Basic light curve models

2012 Sagan Exoplanet Workshop

Working with Exoplanet Light Curves

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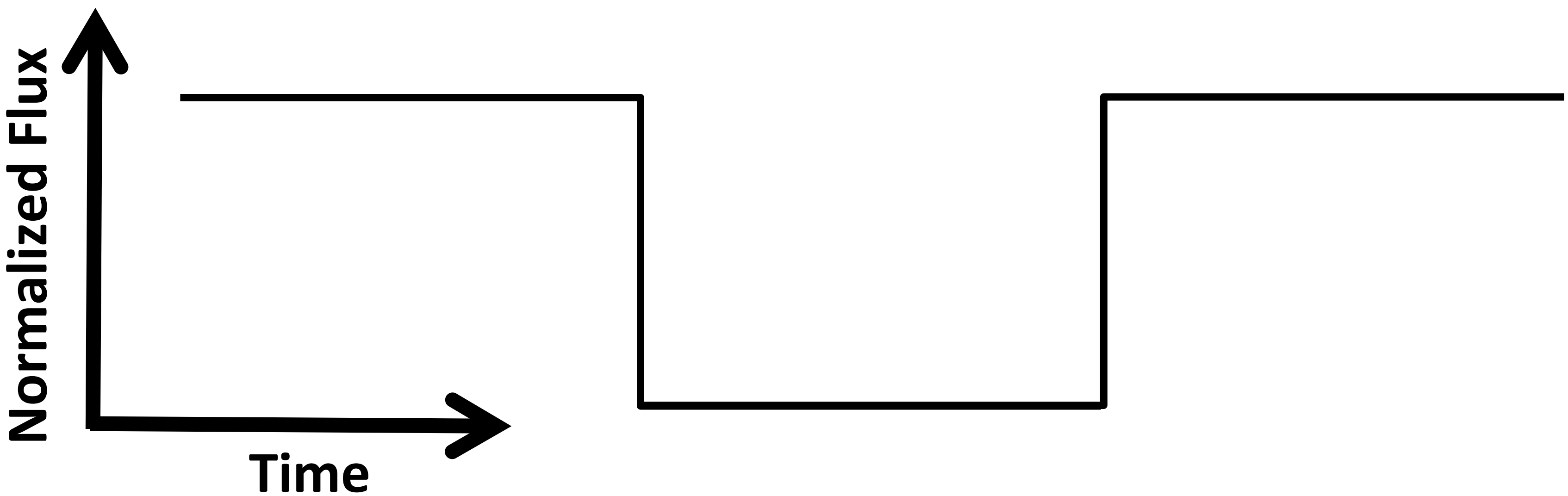
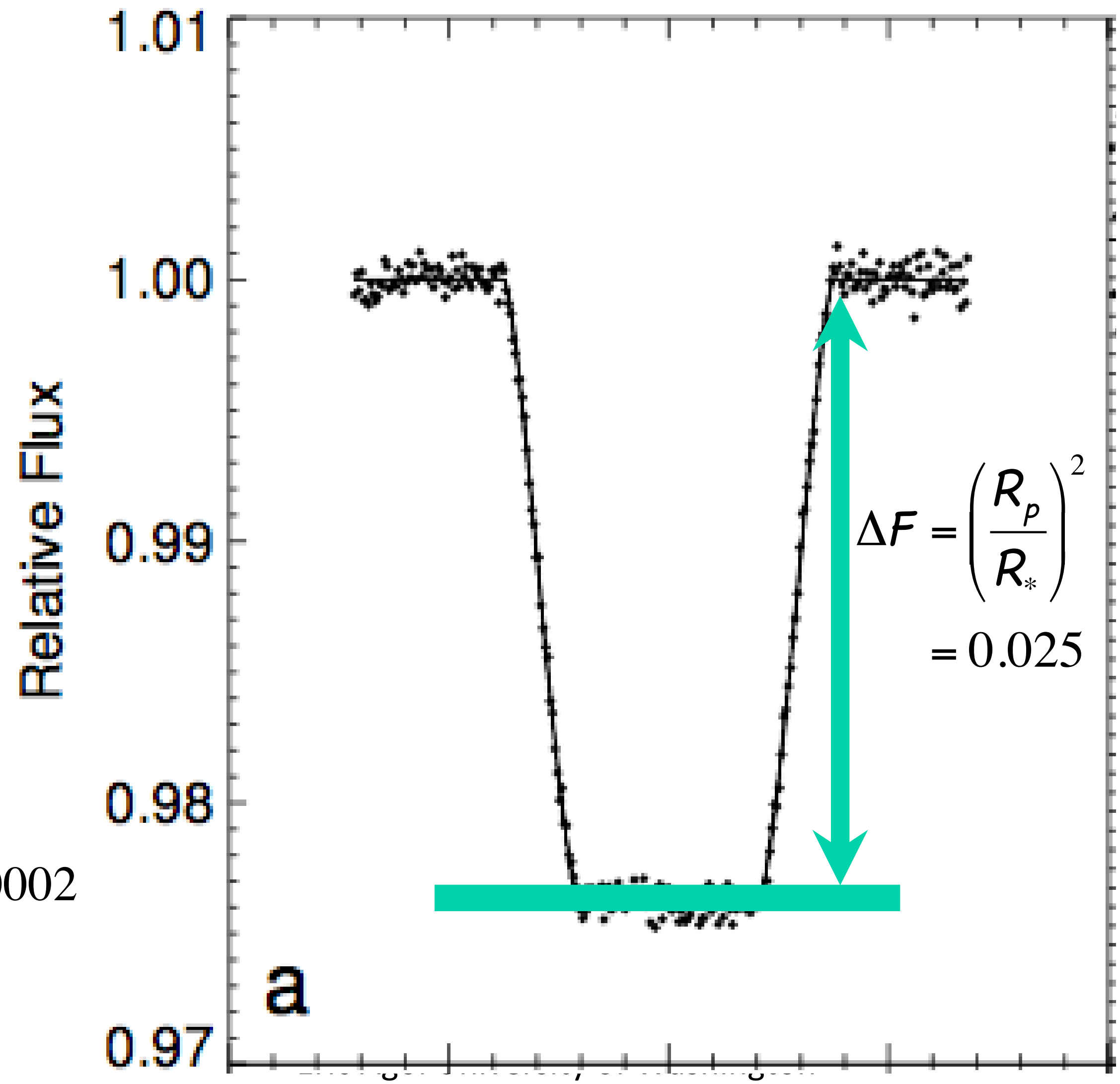
Boxcar Transit Model:



Transits only give us quantities with dimensions of
1) time ; 2) flux ; 3) dimensionless
Box-car/pulse/top-hat transit shape is useful for transit searches, e.g. BLS or QATS

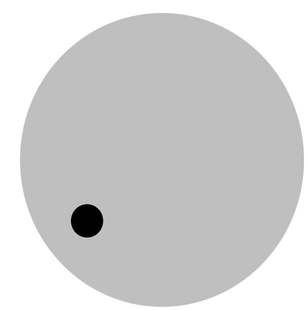
Mid-infrared transit

8 micron transit of HD 189733 observed with Spitzer: almost no limb darkening!

