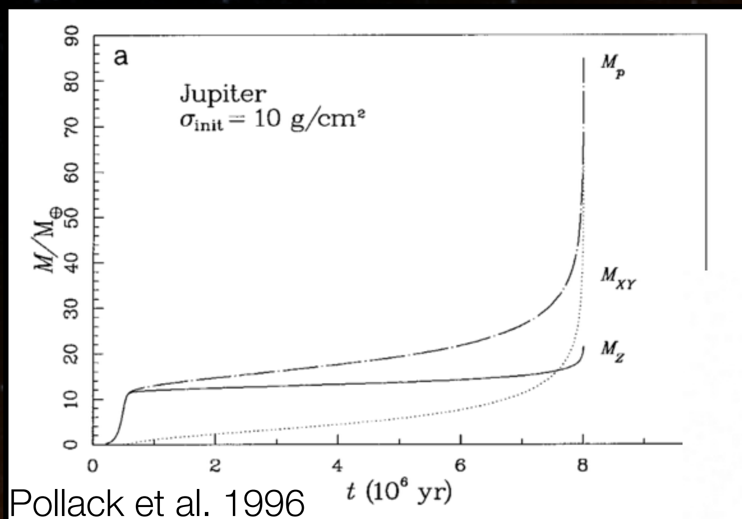
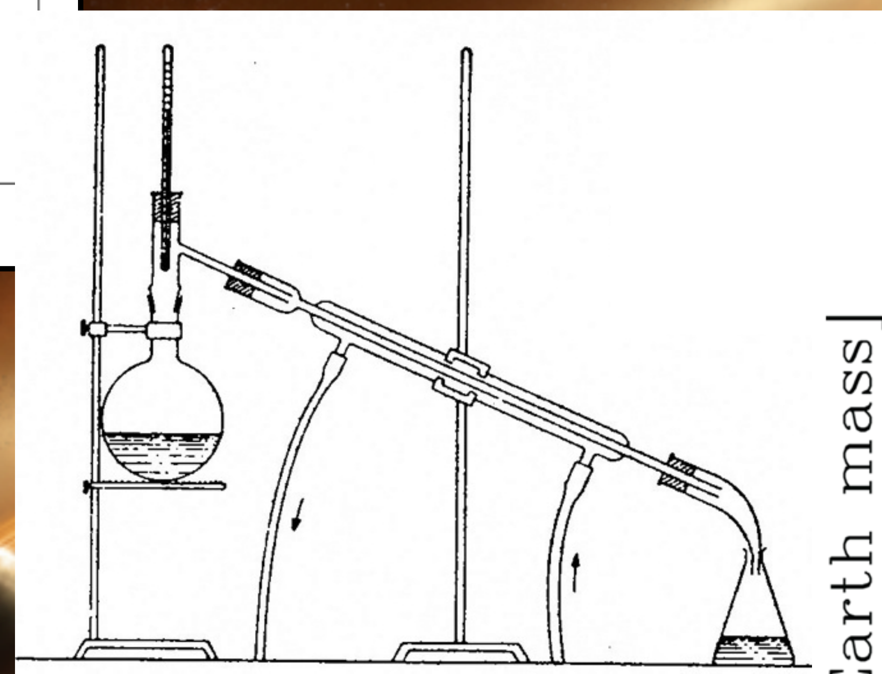
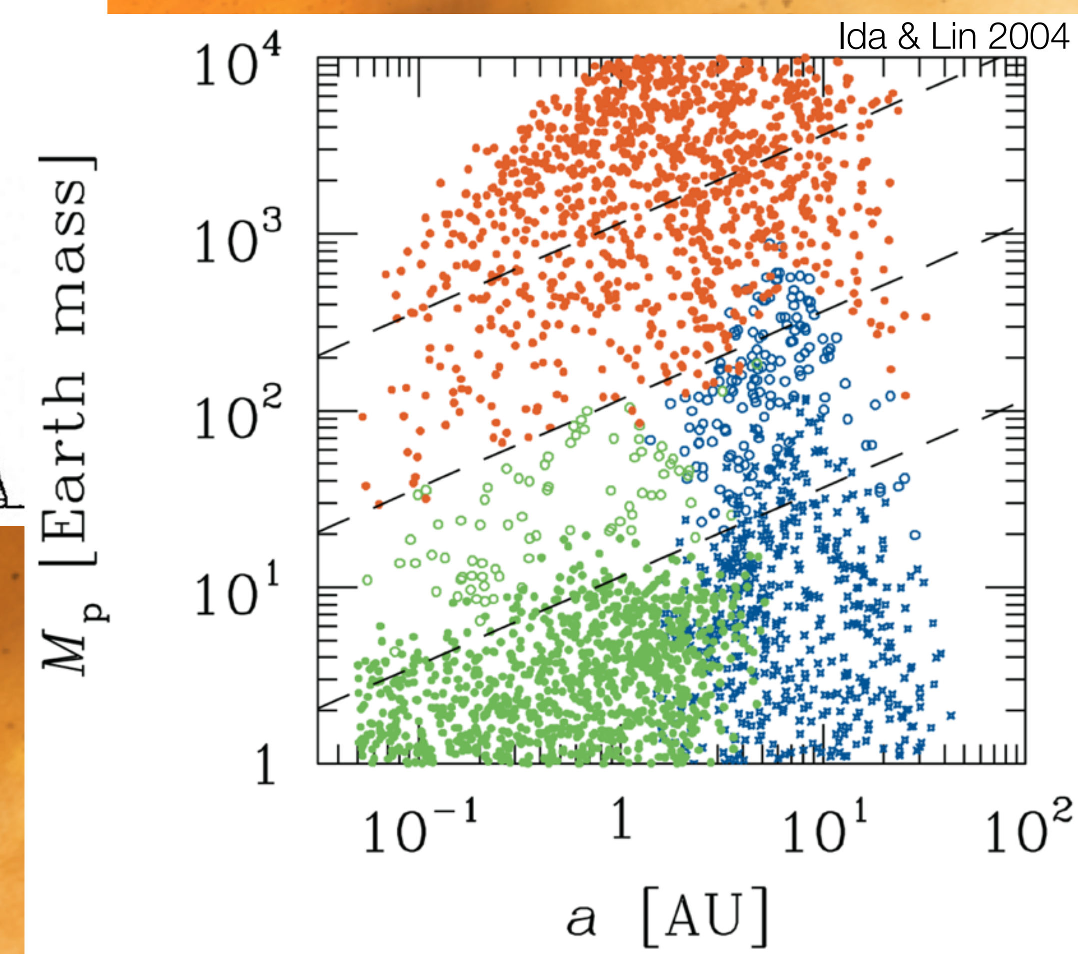
Population synthesis models based on core accretion



Pollack et al. 1996





Ida & Lin 2004

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FNSNF PlanetS

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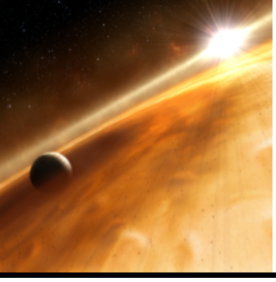
Sagan Summer School
Pasadena, 28.07.2015



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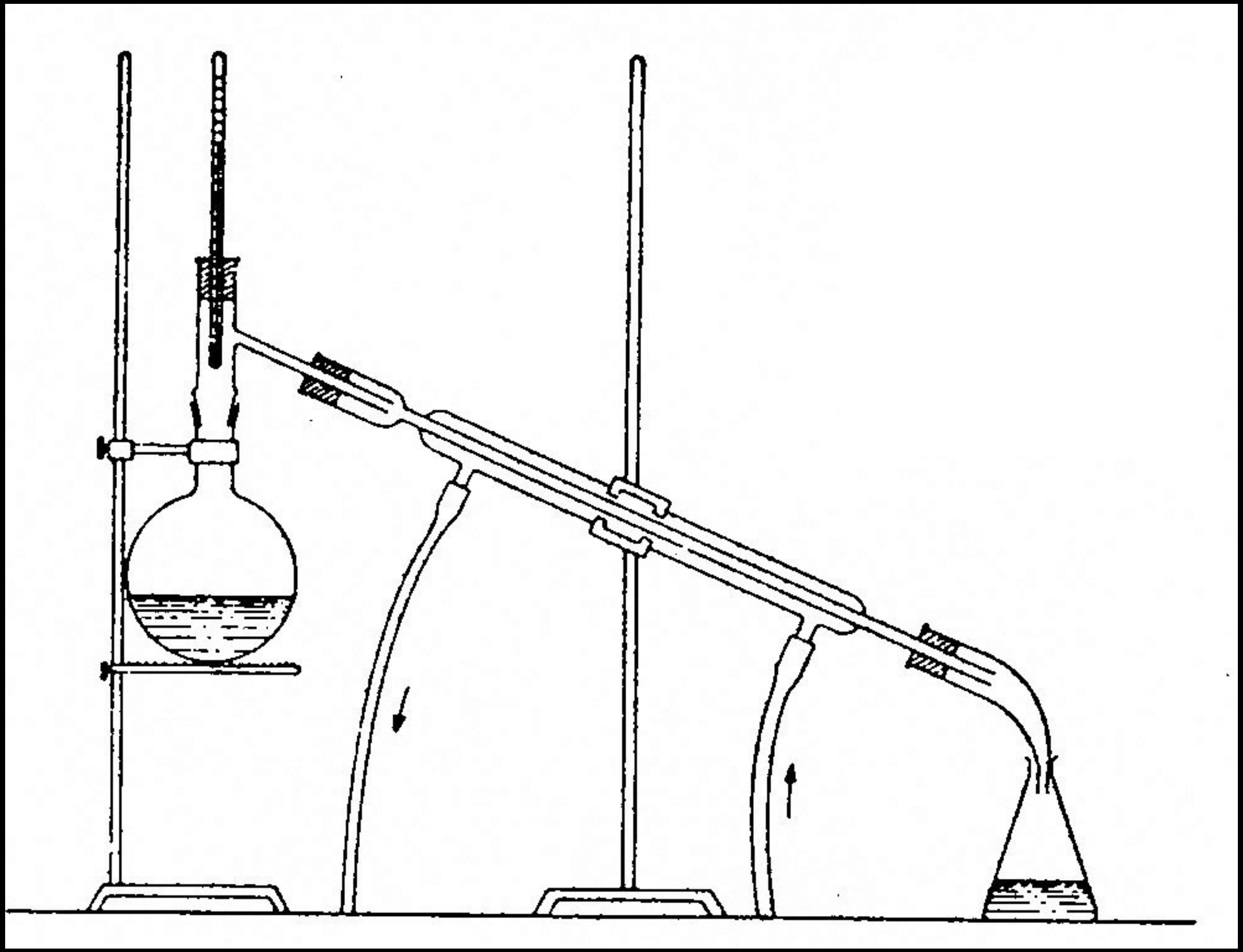
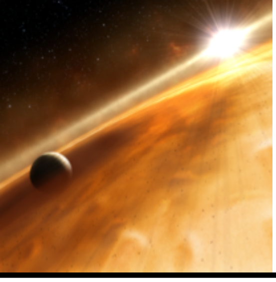


1. Introduction: population synthesis principle



Motivation

Many detections from space and ground (HARPS, Kepler, …). More to come (SPHERE, GPI, TESS, CHEOPS, Gaia, ESPRESSO, PLATO, NGTS, CARMENES, WFIRST, …)

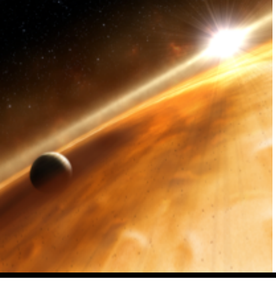
Field observationally driven, theory struggles to keep up. Improve theoretical understanding by comparing theory and observation.

Difficulty: planet formation theory difficult to test directly with observations: specific physical process convoluted with many other.

But: high number of exoplanets: can be treated as a population.
- -statistical constraints
- -data from many different techniques ⇒much more stringent constraints on theoretical models by combining M, a, e, R, L, spectra, …

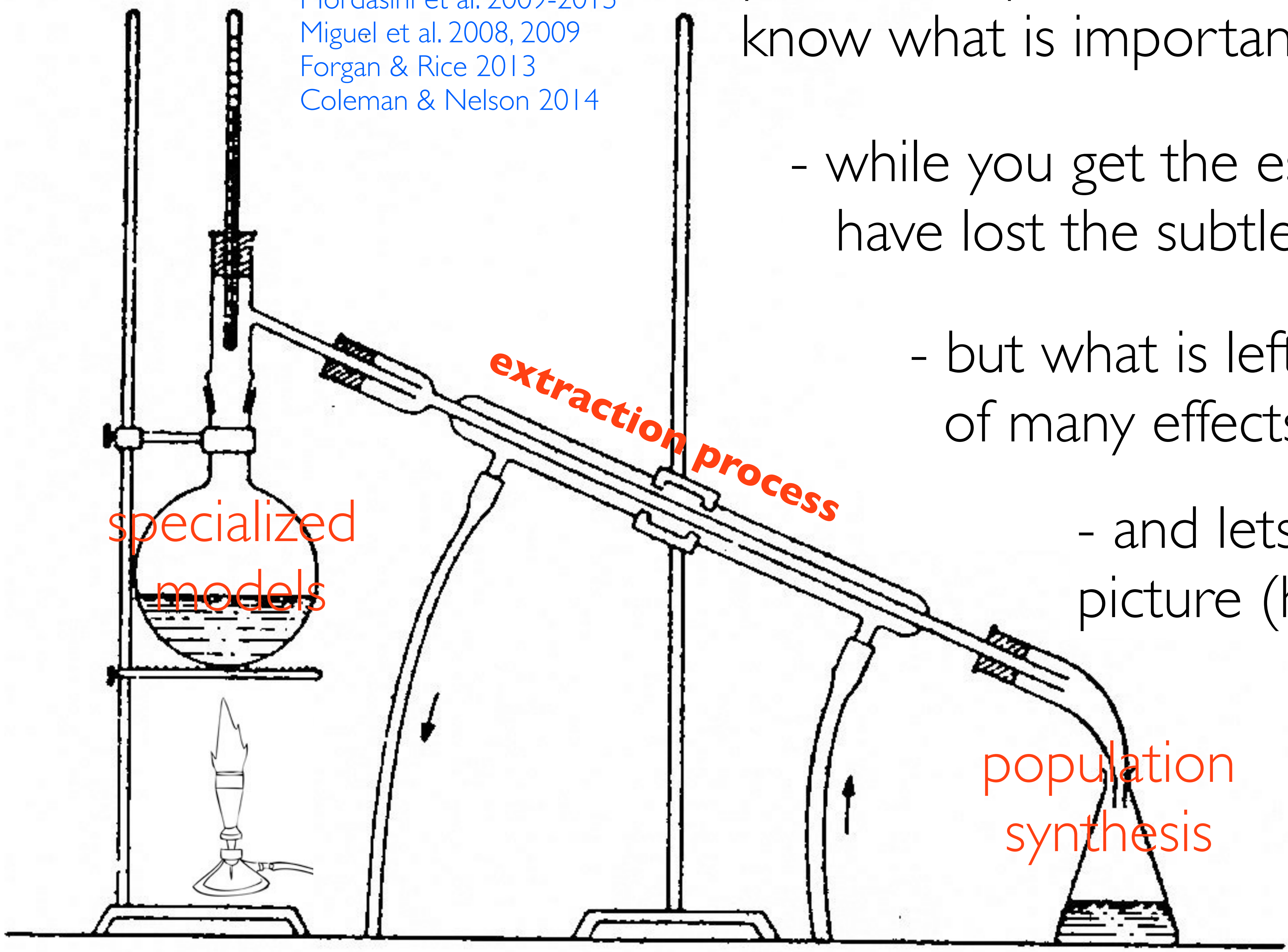
With population synthesis, we can use this wealth of constraints.

Essence of the method: a global model of planet formation and evolution, combining simplified descriptions of the essential physical mechanisms.

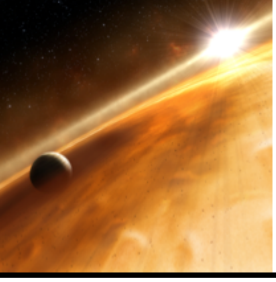


The essence of the method

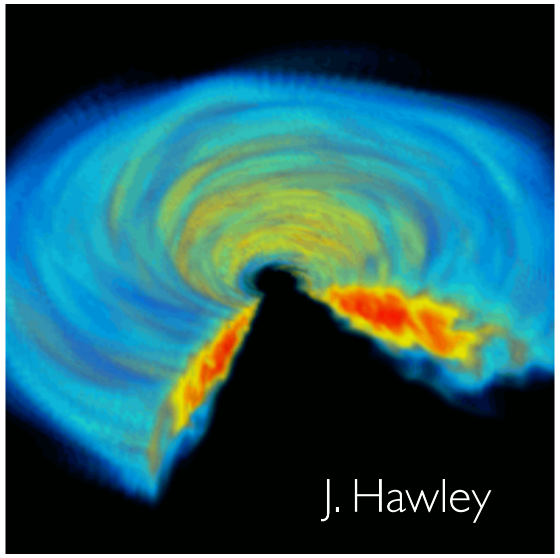
Ida & Lin 2004-2013
Thomes et al. 2008
Mordasini et al. 2009-2015
Miguel et al. 2008, 2009
Forgan & Rice 2013
Coleman & Nelson 2014



- you need specialized models to know what is important
- while you get the essence, you have lost the subtlety of the original
- but what is left is a concentrate of many effects
- and lets you see the big picture (hopefully)



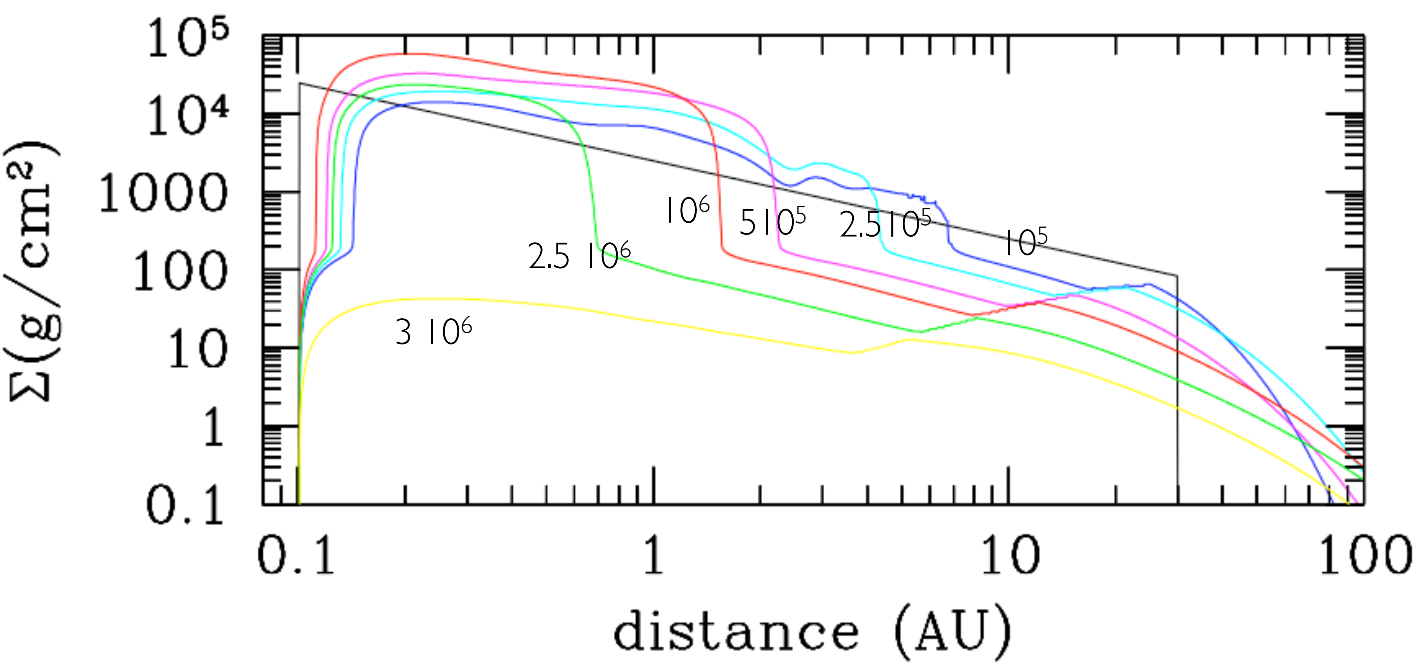
Distill how strongly?



$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + (\rho \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(P + \frac{B^2}{8\pi}\right) - \rho \nabla \Phi + \left(\frac{\mathbf{B}}{4\pi} \cdot \nabla\right)\mathbf{B} + \eta_V\left(\nabla^2 \mathbf{v} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{v})\right)$$

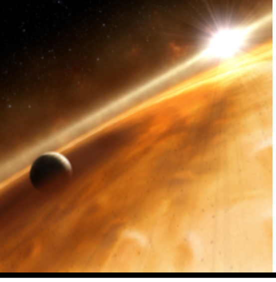
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta_B \nabla \times \mathbf{B})$$



$$\frac{d\Sigma}{dt} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\tilde{\nu}\Sigma r^{1/2}\right] + \dot{\Sigma}_w(r)$$

$$\Sigma(r) = \Sigma_0 \left(\frac{r}{r_0}\right)^{-\alpha} e^{-t/\tau}$$

How simple is still good enough?



