40^\circ$



# The Kozai-Lidov Formalism

The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

Conservation of the z component of angular momentum for both the inner outer orbits

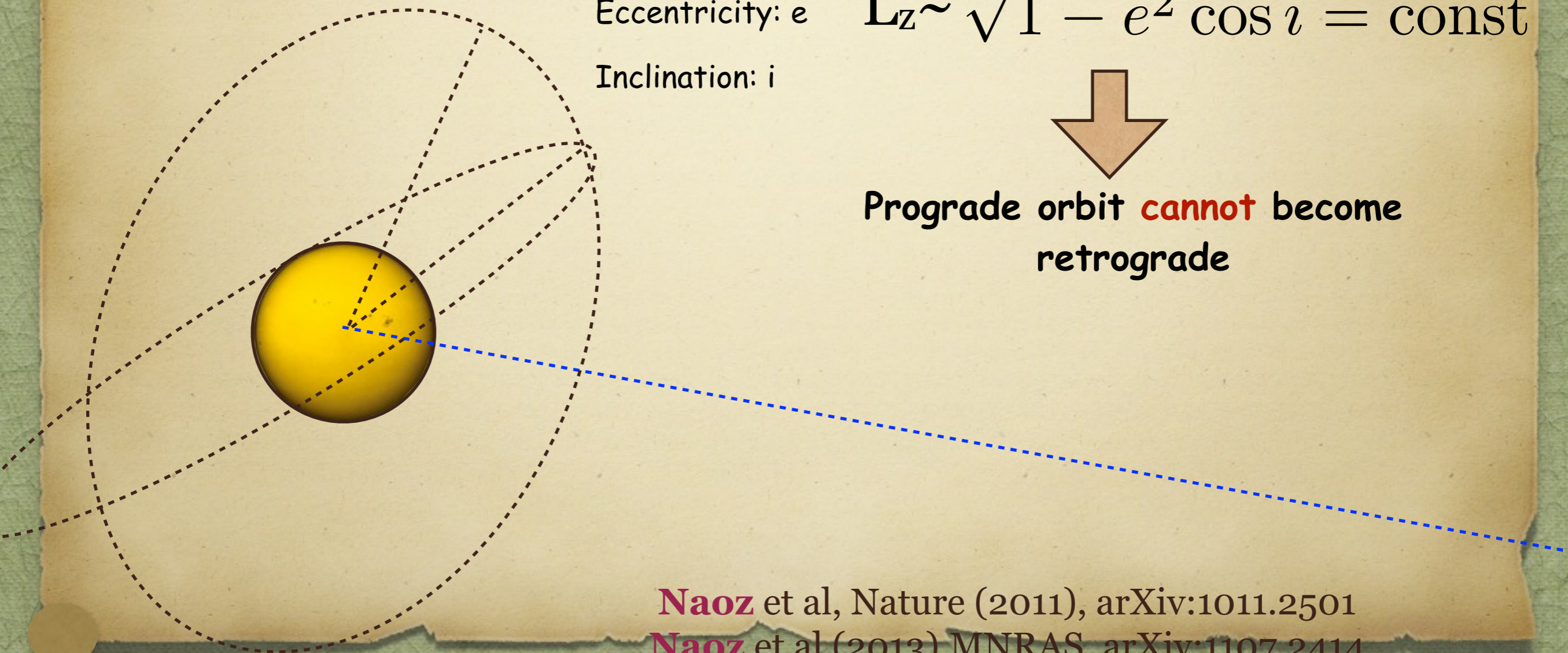
The orbital elements:

Eccentricity:  $e$       $L_z \sim \sqrt{1 - e^2} \cos i = \text{const}$

Inclination:  $i$



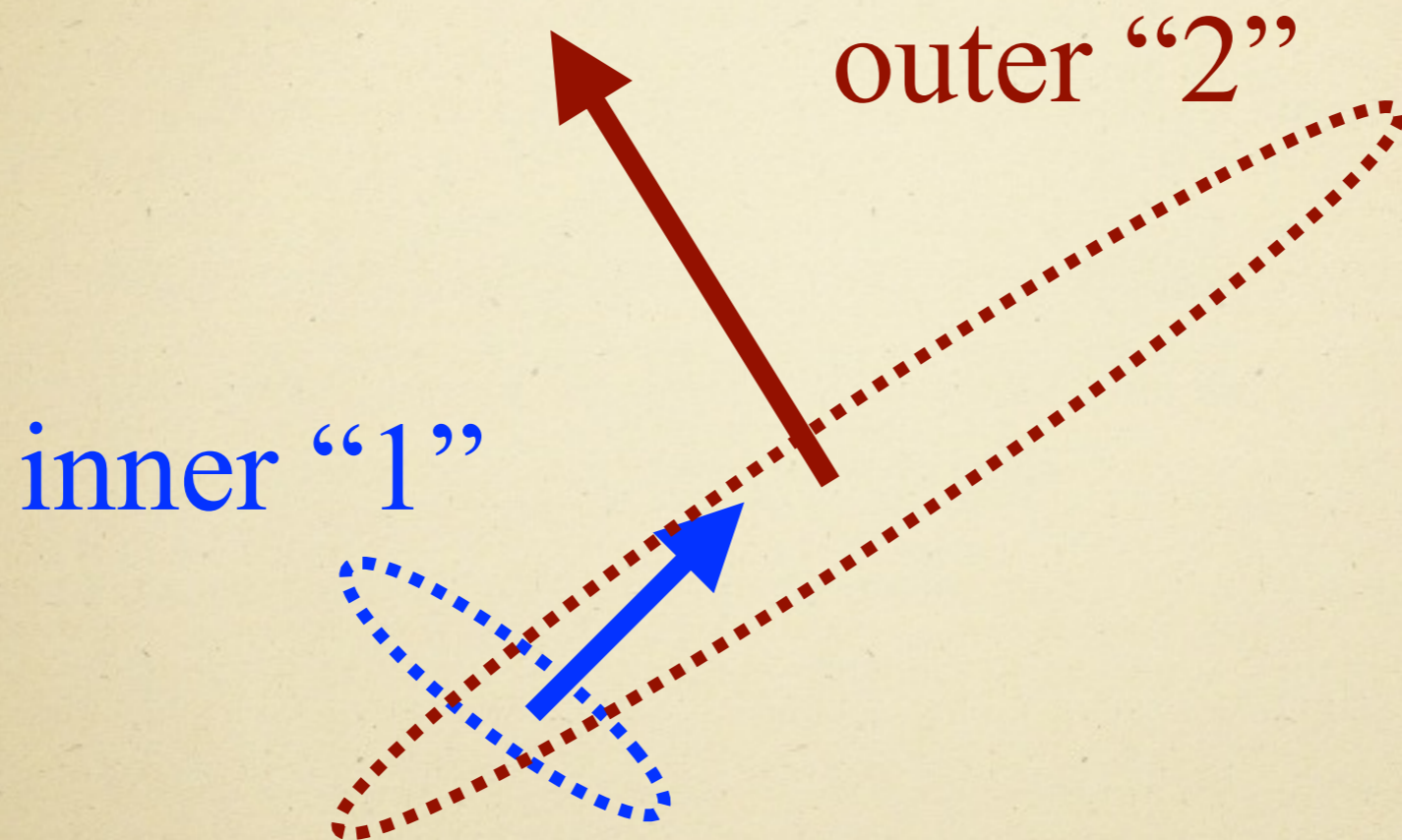
Prograde orbit **cannot** become retrograde



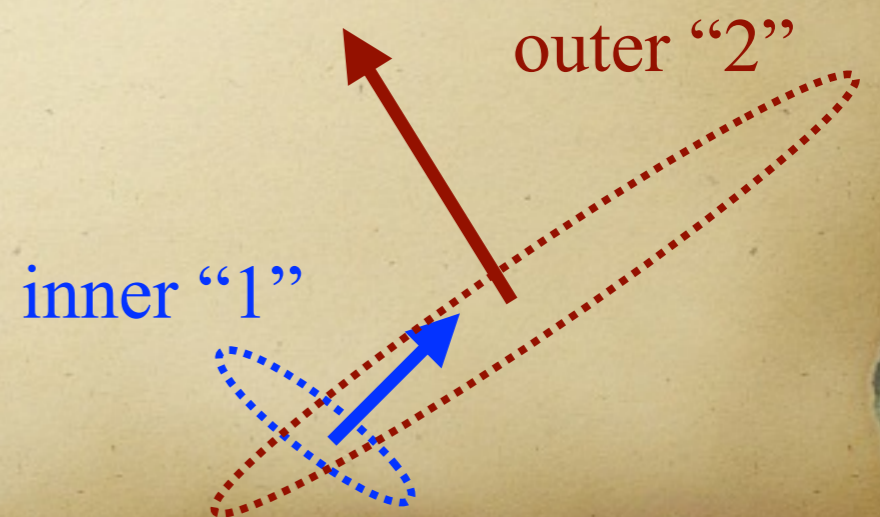
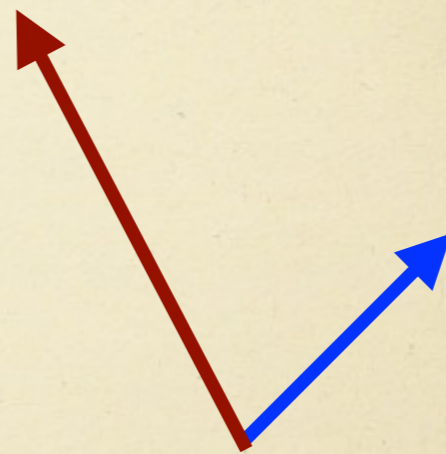
**Naoz** et al, Nature (2011), arXiv:1011.2501

**Naoz** et al (2013), MNRAS, arXiv:1107.2414

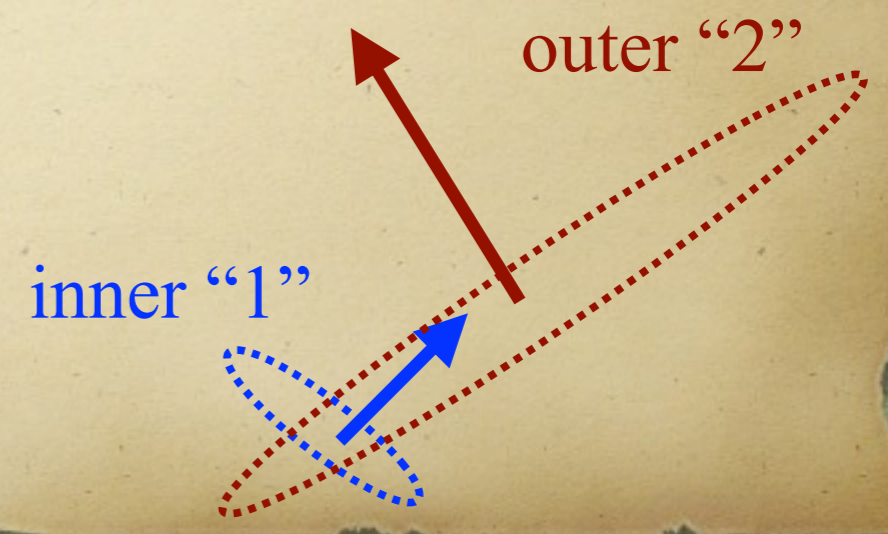
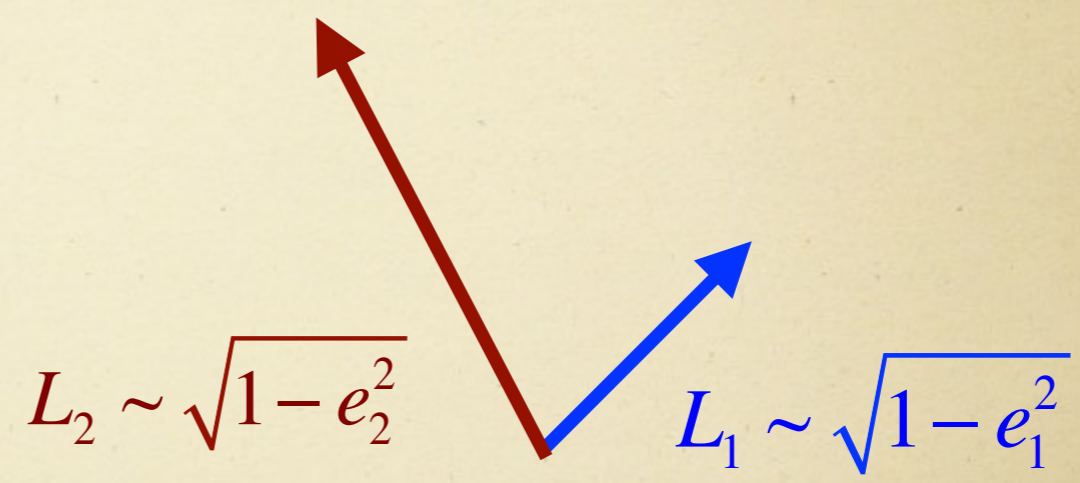
Is it constant?



Is it constant?