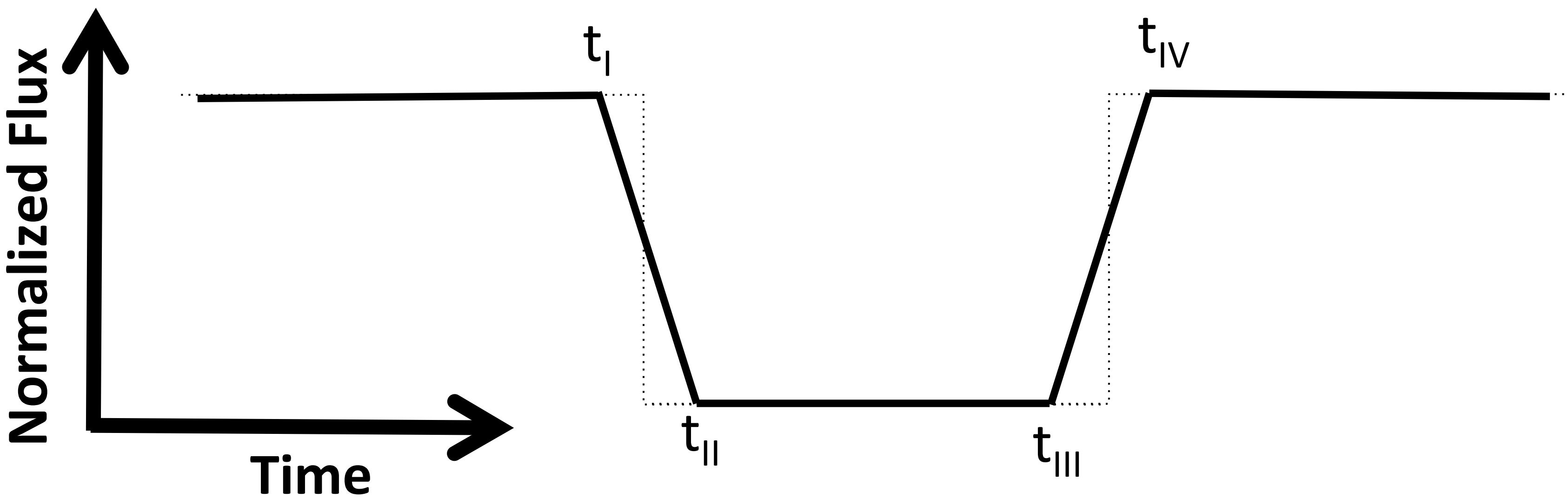


Knutson et al. 2007; Agol et al. 2010

$$\frac{R_p}{R_*} = 0.1545 \pm 0.0002$$



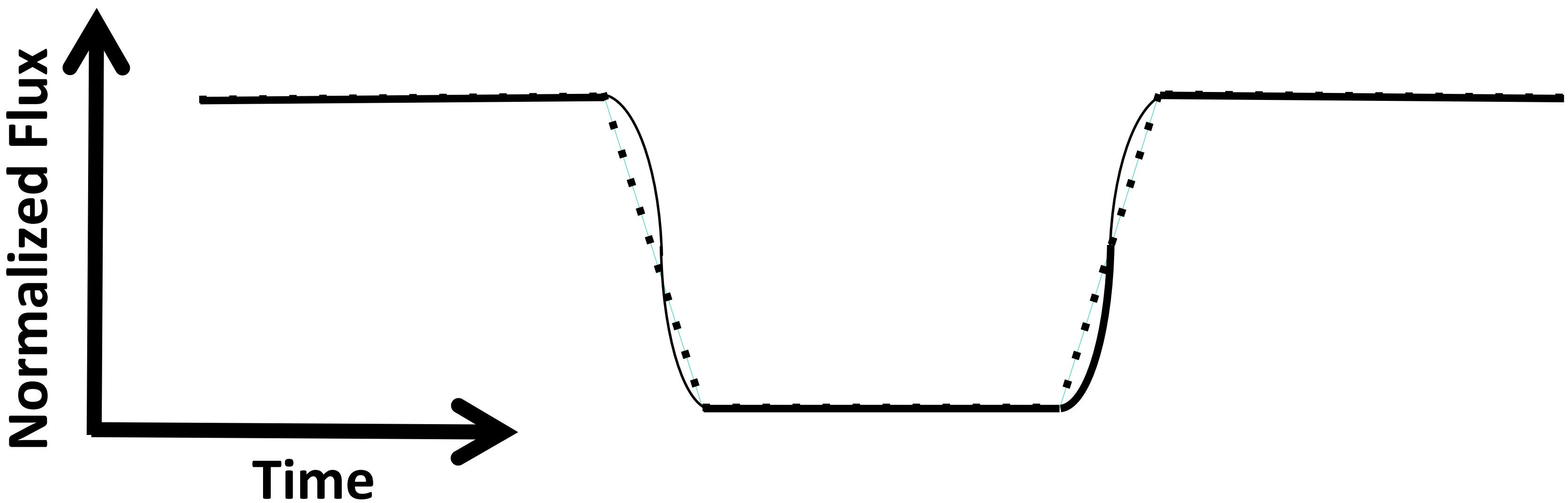
Trapezoidal Transit Light Curve:



t_I: 1st contact, start ingress
t_{II}: 2nd contact, end ingress
t_{III}: 3rd contact, start egress
t_{IV}: 4th contact, end egress

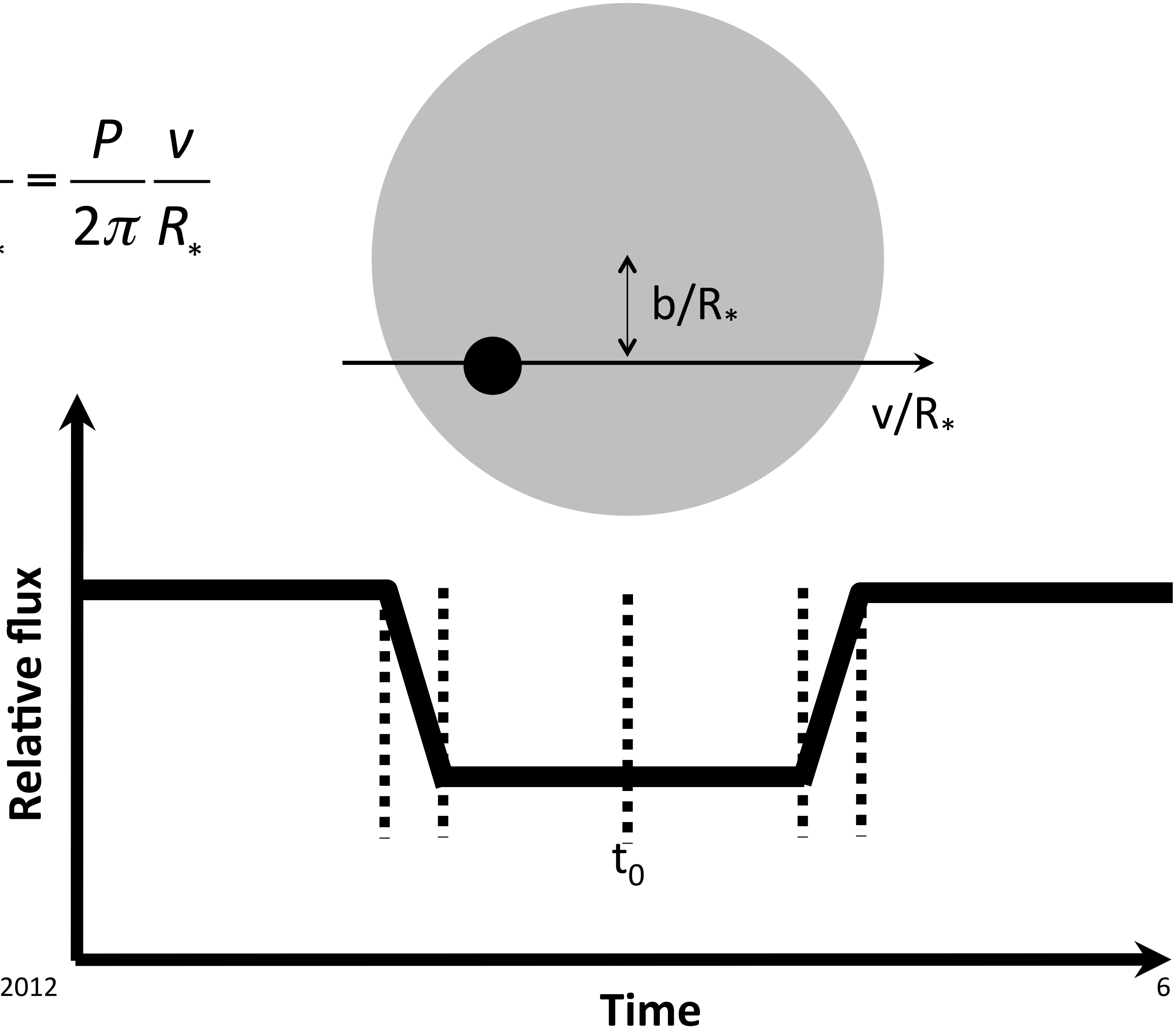
1. Ingress duration: $t_{II}-t_I$
2. Transit duration: $t_{IV}-t_I$
3. Transit 'time': $(t_{II}+t_{III})/2$
4. Orbital period

Uniform Transit Light Curve:



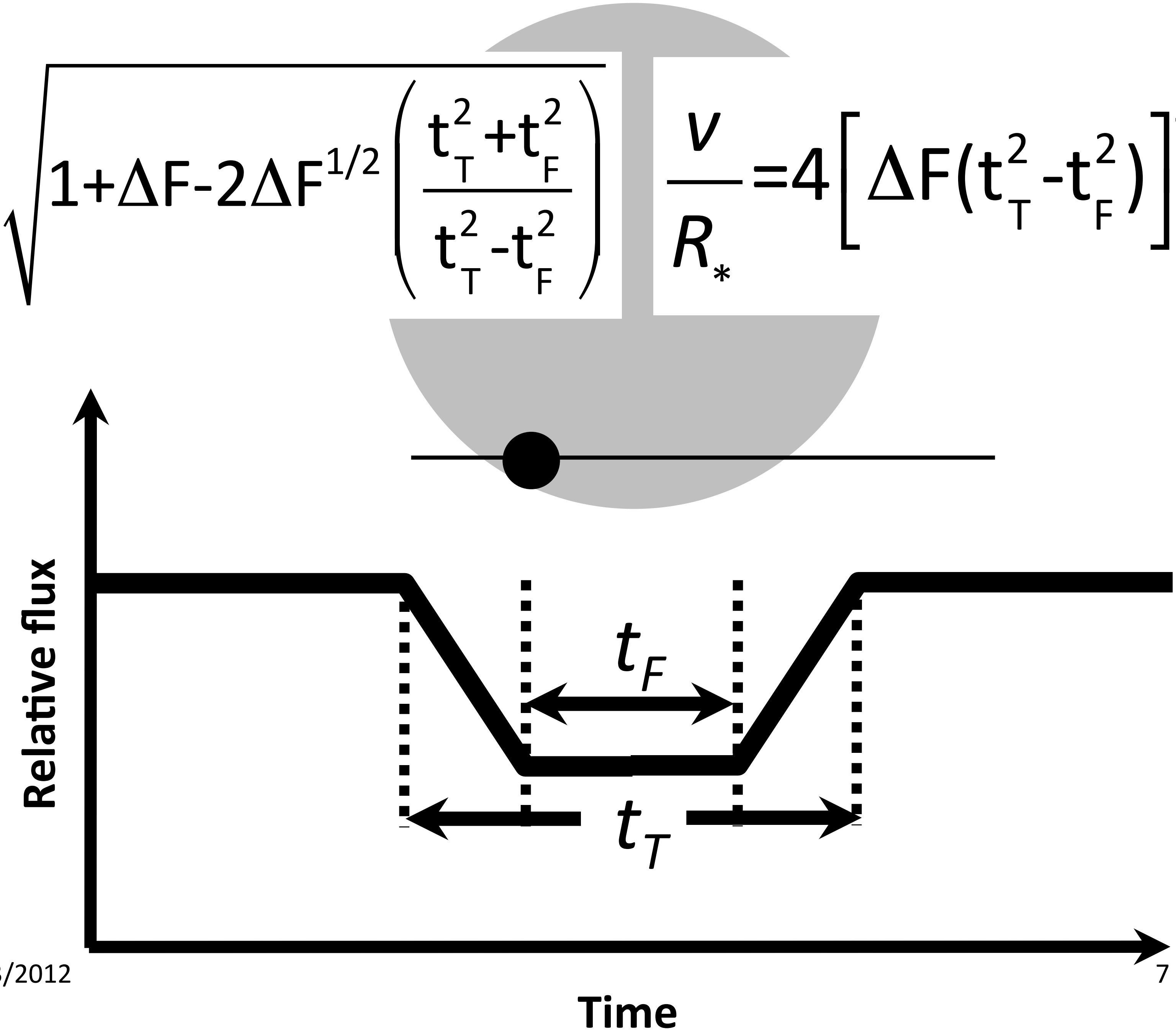
For circular orbit: impact parameter (b/R_*), velocity of planet across star (v/R_*), central time of transit (t_0), and radius ratio (R_p/R_*). With period, find semi-major axis (a/R_*)

$$\frac{a}{R_*} = \frac{P}{2\pi}\frac{v}{R_*}$$



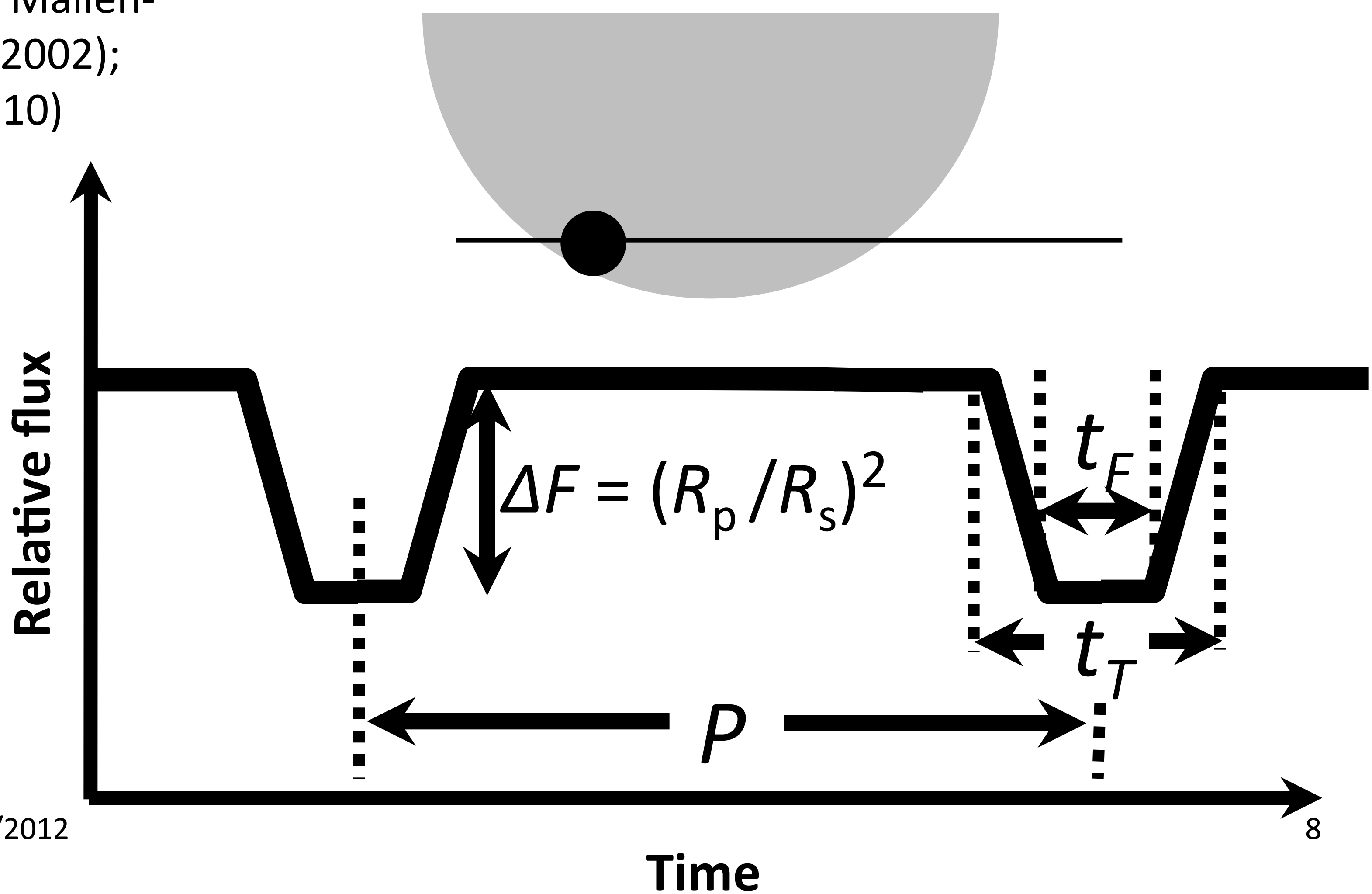
$$\frac{b}{R_*} = \sqrt{1 + \Delta F - 2\Delta F^{1/2}\left(\frac{t_T^2 + t_F^2}{t_T^2 - t_F^2}\right)}$$

$$\frac{v}{R_*} = 4\left[\Delta F(t_T^2 - t_F^2)\right]^{-1/2}$$



$$\rho_* = \frac{24}{\pi^2} \frac{P \Delta F^{3/4}}{G(t_T^2 - t_F^2)^{3/2}}$$

Seager & Mallen-Ornelas (2002); Winn (2010)

