Crowded Field Photometry and Difference Imaging

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Outline

- Motivation: Why crowded fields?
- Astronomical image formation and pixel sampling
- Effects of object crowding in microlensing surveys
- From conventional PSF fitting to image differencing
- Alard & Lupton algorithm for PSF matching
    - Constant PSF-matching kernels
    - Handling differential background
    - Spatially variable kernels
    - Flux conservation
- From images to light curves: implementation details
- Science examples

Why crowded fields?

- Pack enough objects along the line of sight to get a good probability of chance alignments (microlensing)
- Study inherently crowded objects: stellar clusters, but also GRB, SN and other transients against their extended hosts
- Accumulate “critical mass” of your favorite objects per exposure
- Avoid observing empty sky

Astronomical (CCD) image formation

1. "True" above atmospheric image

$$O(x,y) = \delta_2(x,y) = \delta(x)\delta(y)$$

2. Convolve with seeing (air turbulence, optics, tracking)

$$O * S(x,y) \equiv \int O(u,v)S(x-u,y-v)\,du\,dv$$

3. Convolve with pixel response function (top hat ~ OK)

$$S * P(x,y)$$

4. Sample at regularly spaced points, i.e. multiply by a series of deltas

$$\mathrm{III}_2(x,y) \equiv \sum_{m,n} \delta_2(x-m,y-n) = \sum_{m,n} \delta(x-m)\delta(y-n)$$

