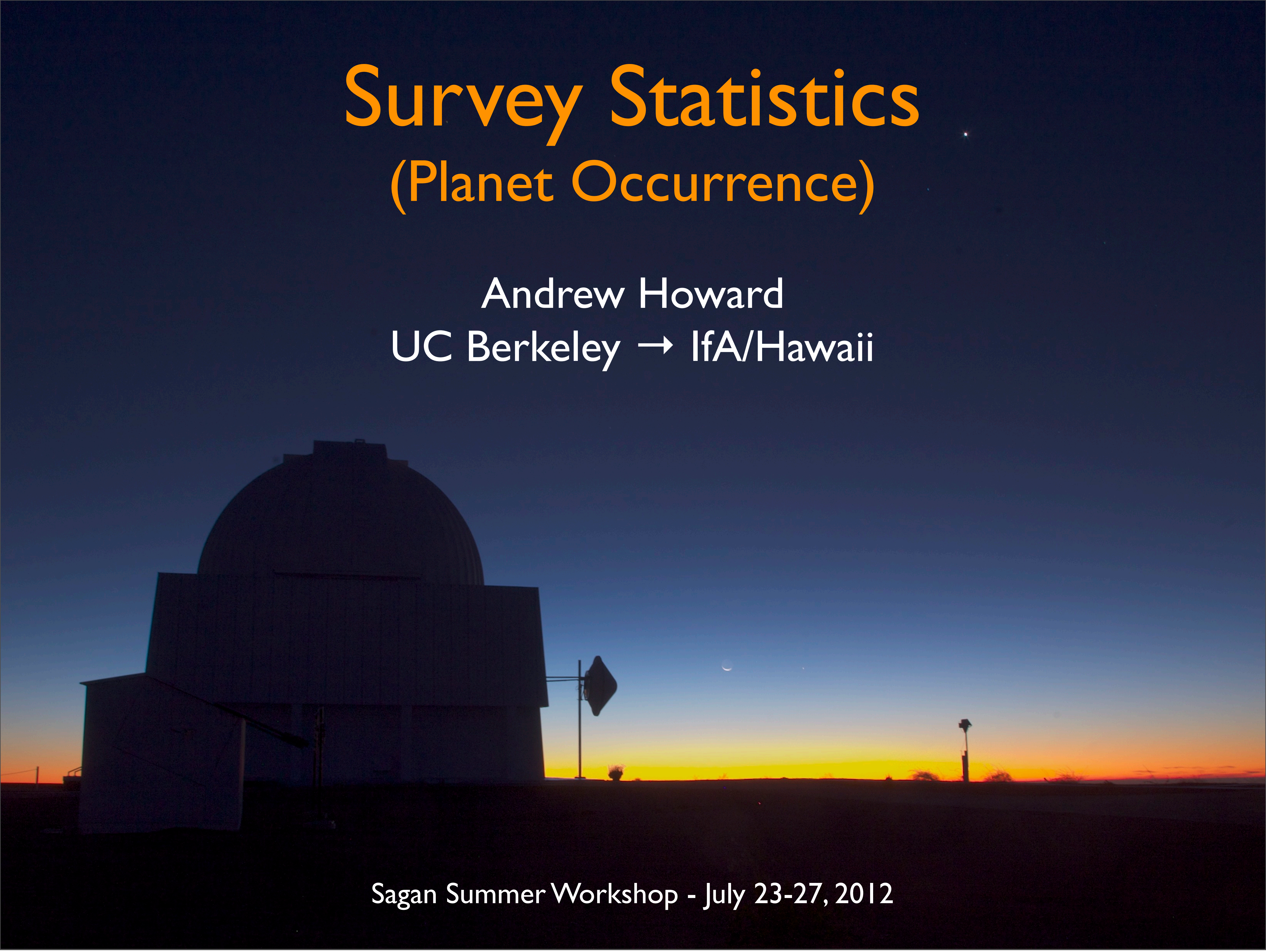


Survey Statistics
(Planet Occurrence)

Andrew Howard
UC Berkeley → IfA/Hawaii

Sagan Summer Workshop - July 23-27, 2012

Outline

Planet Occurrence - what can we measure?

Planet-Metallicity Correlation

Doppler Surveys - Eta-Earth Survey

Transit Survey Completeness

Kepler Planet Occurrence

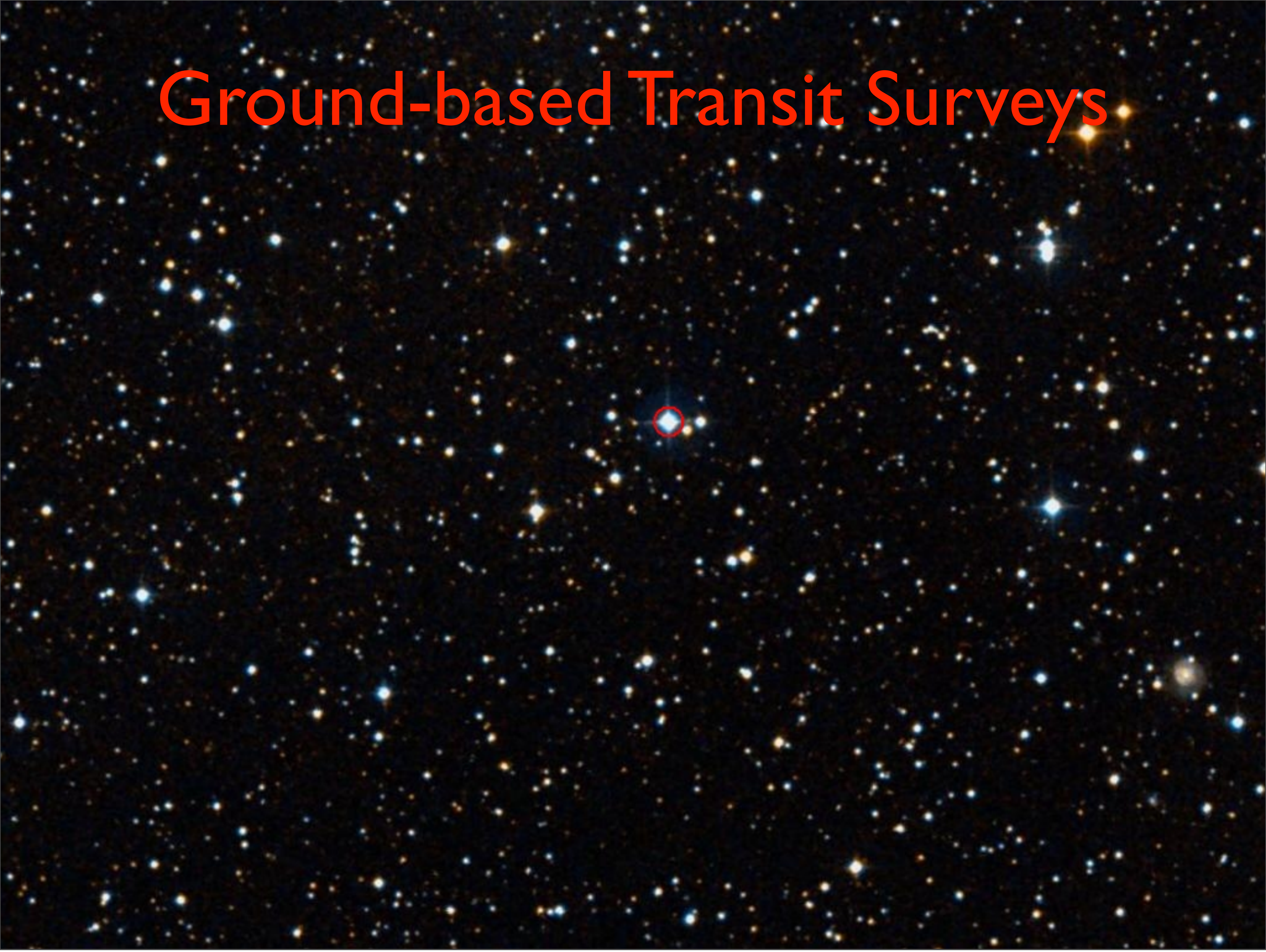
Ground-based Transit Surveys

- Define planet parameters of measurement (*M*, *R*, *P*, e, etc.)
- Set planet detection threshold
- Incompleteness — correct for missed planets

$$\text{Occurrence} = \frac{\text{Number of Planets}}{\text{Number of Stars}}$$

- Define stellar parameters of measurement (*M*, *R*, *Fe/H*, *Teff*, *logg*, etc.)

Ground-based Transit Surveys



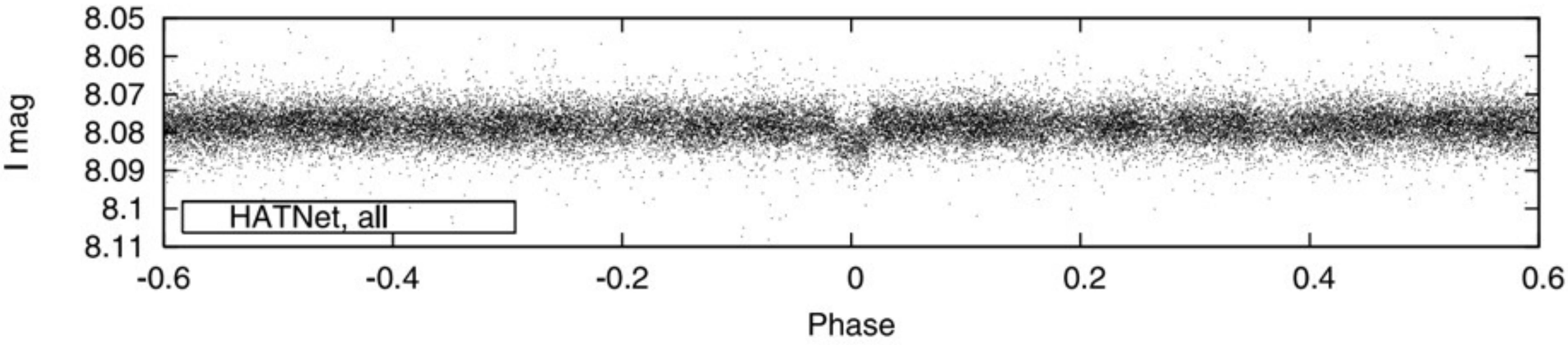
Ground-based Transit Surveys

- Define planet parameters of measurement (*M*, *R*, *P*, e, etc.)
- Set planet detection threshold
- High incompleteness — correct for missed planets

$$\text{Occurrence} = \frac{\text{Number of Planets}}{\text{Number of Stars}}$$

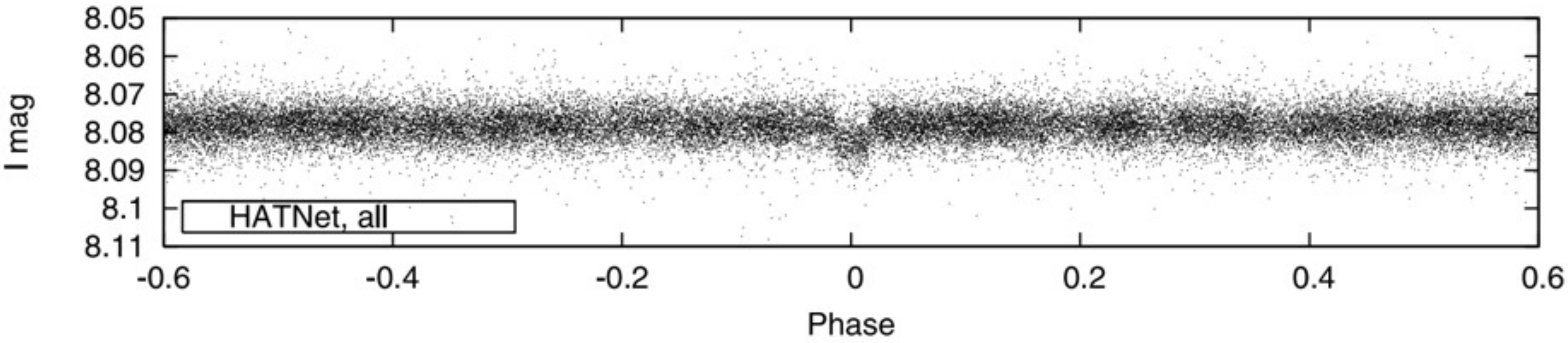
- Define stellar parameters of measurement (*M*, *R*, *Fe/H*, *Teff*, *logg*, etc.)

Transit Survey Completeness



von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness

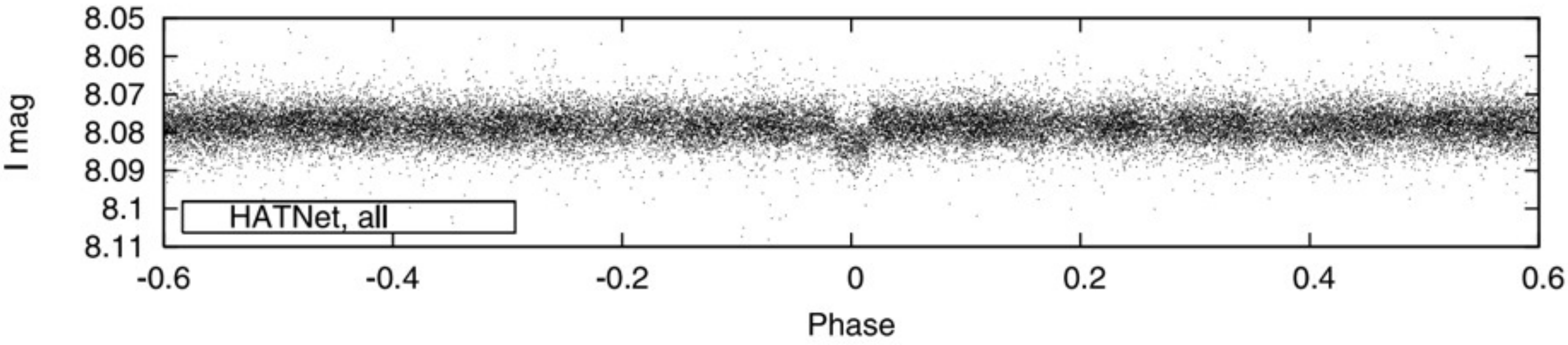


White noise

$$\boxed{S/N_{\mathrm{transit}} = \frac{\mathrm{depth}}{\sigma}\sqrt{n}}$$

von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness



White noise

$$\mathrm{S/N_{transit}} = \frac{\mathrm{depth}}{\sigma}\sqrt{n}$$

Red+white noise

$$\begin{aligned}\mathrm{S/N_{transit}} &= \frac{\mathrm{depth}}{\sqrt{\frac{1}{n^2}\sum_{i,j}\mathrm{cov}[i;\, j]}} \\ &= \frac{\mathrm{depth}}{\sqrt{\frac{\sigma^2}{n} + \frac{1}{n^2}\sum_{i\neq j}\mathrm{cov}[i;\, j]}}\end{aligned}$$

von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness

Red+white noise

$$\mathrm{S/N_{transit}} = \frac{\mathrm{depth}}{\sqrt{\frac{1}{n^2}\sum_{i,j}\mathrm{cov}[i;\,j]}} = \frac{\mathrm{depth}}{\sqrt{\frac{\sigma^2}{n} + \frac{1}{n^2}\sum_{i\neq j}\mathrm{cov}[i;\,j]}}$$

von Braun et al. (2009); Pont et al. (2006)

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↓

$$\mathrm{S/N_{transit}} = \sqrt{\frac{(\mathrm{depth}\cdot n)^2}{\sum_{k=1}^{N_{\mathrm{tr}}}\left[n_k^2\left(\frac{\sigma_w^2}{n_k} + \sigma_r^2\right)\right]}},$$

von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness

Red+white noise

$$\mathrm{S/N}_{\mathrm{transit}} = \frac{\mathrm{depth}}{\sqrt{\frac{1}{n^2}\sum_{i,j}\mathrm{cov}[i;\,j]}}$$

$$= \frac{\mathrm{depth}}{\sqrt{\frac{\sigma^2}{n} + \frac{1}{n^2}\sum_{i\neq j}\mathrm{cov}[i;\,j]}}$$

↓

total number of data points

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Transit Survey Completeness

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↓

total number of data points

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number of data points
in *k*th transit

von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness

Red+white noise

$$\mathrm{S/N}_{\mathrm{transit}} = \frac{\mathrm{depth}}{\sqrt{\frac{1}{n^2}\sum_{i,j}\mathrm{cov}[i;\,j]}} = \frac{\mathrm{depth}}{\sqrt{\frac{\sigma^2}{n} + \frac{1}{n^2}\sum_{i\neq j}\mathrm{cov}[i;\,j]}}$$

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number of data points
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white noise

von Braun et al. (2009); Pont et al. (2006)

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red noise

number of data points
in *k*th transit

white noise

von Braun et al. (2009); Pont et al. (2006)

Transit Survey Completeness

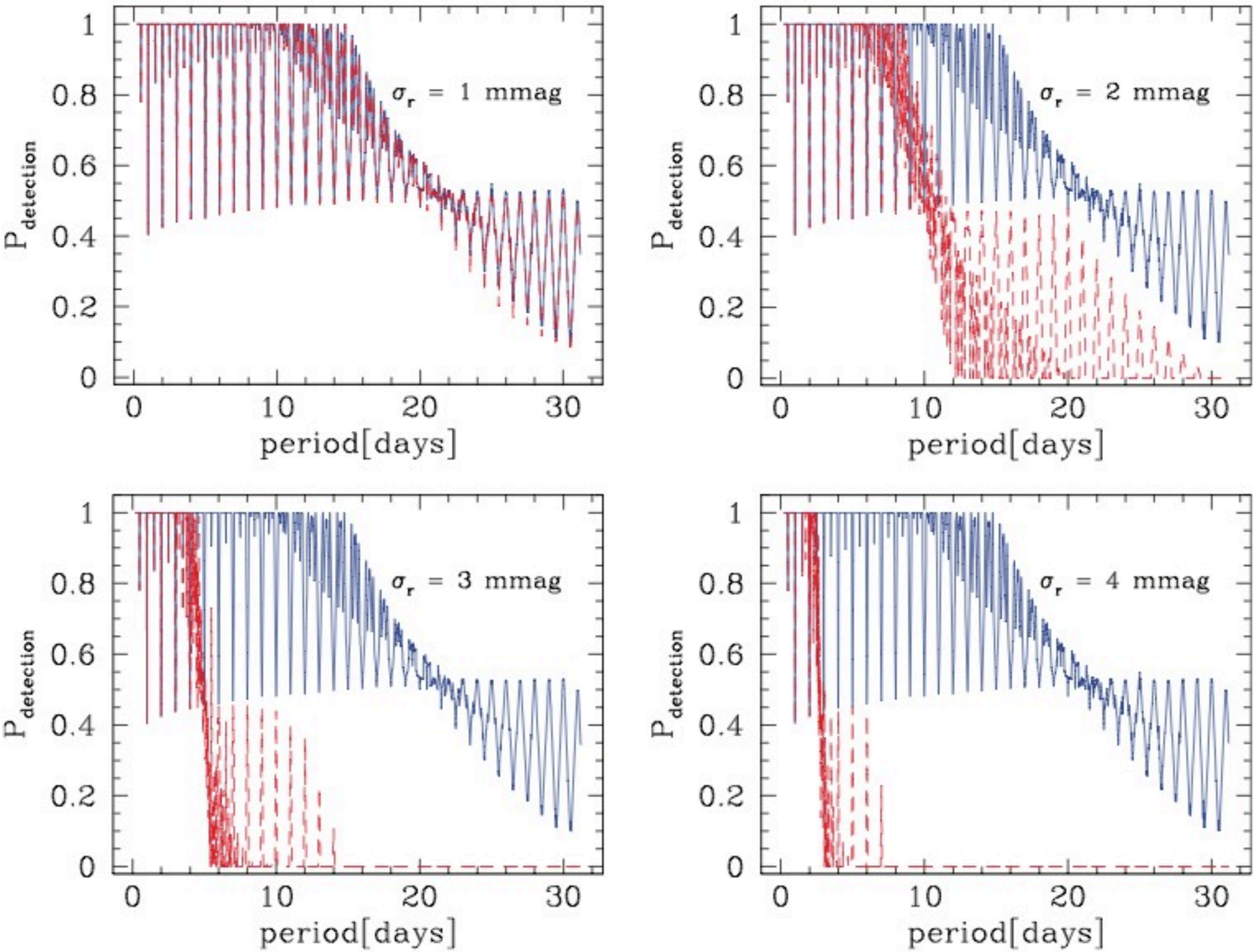
What is the detection efficiency of a transit survey given:

- σ_w (white noise)
- σ_r (red noise)
- night length
- run duration

Requiring 2+ transits

von Braun et al. (2009)

Transit Detection Efficiency: Effect of Red Noise



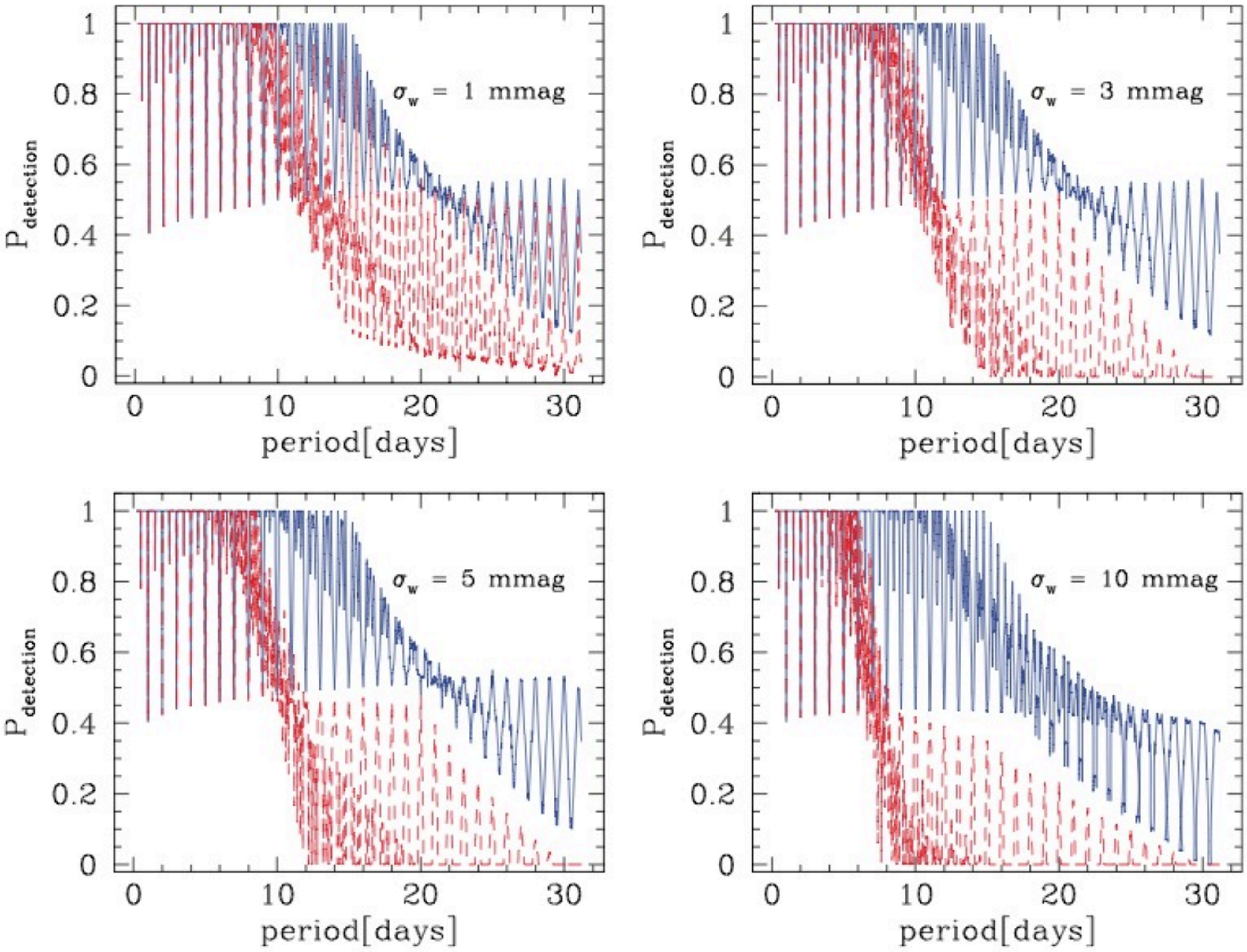
σ_w = 5 mmag

blue → $\sigma_r = 0$ mmag
red → $\sigma_r \neq 0$ mmag

$S/N_{thresh} = 7$
60 nights
8 hr/night
5 min cadence
$R_p/R_s = 0.1$
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of White Noise



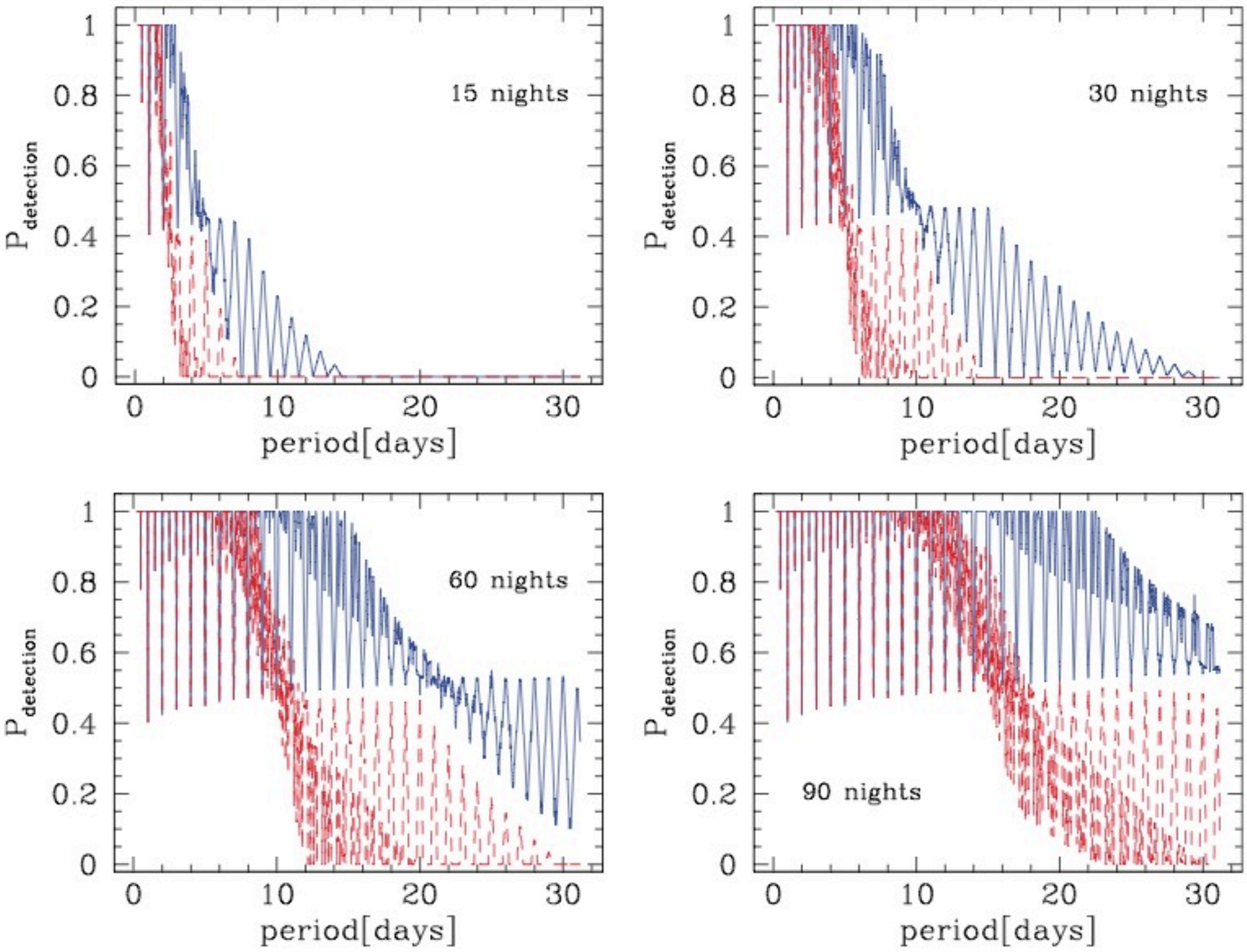
σ_r = 1-10 mmag

blue → σ_w = 0 mmag
red → σ_w = 2 mmag

S/N_{thresh} = 7
60 nights
8 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of Run Length



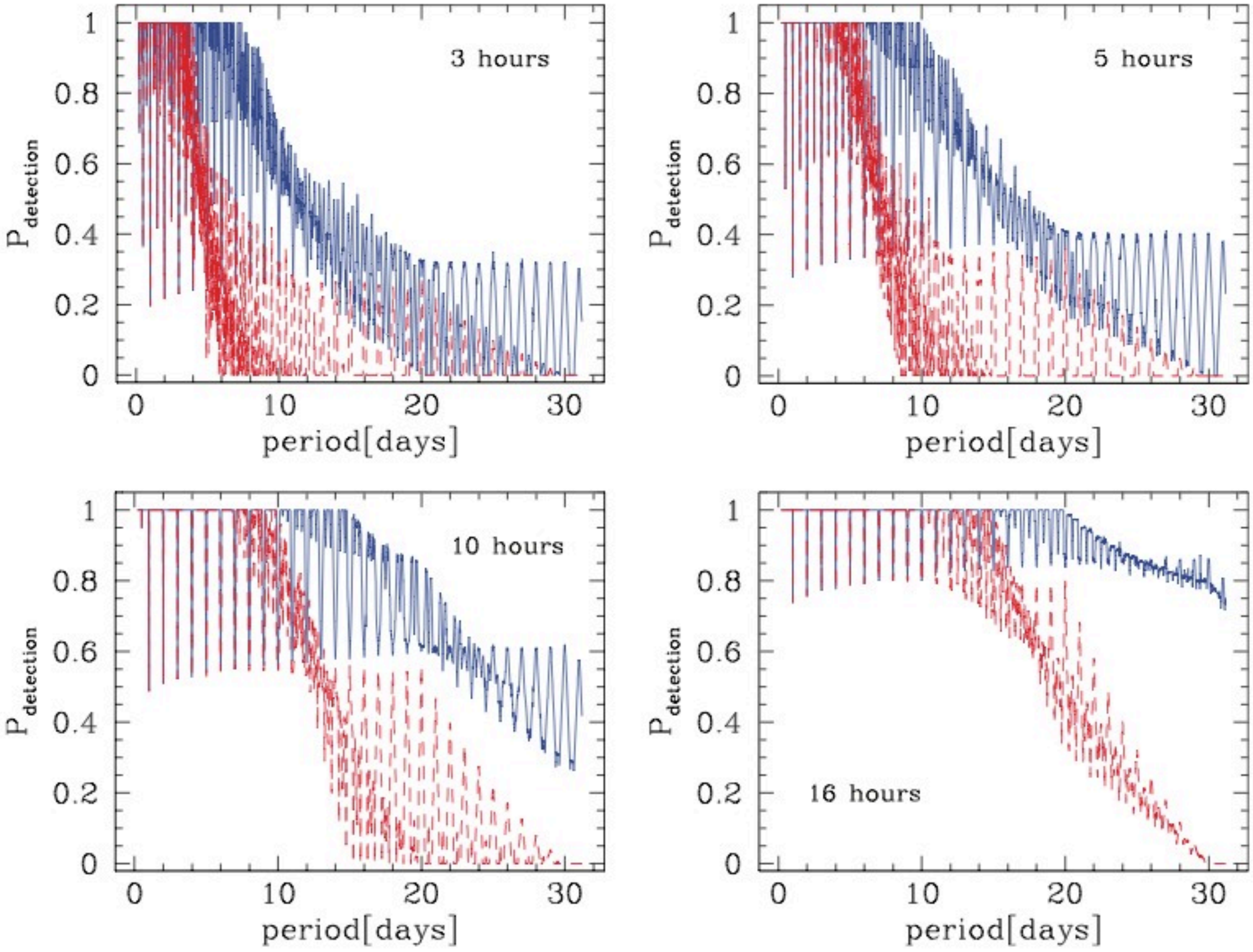
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
15-90 nights
8 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of Night Length