Population synthesis work flow

Formation model

Link disk properties ⇒ planet properties

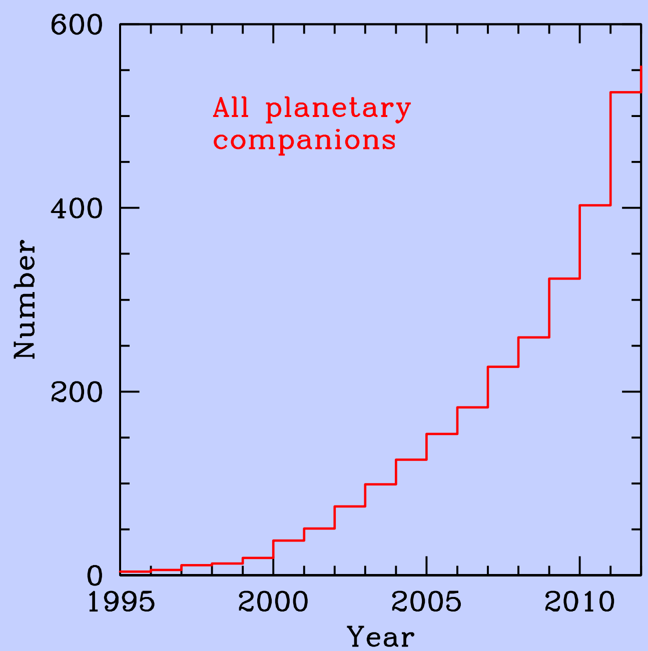
Initial Conditions: Probability distributions & parameters

Disk gas mass
Disk dust mass
Disk lifetime

From observations

Draw and compute synthetic planet population

Apply observational detection bias



Observed population

Comparison:

Observable sub-population
- Distribution of semi-major axis
- Distribution of masses
- Fraction of hot/cold Jupiters
- Distribution of radii

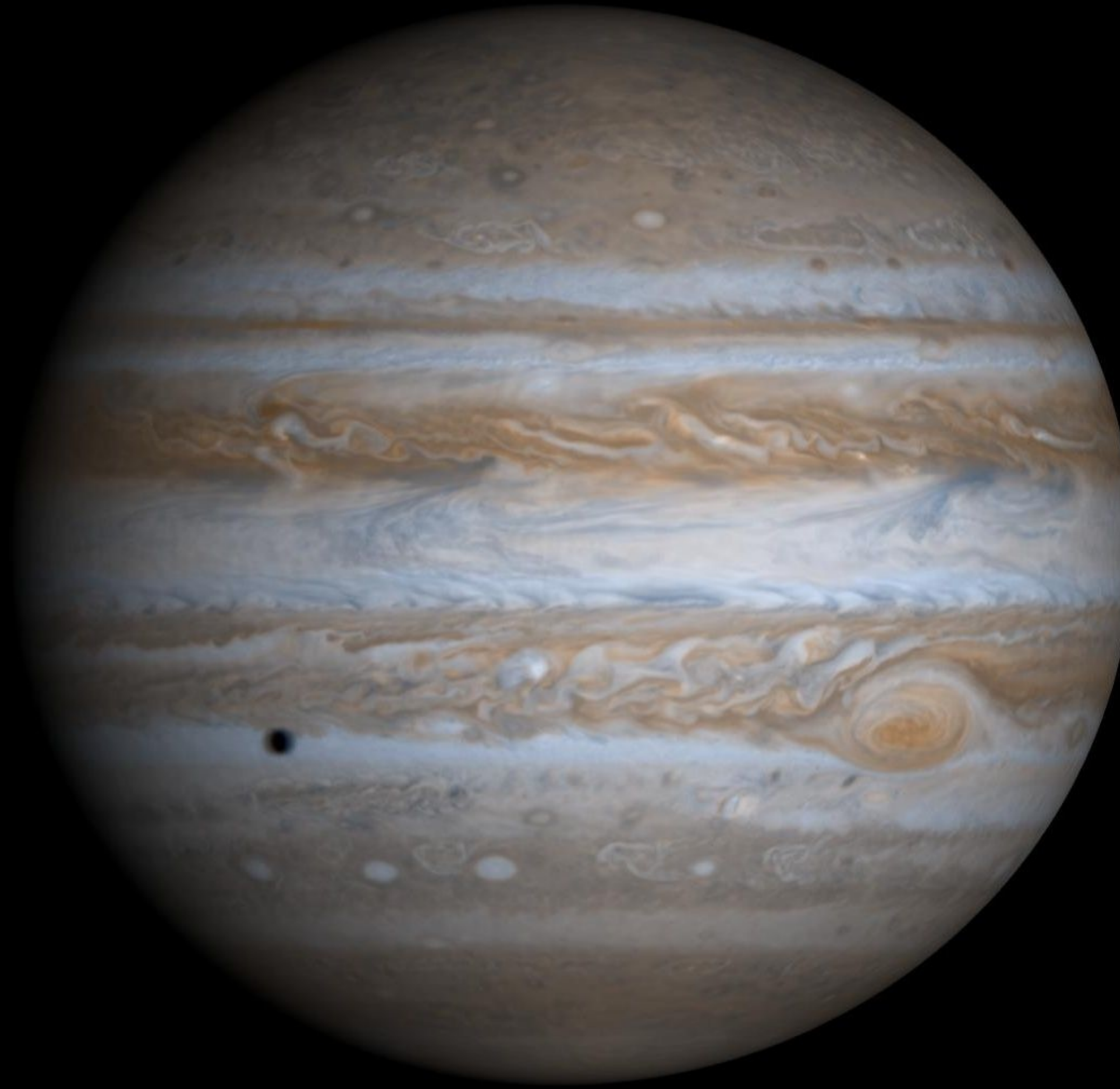
No match: improve, change parameters

Match

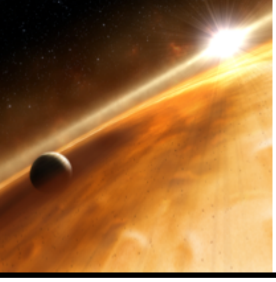
Model solution found!

Predictions

(going back to the full synthetic population)



1.
Input physics

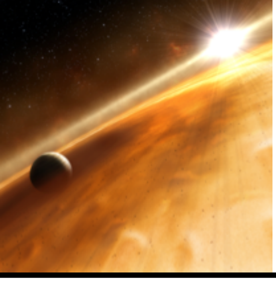


First modern model: Ida & Lin 2004

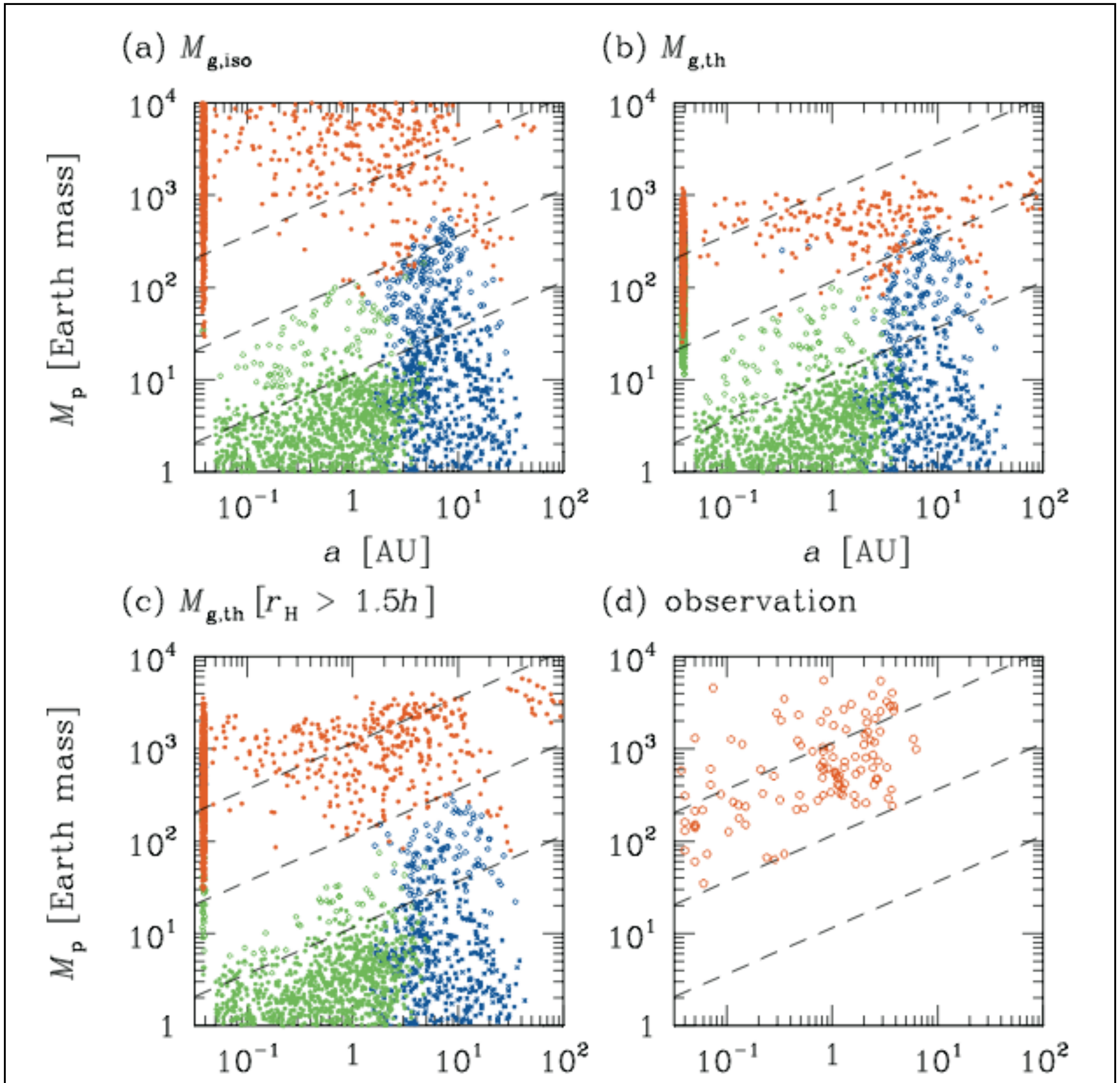
Ida & Lin (2004, 2005, 2008, 2010, 2013) building on Kokubo & Ida 2002, Ida & Makino 1993, …

- Disk model: powerlaw, exponential decrease
- Accretion of solids: Safonov rate equation, isolation mass
  Safronov 1969, Greenzweig & Lissauer 1992, Ida & Makino 1993
- Accretion of gas: Parameterized KH-contraction, fitted M_{crit}
  Perri & Cameron 1974, Mizuno et al. 1978, Ikoma et al. 2000
- Termination of gas accretion: Gap formation, disk dissipation
  Lubow 1999, Kley & Dirksen 2006
- Coalescence of embryos: 1 embryo per disk, later semi-analytical prescription (orbit crossing)
- Orbits: type I and II disk migration
  Goldreich & Tremaine 1979, Lin & Papaloizou 1986, Paardekooper et al. 2010, …
- <u>Monte Carlo variables:</u> position of embryo, disk mass, dust-to-gas ratio, disk lifetime

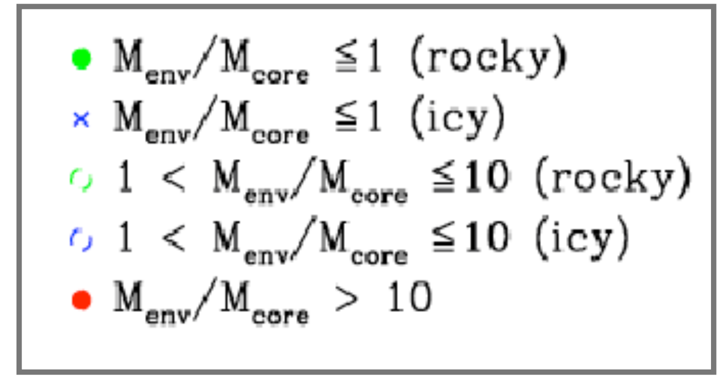
Later several improvements: dead zones, local enhancement of solids, …



First modern pop. synthesis

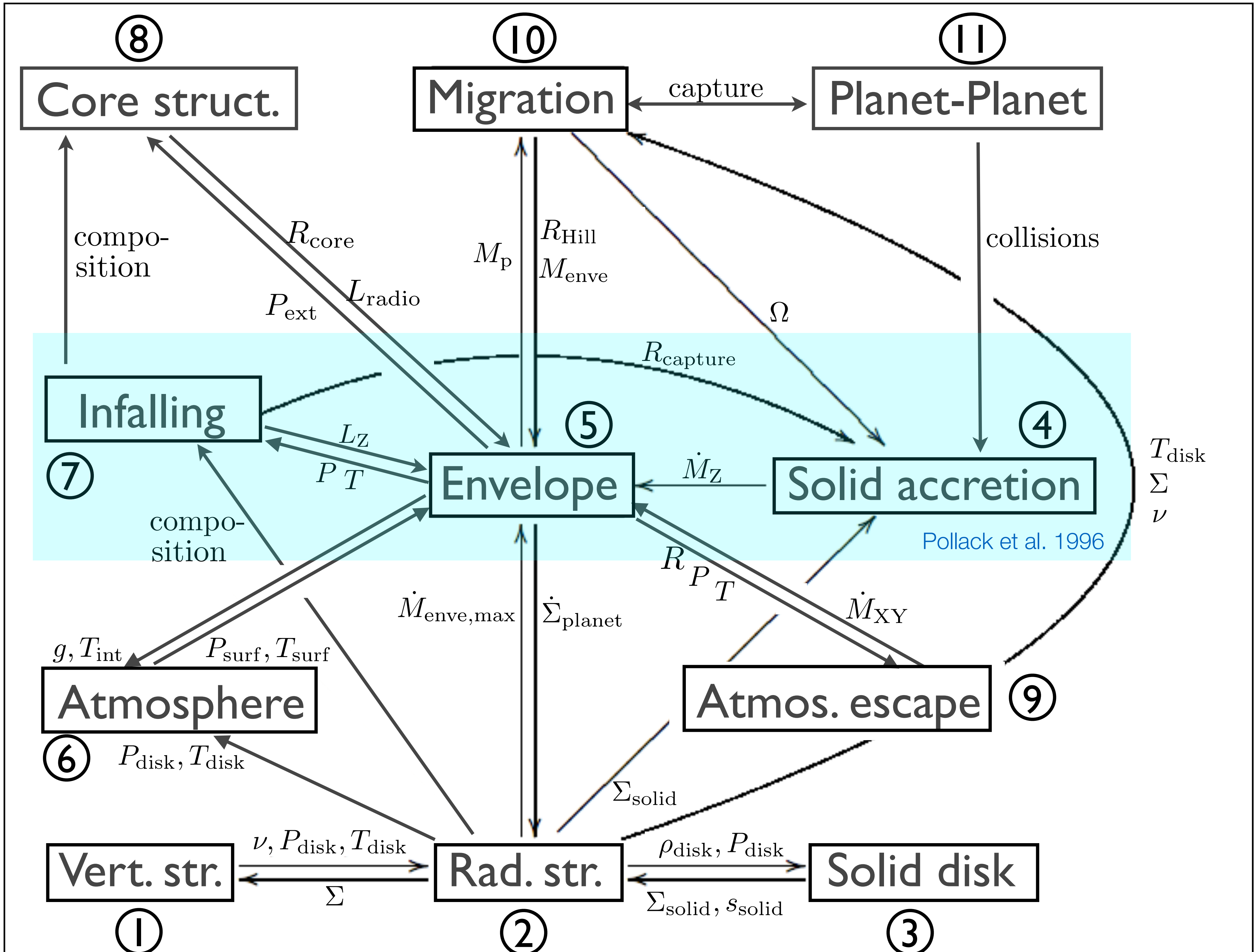


Ida & Lin 2004



- aM: diversity
- Planetary desert
- Metallicity effect (correlation between metallicity and giant planet detection probability)
-termination of gas accretion
-effects of type II migration

Global formation & evolution model



Alibert, Mordasini, Benz 2004; Alibert et al. 2005, Mordasini et al. 2012, Alibert et al. 2013, Sheng & Mordasini 2014,…

Core accretion paradigm

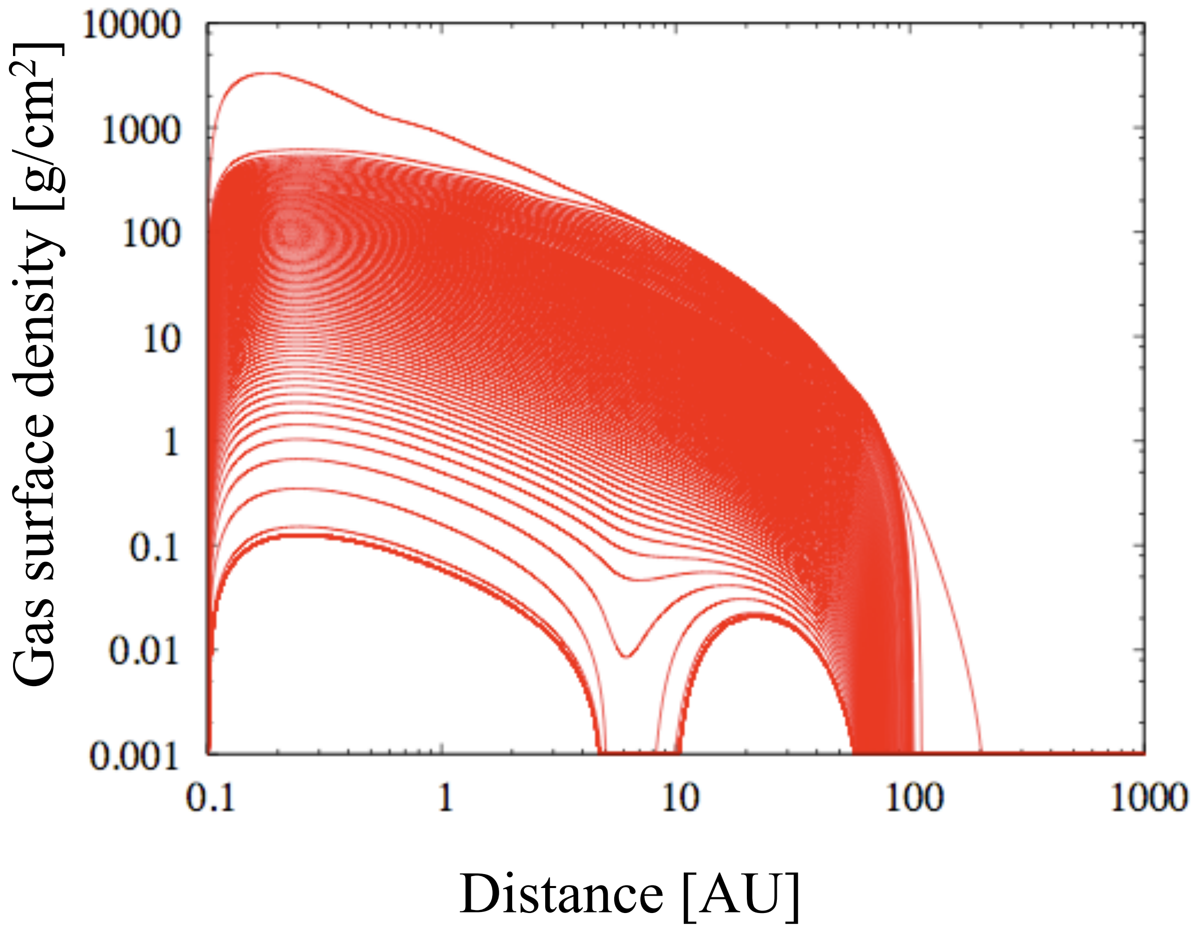
protoplanetary disk evolution
+
Planet model (gas & solid accretion, interior structure)
+
Disk migration (non-isothermal type I & type II)
+
N-body

Simple standard models, but coupled

1+1D α disk model

Evolution of the gas surface density

Lyden-Bell & Pringle 1974



$$\frac{d\Sigma}{dt} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\tilde{\nu}\Sigma r^{1/2}\right] + \dot{\Sigma}_w(r) + \dot{Q}_{\rm planet}(r)$$

Viscosity — Photoevaporation — Planet accretion

Vertical & radial structure.

Shakura & Sunyaev 1973
Chiang & Goldreich 1997

- constant α $\nu = \alpha c_s H$
- stellar irradiation included for temperature

- external photoevaporation

Matsuyama et al. 2003

$$\dot{\Sigma}_{w,\rm ext} = \begin{cases} 0 & \text{for } r < \beta R_{g,\rm I} \\ \dfrac{\dot{M}_{\rm wind,ext}}{\pi(r_{\rm max}^2 - \beta^2 R_{g,\rm I}^2)} & \text{otherwise} \end{cases}$$

- internal photoevaporation

Clarke et al. 2001

$$\dot{\Sigma}_{w,\rm int} = \begin{cases} 0 & \text{for } r < R_{\rm wind} \\ 2c_{s,\rm II} n_0(r) u_{\rm ma} & \text{otherwise} \end{cases}$$

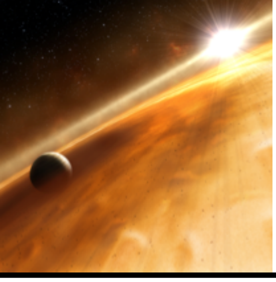
Initial surface density profile: Andrews et al. 2009

$$\Sigma(r, t=0) = \Sigma_0 \left(\frac{r}{R_0}\right)^{-\gamma} \exp\left[-\left(\frac{r}{R_c}\right)^{2-\gamma}\right]$$

Inner disk edge: 0.1 AU (arbitrary)
Outer disk edge: free

- gas accreted by planet taken from feeding zone

$a_{\rm planet} \pm 0.5 \times R_H$ $\dot{Q}_{\rm planet} = -\dot{M}_{XY}$



Planet solid accretion rate

Collisional growth of one big body from small background planetesimals

-Simple Safronov type rate equation for growth of planet's core

$$\frac{dM_Z}{dt} = \Omega \Sigma_p \pi R_{capt}^2 F_G(e,i)$$

-$F_G(e,i)$ 3 Body gravitational focussing factor from Greenzweig & Lissauer 1992

-R_{capt}: Capture radius (envelope effect) > R_{core}

-Random velocity **σ(e,i)** of planetesimals is key parameter (runaway, oligarchic, orderly)

- -original model: (low) random velocities from Pollack et al. 1996
- -updated model (Fortier et al. 2011): equilibrium stirring-gas damping (oligarchic regime)

Ejection of planetesimals by (massive) protoplanets

Ida & Lin 2004

$$\frac{\text{accretion rate}}{\text{ejection rate}} = \left(\frac{V_{\text{esc,disk}}}{V_{\text{surf,planet}}}\right)^4$$

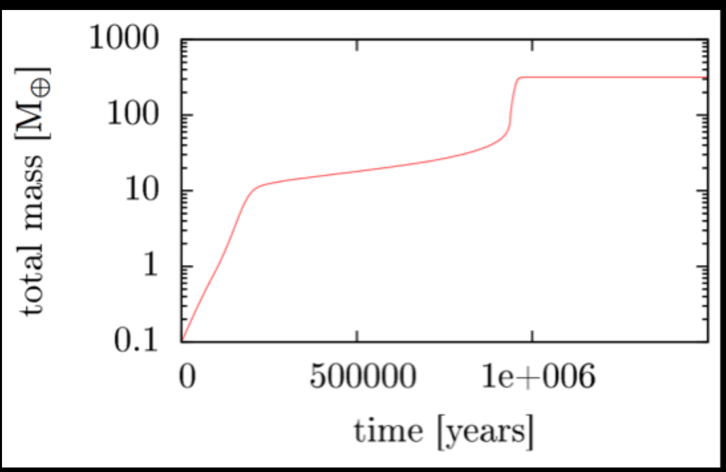
$$V_{\text{esc,disk}} = \sqrt{2\,G\,M_\odot/a_{\text{planet}}}$$