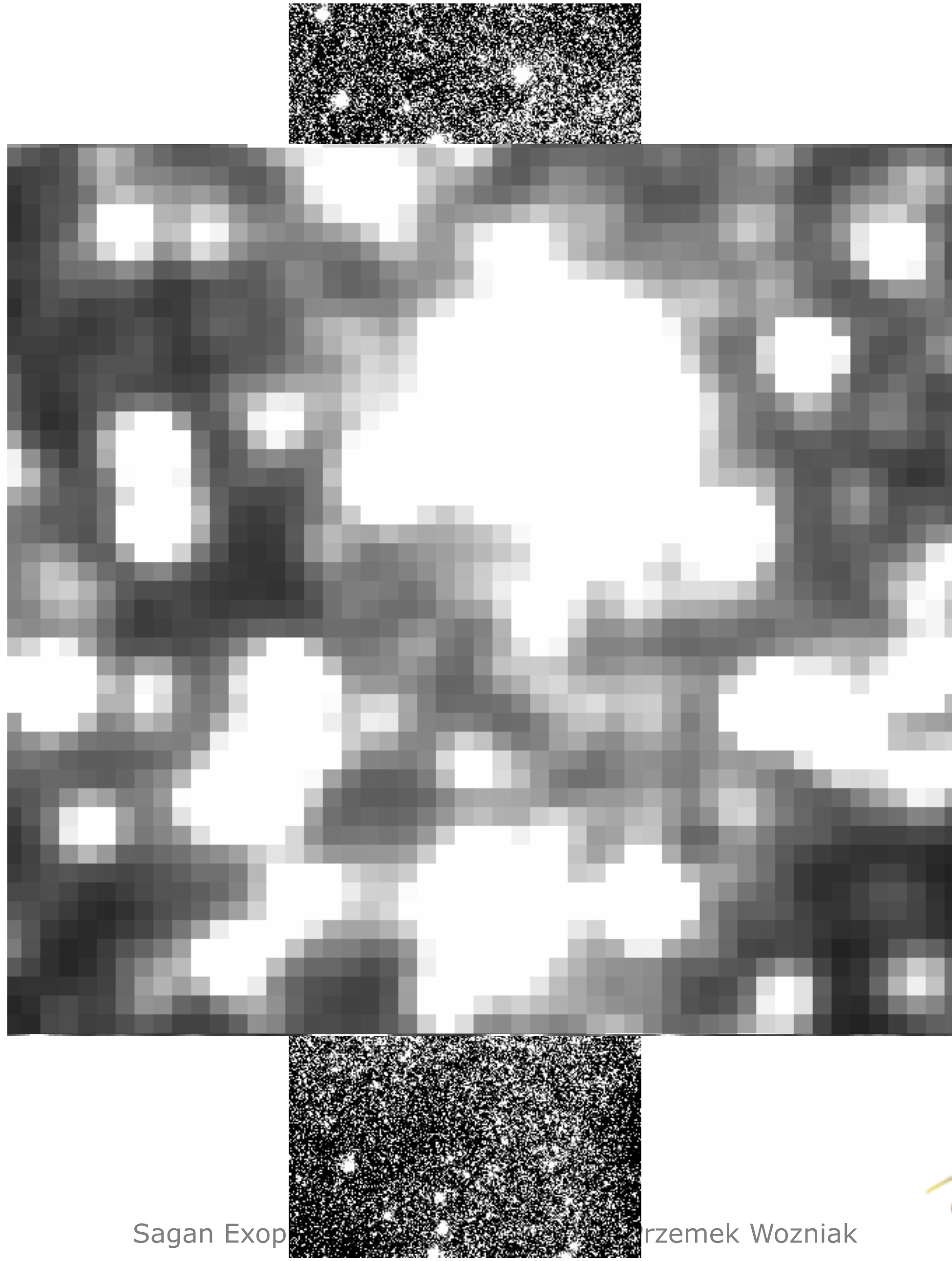
5. For a point source the result is PSF

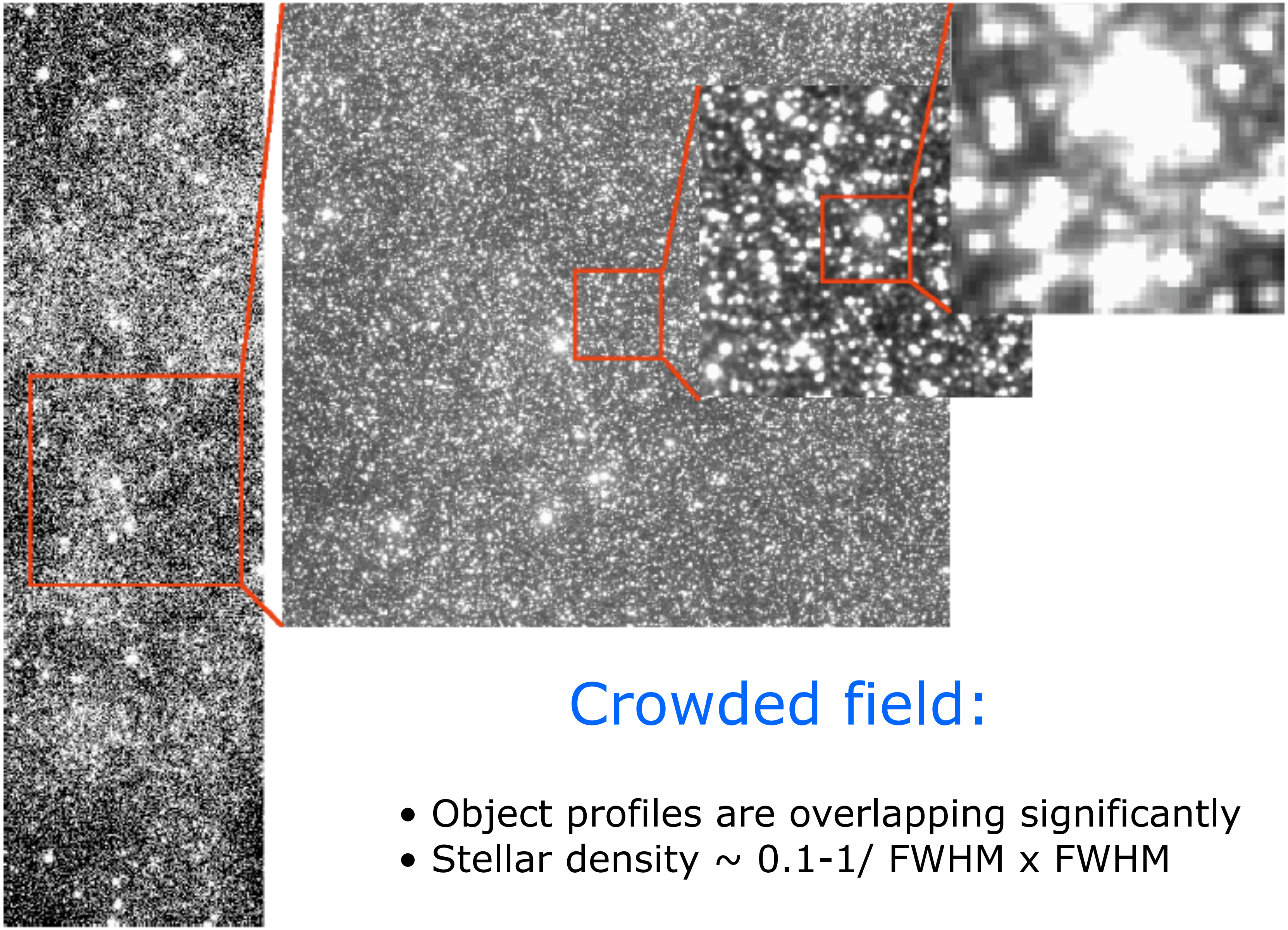
$$PSF(x,y) \equiv \sum_{m,n} S \otimes P(m,n)\delta_2(x-m,y-n)$$