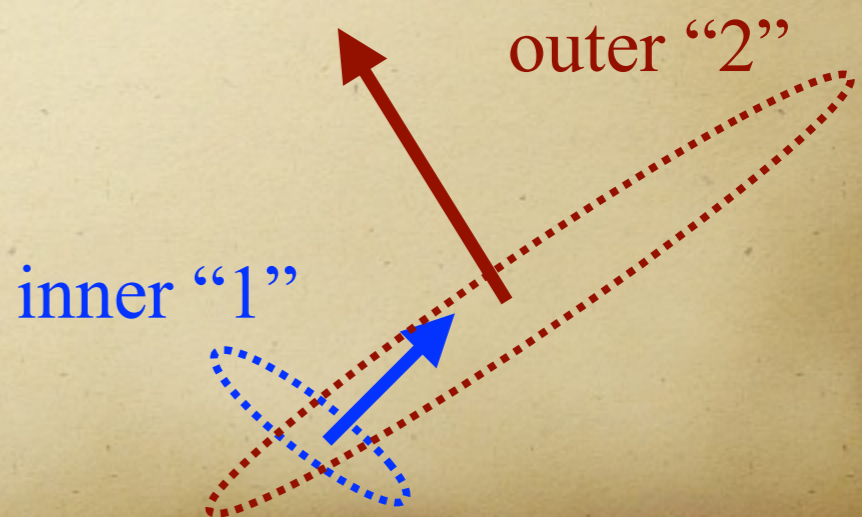
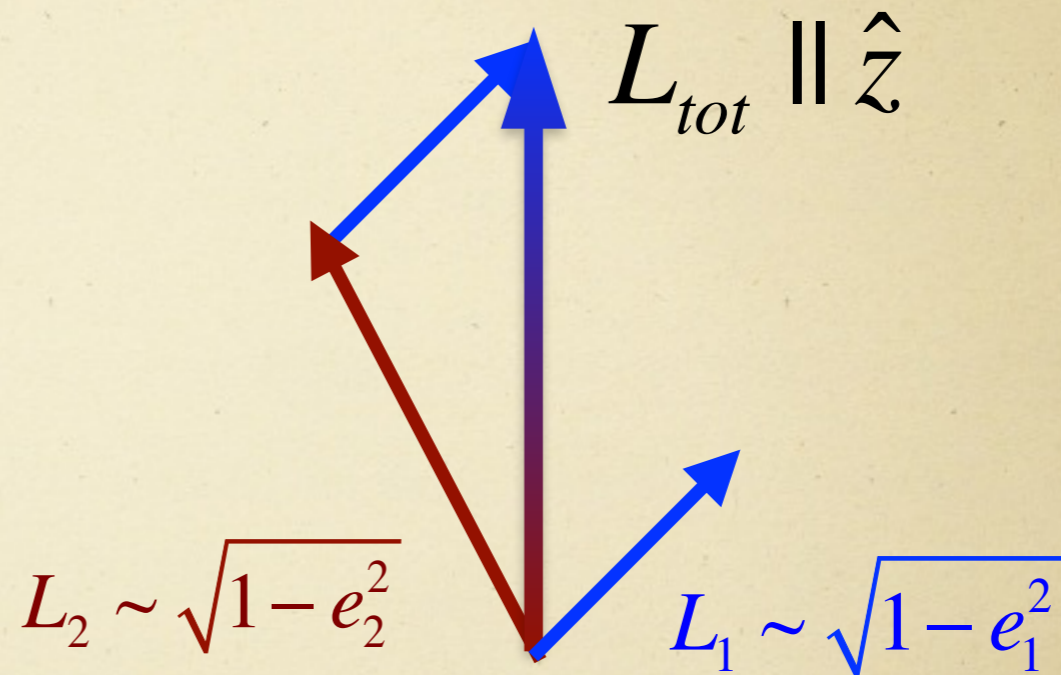
# Is it constant?



# Is it constant?

Adding vector ...

$$\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2$$

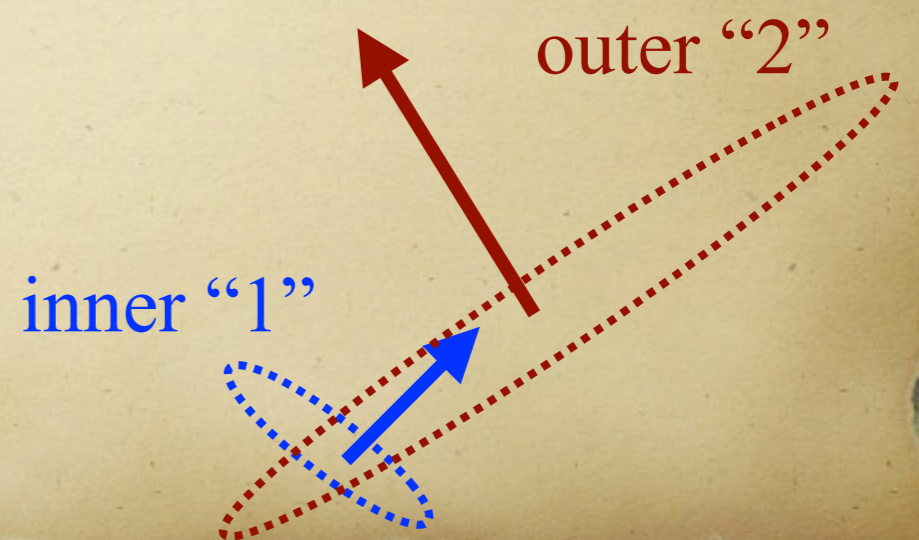
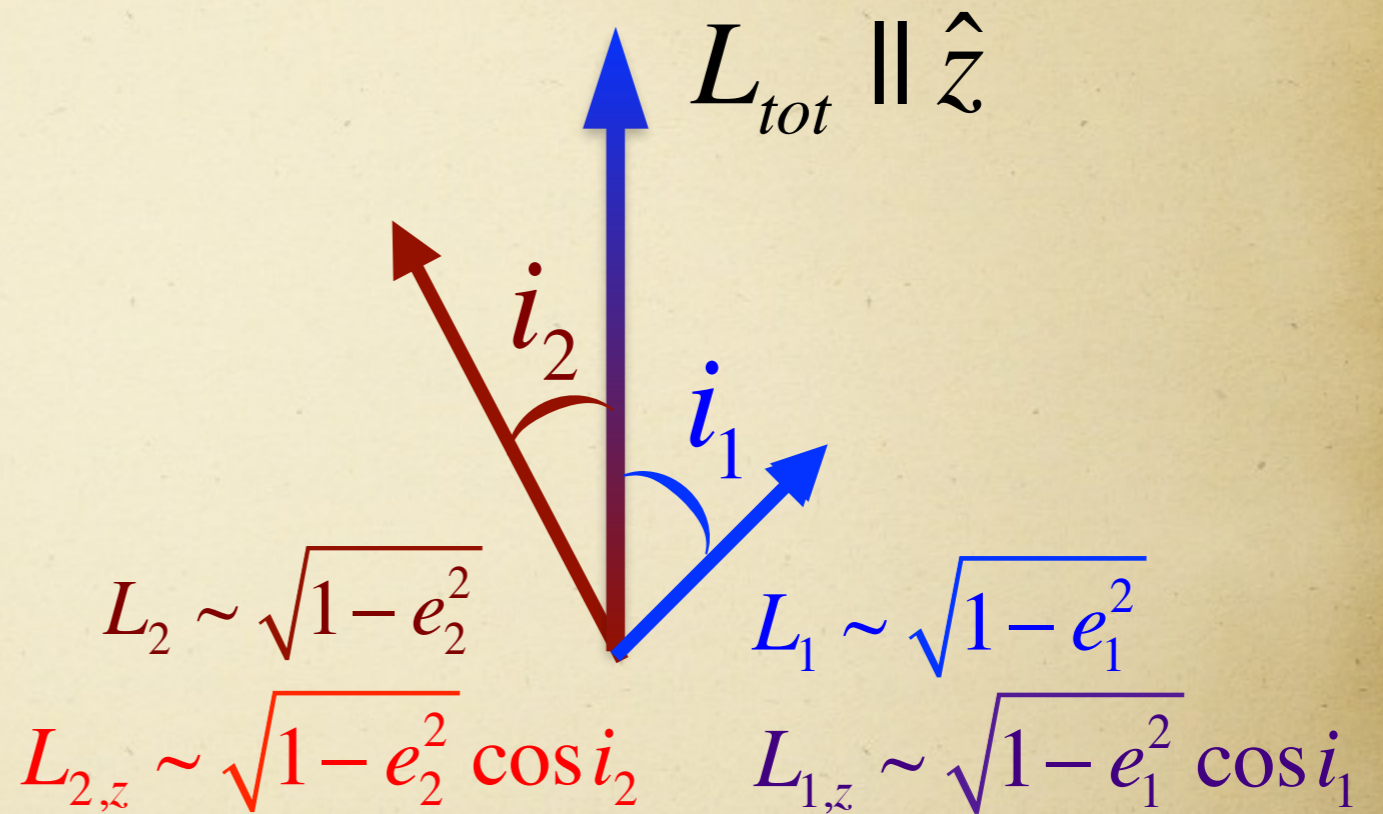


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Adding vector ...

$$\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2$$

$$\vec{L}_2 = \vec{L}_{tot} - \vec{L}_1$$





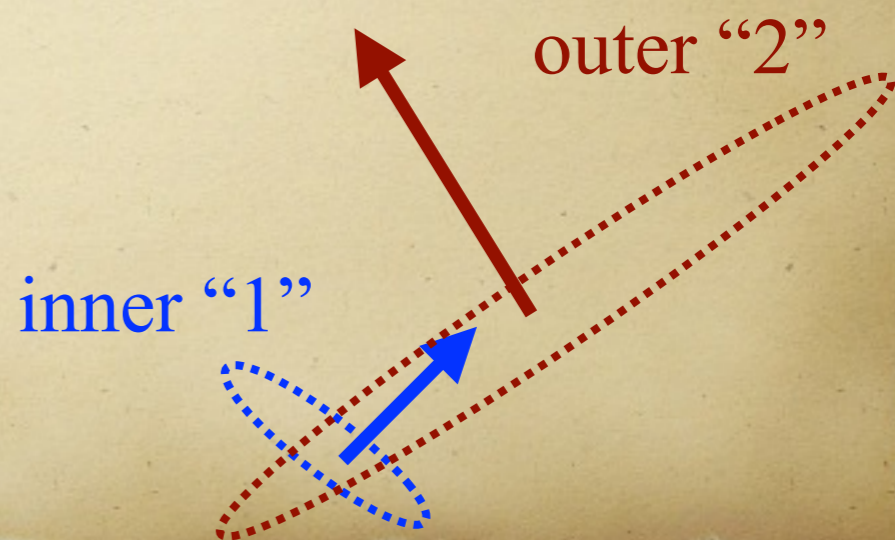
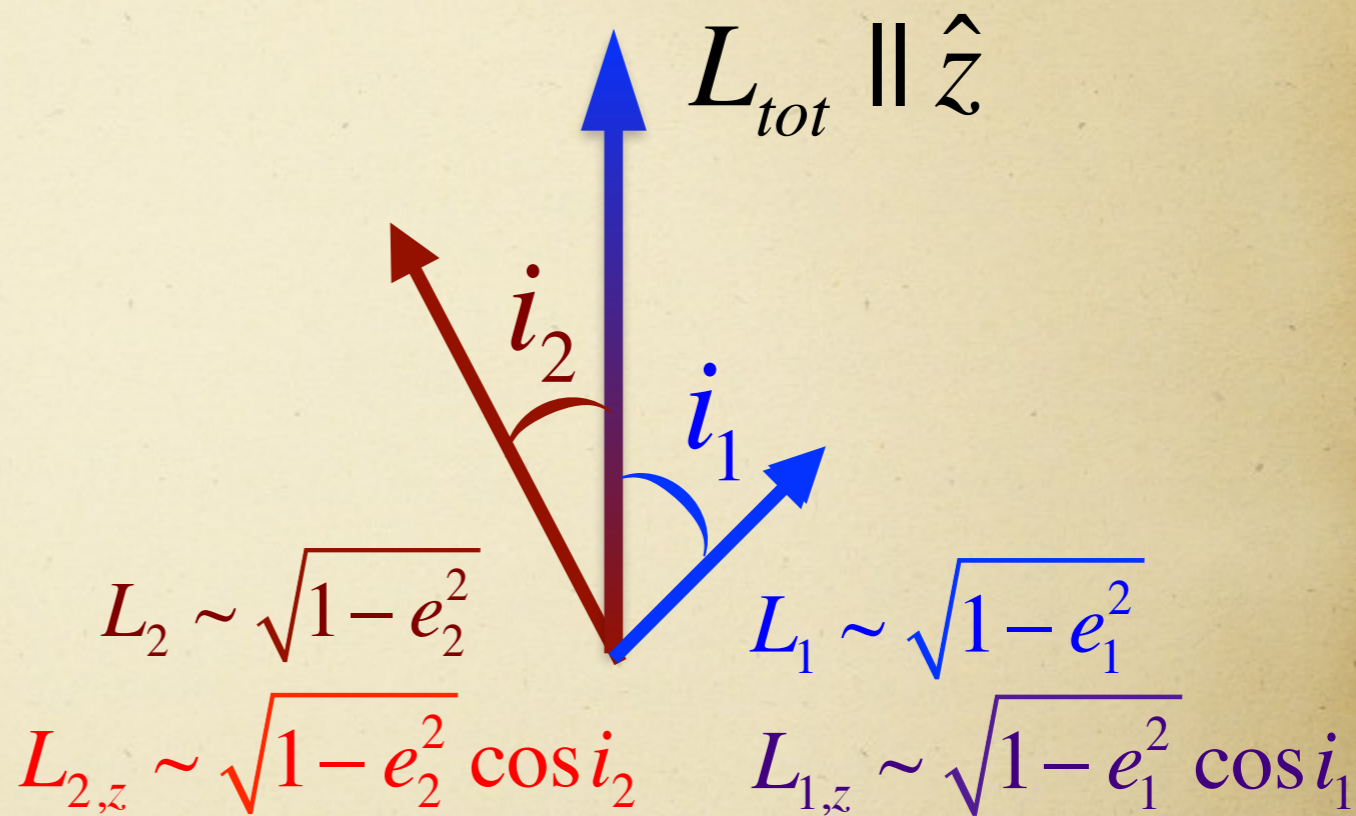
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$$\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2$$

