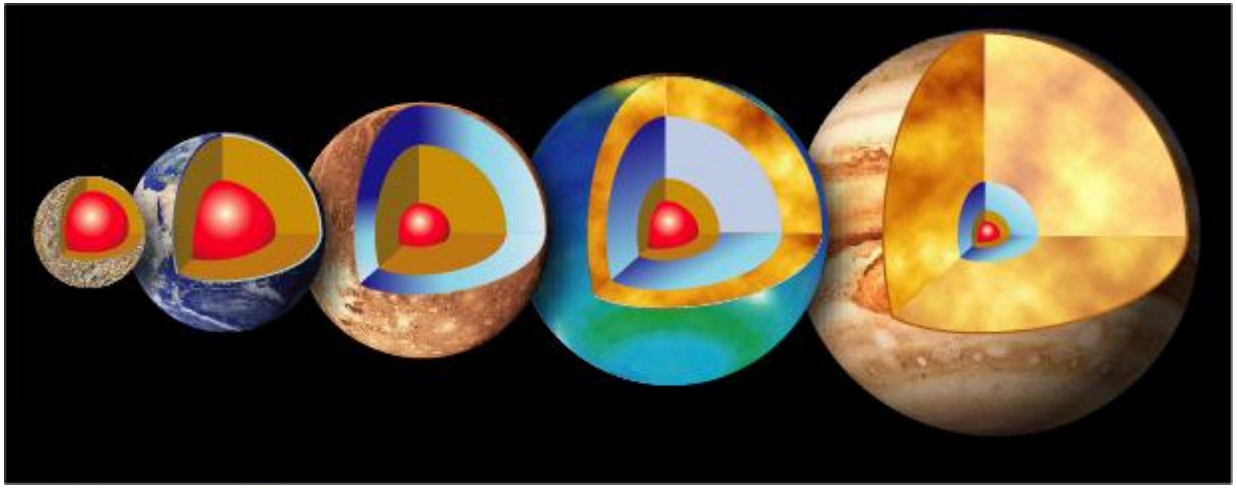
The structure of rocky planets

Christophe Sotin



Iron planets (Mercury)

Terrestrial planets

Ocean / Icy planets

- Icy Moons
- **Uranus and Neptune**

Giant planets

References: Sotin et al. (2010) Terrestrial Planet Interiors; in Exoplanets (Sara Seager, Ed), The Universy of Arizona Press, 375-395.
Leger et al. (2004, 2011); Valencia et al. (2006, 2007,); O'Neil and Lenardic (2007), Sotin et al. (2007); Grasset et al. (2009); Charpinet et al. (2011)

Outline

Introduction

Modeling the interior structure of terrestrial planets
- The Earth’s case
- Equations of state
- Radius versus mass

Application to water-rich planets
- Equation of state of water

Conclusions
- Plate tectonics on terrestrial planets
- What has been found so far ?
- Outstanding questions

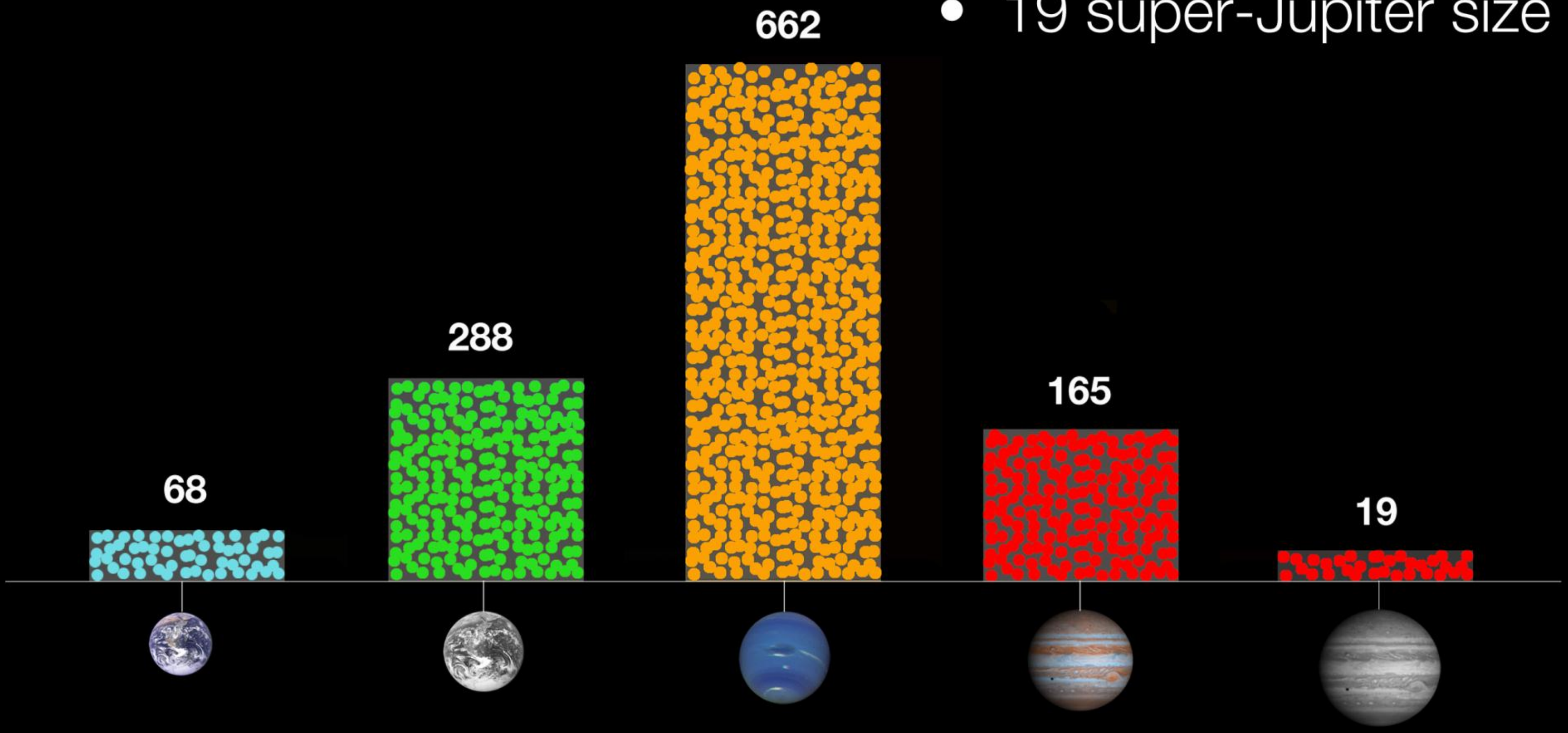
Introduction

Objective 1: Describe how one can obtain the relationship between mass and radius and then use that relationship to determine the class of exoplanet being discovered. Mass and radius are two of the *easiest* parameters to obtain.

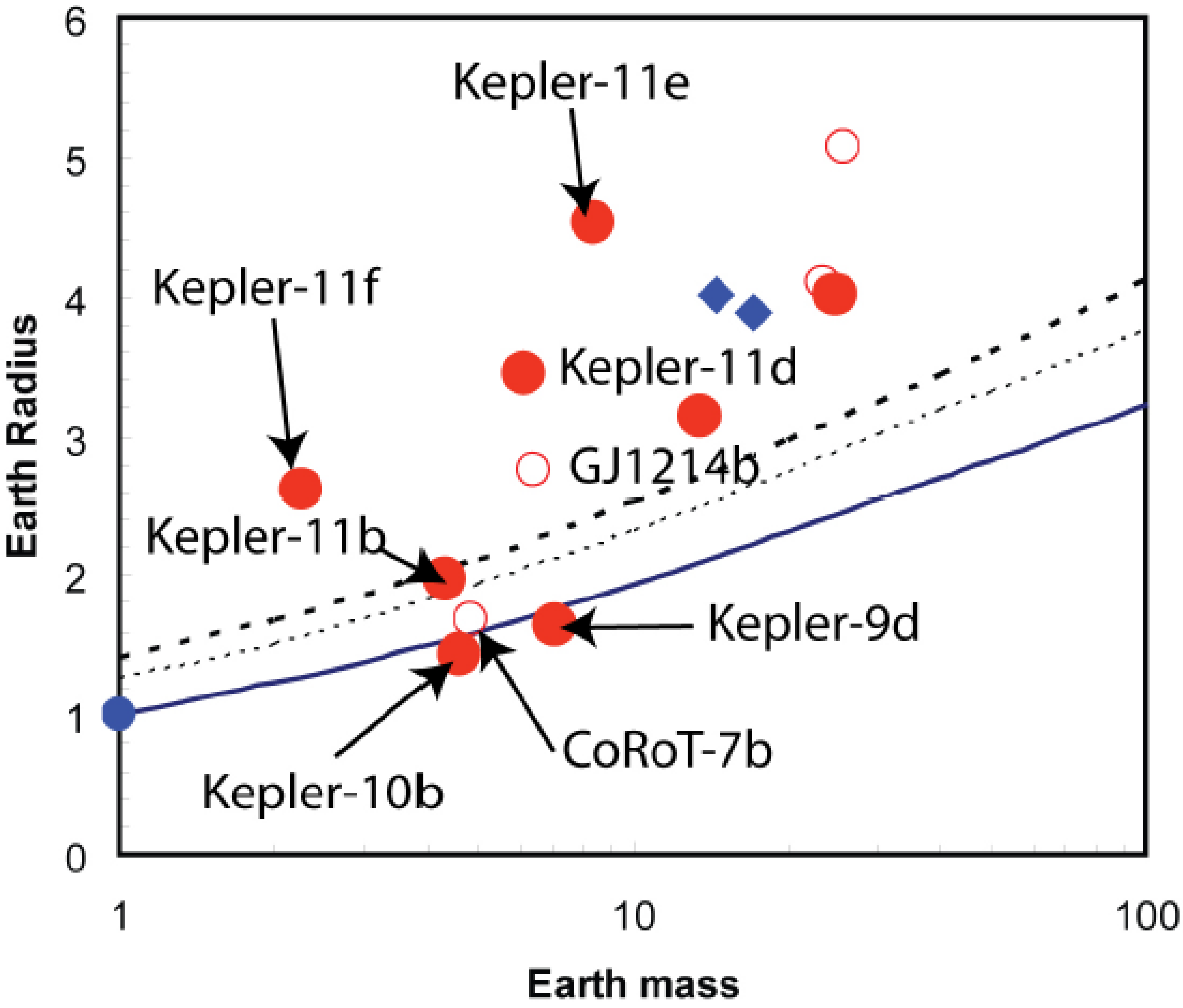
Objective 2: Address some outstanding issues such as the likelihood of plate tectonics on an exoplanet
Plate tectonics is important because volatiles (H2O, CO2) can be recycled into the planet.

Numbers of Planet Candidates

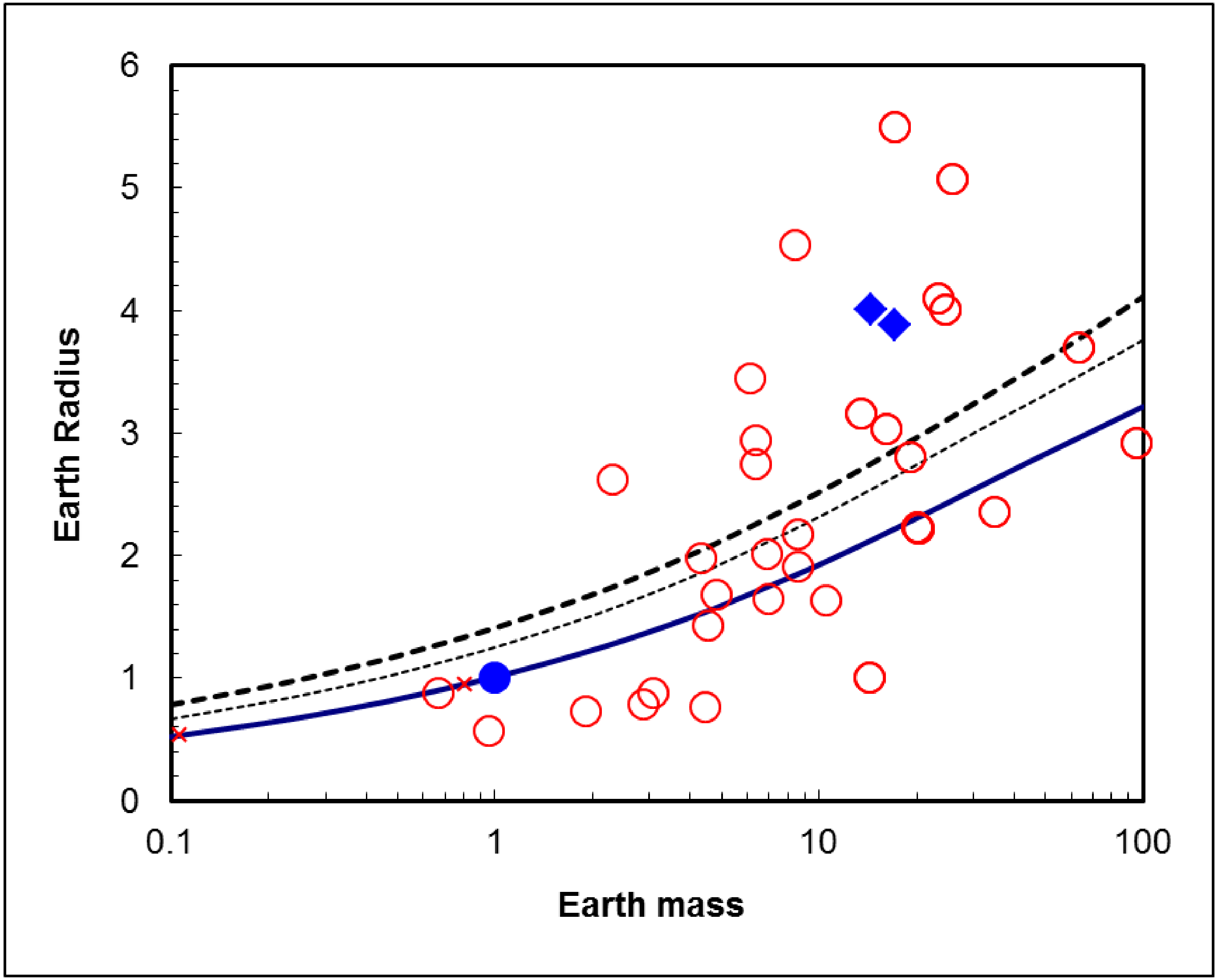
- 68 Earth-size
- 288 super-Earth size
- 662 Neptune size
- 165 Jupiter size
- 19 super-Jupiter size