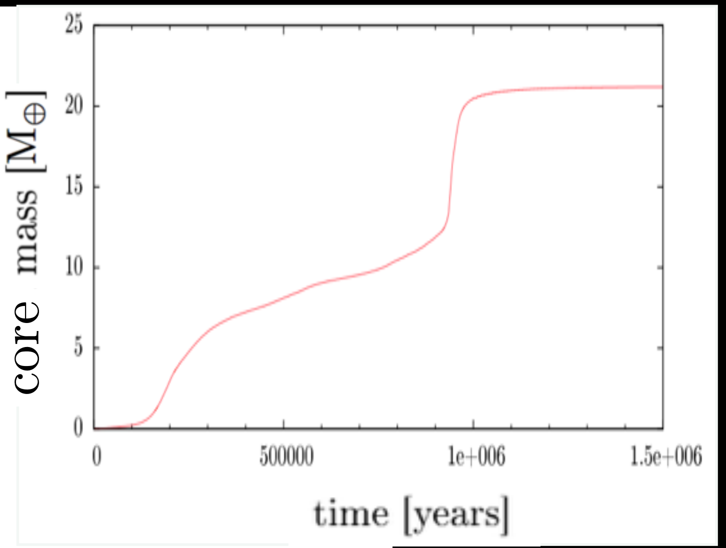
$$V_{\text{surf,planet}} = \sqrt{GM_{\text{planet}}/R_{\text{c}}}$$

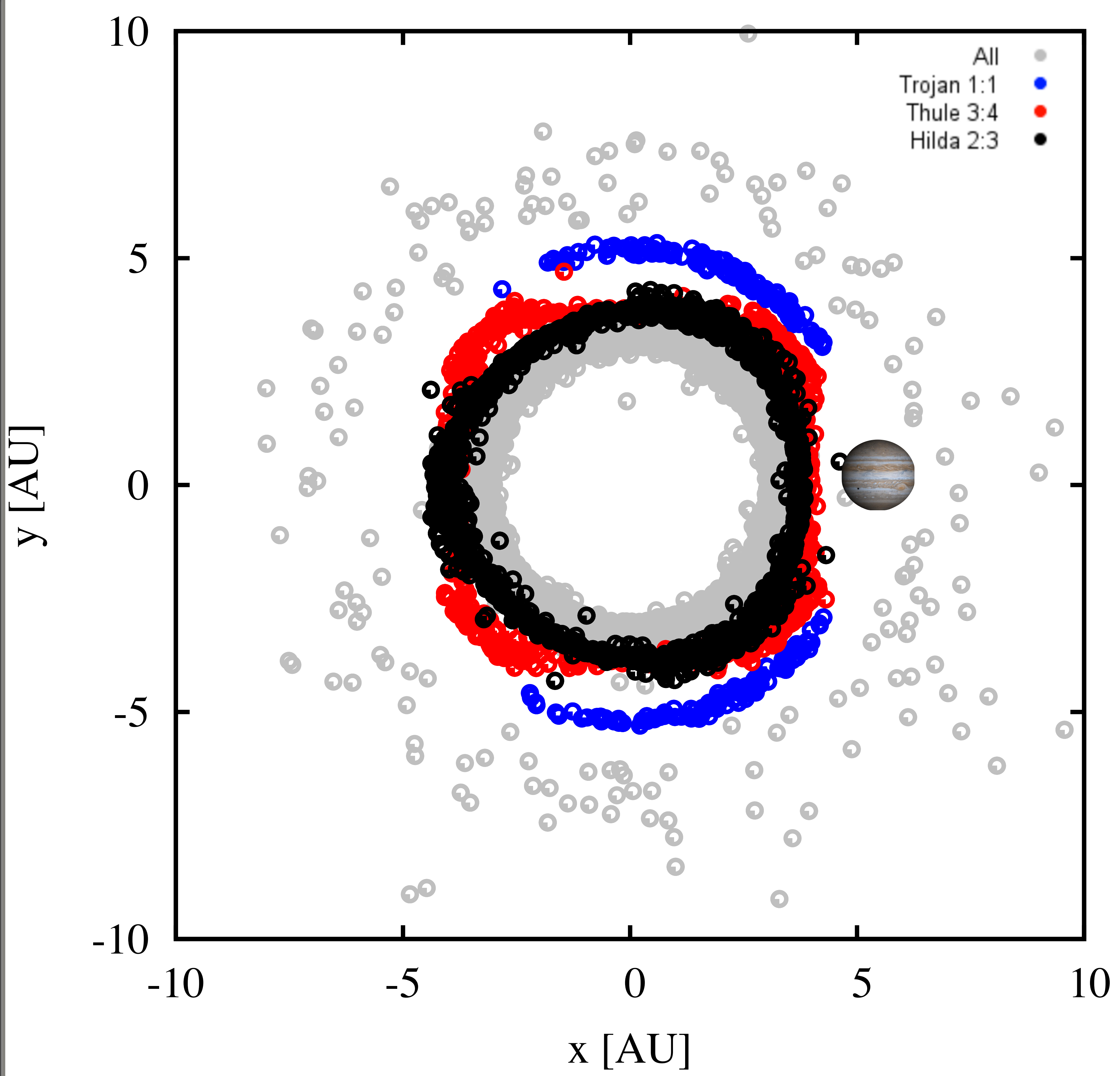
Pebbles: work in progress

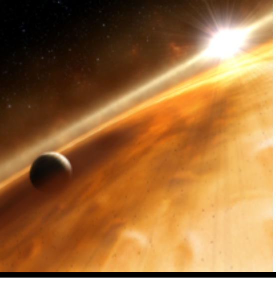
<u>N-Body simulation</u>

- Star, planetesimal swarm & growing planet at 5.2 AU
- Corrotating coord. system
- Planet also accretes gas
- Rapid gas accretion at about 0.9 Myr









Planet gas envelope structure

1-D radial structure equations (similar to stellar structure)

e.g. Bodenheimer & Pollack 1986

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

$$\frac{dl}{dr} = 4\pi r^2 \rho \left(\epsilon - T\frac{\partial S}{\partial t}\right) \qquad \frac{dT}{dr} = \frac{T}{P}\frac{dP}{dr}\nabla$$

Mass conservation
Hydrostat. equilibrium
Energy conservation
Energy transport

$$\nabla = \frac{d\ln T}{d\ln P} = \min(\nabla_{\mathrm{ad}}, \nabla_{\mathrm{rad}}) \qquad \nabla_{\mathrm{rad}} = \frac{3}{64\pi\sigma G}\frac{\kappa l P}{T^4 m}$$

Additional energy source:
-impacting planetesimals
-deuterium burning
-radiogenic heating

Gas accretion rate given by ability to radiate away energy (T_{KH})

Gas accretion rate in runaway/detached phase (M_{core}>~10 M_E)

Accretion rate in the disk
(flow of gas usually towards the star)

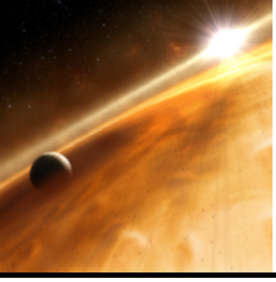
$$\dot{M}_{disk} = 3\pi\tilde{\nu}\Sigma + 6\pi r\frac{\partial\tilde{\nu}\Sigma}{\partial r}$$

Planet cannot accrete more than disk gives

$$\frac{dM_{XY}}{dt} = Min\left[\frac{dM_{struct}}{dt}, k_{\mathrm{Lub}}\dot{M}_{disk}\right]$$

Local reservoir can be accreted at Bondi rate

No external cut-off



Envelope structure: boundary conditions

1. Attached phase

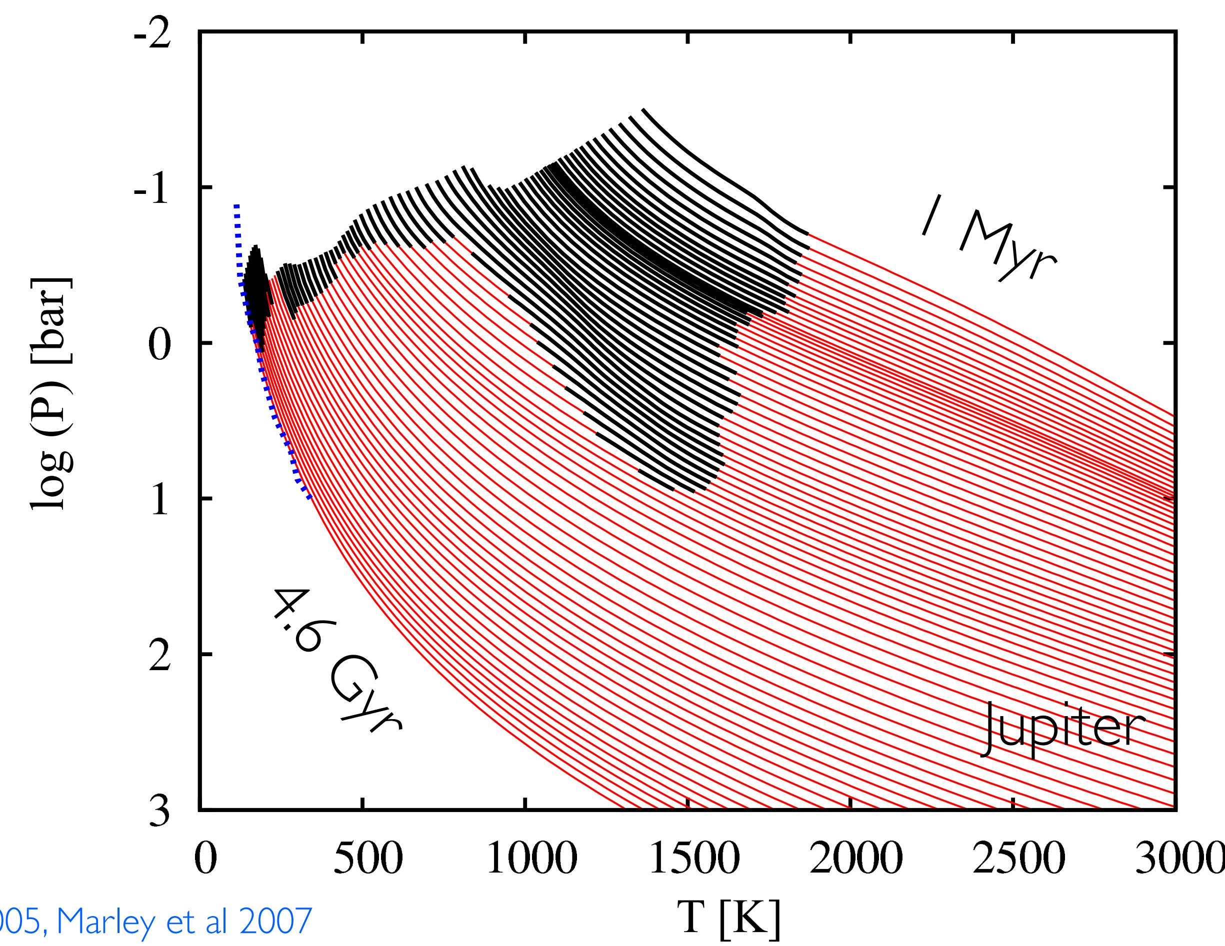
- low mass planets (M_{core}< ca.10-20 M_{Earth})
- pre gas runaway accretion
- structure goes smoothly to Hill or accretion radius
- boundary conditions: background nebula

2. Detached phase

- gas runaway accretion (high mass planets)
- structure has a free outer radius
- rapid collapse of radius from R_{Hills} to ~2 R_J
- upper boundary: accretion shock

3. Thermodynamic evolution M=cst

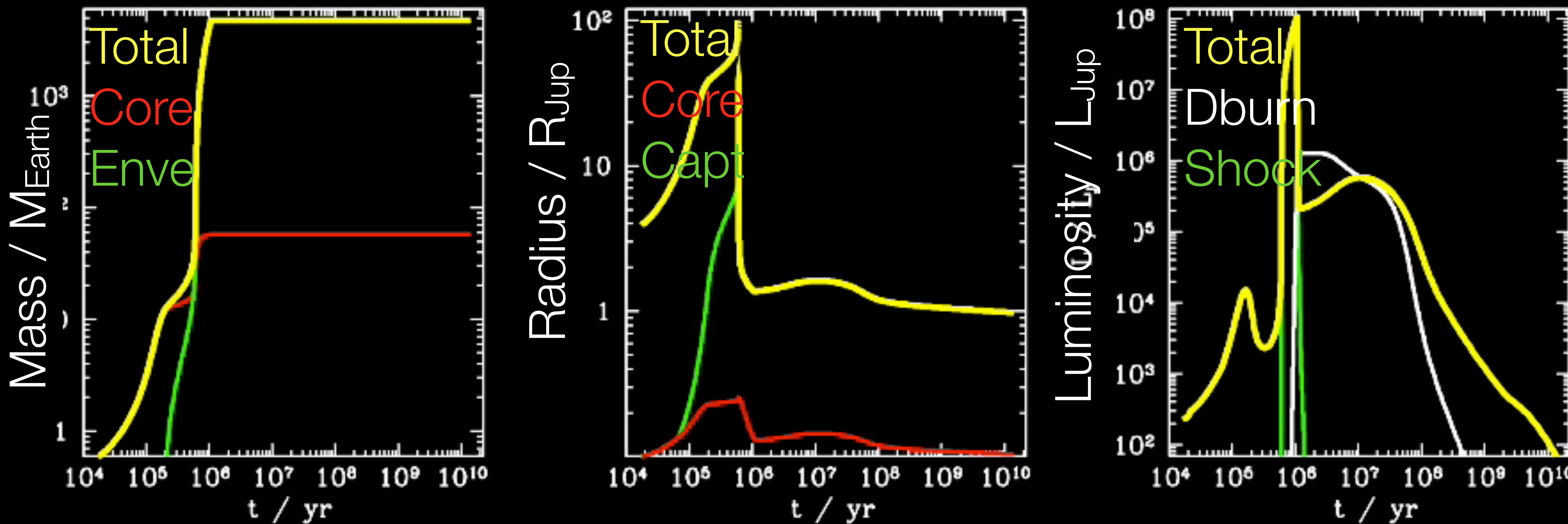
- Eddington approximation (gray atmosphere
- After disk dispersion



Pollack et al. 1996, Papaloizou & Nelson 2005, Stahler et al. 1980, Bodenheimer et al. 2000, Guillot 2005, Marley et al 2007

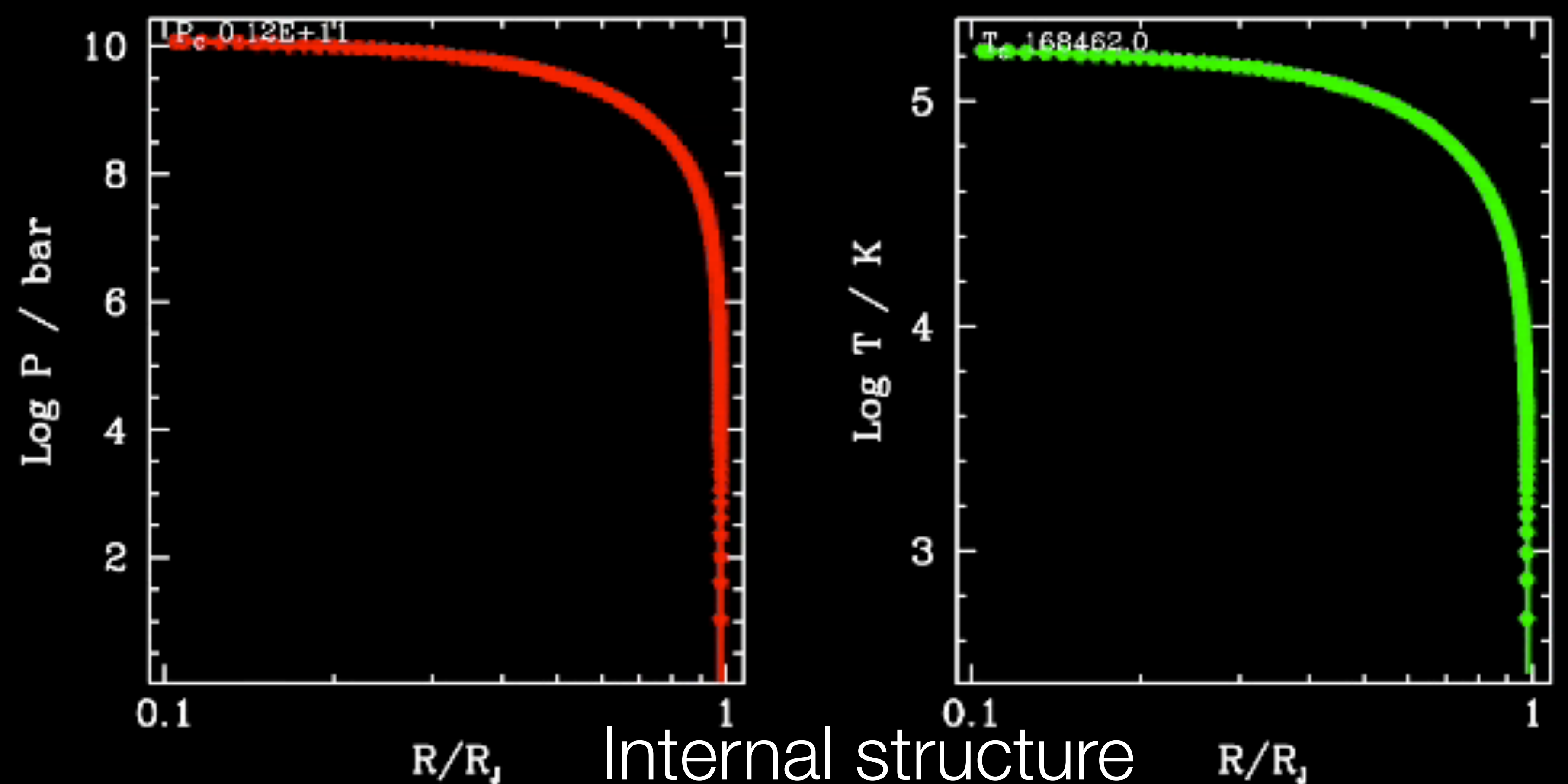
Time 1.39E+10 yrs

Temporal Evolution

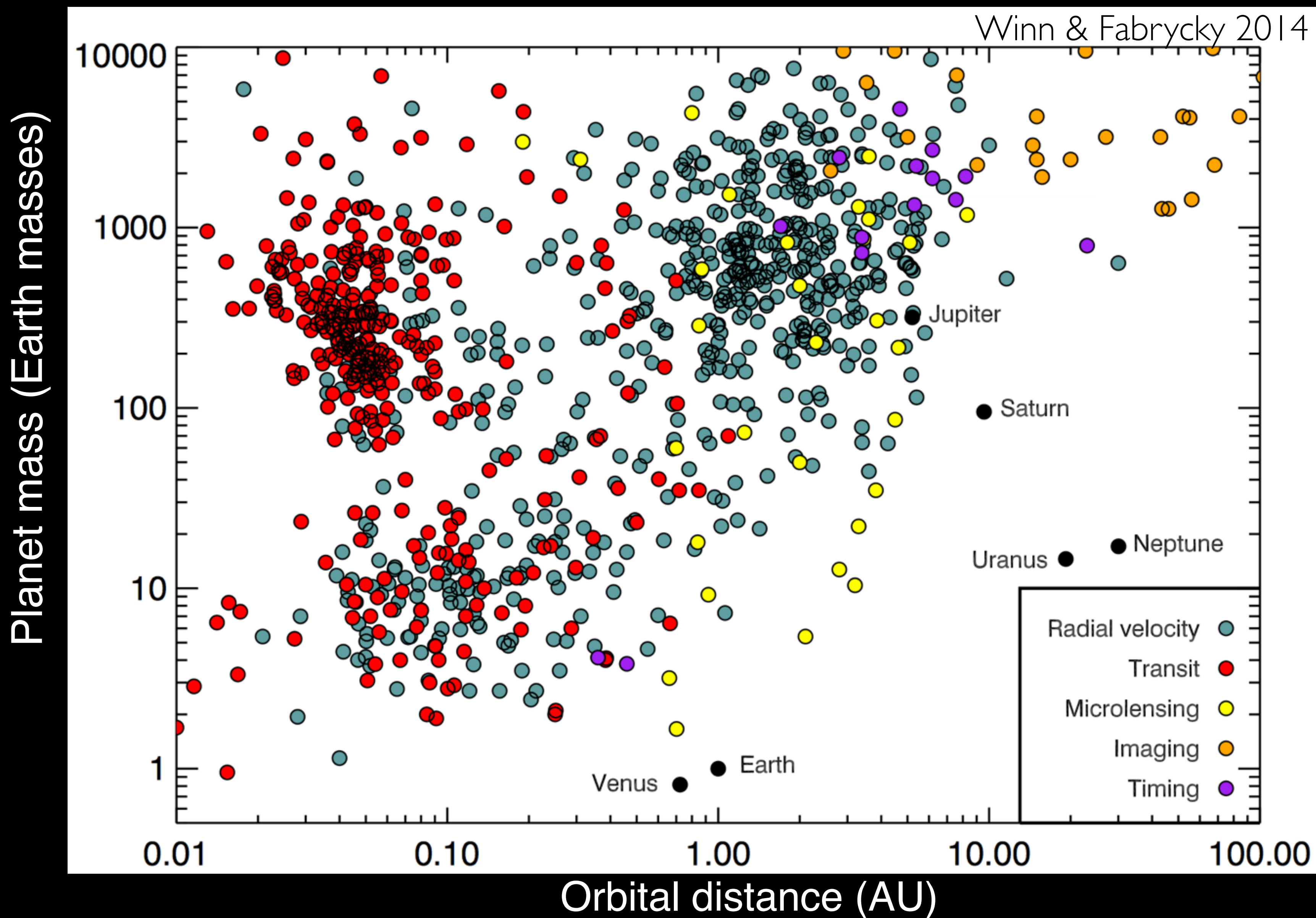


Numerical Data

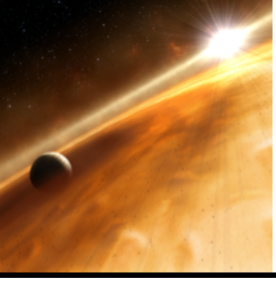
$M_{tot}/M_{\oplus}$	4753.655
$M_{core}/M_{\oplus}$	57.746
$M_{env}/M_{\oplus}$	4695.909
R/R_J	0.98
L/L_J	6.82E+01



Internal structure



3.
Statistical results on masses



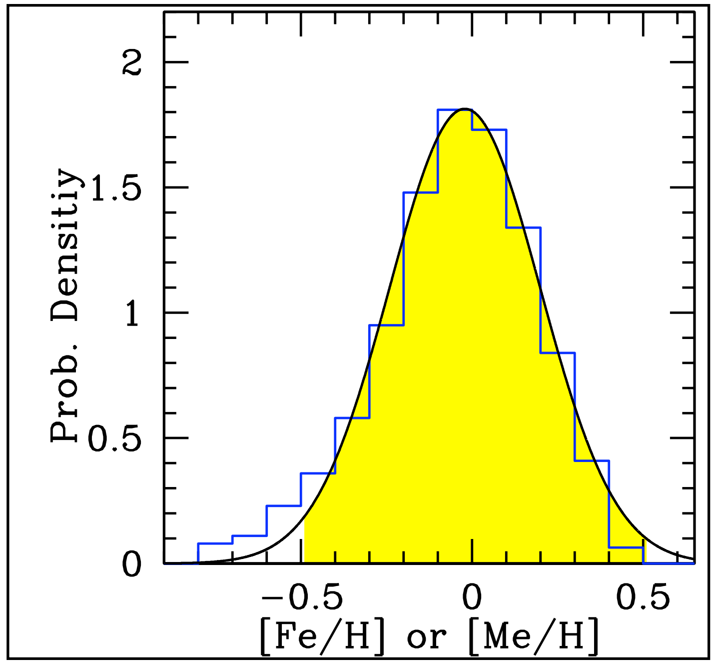
3 Monte Carlo initial conditions

Mordasini et al. 2009

1 Metallicity

assume same in star and disk

Stellar [Fe/H] from spectroscopy. Gaussian distribution for [Fe/H] with μ ~0.0, σ~ 0.2. (e.g. Santos et al. 2003)