$$\vec{L}_2 = \vec{L}_{tot} - \vec{L}_1$$

$$L_2^2 = L_{tot}^2 + L_1^2 - 2L_{tot}L_1 \underbrace{\cos i_1}_{L_{1,z}}$$



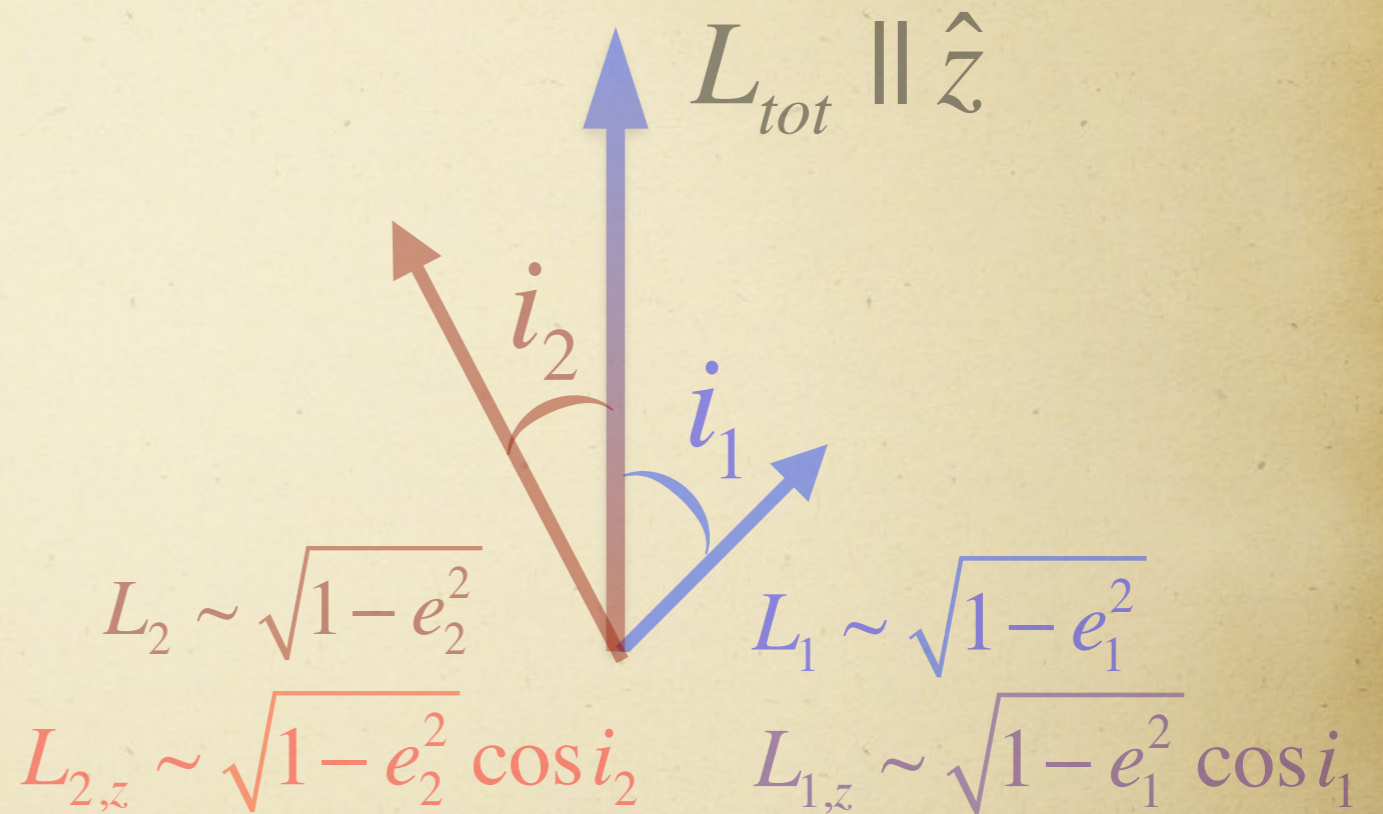
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THE ASTRONOMICAL JOURNAL

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## Secular Perturbations of Asteroids with High Inclination and Eccentricity

YOSHIHIDE KOZAI\*

*Smithsonian Astrophysical Observatory, Cambridge, Massachusetts*

(Received August 29, 1962)

Secular perturbations of asteroids with high inclination and eccentricity moving under the attraction of the sun and Jupiter are studied on the assumption that Jupiter's orbit is circular. After about periodic time

Yoshihide Kozai

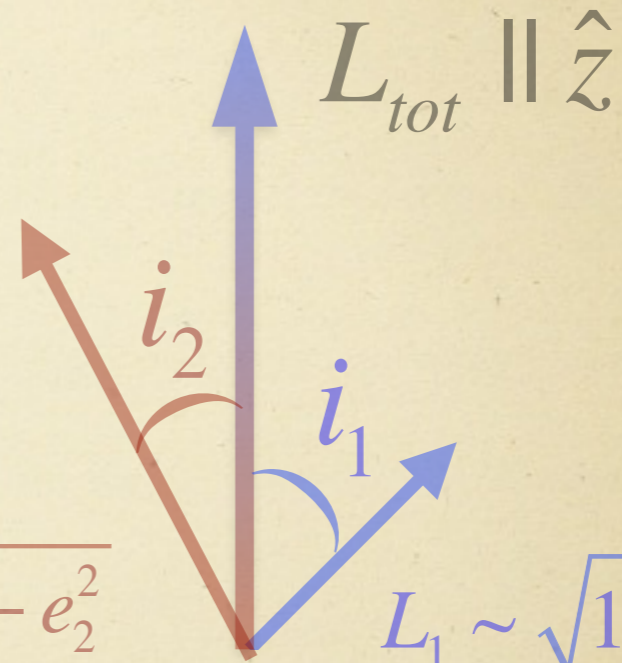
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$$L_2 \sim \sqrt{1 - e_2^2}$$

$$L_1 \sim \sqrt{1 - e_1^2}$$

$$L_{2,z} \sim \sqrt{1 - e_2^2} \cos i_2$$

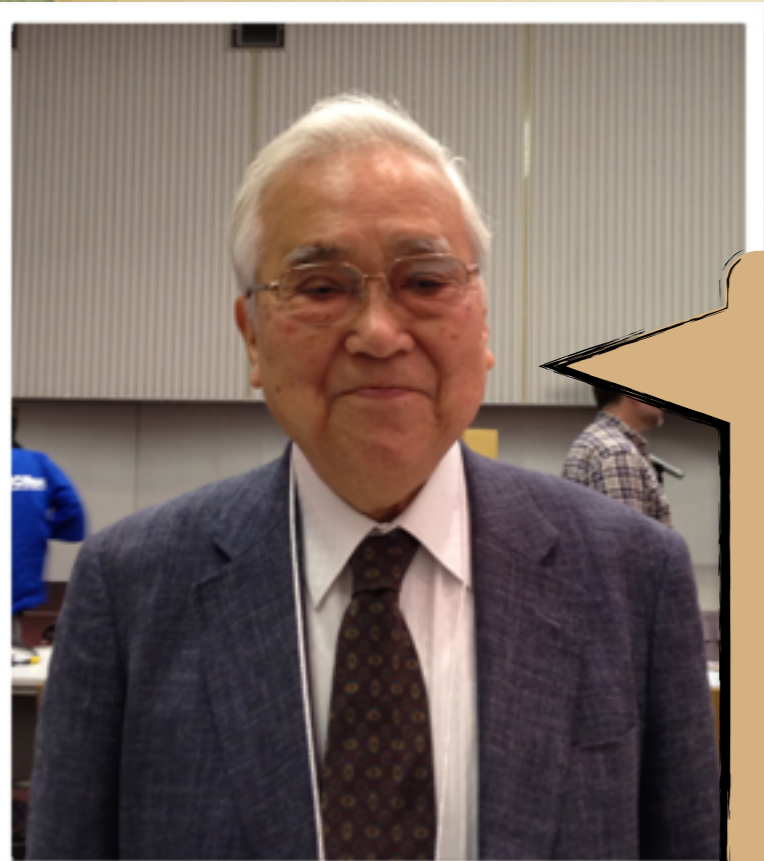
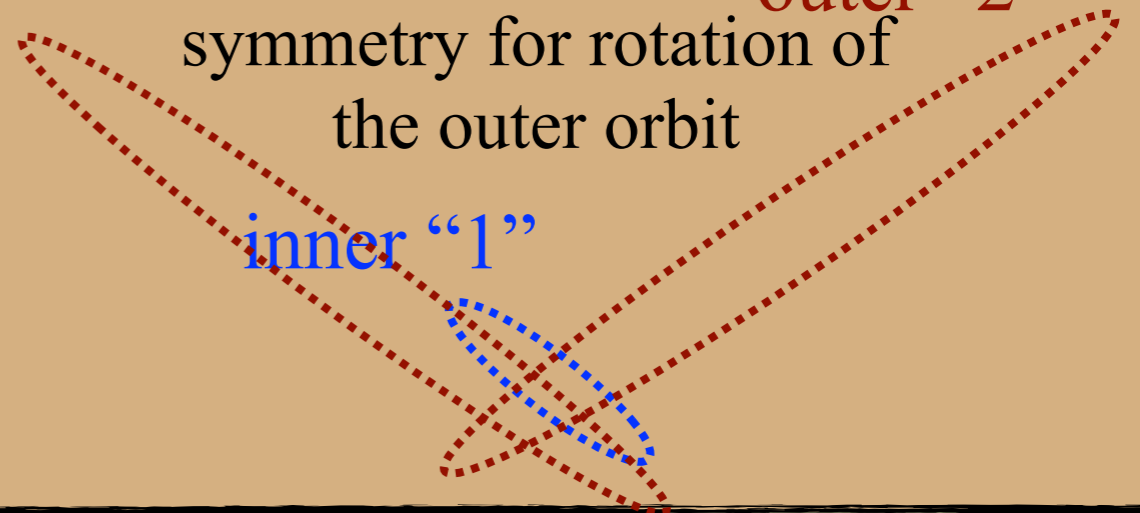
$$L_{1,z} \sim \sqrt{1 - e_1^2} \cos i_1$$

for the quadrupole approx.  $\sim (a_1/a_2)^2$ :

$$L_2 = Const.$$

outer "2"  
symmetry for rotation of  
the outer orbit

inner "1"



Yoshihde Kozai

# Is it constant?

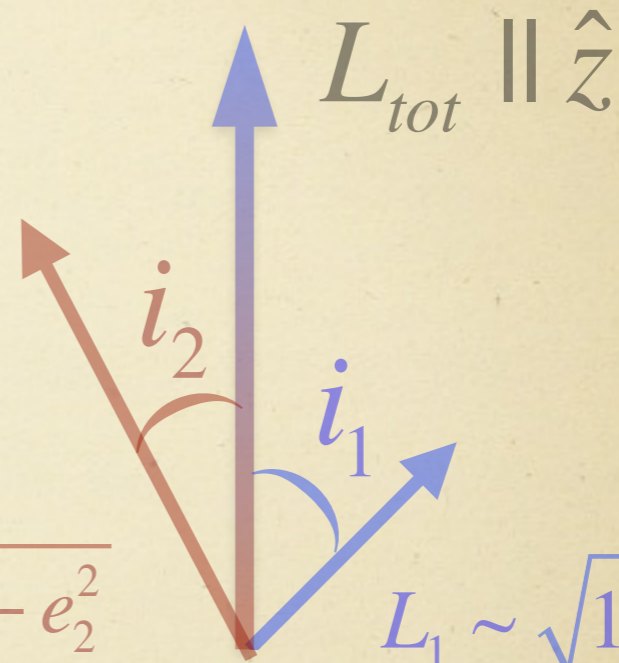
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$$L_2^2 = L_{tot}^2 + L_1^2 - 2 L_{tot} L_1 \cos i_1$$

$$\underbrace{L_1 \cos i_1}_{L_{1,z}}$$



$$L_2 \sim \sqrt{1 - e_2^2}$$

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$$L_{2,z} \sim \sqrt{1 - e_2^2} \cos i_2$$

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for the quadrupole approx.  $\sim (a_1/a_2)^2$ :