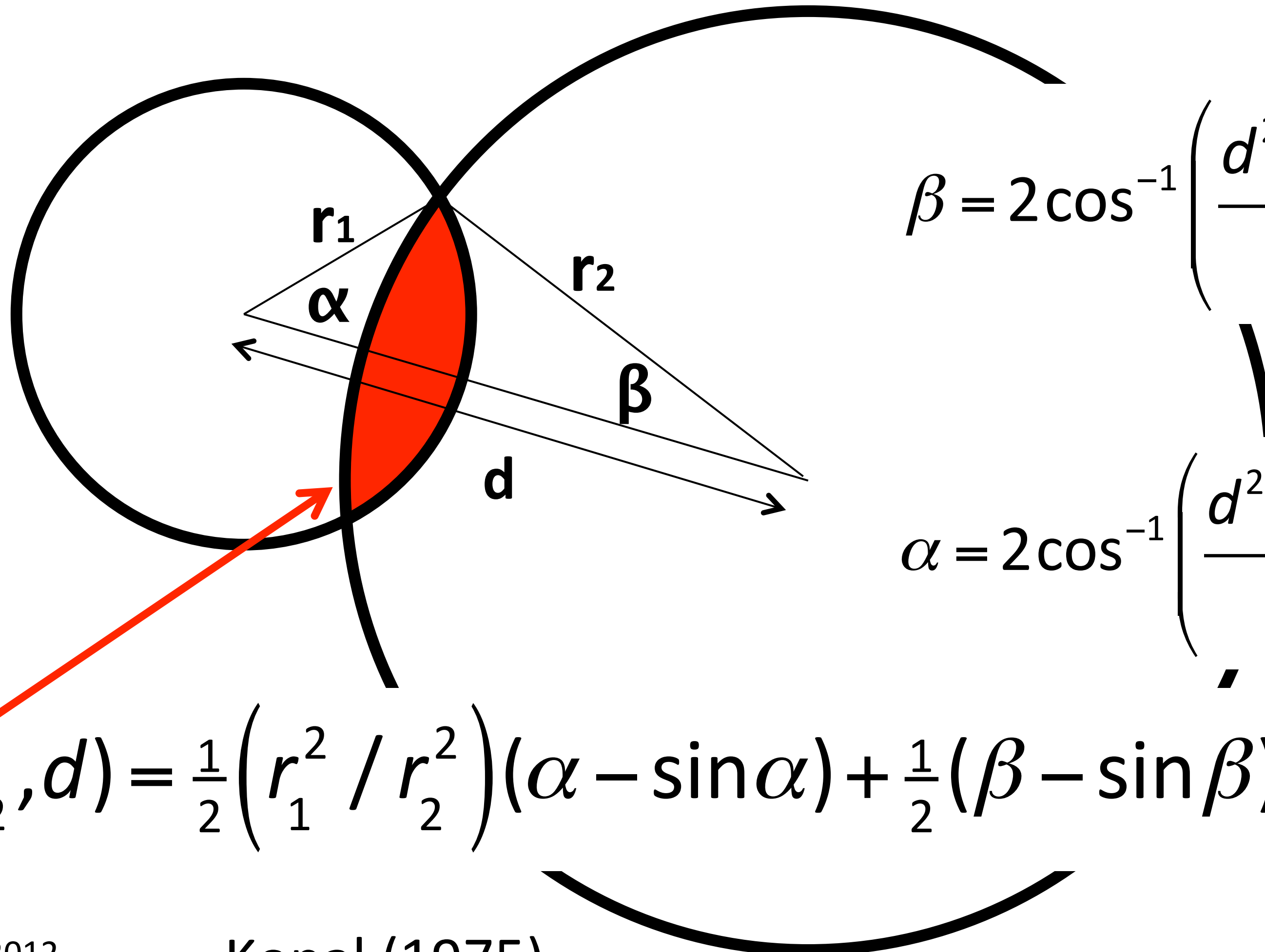


Angular size

- Dimensionless angular sizes can be estimated from light curves (radians are dimensionless)
- For Kepler-36b,c know a_1/R_*, a_2/R_*, R_p/R_*, so $R_p/(a_2-a_1)$ gives angular size at conjunction: ≈2.7 x angular diameter of moon viewed from Earth

Kepler 36c seen from 36b

Area of overlap of two circles: ingress/egress



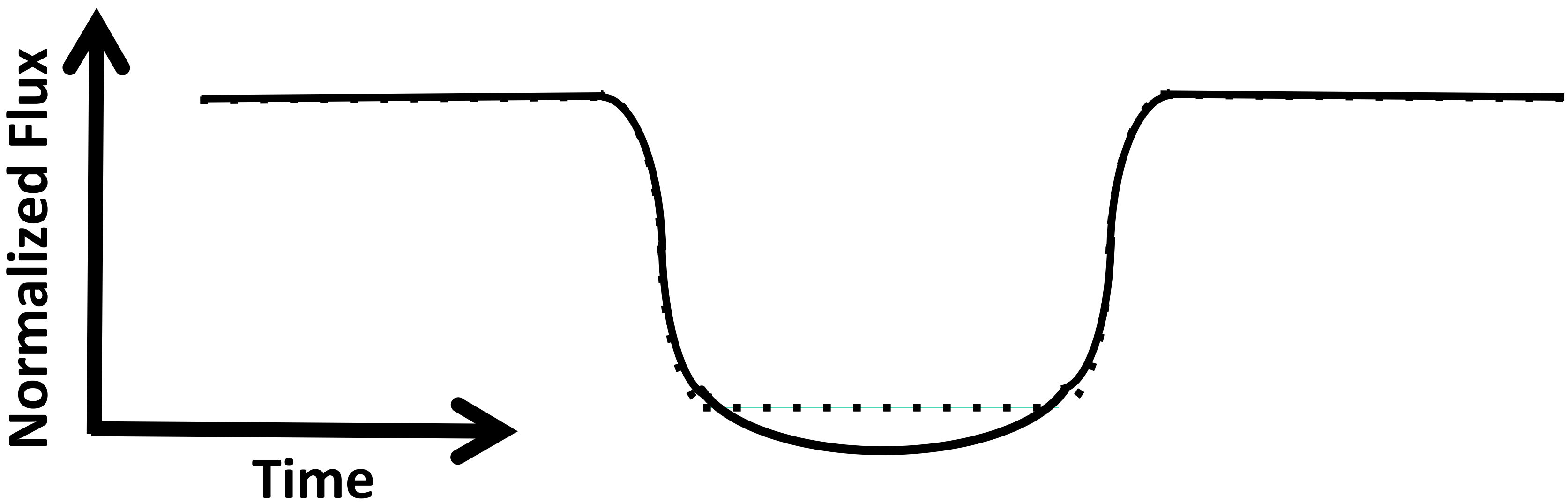
$$\beta = 2\cos^{-1}\left(\frac{d^2 - r_1^2 + r_2^2}{2dr_2}\right)$$

$$\alpha = 2\cos^{-1}\left(\frac{d^2 - r_2^2 + r_1^2}{2dr_1}\right)$$

$$\delta(r_1, r_2, d) = \tfrac{1}{2}\left(r_1^2 / r_2^2\right)(\alpha - \sin\alpha) + \tfrac{1}{2}(\beta - \sin\beta)$$

Kopal (1975)

Limb-darkened Transit Light Curve:

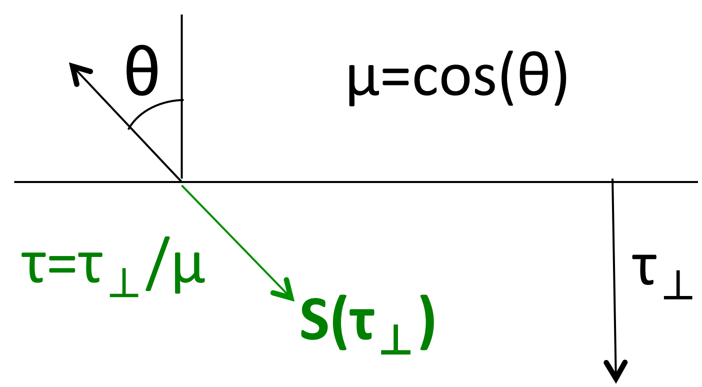


Limb darkening makes life complicated: can cause degeneracy between impact parameter, limb-darkening parameter(s), and radius ratio.

Limb-darkening primer

- Caused by temperature gradient near photosphere: $S(\tau_\perp)=a+b\,\tau_\perp$.
- Glancing angles view higher in atmosphere, weight source function towards low temp:

$$I(0) = \int_0^\infty d\tau S(\tau_\perp) e^{-\tau}$$
$$= \int_0^\infty d\tau (a + b\tau\mu) e^{-\tau}$$
$$= a + b\mu$$

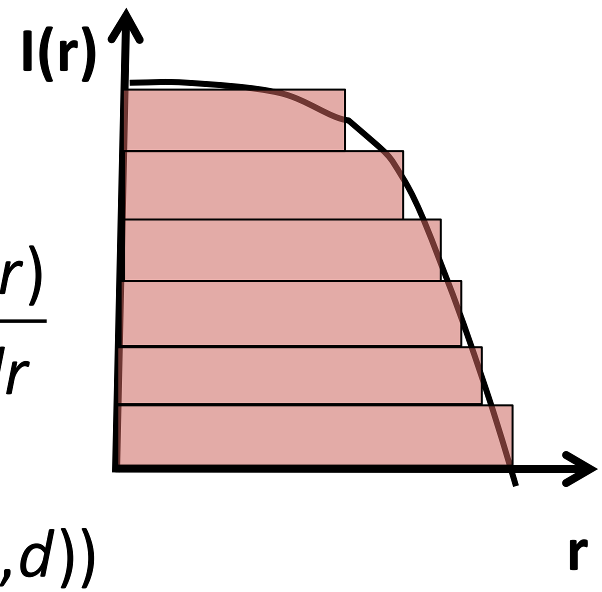


Integration over limb darkening

$$F(r_1, r_2, d, I(r)) = \int_{visible \quad area} r\, dr\, d\phi \cdot I(r)$$

$$= \tfrac{1}{2} \int_{visible \quad area} dr^2\, d\phi \cdot \frac{dI(r)}{2dr}$$

$$= \pi \int_0^{r_2^2} dr^2 \frac{dI(r)}{dr} (1 - \delta(r_1, r, d))$$



Analytic for quadratic & 'non-linear' limb-darkening models (Mandel & Agol 2002; Pal 2008)

Sky separation of planets versus time:

1. Straight line transit: $r_{sky}/R_* = \sqrt{\left(v/R_*\right)^2 (t-t_0)^2 + \left(b/R_*\right)^2}$

– fine for $a/R_* \gg 1$, e small

2. Circular orbit: $r_{sky}/R_* = a/R_* \sqrt{1-\sin^2 i \cos^2\left(2\pi(t-t_0)/P\right)}$
3. Keplerian orbit – requires Kepler solver (m & e → f); 7 parameters (Murray & Dermott):

$$r_{sky}/R_* = a/R_* \sqrt{1-\sin^2 i \sin^2\left(\omega + f\right)}$$

4. N-body integrator (for 3+ bodies, precession, GR, etc.): 7n-1 parameters (Fabrycky)

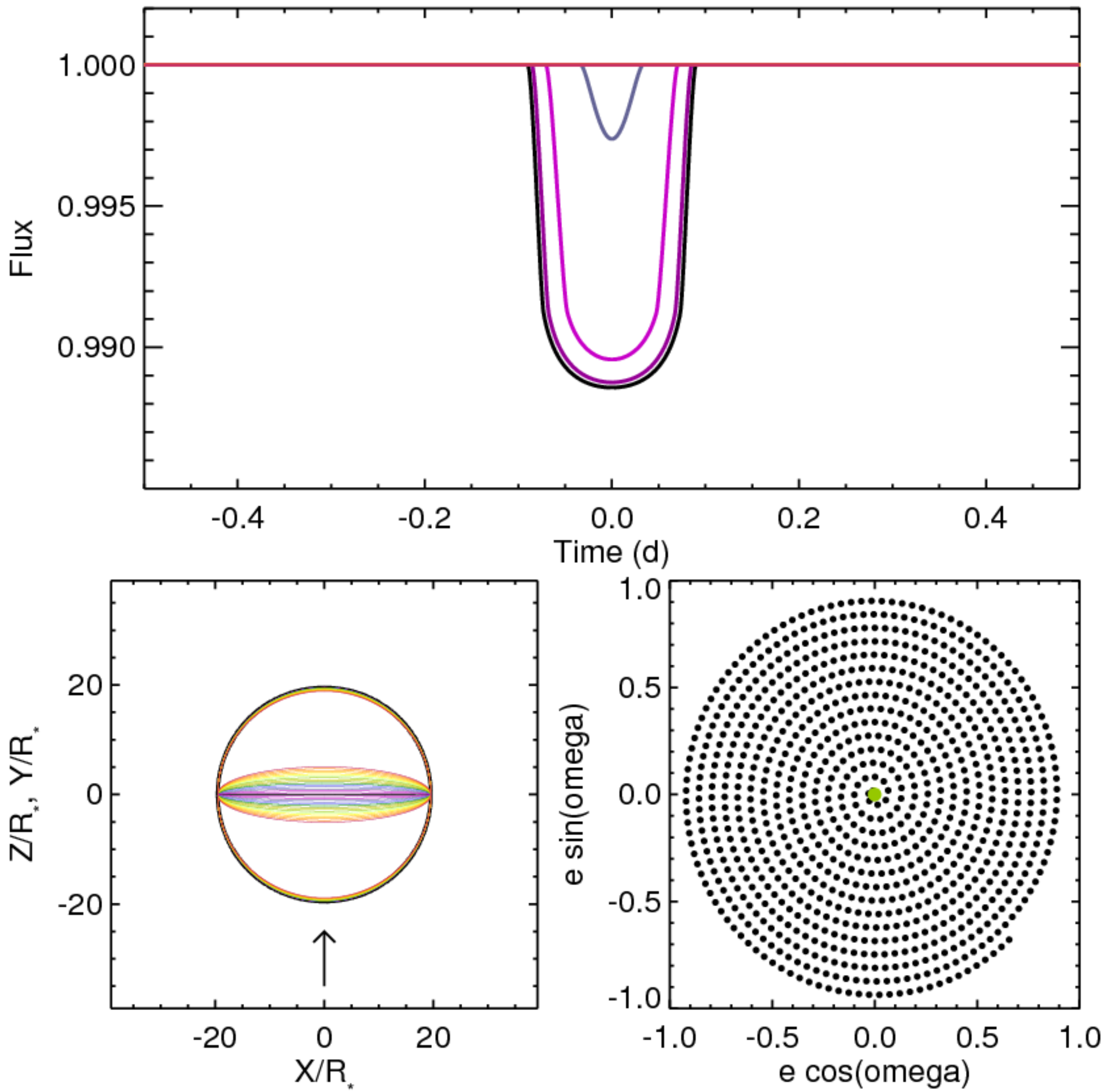
- Integrate over each exposure until converged (Kipping 2010)

Choice of limb-darkening model:

1. If data quality are poor, fix to limb-darkening of atmosphere models (Claret 2000, Sing 2011)
2. If high quality, may let parameters float & fit for them
3. Model limb-darkening do not agree perfectly with data, although 3D atmospheres work well (Hayek et al. 2012)
4. Unnecessary for secondary eclipse (except for high S/N), but need to add in flux from star
5. Small planet approximation: occulted flux ≈ (area of planet-star overlap) x (stellar intensity at center of planet)

Keplerian transits

Ford, Quinn & Veras (2008)



Detrending

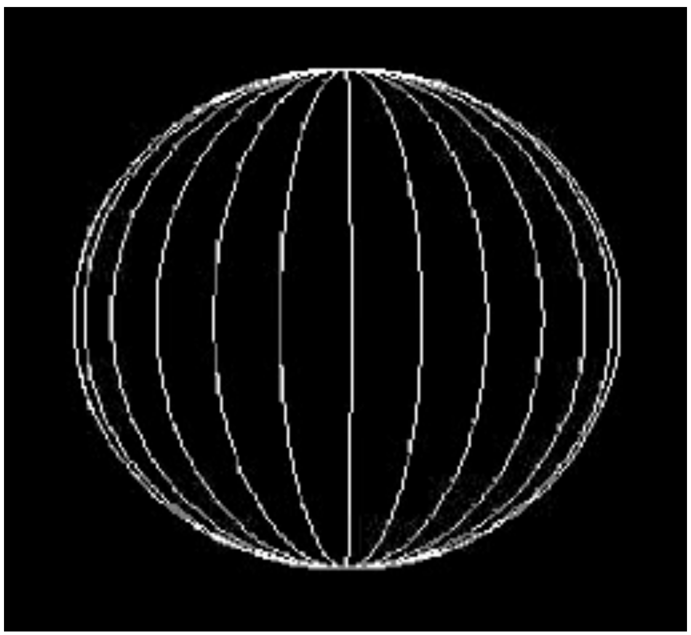
- Best practice:

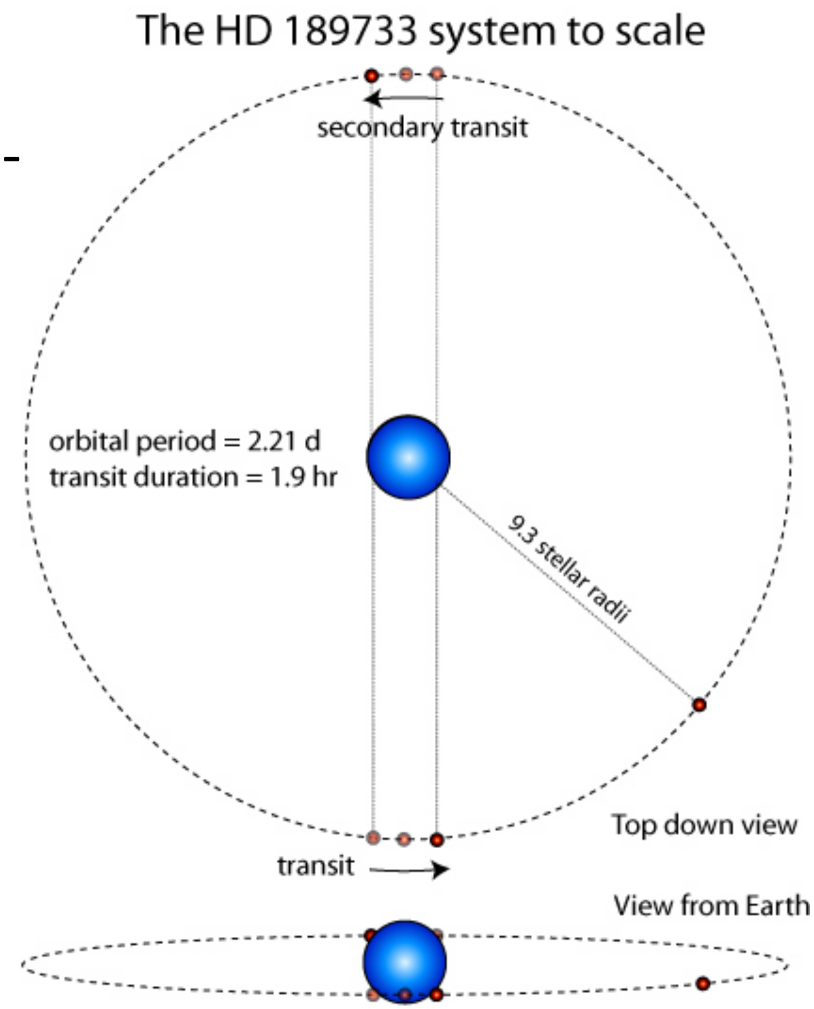
1) compute transit model;
2) divide model into light curve;
3) fit with detrending function (e.g. polynomial), which can be carried out as a local linear optimization (which is *fast*) for each transit separately;
4) repeat 1-3 to optimize non-linear transit parameters (e.g. Levenberg-Marquardt, MCMC)

Phase functions of transiting planets

<u>Assumptions</u>:

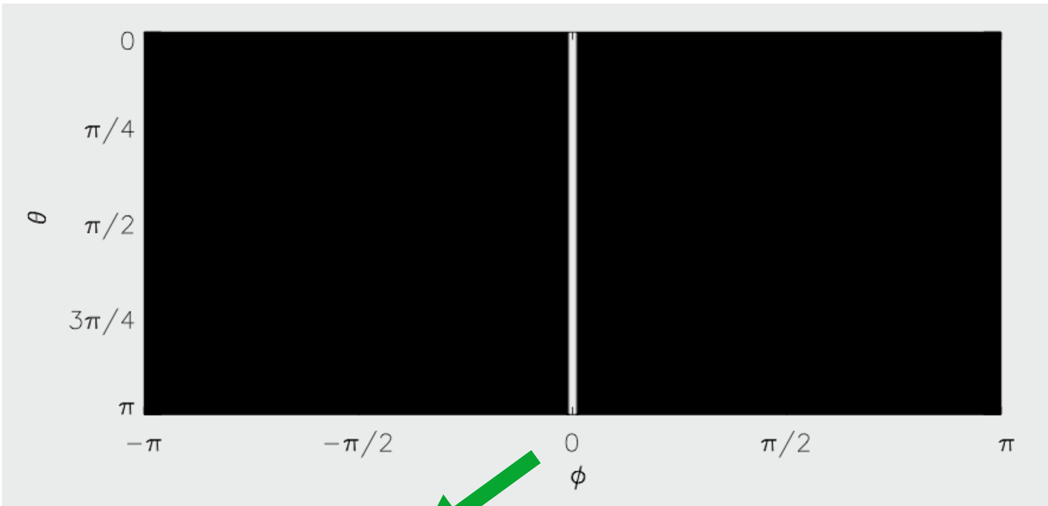
- Time-steady, edge-on, no limb-darkening, negligible stellar variability

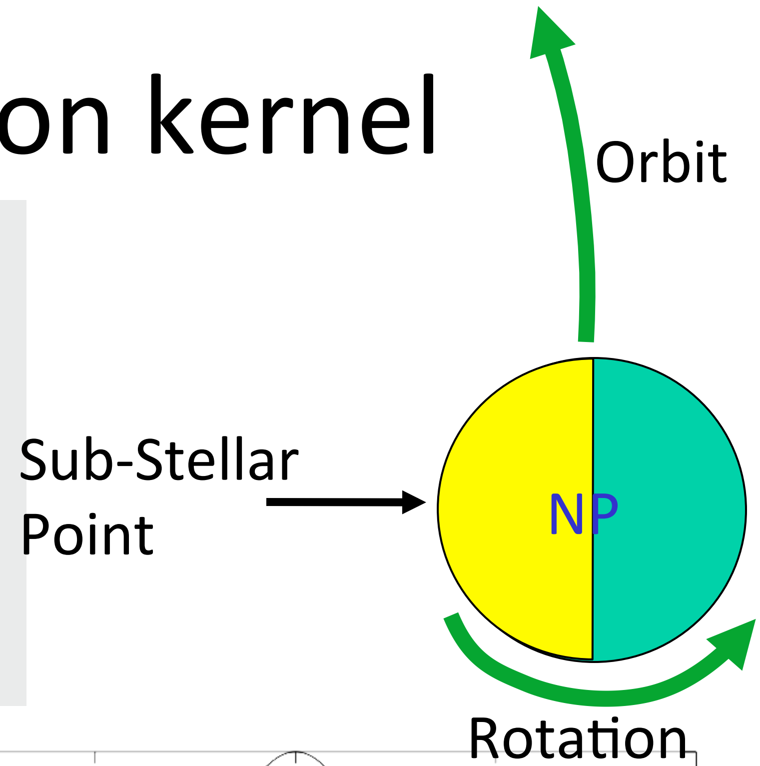


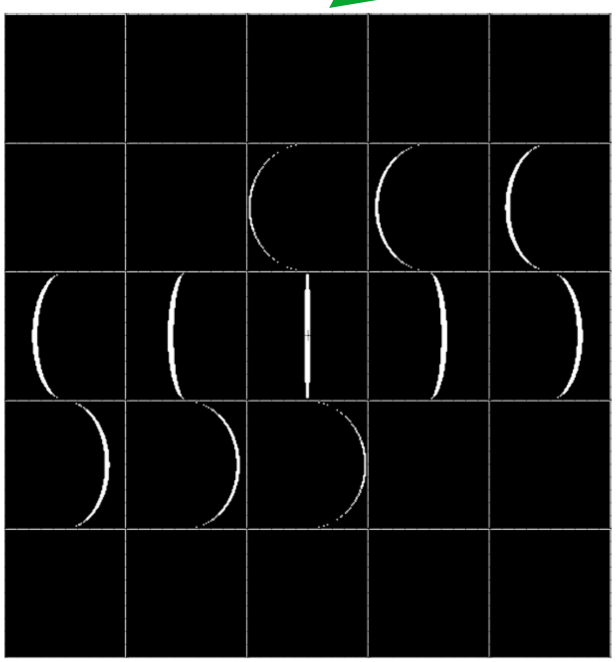


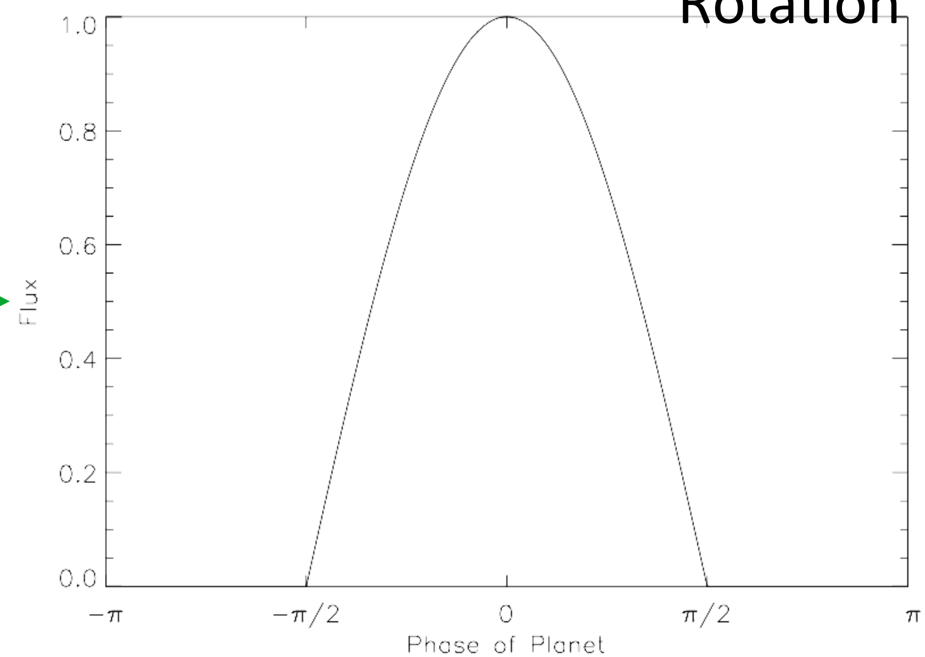
Laughlin (oklo.org)