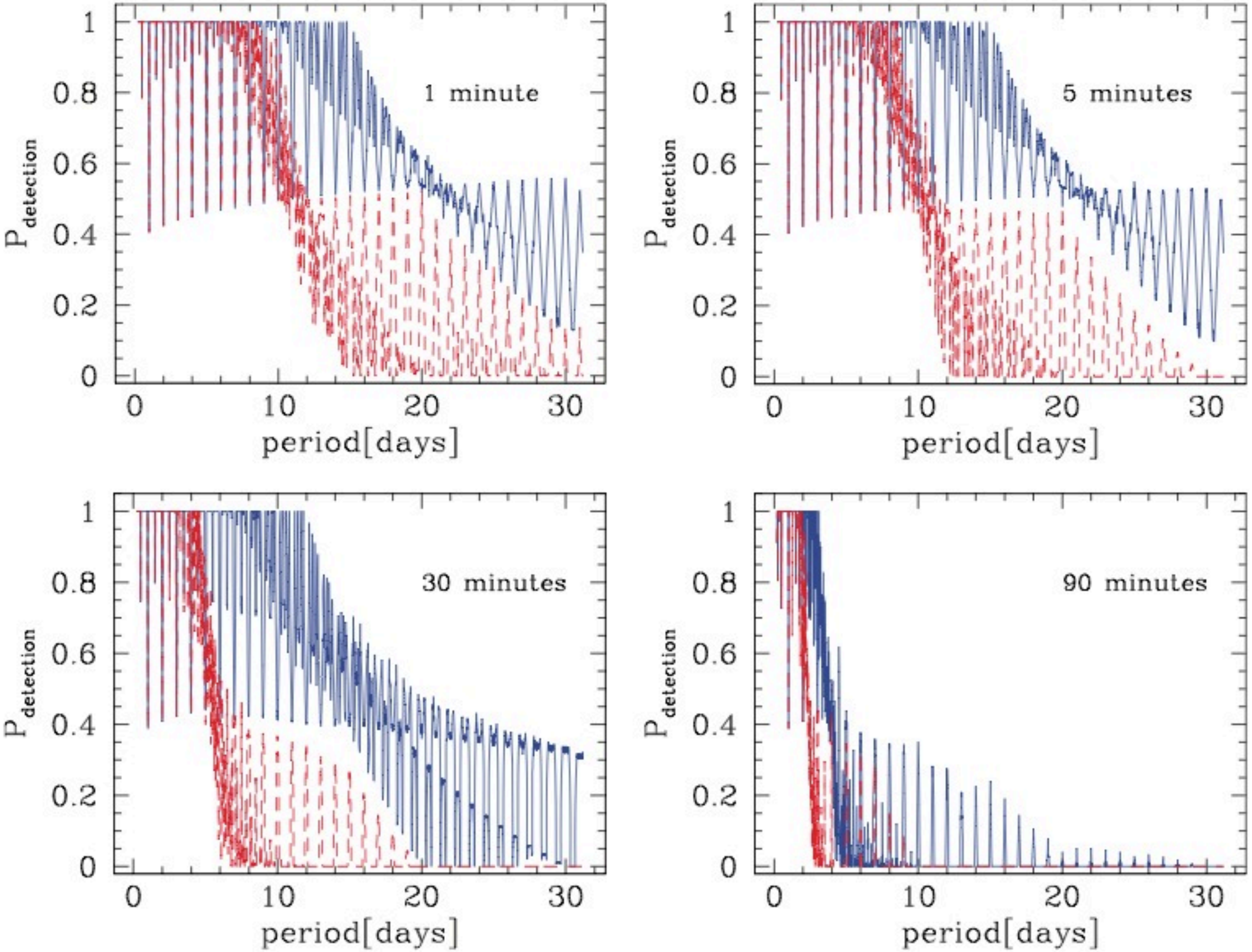
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
60 nights
3-16 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of Cadence



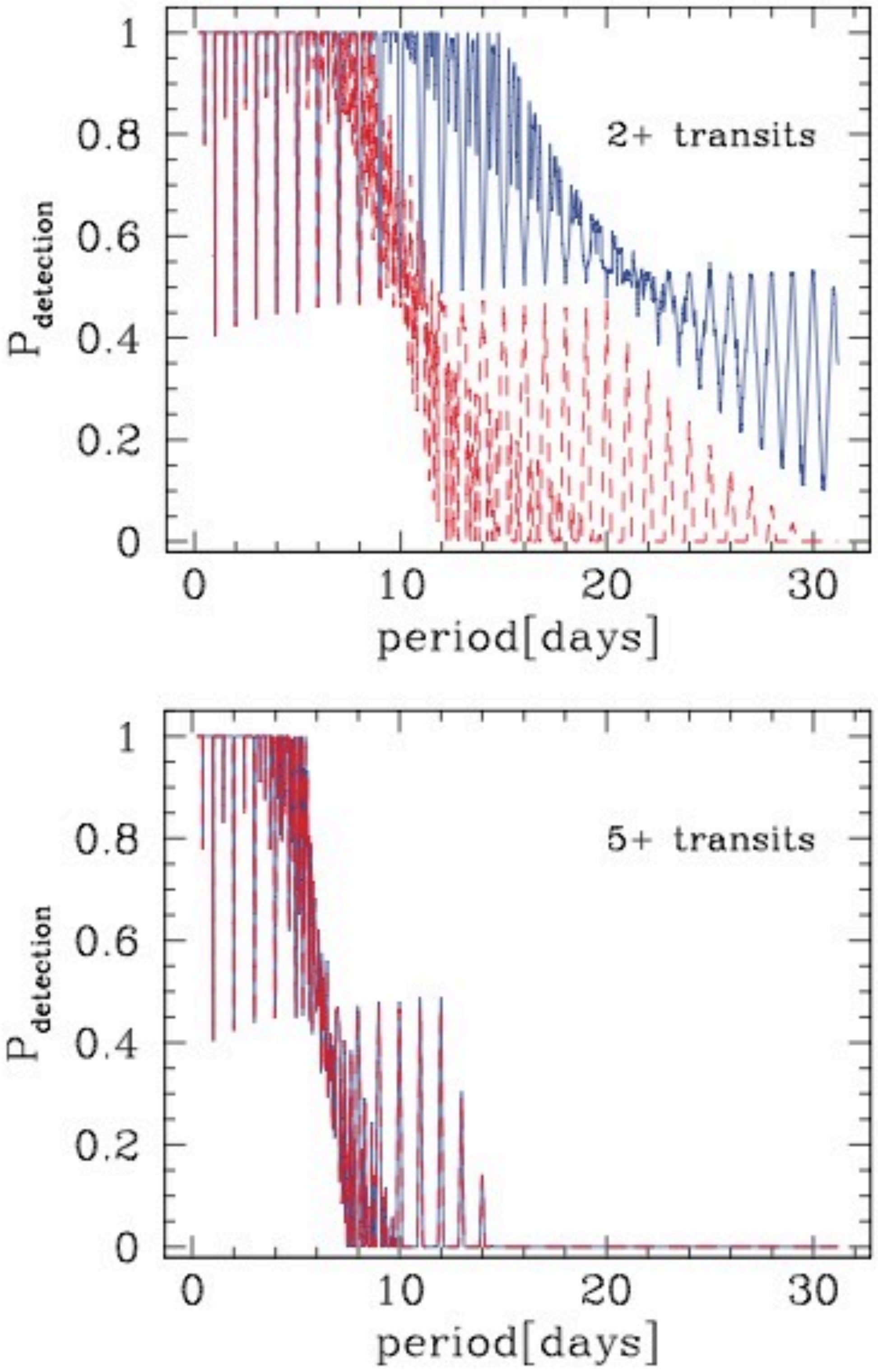
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
60 nights
8 hr/night
1-90 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of N_{transits}



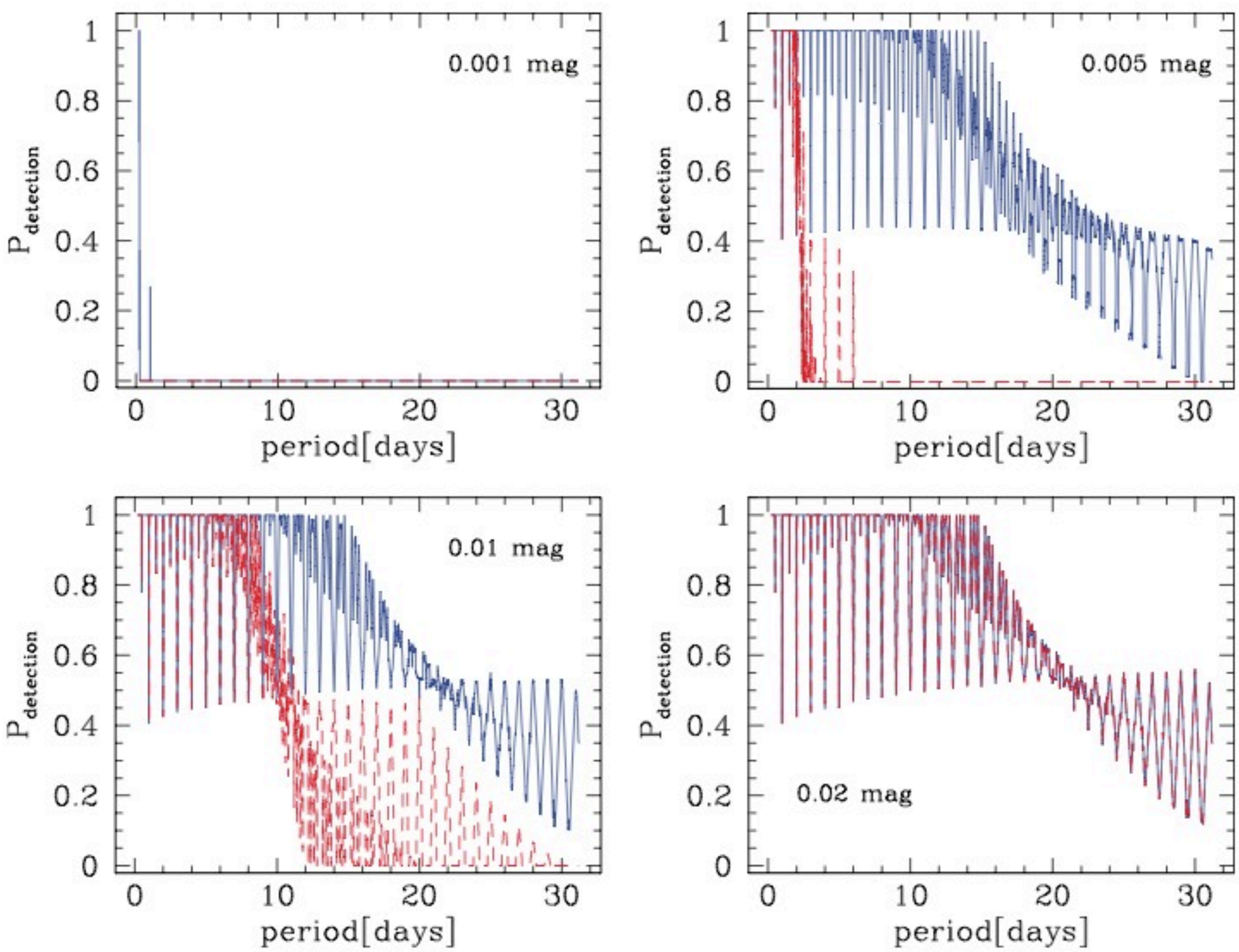
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
60 nights
8 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Effect of depth



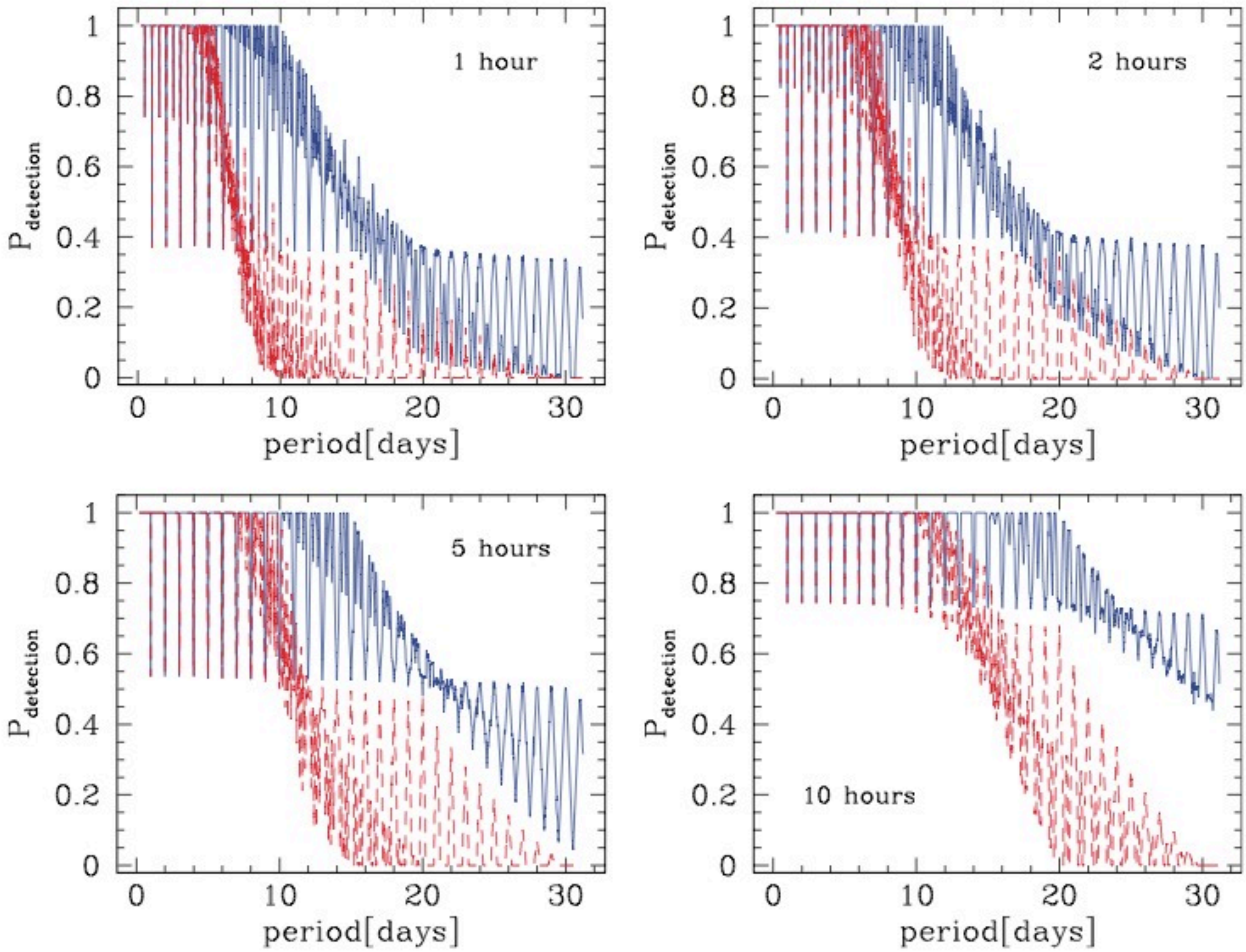
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
60 nights
8 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 1-20 mmag

von Braun et al. (2009)

Transit Detection Efficiency: Effect of transit duration



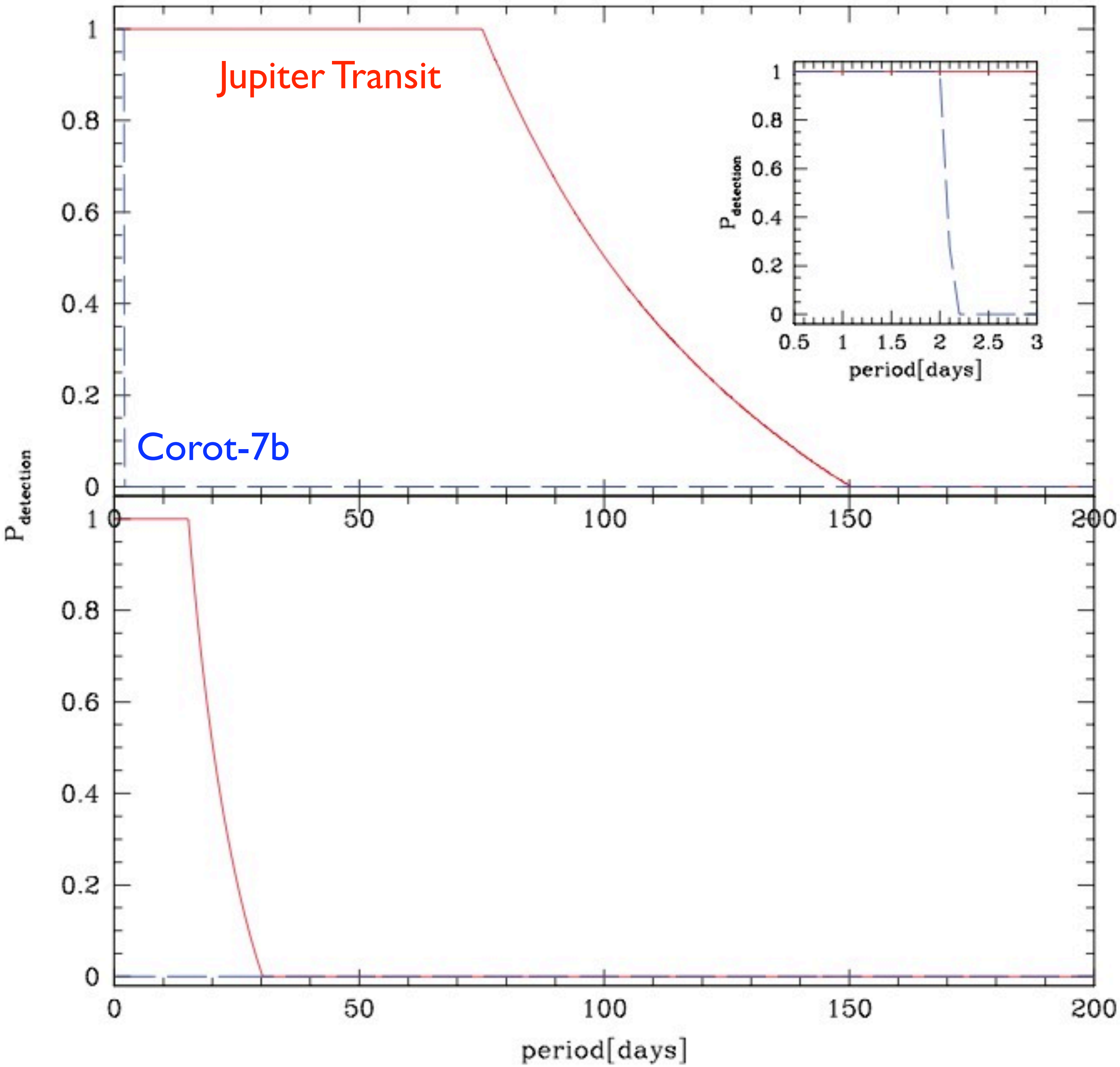
σ_w = 5 mmag

blue → σ_r = 0 mmag
red → σ_r = 2 mmag

S/N_{thresh} = 7
60 nights
8 hr/night
5 min cadence
R_p/R_s = 0.1
depth = 0.01

von Braun et al. (2009)

Transit Detection Efficiency: Space Mission



von Braun et al. (2009)

Outline

Planet Occurrence - what can we measure?

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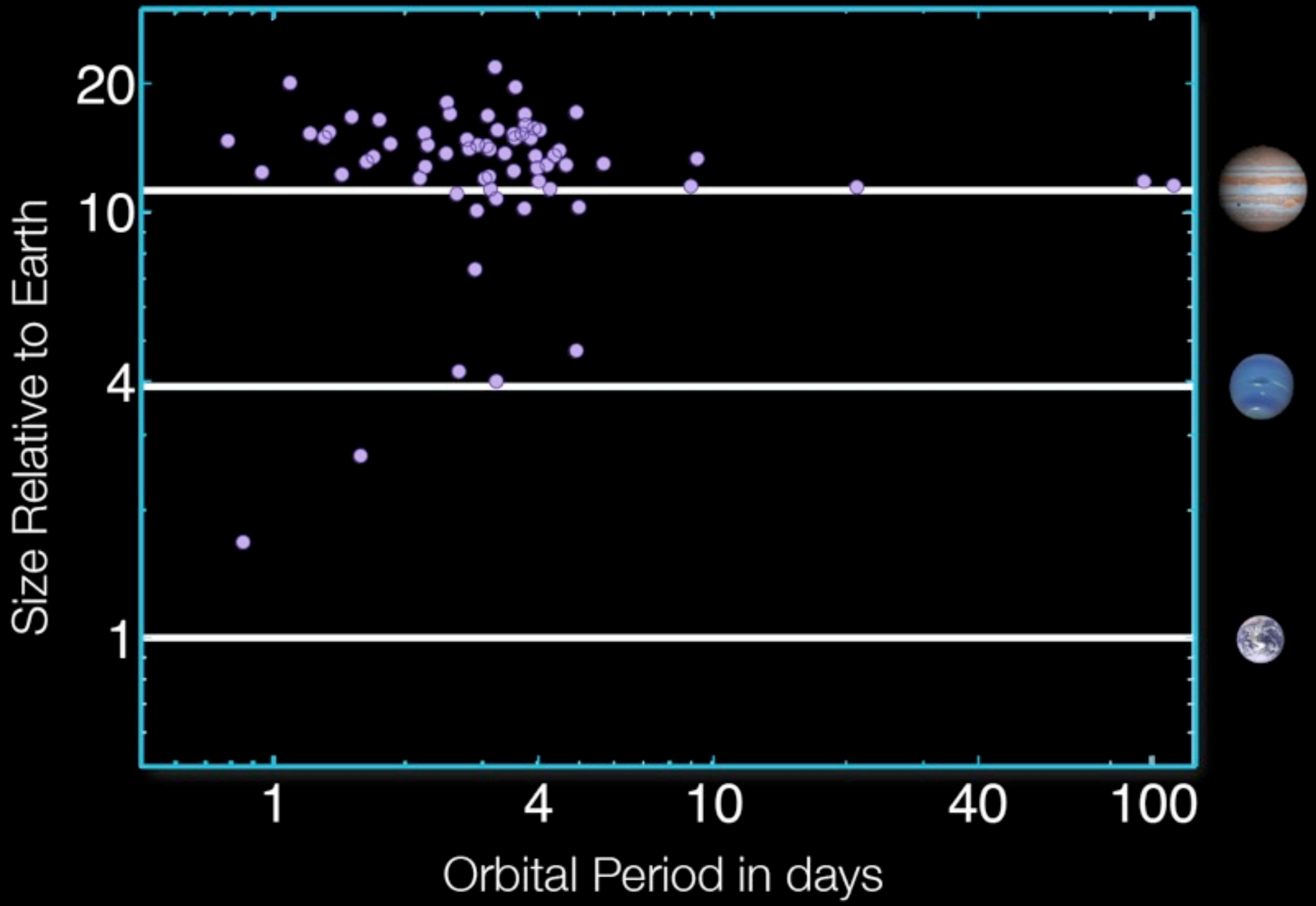
Doppler Surveys - Eta-Earth Survey

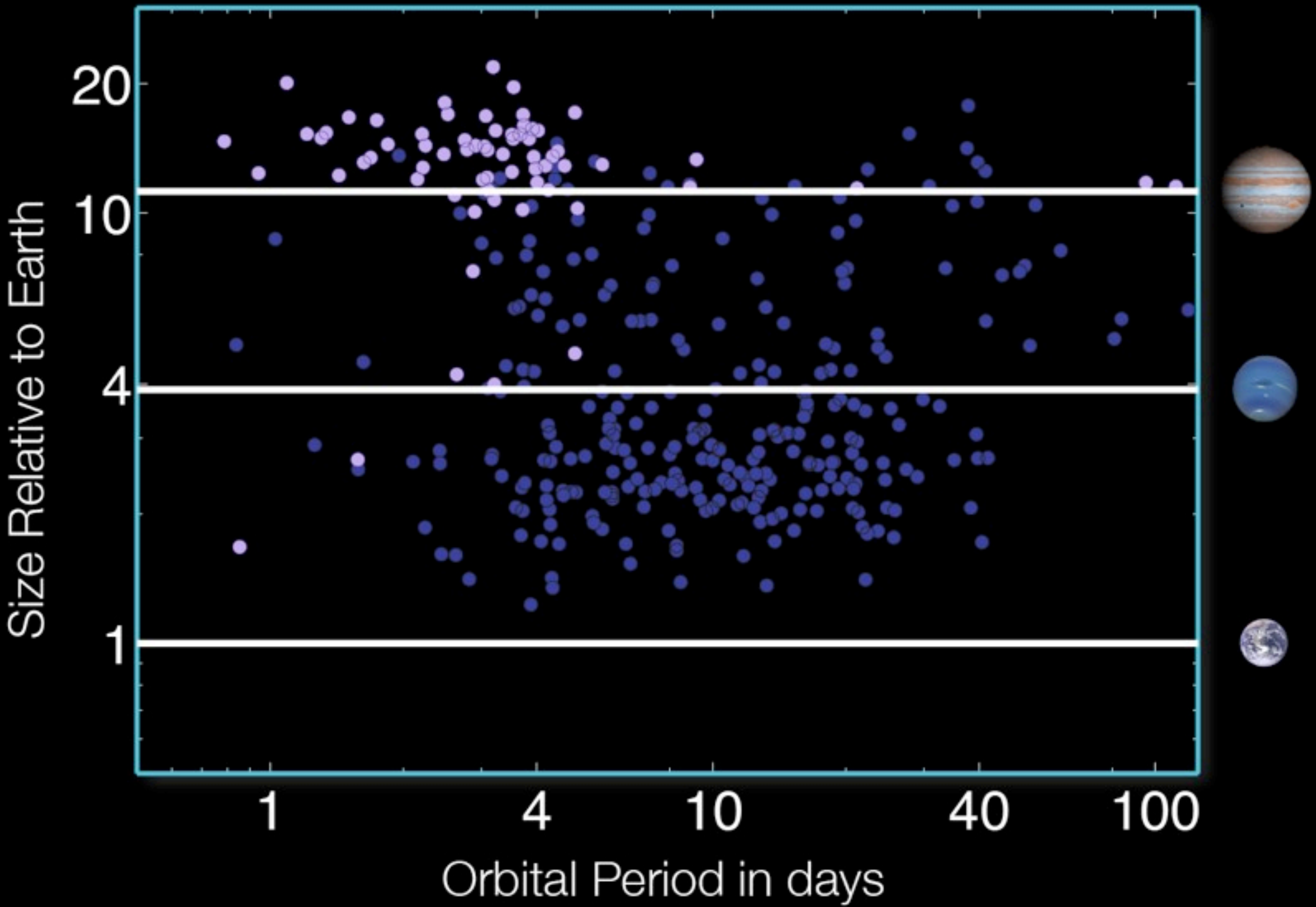
Transit Survey Completeness

Kepler Planet Occurrence

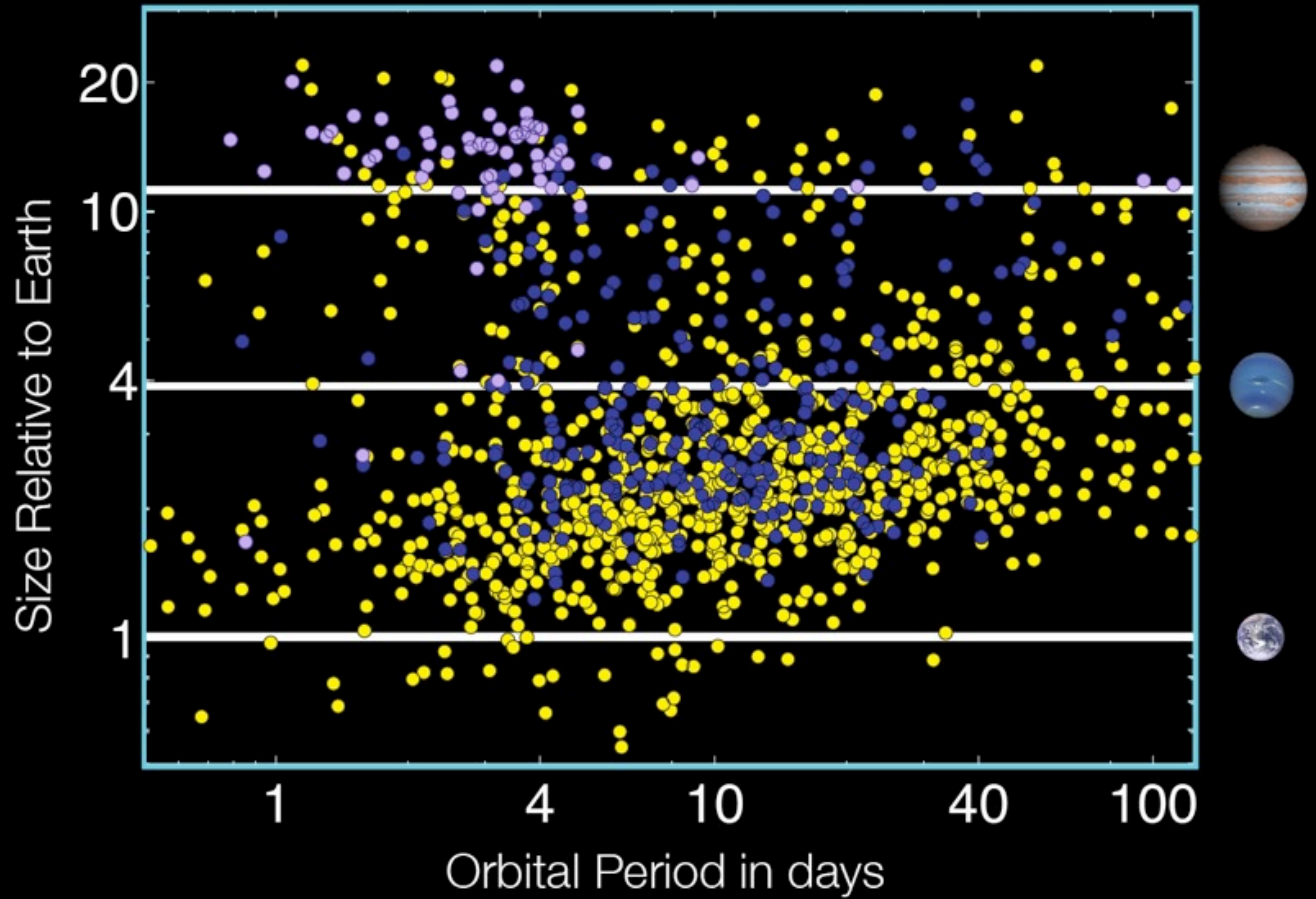
Pre-Kepler Transiting Planets - 2009



Kepler Candidates as of June 2010



Kepler Candidates as of February 1, 2011



Compute Occurrence

$$\text{Occurrence} = \frac{\#\ \text{planets}}{\#\ \text{stars}}$$

Compute Occurrence

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Source of Incompleteness

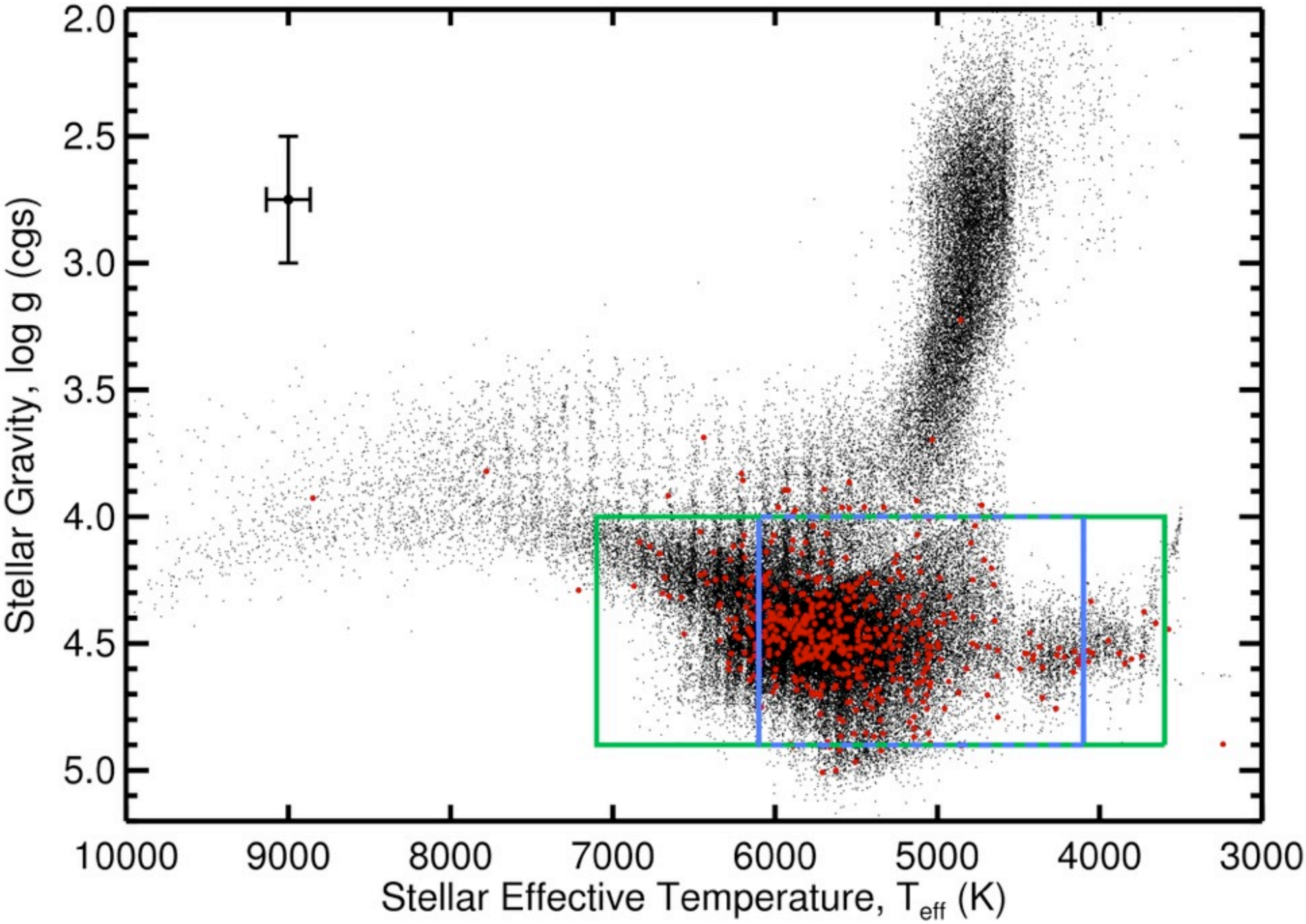
- window function
- S/N completeness
- inclination
- pipeline completeness