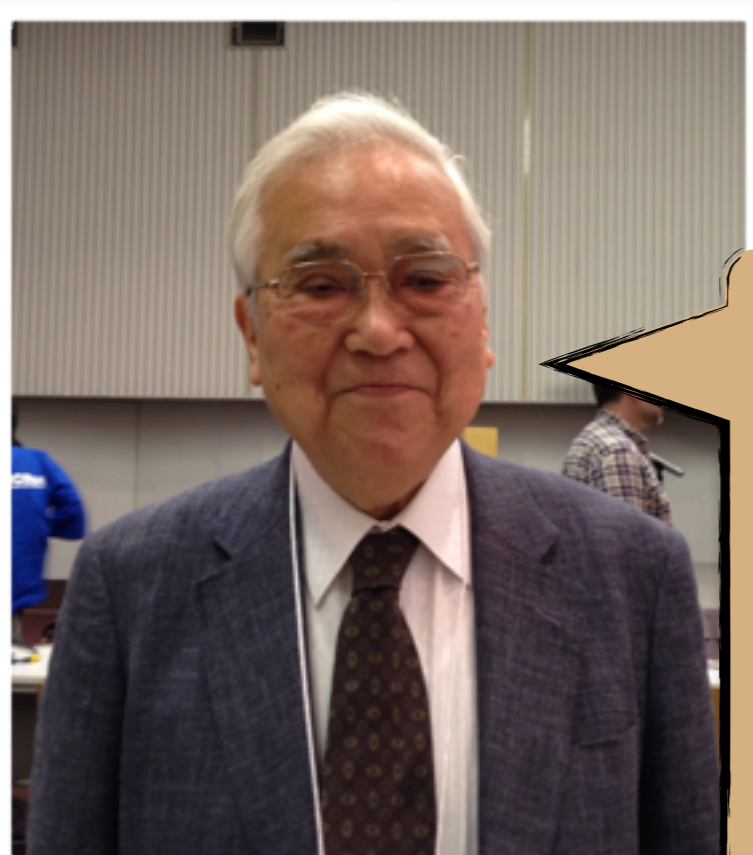
$$L_2 = \text{Const.}$$

$$L_{1,z} = \text{Const.}$$

$$L_{2,z} = \text{Const.}$$

$$L_1 \neq \text{Const.}$$

$$\mathcal{H}_{quad}(\omega_1)$$



Yoshihde Kozai

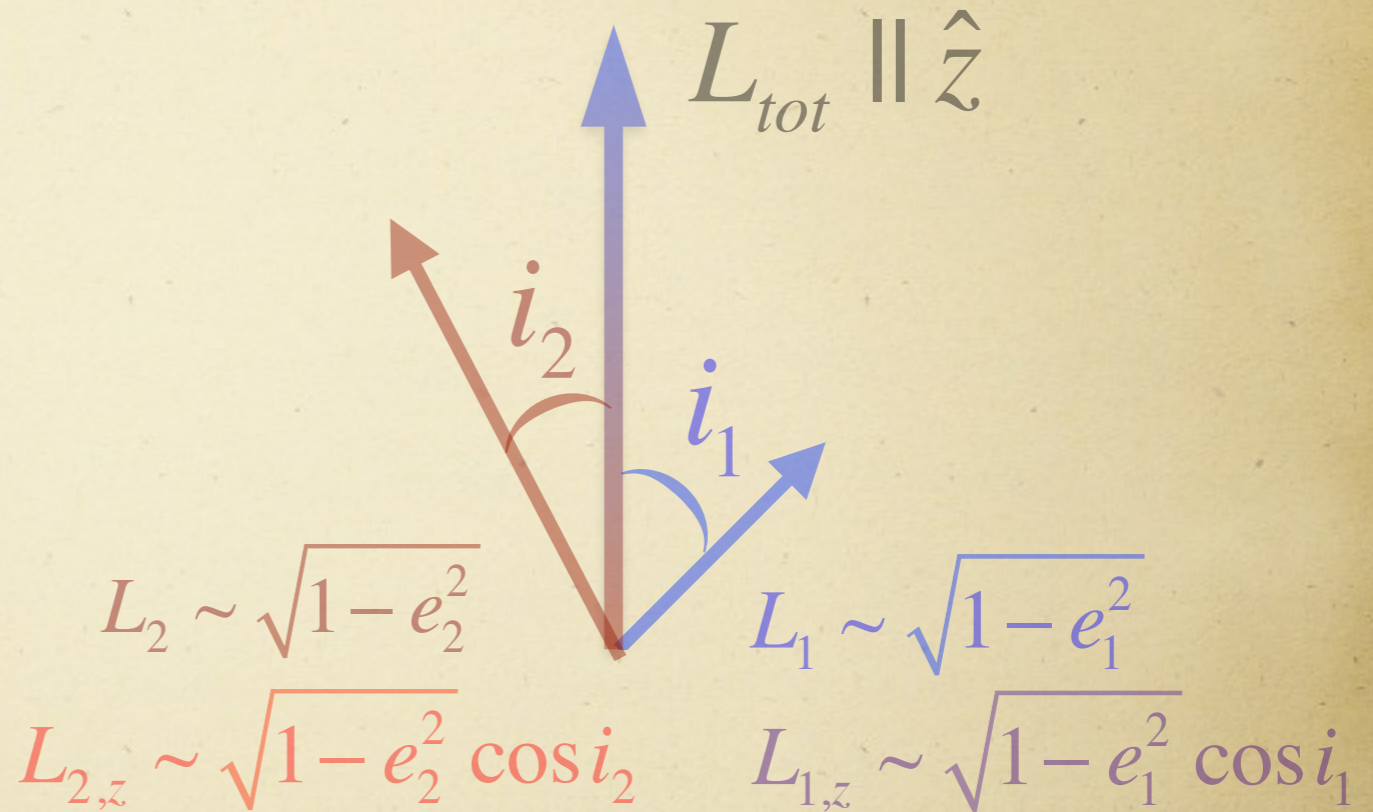
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for the quadrupole approx.  $\sim (a_1/a_2)^2$ :

$$L_2 = \text{Const.}$$

$$L_{1,z} \neq \text{Const.}$$

$$L_{2,z} \neq \text{Const.}$$

$$L_1 \neq \text{Const.}$$

$$\mathcal{H}_{quad}(\omega_1, \underbrace{\Omega_1 - \Omega_2}_{\pi})$$

$$\frac{df}{dx}_{x=2} \neq \frac{df(x=2)}{dx}$$

**Naoz** et al, Nature (2011), arXiv:1011.2501

# The Kozai-Lidov Formalism EKL

The eccentricity and inclination oscillate

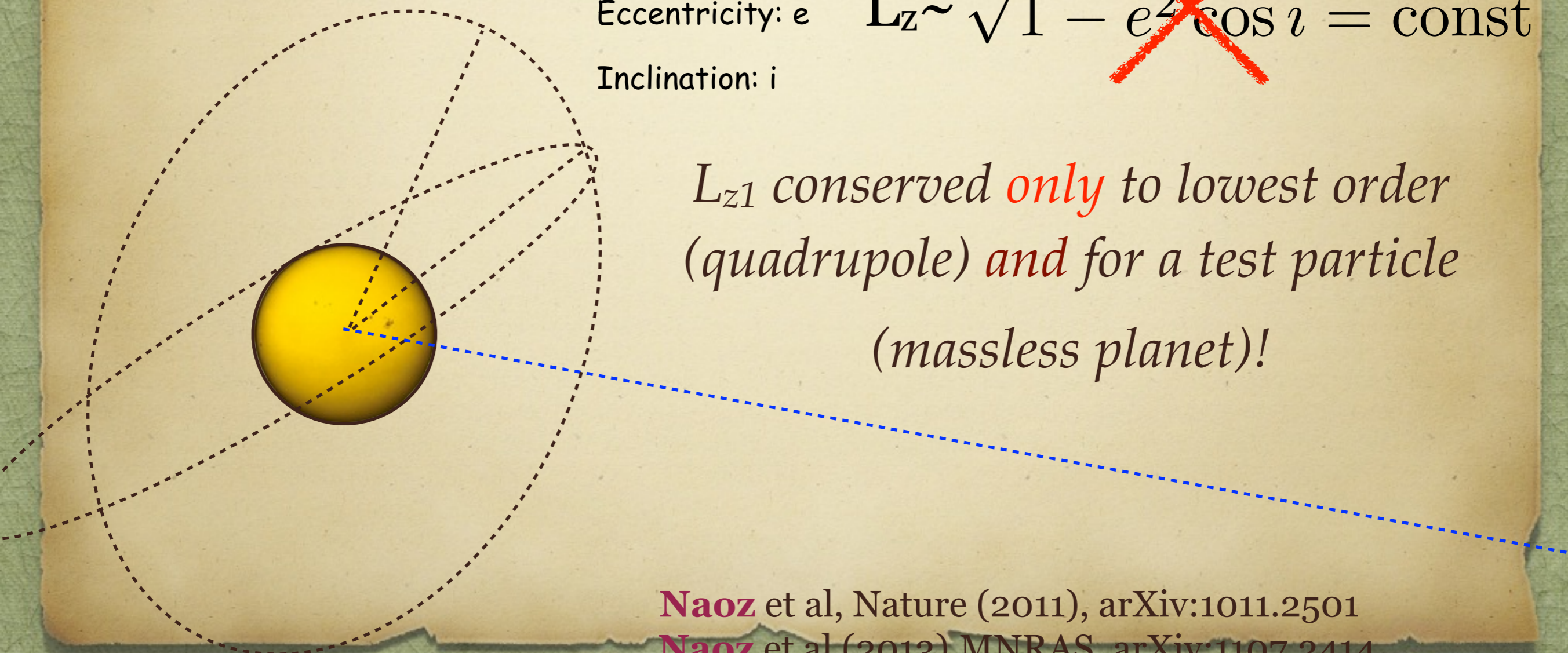
Conservation of ~~the~~ z component of angular momentum for both the inner ~~outer~~ orbits

The orbital elements:

Eccentricity:  $e$     $L_z \sim \sqrt{1 - e^2} \cos i = \text{const}$

Inclination:  $i$

$L_{z1}$  conserved *only* to lowest order (quadrupole) *and* for a test particle (massless planet)!



**Naoz** et al, Nature (2011), arXiv:1011.2501

**Naoz** et al (2013), MNRAS, arXiv:1107.2414

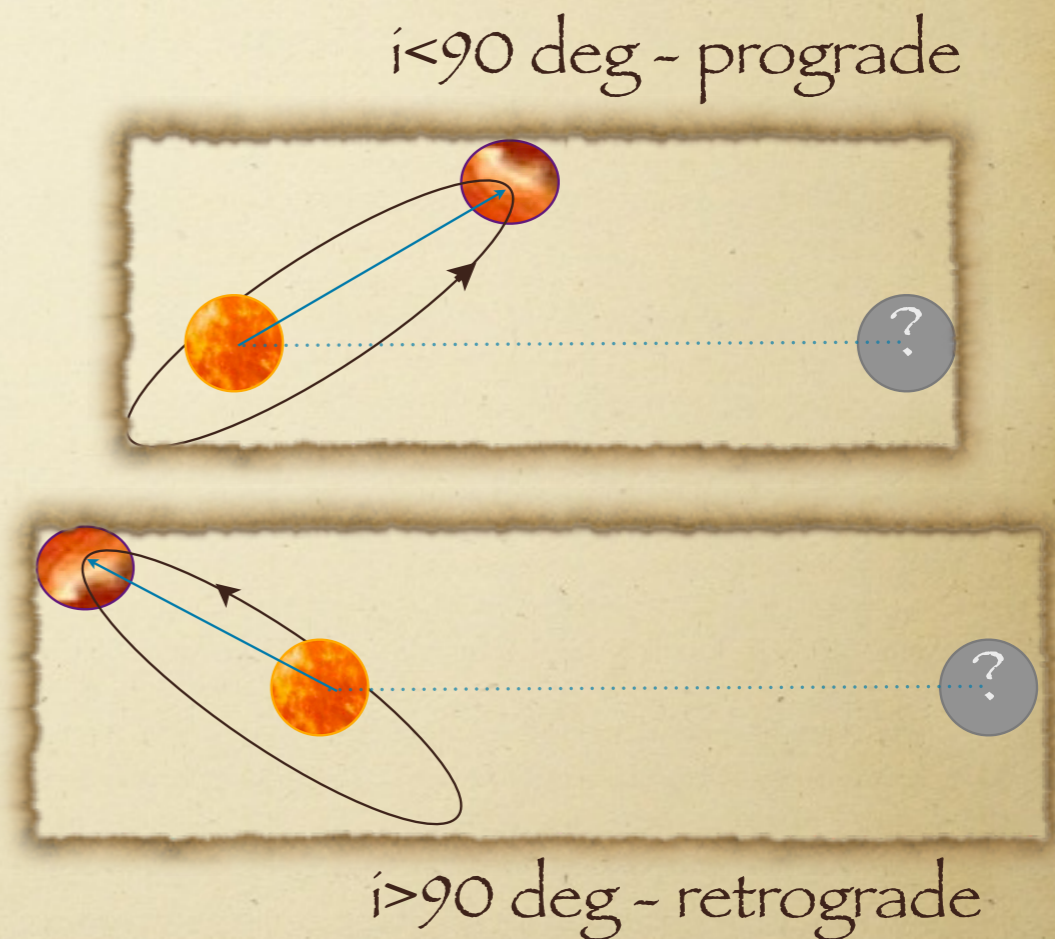
# Our treatment

## The eccentric Kozai-Lidov mechanism - KEL

- Allow for the **z-component** of the angular momenta of the **inner** and **outer** orbit to change - already at the **quadrupole level**
- Expanding the approximation to the **octupole level** (e.g., Ford et al 2000, Blaes et al 2002 - already done before us!!!)
- Both the magnitude and orientation of the angular momentum can change

larger parts of the parameter space

**Naoz** et al, Nature (2011), arXiv:1011.2501  
**Naoz** et al (2013), MNRAS, arXiv:1107.2414



for test particle approx. see:  
Lithwick & **Naoz** (2011), ApJ, arXiv:1106.3329  
Katz, Dong Malhotra (2011), arXiv:1106.3340

Lets...flip the planet





point mass limit

# Lets...flip the planet

Example system:  $a_1=6\text{AU}$ ,  $a_2=100\text{AU}$ ,  $m_1=1.M_{\text{sun}}$   $M_2=1M_j$ ,  $M_3=40M_j$   $i=65$  deg secular dynamics + GR

GR effects: e.g., Ford et al 2000,  
Naoz, Kocsis, Loeb, Yunes 2013

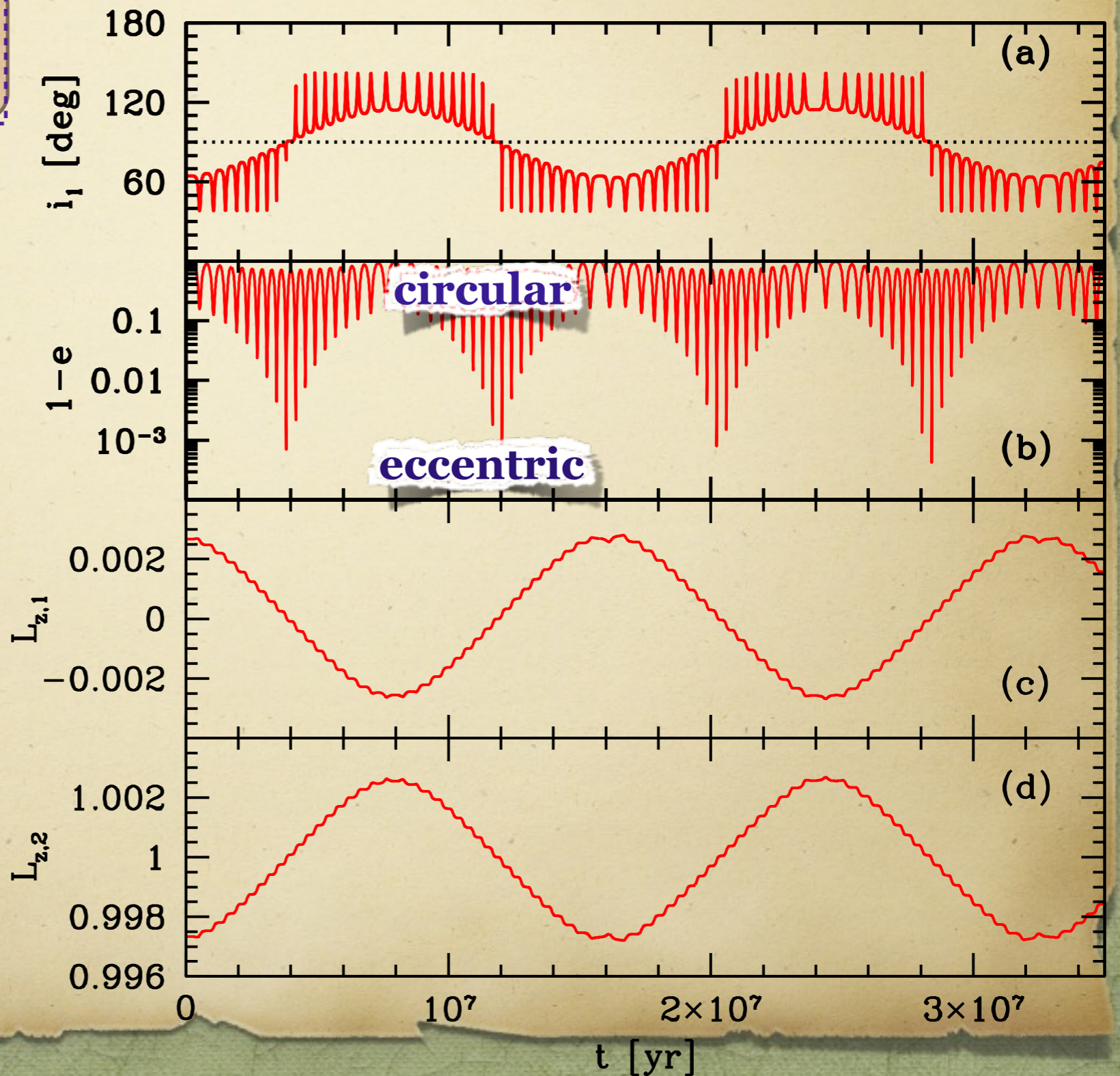
(a) inner orbit inclination

(b) inner orbit eccentricity

(c) inner orbit z-com.  
angular momentum

(d) inner orbit z-com.  
angular momentum

Naoz et al, Nature (2011)