Sampling and interpolation

- Band limited data: have cutoff frequency +/– f_c
- Sampling theorem
- Nyquist rate (or frequency): $2f_c$
- Undersampling breaks interpolation and FFT
- Rule of thumb: 2.5 pix/FWHM
- Examples

  OGLE-II : 0.40” pixels, 1.3’’ median seeing FWHM
  OGLE-III: 0.26” pixels, 1.2” median seeing FWHM





Crowded field:

- Object profiles are overlapping significantly
- Stellar density ~ 0.1-1/ FWHM x FWHM

Approximate development timeline

- 1987 DAOPHOT (Stetson et al.)
- 1992 OGLE and MACHO surveys, modified DoPHOT
- 1993 DoPHOT (Schechter, Mateo & Saha)
- 1996 Pixel lensing with Fourier Division (Tomaney & Crotts)
  PEIDA software for EROS (Ansari)
- 1998 Robust global subtraction algorithm (Alard & Lupton)
- 1999 MACHO DIA analysis (Alcock et al.)
- 2000 Extension of AL algorithm to variable kernels (Alard)
  ISIS package (Alard)
  cdophot (Reid, Sullivan, & Dodd)
- 2001 OGLE DIA package (Wozniak)
- 2002 DIA based std OGLE and MOA pipelines
- 2005 DIAPL extensions/modifications (Pych)
- … DIA pipelines in SDSS, LSST, PanSTARRS

Profile fitting in crowded fields

	DAOPHOT	DoPHOT
PSF model	Empirical PSF (analytical model fit + sub-sampled table of residuals)	Analytical PSF (pseudo-Gaussian)
PSF gradient	Originally fixed PSF shape, then 1st order variation with a weighted sum of 3 fixed PSFs, then …	Originally fixed PSF shape, then 2-D polynomial fit for each shape parameter for an ensemble of stars
Background estimator	Local background estimates based on a large pixel annulus (mode)	Local sky level fitted for each object, then a global polynomial model for the ensemble
Detection	Convolves with a lowered Gaussian filter and identifies local intensity peaks	Finds local intensity peaks between a pair of progressively fainter flux thresholds
Pixel value	Integrates PSF over square pixels	Evaluates PSF at each pixel

Profile fitting in crowded fields

	DAOPHOT	DoPHOT
Deblending	Examines significance and flux contributions of stars in PSF group	Classifies extendedness, goodness of fit test with 2 x PSF model
Algorithm	Simultaneous fitting of relatively isolated and self-contained groups of stars	An iterative fitting and subtraction of progressively fainter stars with parameter refinement
Optimization	Linearized least squares fit with non-linear model	Non-linear least squares
Warm starts	Modular enough to enable	Warm start and fixed position mode

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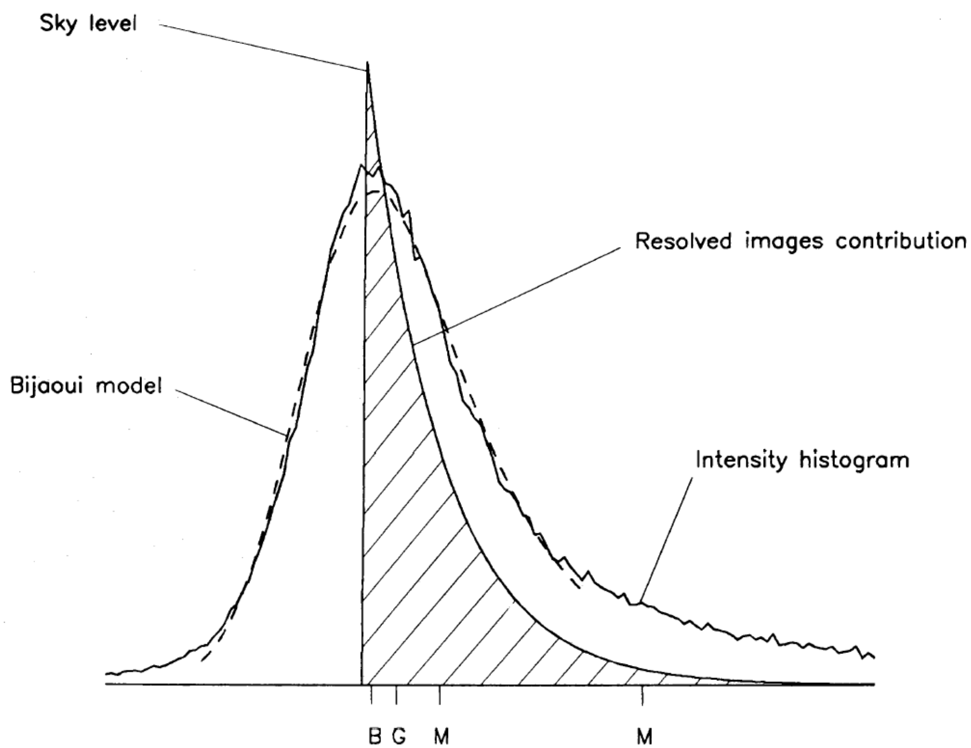
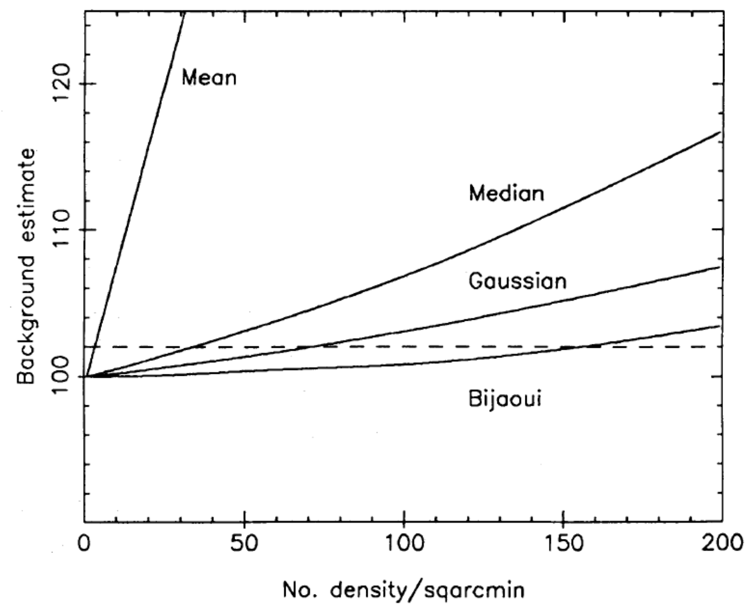
Effects of crowding: background estimators