Compute Occurrence

$$\text{Occurrence} = \frac{\#\ \text{planets}}{\#\ \text{stars}}$$

Choose stars carefully:

- Limit T_{eff} and $\log g$
- Kp cut (bright stars only)
- Use SNR cut based on RMS



Howard et al. (2012)

Computing Occurrence

$$\text{Occurrence} = \frac{\#\ \text{planets}}{\#\ \text{stars}}$$

Choose stars carefully:

- Limit T_{eff} and log$g$
- Kp cut (bright stars only)
- Use SNR cut based on RMS

Computing Occurrence

Choose planets carefully:

- Use SNR cut based on RMS
- Only consider $R > 2\ R_E$
- Correct for inclinations

$$\text{Occurrence} = \frac{\#\ \text{planets}}{\#\ \text{stars}}$$

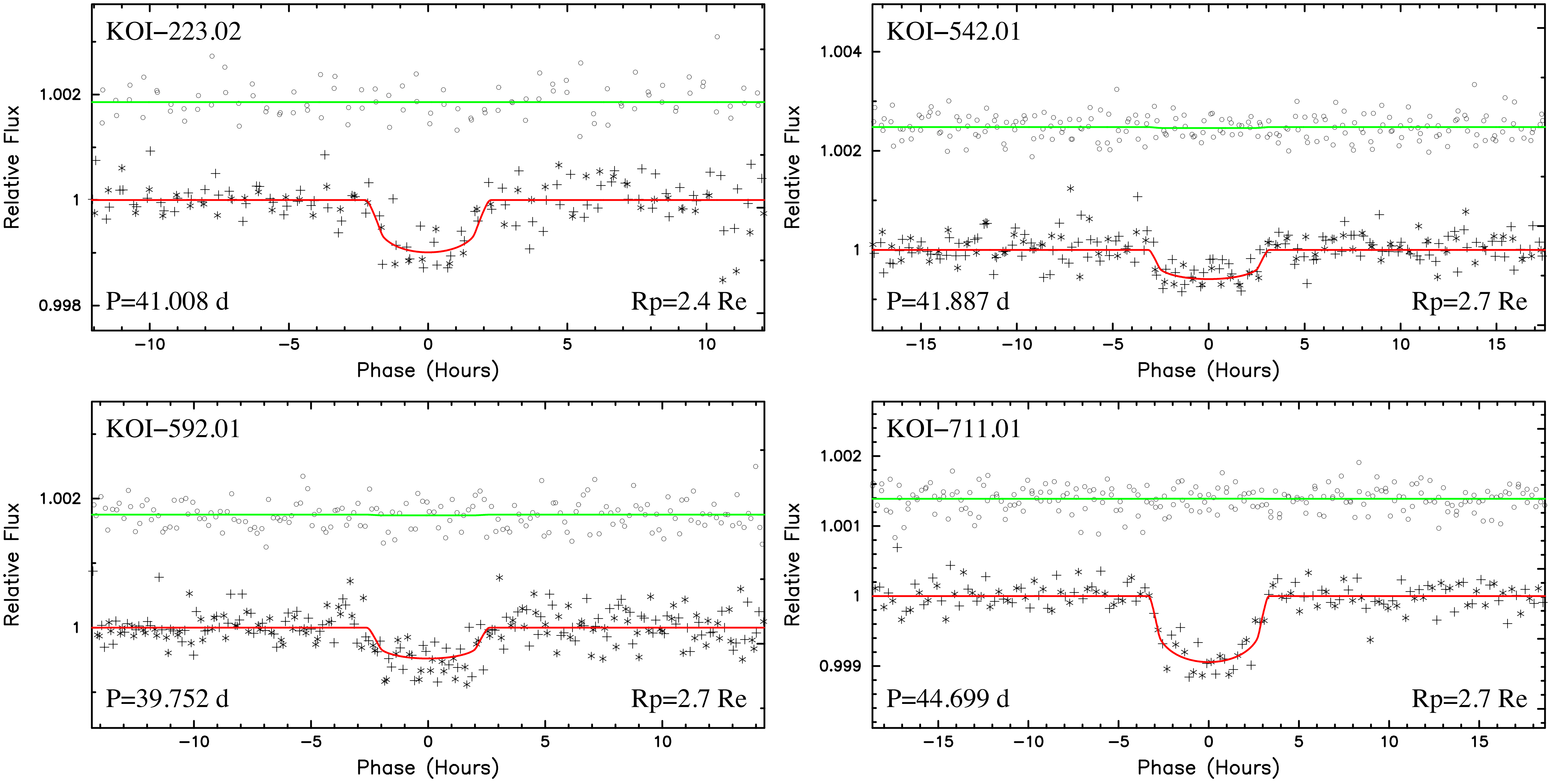
Choose stars carefully:

- Limit $T_{\rm eff}$ and $\log g$
- *Kp* cut (bright stars only)
- Use SNR cut based on RMS

Apply all cuts consistently to planets and stars

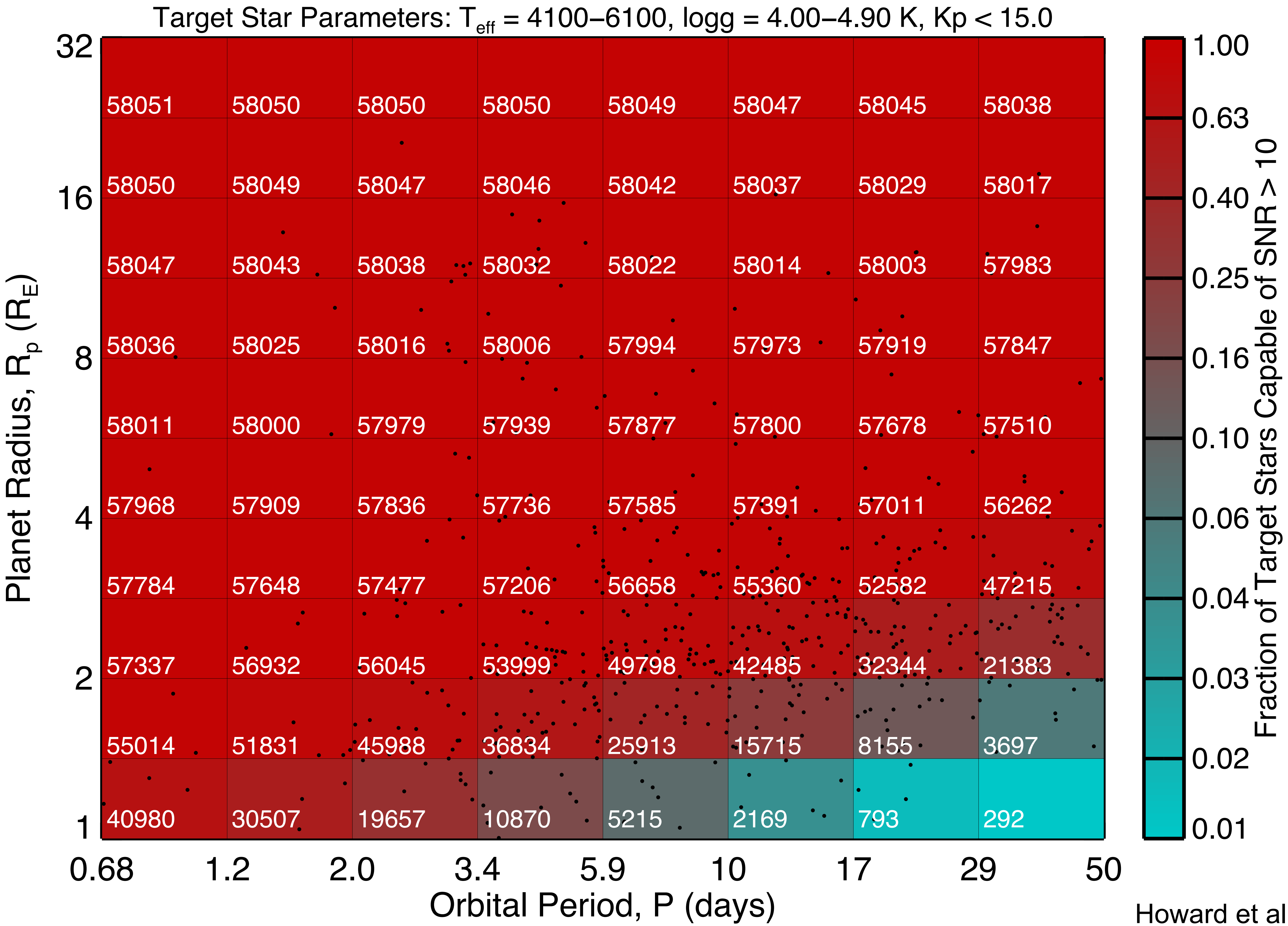
Kepler Transits with SNR ≈ 10

$$S/N = \frac{\delta}{\sigma_{\mathrm{CDPP}}}\sqrt{\frac{n_{\mathrm{tr}} \cdot t_{\mathrm{dur}}}{3\,\mathrm{hr}}}$$



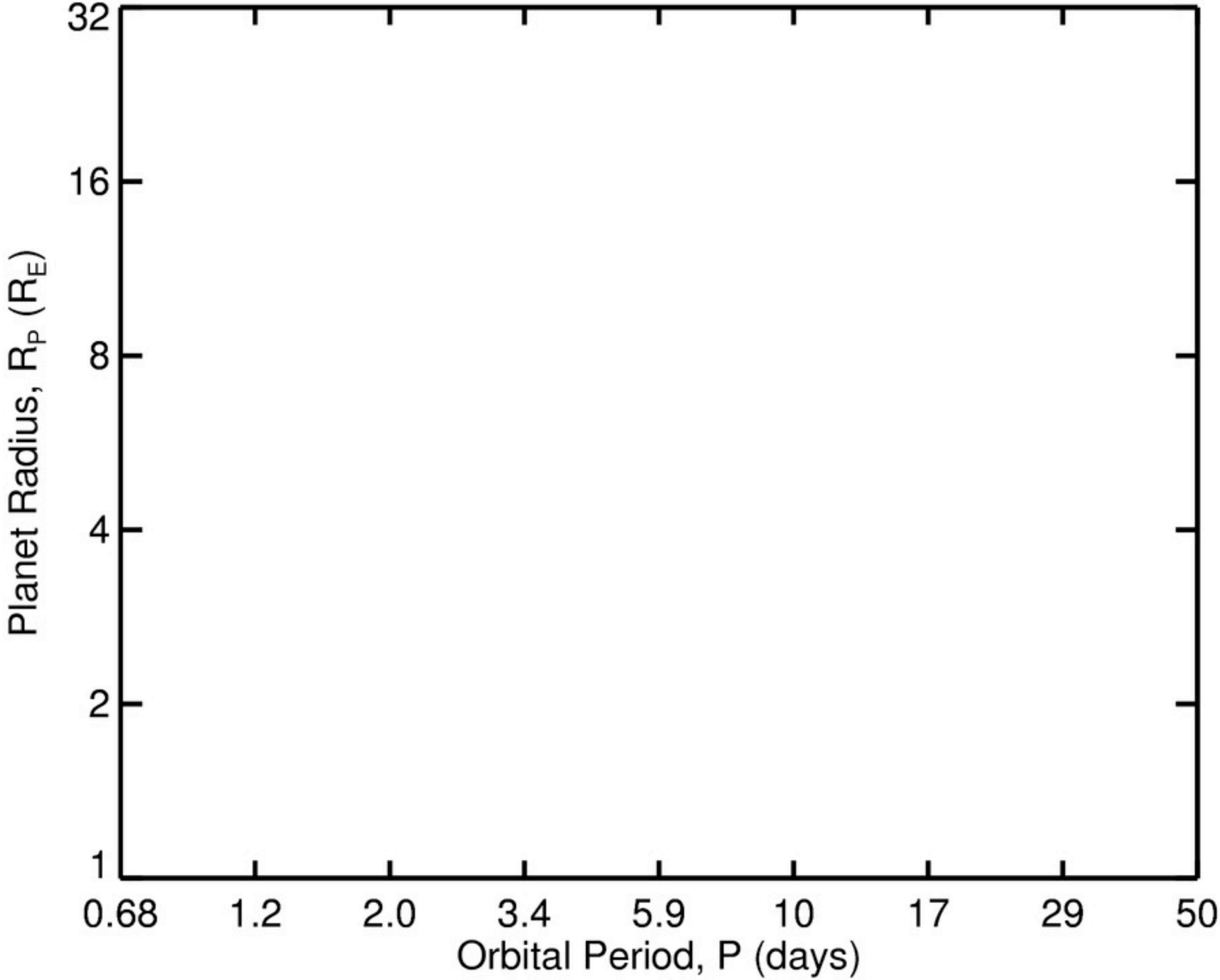
Howard et al. (2012)

Detection Completeness
SNR > 10 in 90-day quarter

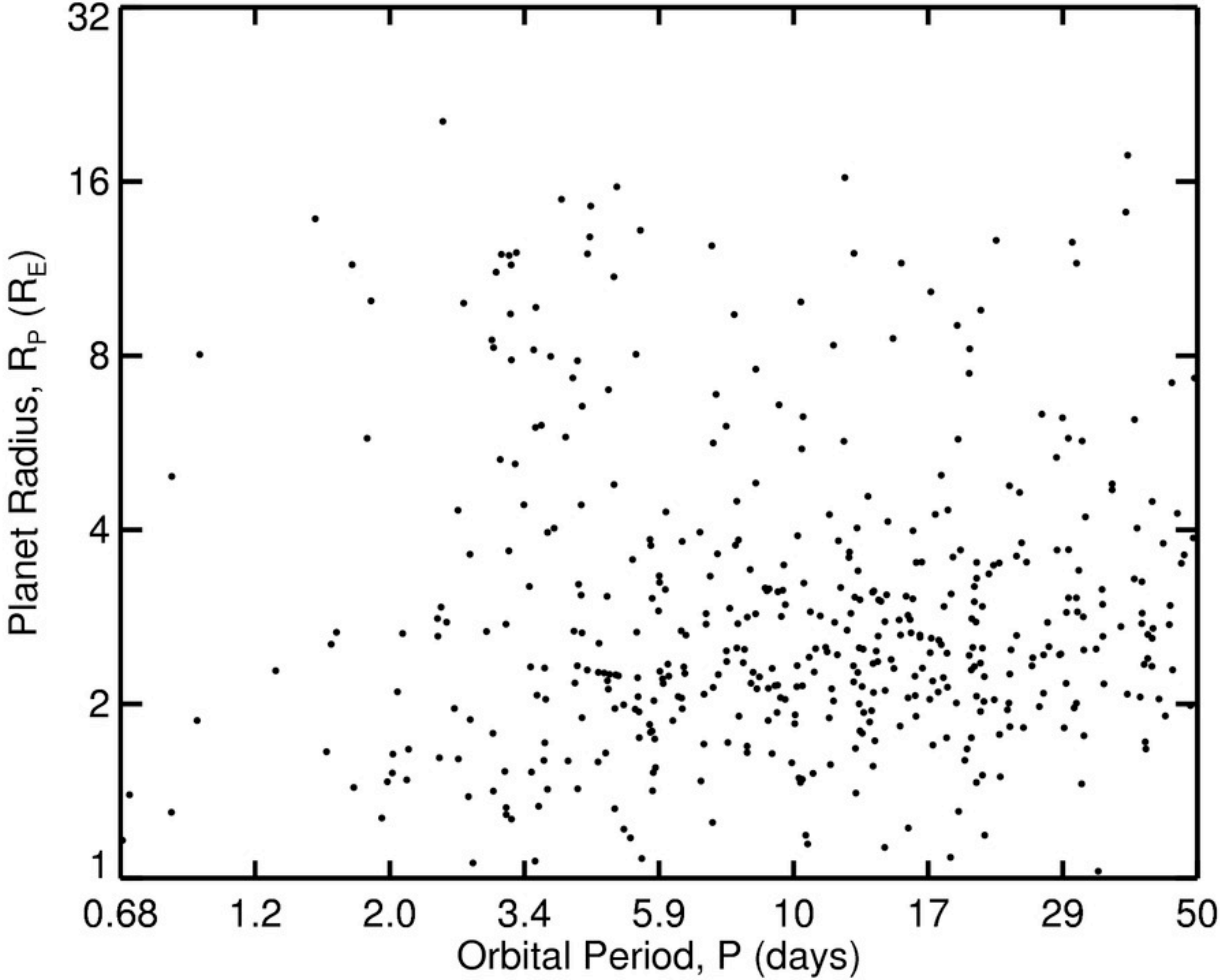


58051	58050	58050	58050	58049	58047	58045	58038
58050	58049	58047	58046	58042	58037	58029	58017
58047	58043	58038	58032	58022	58014	58003	57983
58036	58025	58016	58006	57994	57973	57919	57847
58011	58000	57979	57939	57877	57800	57678	57510
57968	57909	57836	57736	57585	57391	57011	56262
57784	57648	57477	57206	56658	55360	52582	47215
57337	56932	56045	53999	49798	42485	32344	21383
55014	51831	45988	36834	25913	15715	8155	3697
40980	30507	19657	10870	5215	2169	793	292

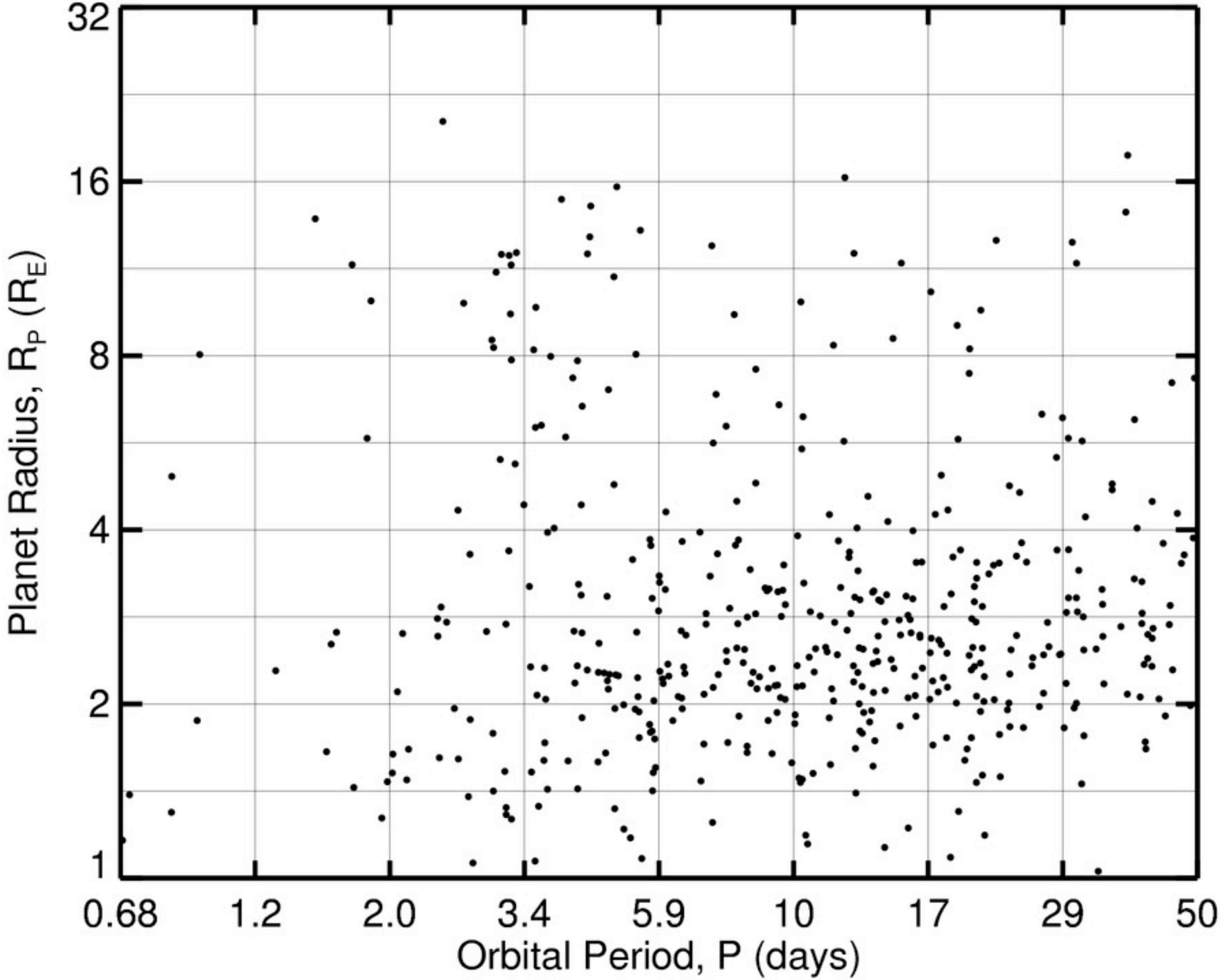
Howard et al. (2012)



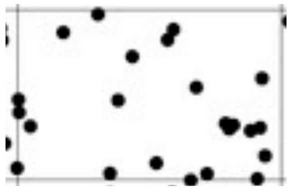
Howard et al. (2012)



Howard et al. (2012)



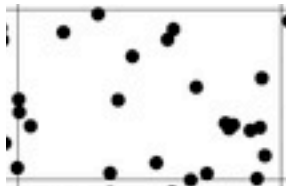
Howard et al. (2012)



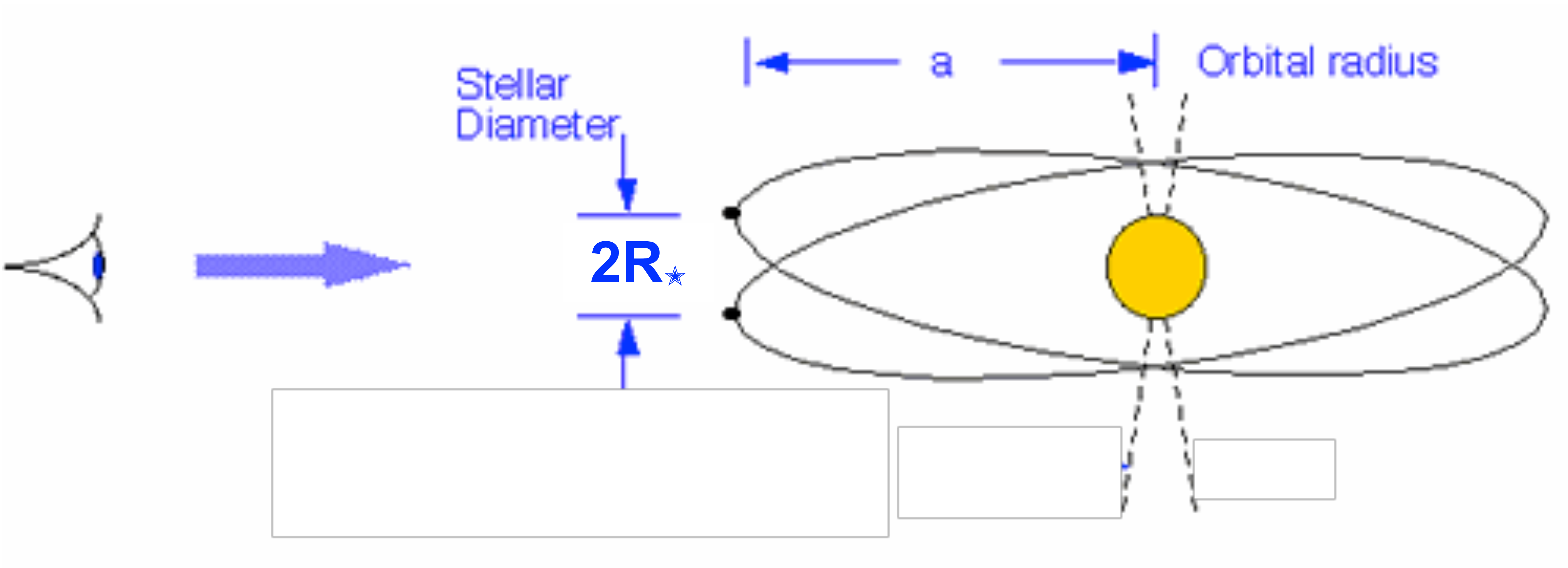
Howard et al. (2012)

For each detected planet, we know:

$p_{transit} = R_{\star}/a$ - transit probability



Howard et al. (2012)

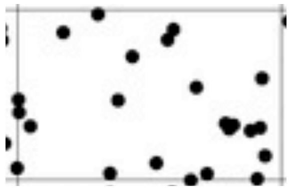


Probability of Transit = $R_\star/a$

For each detected planet, we know:

$p_{transit} = R_\star/a$ - transit probability

$n_\star$ - number of stars around which that planet could have been detected with SNR > 10



Howard et al. (2012)

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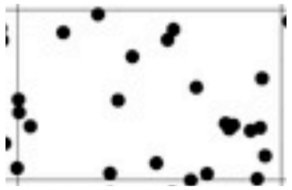


For each detected planet

there are actually $1/p_{transit}$ planets

in all orbital inclinations orbiting $n_\star$ stars

(augment detected planets)

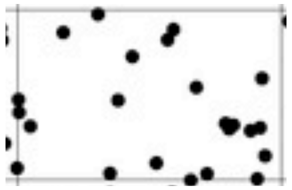
Howard et al. (2012)

For each detected planet, we know:

$p_{transit} = R_\star / a$ - transit probability

$n_\star$ - number of stars around which that planet could have been detected with SNR > 10

$$\boxed{\text{Occurrence} = \sum (1/p_{transit}) \,/\, n_\star}$$



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Howard et al. (2012)