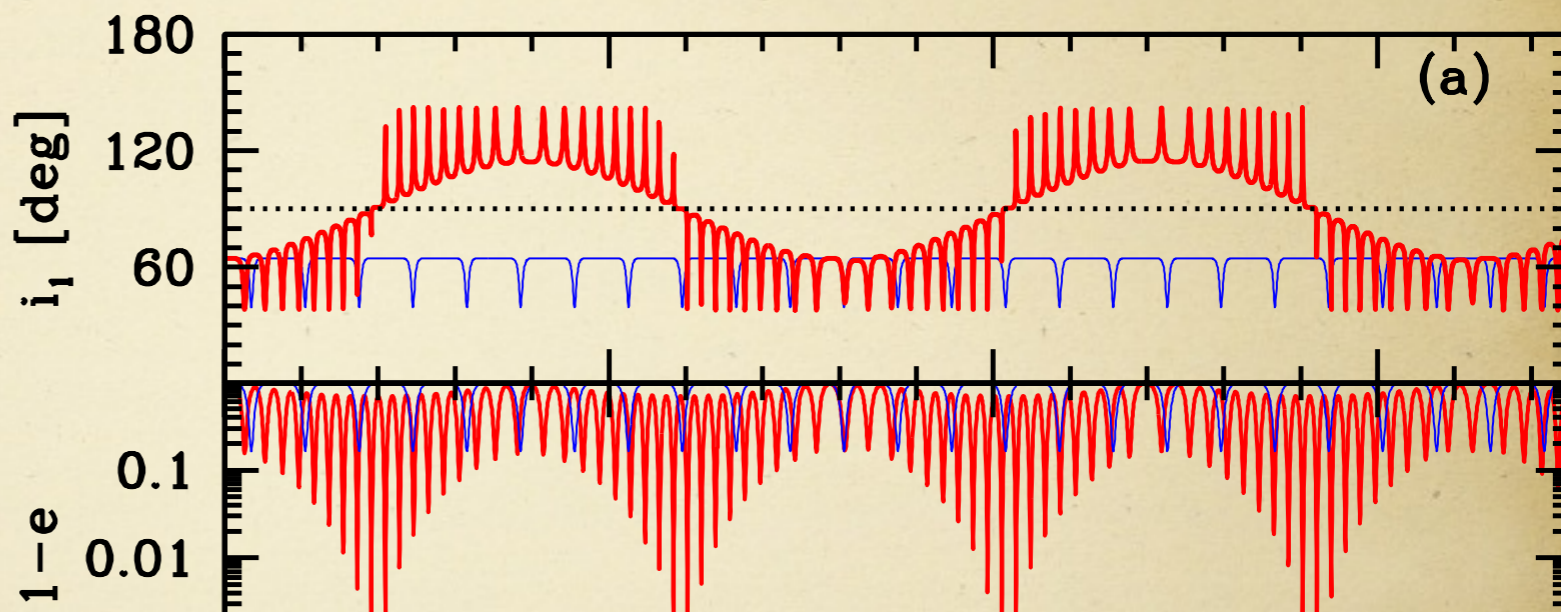
point mass limit

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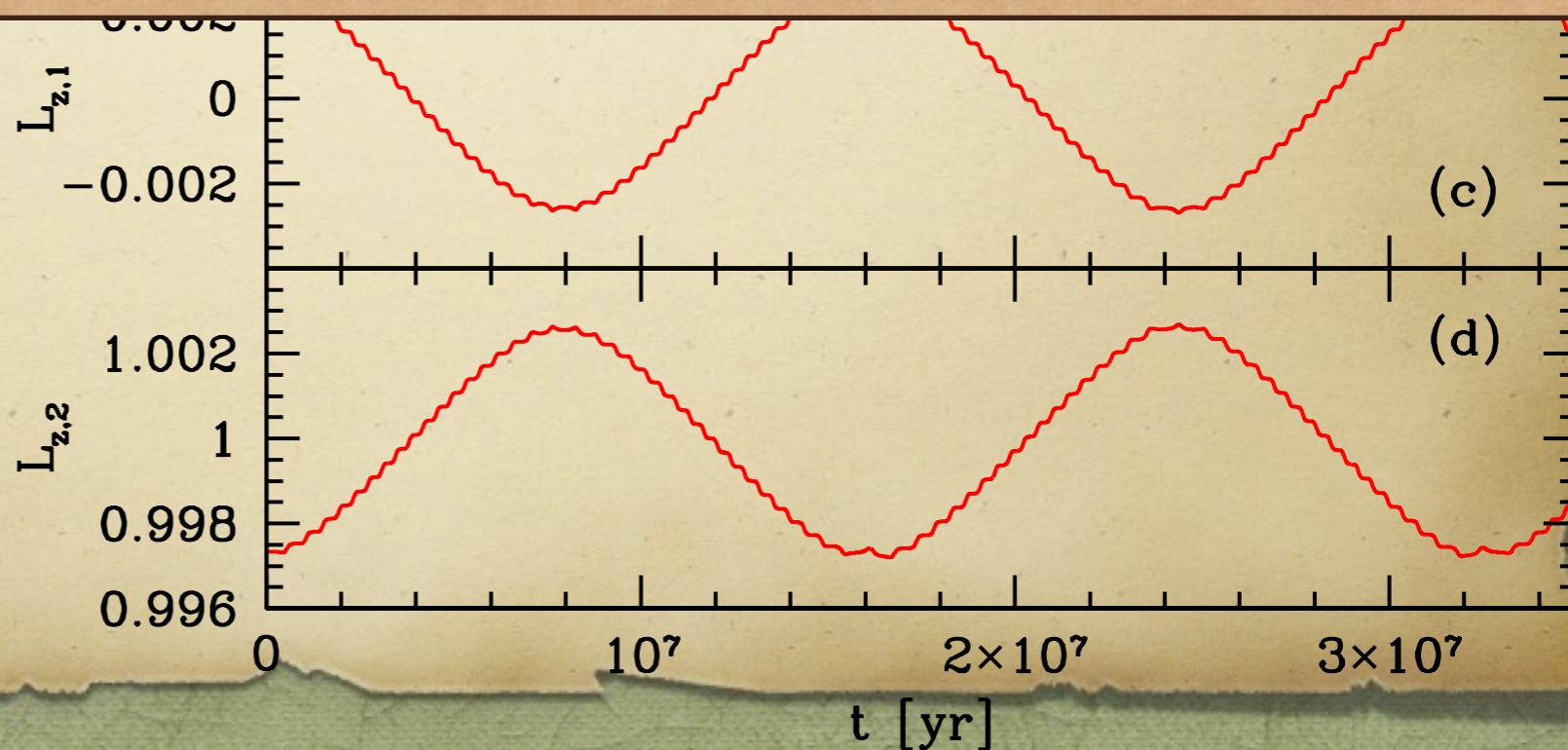
(a) inner orbit inclination



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(c) inner orbit z-com.  
angular momentum

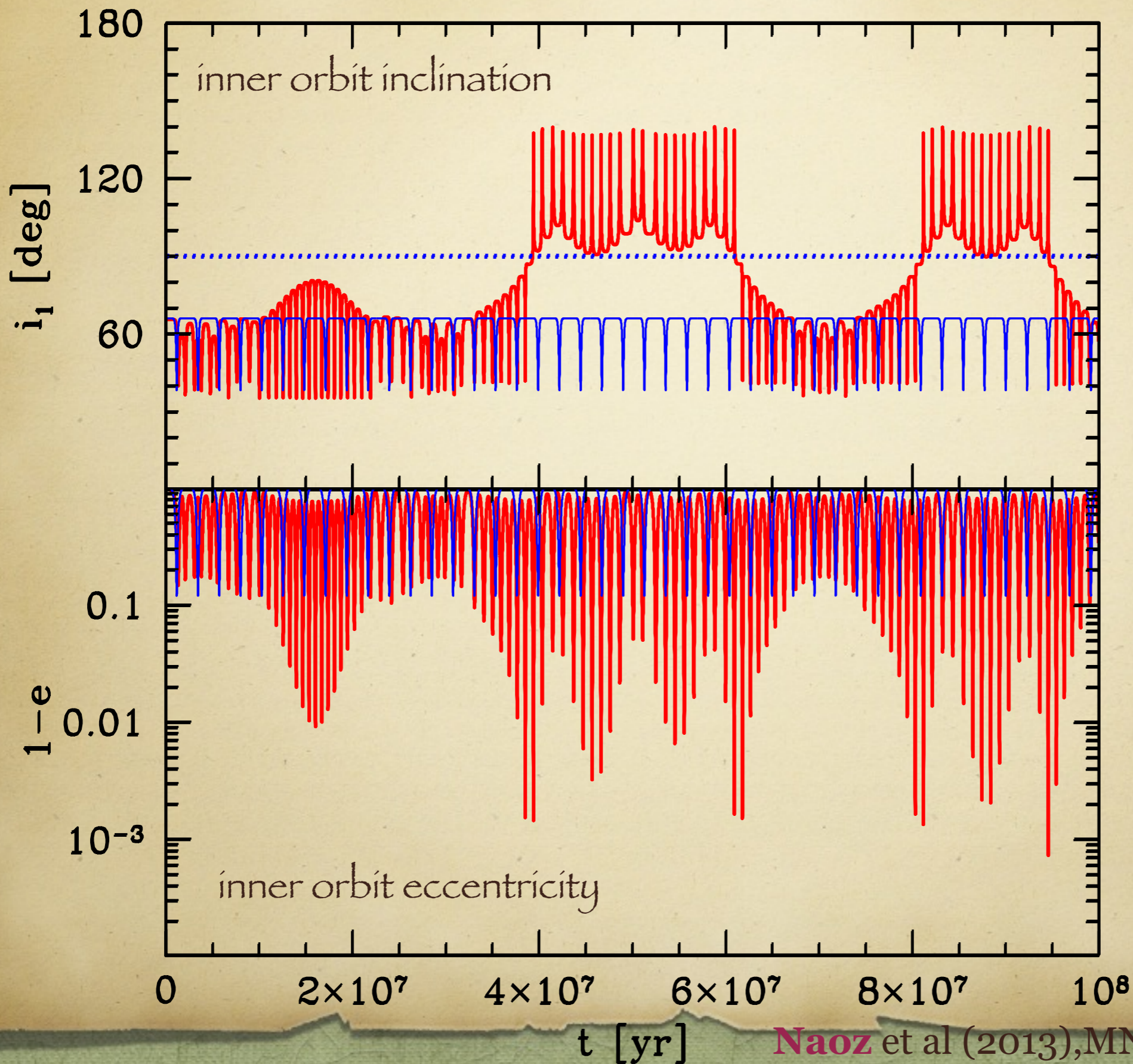
Compare to: "Standard" (quadrupole) Kozai



(d) inner orbit z-com.  
angular momentum

Naoz et al, Nature (2011)

# EKL



$$M_1 = 1 M_\odot$$
$$M_2 = 1 M_J$$
$$M_3 = 4 M_J$$

$$a_1 = 5 \text{ AU}$$
$$a_2 = 51 \text{ AU}$$

$$i = 71^\circ$$

# Question

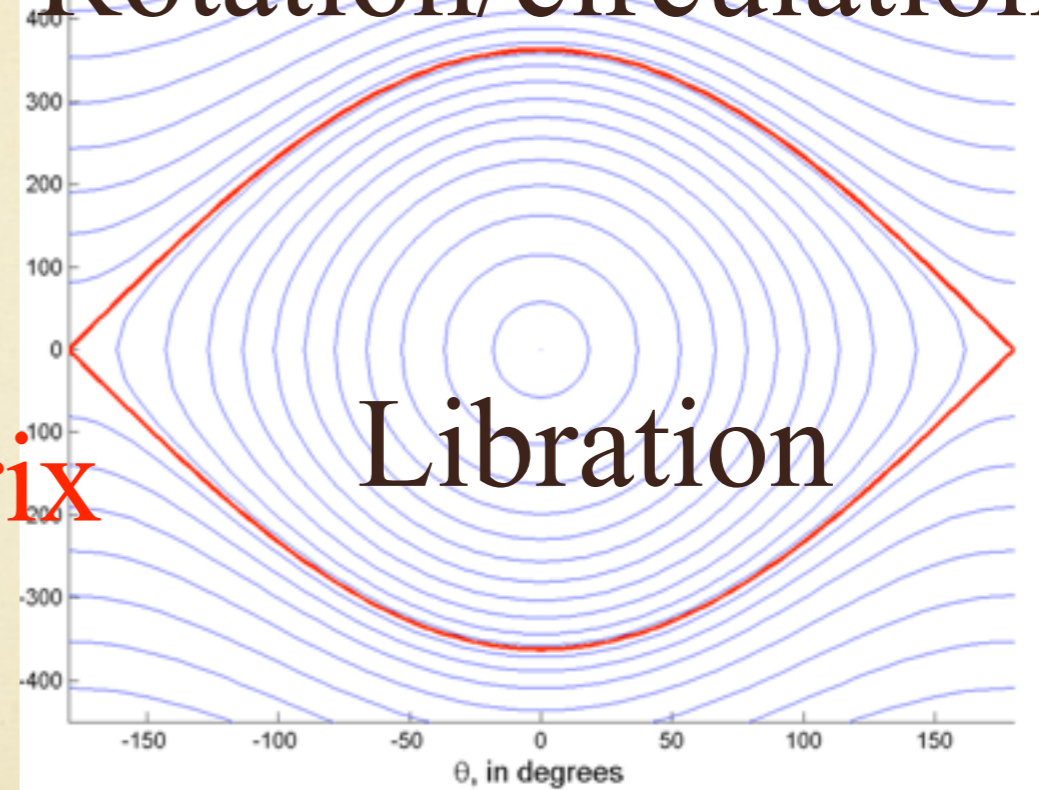
- Why high inclination  $>40^\circ$ ?
- Is high inclination required also in the EKL mechanism?
- What about chaos?