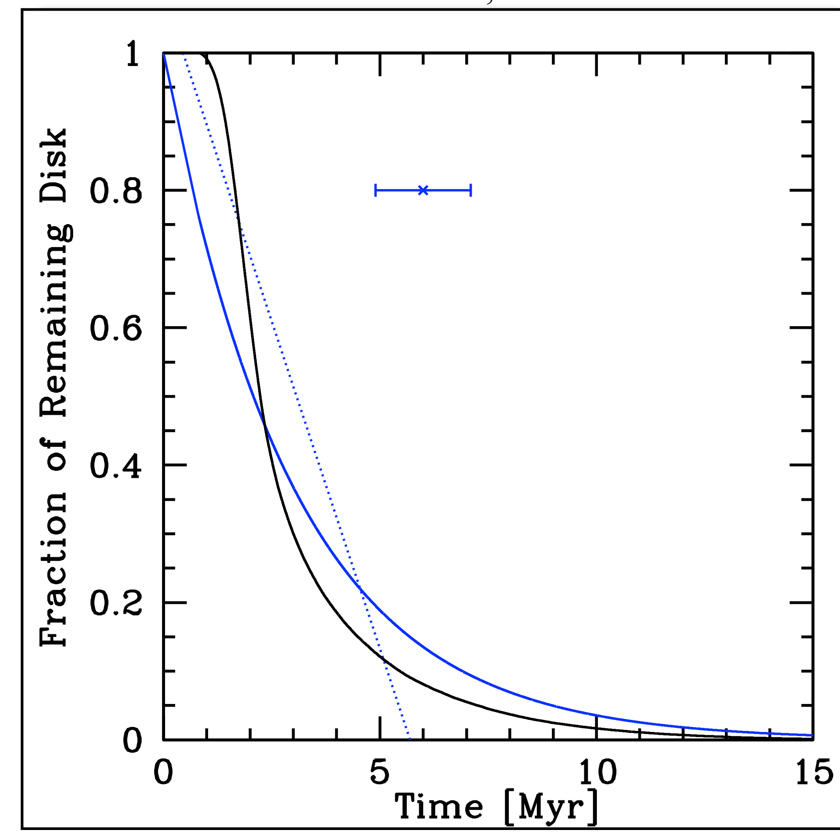


3 Disk lifetime

IR excess

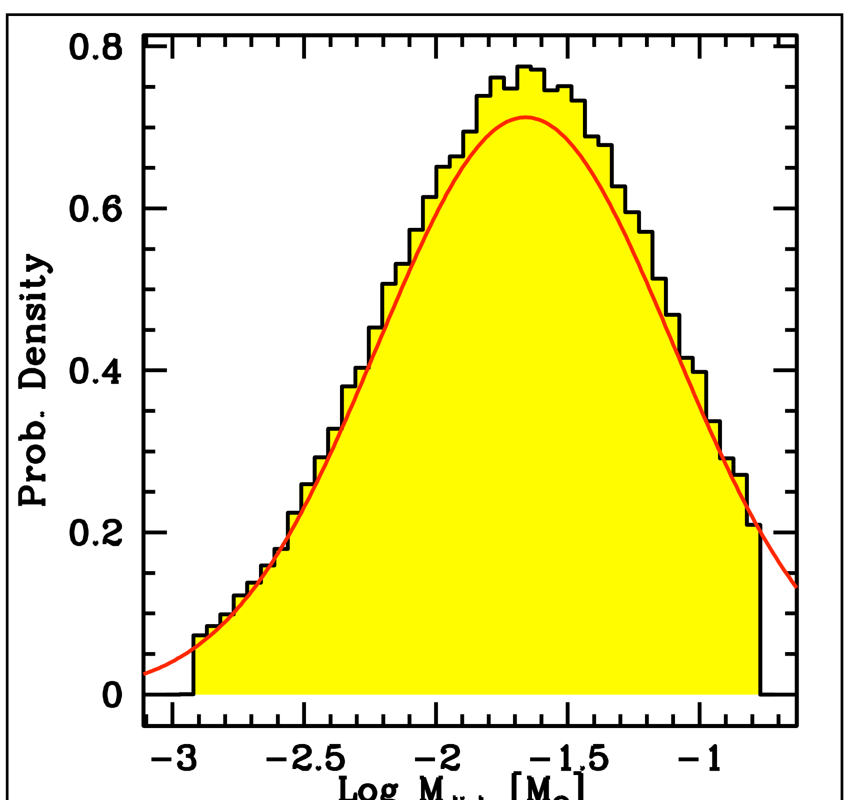
vary lifetime via photoevaporation rate

Haisch et al. 2001, Fedele et al. 2010



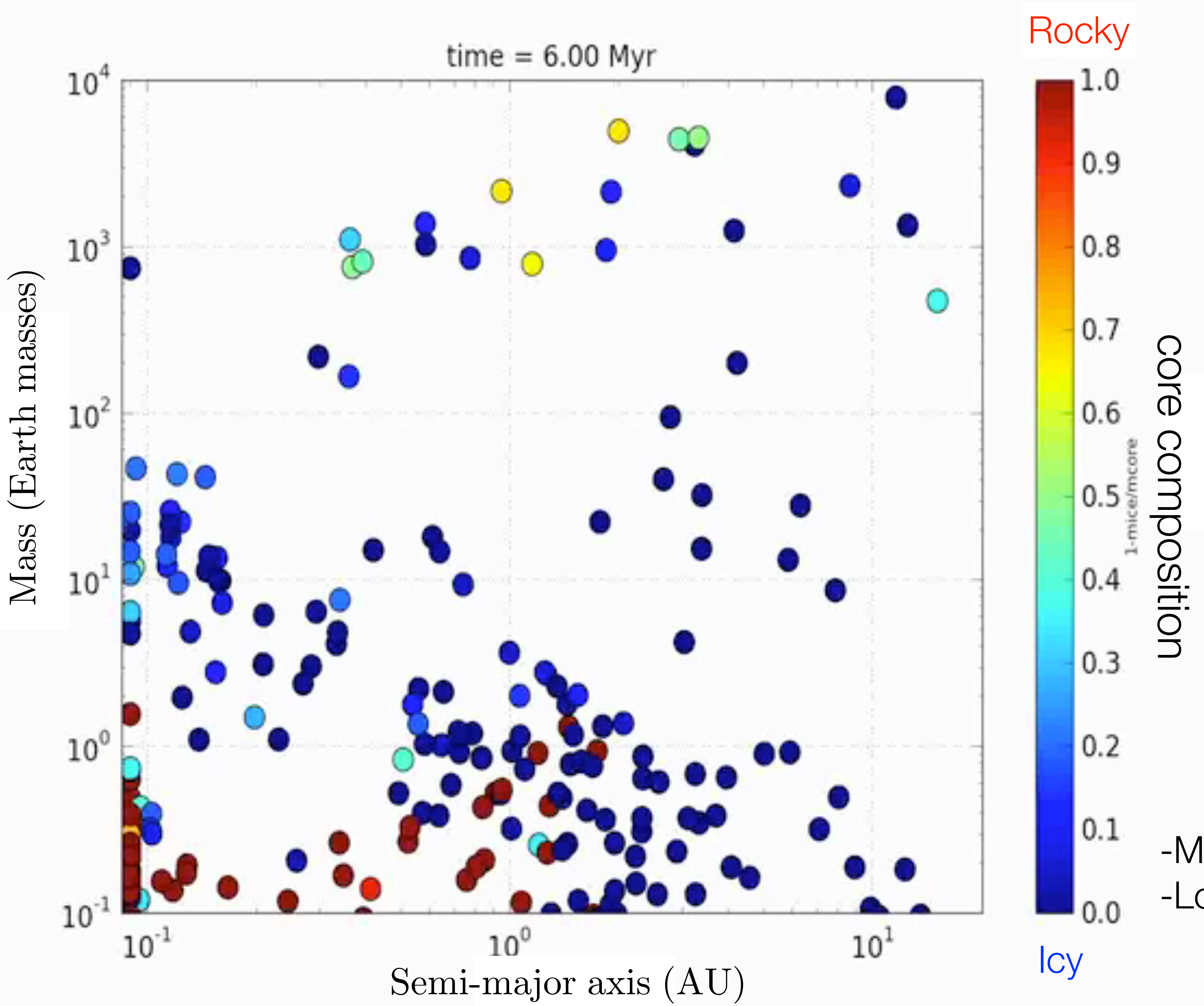
2 Disk (gas) masses

Thermal continuum emission from cold dust at mm and submm wavelengths (Ophiuchus nebula).



Draw initial conditions in Monte Carlo way to calculate synthetic population

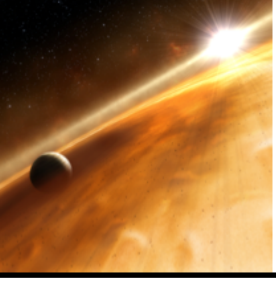
Formation tracks



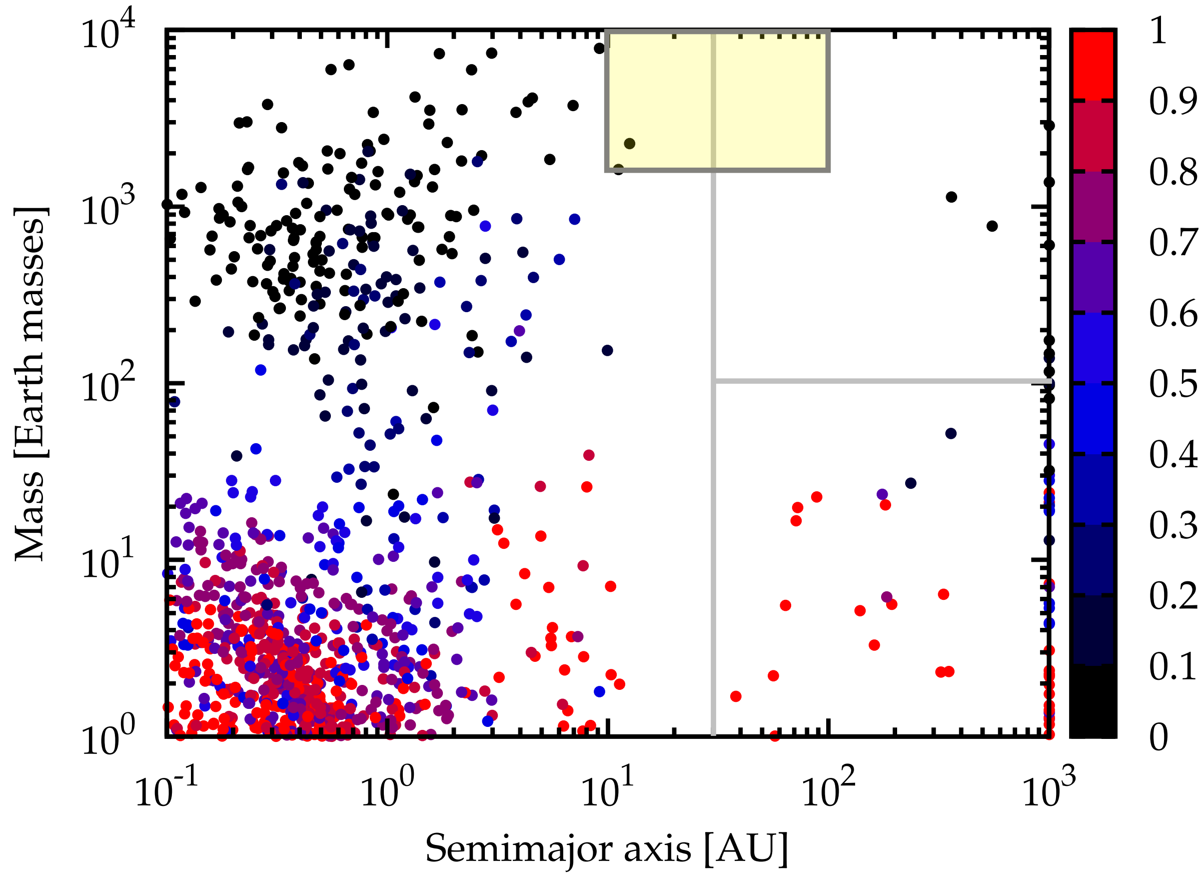
- 10 Embryos/disk
- $M_{ini} = 0.01\ M_\oplus$
- Full N-body
- $M_{star} = 1\ M_\odot$
- Migration (Type I and II)
- $\alpha = 2 \times 10^{-3}$

-Mostly compact systems
-Low eccentricities

Alibert et al. 2013



Planets at large distances

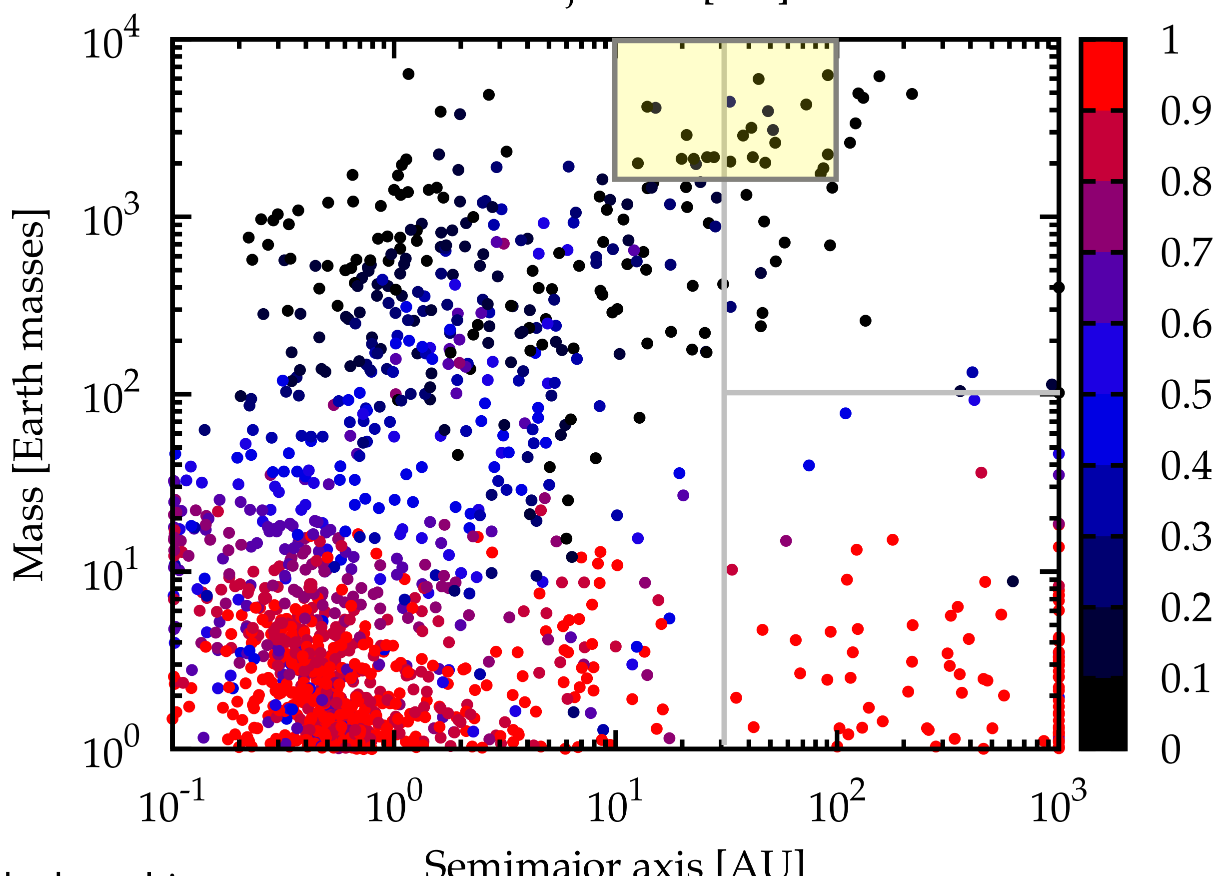


Fraction of stars with scattered or ejected planets (a>30 AU)

$\tau_{e,i}$=0.1 τ_{mig} — 500 stars with 10 embryos

- 2 far giant planets 0.4 %
- 7 ejected giant planets 1.4 %
- 2 giants: frequency **0.4 %**

- 18 far low-mass planets 3.6%
- 33 ejected low-mass planets 6.6 %



longer $\tau_{e,i}$ — 659 stars with 10 embryos

(Cresswell & Nelson 2008)

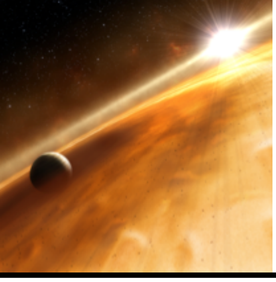
- 38 far giant planets 5.8 %
- 24 giants: frequency **3.6 %**
- 2 ejected giant planets 0.5 %

Brandt et al. 2014:
5-70 MJ
10-100 AU
1.0 - 3.1 %

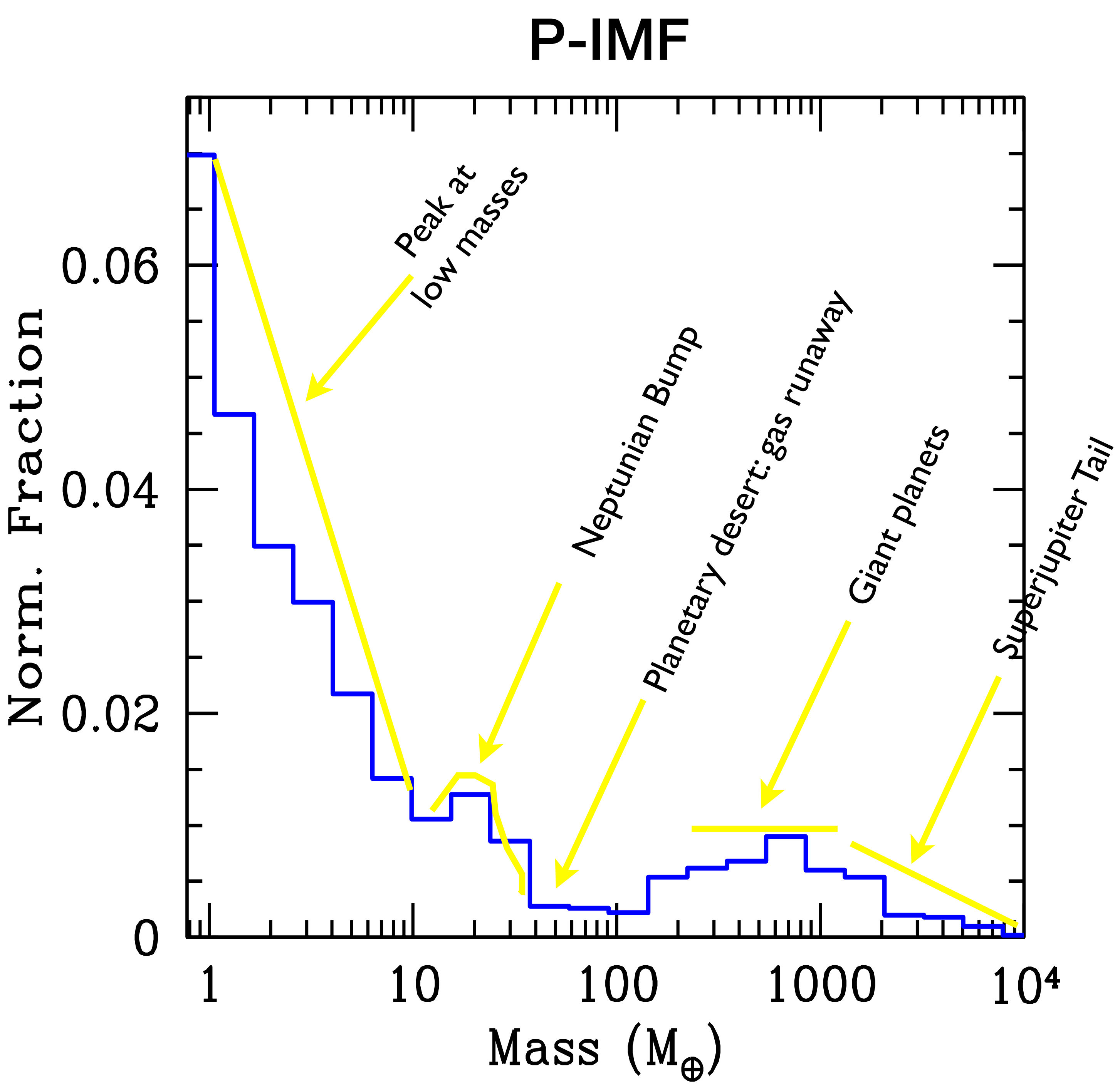
- 48 far low-mass planets 7.3 %
- 46 ejected low-mass planets 7.0 %

Swoboda et al. in prep

$M_{star} = 1\ M_\odot$



Planetary initial mass function



10 embryos/disk (full N-body), start mass: 0.01 M_{Earth}
M_{star}=1$M_\odot$, full non-isothermal type I, alpha= 2 x10^{-3}

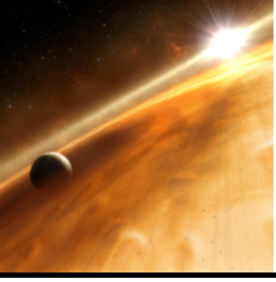
Type	Mass (M	% (of M>1 M
(Super)-Earth	< 7	61
Neptunian	7-30	17
Intermediate	30-100	3
Jovian	100-1000	13
Super-Jupiter	> 1000	5

- *Complex structure, dominated by low mass planets*
- *Consistent w. non-detection of Jupiters around ~90% stars.*