Low mass exoplanets in March 2011



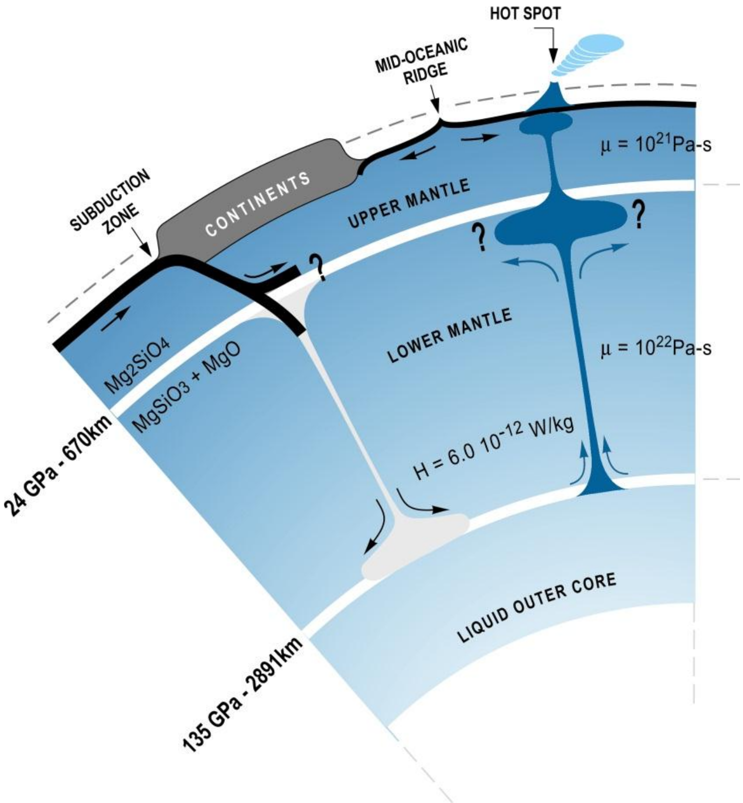
Low mass exoplanets – July 2012



Modeling the interior structure of terrestrial planets

- The Earth’s case
- Equations of state
- Radius versus mass

Internal structure of the Earth - composition



	EEH Earth model	PUM	LM	Core
O	30,28	44,76	43,8	1,61
Fe	33,39	5,89	12,69	80,25
Si	19,23	21,35	24,28	10,34
Mg	12,21	23,21	16,18	0
Total	**95,11**	**95,21**	**96,95**	**92,2**
Ni	2,02	0,25	0,71	4,99
Ca	1,01	2,32	1,2	0
Al	0,93	2,13	1,1	0
S	0,85	0,01	0,01	2,57
Total	**99,92**	**99,92**	**99,97**	**99,76**

O	30,28	44,76	43,8	1,61
Fe	35,41	6,14	13,4	85,24
Si	19,69	22,41	24,83	10,34
Mg	13,68	26,59	17,93	0

Core : Iron + light element (S, O, other).

Mantle : $(Mg,Fe)_2Si_2O_6$, $Ca(Mg,Fe)Si_2O_6$, $(Mg,Fe)_2SiO_4$ and Al phase / $(Mg,Fe)SiO_3$, $(Mg,Fe)O$ and Al phase

Input parameters

	EEH	PUM	LM	Core	2/3LM+1/3Core
Fe/Si	**0,909**	0,138	0,273	4,166	0,944
Mg/Si	**0,803**	1,372	0,835	0,000	0,691
Fe/(Fe+Mg)	0,531	0,092	0,246	1,000	0,577

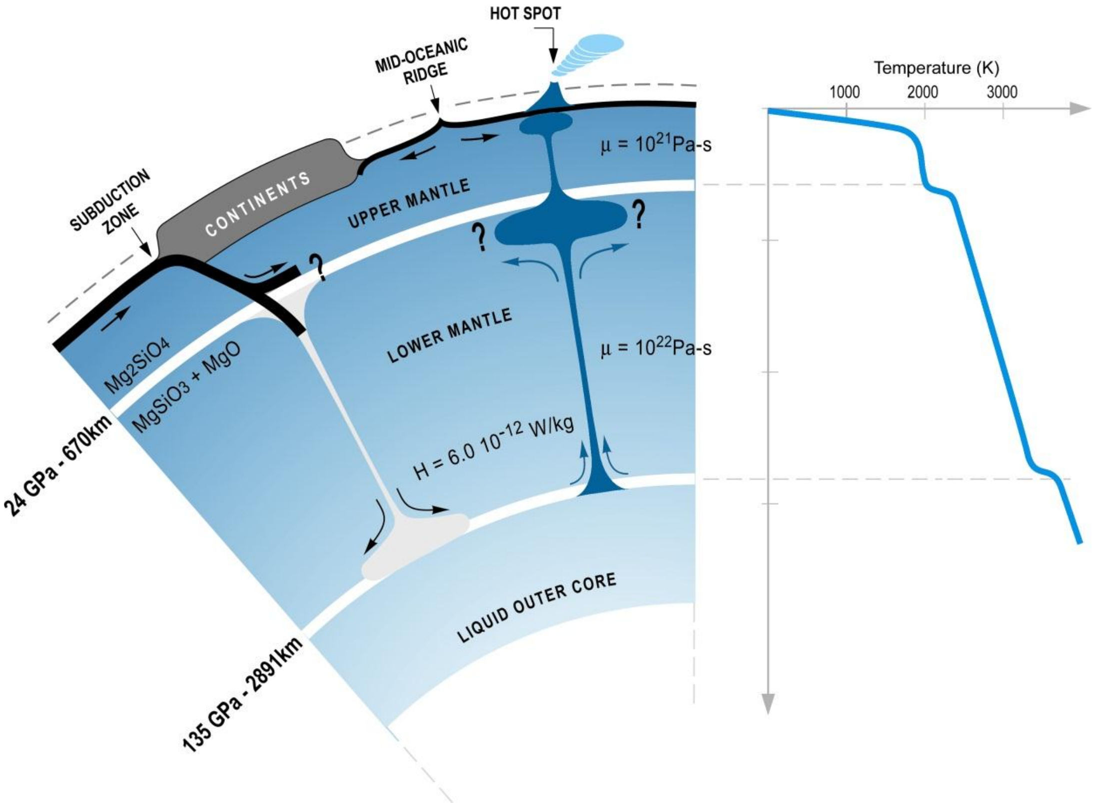
Five parameters are required:

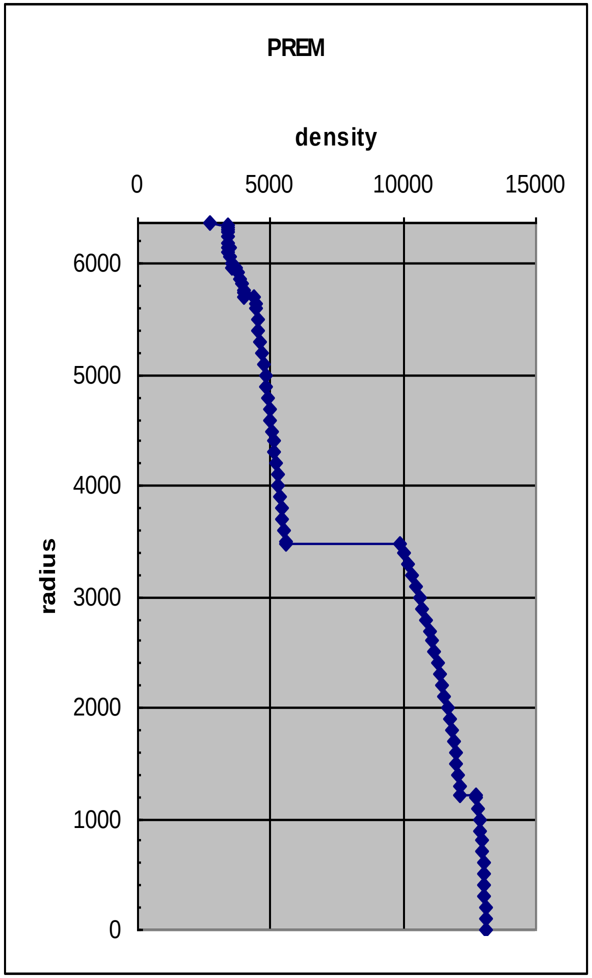
1) Total mass of the planet
2) Fe/Si (Stellar)
3) Mg/Si (Stellar)
4) Water mass fraction (Earth like / Ocean planet)
5) Mg# = Mg/(Mg+Fe)

Solar values		
	Mg,Fe,Si	+Ni,Ca,Al,S'
Fe/Si	0,977	**0,986**
Mg/Si	1,072	**1,131**
Fe/(Fe+Mg)	0,477	0,466

Large uncertainties on the composition of the Earth – how does it influence the M(R) law

Internal structure of the Earth





Mass = 6 10^{24} kg : 1/3 core and 2/3 mantle

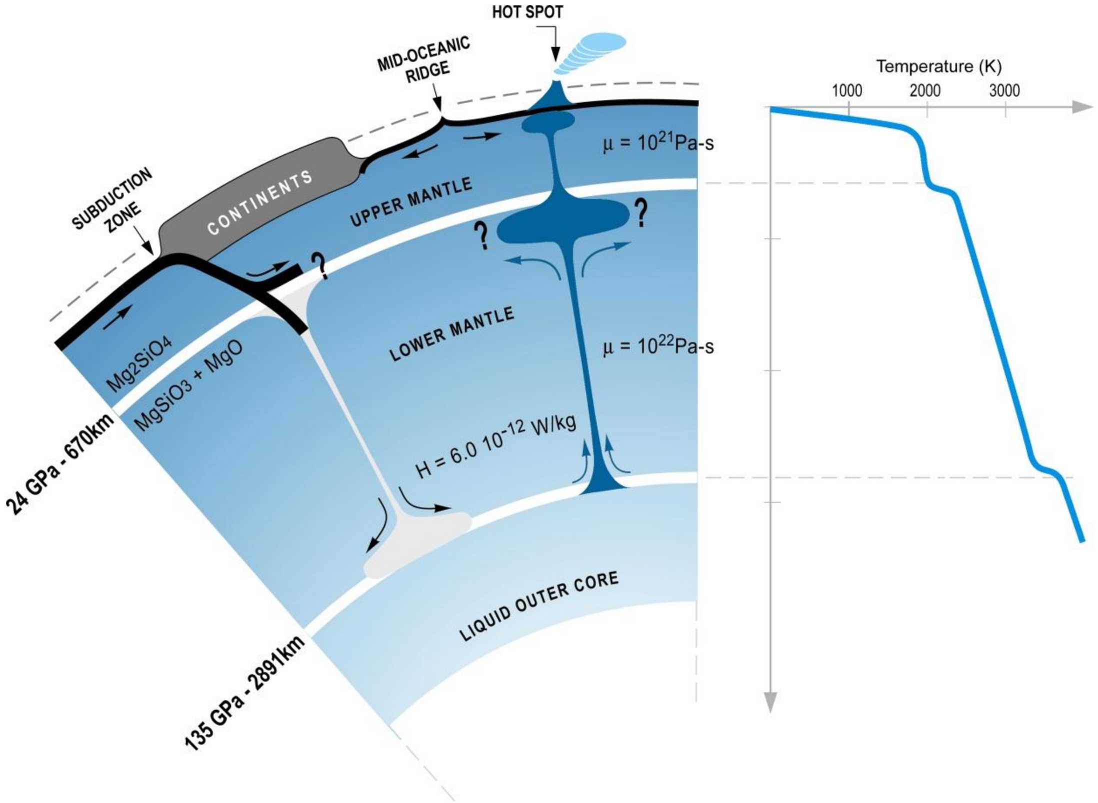
Upper and Lower mantle

Subsolidus Convection in the mantle

Core : Iron + light element (S, O, other).