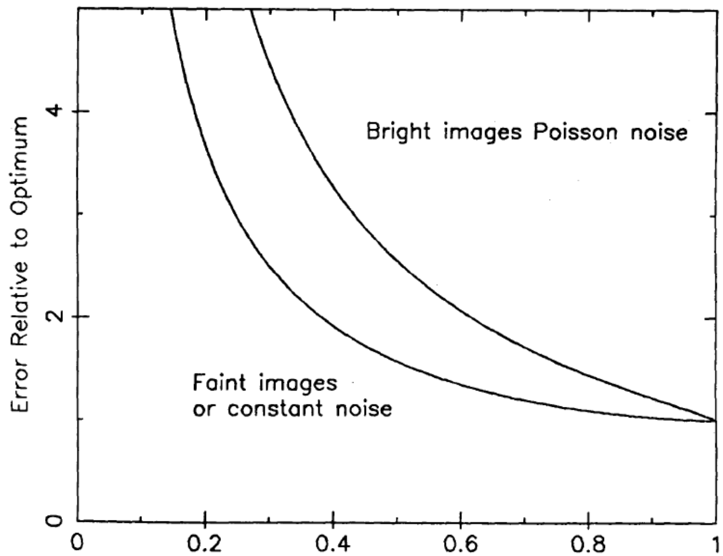




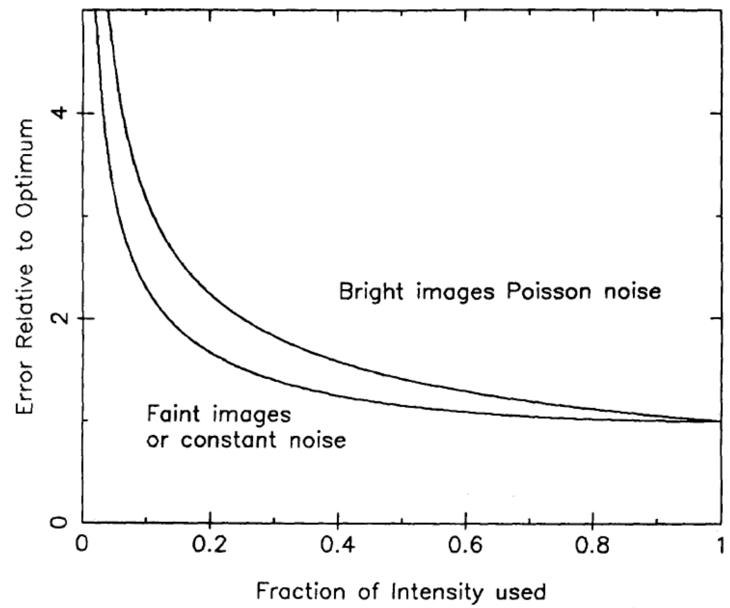
- Background level set by merging PSF wings and faint cores
- Confusion limit sets the detection threshold (local !)
- Biased and noisy background estimates