Planets with M < 30 M_{Earth} : over 75% of all planets

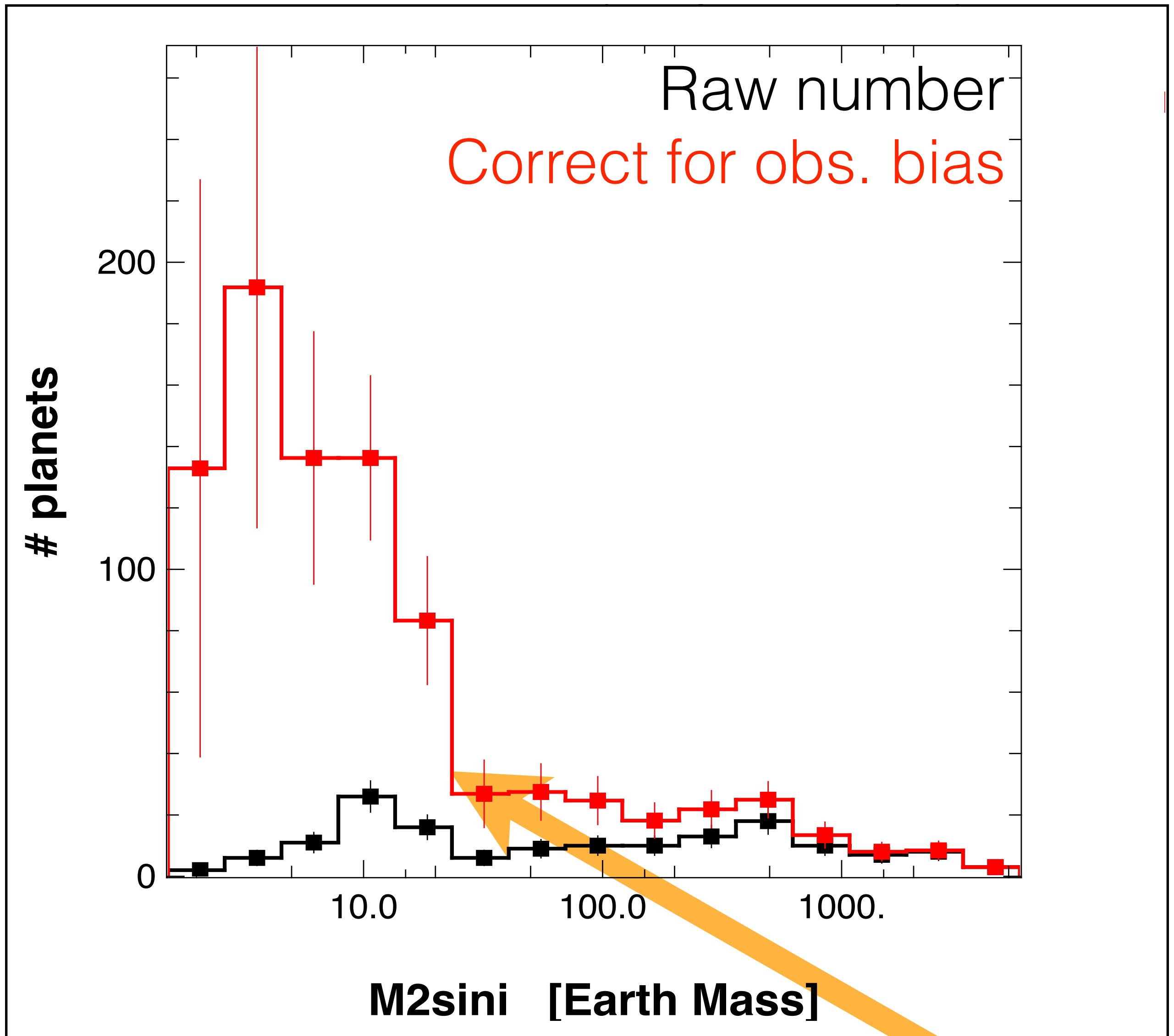
Giant planets = tip of the iceberg

Benz et al. 2014



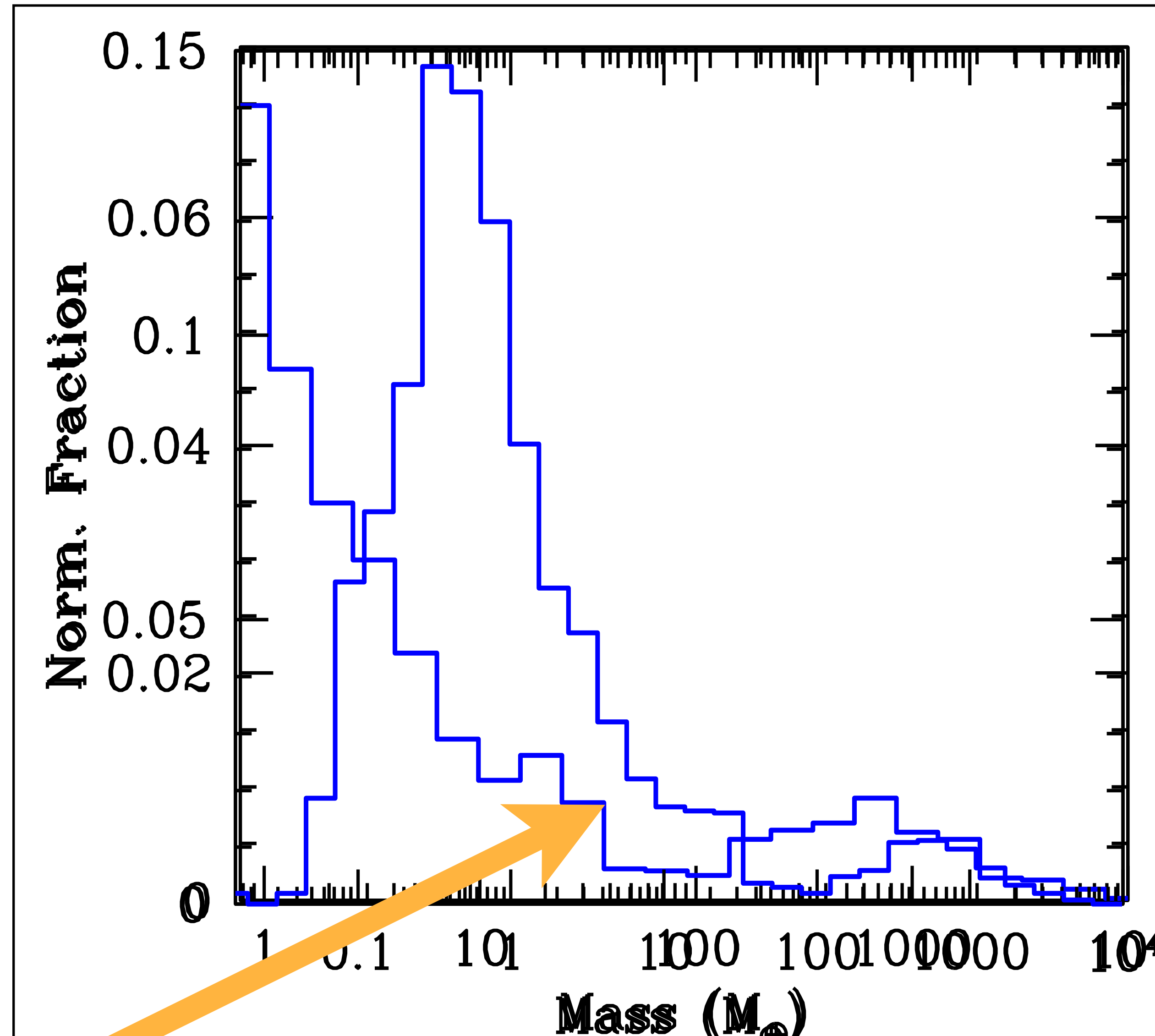
Comparison with observations

Observations



Mayor et al. 2011

Synthetic

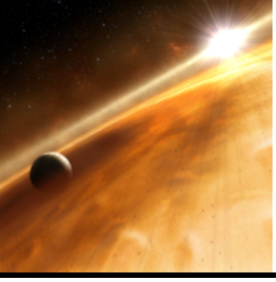


Benz et al. 2014

Sudden increase

Typical for core accretion. Constraint on M_{crit} & gas accretion rate

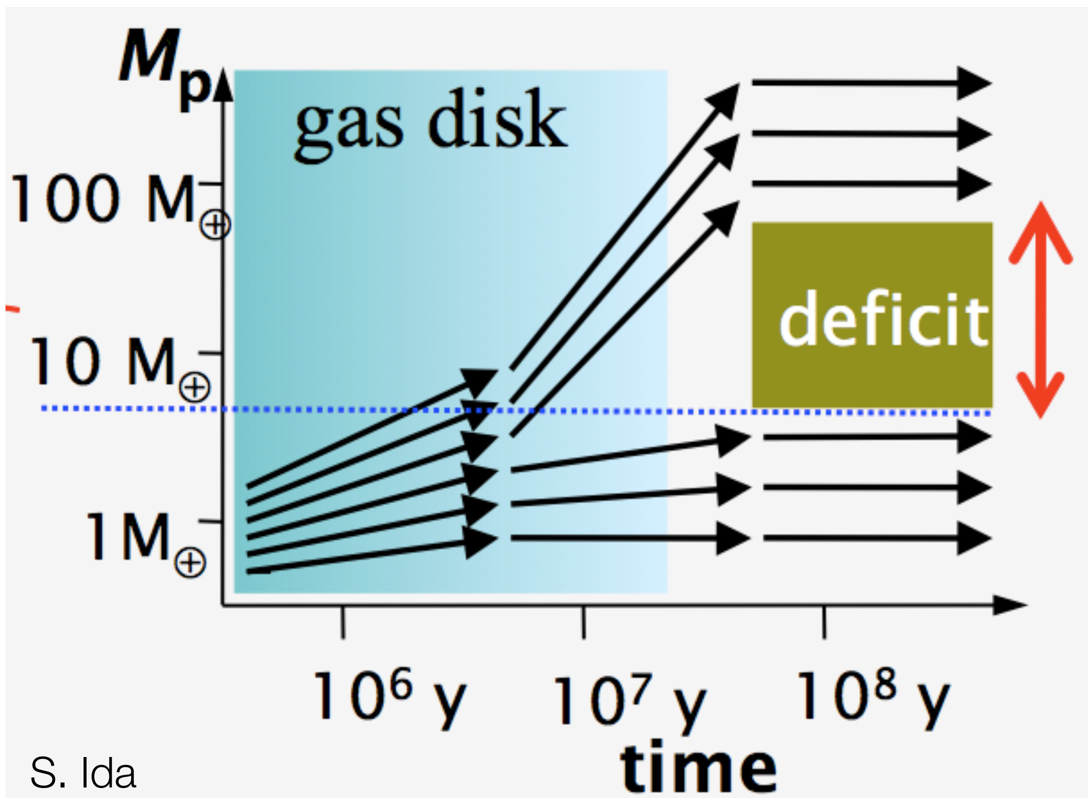
Many low-mass planet - much remains to be discovered

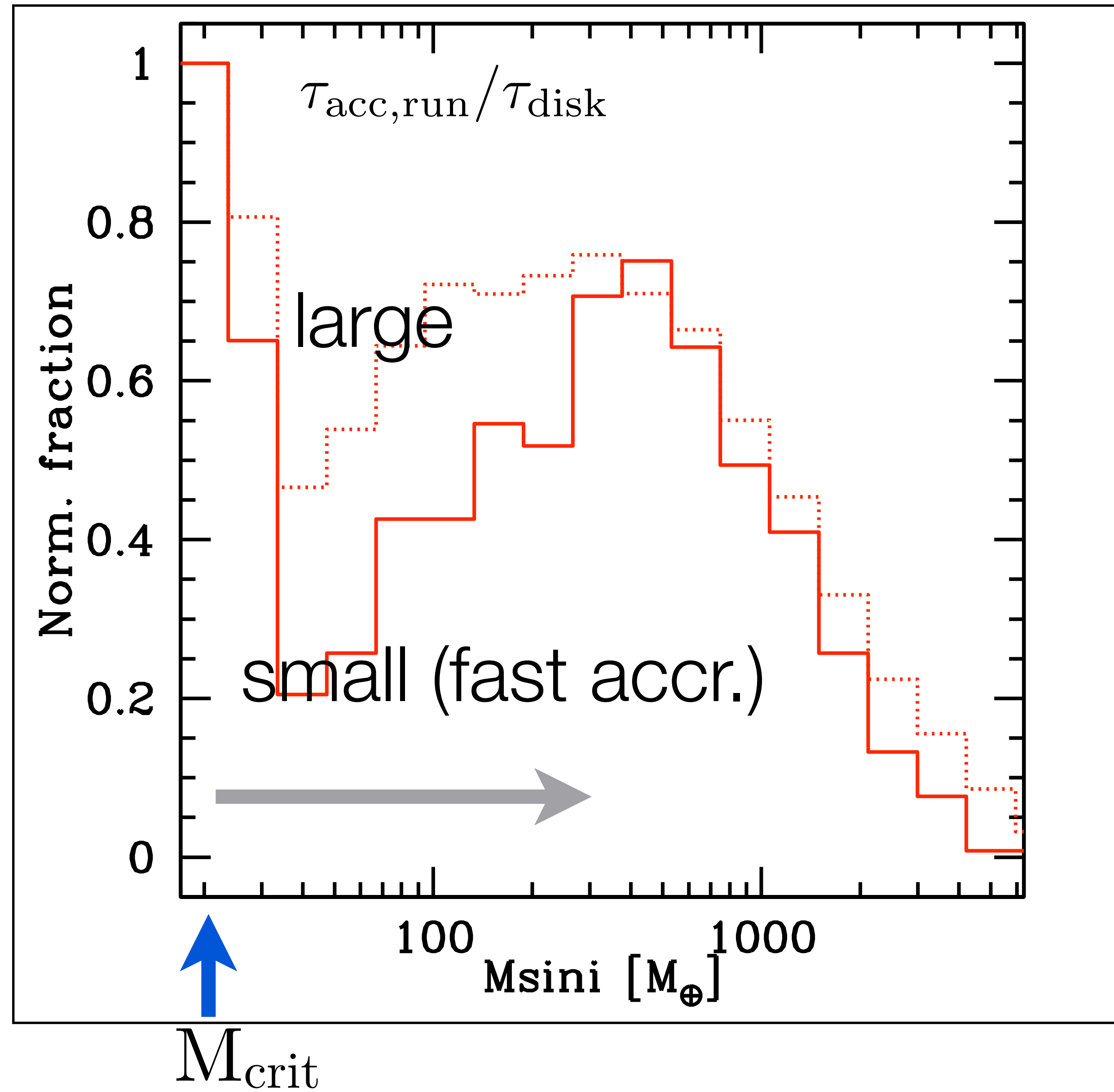


Constraints in the P-IMF: transition

M_{crit}: depends on luminosity, opacity and gas composition ~5-15 M_E

Once M_{crit} is reached, rapid gas accretion begins.

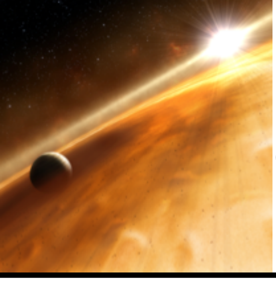




If $\tau_{acc,run}/\tau_{disk}$ << 1, a "planetary desert" can form.

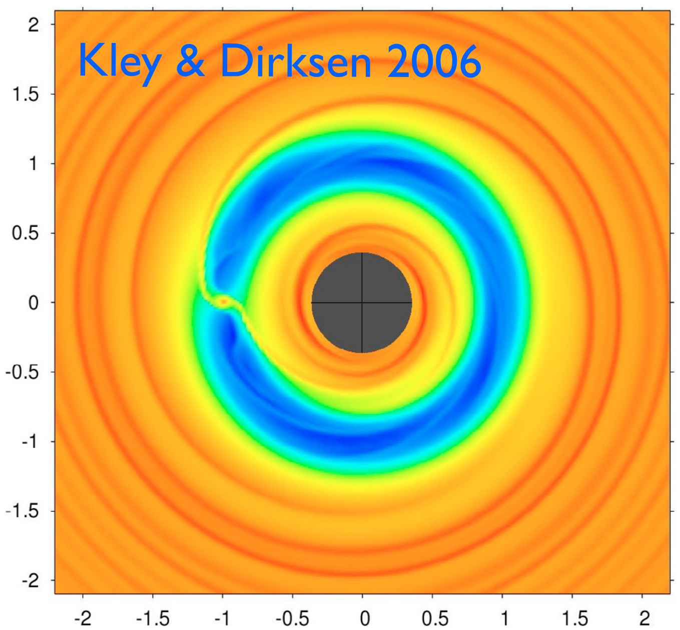
Depending on $\tau_{acc,run}/\tau_{disk}$ P-IMF slope can be positive, flat, negative.

Controlled by: local gas mass, viscous transport, Bondi rate, T_{acc}~T_{KH} (M)

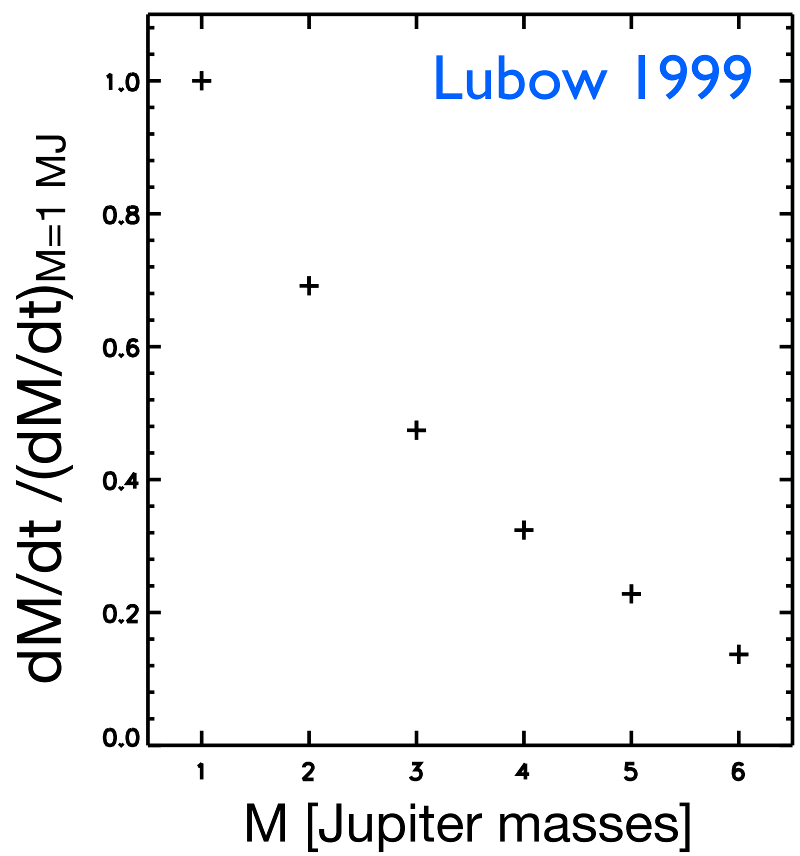


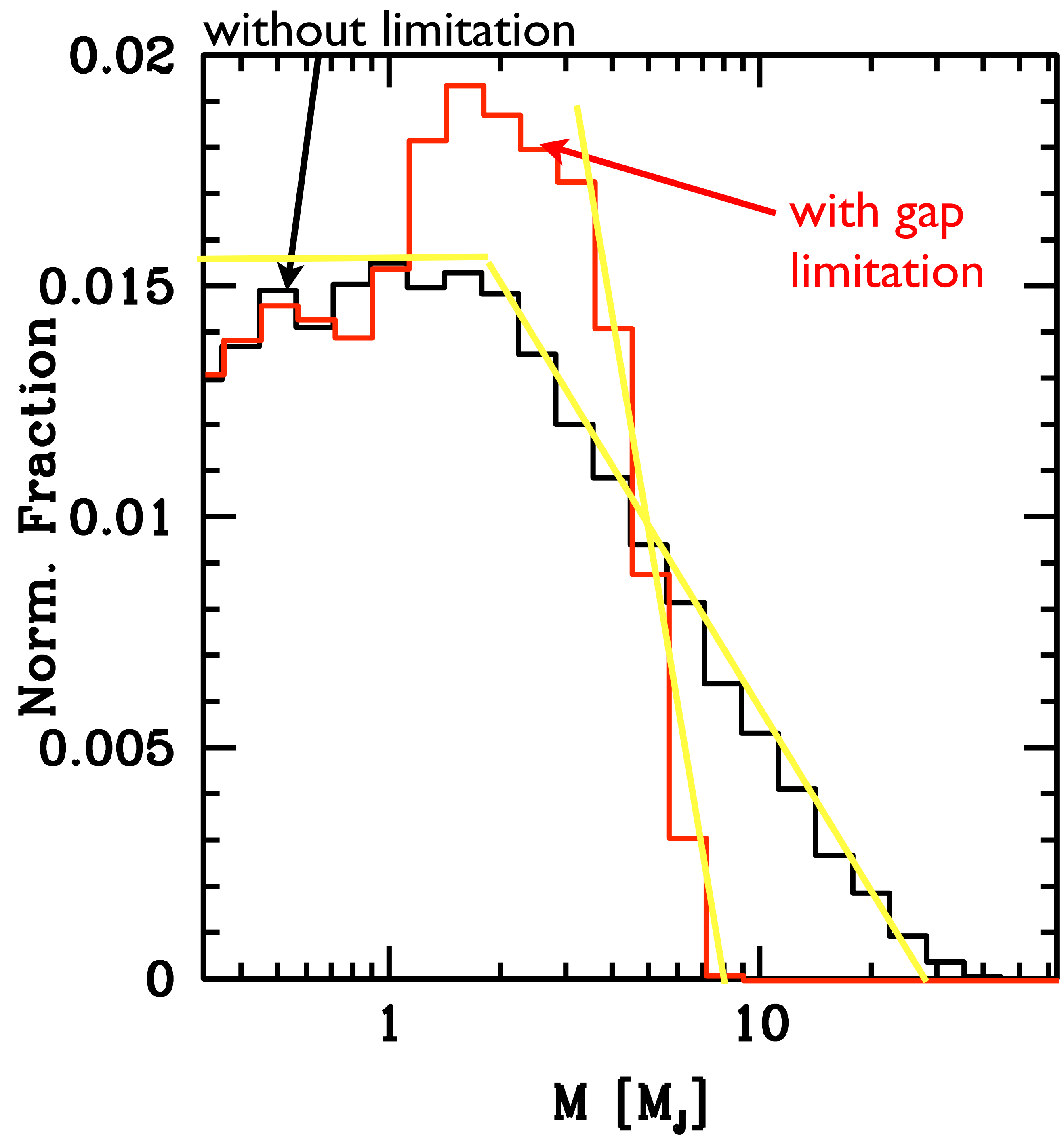
Constraints in the P-IMF: upper end

Upper end of the P-IMF: controlled by disk mass & lifetime distributions, and gap formation.