Mantle : $(Mg,Fe)_2Si_2O_6$, $Ca(Mg,Fe)Si_2O_6$, $(Mg,Fe)_2SiO_4$ and Al phase / $(Mg,Fe)SiO_3$, $(Mg,Fe)O$ and Al phase

Modeling the mass - radius relationship. Temperature



$$\frac{dT}{dP} = \frac{\alpha T}{\rho C_P} = \frac{\gamma T}{\rho \Phi}$$

$$\Phi = \frac{K_S}{\rho} = \frac{dP}{d\rho}$$

$$\gamma = \gamma_0 \left(\frac{\rho_0}{\rho} \right)^q$$

T : Temperature
P : Pressure
ρ: density (kg/m^3)
α: thermal expansion coefficient (K^{-1})
C_p : specific heat (J/kg/K)

Relationship between radius and mass

$$M = 4\pi \int_0^R r'^2 \rho(r') dr'$$

$$\frac{dP}{dr} = -\rho(r) g(r)$$

$$g(r) = \frac{4\pi G}{r^2} \int_0^r r'^2 \rho(r') dr'$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C p}$$

$$P_{th} = \int_{T0}^{T} \alpha K_T \, dT$$

Mass and radius are two of the few parameters.

They are related to each other through a simple equation in a 1D model.

Density depends on composition (elementary and molecular), pressure, and temperature

In the calculations, the main parameters are:

- Amount of volatiles (H_2O)
- The amount of Fe
- Distribution of Fe between iron core and mantle

We need an Equation of State (EoS) which relates density to pressure and temperature.

Example of the Birch-Murnagham EoS :

$$P = \frac{3K_{0T}}{2}\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{5}{3}}\right]\left\{1 + \frac{3}{4}(K'_{0T} - 4)\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}} - 1\right]\right\}$$

The 3rd order Birch-Murnagham EoS

$$\begin{cases} P(\rho,T)=\frac{3}{2}K^{0}_{T,0}\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{7/3}-\left(\frac{\rho}{\rho_{T,0}}\right)^{5/3}\right]\left\{1-\frac{3}{4}\left(4-K^{'}_{T,0}\right)\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{2/3}-1\right]\right\} \\ K^{0}_{T,0}=K_{0}+a_{P}\left(T-T_{0}\right) \\ K^{'}_{T,0}=K^{'}_{0} \\ \rho_{T,0}=\rho_{0}\exp\left(\int_{300}^{T}\alpha_{T,0}\,dT\right) \\ \alpha_{T,0}=a_{T}+b_{T}.T-c_{T}.T^{-2} \end{cases}$$

Used for the upper mantle

8 parameters known at ambient pressure:

- T_0: the reference temperature
- ρ_0: density
- K_0: bulk modulus
- $K'_{T,0}$ α_P: pressure and temperature derivatives of bulk modulus
- a_T, b_T, c_T: thermal expansion coefficients

The Mie-Grüneisen-Debye formulation

$$\begin{cases} P(\rho,T)=P(\rho,T_0)+\Delta P_{th} \\ P(\rho,T_0)=\frac{3}{2}K_0\left[\left(\frac{\rho}{\rho_0}\right)^{7/3}-\left(\frac{\rho}{\rho_0}\right)^{5/3}\right]\left\{1-\frac{3}{4}\left(4-K_0^{'}\right)\left[\left(\frac{\rho}{\rho_0}\right)^{2/3}-1\right]\right\} \\ \Delta P_{th}=\left(\frac{\gamma}{V}\right)\left[E(T,\theta_D)-E(T_0,\theta_D)\right] \\ E=9nRT\left(\frac{T}{\theta_D}\right)^3\int_0^{\theta_D/T}t^3dt/\left(e^t-1\right) \\ \theta_D=\theta_{D0}\left(\frac{\rho}{\rho_0}\right)^{\gamma} \\ \gamma=\gamma_0\left(\frac{\rho}{\rho_0}\right)^{-q} \end{cases}$$

Used for the lower mantle and core

Thermal and static pressure are dissociated. 8 parameters :

- T_0: the reference temperature
- ρ_0 : density
- K_0 and $K'_{T,0}$: bulk modulus and its pressure derivative
- θ_{D0}: reference Debye temperature
- n : number of atoms per chemical formula
- q and γ_0: scaling exponents

Other formulations & comparisons

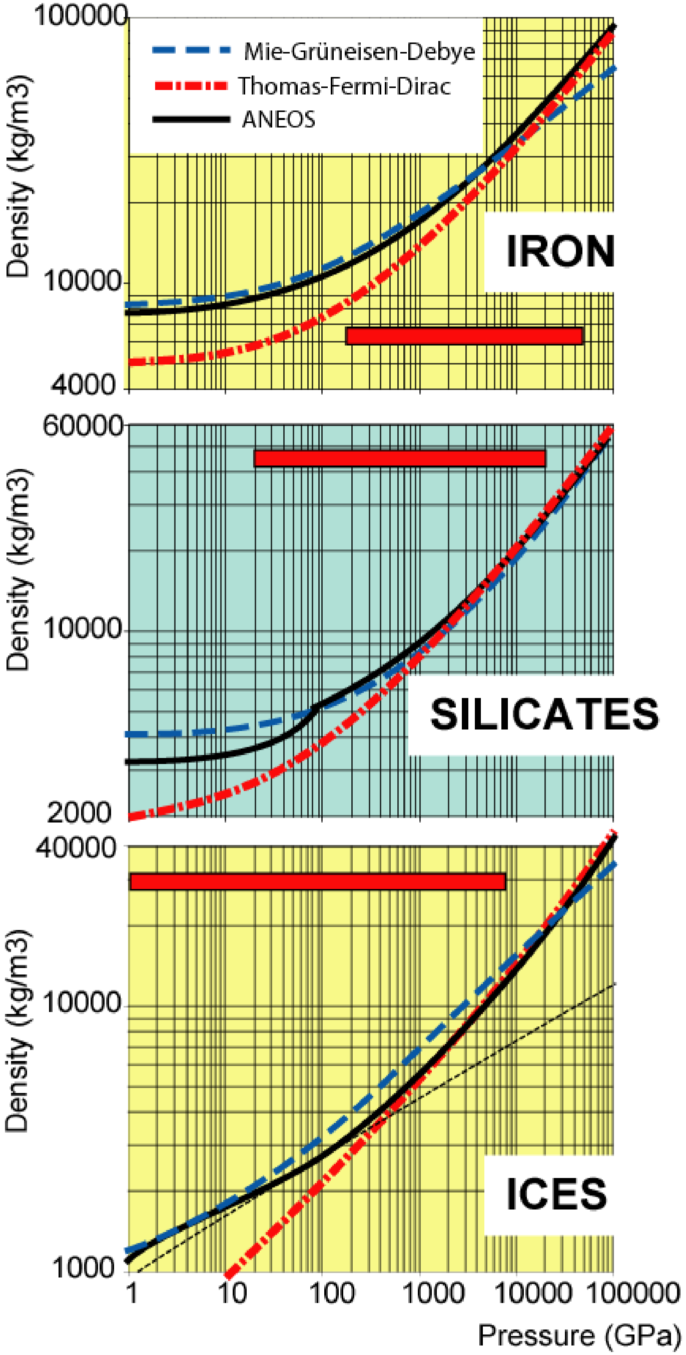
Birch-Mürnhagan EOS
- Liquid layer
- Upper silicate mantle

Mie-Grüneisen-Debye EOS
- Lower silicate mantle

Thomas-Fermi-Dirac
- Icy mantle
- Metallic core (P> 10 TPa)

Vinet EoS

ANEOS (Thompson, 1990)



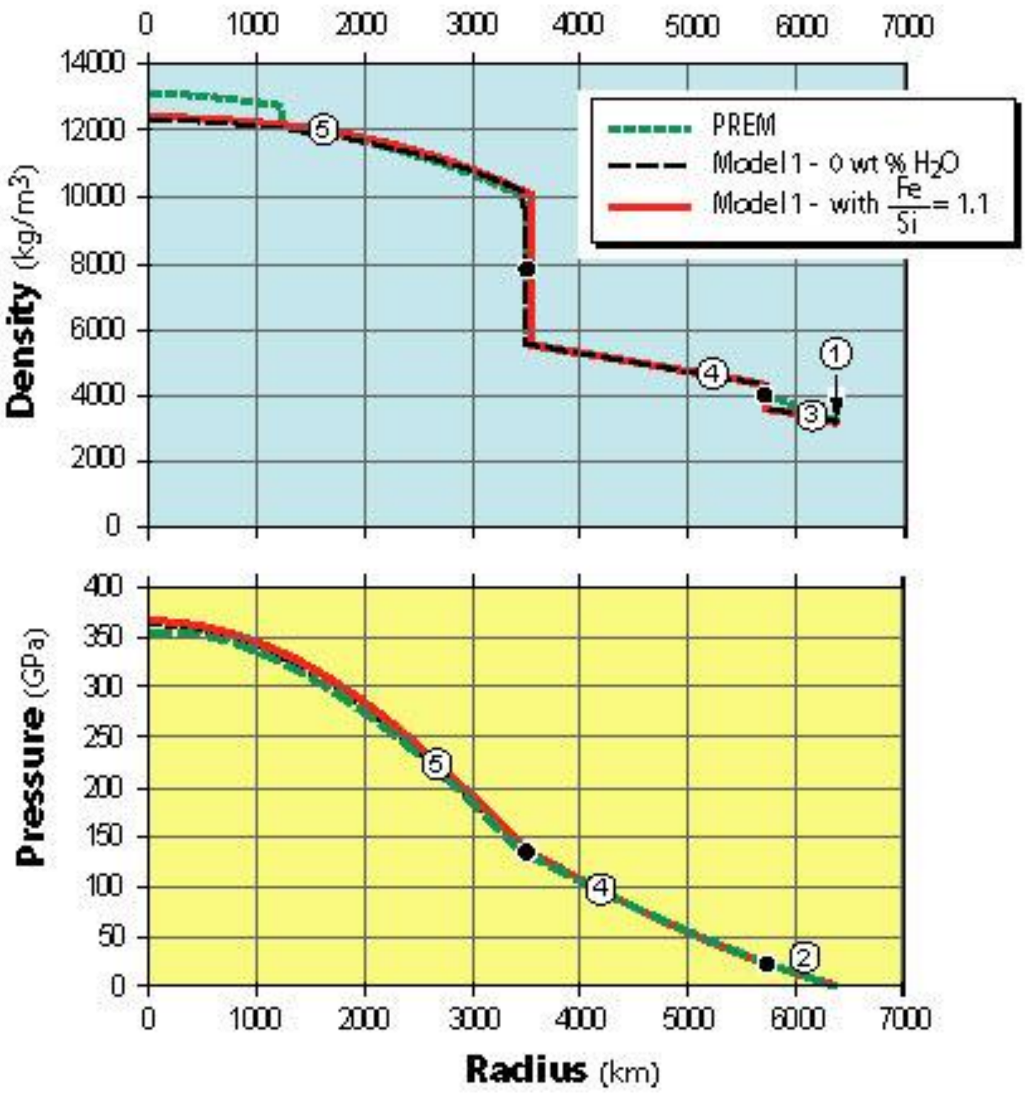
Results : Validation of the model - Earth

Model :

- Fe/Si = 0.987
- Mg/Si = 1.136
- Mg# = 0.9
- H_2O: 0.01 wt %

$M = M_{Earth}$

R=6414 km (0.6%)