Phase function kernel



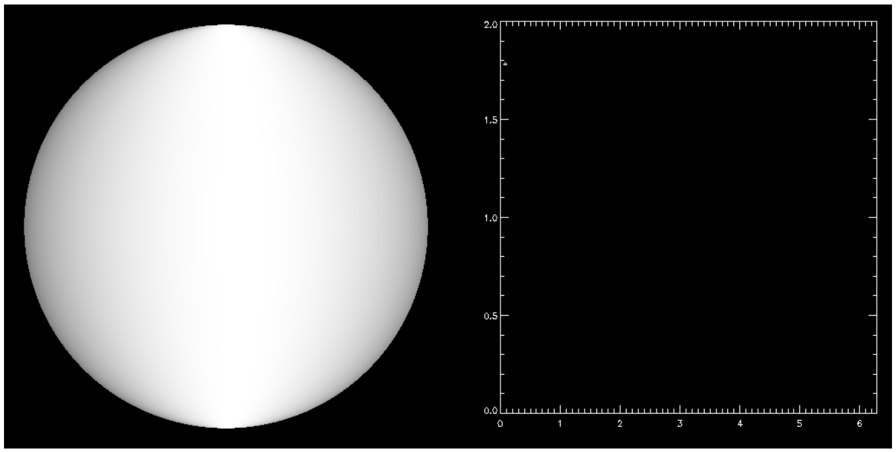




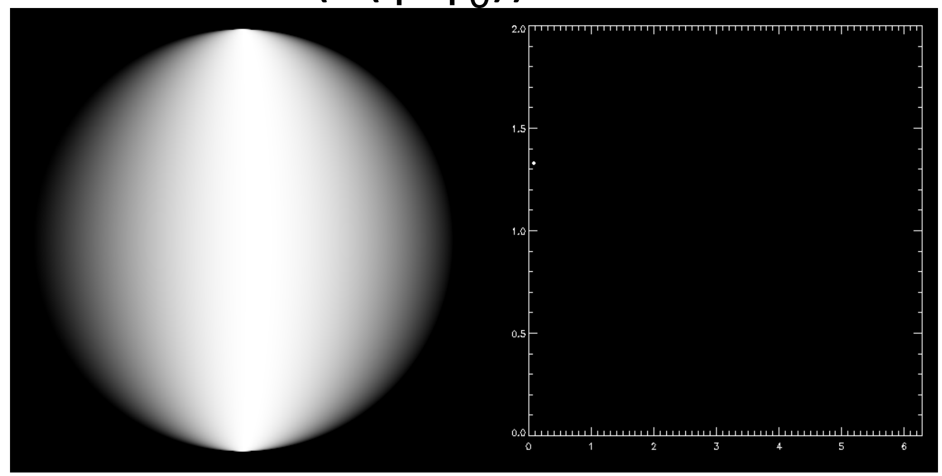


Fourier decomposition

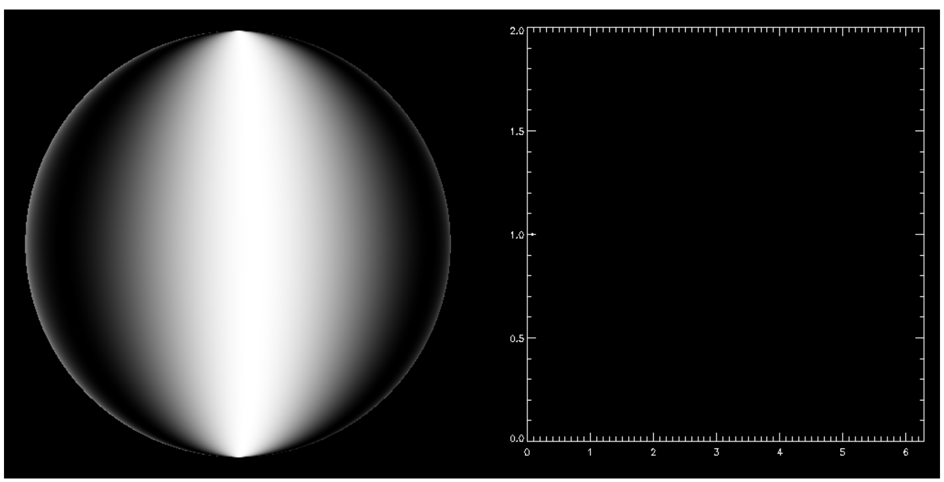
$\cos(\phi-\phi_0)$



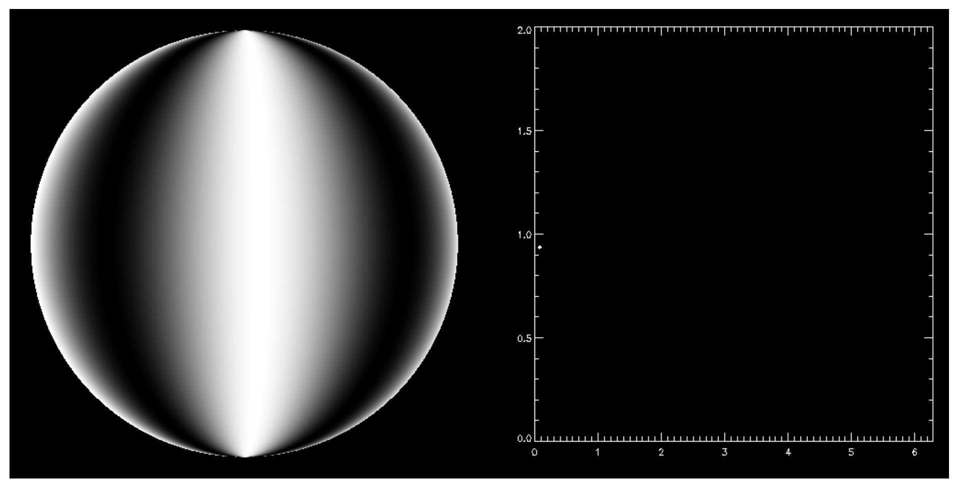
$\cos(2(\phi-\phi_0))$



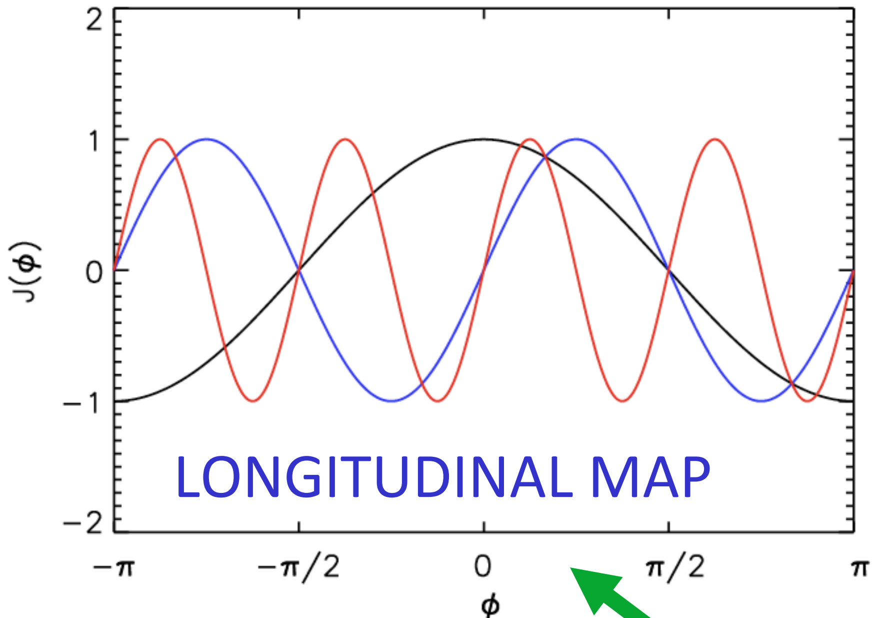
$\cos(3(\phi-\phi_0))$



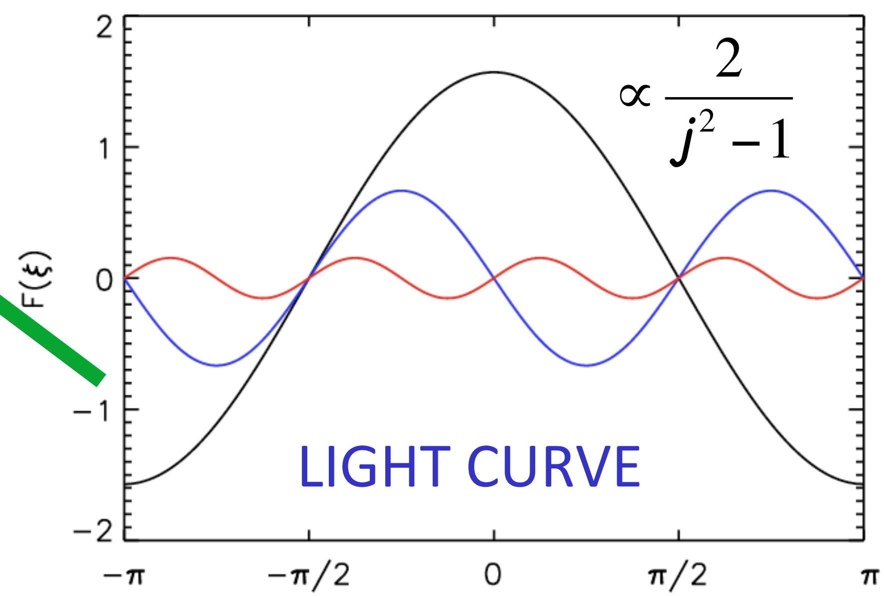
$\cos(4(\phi-\phi_0))$



Sinusoidal Longitudinal Mapping



Higher Frequencies get damped out



Easy for j=1 and even modes (j=2,4,…)

No information for odd modes with j>1!

Cowan & Agol (2008)

Phase function approximation

- Longitudinal mapping of planet is limited by j=3 frequency (invisible); j=4 is suppressed (too small for JWST), so can be fit with 5 parameters:

$$F(\xi) = F_0 + F_1 \cos(\xi - \xi_1) + F_2 \cos(2[\xi - \xi_2])$$

Global Average Temperature

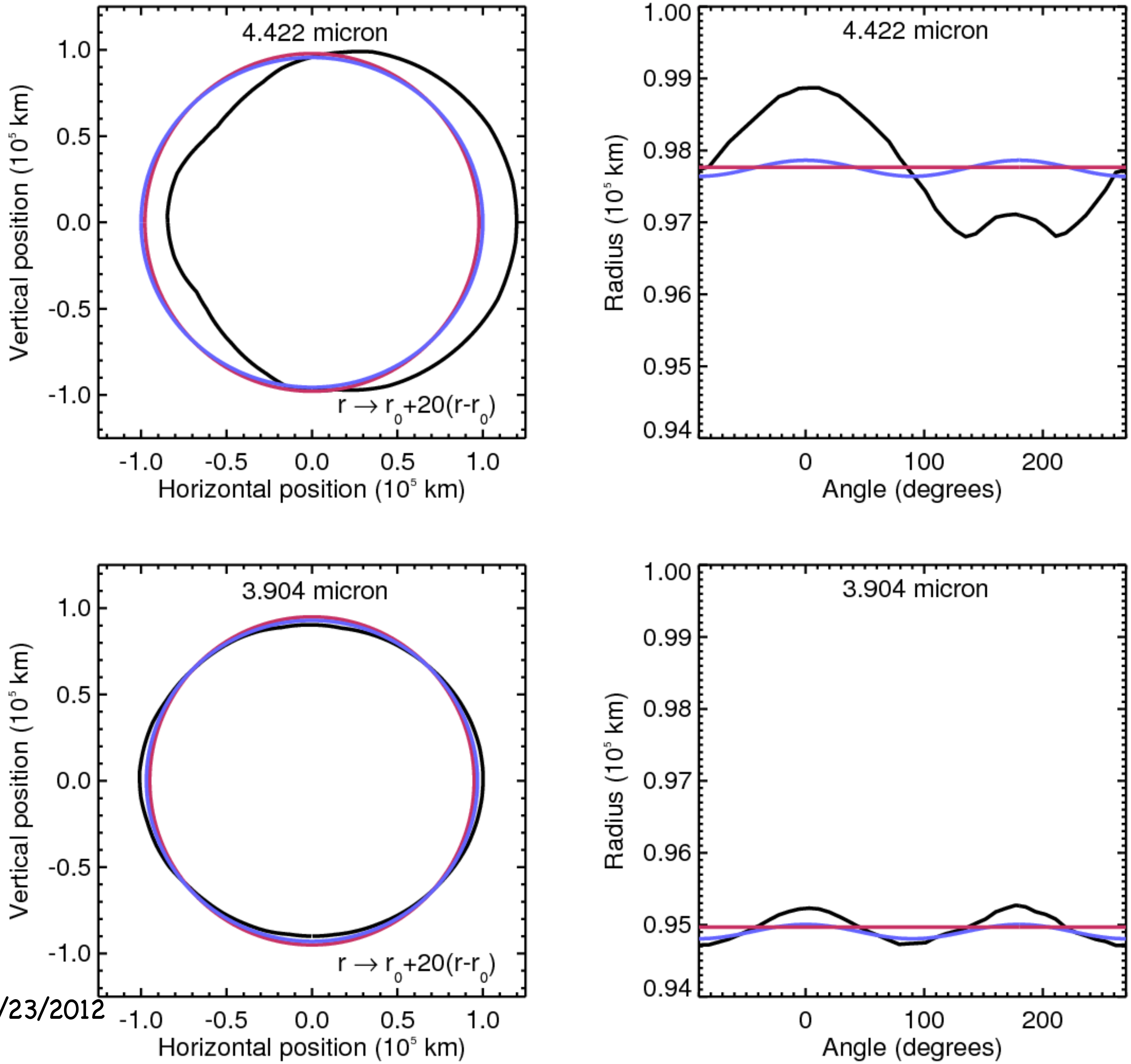
Big Equatorial Hot/Cold Spots

Small Equatorial Hot/Cold Spots

- If assumptions violated, different formulae apply (e.g. limb-darkening, reflected light, obliquity, etc.)

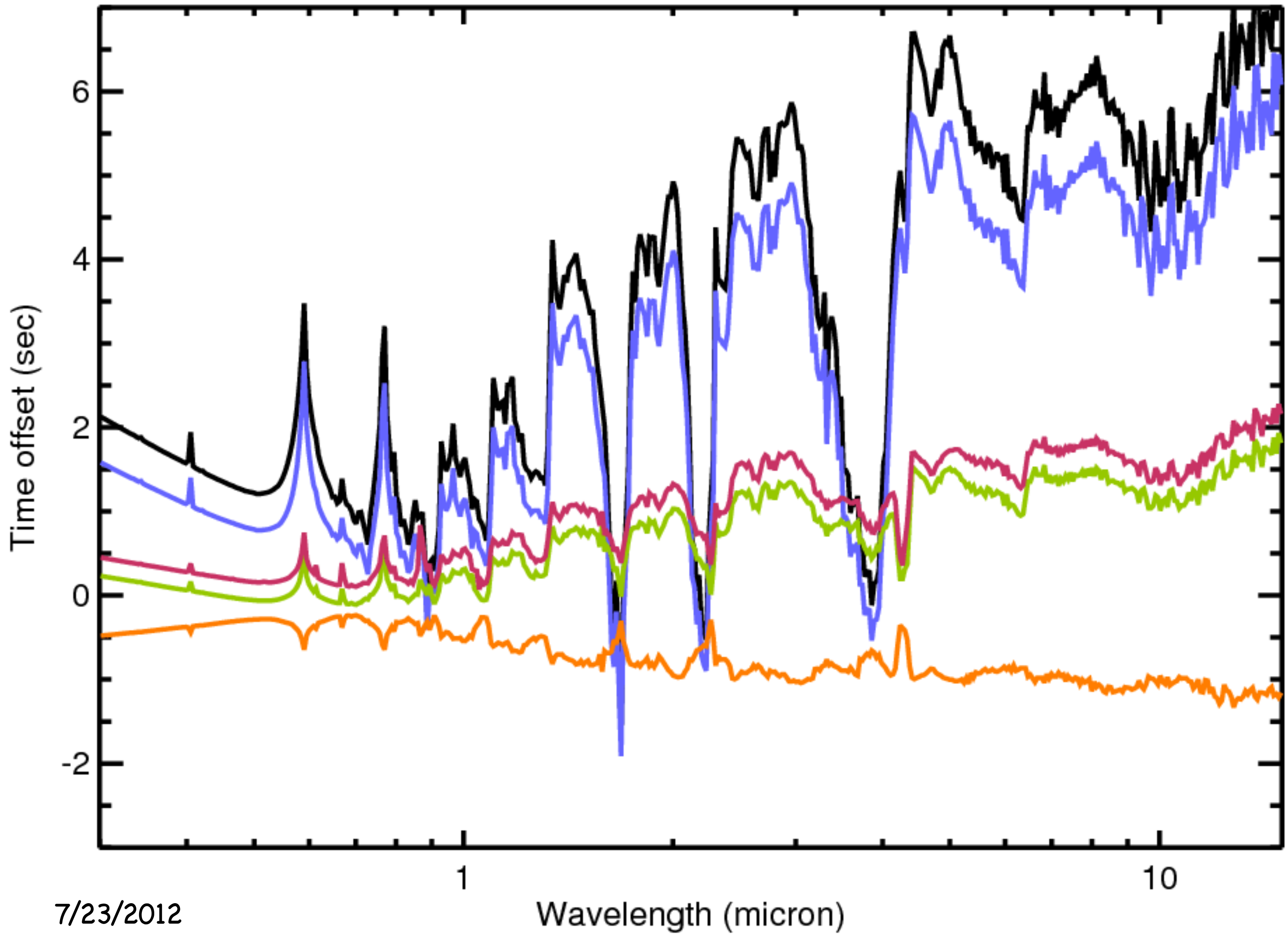
Beyond ‘basic’ light curves

- Planet asymmetry: rotational & thermal oblateness (Carter & Winn 2010; Dobbs-Dixon et al. 2012)
- Wavelength dependence (Knutson, Bean)
- Moons, rings (Kipping, Barnes)
- Secondary eclipse mapping (Majeau et al. 2012)
- Refraction (longer periods; Sidis & Sari 2011), gravitational lensing (irrelevant)
- Star spots, granulation, flares, gravity darkening (Sanchis-Ojeda, Winn)
- Light travel time, Doppler effects, relativistic effects (Avi Shporer)
- Reflected light, polarization, mutual events
- Duration variations (Miralda-Escude 2002)



Dobbs-Dixon, Agol, Burrows 2012

Wavelength-dependent time offset



Advice from the trenches

- Estimate error bars multiple ways: MCMC, delta chi-square, boot-strap simulation, etc. Error analysis takes an order of magnitude longer than initial fitting!
- Be cautious (avoid Type I, false positive error):
  i. analyze data multiple ways;
  ii. make sure results are robust;
  iii. use physics as a guide;
  iv. explore systematic errors
- But, be willing to trust the data (avoid Type II, false negative error):
  i. rule out alternatives;
  ii. try to model new phenomena

Advice from the trenches

- Be cautious using results in the literature:
  i. some errors underestimated;
  ii. orbital element definitions differ (e.g. periastron of star vs. planet); sky plane vs. SS plane
  iii. values of physical constants differ
  iv. Some time units differ (Eastman & Gaudi)
- Double-check your work carefully; take your time (most mistakes happen due to haste). It's better to be correct than to be first.
- Use uncorrelated quantities that describe features in the light curve: e.g. use first & last transit time rather than epoch & period; use transit duration & impact parameter (b), rather than sky velocity & b

Exercises:

1. Derive the relations: ($M_p << M_*$, chord across star is straight, circular orbit, no limb-darkening)

$$b=\sqrt{1+\Delta F-2\Delta F^{1/2}\left(\frac{t_T^2+t_F^2}{t_T^2-t_F^2}\right)} \qquad \rho_* = \frac{24}{\pi^2}\frac{P\Delta F^{3/4}}{G(t_T^2-t_F^2)^{3/2}} \qquad v=4\left[\Delta F(t_T^2-t_F^2)\right]^{-1/2}$$

2. Show that the Fourier transform of the function $f(\phi)=\max(\cos(\phi+\xi),0)$, with ξ a constant, is zero for coefficients of odd values of $j > 1$ for the terms $\cos(j\phi)$ and $\sin(j\phi)$

References

1. Carol Haswell – Transiting exoplanets
2. Michael Perryman – Exoplanet Handbook
3. Sara Seager et al. – Exoplanets