Effects of crowding: working with PSF cores

Centroid



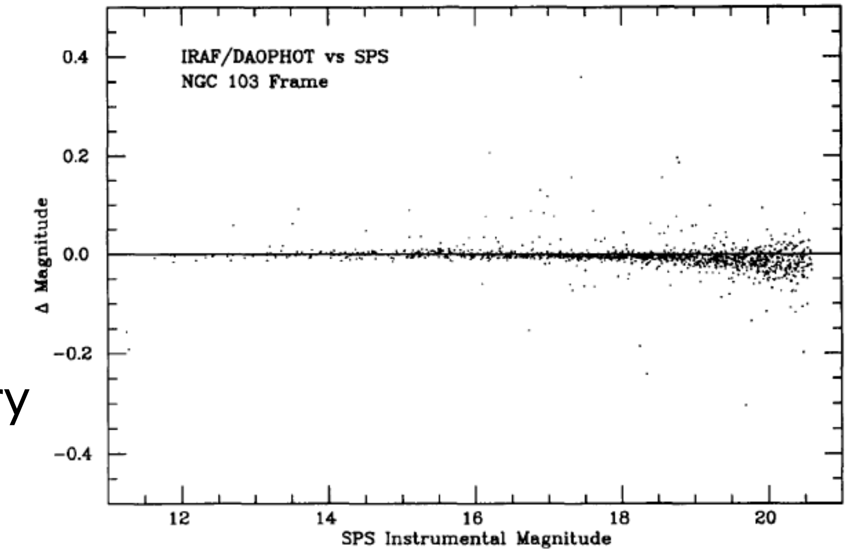
Flux

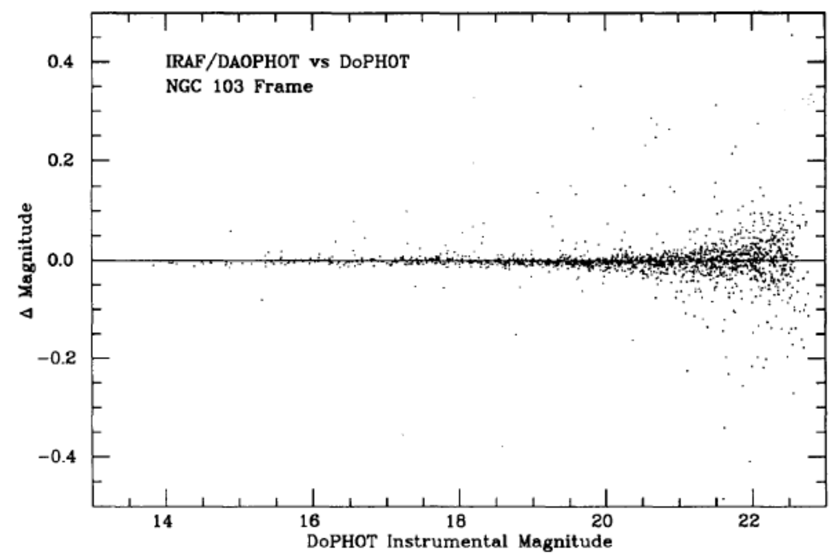


- Parameter estimation based on inner PSF core
- Biased and noisy centroid and flux estimates
- Broader effective PSF