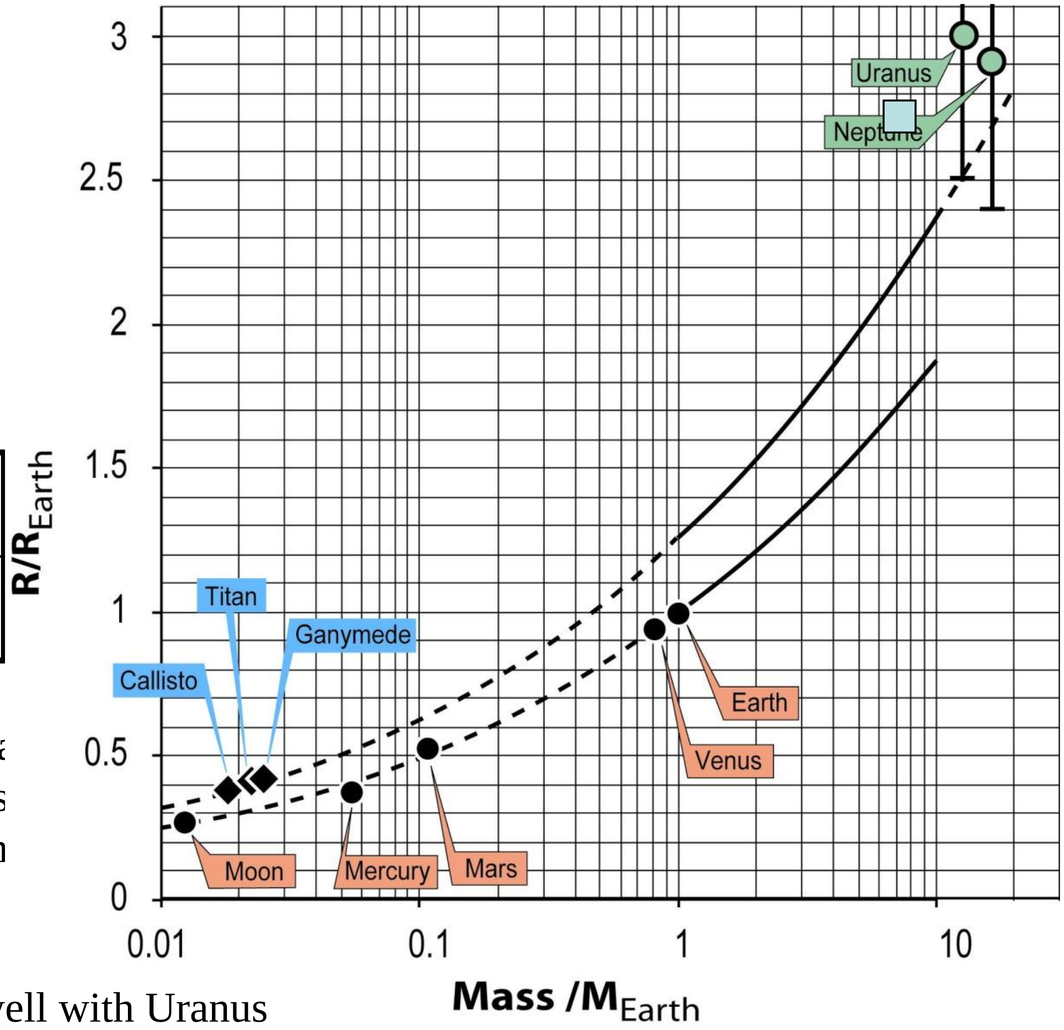


Results : Validation of the model – Solar system

$$\frac{R}{R_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.274}$$

	Earth-like		Ocean/Icy	
0.01-1	1.00	0.306	1.258	0.302
1-10	1.00	0.274	1.262	0.275

A planet with 50% water is 26% larger than
planet without water (for the same total mass
The points Uranus and Neptune have 1 Earth
radius of atmosphere removed.



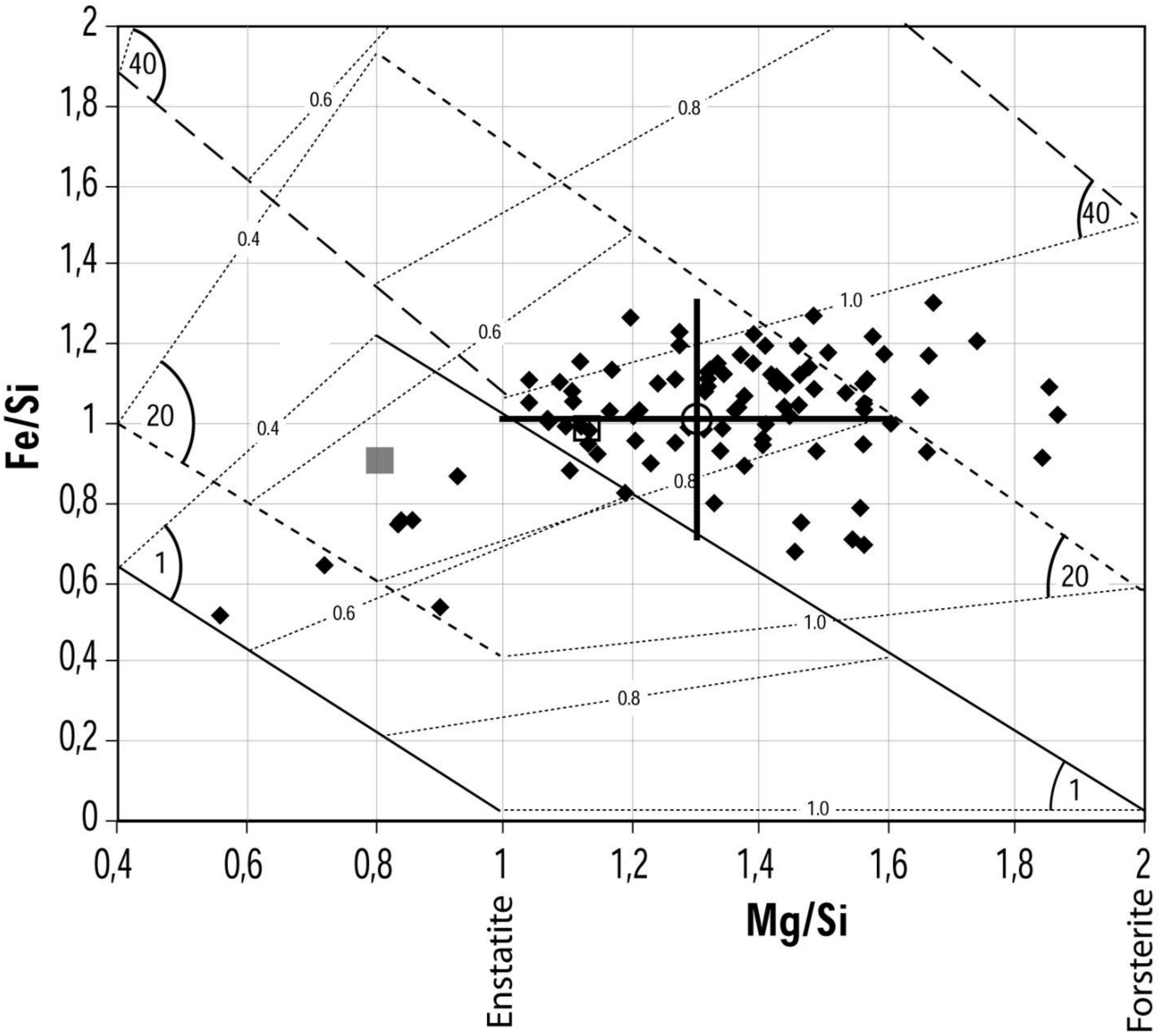
GJ1214 has more than 50% ice in it. It fits well with Uranus and Neptune without their H2/He atmosphere

What do we know about extra-solar planets?

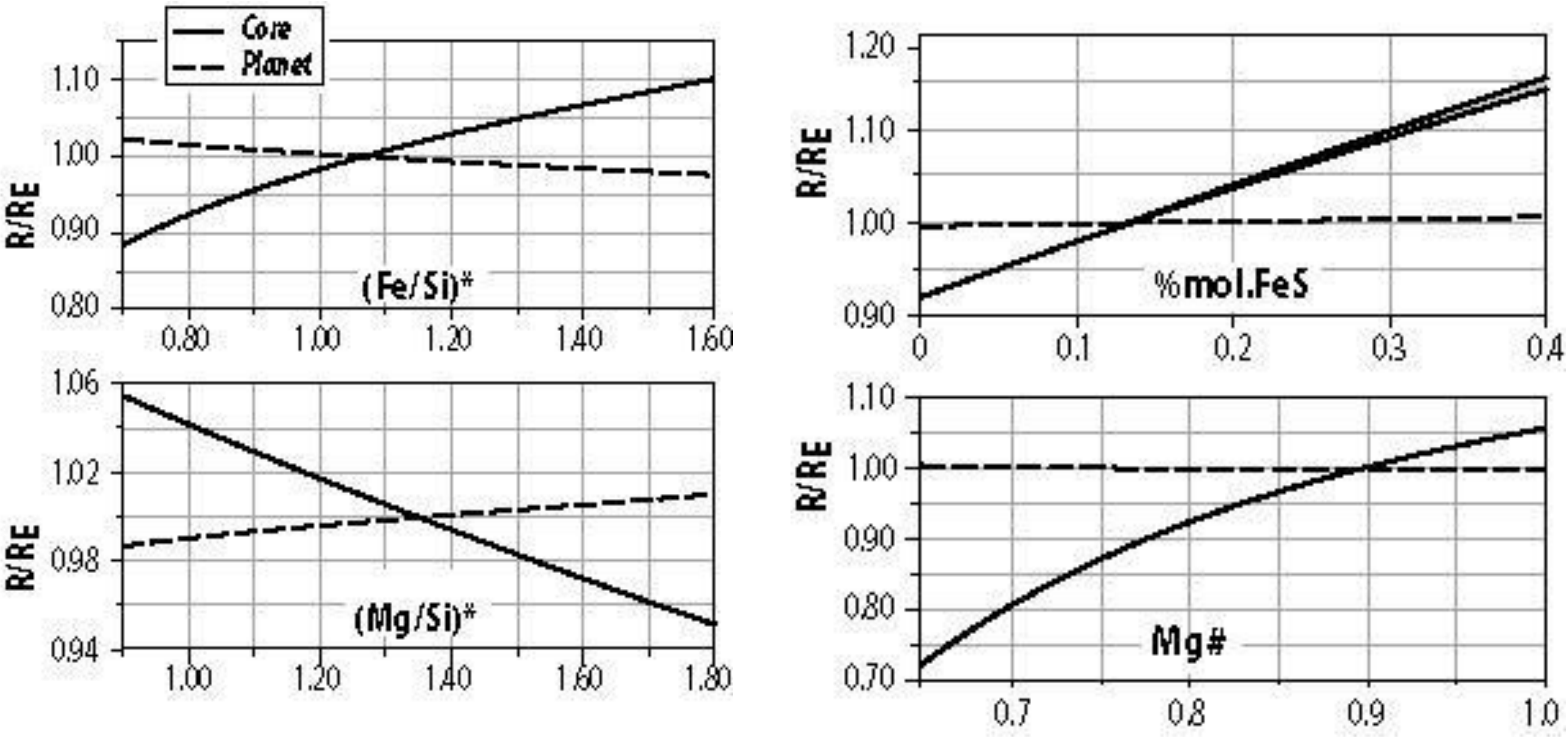
Composition:
Data from Beirao et al. (2006) and Gilli et al. (2006)

Empty square is solar composition.
Filled square is the enstatite end-member composition for the Earth’s mantle.
Empty circle is barycenter of all the stellar compositions.
The large cross is typical uncertainties

Lines are values of Mg#
Areas give mass fraction of the core



Radius versus composition (1 M_E < M < 10 M_E)

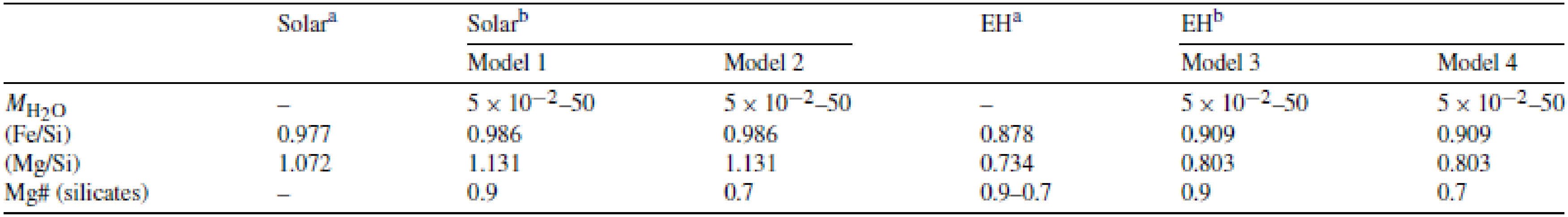


Total radius does not vary significantly with on the composition
The amount of Fe plays a significant role for the radius of the core