# EKL and the Pendulum

## The Pendulum

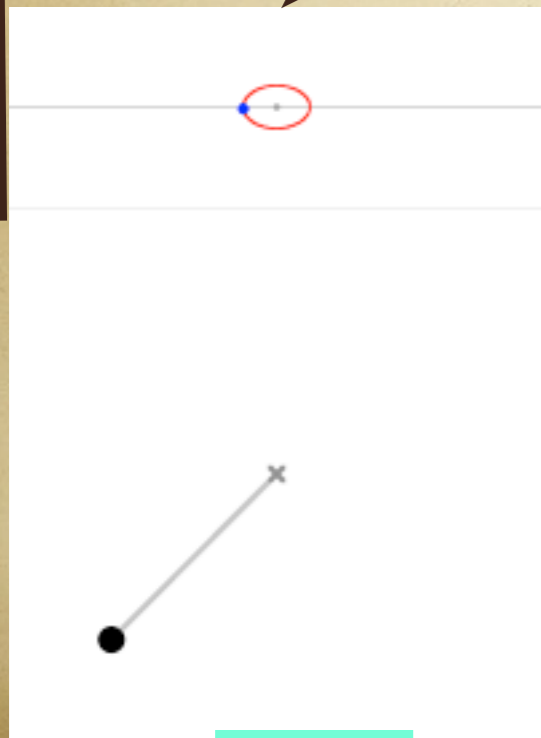
$$H(\theta, p) = \frac{p^2}{2mL} - mgL \cos \theta$$

Rotation/circulation

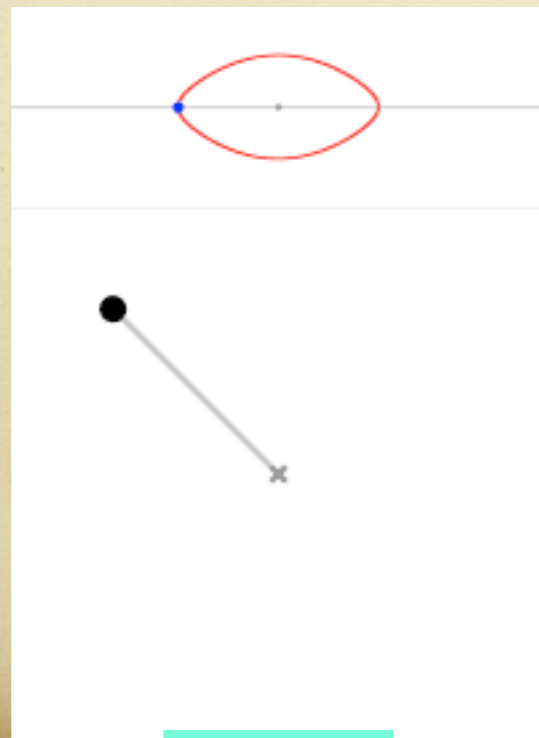


The separatrix

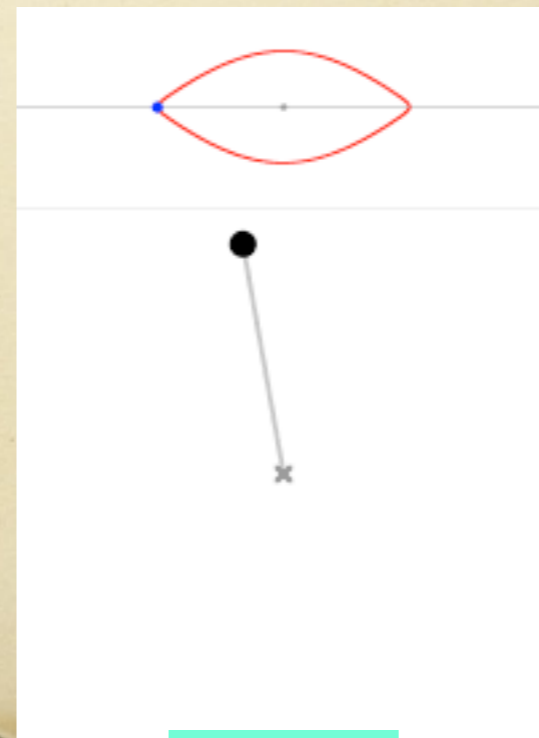
Libration



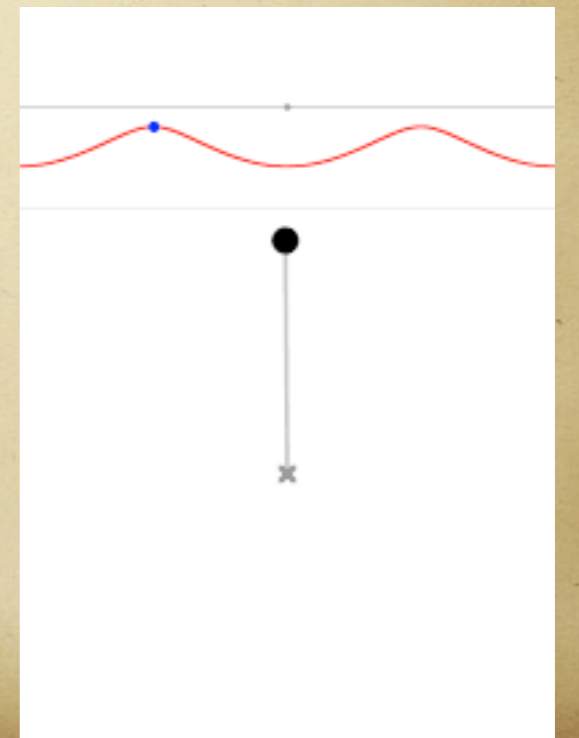
$\theta_0=45^\circ$



$\theta_0=135^\circ$



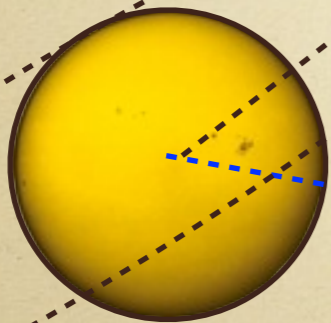
$\theta_0=170^\circ$



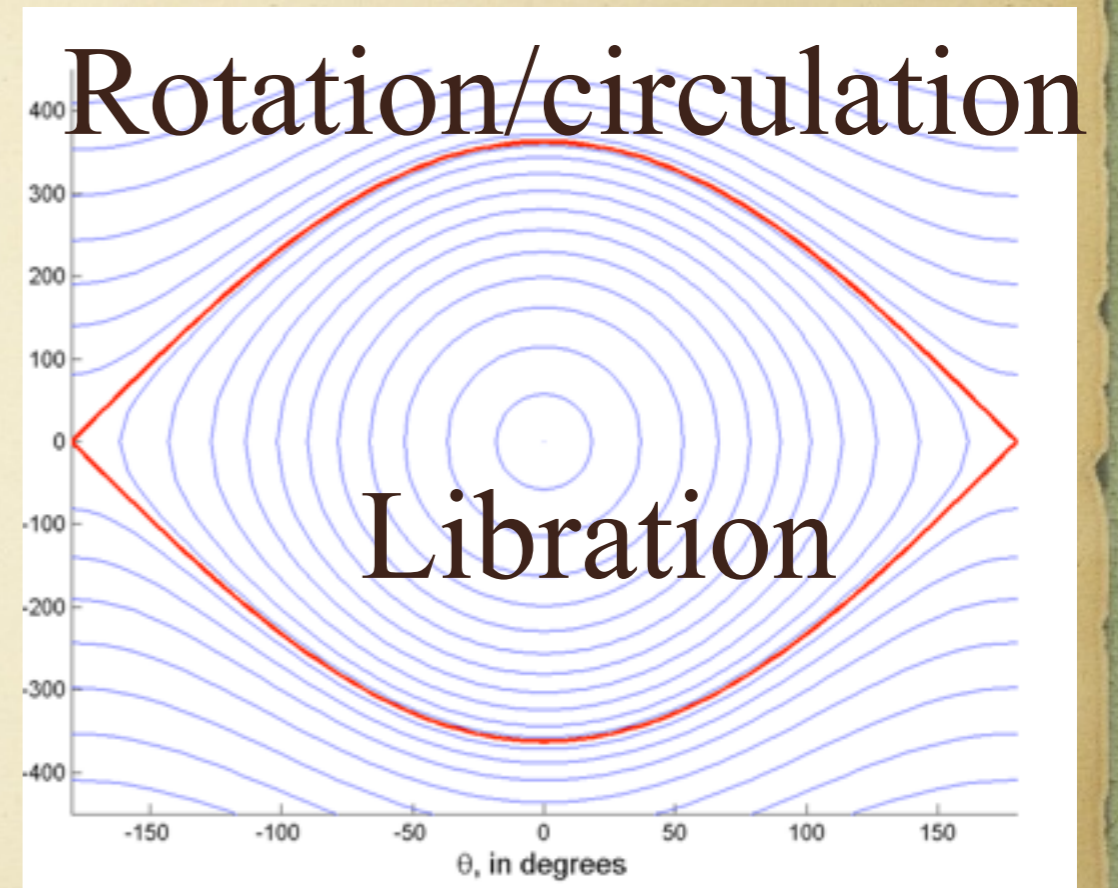
# EKL and the Pendulum

Quadrupole test particle limit:

$$e_0 = 0$$

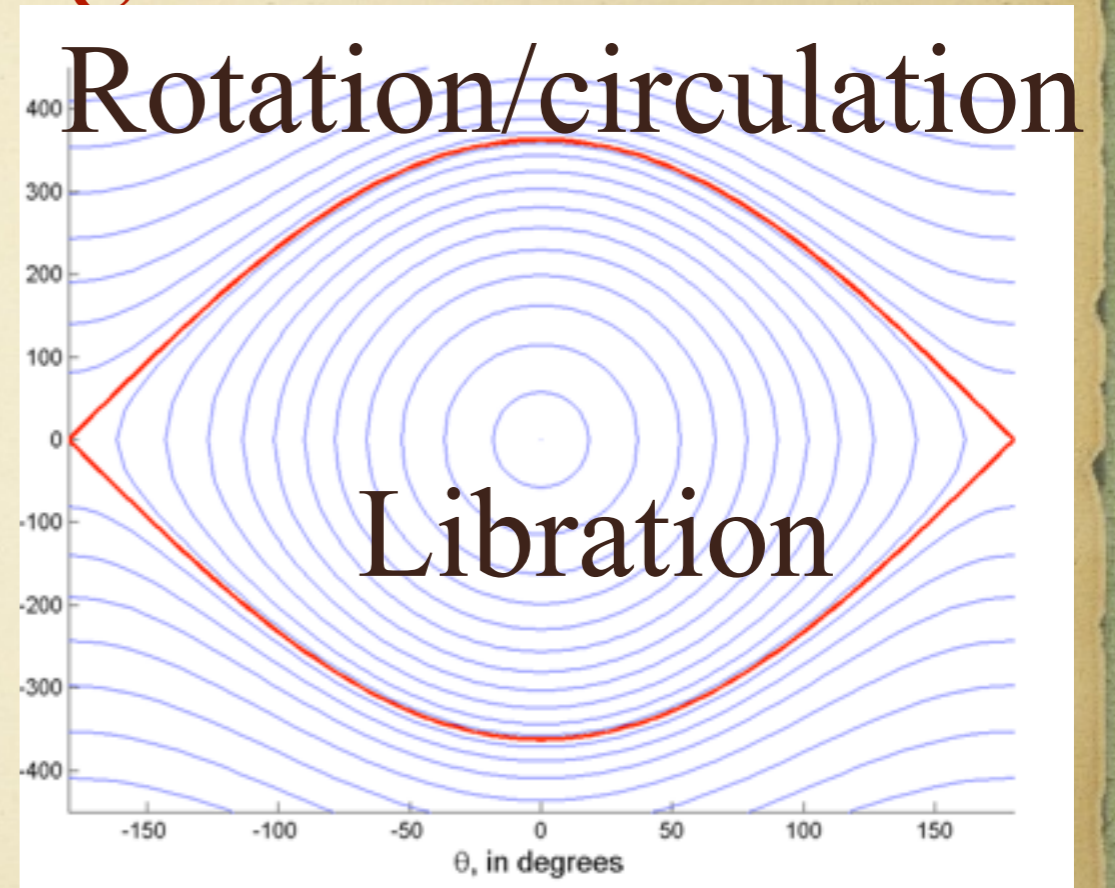
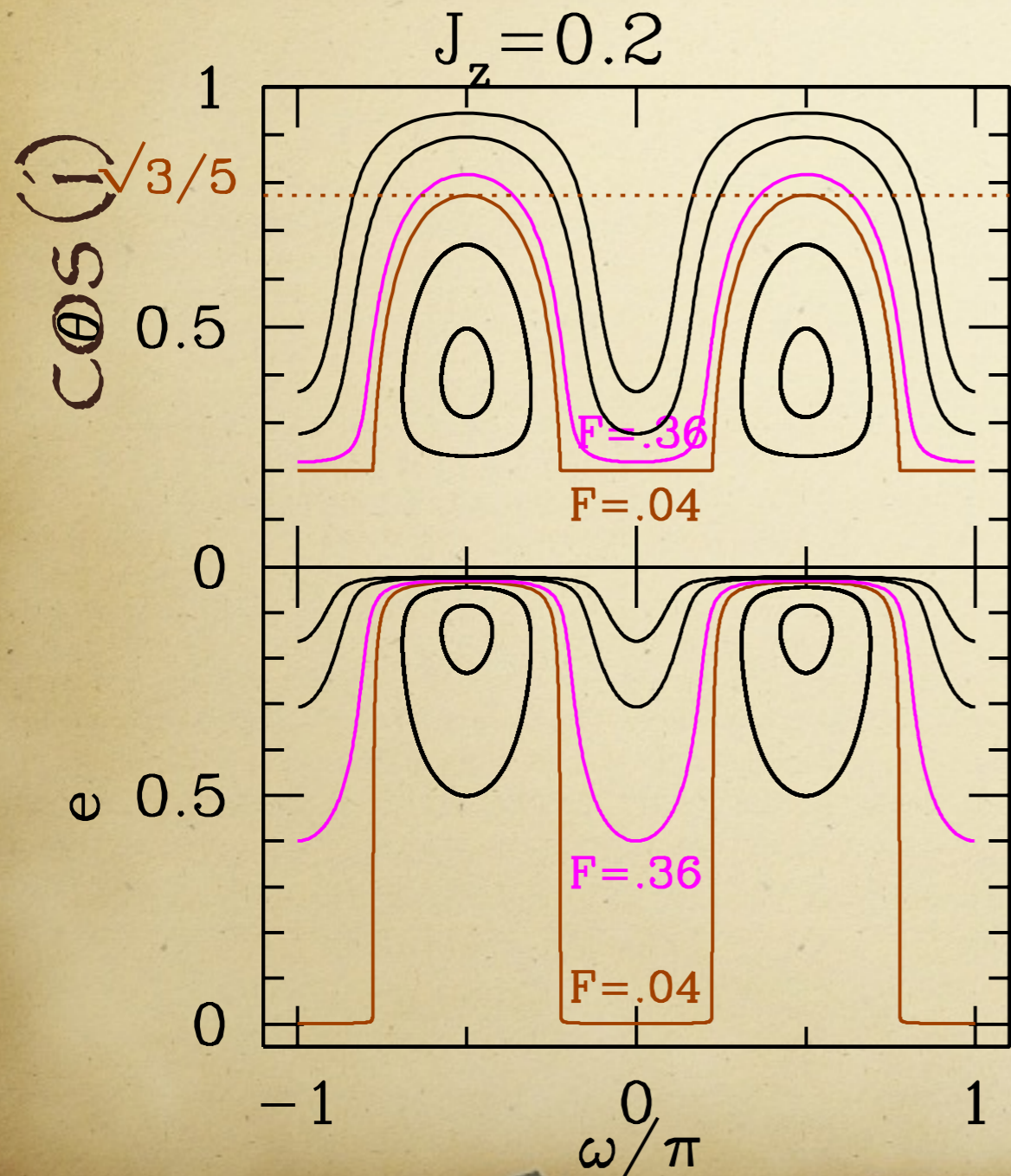