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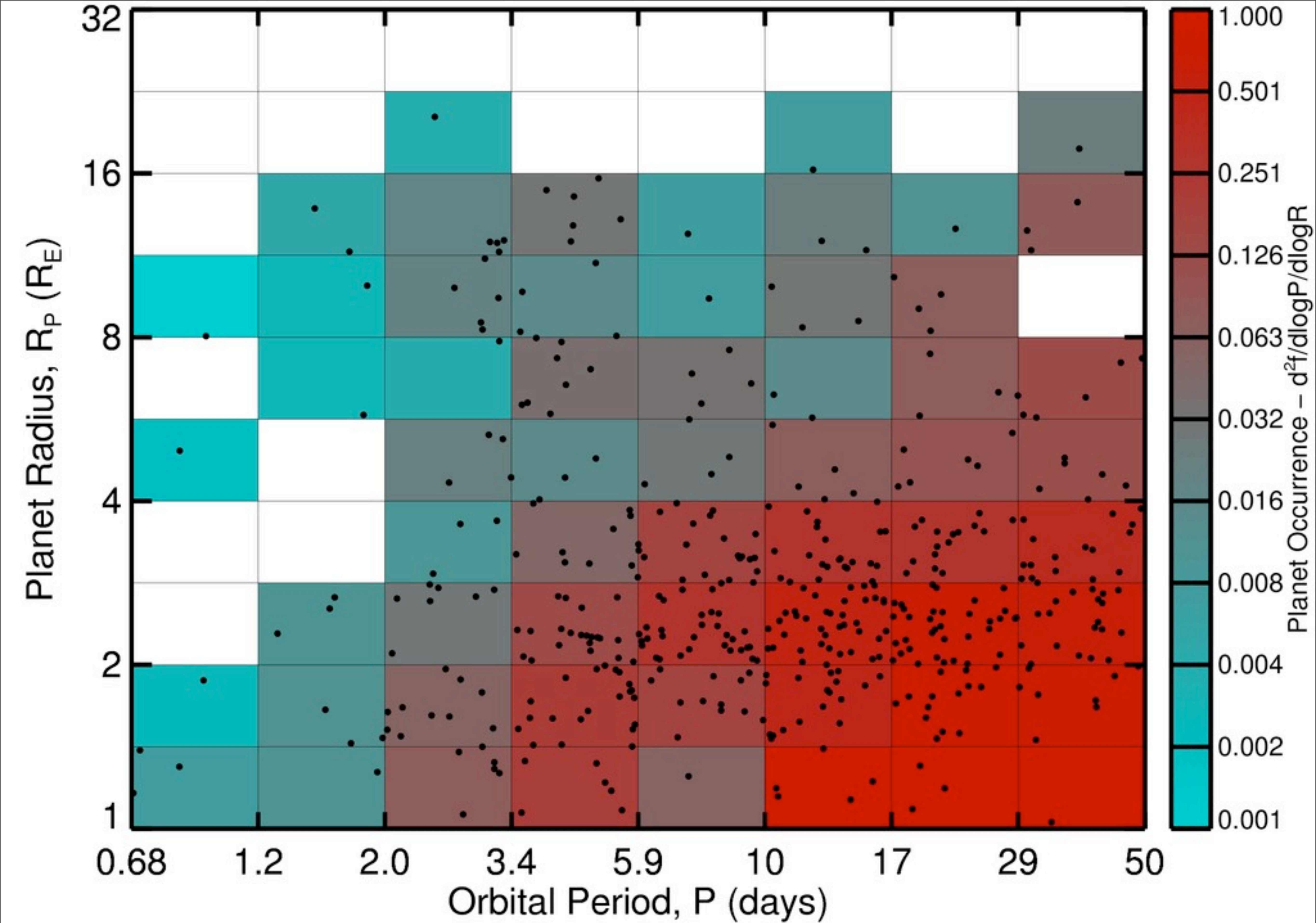
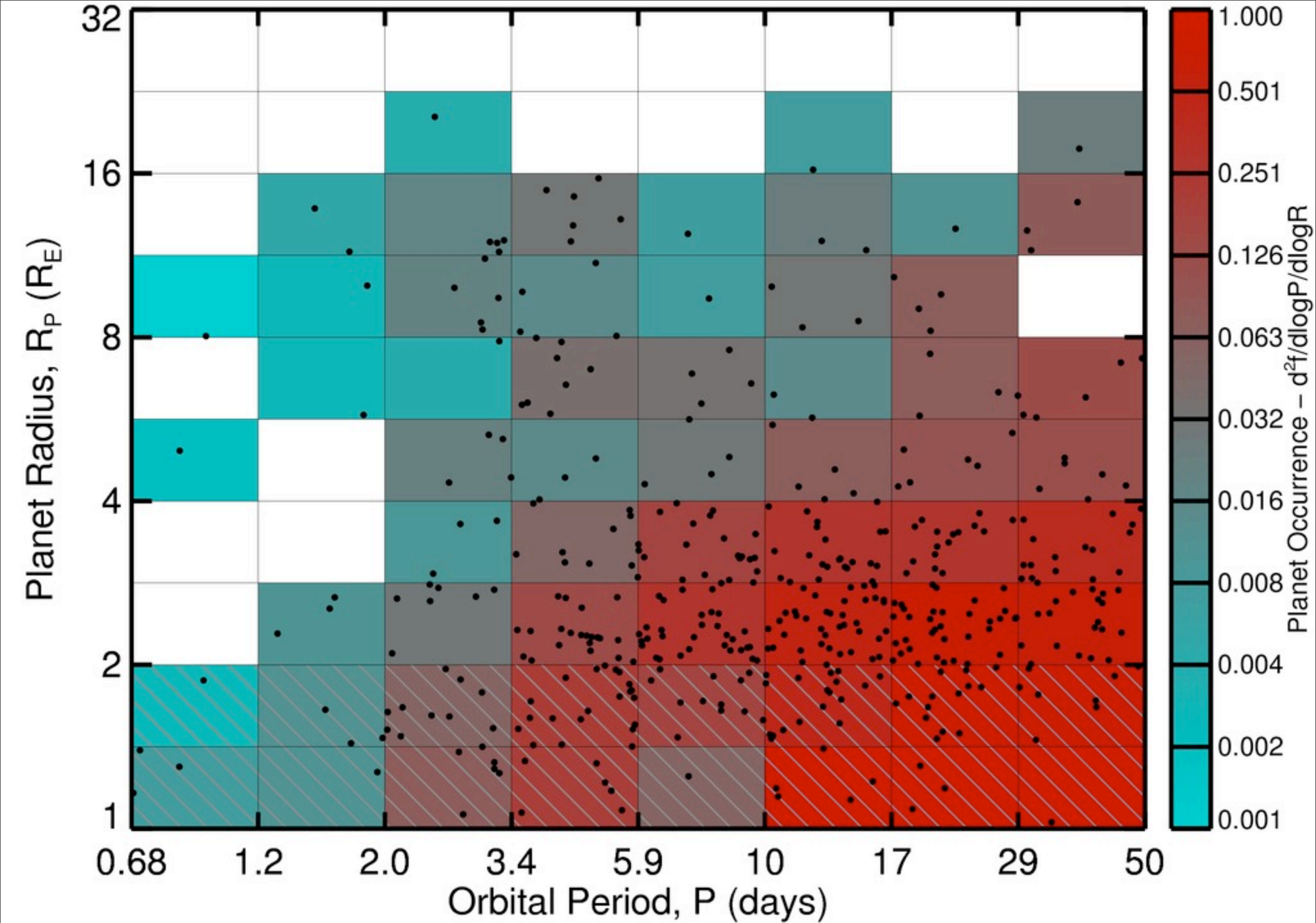
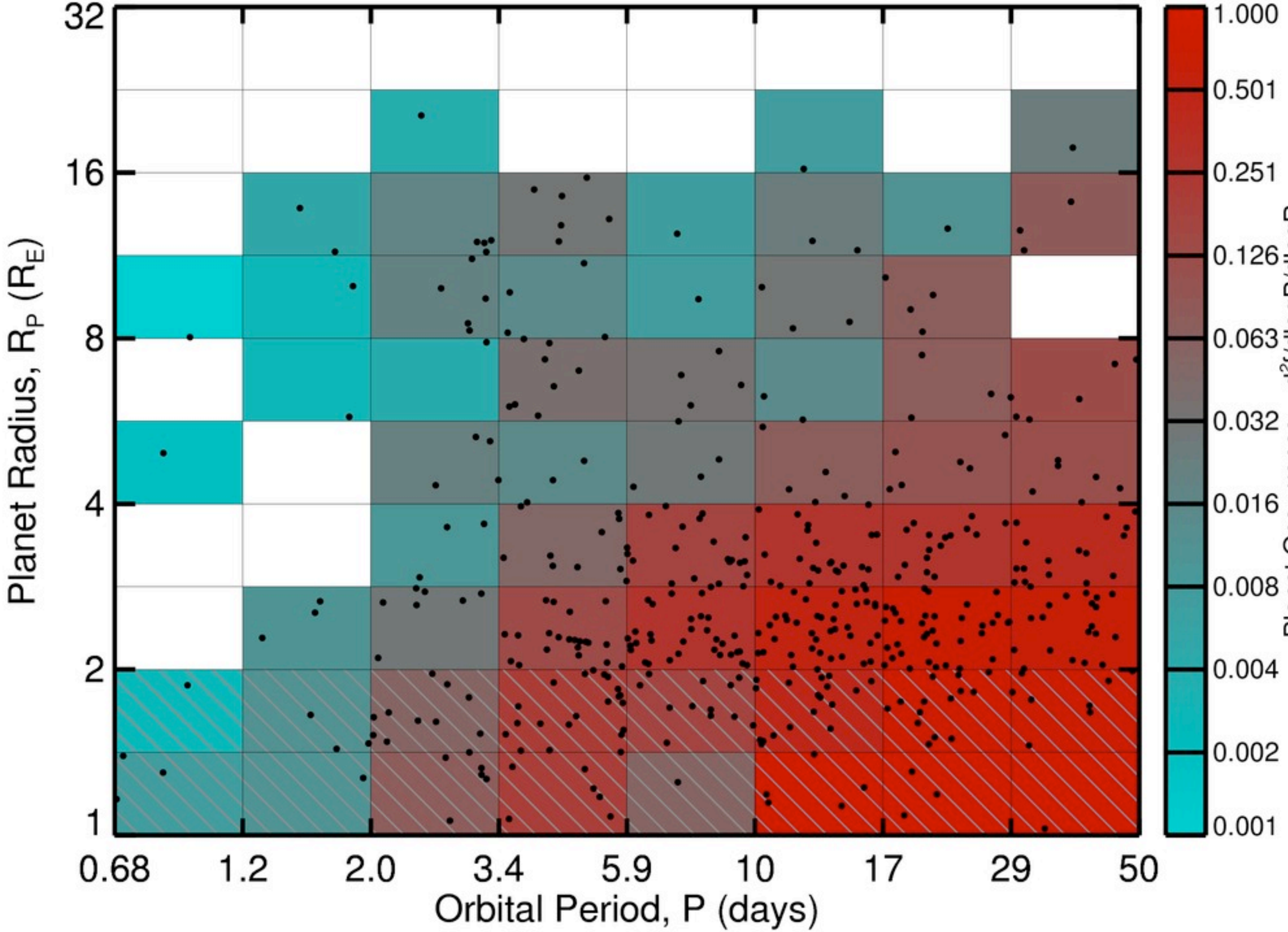


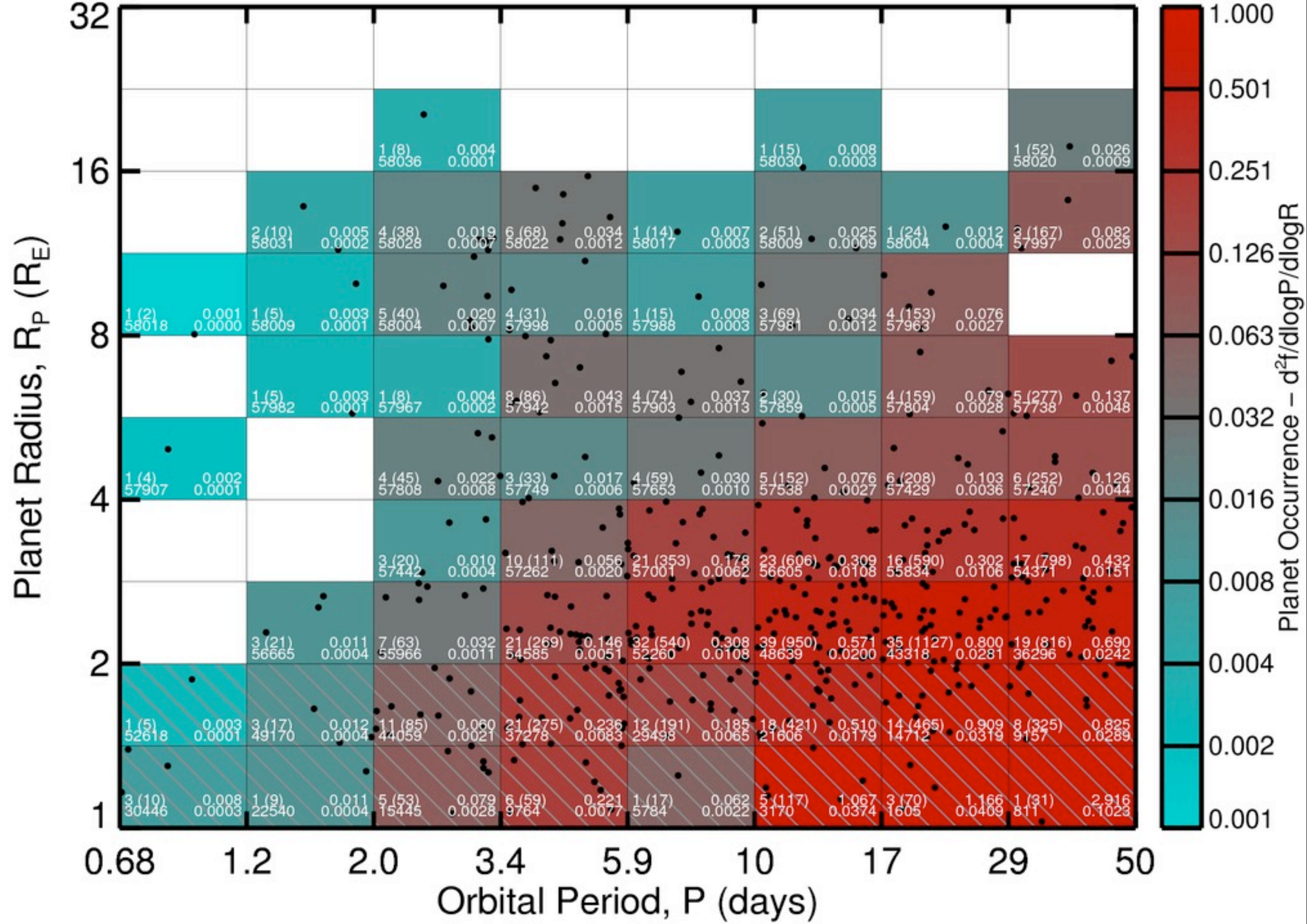
Howard et al. (2012)



Howard et al. (2012)

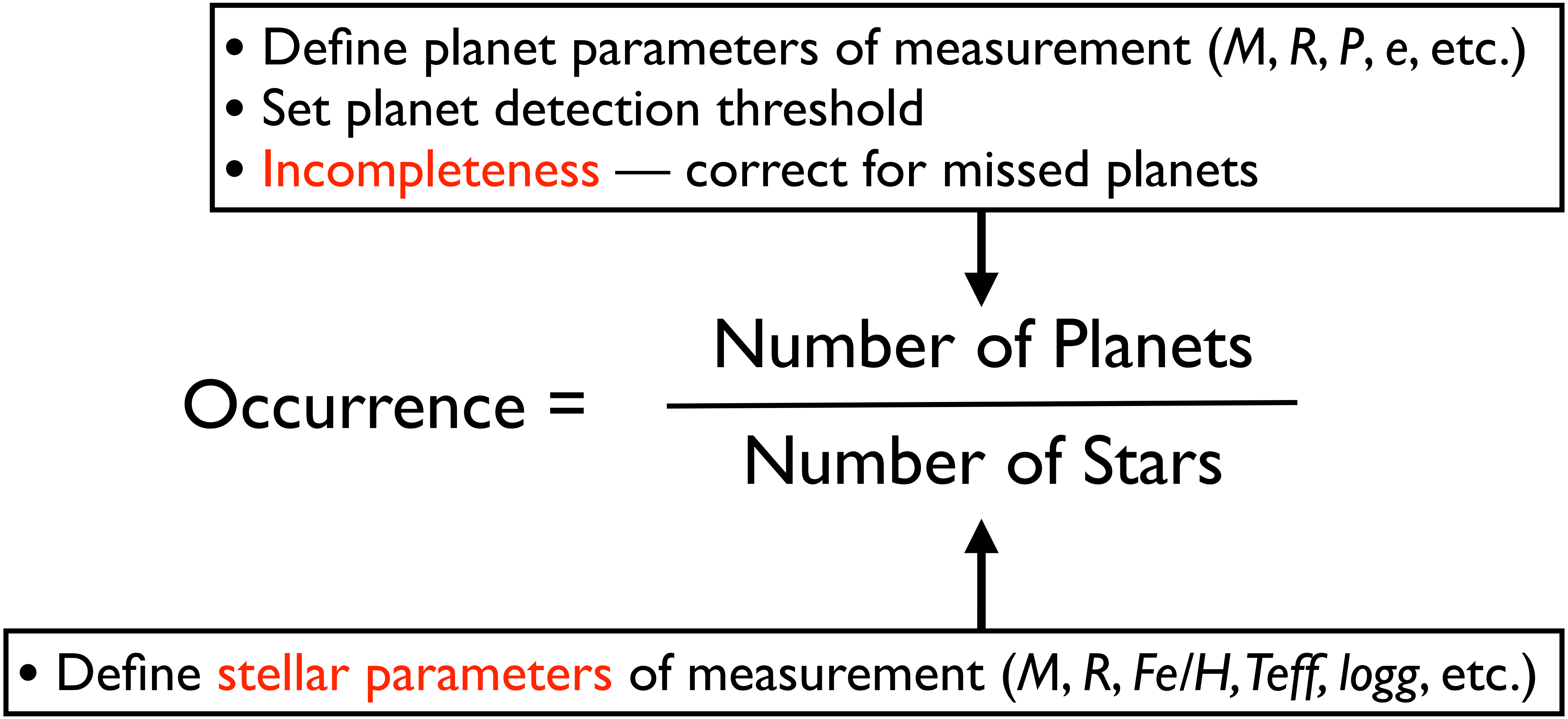


Howard et al. (2012)



Howard et al. (2012)

Kepler Occurrence



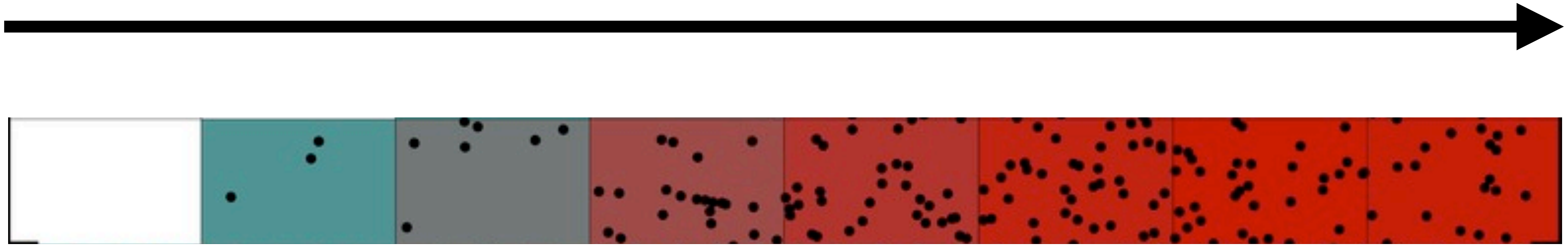
- Define planet parameters of measurement (*M*, *R*, *P*, e, etc.)
- Set planet detection threshold
- Incompleteness — correct for missed planets

$$\text{Occurrence} = \frac{\text{Number of Planets}}{\text{Number of Stars}}$$

- Define stellar parameters of measurement (*M*, *R*, *Fe/H*, *Teff*, *logg*, etc.)

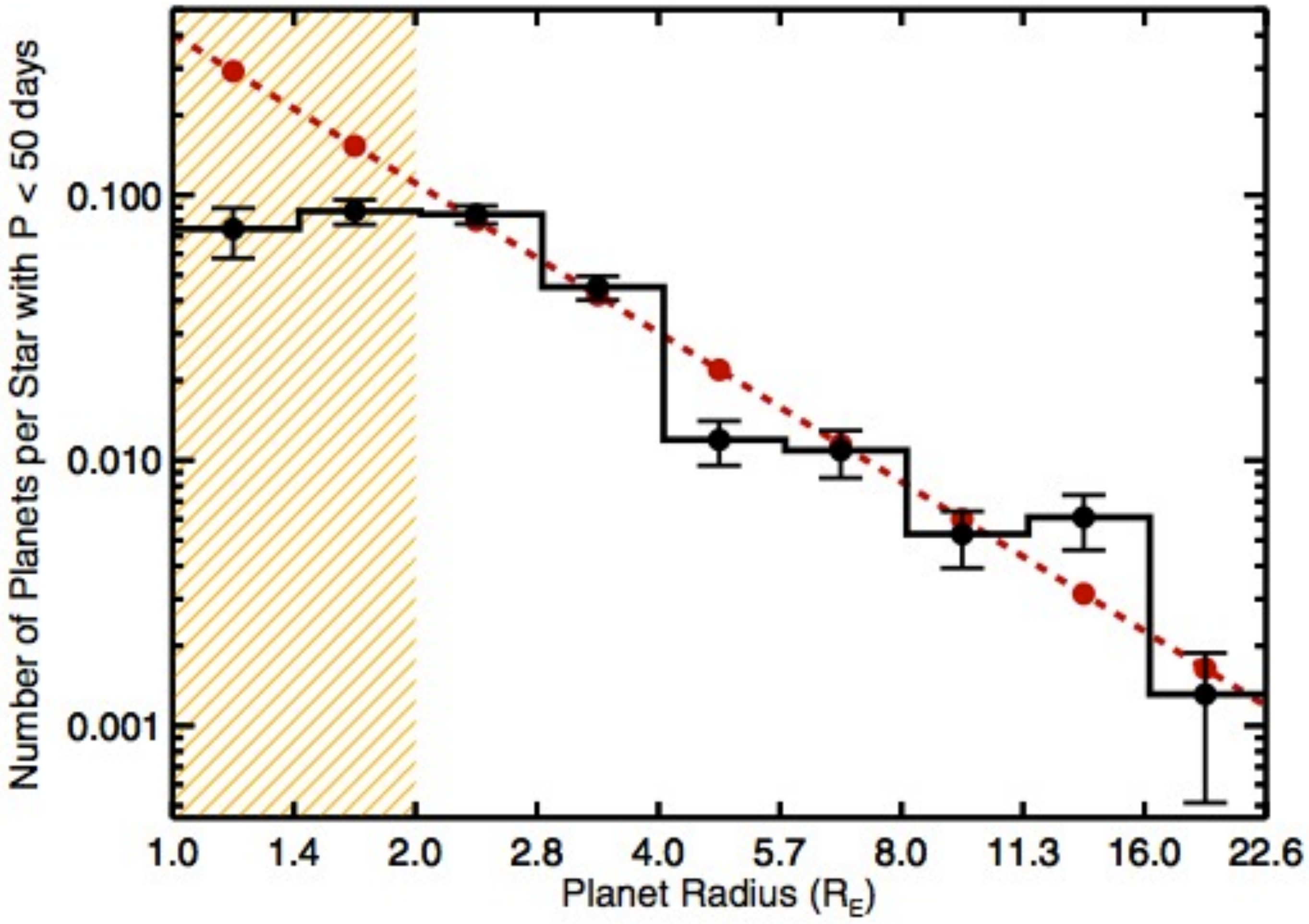
Compute Occurrence vs. Planet Radius

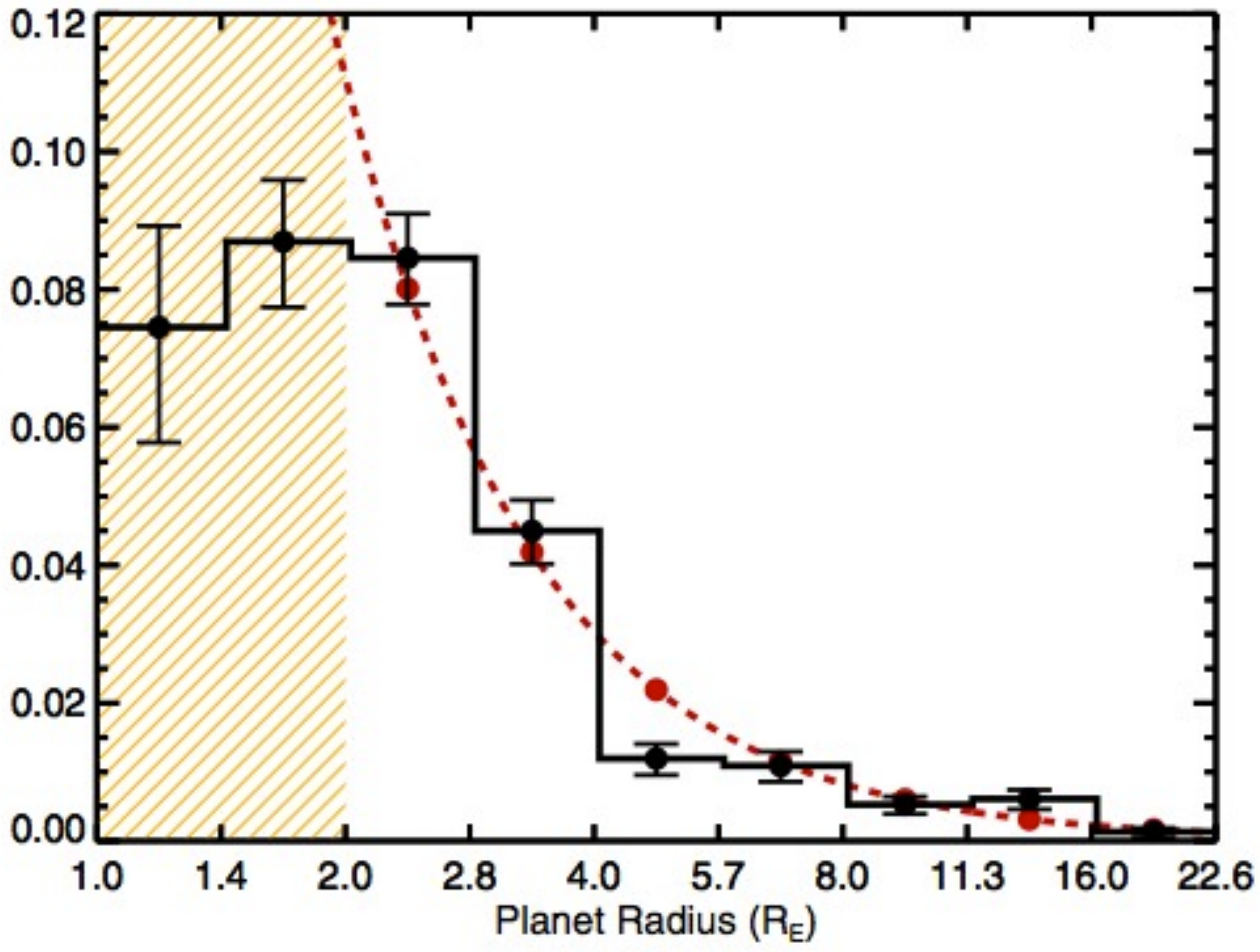
Sum *Occurrence*
for all Periods
in R + ΔR



Howard et al. (2012)

Planet Radius Distribution





Power law:

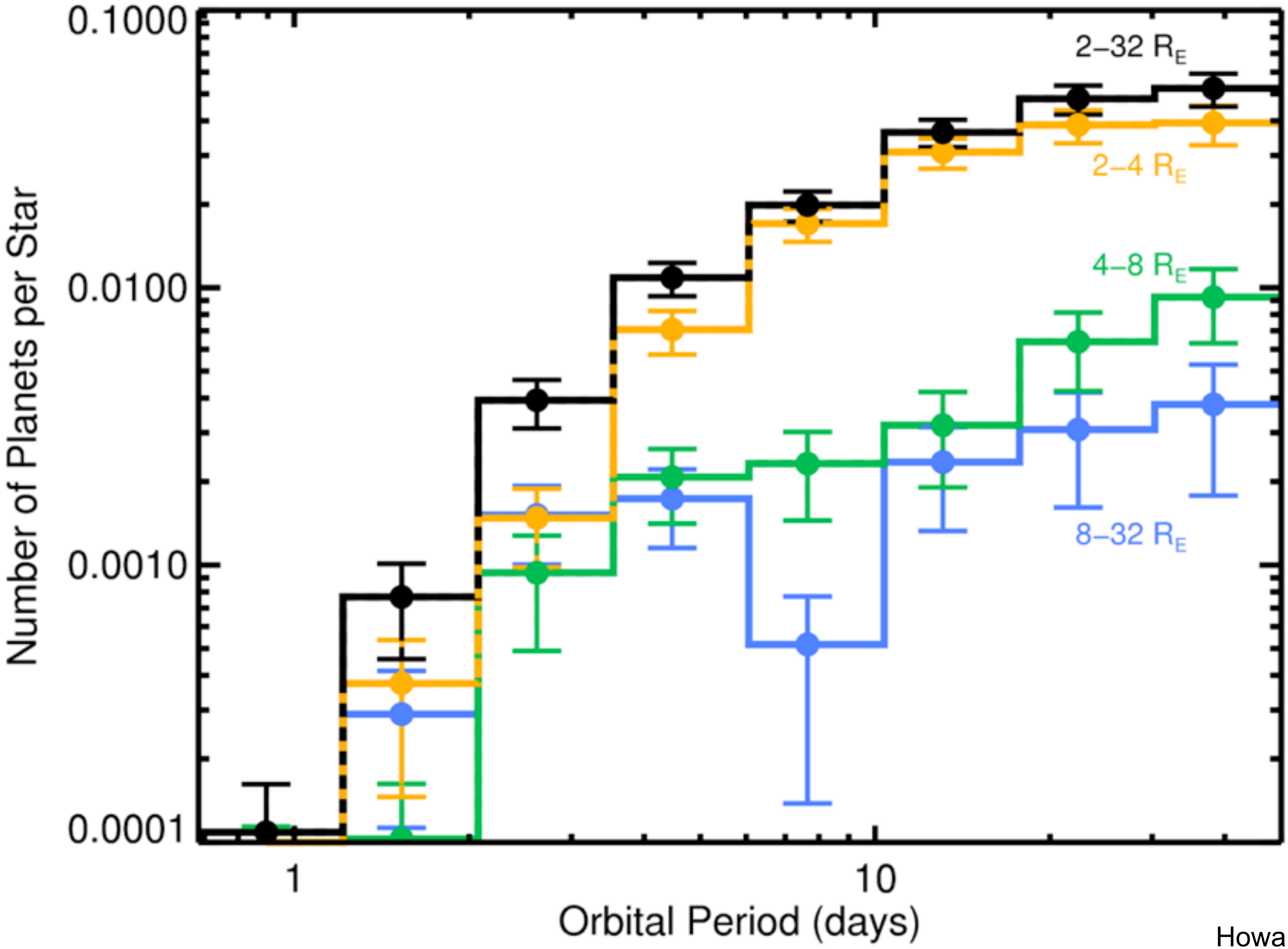
$dN/d\log R = kR^{\alpha}$

$k = 2.9 \pm 0.5$

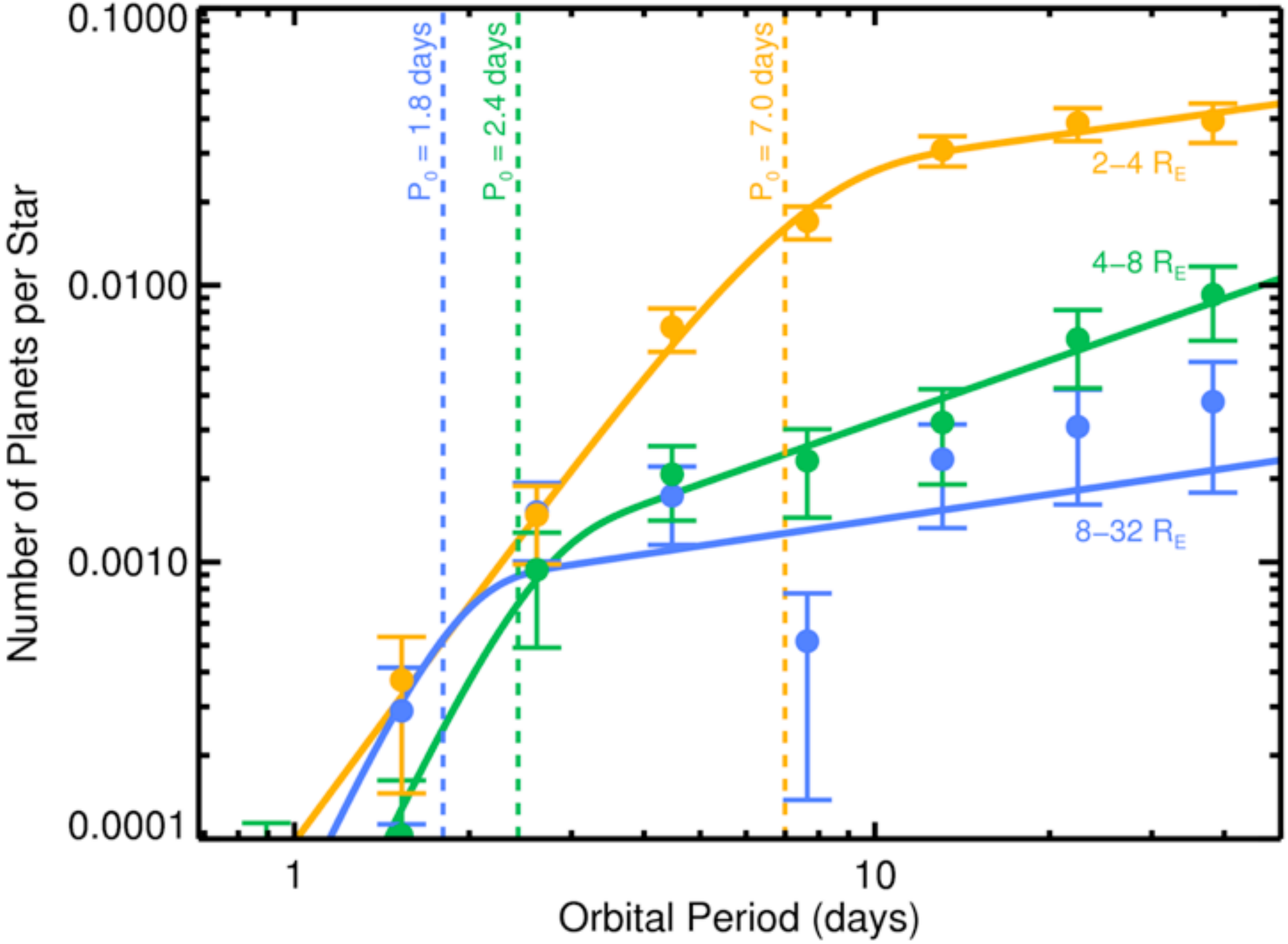
$\alpha = -1.92 \pm 0.11$

Howard et al. (2012)