Application to water-rich planets

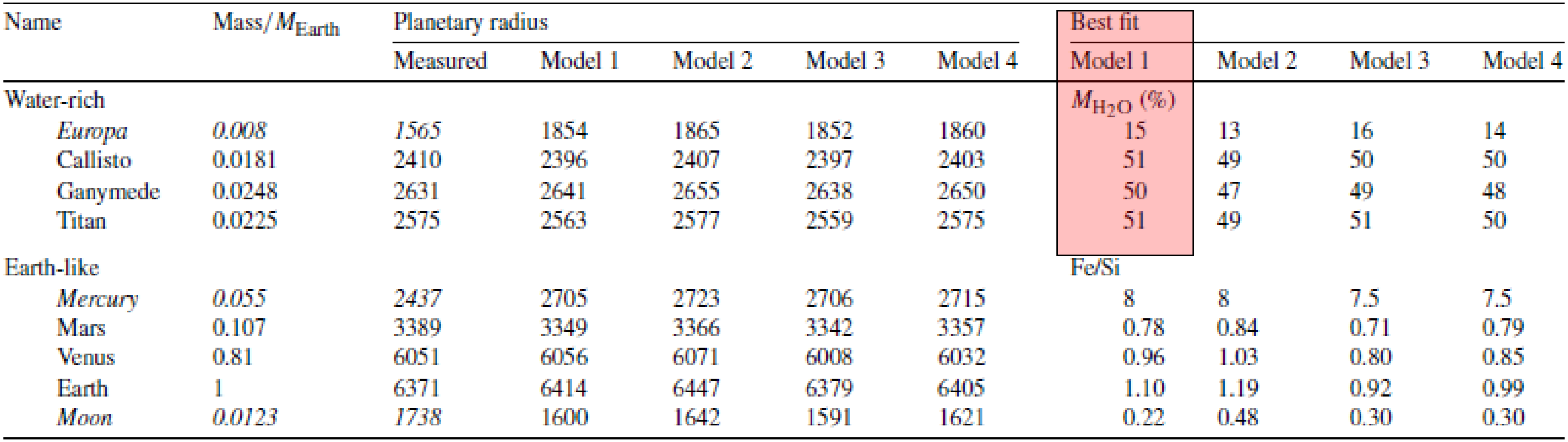
- How much water to add
- Equation of state of water/ice
- Radius versus mass for water-rich planets

How much water to add?



	Solar[a]	Solar[b]		EH[a]	EH[b]	
		Model 1	Model 2		Model 3	Model 4
M_{H_2O}	–	5×10^{-2}–50	5×10^{-2}–50	–	5×10^{-2}–50	5×10^{-2}–50
(Fe/Si)	0.977	0.986	0.986	0.878	0.909	0.909
(Mg/Si)	1.072	1.131	1.131	0.734	0.803	0.803
Mg# (silicates)	–	0.9	0.7	0.9–0.7	0.9	0.7

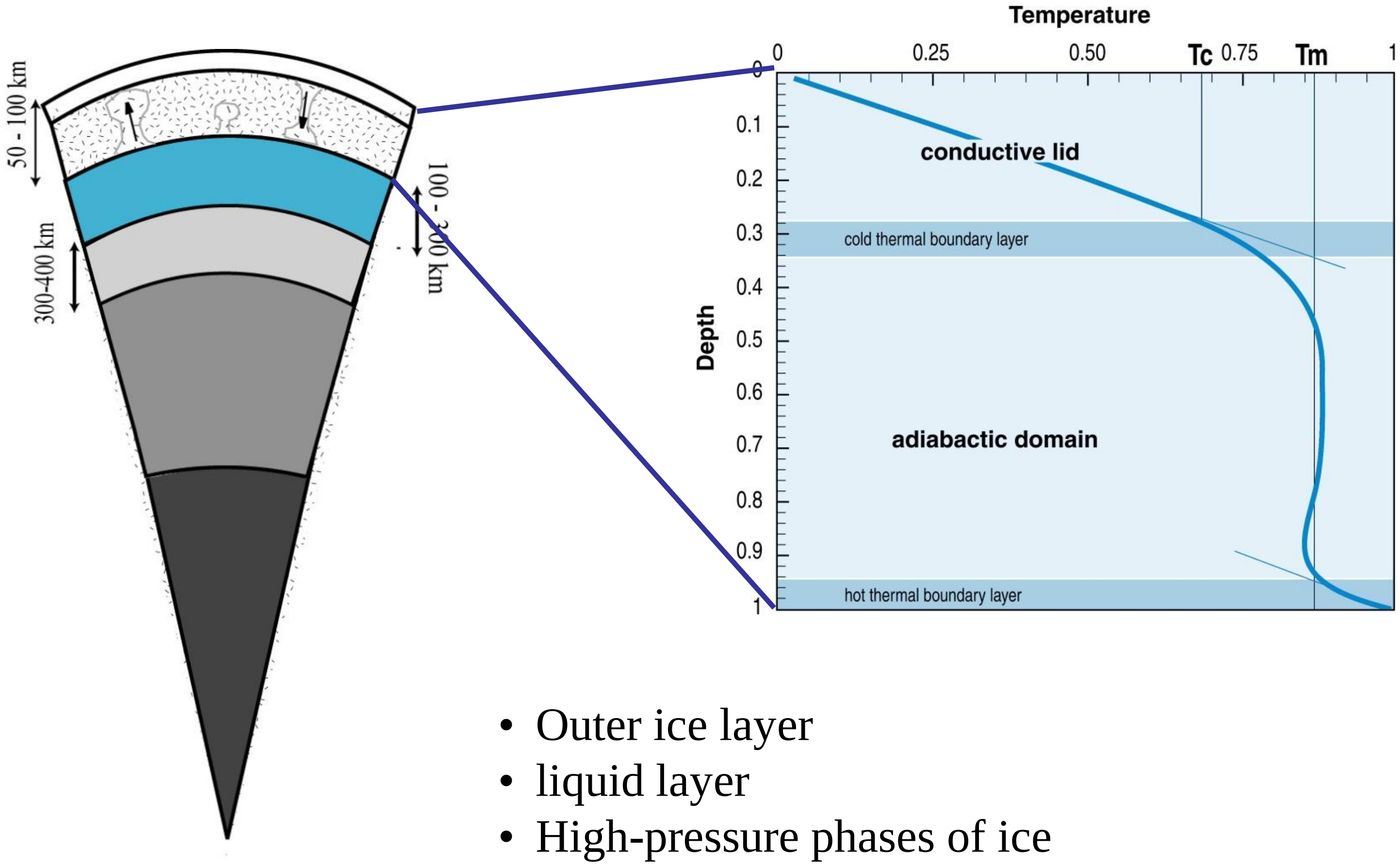
Four models: Solar and Enstatite and two different Mg#.



Name	Mass/M_{Earth}	Planetary radius					Best fit			
		Measured	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Water-rich							M_{H_2O} (%)			
Europa	*0.008*	*1565*	1854	1865	1852	1860	15	13	16	14
Callisto	0.0181	2410	2396	2407	2397	2403	51	49	50	50
Ganymede	0.0248	2631	2641	2655	2638	2650	50	47	49	48
Titan	0.0225	2575	2563	2577	2559	2575	51	49	51	50
Earth-like							Fe/Si			
Mercury	*0.055*	*2437*	2705	2723	2706	2715	8	8	7.5	7.5
Mars	0.107	3389	3349	3366	3342	3357	0.78	0.84	0.71	0.79
Venus	0.81	6051	6056	6071	6008	6032	0.96	1.03	0.80	0.85
Earth	1	6371	6414	6447	6379	6405	1.10	1.19	0.92	0.99
Moon	*0.0123*	*1738*	1600	1642	1591	1621	0.22	0.48	0.30	0.30

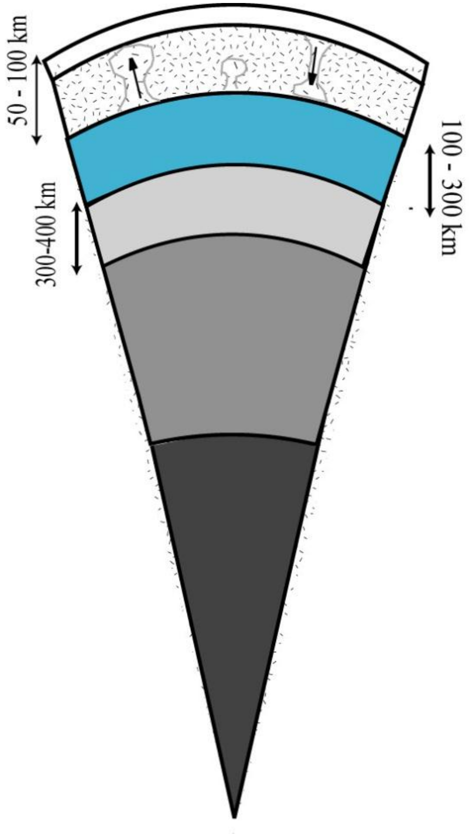
The right part of the table indicates the required value of M_{H_2O} (ocean-planet) or Fe/Si (Earth-like planet) in order to get the value of the measured radius for each body and for each of the four models described in Table 2.

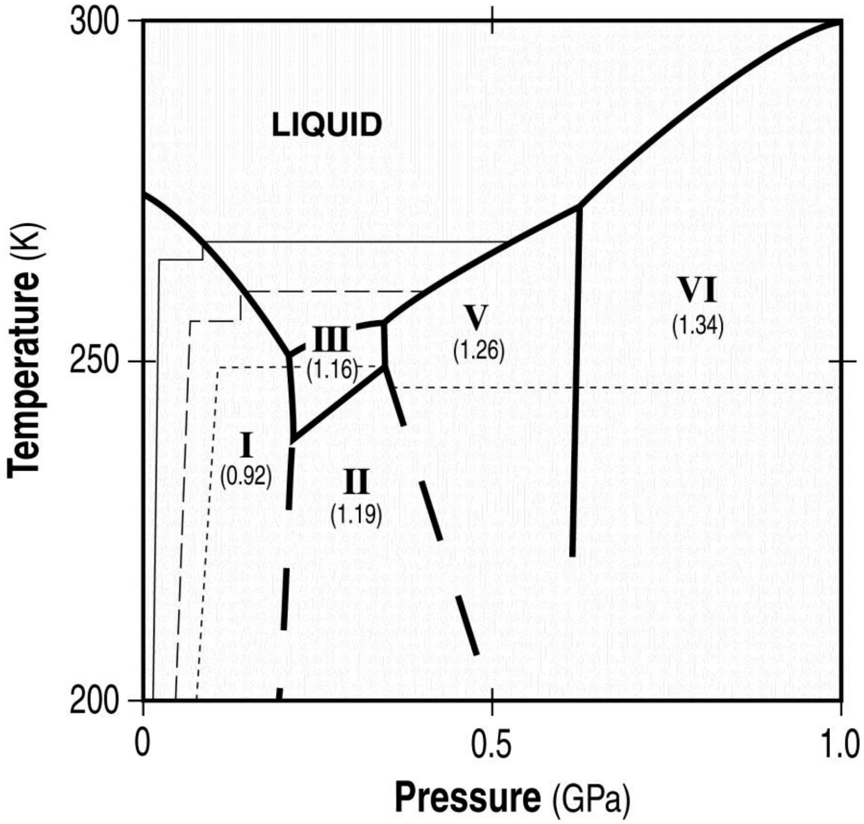
Internal structure of large icy satellites: a model for icy exoplanets (ocean exoplanets)



- Outer ice layer
- liquid layer
- High-pressure phases of ice
- Silicate layer
- Iron core

Internal structure of large icy satellites (2/2)