circular outer orbit



Q: Why 40 - 140 degrees limits?

Quadrupole test particle limit:



A: The separatrix has:

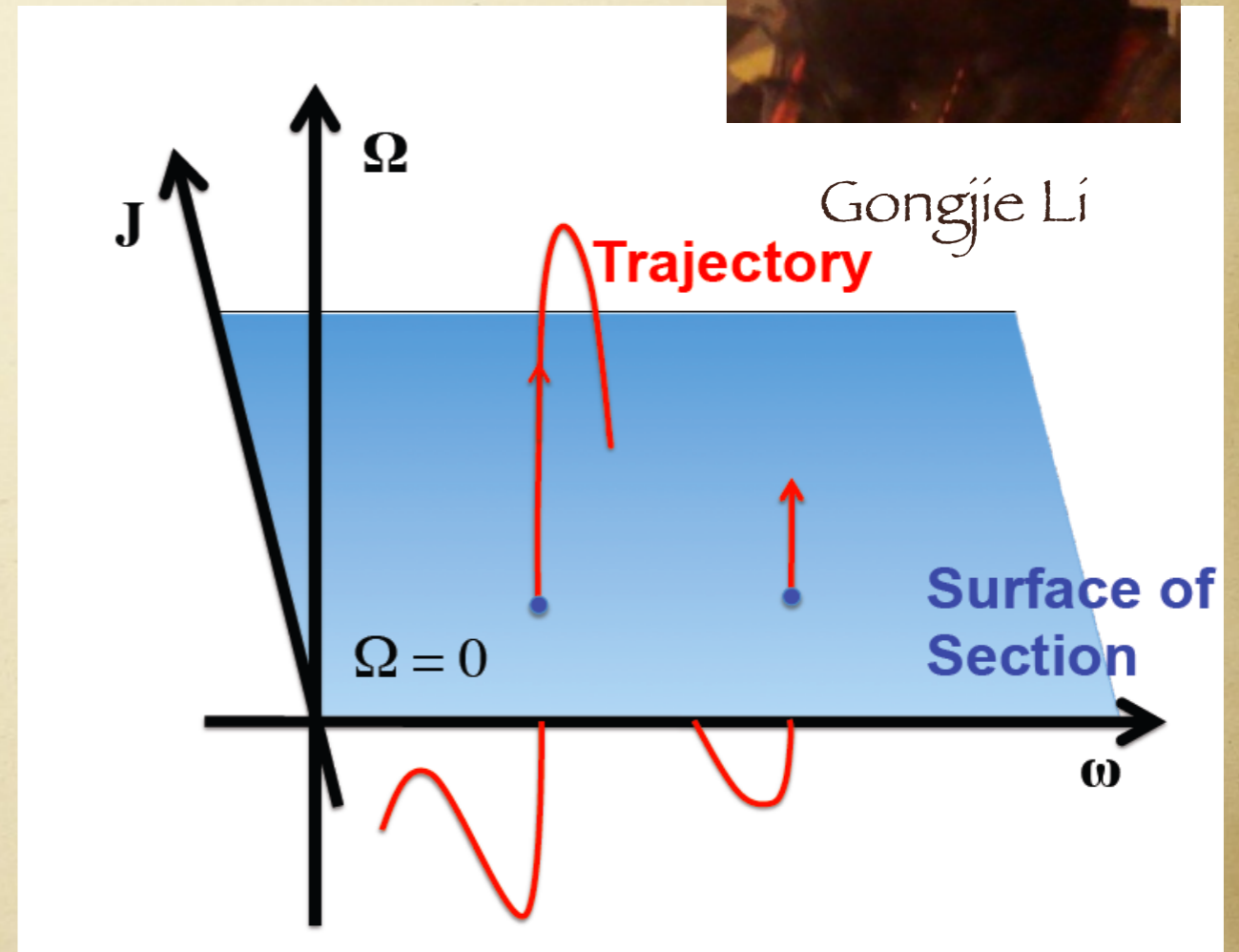
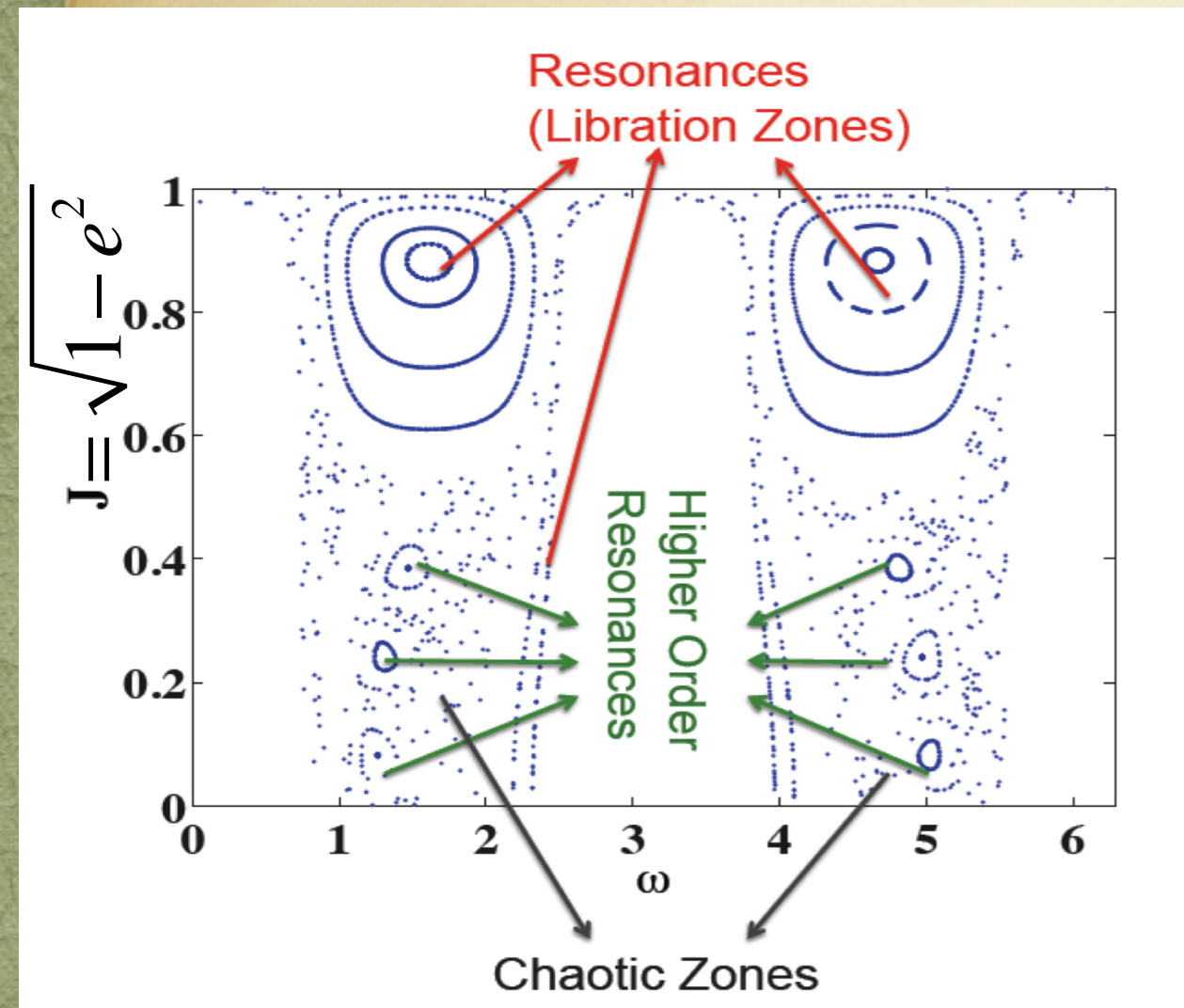
$$e_0 = 0, \cos i_0 = \sqrt{\frac{3}{5}}$$

Q: Is the 40 - 140 degrees limits hold?

A: No



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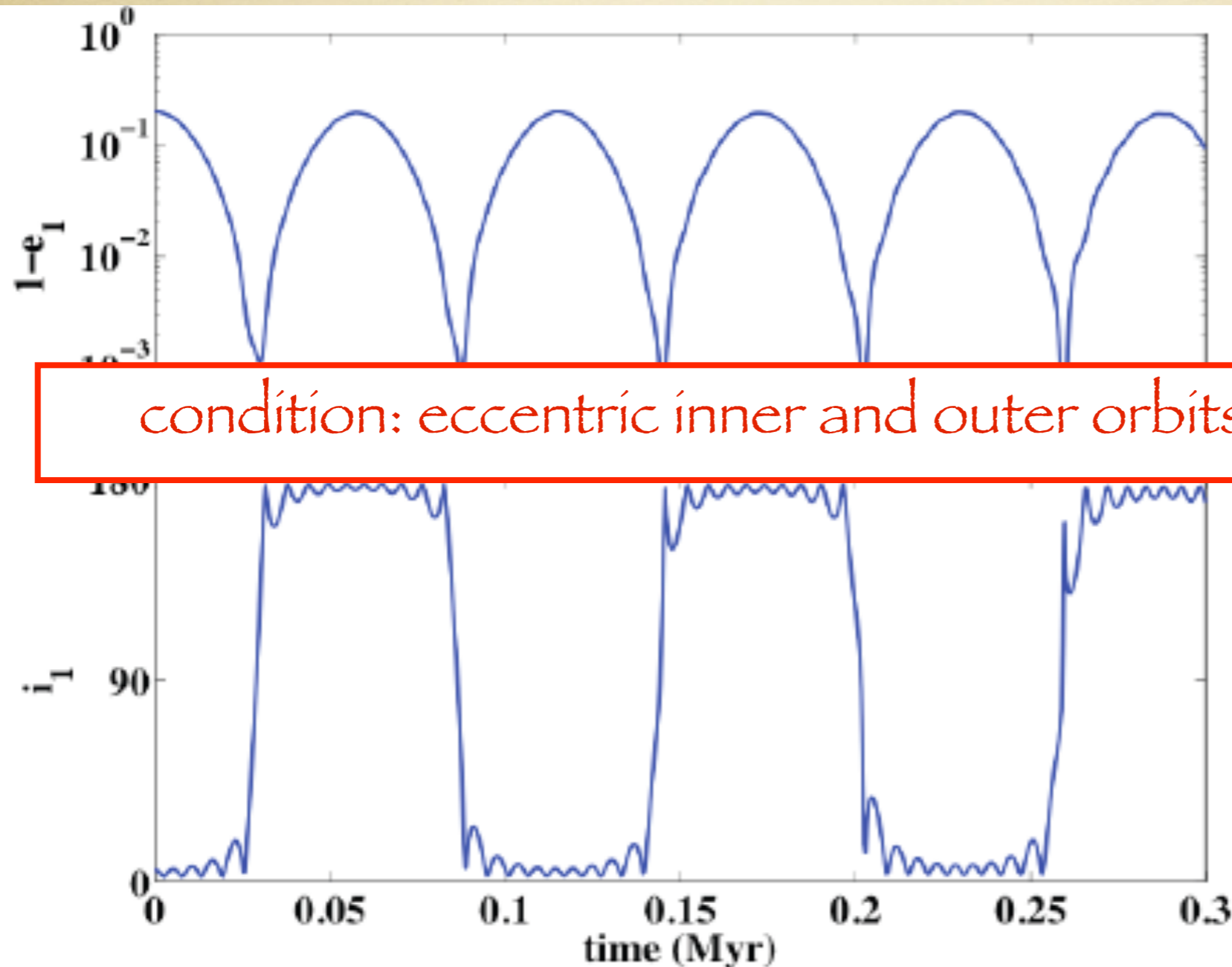


Q: Is the 40 - 140 degrees limits hold?