Planet Occurrence vs. Orbital Period



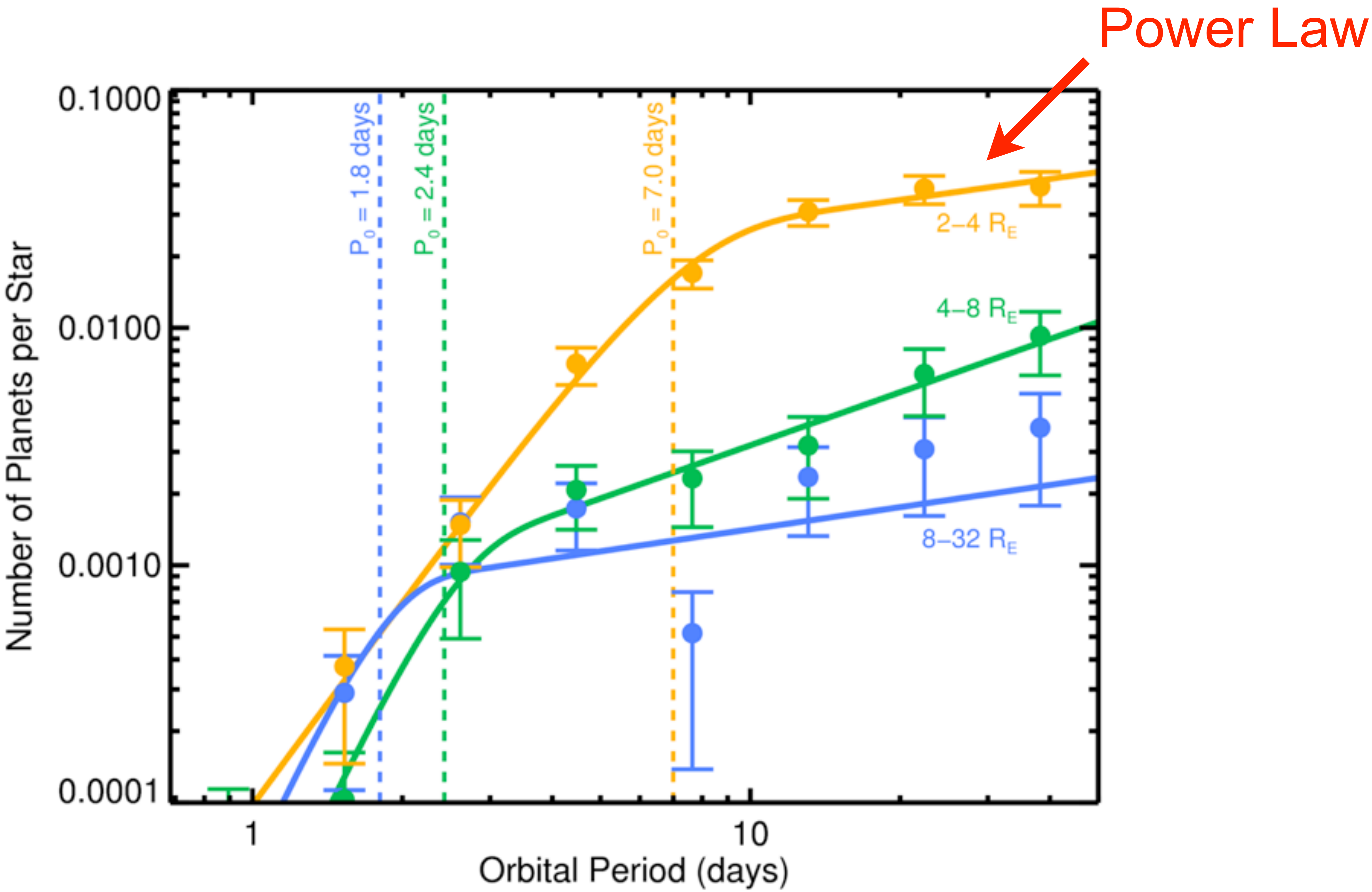
Howard et al. (2012)



Power law with Exponential Cutoff:

$$dN/d\log P = kP^{\beta}\,(1-\exp((P/P_0)^{\gamma}))$$

Howard et al. (2012)



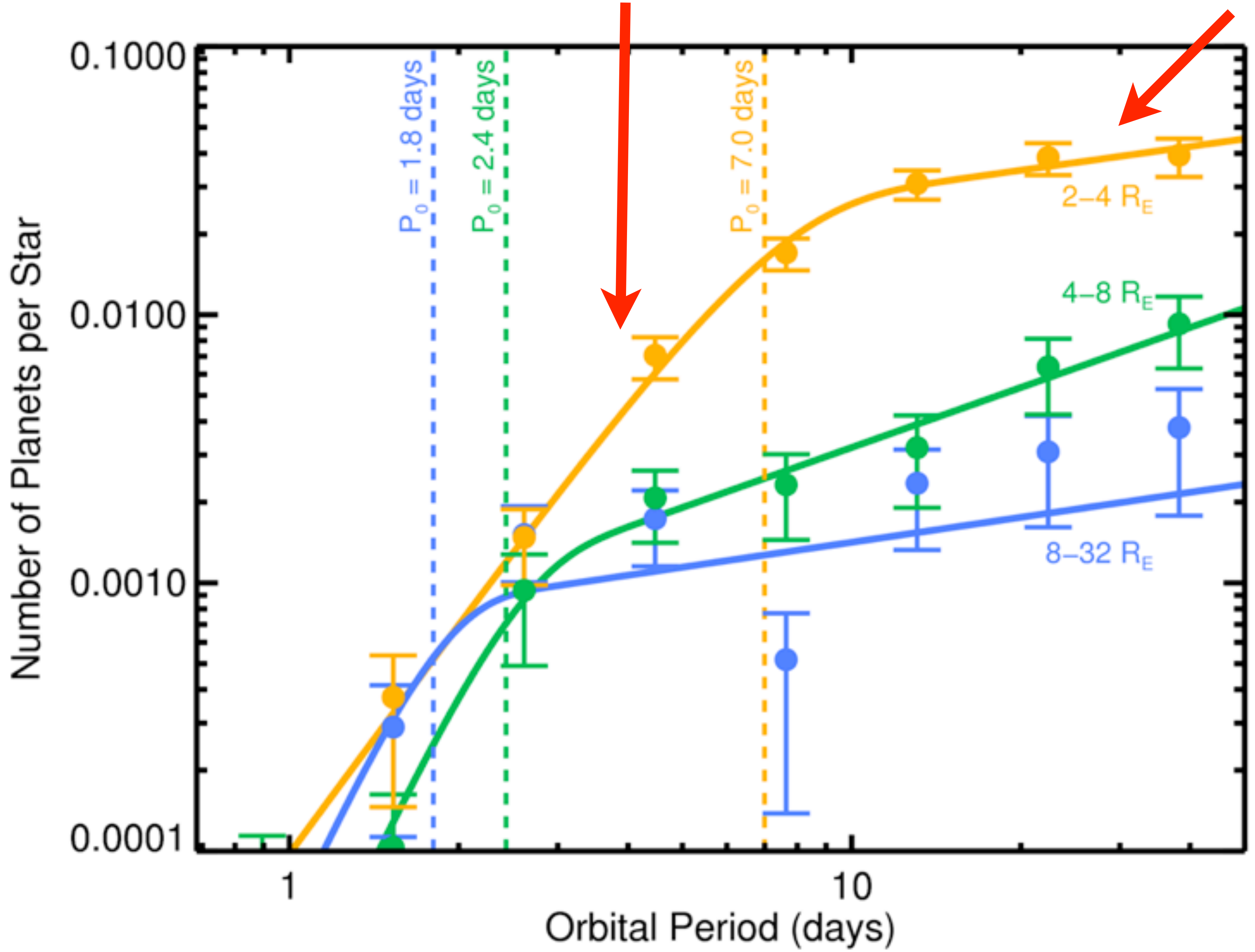
Power law with Exponential Cutoff:

$$dN/d\log P = kP^{\beta}(1-\exp((P/P_0)^{\gamma}))$$

Howard et al. (2012)

Exponential Cutoff

Power Law



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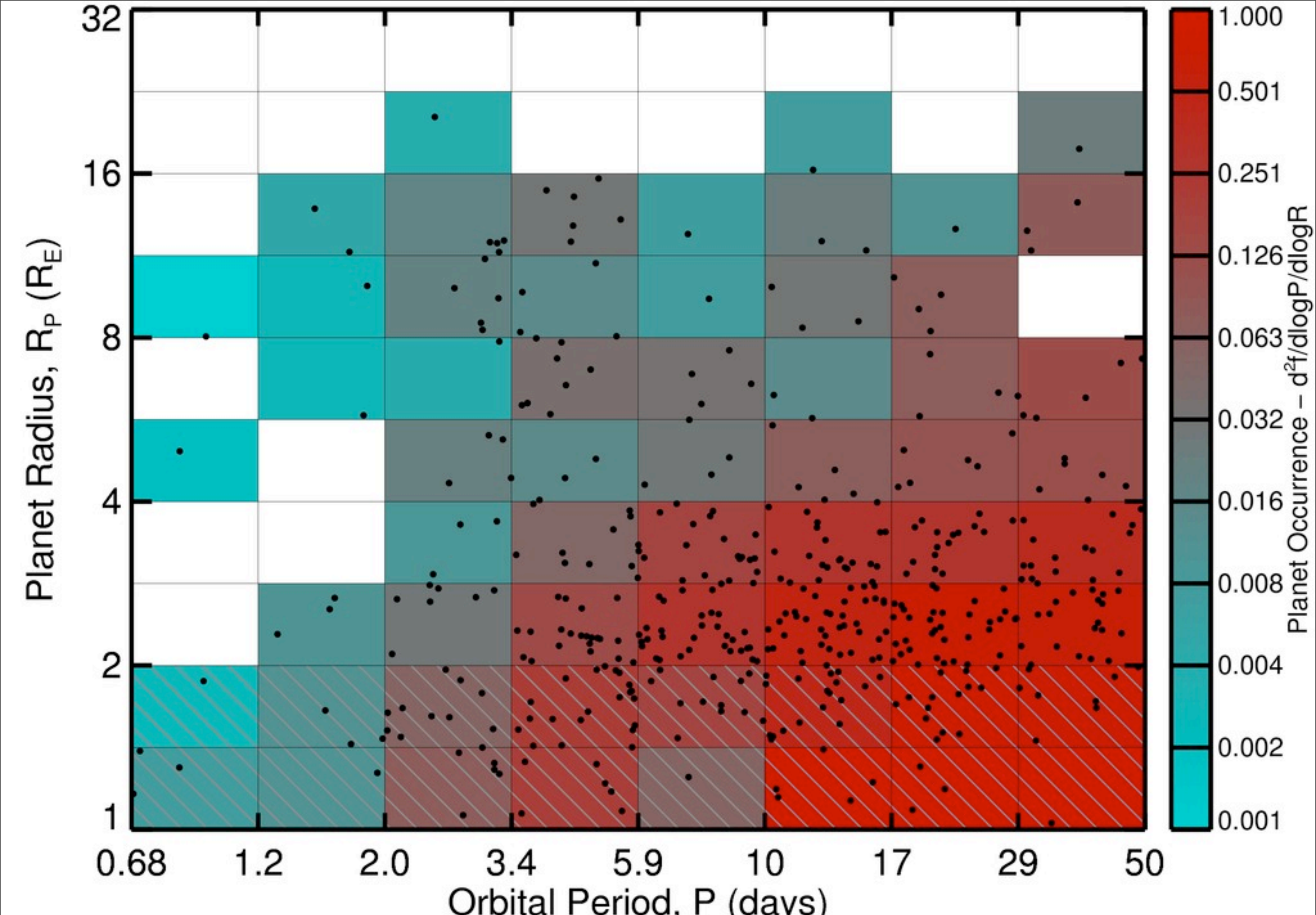
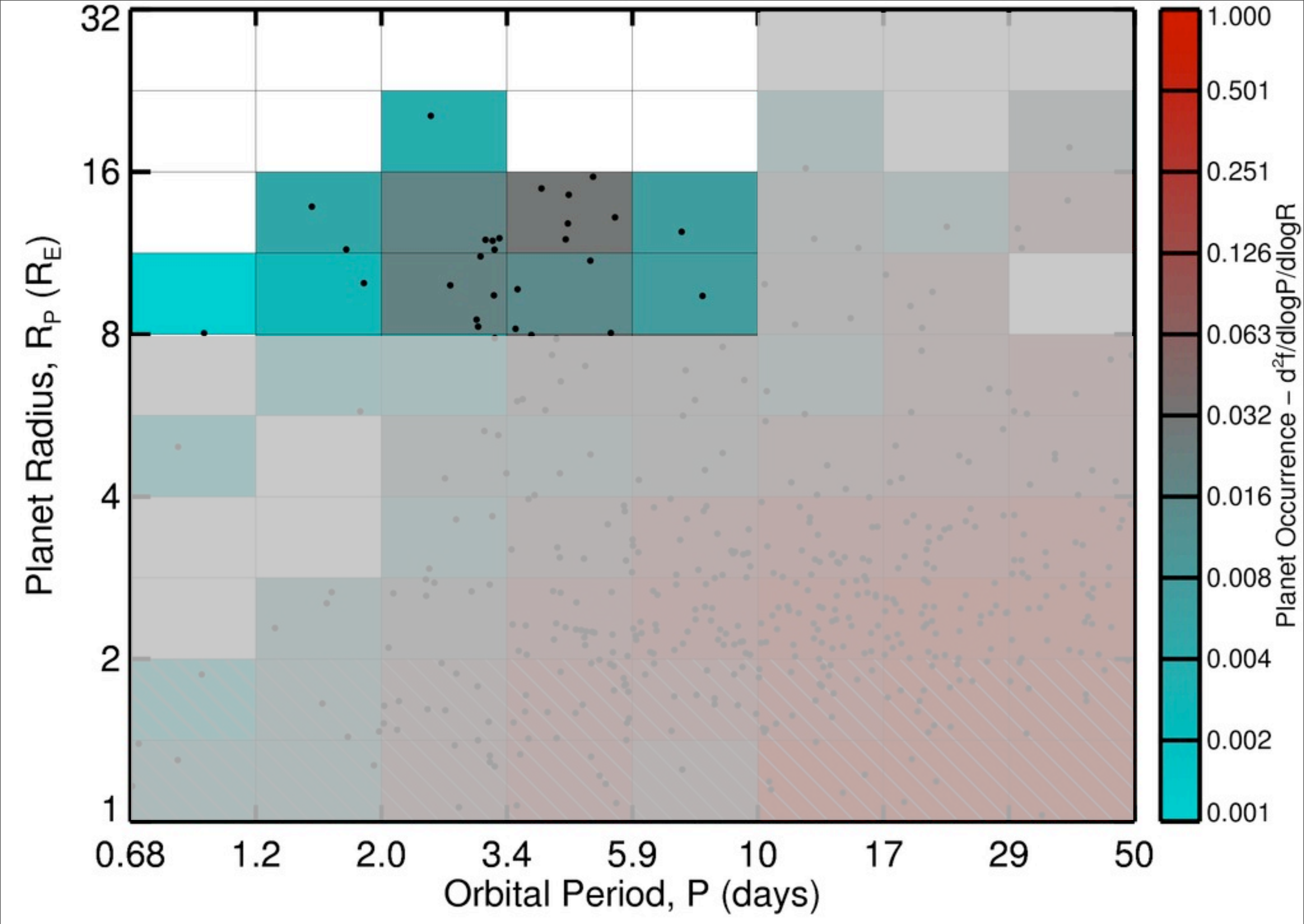
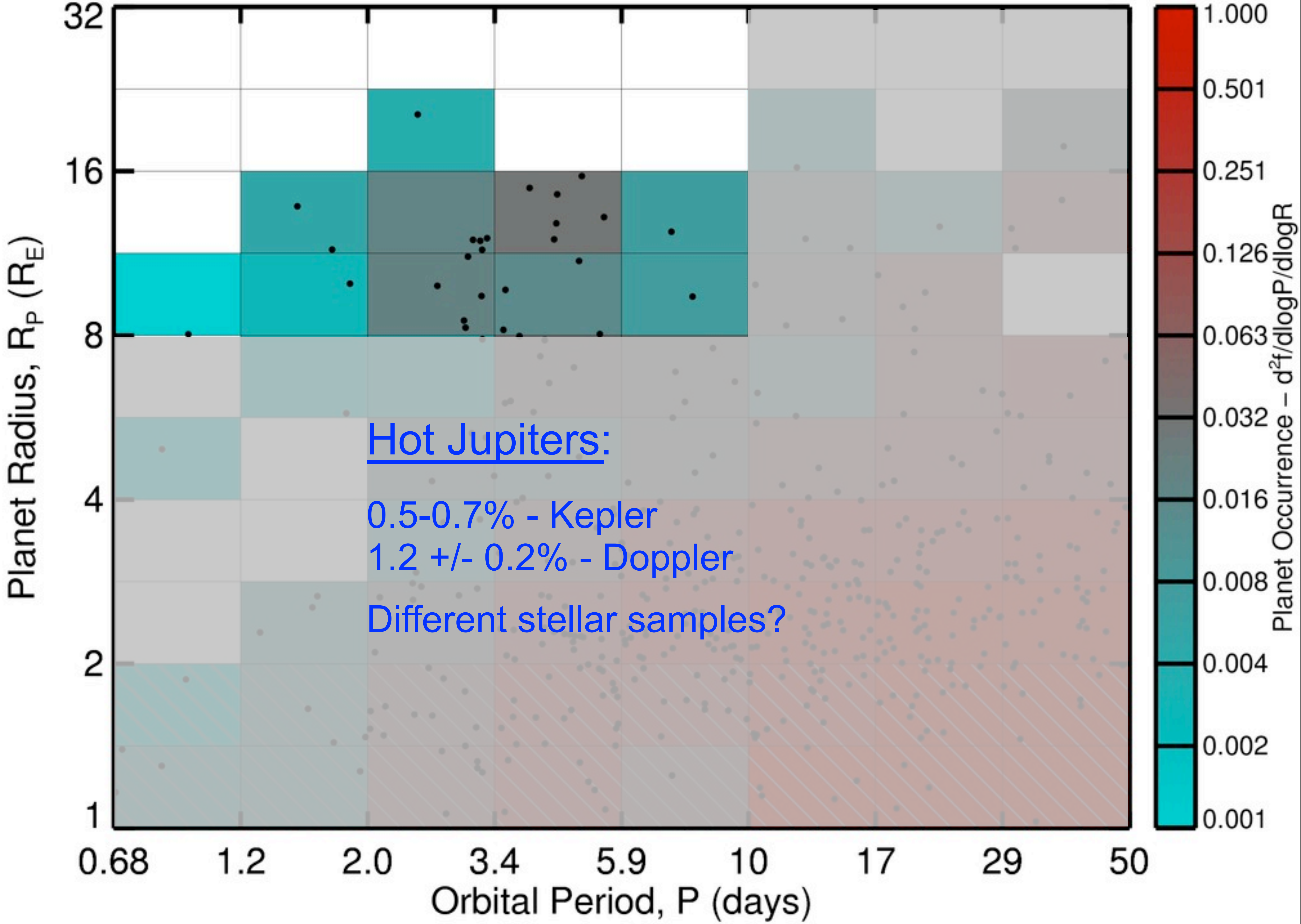


Howard et al. (2012)



Howard et al. (2012)



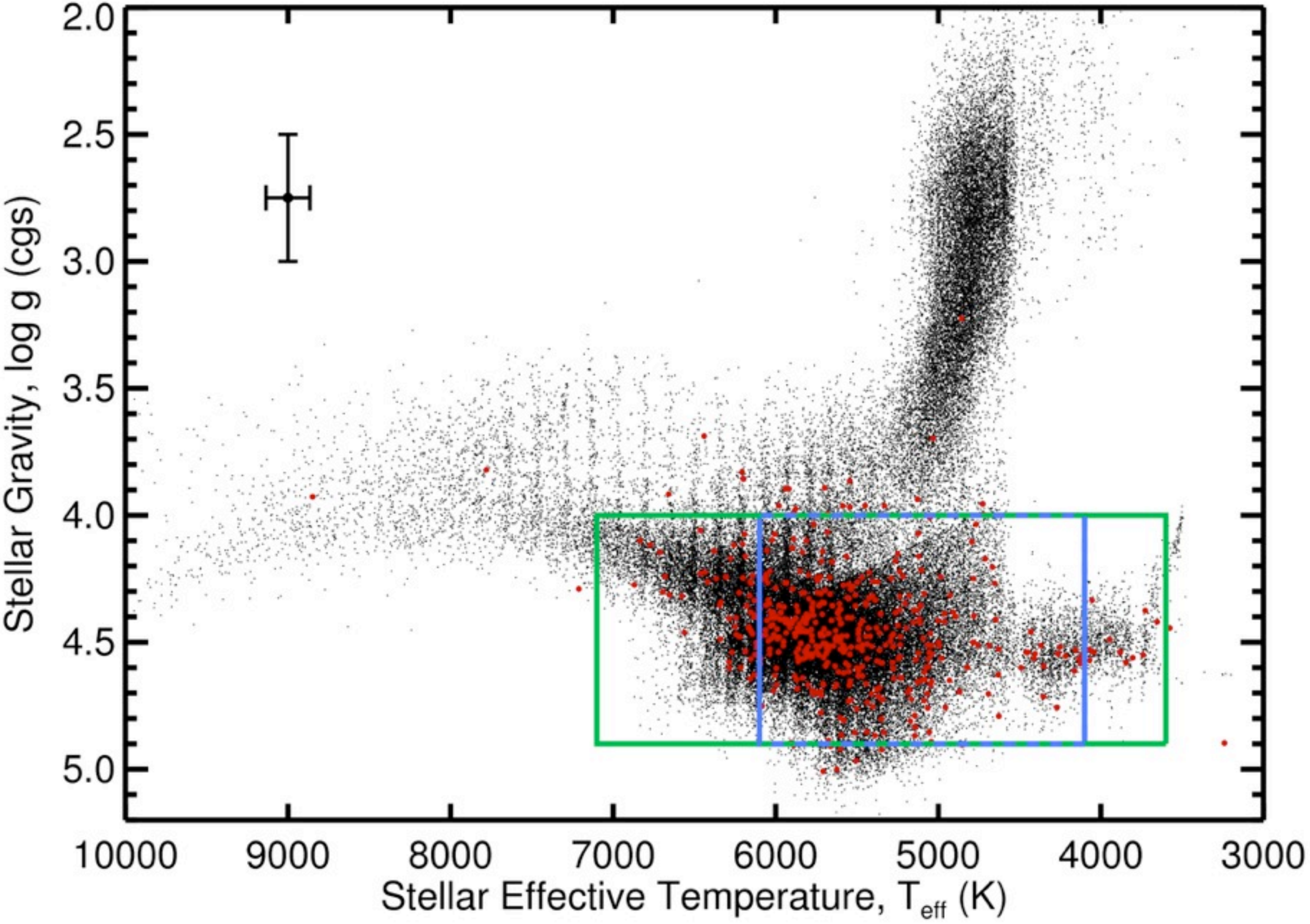
Hot Jupiters:

0.5-0.7% - Kepler
1.2 +/- 0.2% - Doppler

Different stellar samples?

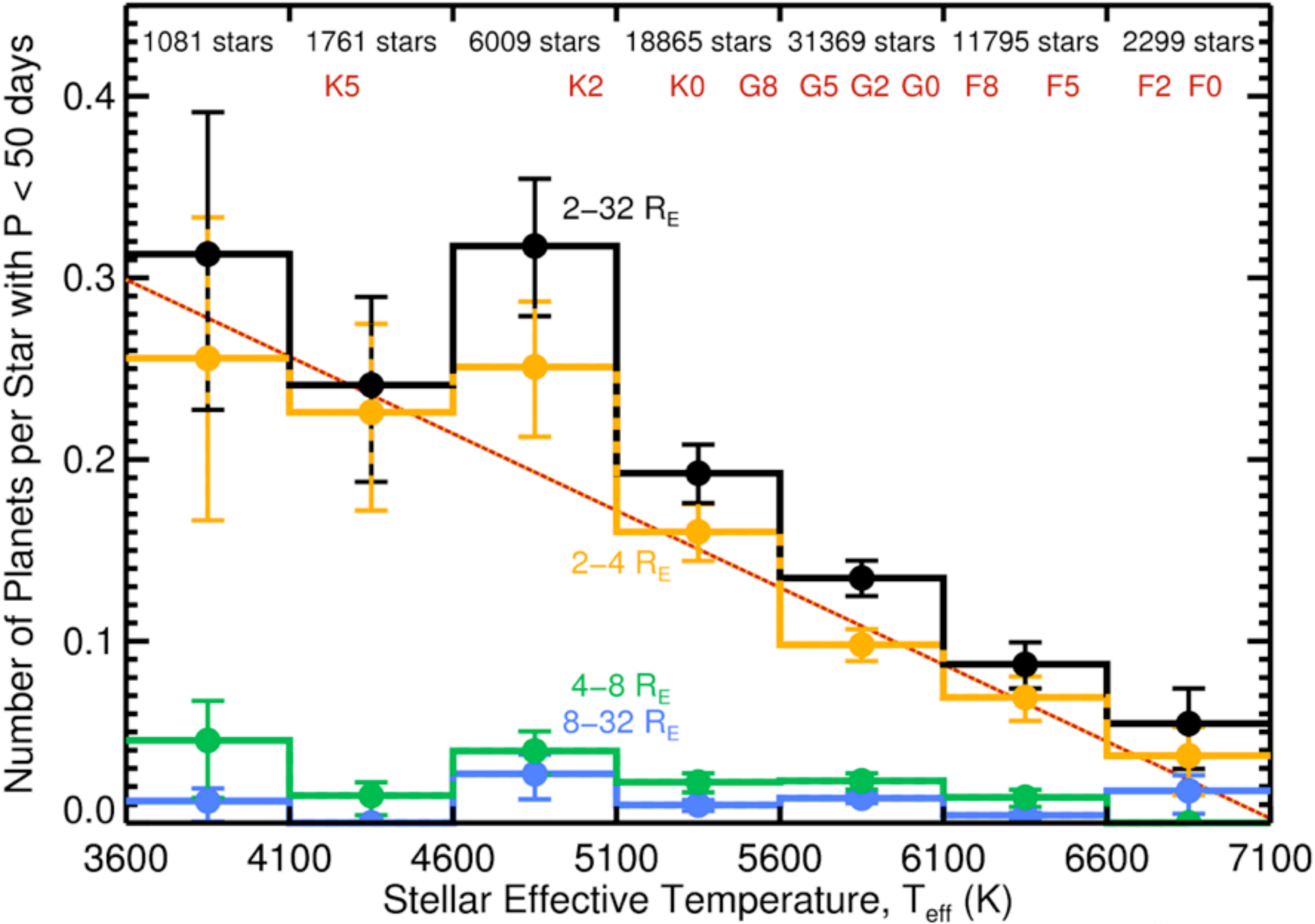
Howard et al. (2012)

Expanded Stellar Sample



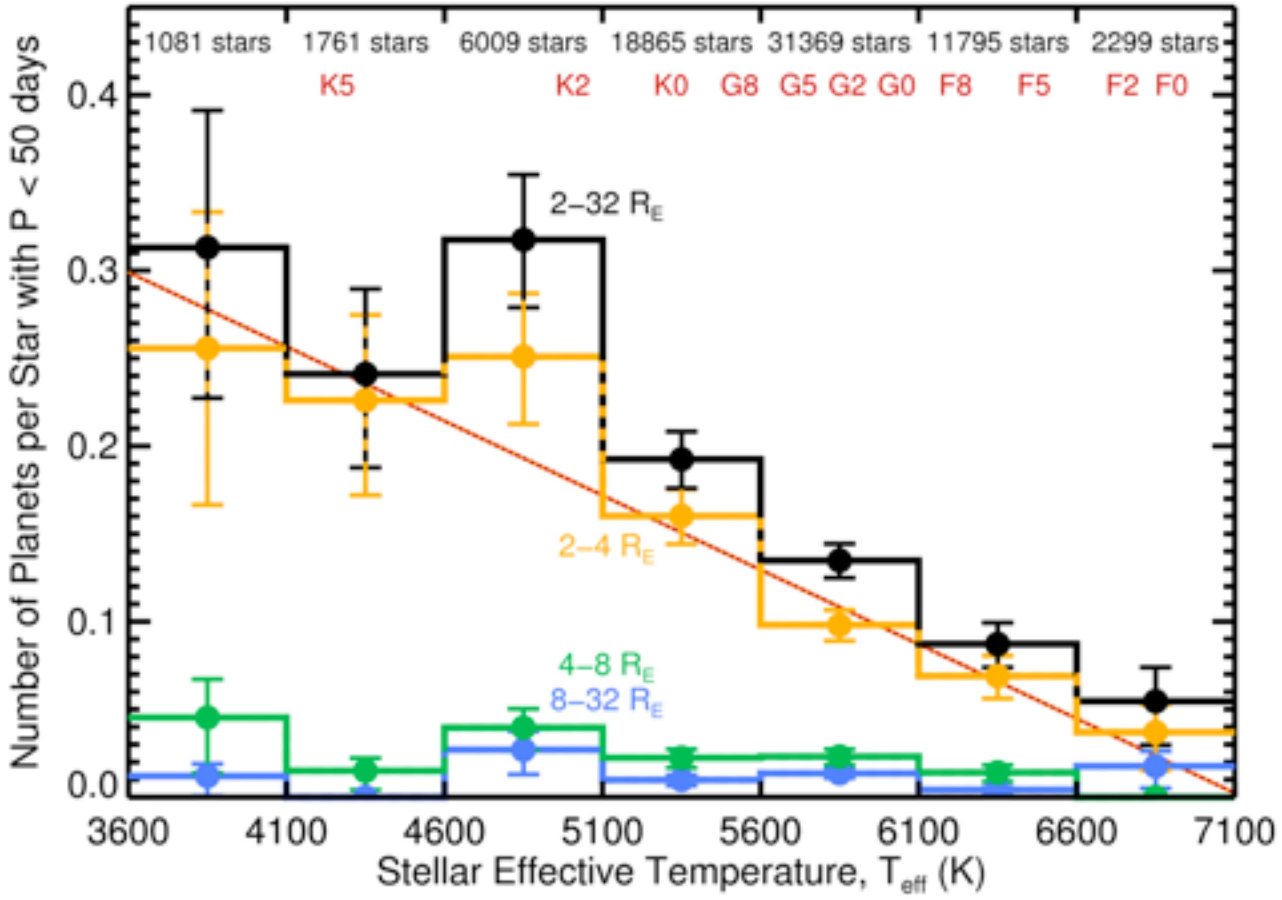
Howard et al. (2012)