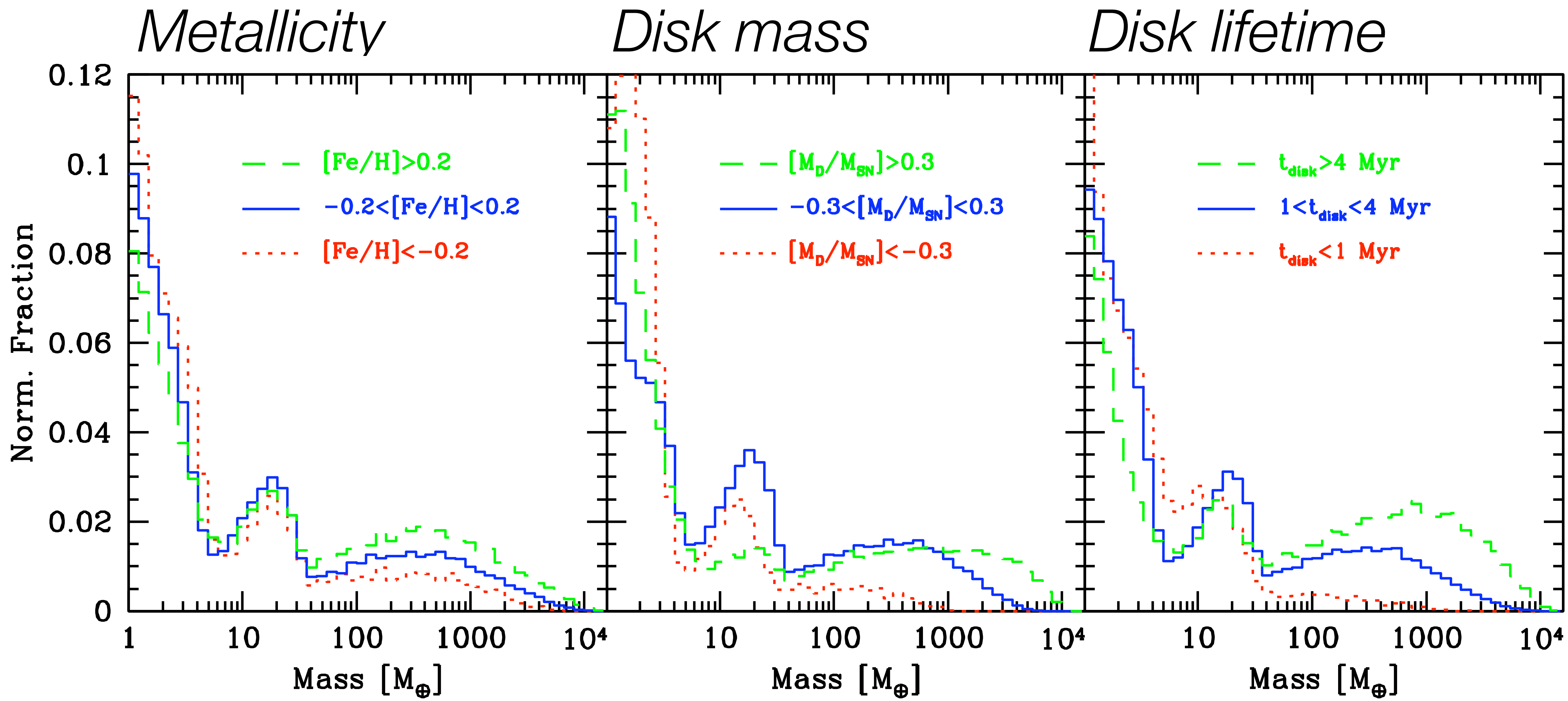


Gap:
Auto-regulatior
of maximal
planetary
masses?





P-IMF: impact of disk properties



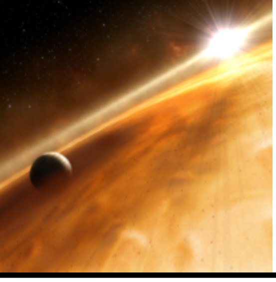
- higher number of giants
- but not more massive
- Threshold mass (M_{crit})

- correlation disk mass giant planet mass

Long living disks: giants

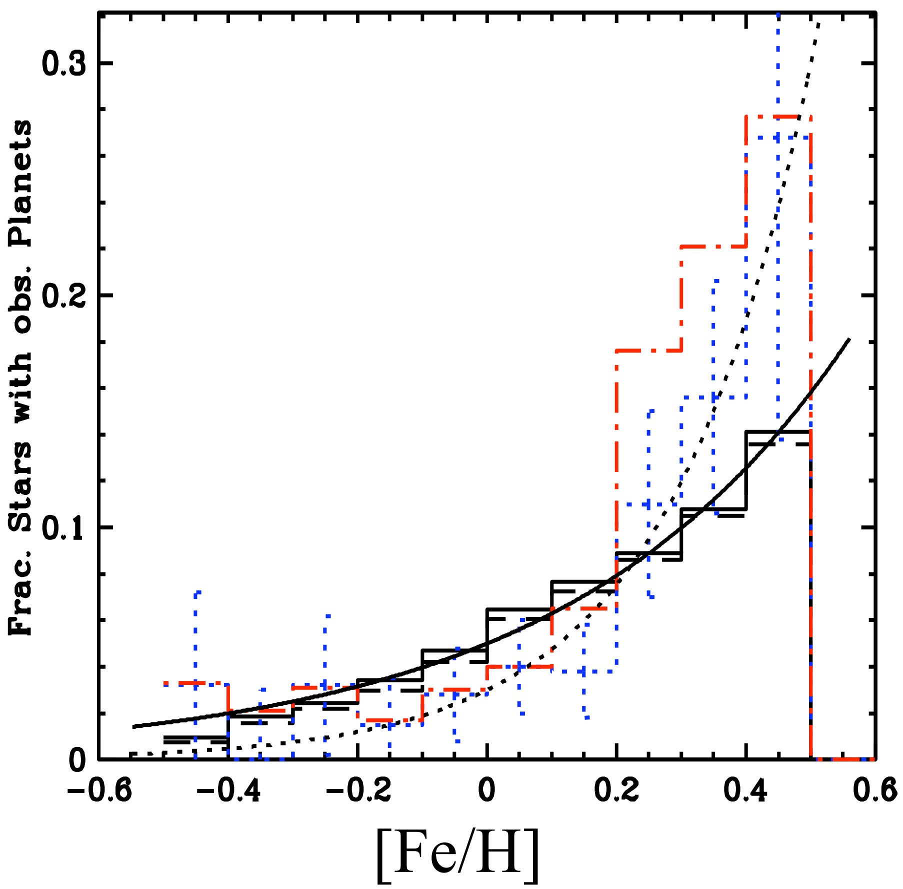
- more numerous and
- higher mass
  - -Correlation with M_D

Mordasini et al. 2012



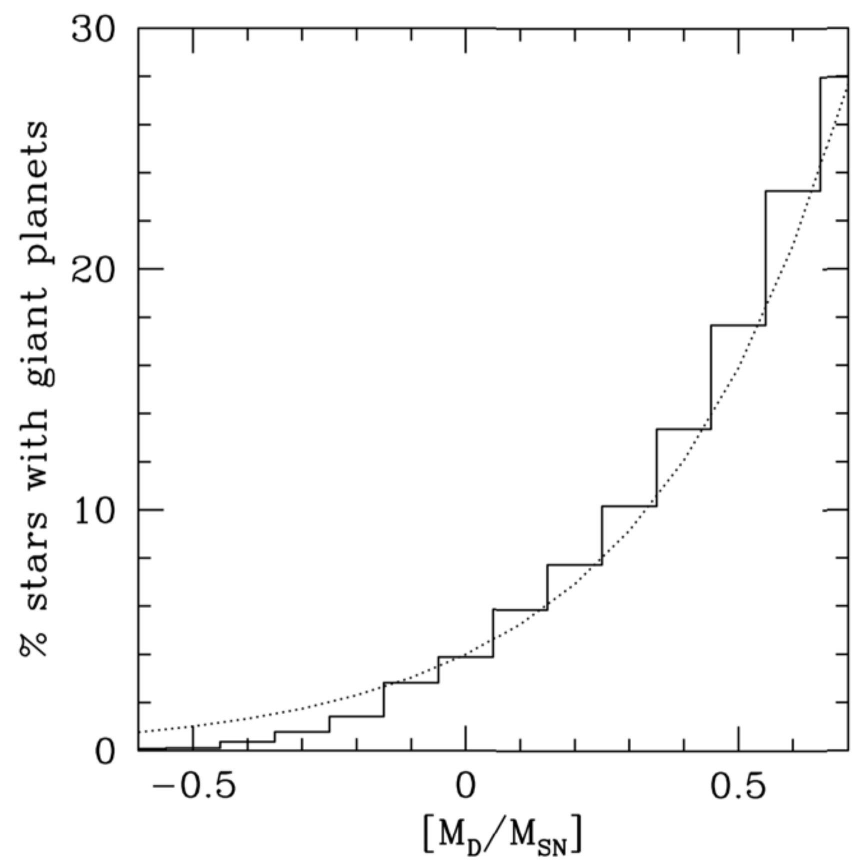
Giant planet frequency

Metallicity



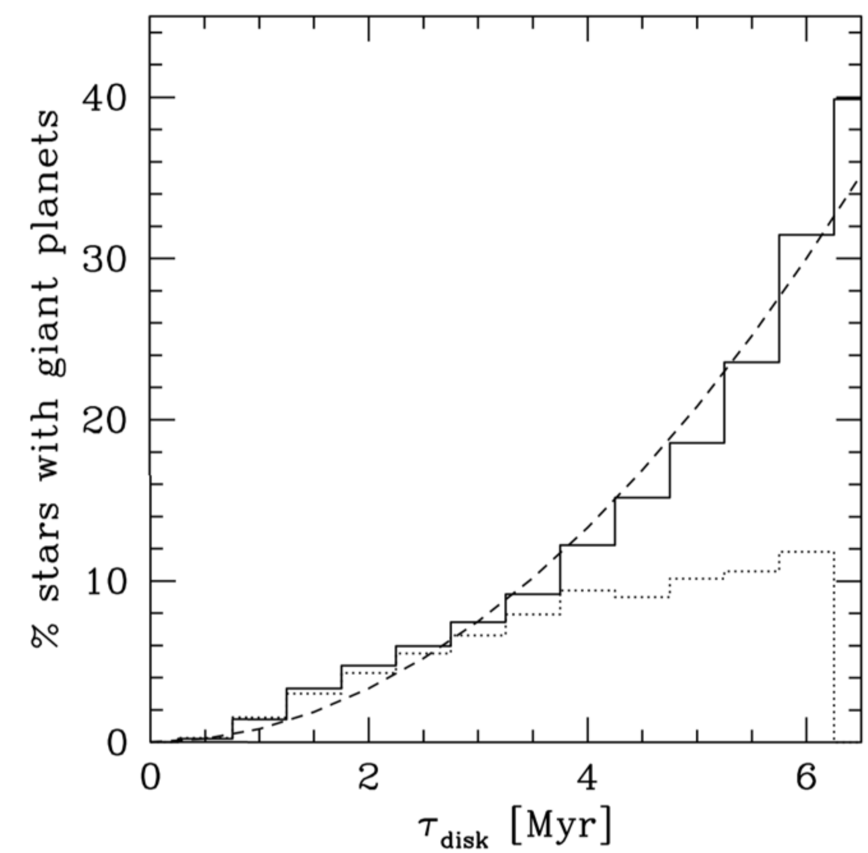
- Trend as observation, but weaker dependency
- Argument in favor of core accretion

Disk mass



- Approximately linear
- 4 x (M_{disk}/0.017 M⊙)$^{1.2}$ %

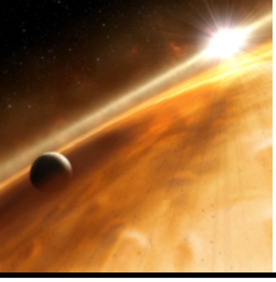
Disk lifetime



- Approximately quadratic
- 8 x (T_{disk}/3 Myr)^2 %

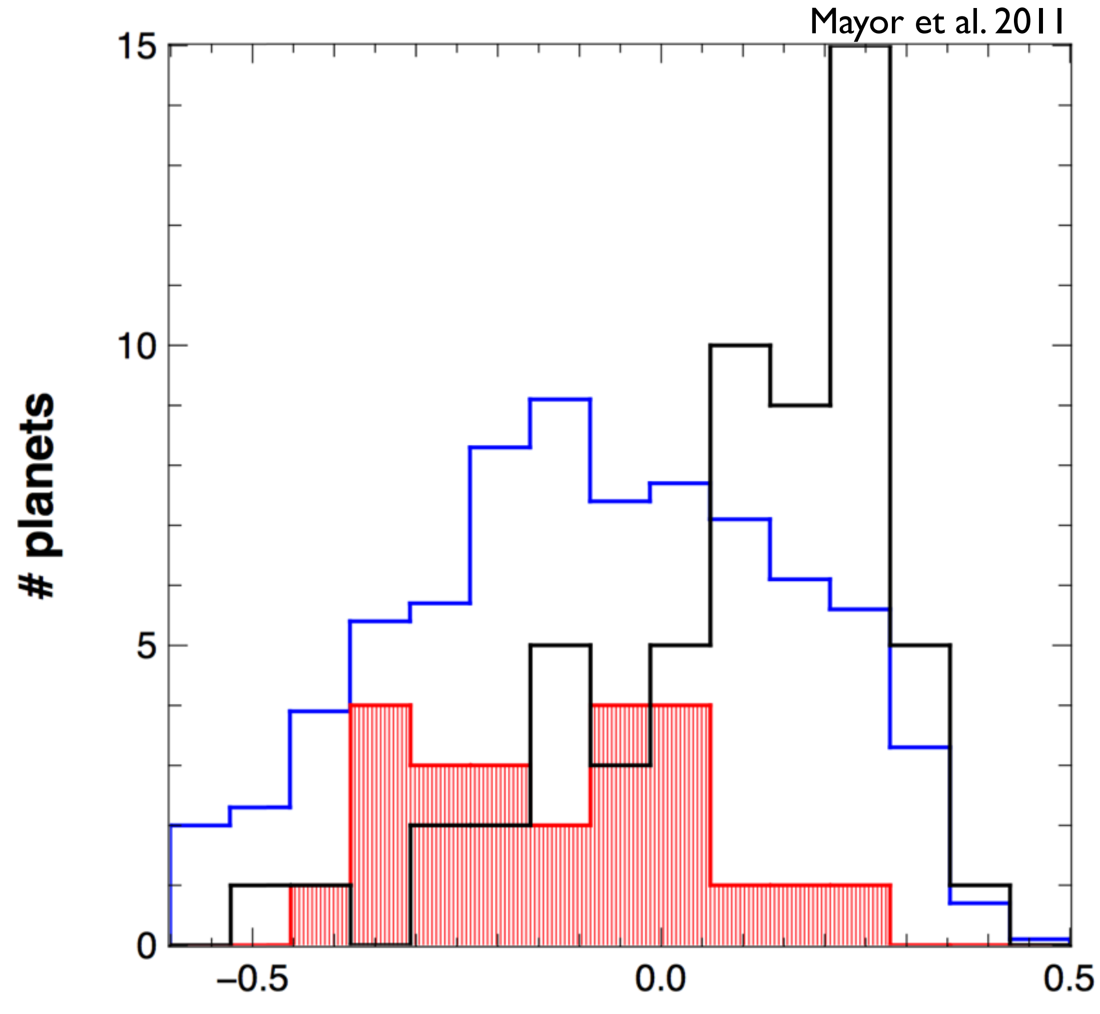
Blue: Observation (Fischer & Valenti 2005)
Red: Observation (Udry & Santos 2007)
Black: Observable synthetic planets

Mordasini et al. 2012



Host star [Fe/H] of giant and low-mass planets

Observations



All stars

Giant planet host stars

Low-mass planet host stars

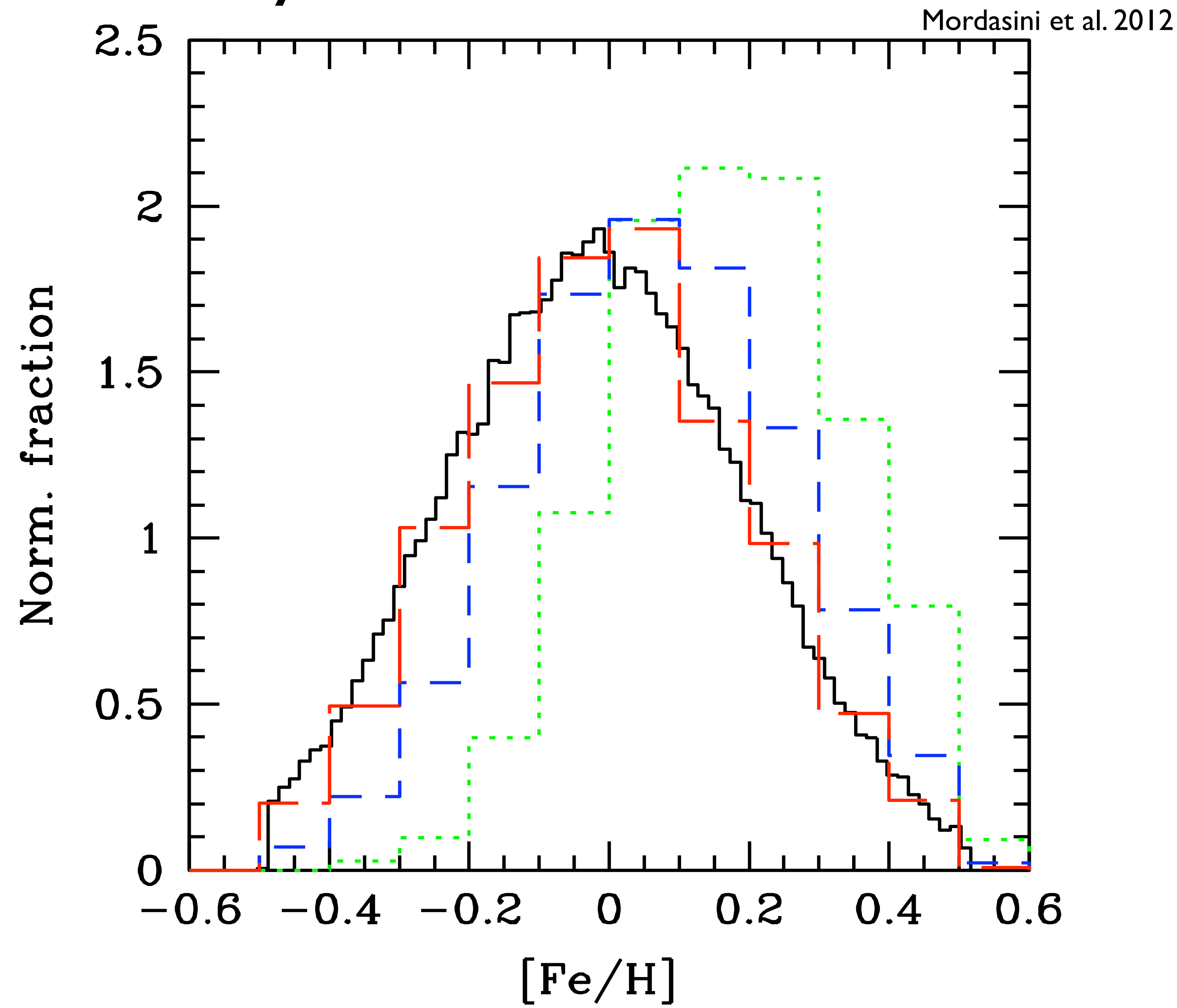
(M<30 M_{Earth})

Giant planet cores need to from early and massive: easier at high [Fe/H]

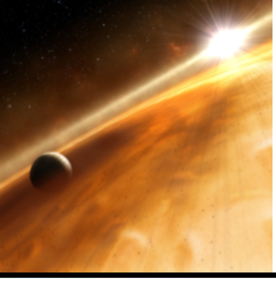
Low-mass planets can also form with lower [Fe/H]

Synthetic

M_{star}=1 $M_{\odot}$ a<0.1 AU



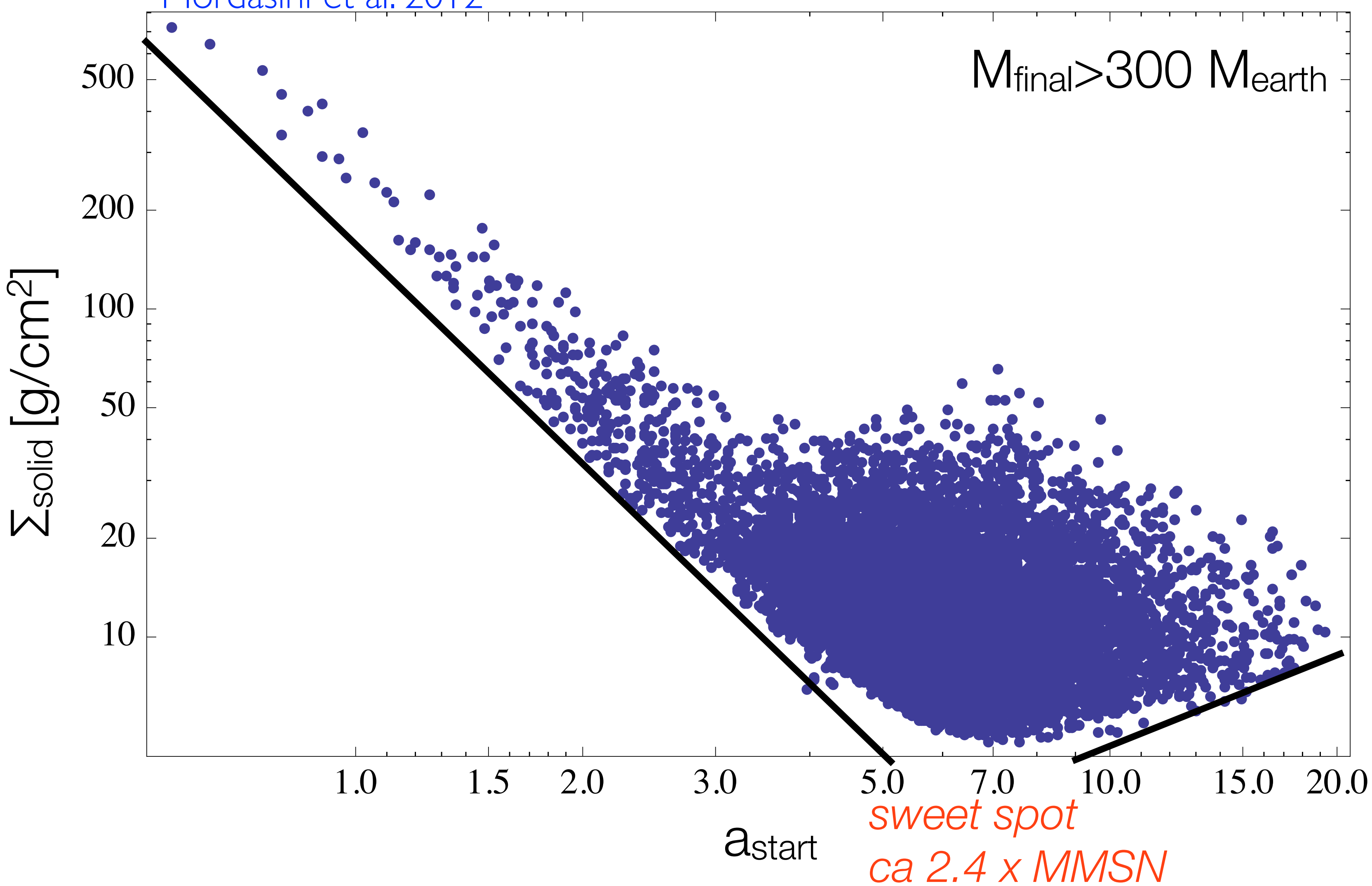
Mean [Fe/H]	$f_I = 0.1$
All planets	-0.02
Hot, $M/M_{\oplus} < 6$	0.00
Hot, $6 < M/M_{\oplus} < 100$	0.06
Hot, $M/M_{\oplus} > 100$	0.17