A: No



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condition: eccentric inner and outer orbits

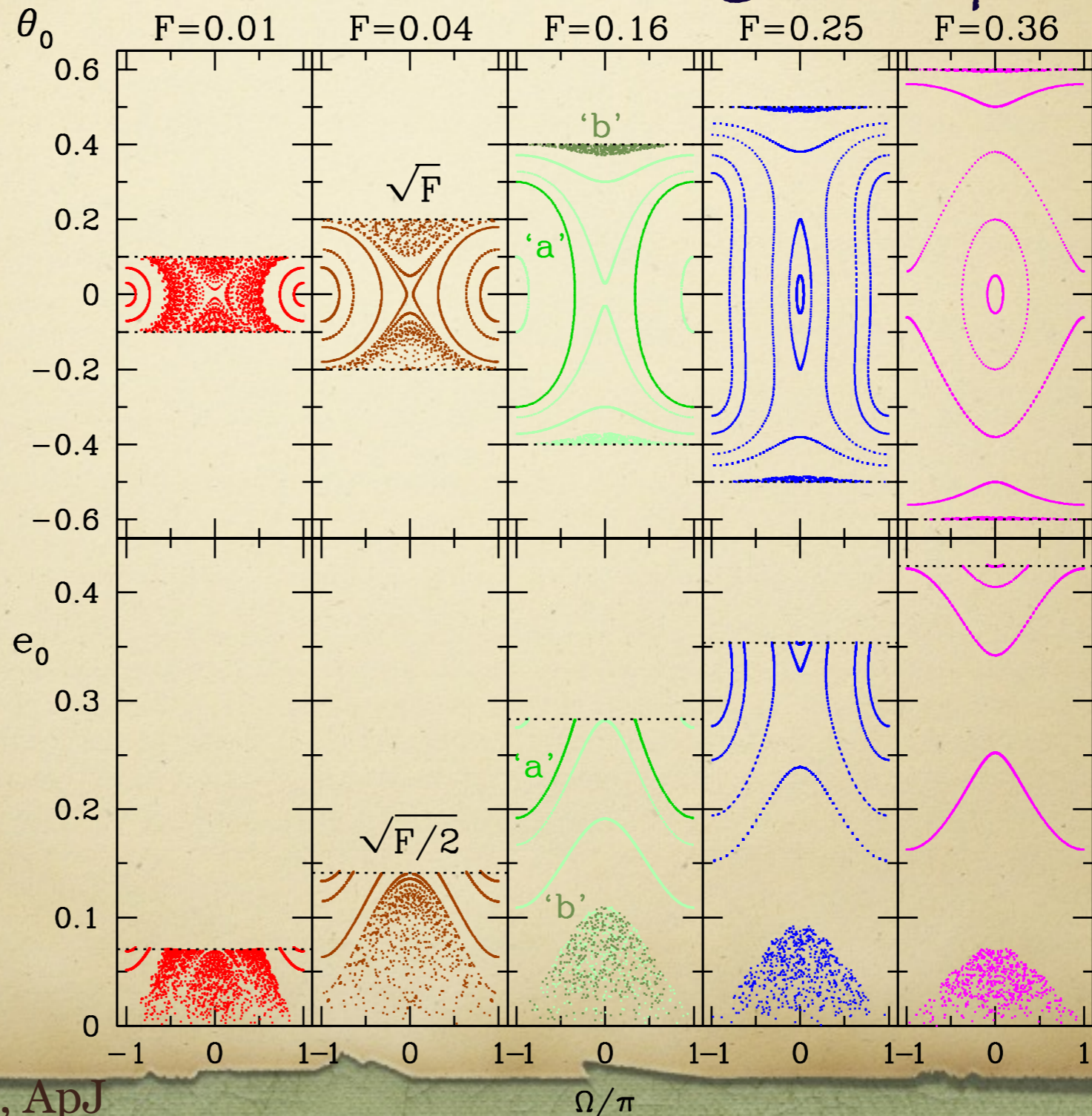
$\omega_1 = 0^\circ, \Omega_1 = 180^\circ,$   
 $e_2 = 0.6, a_1 = 4AU, a_2 = 50AU$   
 $e_1 = 0.8, i = 5^\circ$   
 $m_1 = 1M_\odot, m_2 = 1M_J, m_3 = 0.3M_\odot$

Li, **Naoz**, Kocsis, Loeb 2014, ApJ arXiv:1310.6044

Li, **Naoz**, Holman, Loeb 2014, ApJ arXiv:1405.0494

# Q: Why Chaos?

A: Octupole - chaotic behavior crossing the separatrix



chaos in these  
systems: Holman, Touma  
& Tremaine (1997)

# Maximum eccentricity and initial conditions

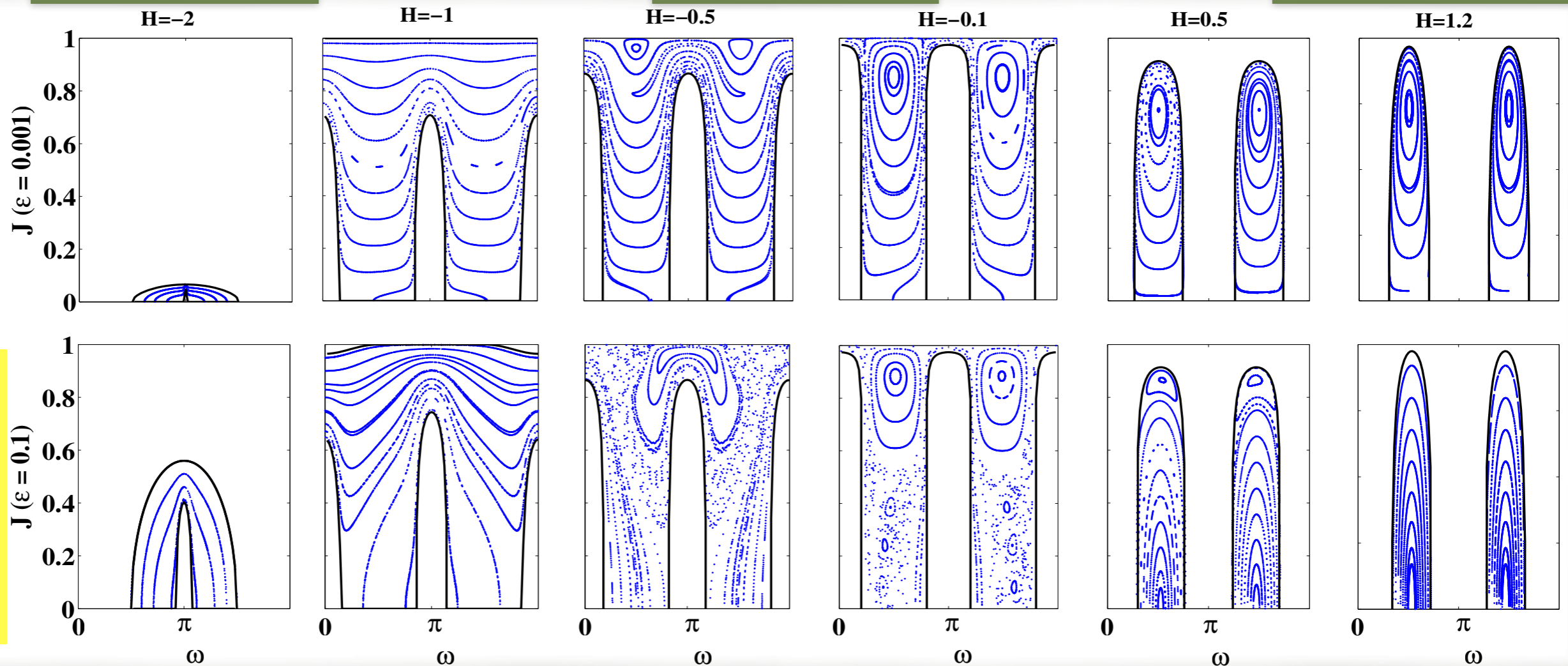


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Low  $i$  High  $e$

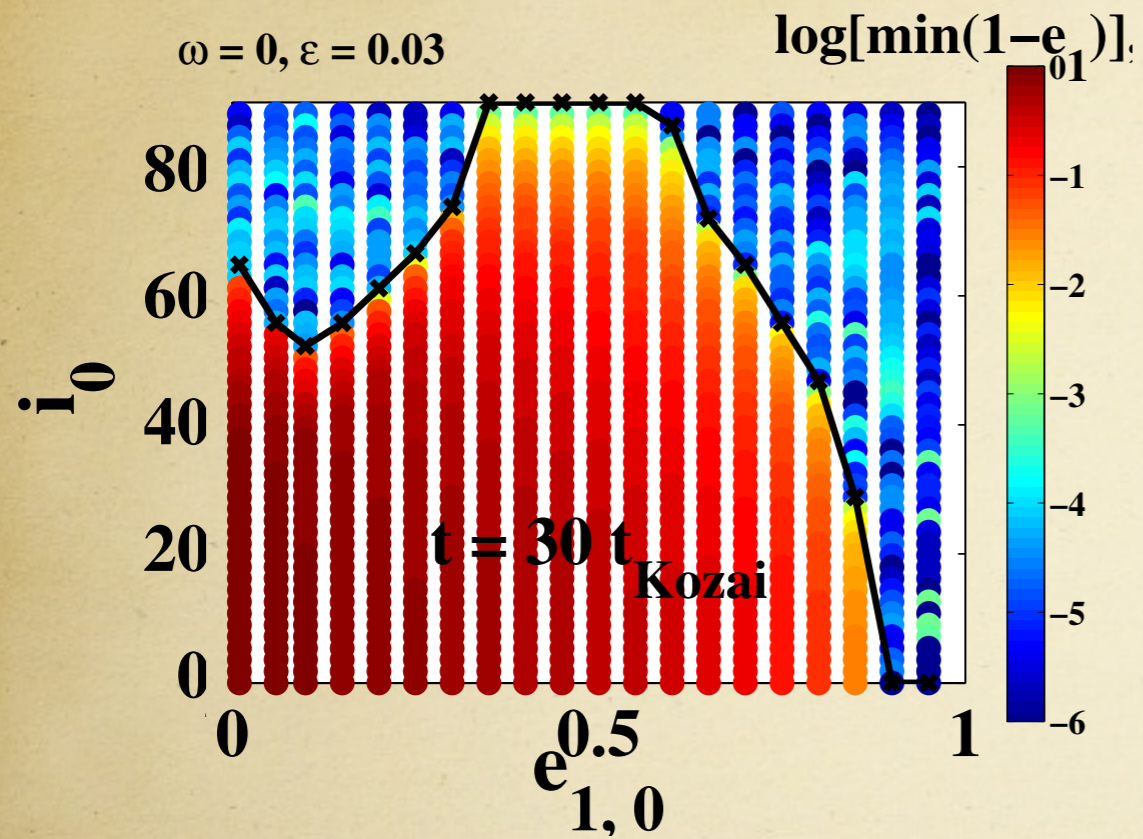
High  $i$  Low  $e$

High  $i$  High  $e$



octupole plays important role

# Eccentricity spikes



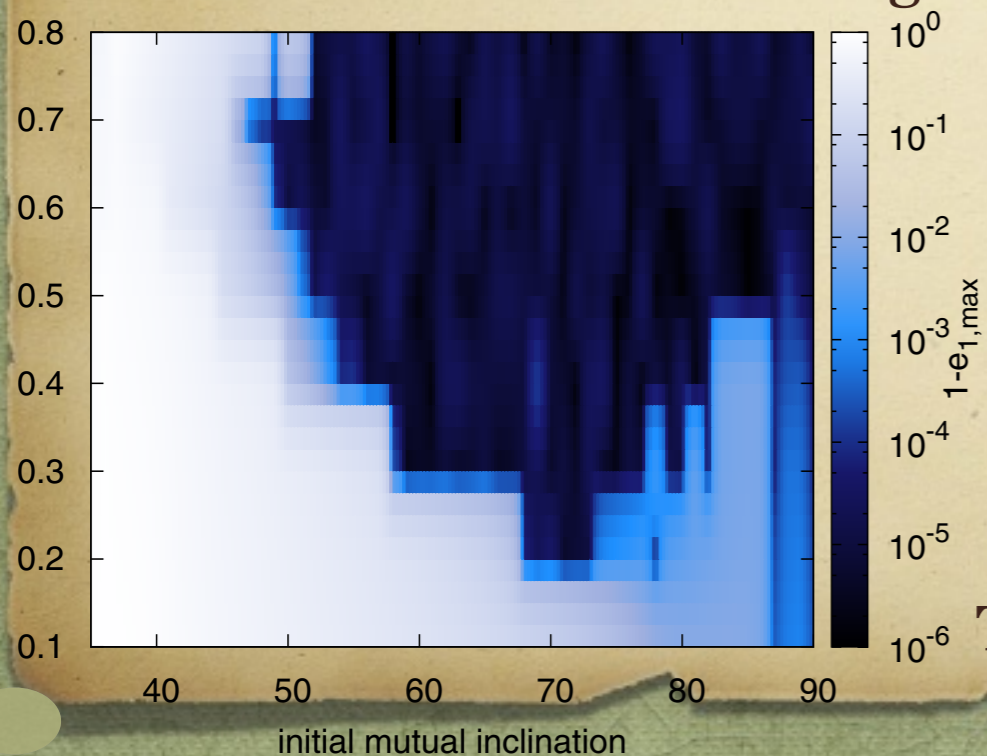
Maximum eccentricity at the **test particle** regime

Li, **Naoz** et al, (2014), ApJ 785, 116 + ApJ 791, 86



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Maximum eccentricity **outside** the test particle regime



Teyssandier, **Naoz**, Lizarraga Rasio (2013), ApJ 779, 166



Ian Lizarraga



Jean Teyssandier

Astrodynamics is alive!