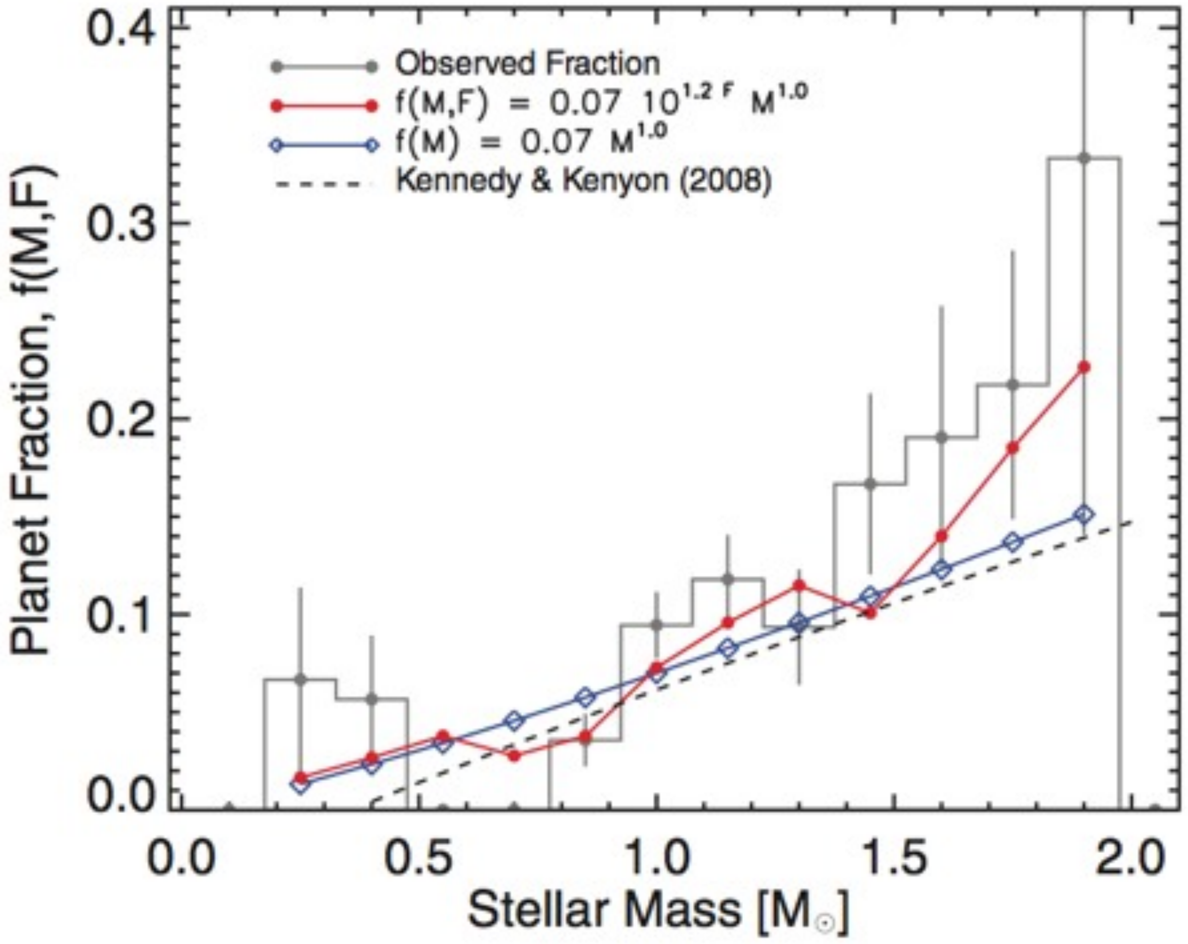
Planet Occurrence vs. Stellar Temperature



Howard et al. (2012)

Planet Occurrence vs. $M_\star$





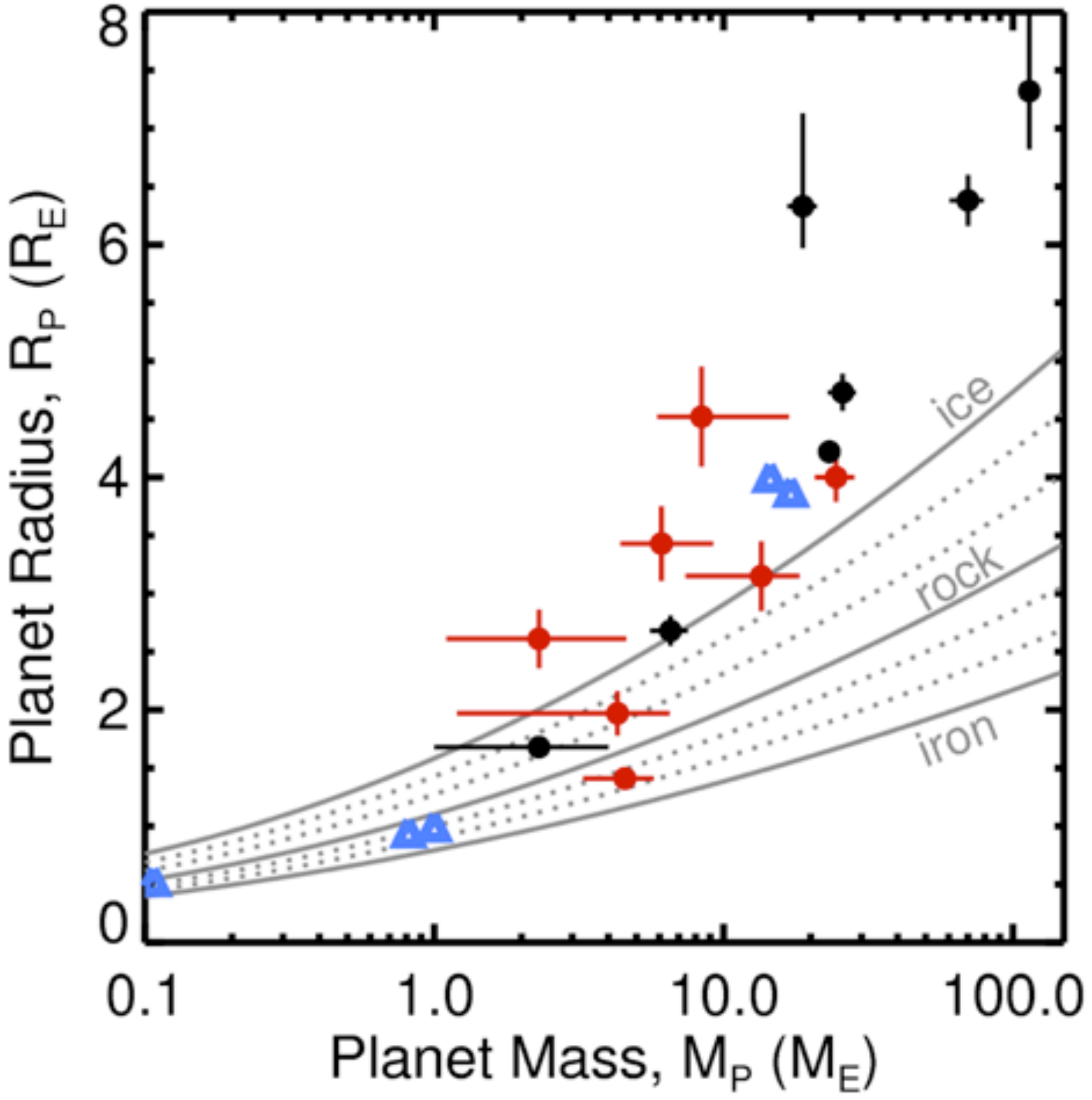
Occurrence within 0.25 AU
of small planets
decreases with $M_\star$

Occurrence within 2.5 AU
of giant planets
increases with $M_\star$

Howard et al. (2012)

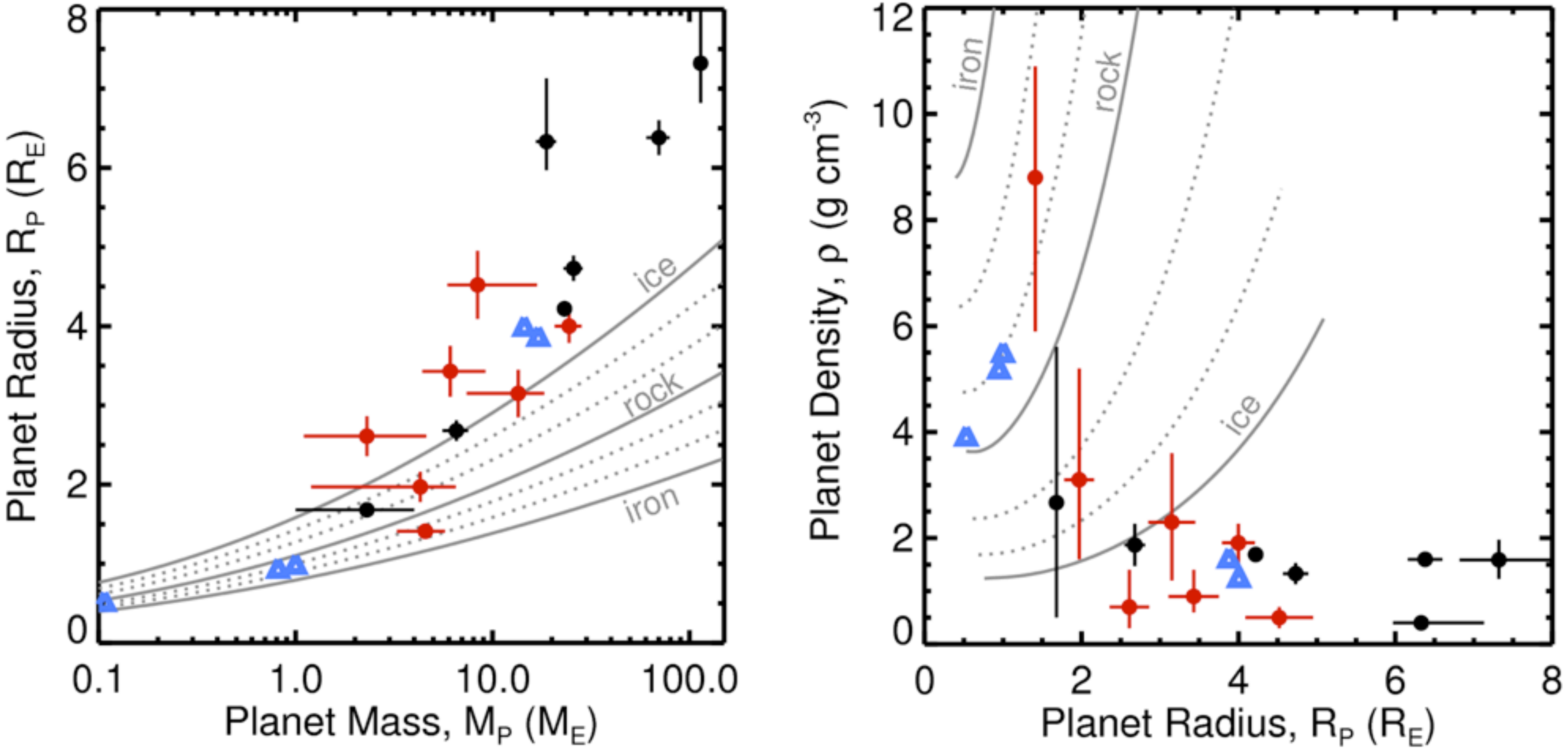
Johnson et al. (2010)

Planet Densities



Howard et al. (2012)

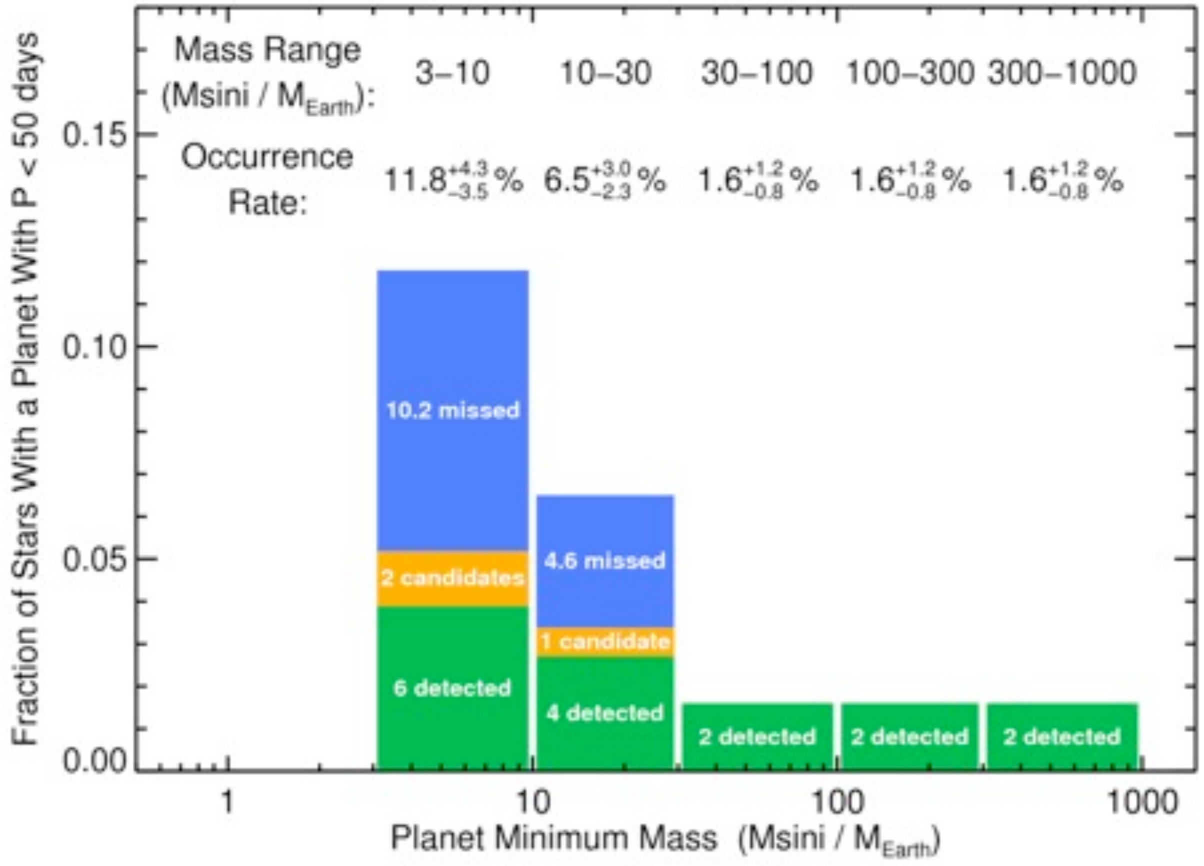
Planet Densities



Howard et al. (2012)

Planet Mass Distribution Eta-Earth Survey (*Doppler*)

Howard et al. (2010)



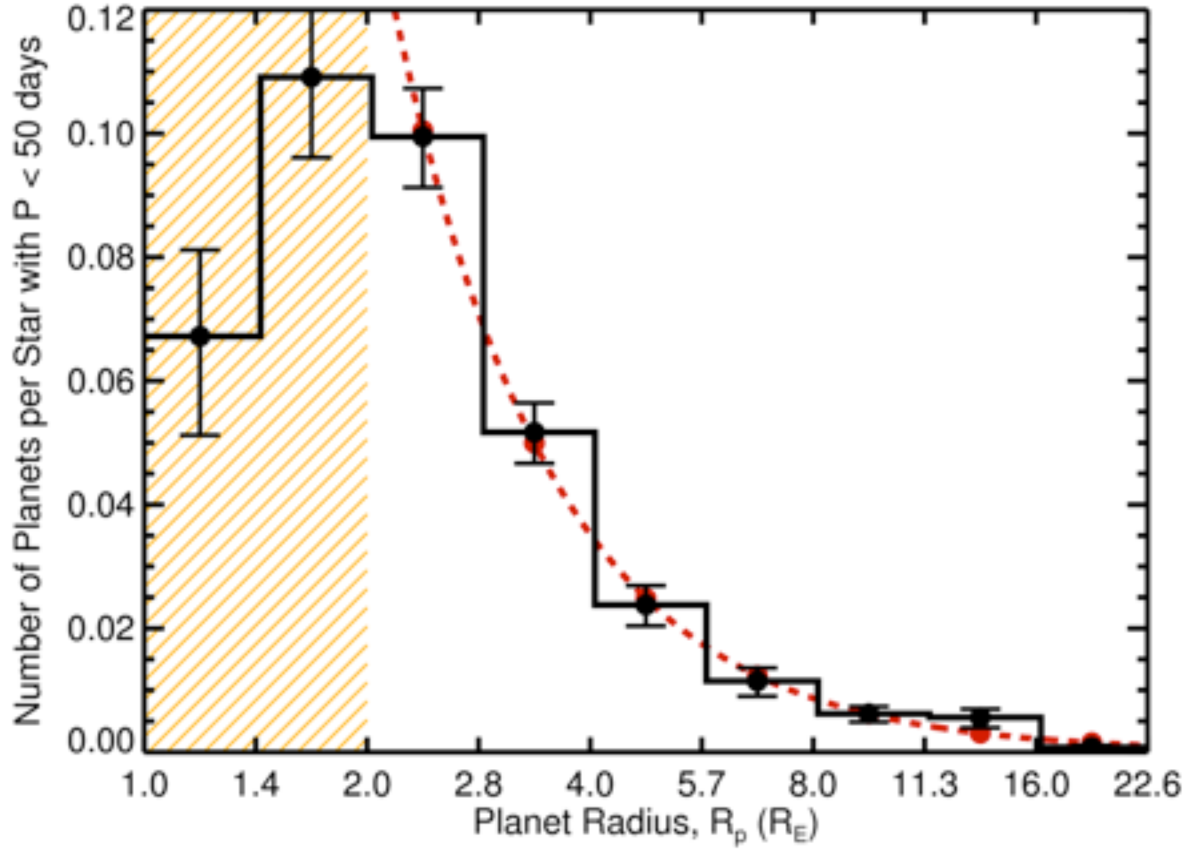
Power Law Mass Function

$df/d\log M = kM^{\alpha}$

$k = 0.39^{+0.27}_{-0.16}$, $\alpha = -0.48^{+0.12}_{-0.14}$

Planet Radius Distribution *Kepler*

Howard et al. (2012)

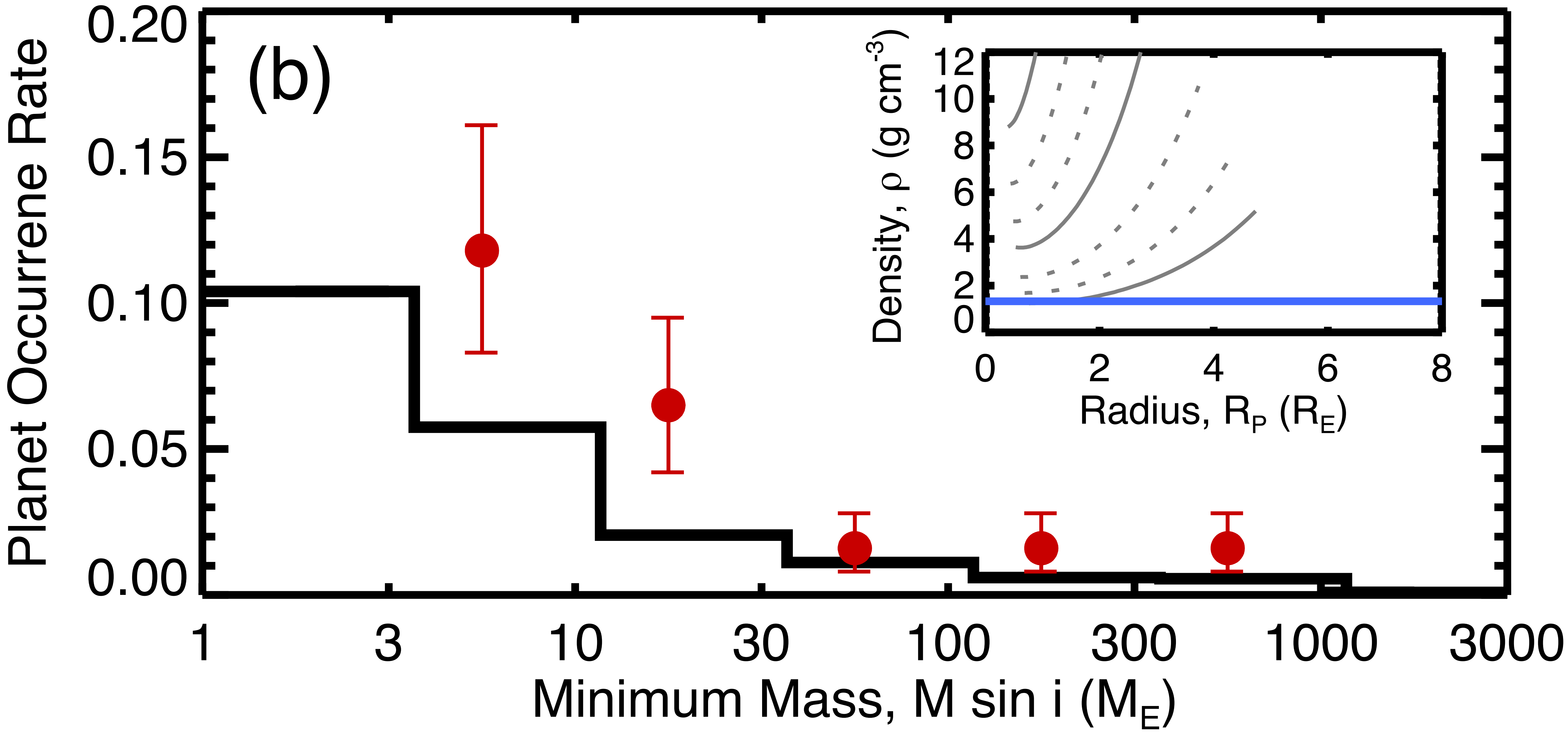


Power Law Radius Function

$df/d\log R = kR^{\alpha}$

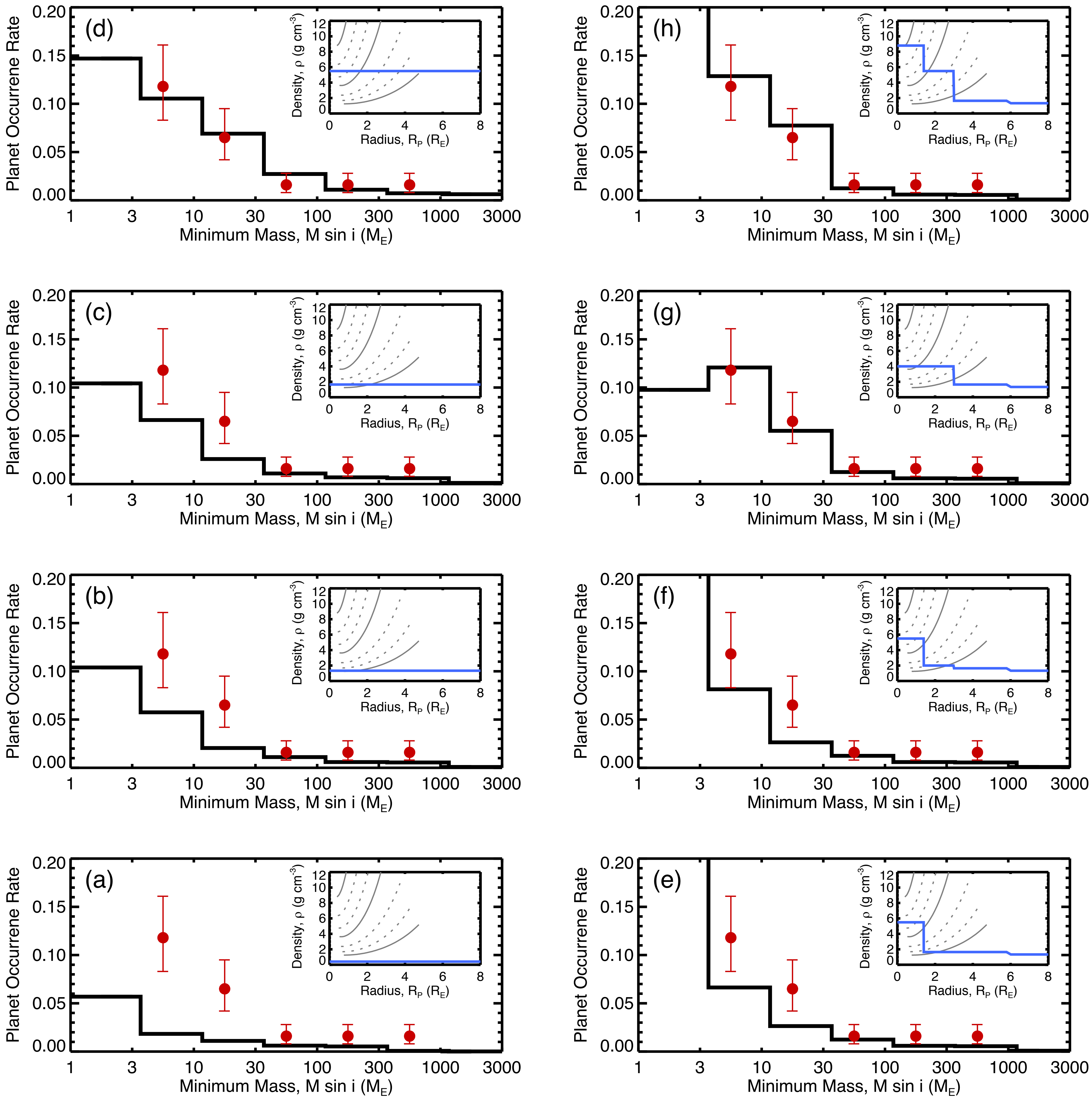
$k = 2.9 \pm 0.5$, $\alpha = -1.92 \pm 0.11$

Planet Densities

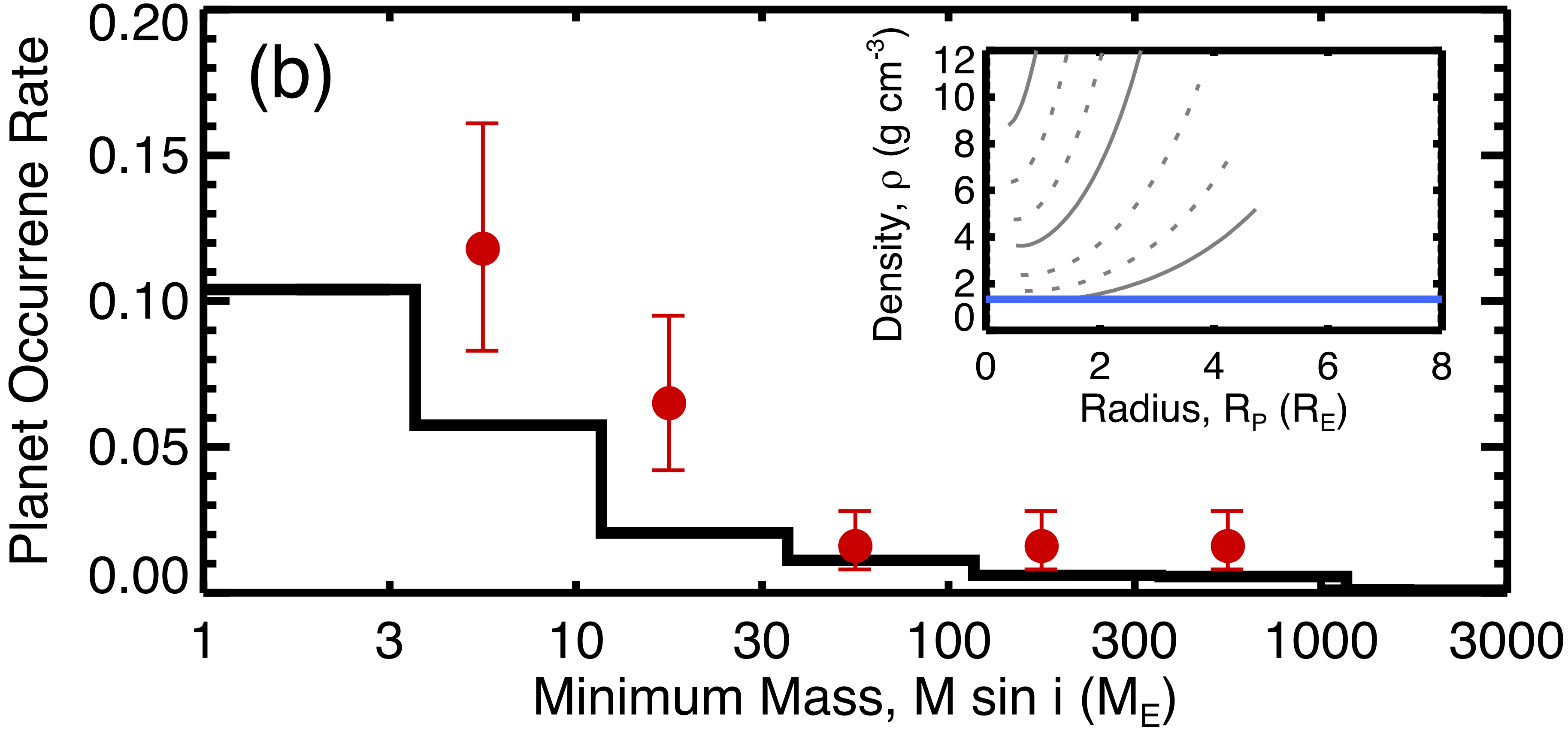


Howard et al. (2012)

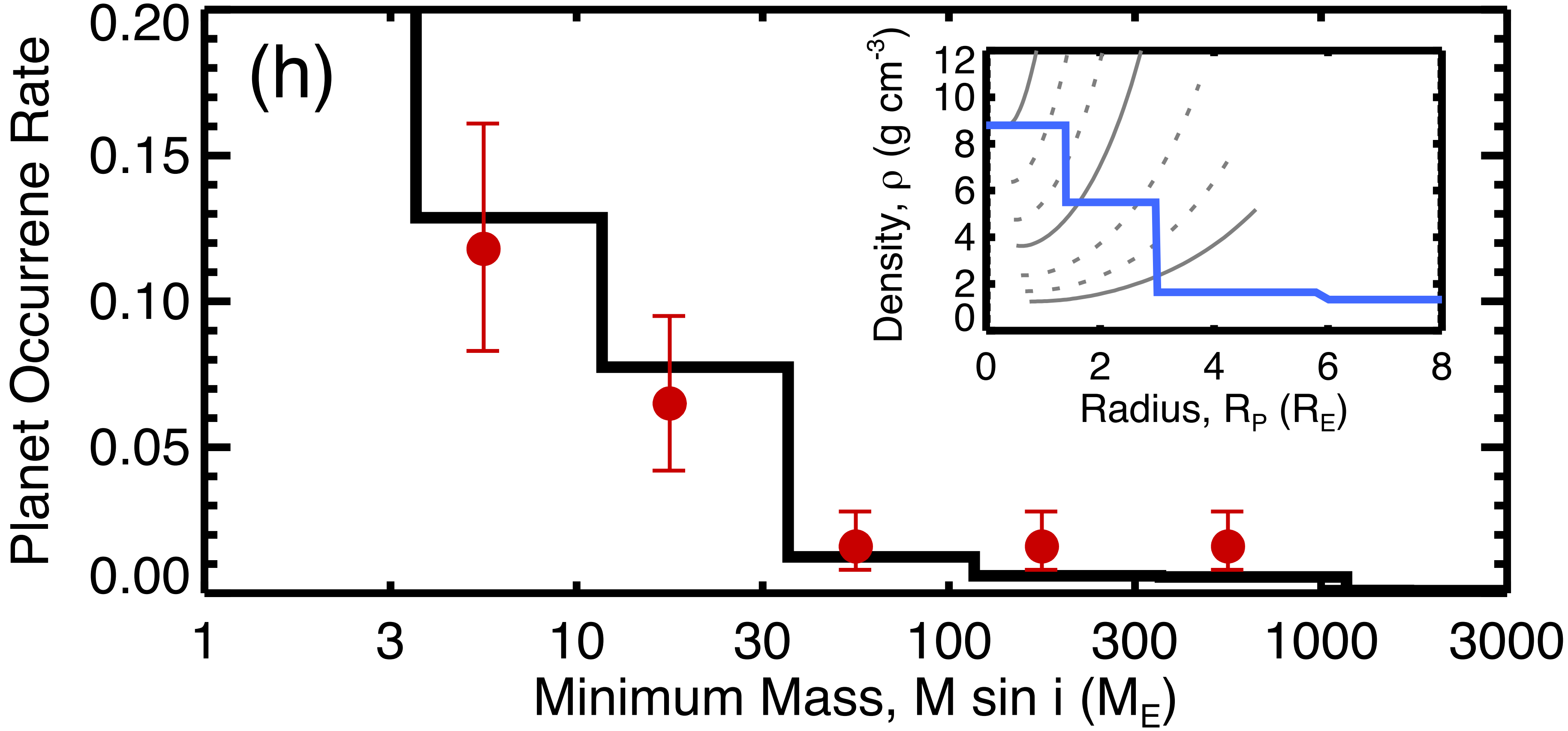
Planet Densities



Howard et al. (2012)