Magnitude scatter vs. bias

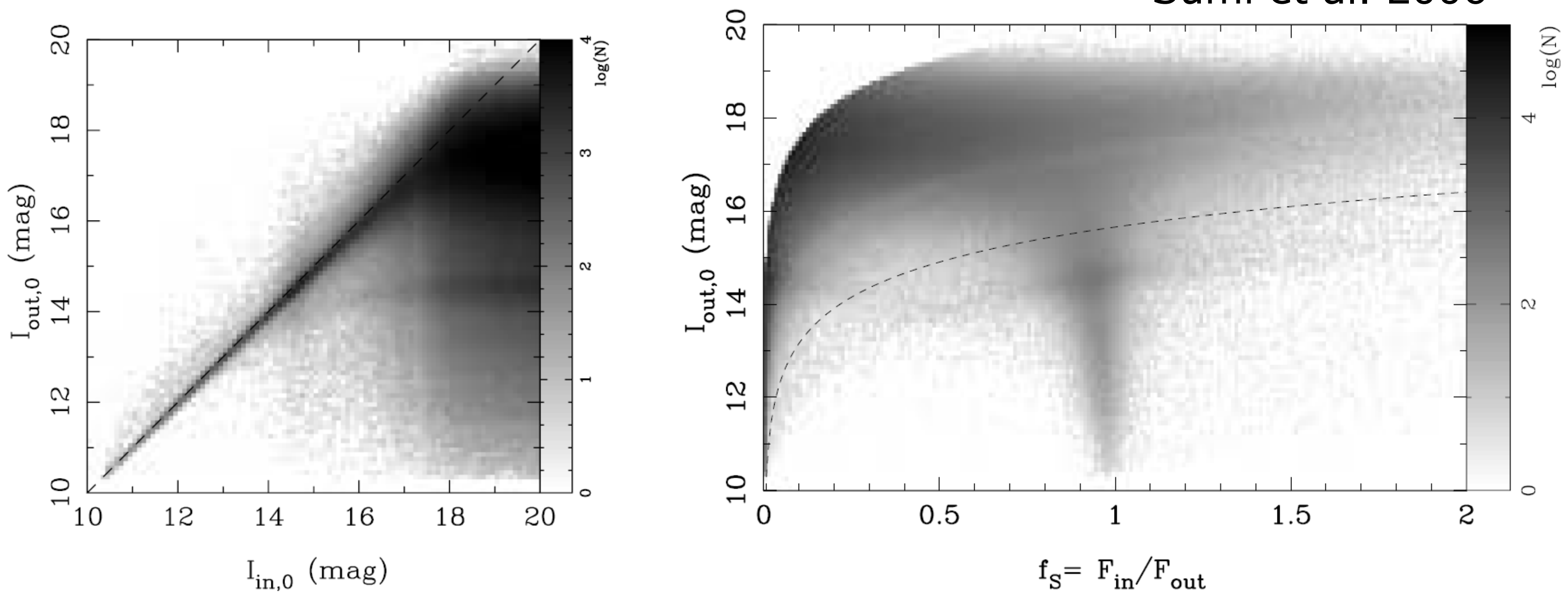
Problems with nonlinear photometry near detection threshold





Blending: Luminosity Function

Sumi et al. 2006



- Undetected sources (failure to deblend)
- Spurious sources around bright objects (variable PSF residuals)
- Luminosity Function (LF) changes both norm and shape

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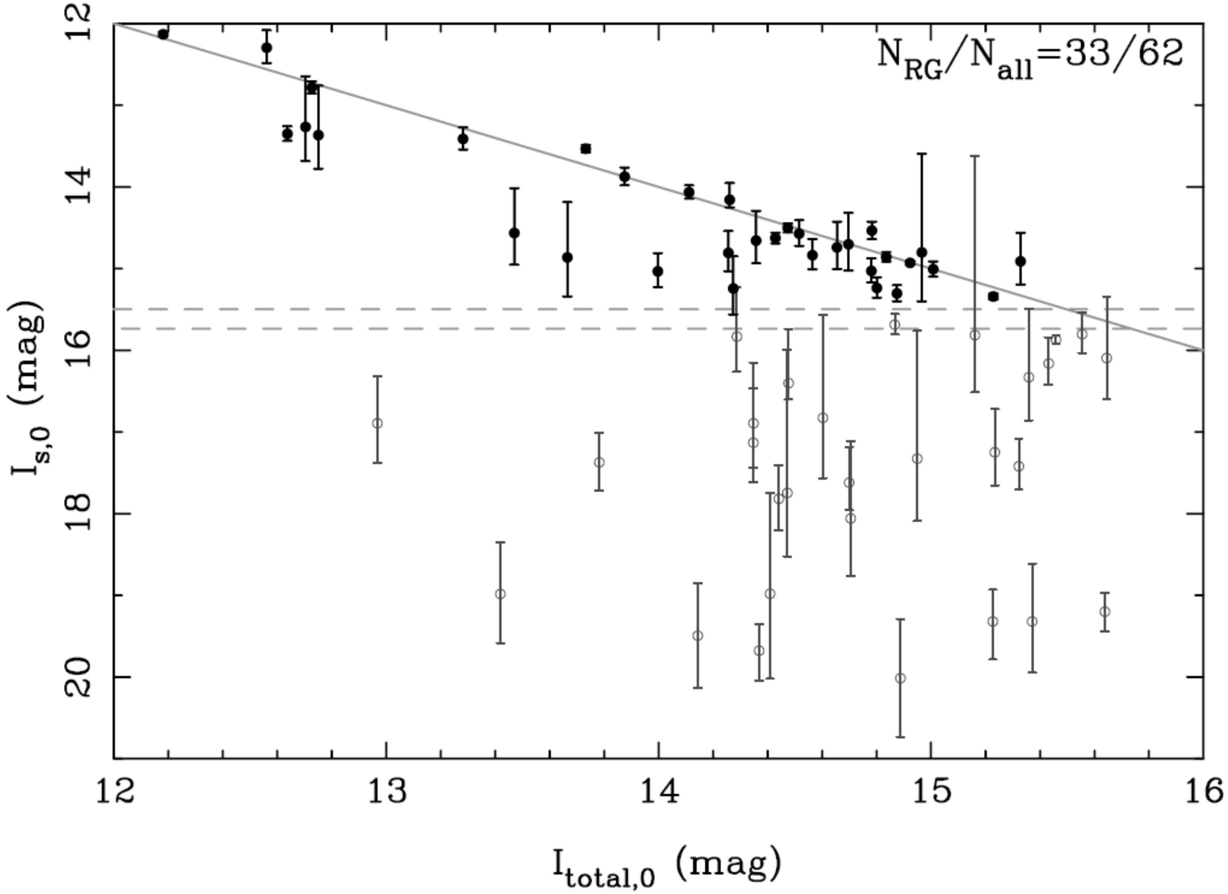
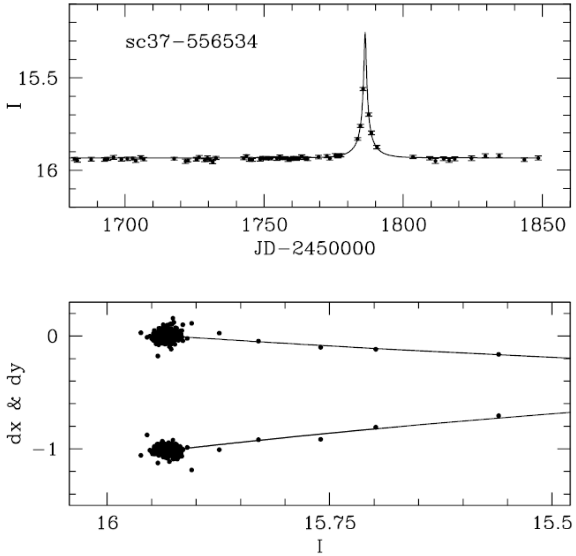
Blending: event baselines