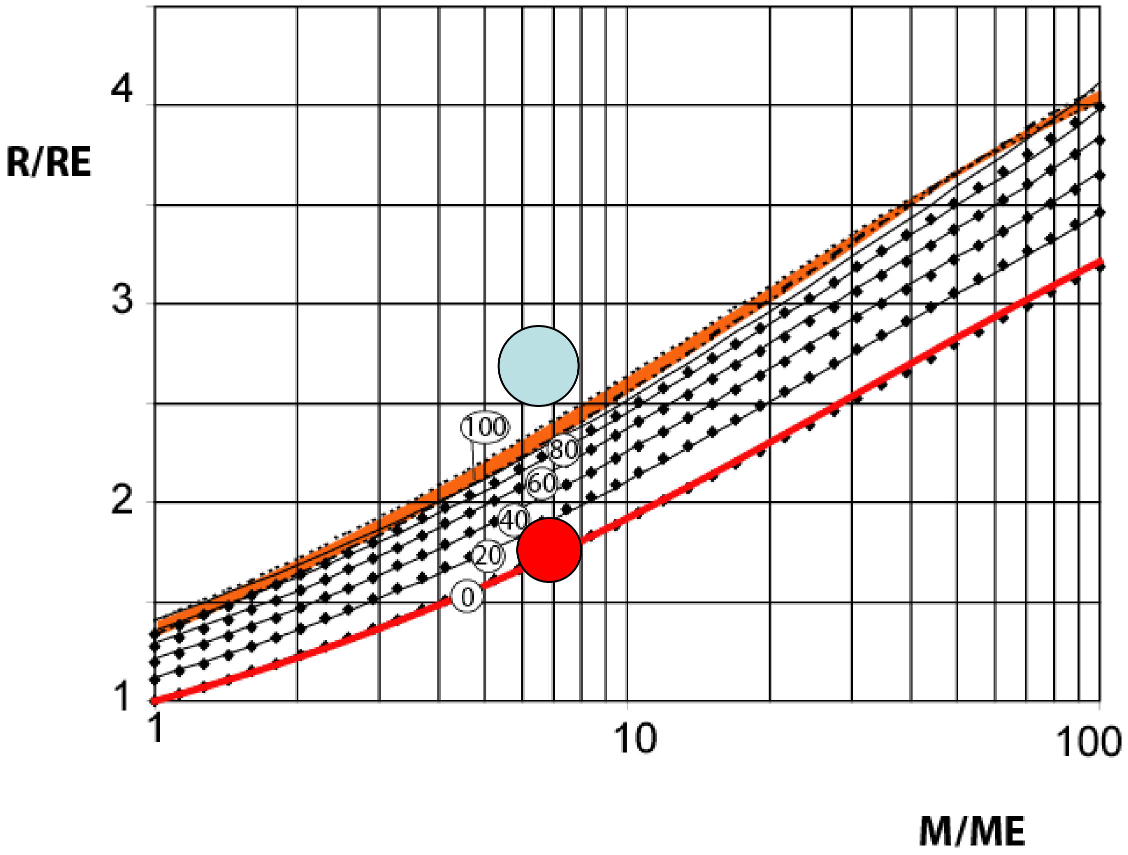
Results : Extrapolation to larger planets

$$\frac{R}{R_{Earth}}=\left(\frac{M}{M_{Earth}}\right)^{0.274}$$

<u>Reference Case</u> :

- **Fe/Si = 1.10** *
- **Mg/Si = 1.25** *
- **Mg# = 0.8**
- **H_2O: 0.01 wt %**

* Averaged from Gilli et al., A&A, 2006



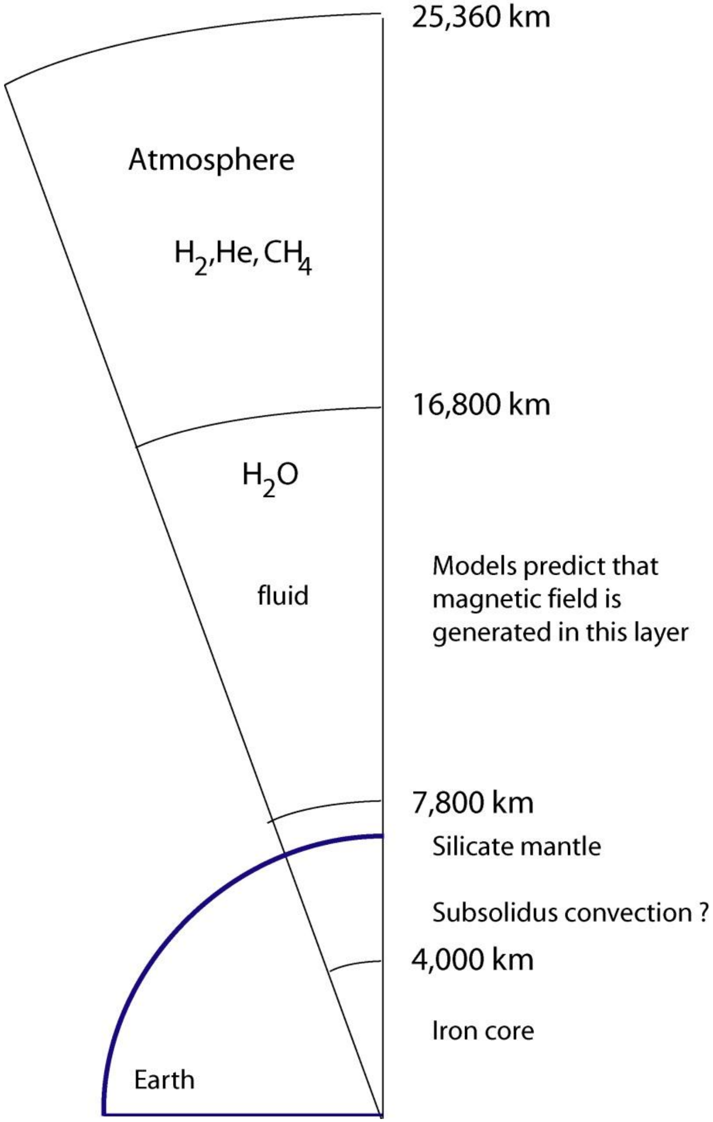
$$\log\left(\frac{R}{R_E}\right)=\log(\alpha)+\left(\beta+\gamma\frac{M}{M_E}+\varepsilon\left(\frac{M}{M_E}\right)\right)\log\left(\frac{M}{M_E}\right)$$

Each coefficient depends on the amount of water (X)

$$\xi=\sum_{i=0}^{2}\xi_i\,X_w^{i-1}$$

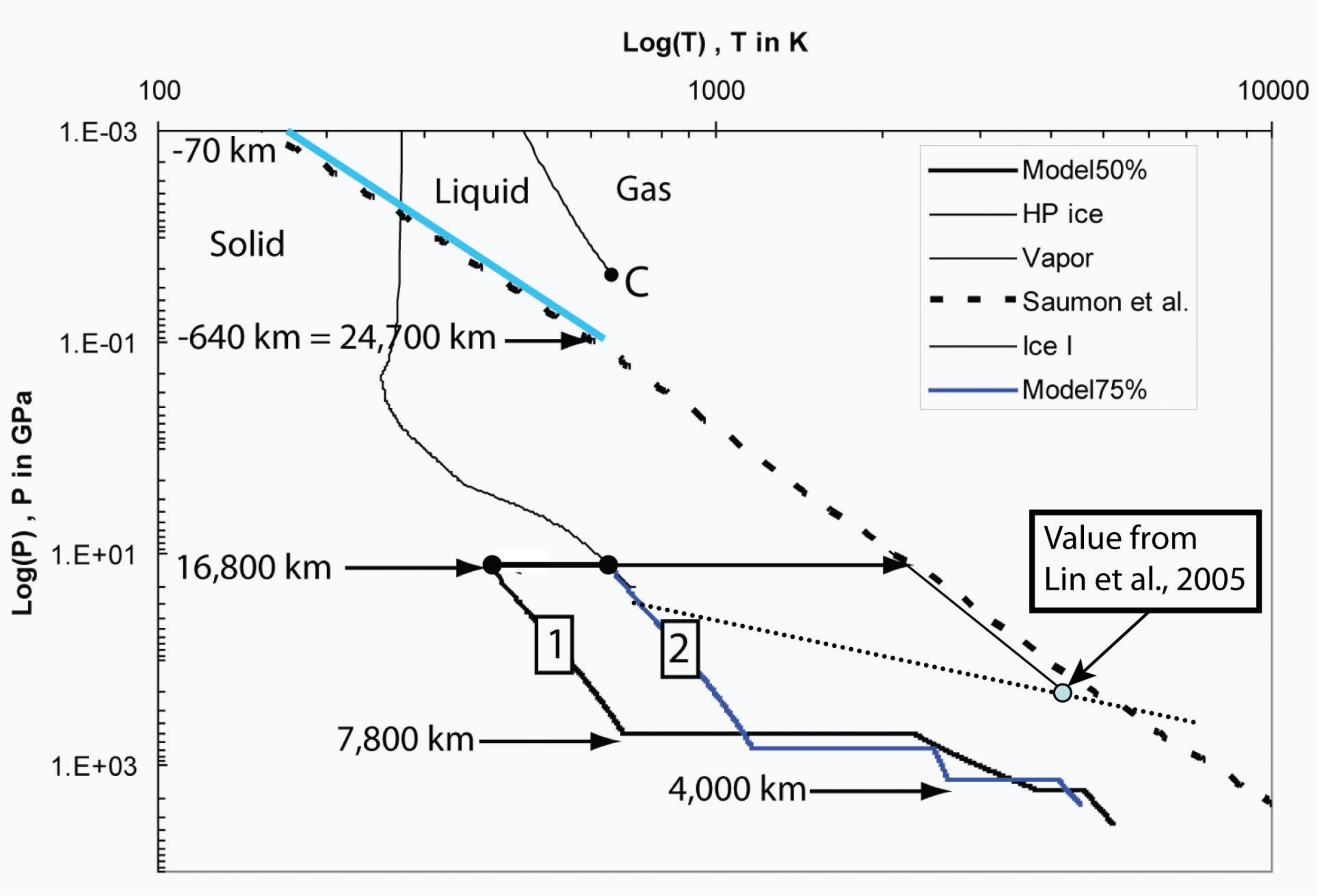
Uranus and Neptune / Earth

	Uranus	**Neptune**
Mass (10^{24} kg)	86.832 (**14.5**)	102.435 (**17.1**)
Volumetric radius (km)	25,364 (**3.98**)	24,625 (**3.87**)
Mean density (kg/m^3)	1,270 (2)	1,638(4)
Albedo	0.300(49)	0.290(67)
Absorbed power (x 10^{15} W)	5.26(37)	2.04(19)
Emitted power (x 10^{15} W)	5.60(11)	5.34(29)
Intrinsic power (x 10^{15} W)	0.34(38)	3.30(35)
Intrinsic flux (W/m^2)	0.042(47)	0.433(36)
Black-body temperature (K)	59.1	59.3
1-bar temperature^b (K)	76 (2)	72 (2)
$J_{2,0}$ (x 10^{-6})	3,516(3)	3,539 (10)
$J_{4,0}$ (x10^{-6})	-35.4 (4.1)	-28(22)
$Q=\omega^2R^3/GM$	0.02951 (5)	0.02609(26)
Moment of inertia (I/MR2)	0.230	0.241



Models predict that magnetic field is generated in this layer

Uranus and Neptune / Earth



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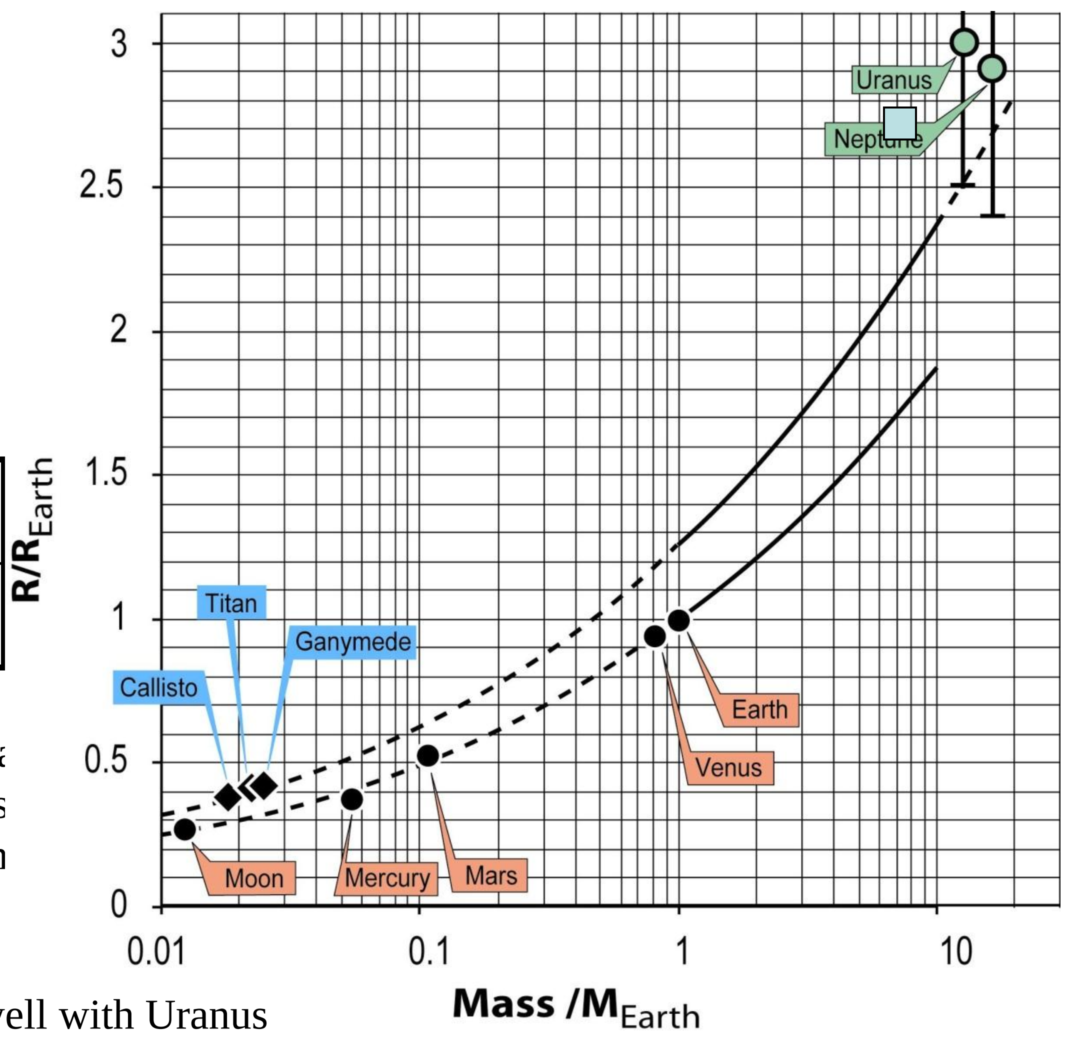