$\rho = 1.6$ g/cm^3
(constant)



ρ(R) rises with
decreasing R

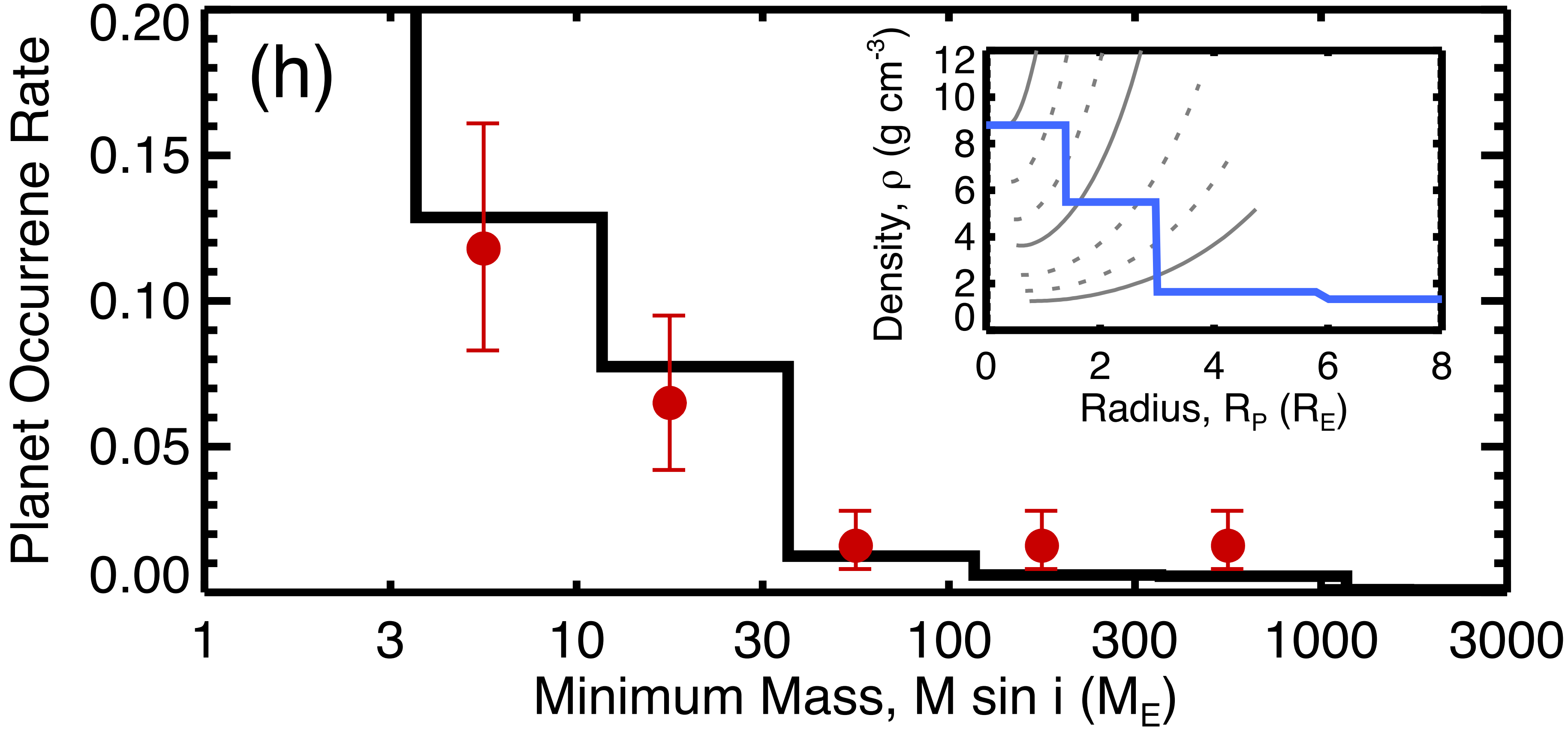
Howard et al. (2012)

Face Value Conclusions:

- On average, planets smaller than ~3 R_E have bulk densities ≳ 4 g cm^{-3}
- Terrestrial composition ?!

Complications:

- Multiple planets per system
- Different stellar samples?
- Not one-to-one mapping from radius to mass



ρ(R) rises with decreasing R

Howard et al. (2012)

Summary: Planet Occurrence

- Define planet parameters of measurement (*M*, *R*, *P*, e, etc.)
- Set planet detection threshold
- Incompleteness — correct for missed planets

$$\text{Occurrence} = \frac{\text{Number of Planets}}{\text{Number of Stars}}$$

- Define stellar parameters of measurement (*M*, *R*, *Fe/H*, *Teff*, *logg*, etc.)
- Check that results don't depend on stellar param boundaries

Questions?

Extra slides

Patterns of Planet Occurrence Reveal Mechanisms of Planet Formation:

1. Population synthesis models incorrectly predicted planet desert

new physics needed in model?
better models of migration & planet-planet interactions needed?
in situ formation ("migration then assembly") ?

2. Planet radius distribution

small planets are more common
limited by 35% errors in stellar radii
precise $R_\star$ will reveal details of R_P distribution

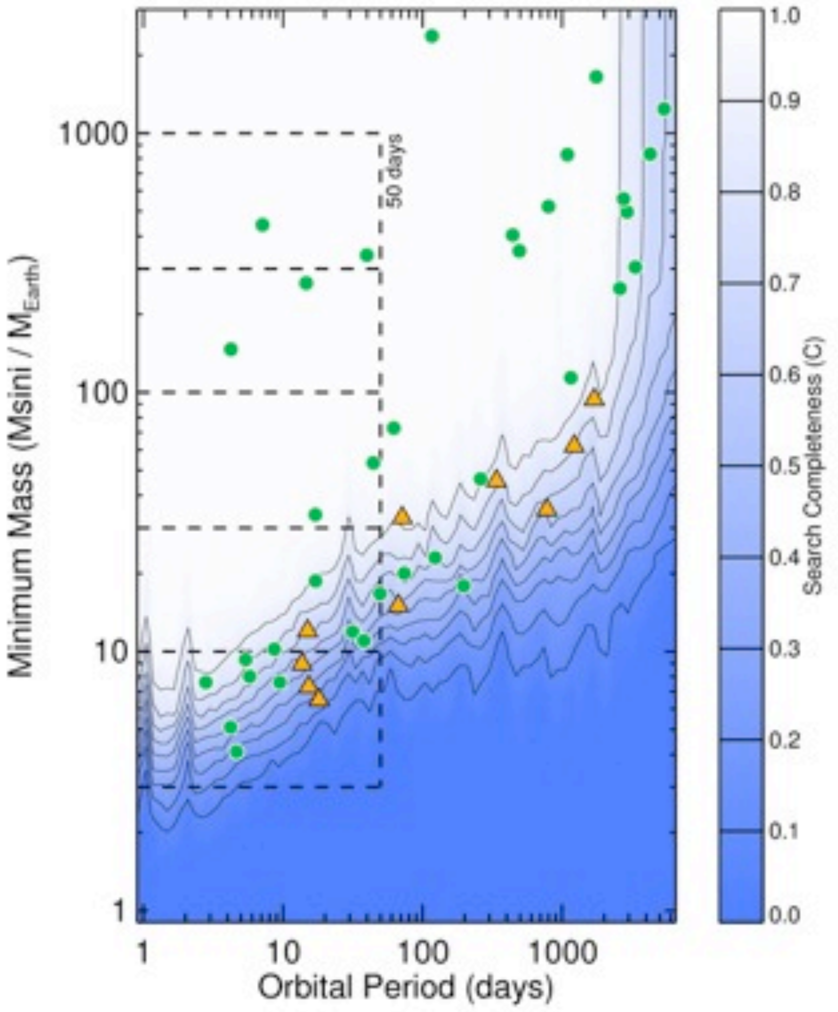
3. Planet period distribution

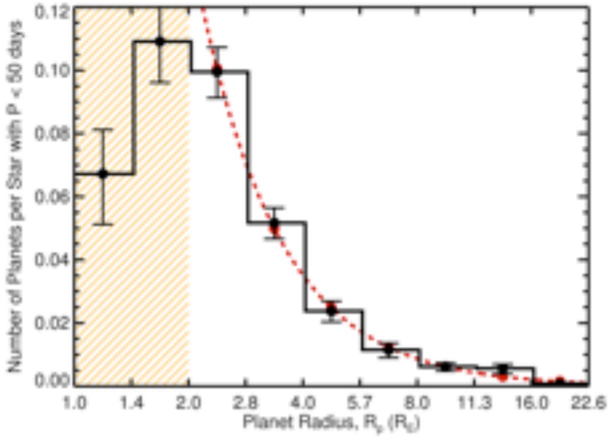
planet occurrence increases with orbital distance (per log *P*)
parking distance varies with planet size

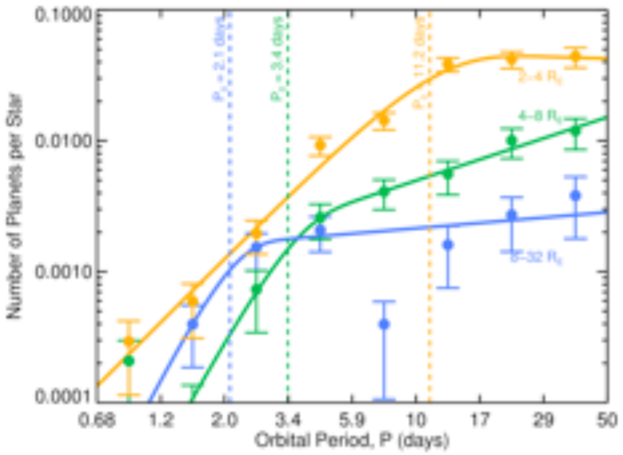
4. Planet occurrence vs. stellar mass

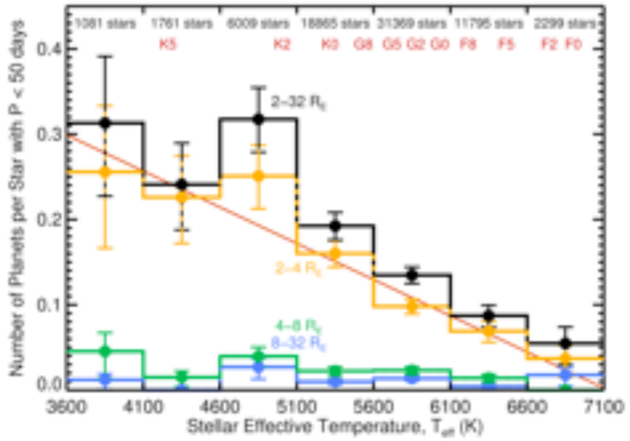
occurrence of close-in sub-Neptune planets decreases with $M_\star$

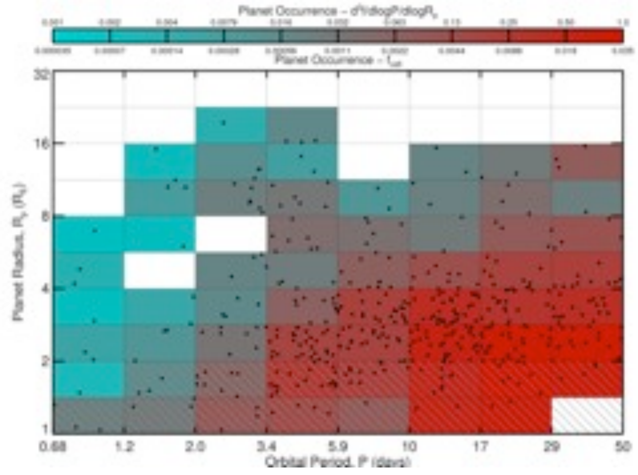
jovian planet occurrence (out to ~2AU) has opposite trend
signature of migration, formation, something else?







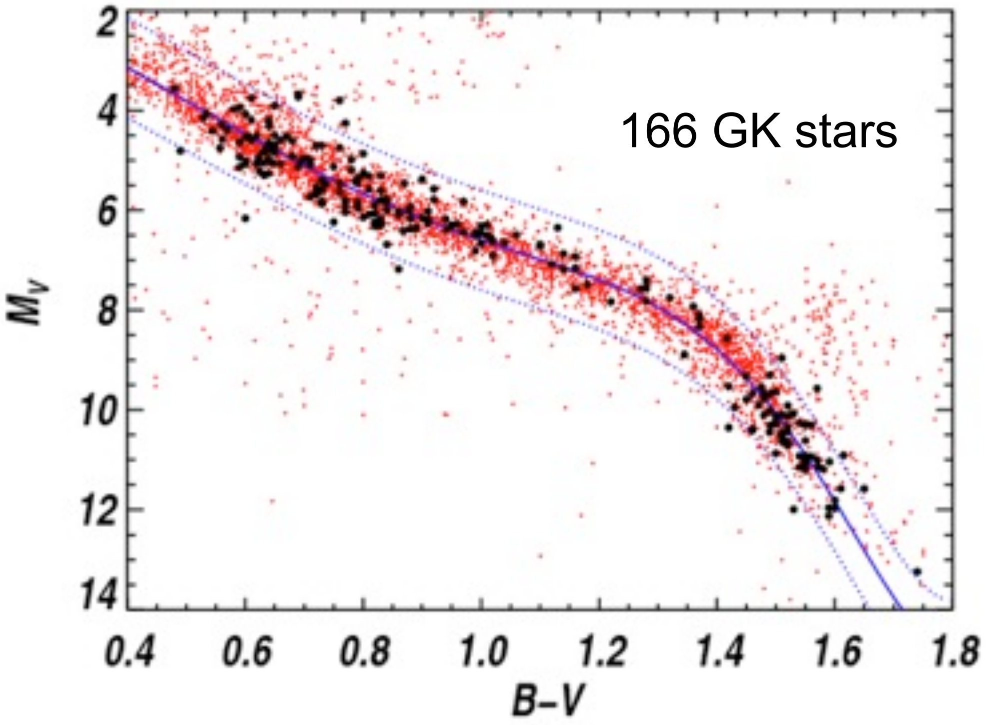


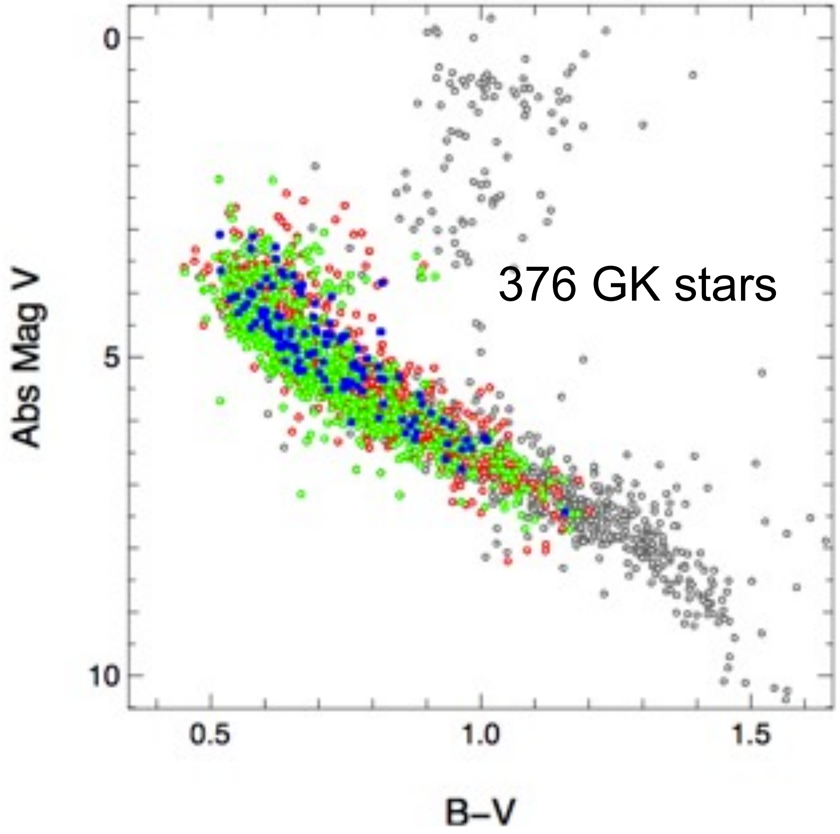


HARPS + CORALIE Volume-limited Survey

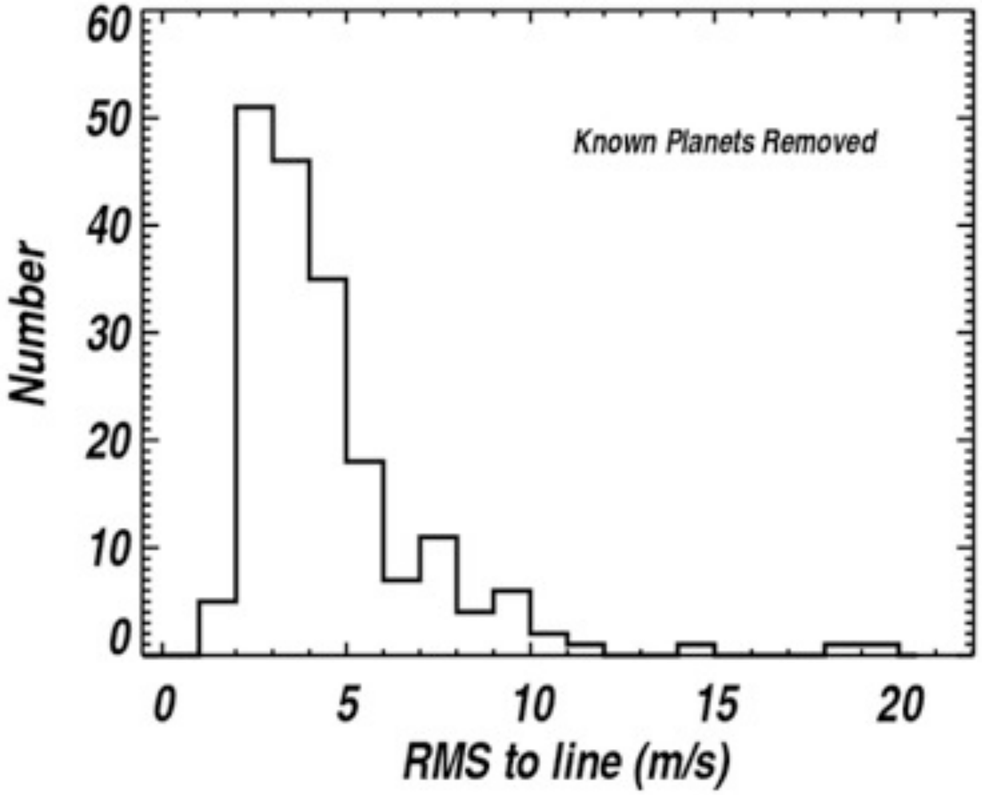
Mayor et al. (2011)

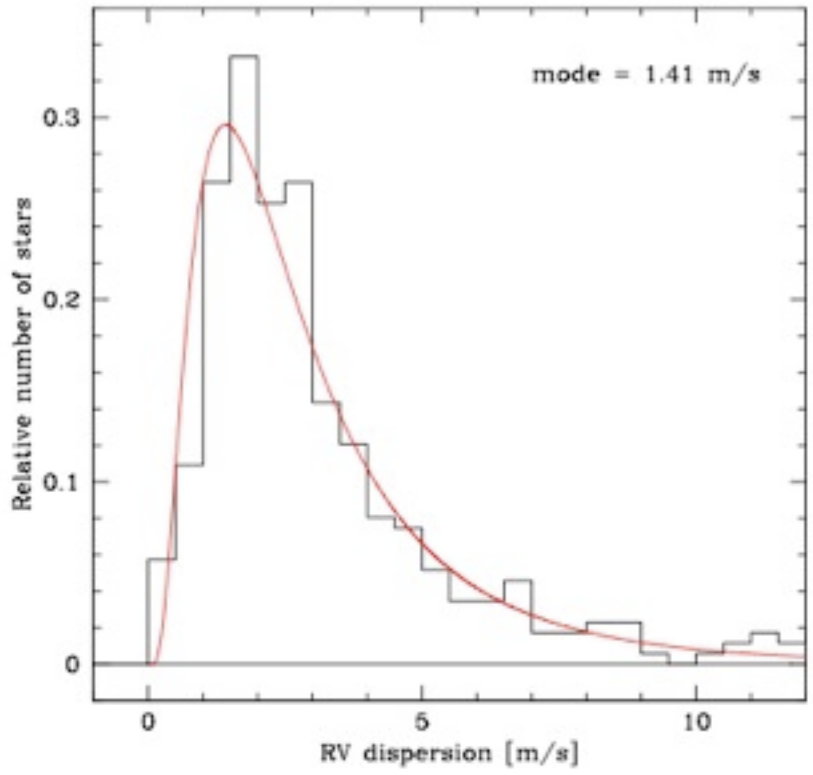
Keck/HIRES





HARPS

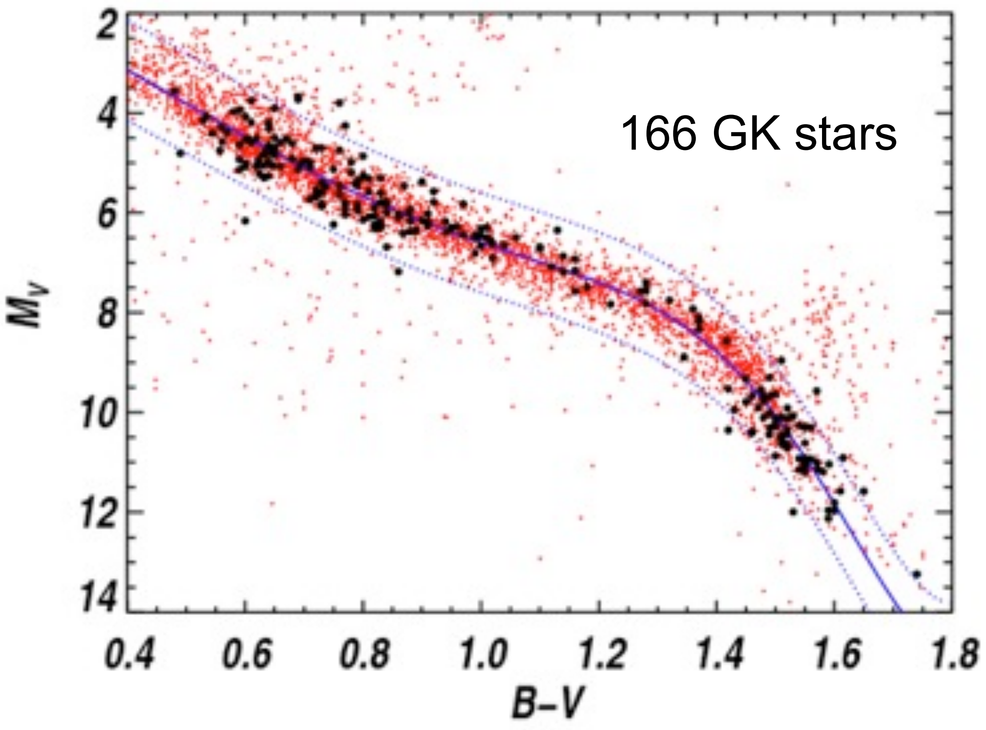




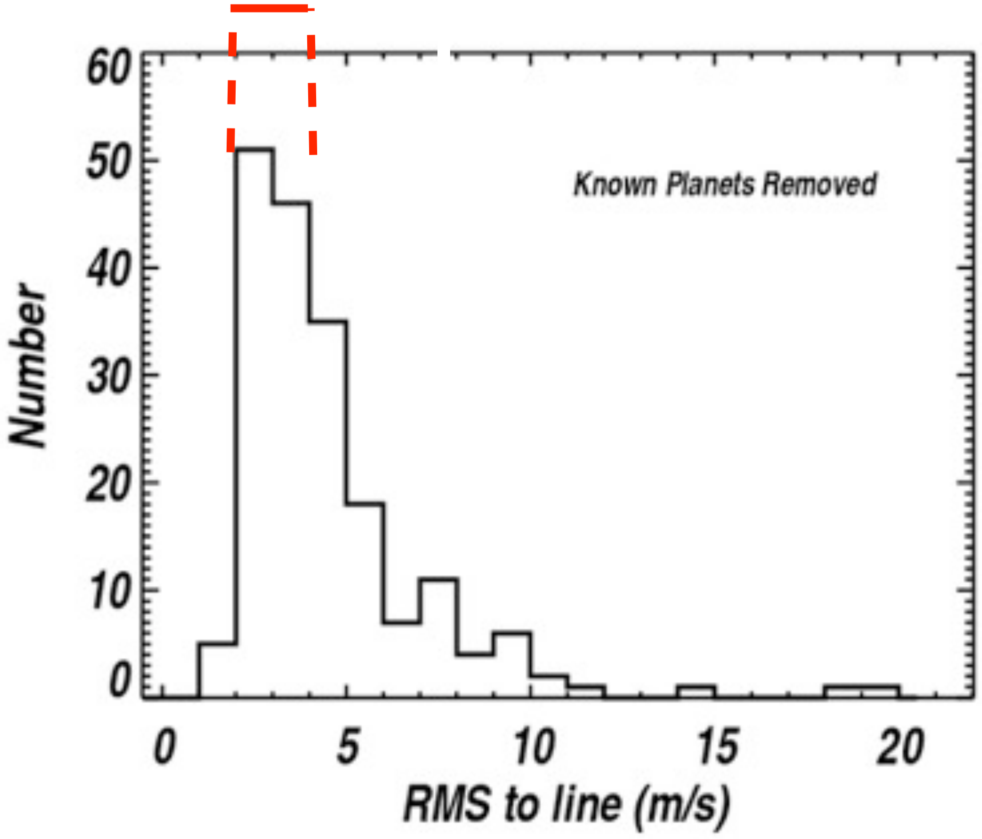
HARPS + CORALIE Volume-limited Survey

Mayor et al. (2011)