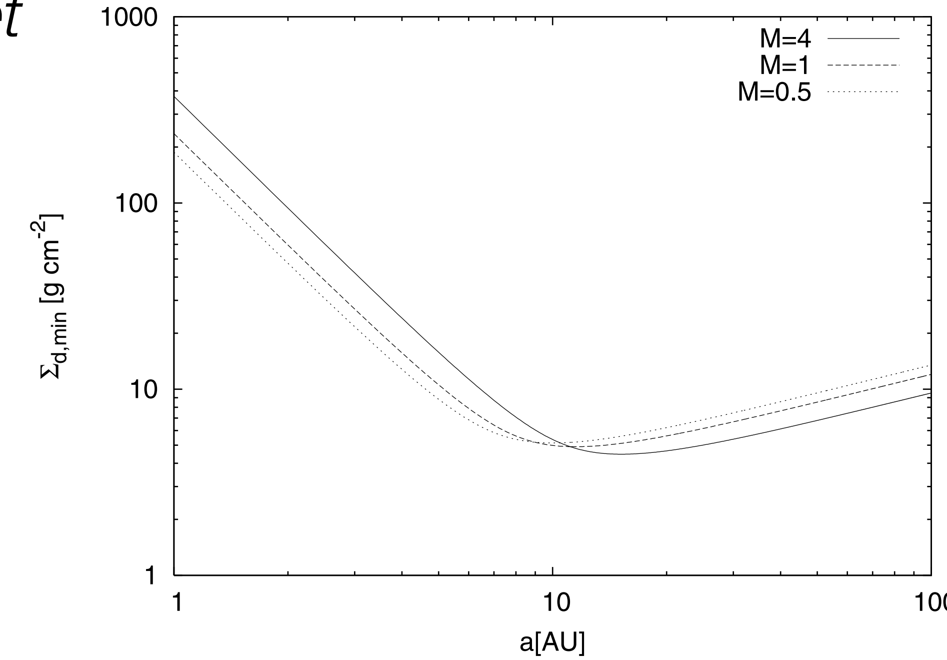
Preconditions for giant planets I

Study a posteriori which initial condition lead to a giant planet

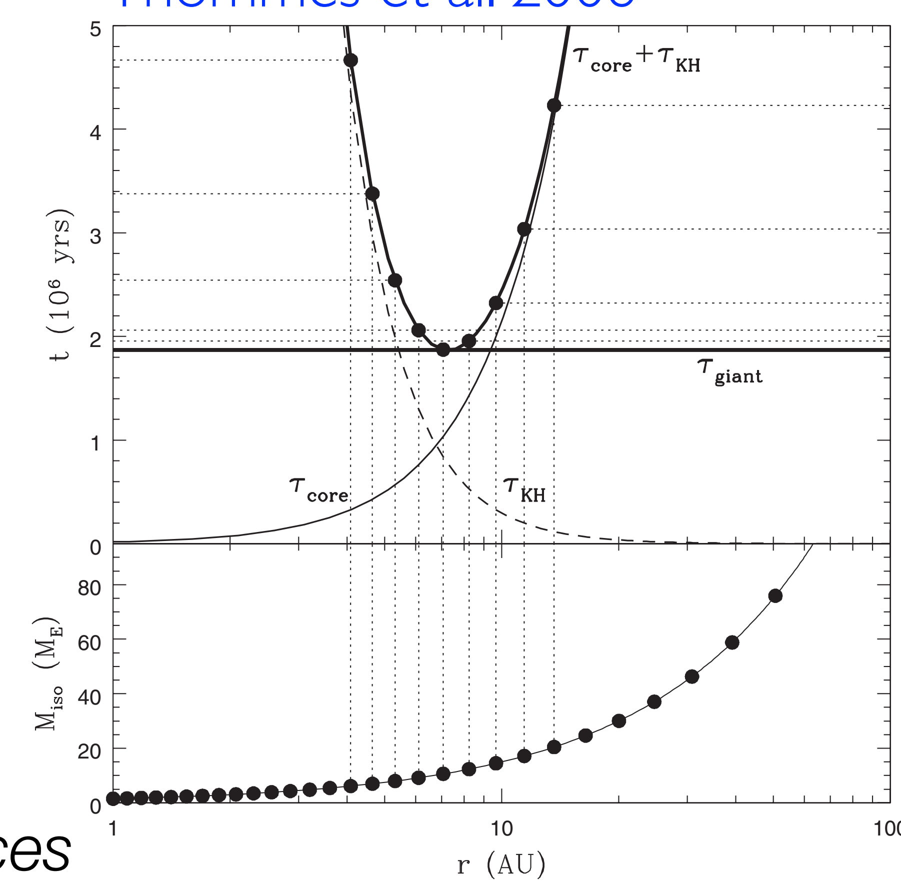
Mordasini et al. 2012



Kornet et al. 2006



Thommes et al. 2008



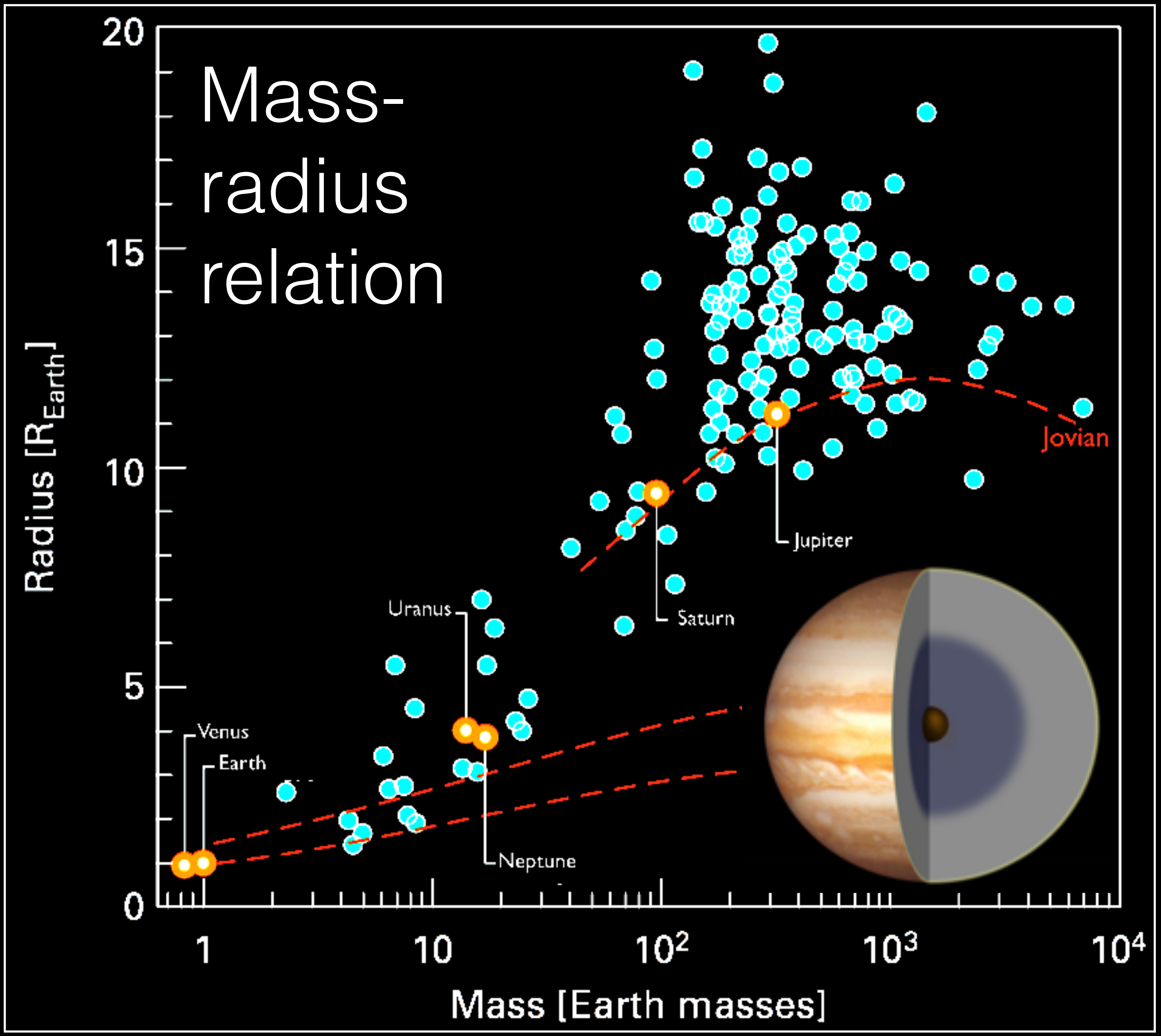
Minimal necessary local planetesimal surface density.

Inside: available mass criterion

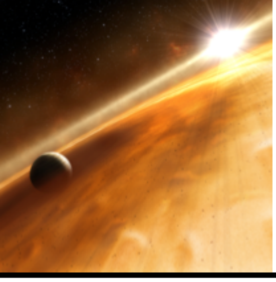
-Migration relaxes the condition somewhat

Outside: timescale criterion

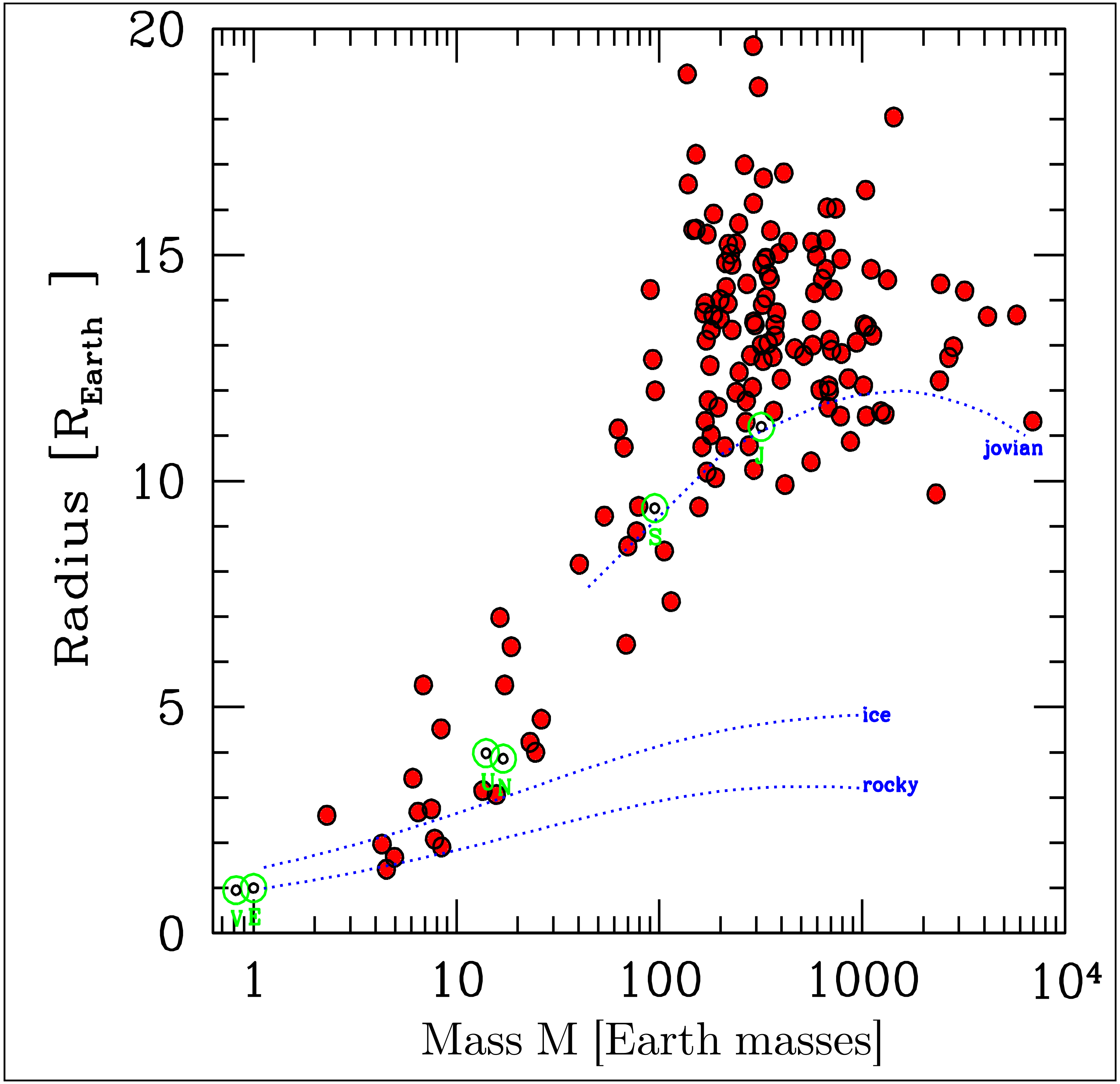
-Only long living disk make giants at low Σ_{solid} at large distances



4.
Statistical results on radii



Mass-radius relation



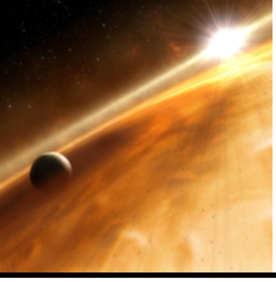
- M-R: First geophys. characterisation: rocky, icy, gaseous

- General trends
- Large diversity
- Inflated giant planets
- Empty regions

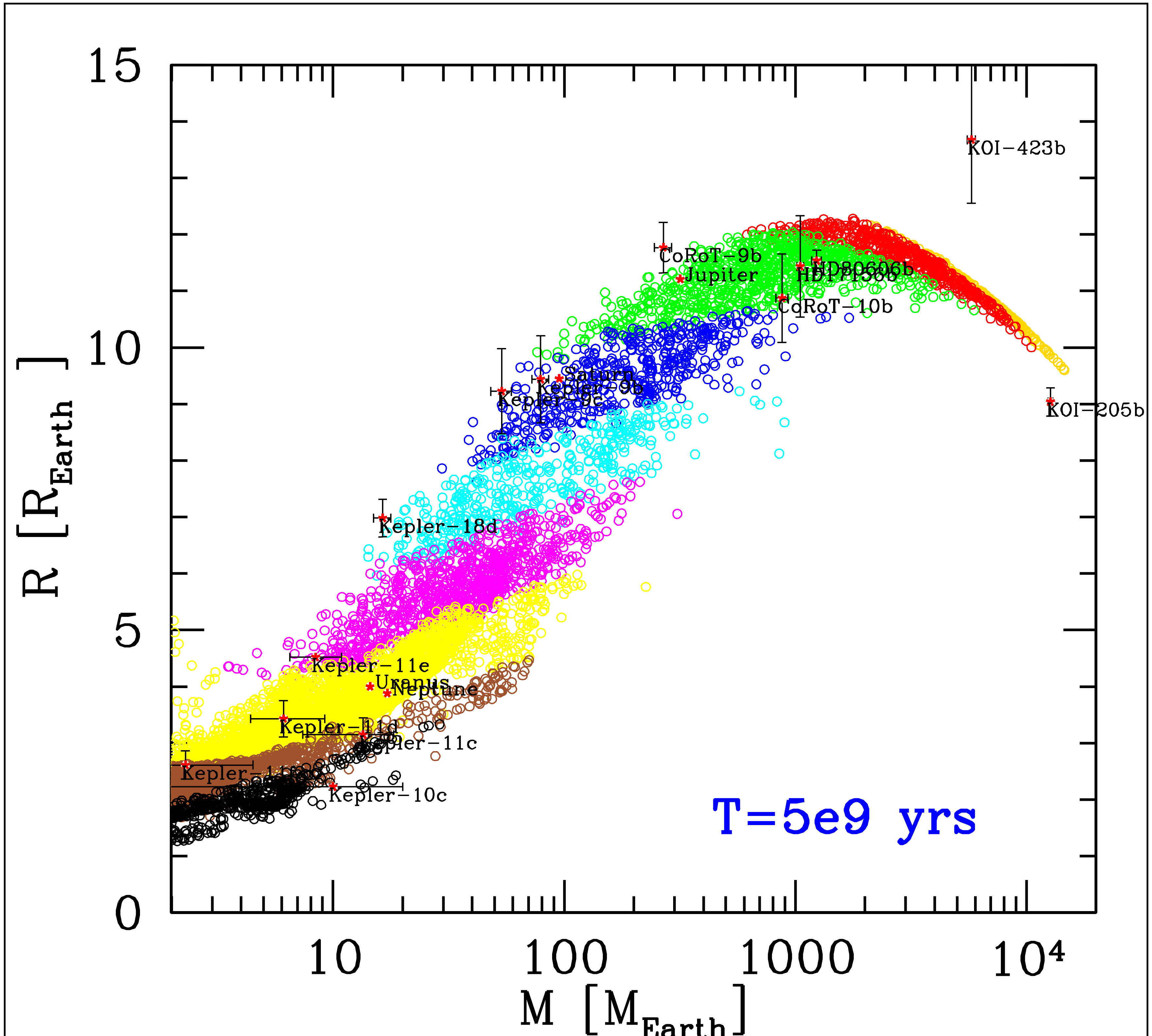
- Understandable with theoretical models?
- Constraints for formation theory beyond the a-M:

Transition solid-gas dominated planets: efficiency of H/He accretion & loss: opacity in protoplanetary atmosphere, atmospheric escape

- Must combine formation and evolution



Formation of the M-R relationship



Mordasini et al. 2012

Fraction Z of solids (rest H/He)

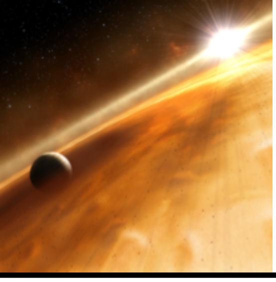
- Orange: $Z \leq 1\%$
- Red: $1 < Z \leq 5\%$
- Green: $5 < Z \leq 20\%$
- Blue: $20 < Z \leq 40\%$
- Cyan: $40 < Z \leq 60\%$
- Magenta: $60 < Z \leq 80\%$
- Yellow: $80 < Z \leq 95\%$
- Brown: $95 < Z \leq 99\%$
- Black: $Z > 99\%$

Rapid collapse at ~0.2 MJ when Z≈ 0.5 (runaway gas accretion)

After disk dispersal (T>10 Myrs), slow contraction.

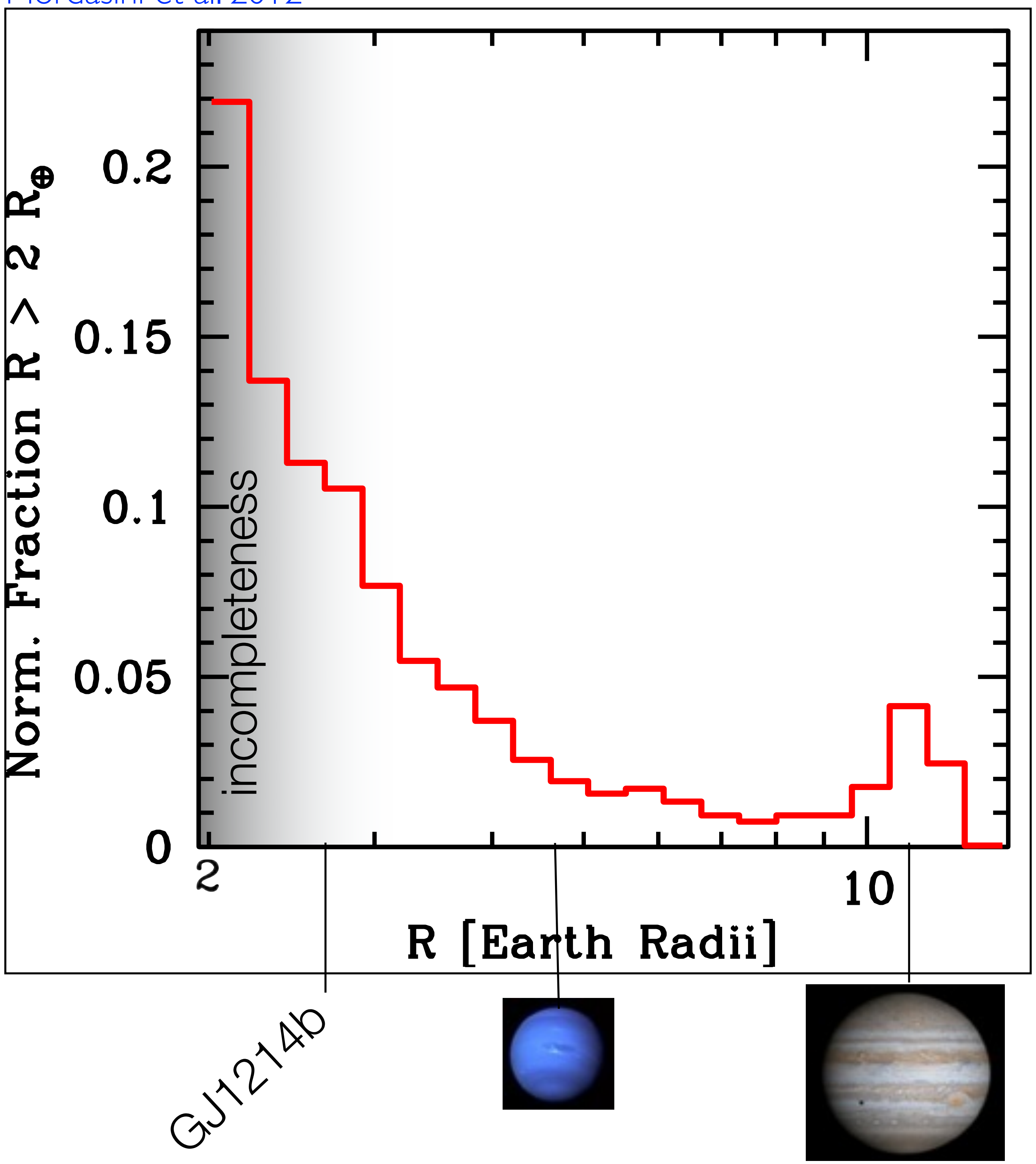
Characteristic S shape

M_{star}=1 M_{sun}. a>0.1AU. Non-isothermal type I. cold accretion. 1 embryo/disk, no special inflation mechanisms, no evap.