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Conclusions

- Plate tectonics on terrestrial planets
- What has been found so far ?
- Outstanding questions

Plate tectonics on large Earths

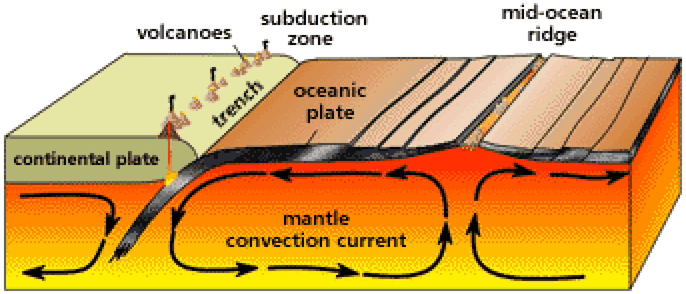
Plate tectonics provides a recycle of volatiles on geological timescales that may be important for the development of life

Two papers came out at the same time with two different conclusions

Valencia et al., ApJ, 2007
O'Neill and Lenardic, GRL, 2007

Valencia et al. : …. We demonstrate that as **planetary mass increases**, the **shear stress** available to overcome resistance to plate motion **increases** while the **plate thickness decreases**, thereby enhancing plate weakness.
These effects contribute favorably to the subduction of the lithosphere, an essential component of plate tectonics.
Moreover, uncertainties in achieving plate tectonics in the one earth-mass regime disappear as mass increases: super-Earths, even if dry, will exhibit plate tectonic behavior.
O'Neil and Lenardic : … mantle convection simulations have been carried out to show that simply **increasing planetary radius** acts to **decrease** the ratio of driving to resisting stresses, and thus super-sized Earths are likely to be **in an episodic or stagnant lid regime**.

Calculation of lithosphere thickness and stress



The deviatoric horizontal normal stress (σ) responsible for causing failure on the plate is (to first order) balanced by the shear stress (τ) applied over the base of the plate.

$$\sigma \approx \tau \frac{L}{\delta}$$

The thickness of the lithosphere or boundary layer (δ) depends on the Rayleigh number (Ra) – a parameter governing convection.

$$\frac{\delta}{D} = \left(\frac{Ra}{Ra_c} \right)^{S}$$

With S=-1/4

$$Ra = \frac{\alpha \rho g D^4 q}{k \kappa \eta} \qquad \tau \approx \eta \frac{u}{D} \qquad u \approx \frac{\kappa}{D} (Ra)^{0.5} \qquad L \approx u \frac{\delta^2}{\kappa}$$

q is heat flux (radiogenic and cooling rate)

τ is the deviatoric stress
η is viscosity (Pa.s)
U is velocity
D is thickness of the convective layer

Results

$$\tau \approx \eta \frac{u}{D}$$

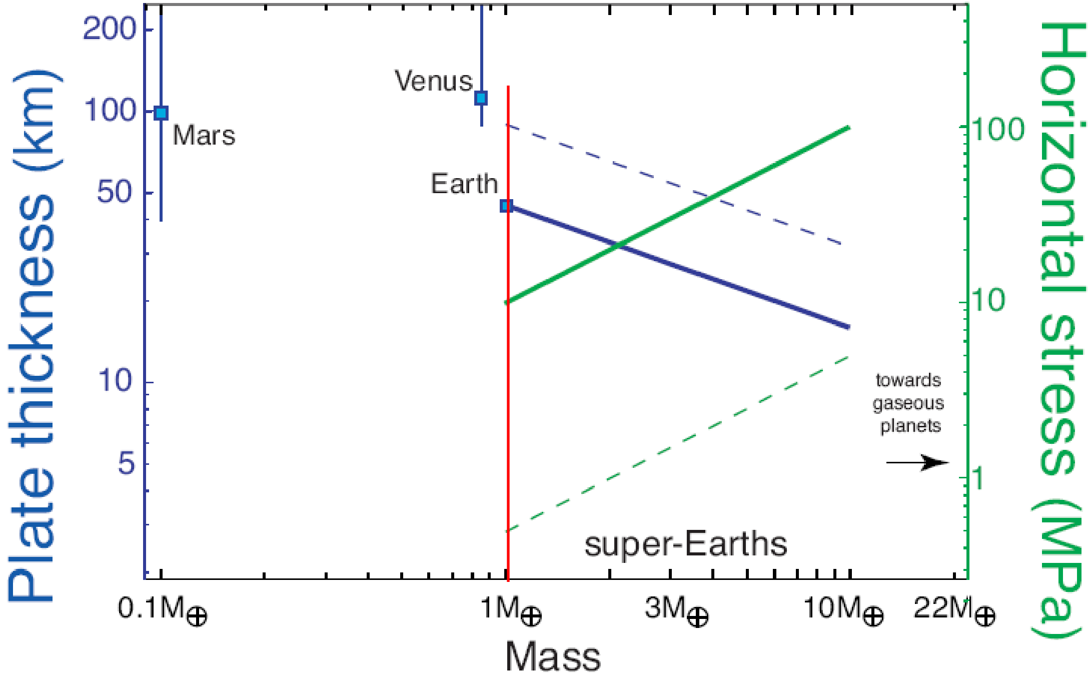
$$Ra = \frac{\alpha \rho g D^4 q}{k \kappa \eta}$$

$$\frac{\rho}{\rho_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.2}$$

$$\frac{g}{g_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.5}$$

$$\frac{D}{D_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.28}$$

Fig. 1



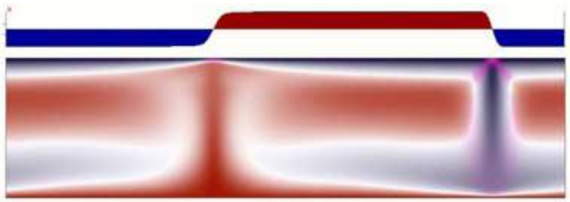
q is proportional to M

$\delta \approx M^{-0.45}$ and $\sigma \approx M^{1.0}$

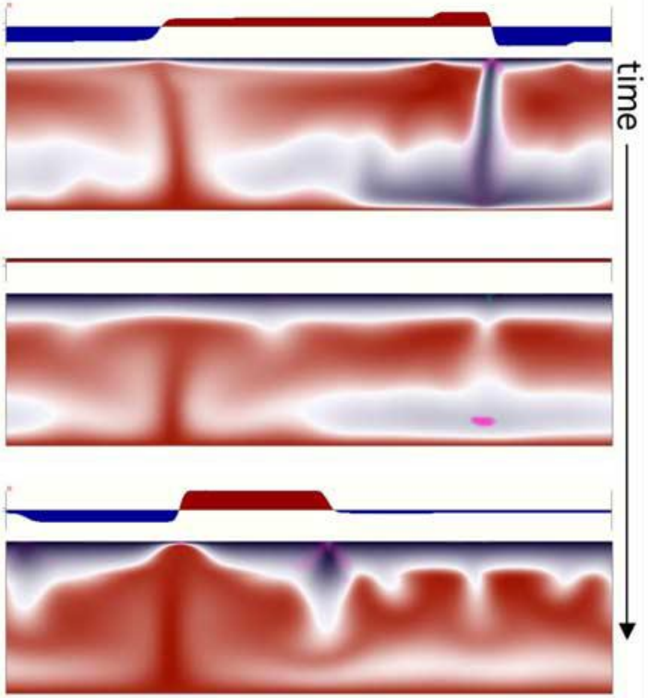
Given that Earth’s convective state leads to plate tectonics, the more favorable conditions experienced by super-Earths will inevitably lead to plate tectonics.

“Geological consequences of super-sized Earths”

by C. O’Neill and A. Lenardic



Scaling Factor =1.00 ; Active



Scaling Factor =1.07 ; Episodic

Scaling Factor =1.10 ; Stagnant

mantle convection simulations have been carried out to show that simply **increasing planetary radius** acts to **decrease** the ratio of driving to resisting stresses, and thus super-sized Earths are likely to be **in an episodic or stagnant lid regime**.

$$\frac{\tau}{\sigma}\downarrow$$

“Geological consequences of super-sized Earths”