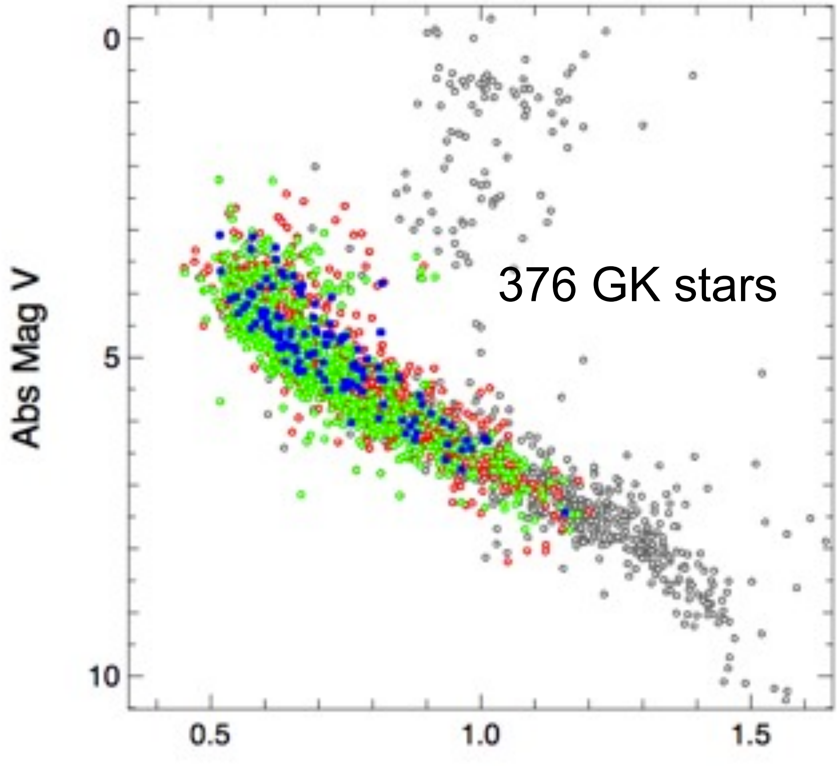
Keck/HIRES



2-4 m/s

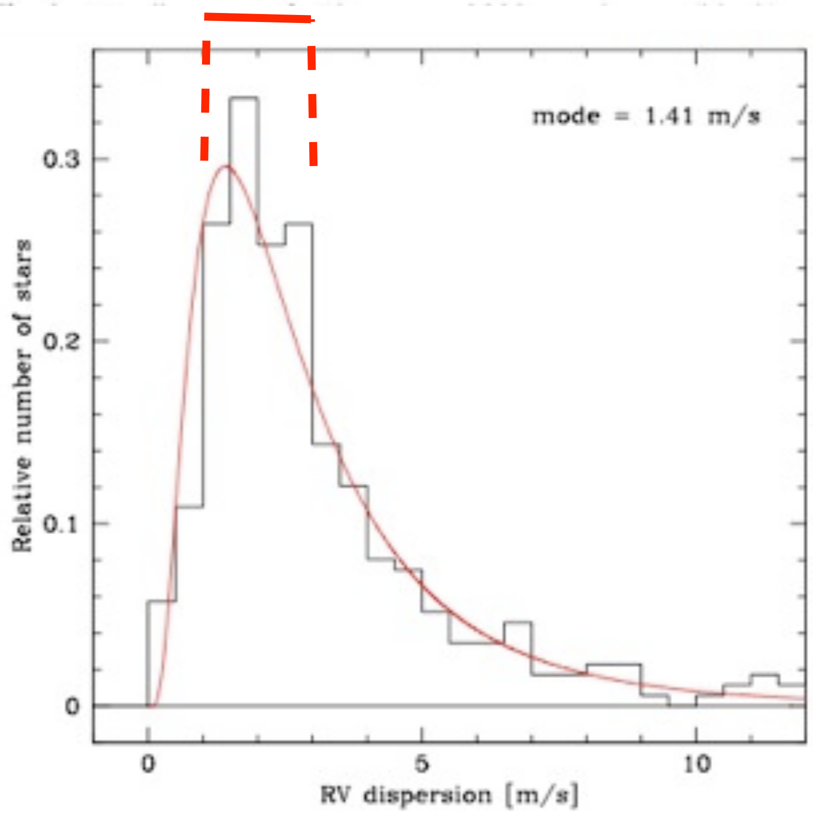


HARPS



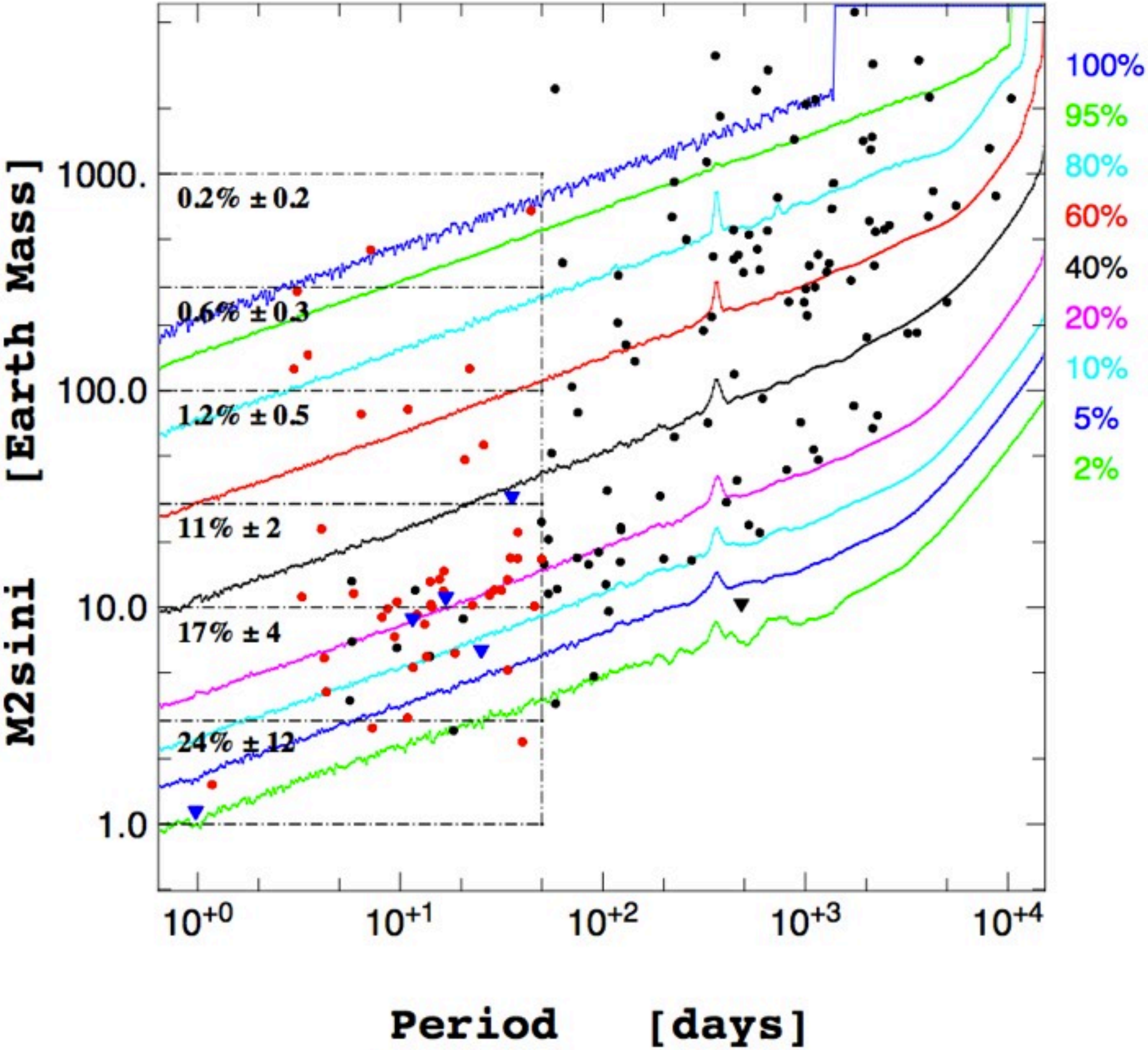
1-3 m/s

B–V



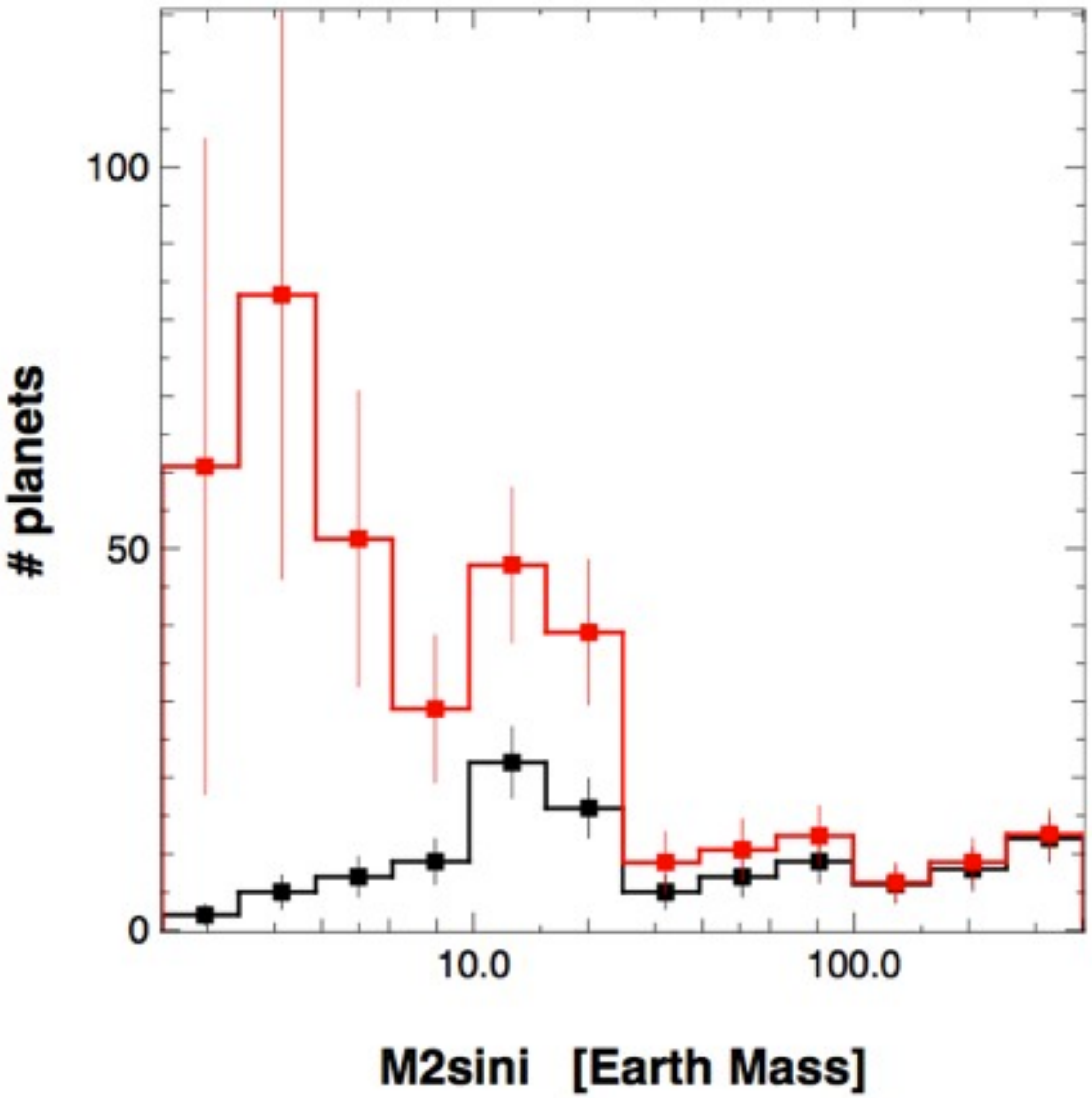
HARPS + CORALIE Volume-limited Survey

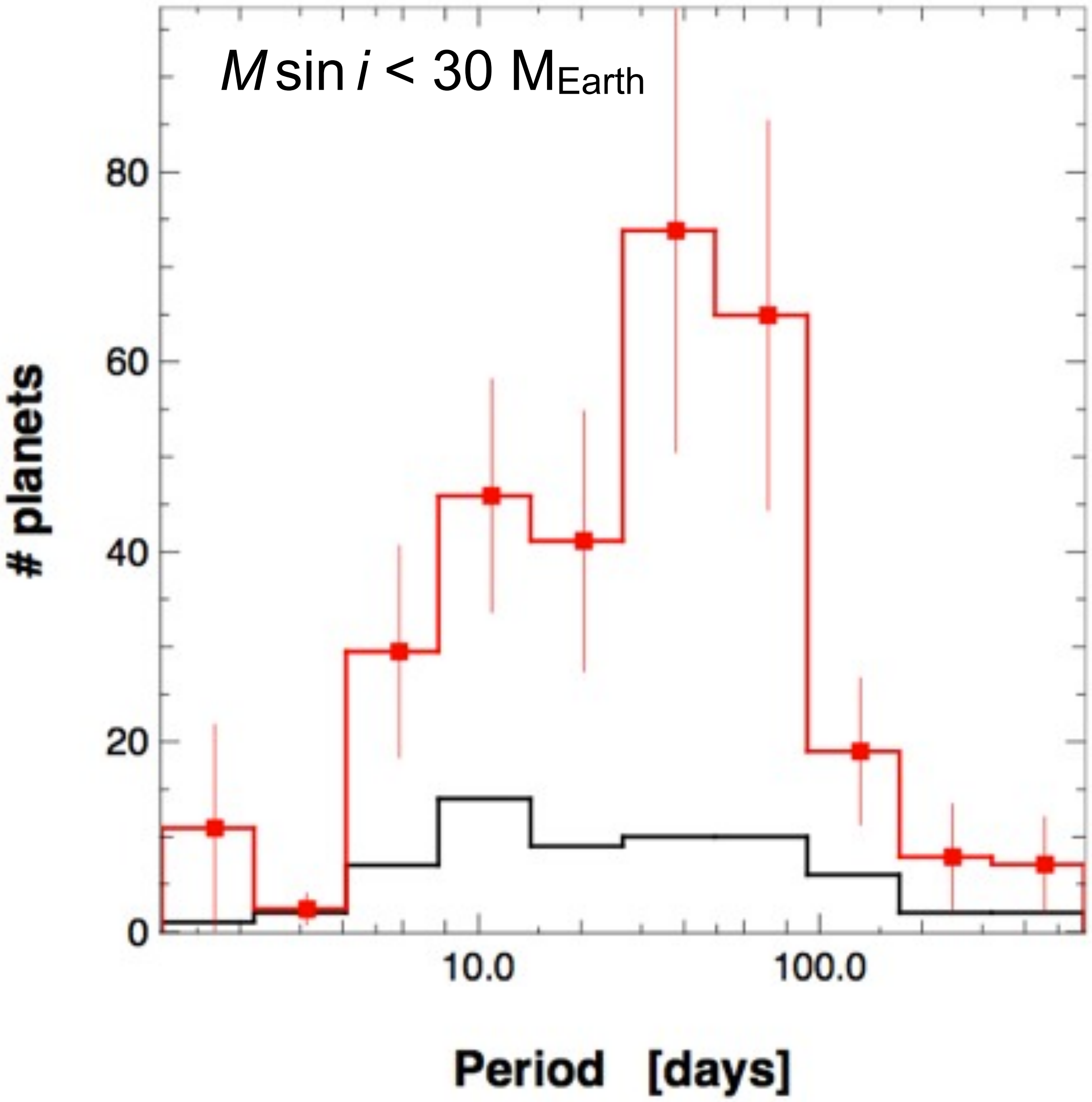
Mayor et al. (2011)



HARPS + CORALIE Volume-limited Survey

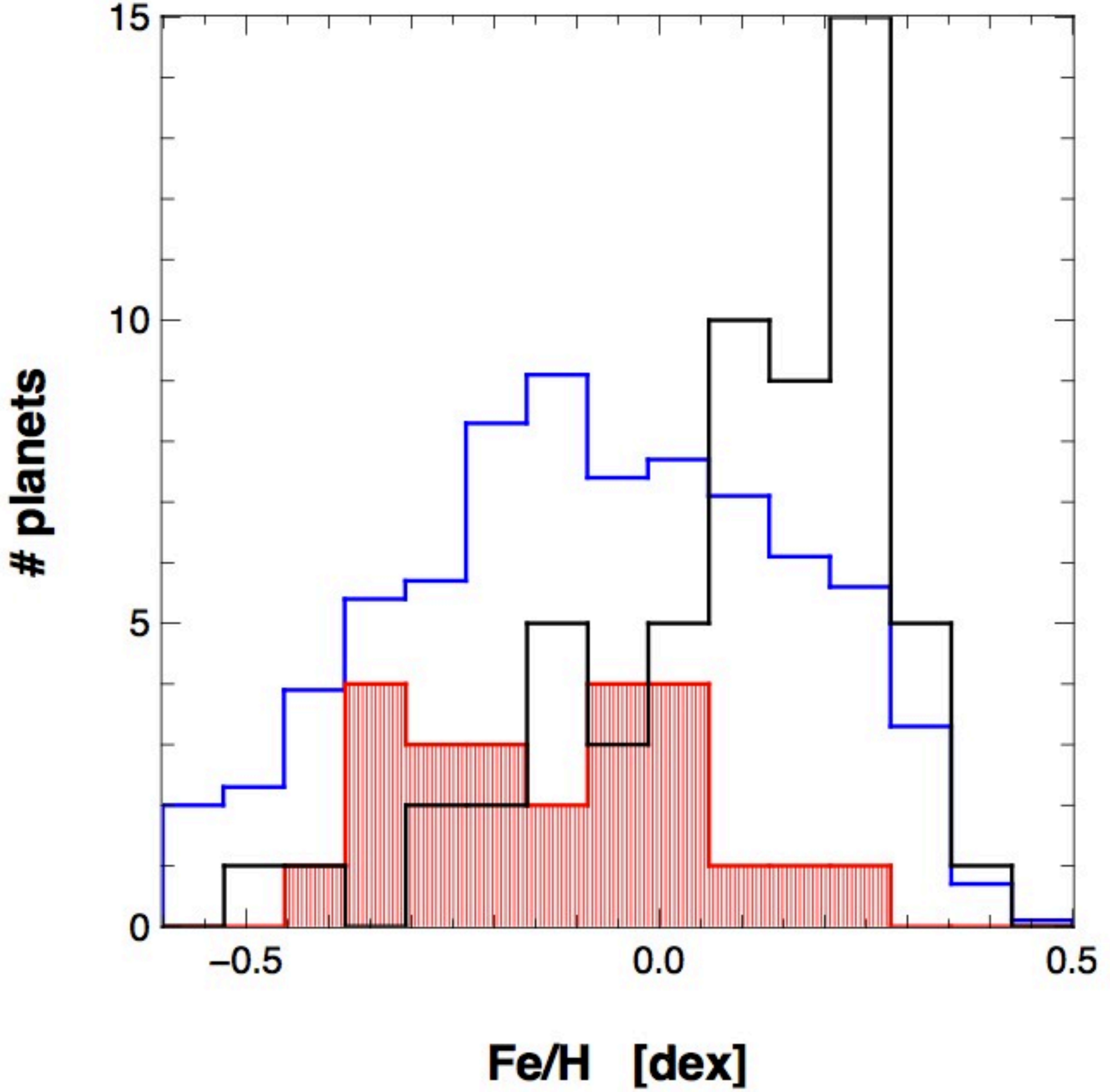
Mayor et al. (2011)

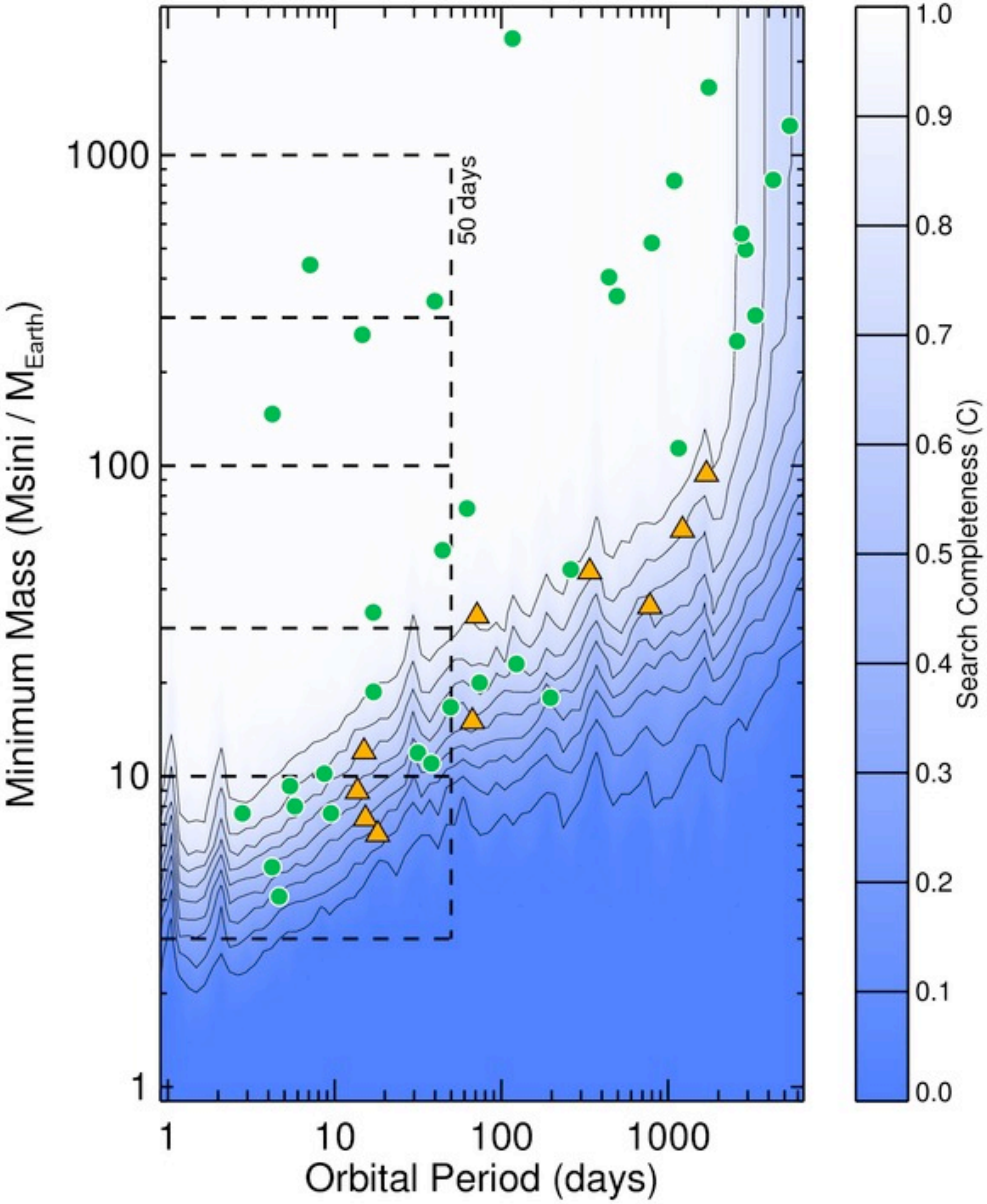




HARPS + CORALIE Volume-limited Survey

Mayor et al. (2011)





Howard et al. (2010)
Mayor et al. (2011)

Keck-HIRES
HARPS/CORALIE

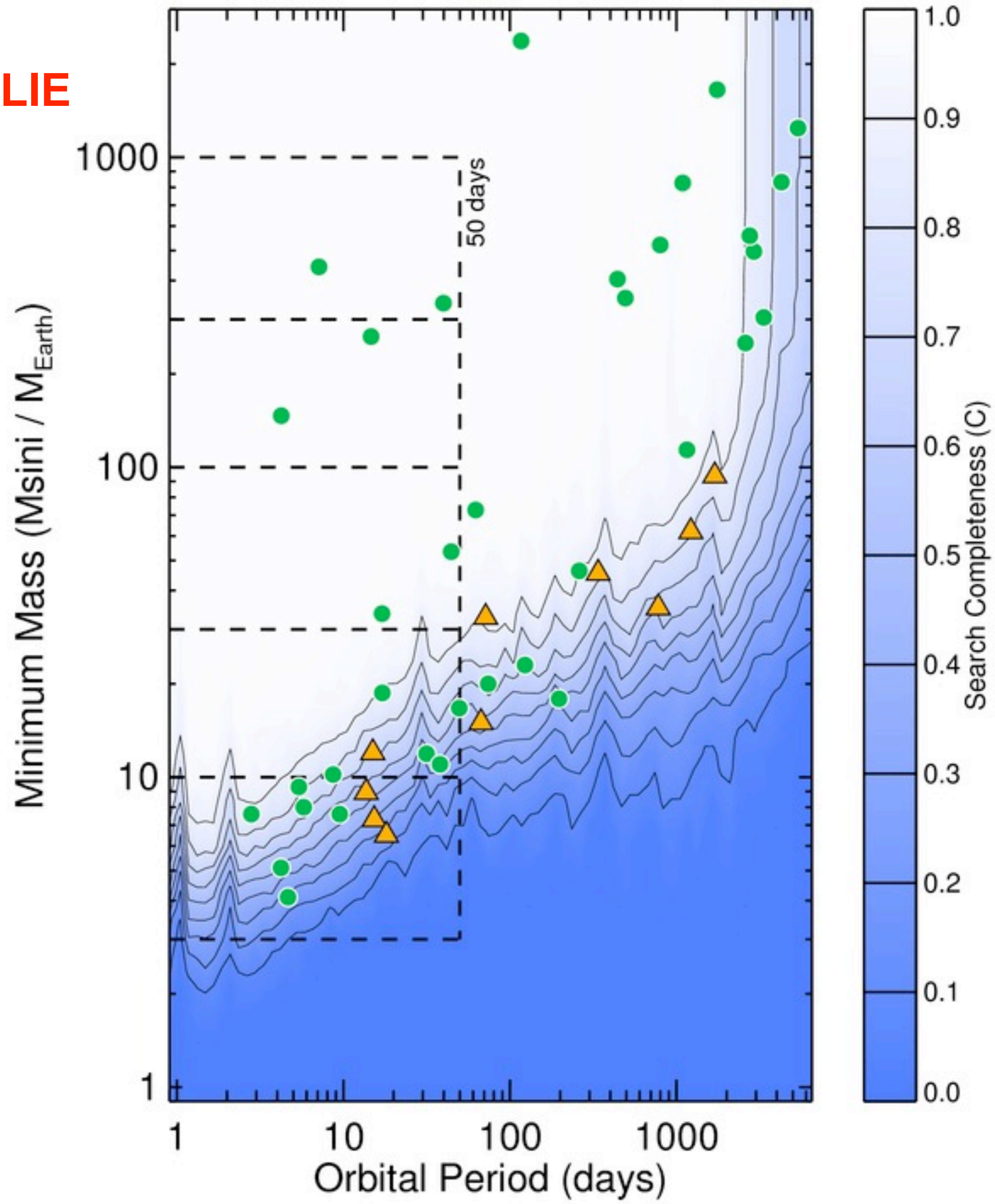
1.6 ± 1.2 %
0.24 ± 0.17 %

1.6 ± 1.2 %
0.58 ± 0.29 %

1.6 ± 1.2 %
1.17 ± 0.52 %

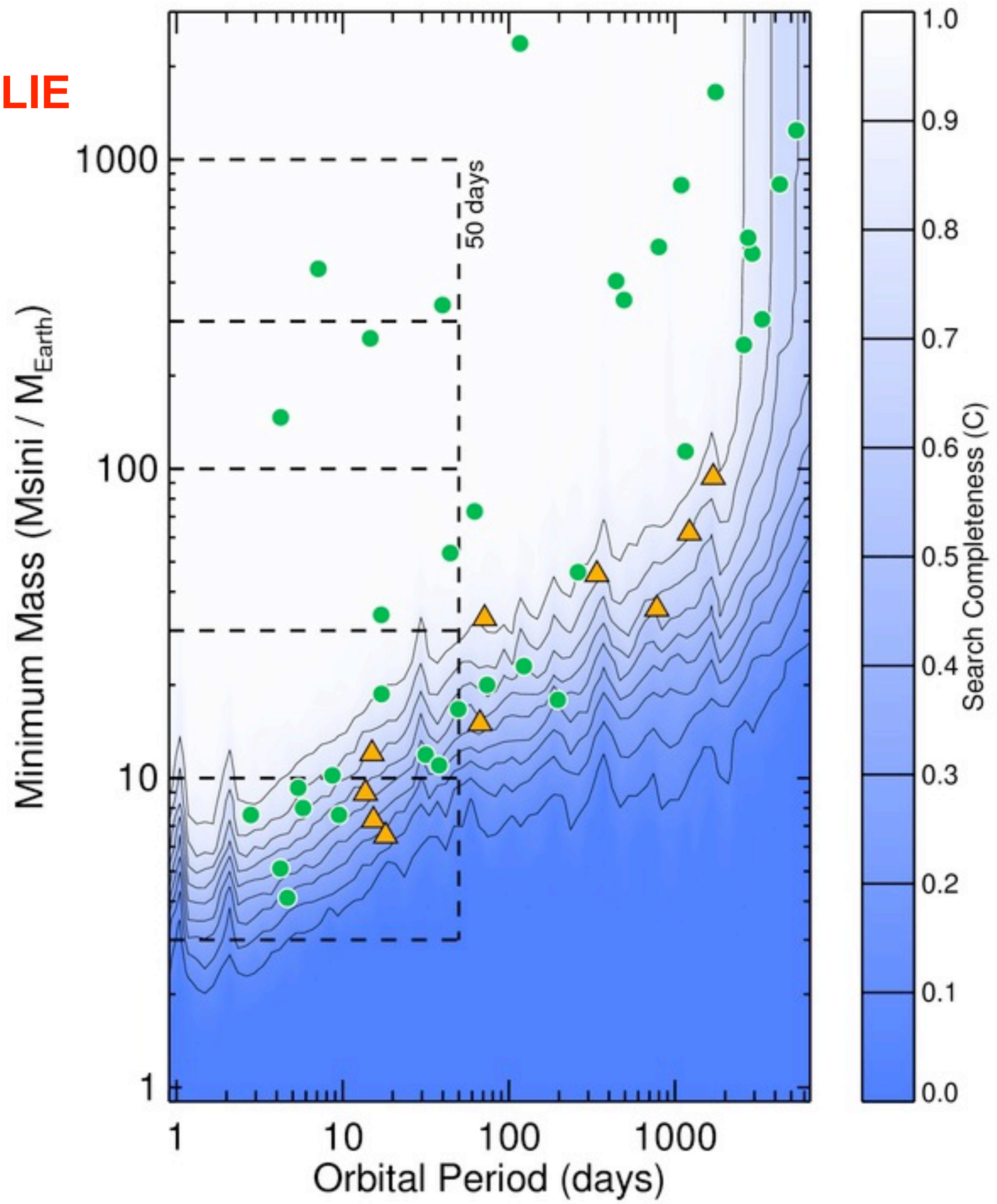
6.5 ± 3.0 %
11.1 ± 2.4 %

11.8 ± 4.3 %
16.6 ± 4.4 %



Howard et al. (2010)
Mayor et al. (2011)

Difference	Keck-HIRES	HARPS/CORALIE
+1.1σ	1.6 ± 1.2 %	0.24 ± 0.17 %
+0.8σ	1.6 ± 1.2 %	0.58 ± 0.29 %
+0.3σ	1.6 ± 1.2 %	1.17 ± 0.52 %
+1.2σ	6.5 ± 3.0 %	11.1 ± 2.4 %
+0.8σ	11.8 ± 4.3 %	16.6 ± 4.4 %



Howard et al. (2010)
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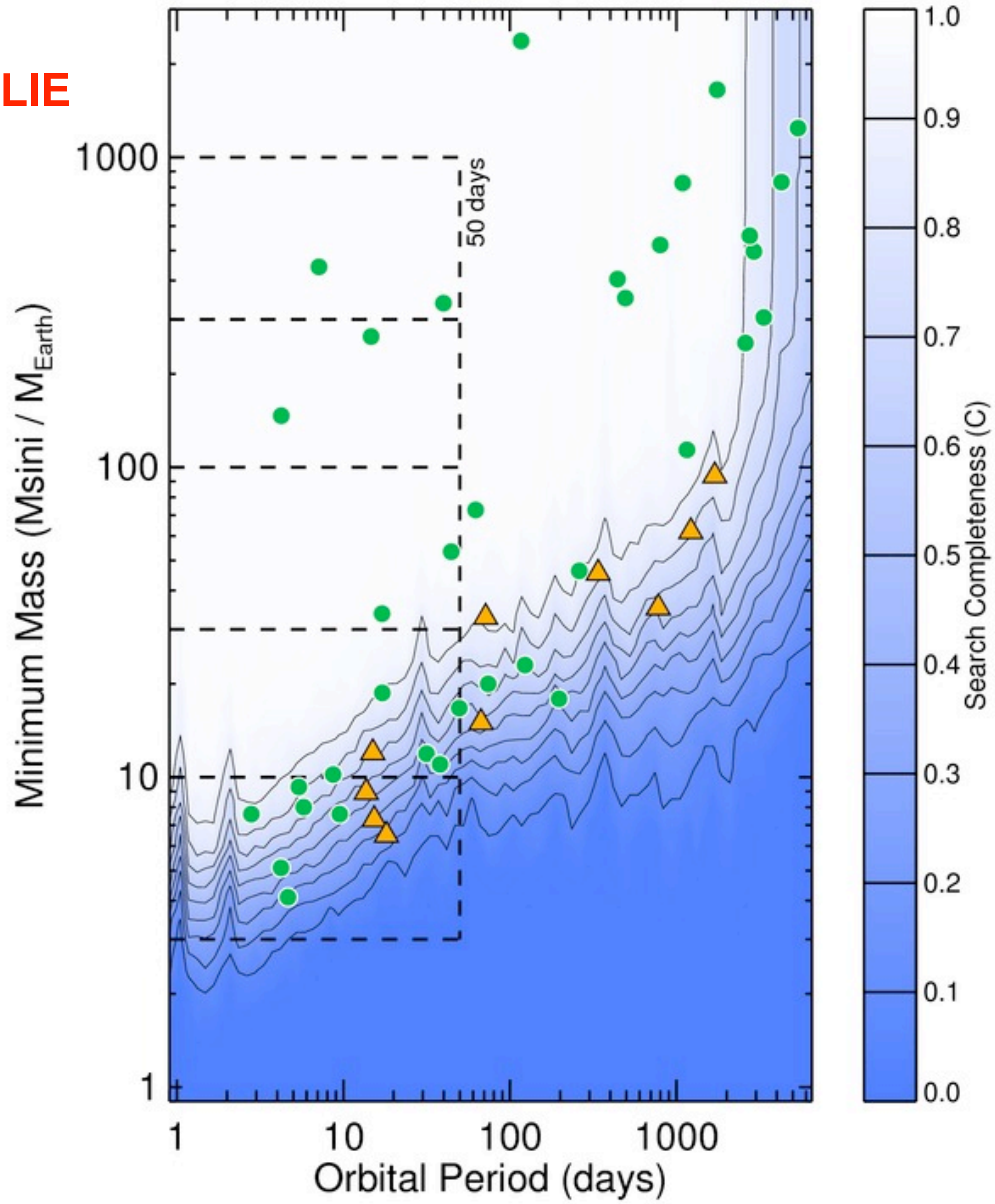
Difference	**Keck-HIRES**	**HARPS/CORALIE**
+1.1σ	**1.6 ± 1.2 %**	**0.24 ± 0.17 %**
+0.8σ	**1.6 ± 1.2 %**	**0.58 ± 0.29 %**
+0.3σ	**1.6 ± 1.2 %**	**1.17 ± 0.52 %**
+1.2σ	**6.5 ± 3.0 %**	**11.1 ± 2.4 %**
+0.8σ	**11.8 ± 4.3 %**	**16.6 ± 4.4 %**
		24 ± 12 %

Howard et al. (2010)
Mayor et al. (2011)



Kepler

20%

Kepler

$R_p > 2\ R_E$

Keck-HIRES

HARPS/CORALIE

23.1%

29.7 %

Doppler

*M*sin*i* = 3-1000 M_E