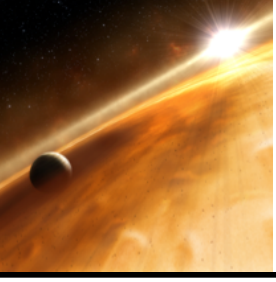
Planetary radius distribution

Mordasini et al. 2012

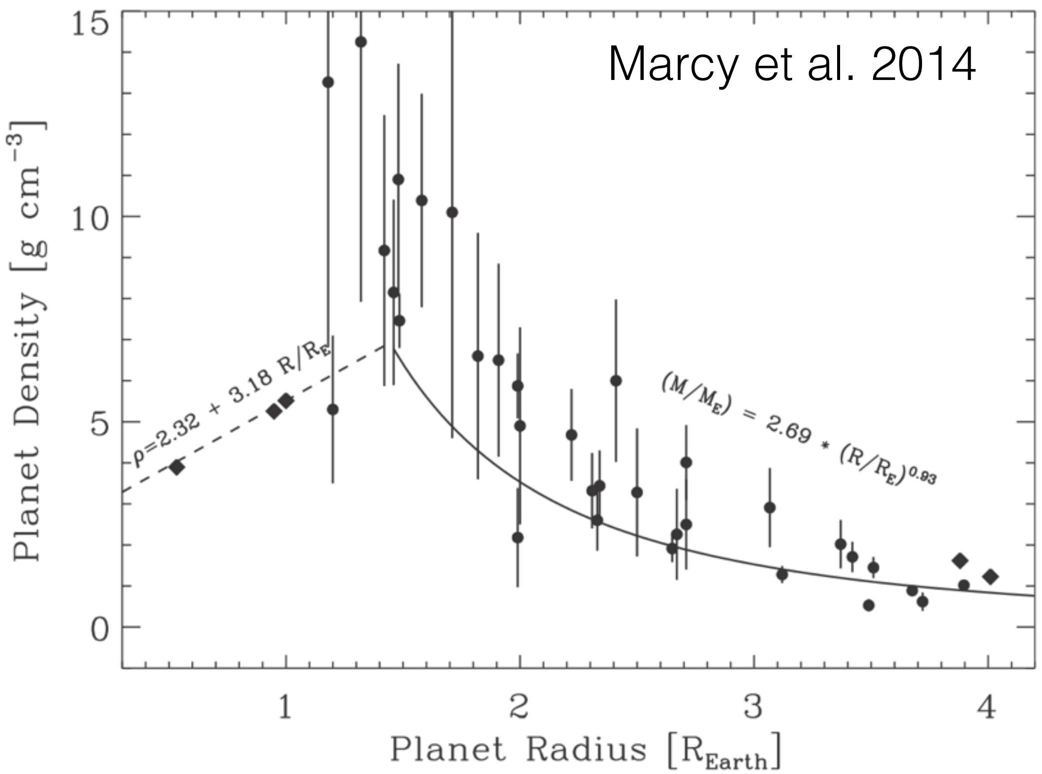


- Peak at lowest radii. High detection rate of Kepler.
- Second peak at ~ 1 R_J ⇨ Giant planets have all approx. *the same radius independent of mass* (degeneracy!)
- Peak: prediction for larger orbital periods (but over-predicted here: one opacity, one stellar mass, no bloating)

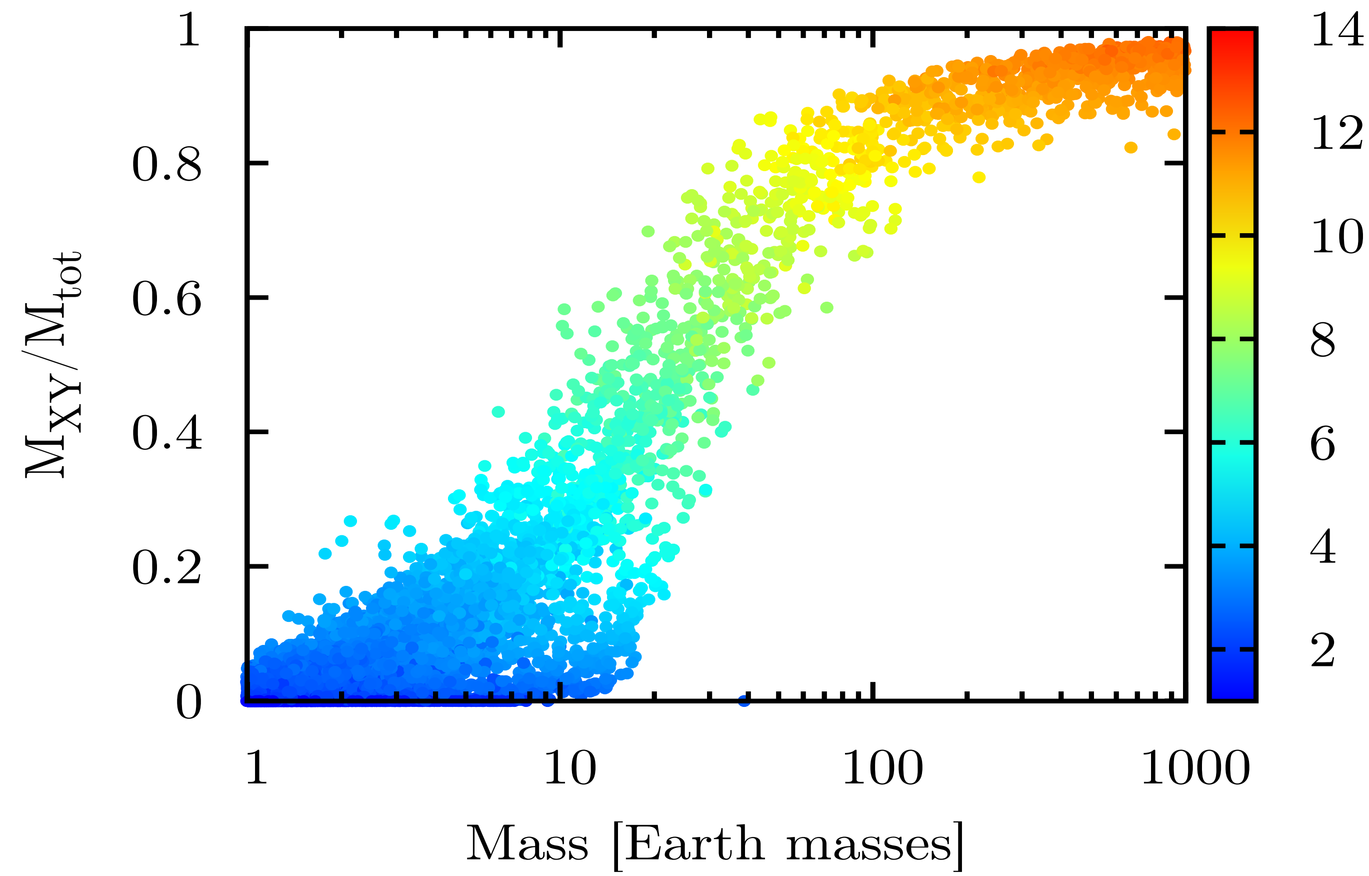
M_{star}=1 M_{sun}. a>0.1AU. Non-isothermal Type I. Cold accretion. 1 embryo/disk, no special inflation mechanisms, no evap.



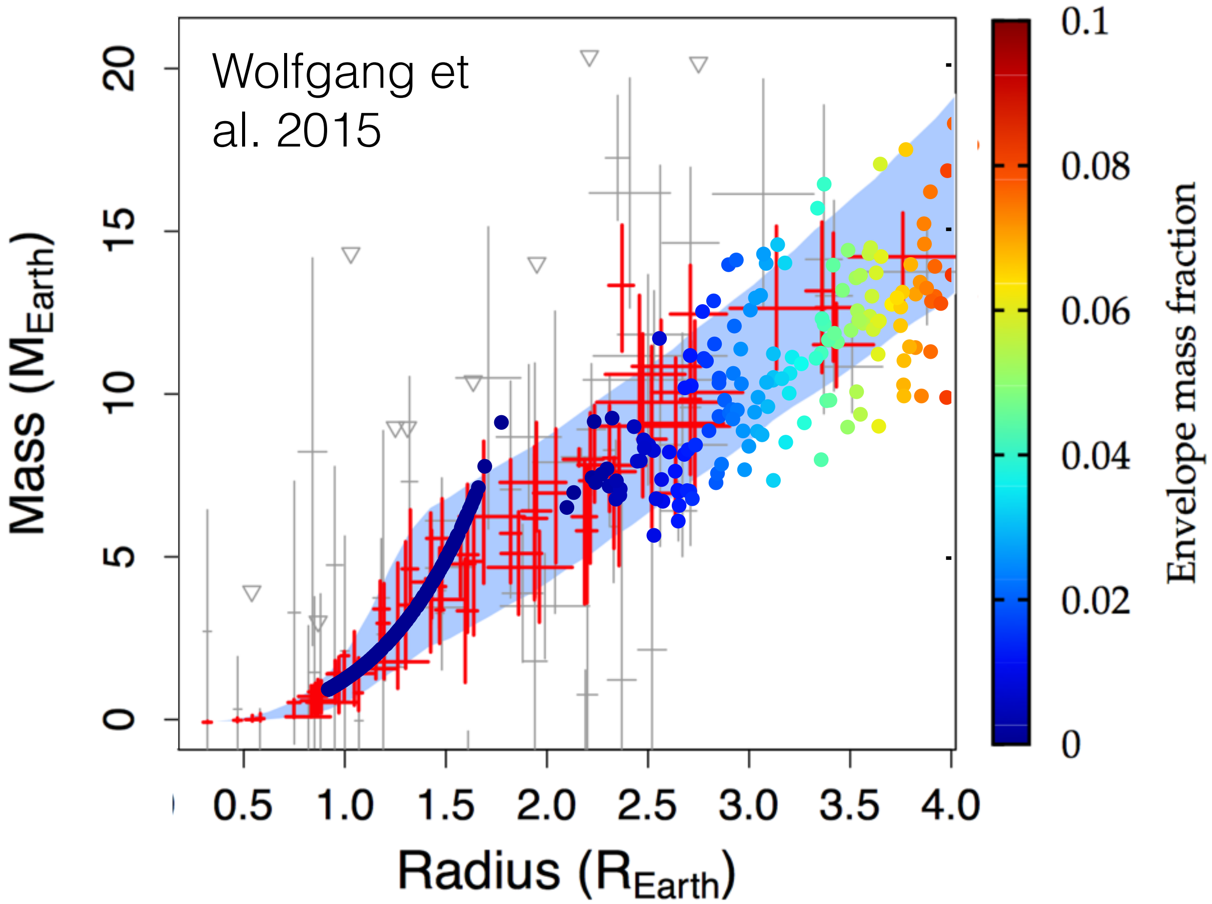
Constraints on H/He fraction



Synthetic, grain opacity: 0.003 x ISM

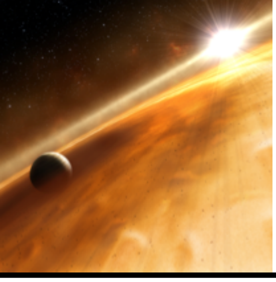


Increasing H/He mass fraction with mass



Theoretical result: dependency on grain opacity in protoplanetary atmosphere during formation

Podolak 2003, Movshovitz et al. 2010: numerical grain dynamics & opacity model
Mordasini 2014, Ormel 2014: analytical models



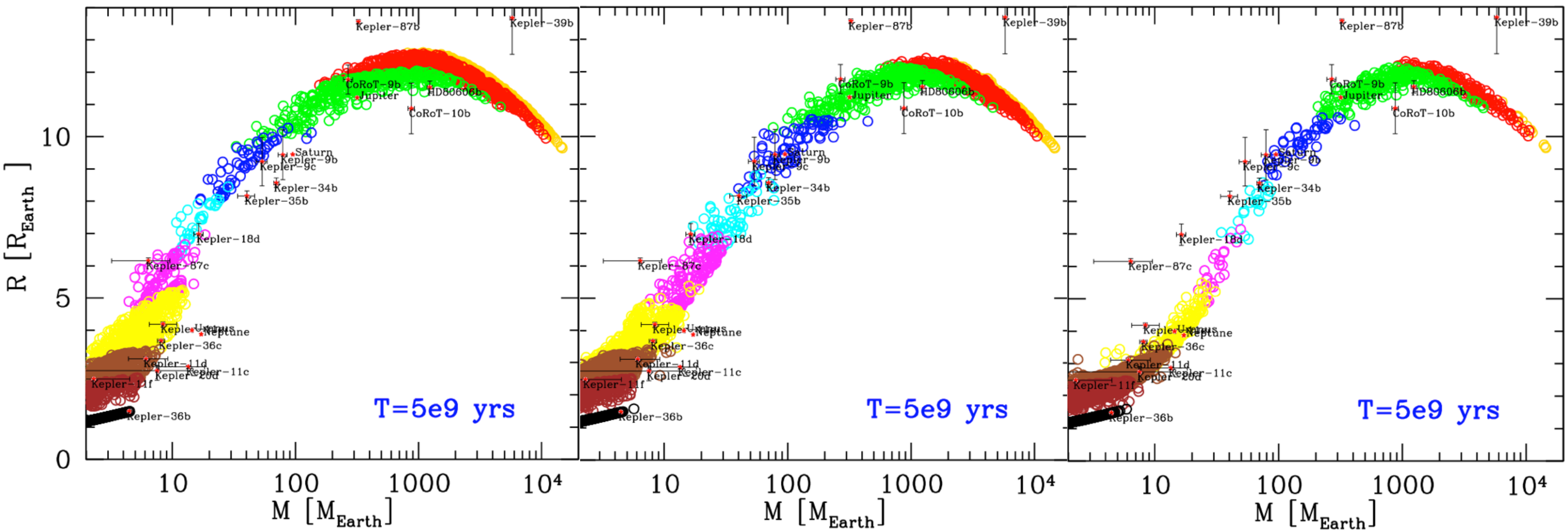
Mass-radius relationship

Mordasini et al. 2014

0.1<a/AU<1

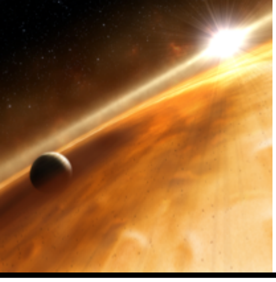
Compare synthetic and observed M-R

Grain free (f_{opa}=0)	f_{opa}=0.003	ISM (f_{opa}=1)

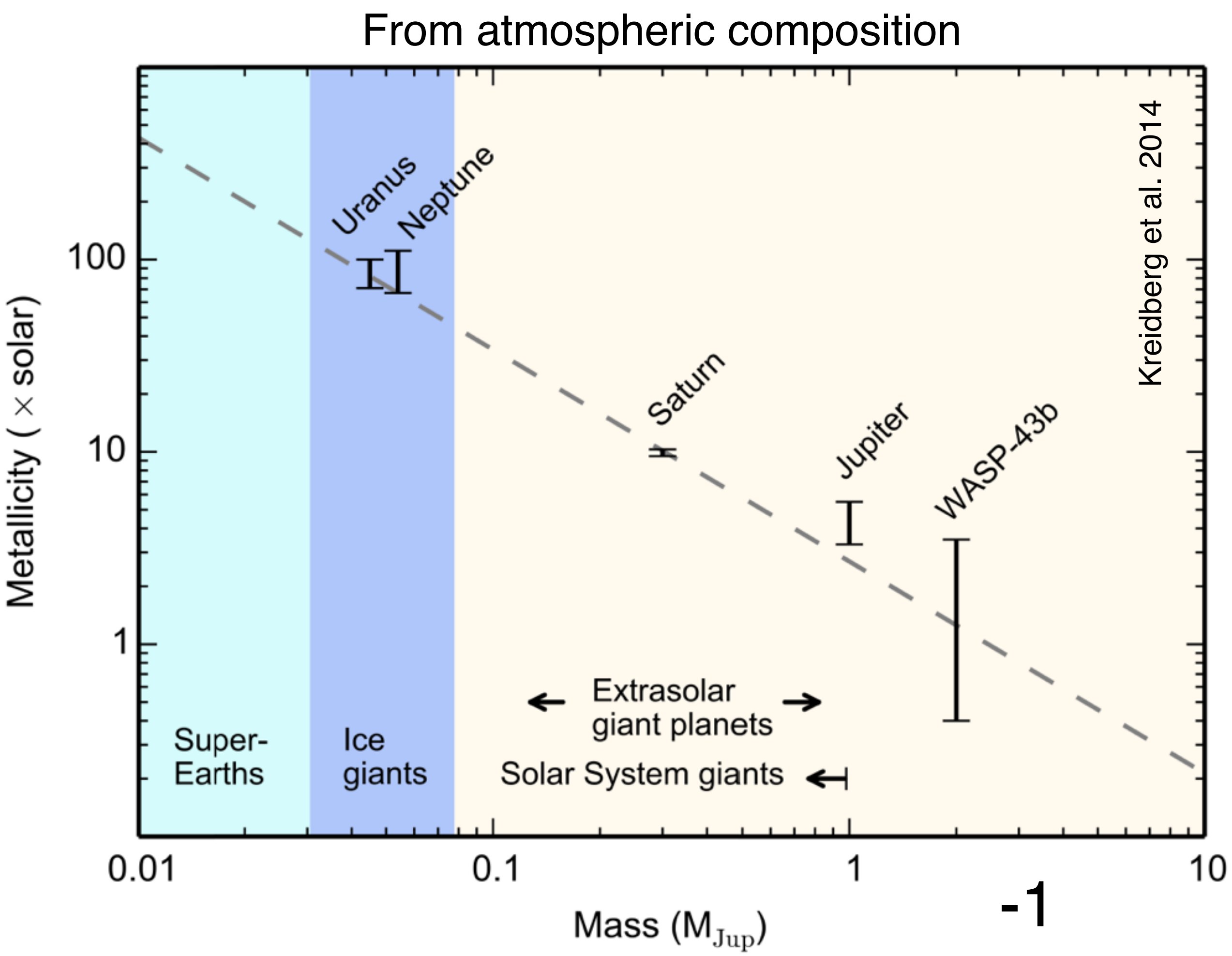


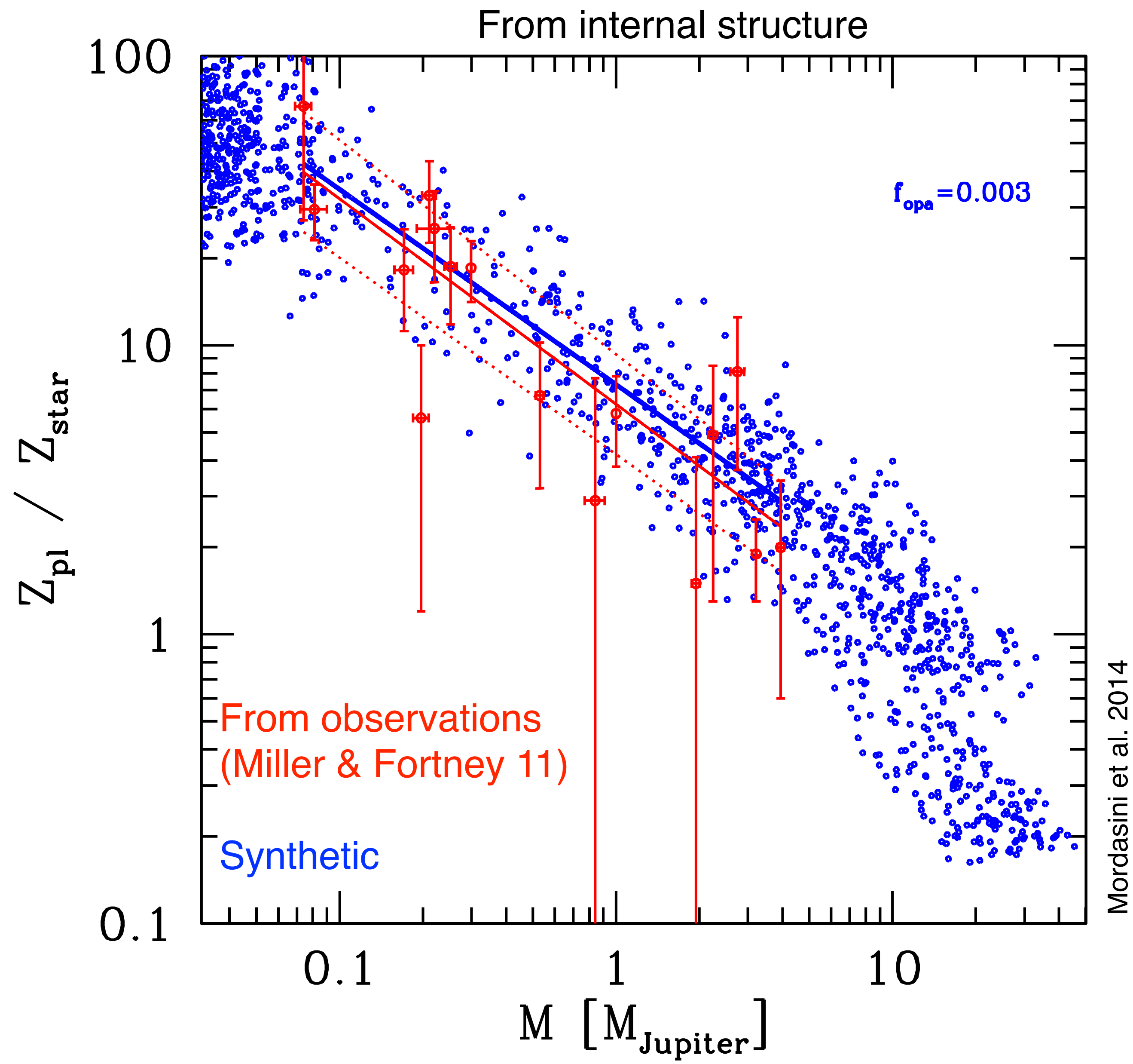
too large too much H/He	radii similar as observed	too small too little H/He

Observational constraints from M-R relation on microphysical grain models.



Enrichment relative to host star





$$\frac{Z_{pl}}{Z_{star}} = \beta \left(\frac{M}{M_{2\!\!+}} \right)^{\alpha}$$

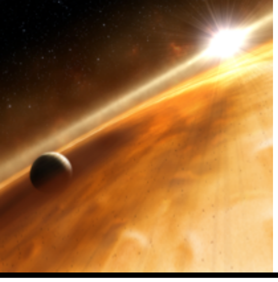
α:
- -1 simplest CA
- -2/3 feeding zone
- 0 grav. instability (?)

β: 2-9 (opacity)

Data set	α	β
Miller & Fortney (2011)	-0.71 ± 0.10	6.3 ± 1.0
$f_{opa} = 0$	-0.73	3.5
$f_{opa} = 0.003$	-0.68	7.2
$f_{opa} = 1$	-0.72	8.5
$f_{opa} = 0.003$, in situ, $\dot{M}_{Z,run} = 0$	-0.88	2.4

4. Conclusions

- Population synthesis is a tool to compare theory and observation to improve understanding of planet formation
  - use full wealth of observational constraints
  - put detailed models to the test
  - see global statistical consequences
- Observational constraints on many processes
  - solid and gas accretion rate (T_{KH})
  - grain dynamics
  - orbital migration rate
- See link between disk and planetary properties
- Predict yield of future instruments/space missions
- Continuously evolving models
  - population syntheses depend on progress of formation theory as a whole
  - a lot to do



DACE

Online demonstration

www.dace.unige.ch