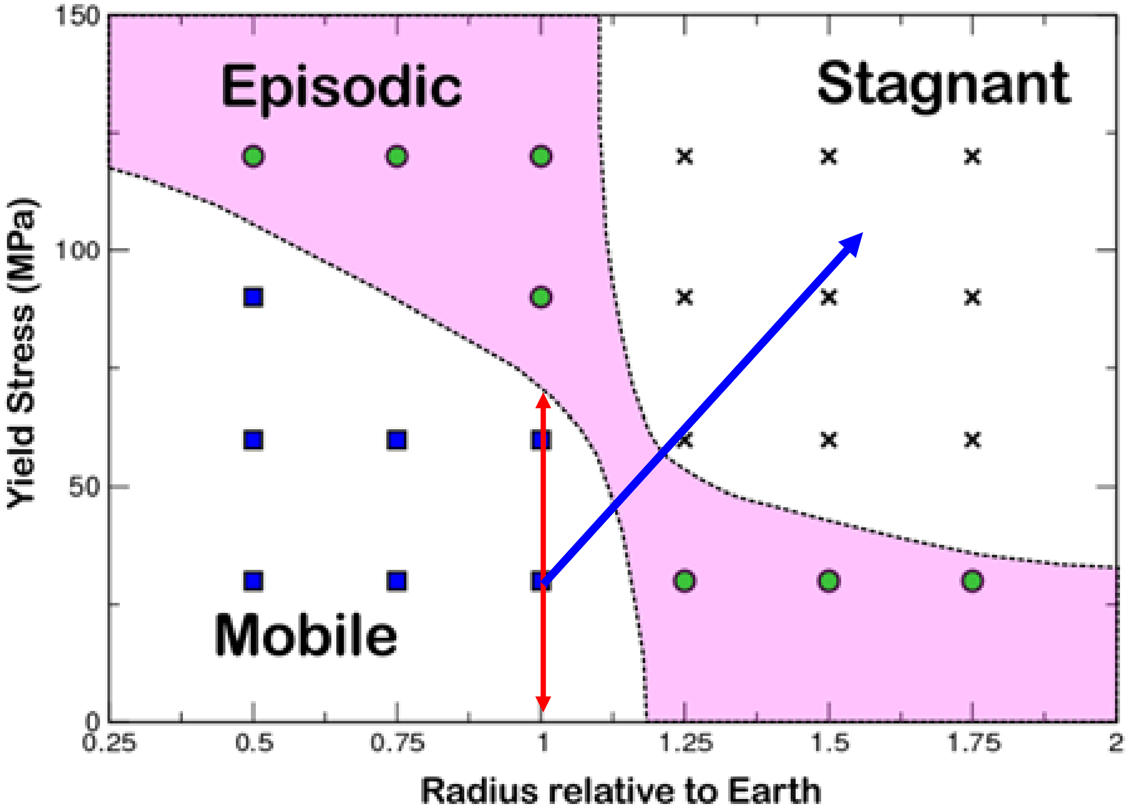
by C. O’Neill and A. Lenardic

$$\sigma \approx \tau \frac{L}{\delta}$$

$$\tau \approx \eta \frac{u}{D}$$

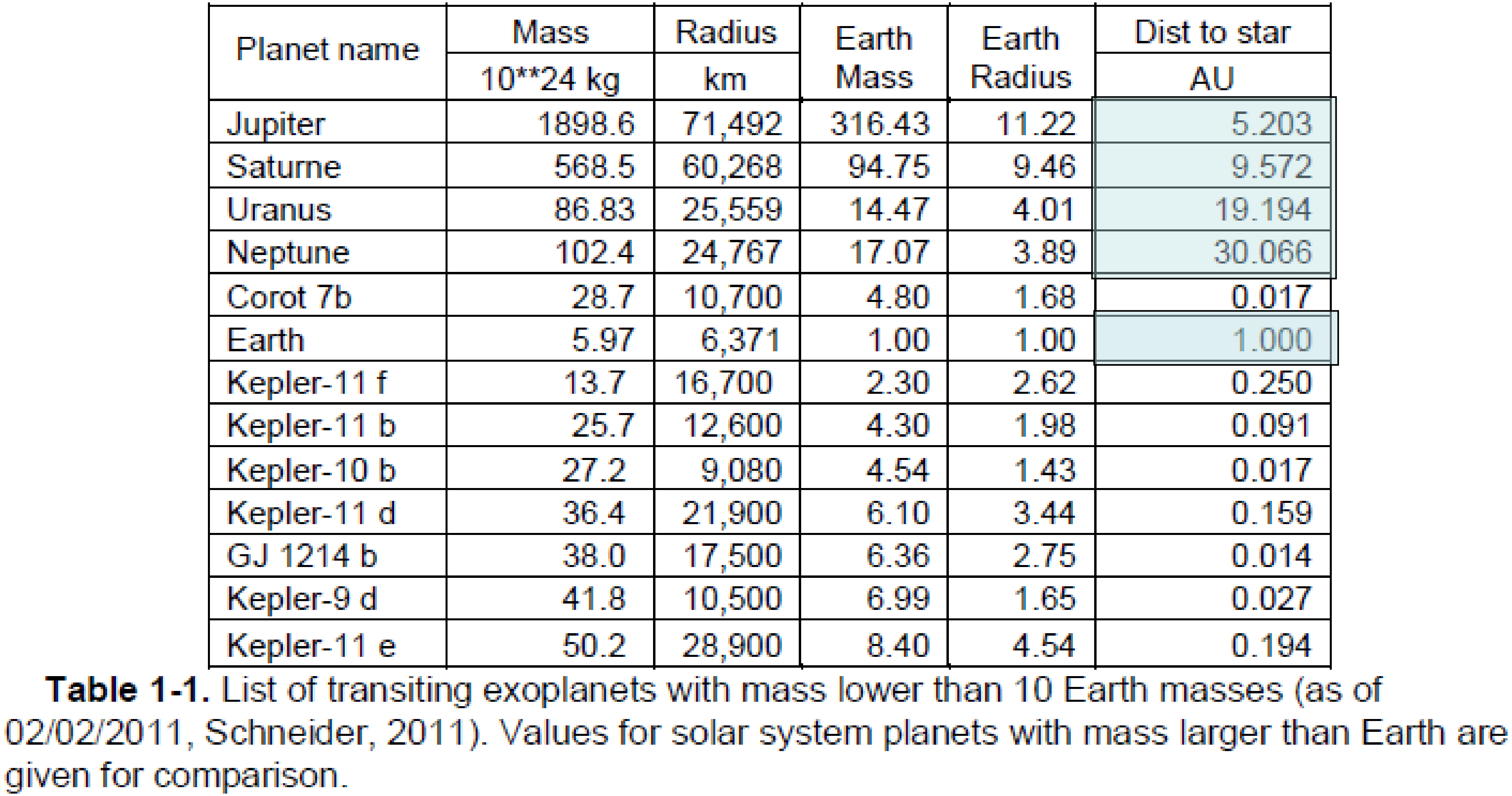
$$\tau \approx M^{0.27}$$

$$u \approx M^{1.19}$$



Why is there no plate tectonics on Mars and Venus? Water?

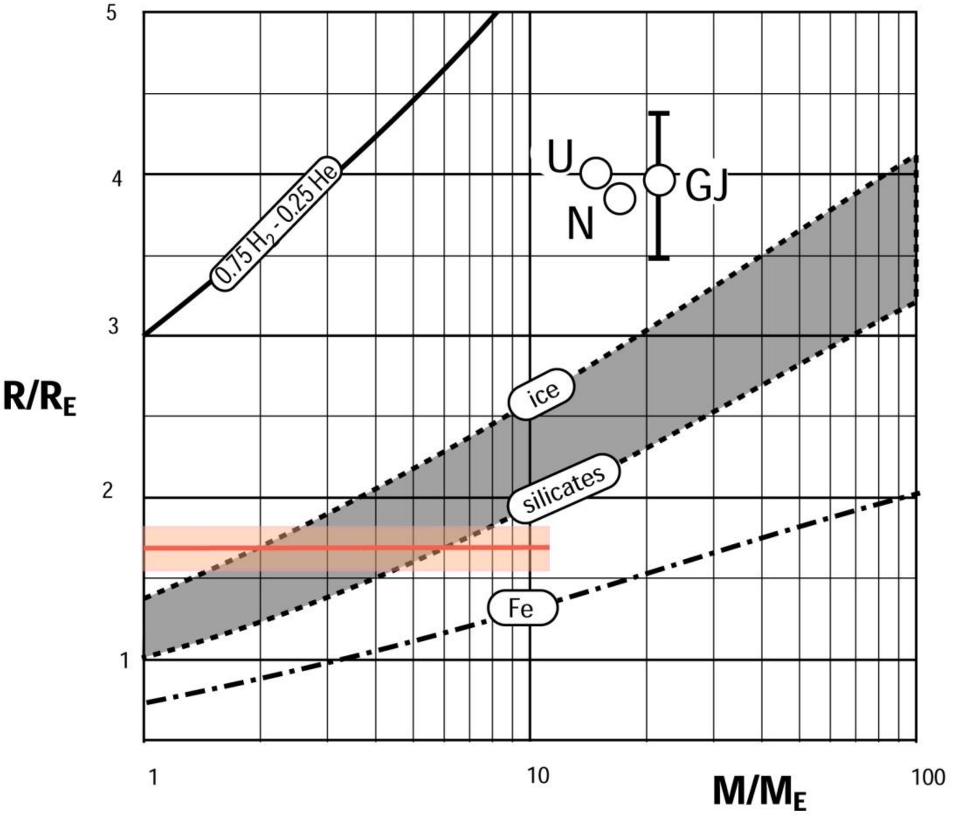
Low mass exoplanets

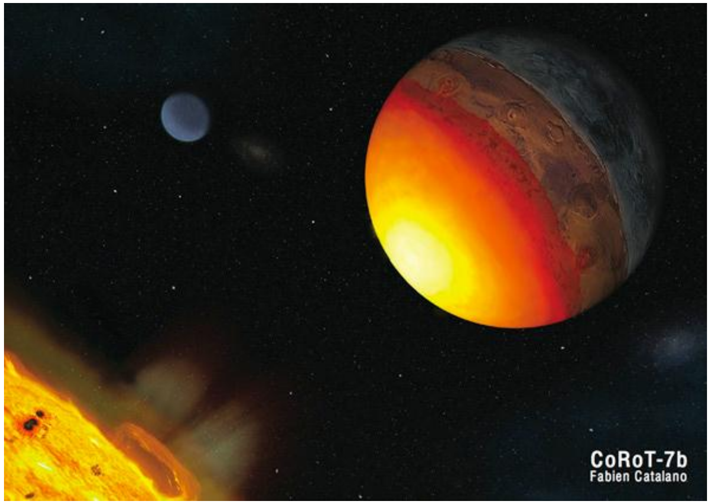


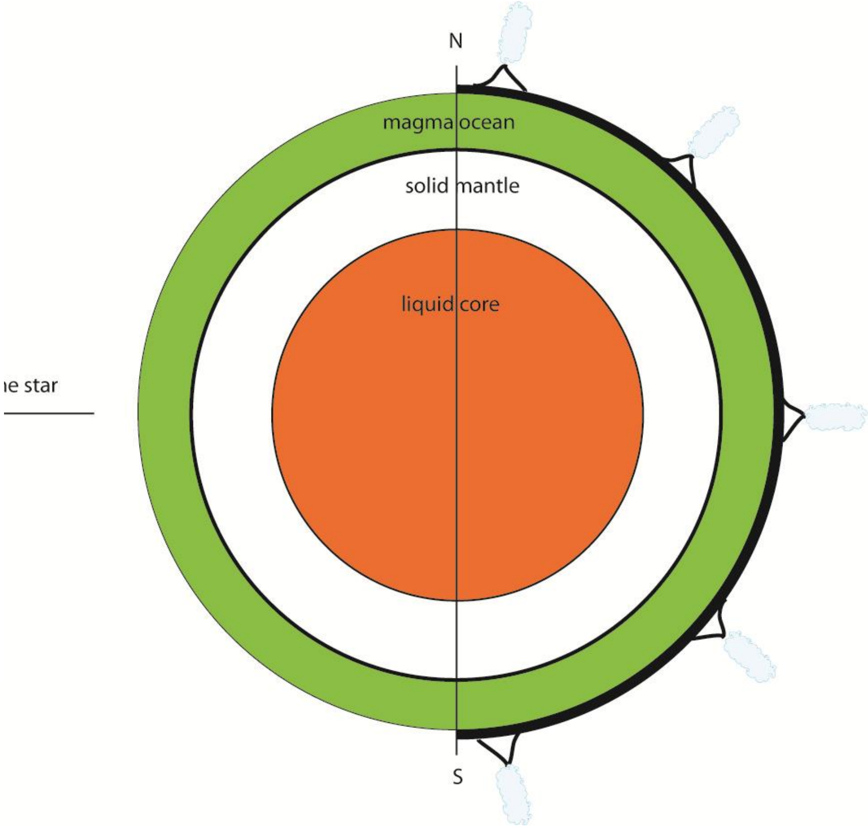
Planet name	Mass	Radius	Earth Mass	Earth Radius	Dist to star
	10**24 kg	km			AU
Jupiter	1898.6	71,492	316.43	11.22	5.203
Saturne	568.5	60,268	94.75	9.46	9.572
Uranus	86.83	25,559	14.47	4.01	19.194
Neptune	102.4	24,767	17.07	3.89	30.066
Corot 7b	28.7	10,700	4.80	1.68	0.017
Earth	5.97	6,371	1.00	1.00	1.000
Kepler-11 f	13.7	16,700	2.30	2.62	0.250
Kepler-11 b	25.7	12,600	4.30	1.98	0.091
Kepler-10 b	27.2	9,080	4.54	1.43	0.017
Kepler-11 d	36.4	21,900	6.10	3.44	0.159
GJ 1214 b	38.0	17,500	6.36	2.75	0.014
Kepler-9 d	41.8	10,500	6.99	1.65	0.027
Kepler-11 e	50.2	28,900	8.40	4.54	0.194

Table 1-1. List of transiting exoplanets with mass lower than 10 Earth masses (as of 02/02/2011, Schneider, 2011). Values for solar system planets with mass larger than Earth are given for comparison.

CoRoT Exo-7b

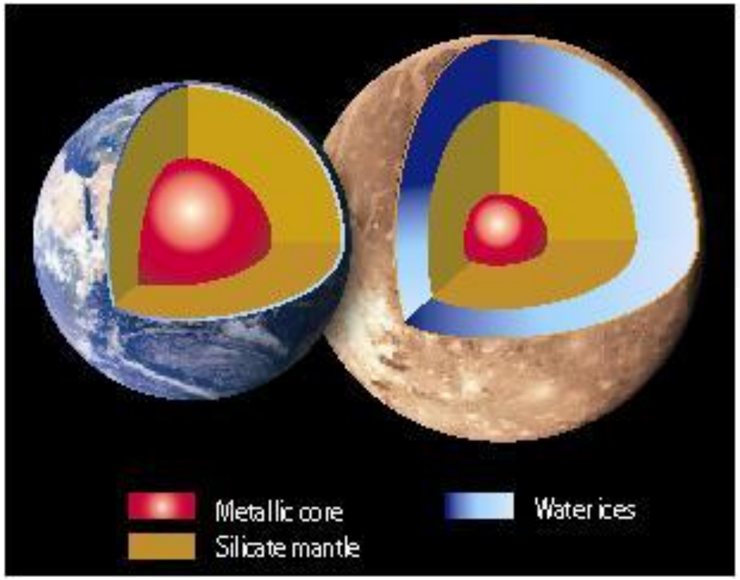






Leger et al., 2009, 2011

Conclusions



- Models give very good prediction of radii
- Amount of water is a first order parameter
- **Radius is 26 % larger for an Ocean planet with 50 %wt of ices**
- Temperature is a second order parameters.
- Composition and Mg# control the size of the core.
- **If Mass and Radius are perfectly known, the amount of water can be known at ± 4.4 %**
- **If 10% uncertainty of mass and radius, then the amount of water can be known at ± 20 %**
- Number of terrestrial planets is increasing
- YES, super-Earths and mini-Neptunes can be distinguished
- BUT [Super-Earths with H2/He atm] give same values than mini-Neptunes