Centroid shifts in variable sources

Smith et al. 2007

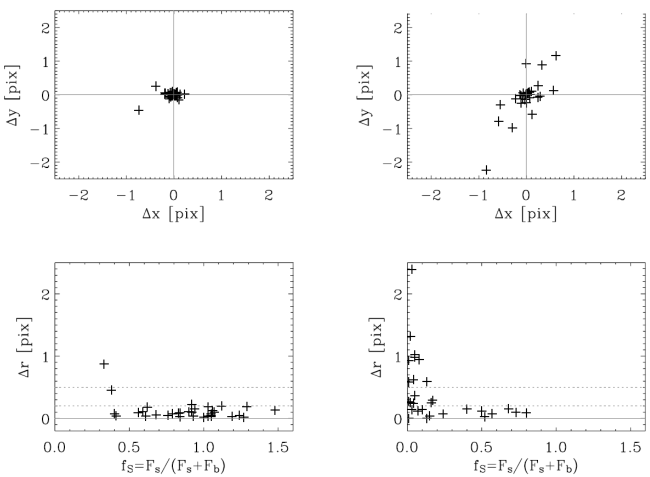


Source and blend fraction: $f_s = F_s/F_0,\ 1 - f_s = F_b/F_0$

Mean light centroid: $$\mathbf{r}(t) = \frac{\mathbf{r}_s f_s A(t) + \mathbf{r}_b(1-f_s)}{A(t) f_s + (1-f_s)}$$

Same at baseline: $$\mathbf{r}_0 = \mathbf{r}_s f_s + \mathbf{r}_b(1-f_s)$$

Motion: $$\Delta\mathbf{r} = \frac{\mathbf{r}_s f_s A(t) f_s + \mathbf{r}_b(1-f_s)}{f_s A(t) + (1-f_s)} - \mathbf{r}_0 = (\mathbf{r}_s - \mathbf{r}_b)\frac{f_s(1-f_s)(A(t)-1)}{f_s A(t) + (1-f_s)}$$

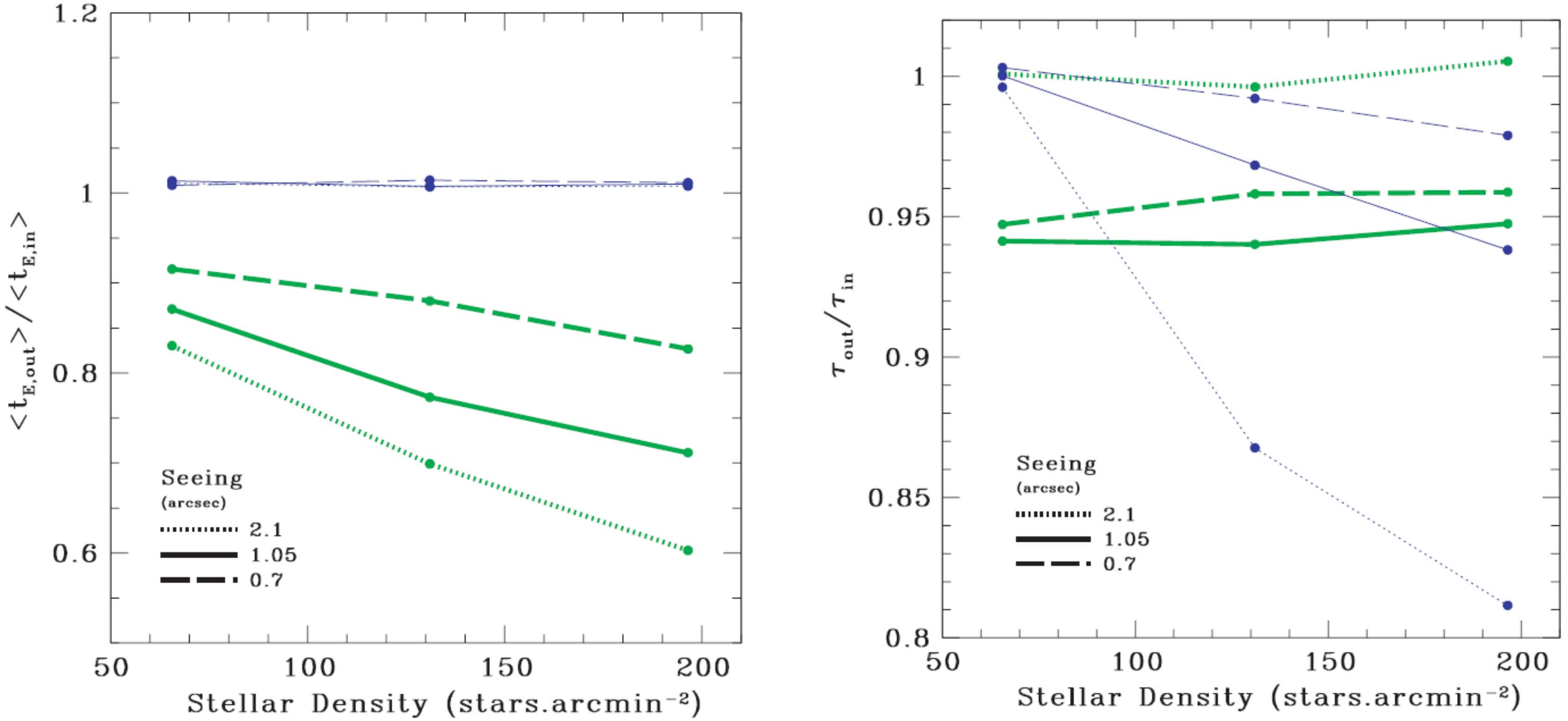


Events with lower source fractions and high magnifications tend to show large centroid shifts

Sumi et al. 2006

Crowding induced microlensing biases: time-scales and optical depth

$$\tau \sim N^{-1} \sum_i t_{\mathrm{E,i}} / \varepsilon(t_{\mathrm{E,i}})$$



Smith et al. 2007

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Image subtraction: Limitations of Fourier Division

Find a PSF-matching kernel in Fourier space:

$FFT(Ker) = FFT(PSF_1)/FFT(PSF_2)$

Issues:

- In crowded fields PSF is ill defined
- Relies on availability of isolated stars
- With noise, no good way to enforce that the end result makes sense
- Noise dominates the PSF wings, where the game is
- Requires very high S/N
- Hard to handle spatially variable solutions and find enough “clean” information in the image
- Sky backgrounds have to be matched separately
- Very sensitive to under-sampling and aliasing

Can be stabilized with:

- Real data in the core + smooth model in the wings

Image subtraction:
Alard & Lupton method

Alard & Lupton (1998), Alard (2000)

- Forget FFT and do it in real space
- Insist on linear kernel decomposition
- Propose a particular basis for the kernel that works with a wide range of images

AL image decomposition

- model as convolution

$$I \models K \otimes R$$

$$I(\mathbf{r}) \models (K \otimes R)(\mathbf{r}) \equiv \int K(\mathbf{r} - \mathbf{r}')R(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\mathbf{r} \equiv (x, y); \qquad \mathbf{r} - \mathbf{r}' \equiv (u, v)$$

- assume linear kernel basis

$$K(u, v) = \sum_i a_i K_i(u, v)$$

- rearrange operator order

$$K \otimes R = \left(\sum_i a_i\, K_i\right) \otimes R = \sum_i a_i (K_i \otimes R)$$

- model as linear combination of images

$$I(x, y) = \sum_i a_i C_i(x, y) \qquad C_i = K_i \otimes R$$

Reducing to linear least squares

- minimize cost function

$$\chi^2 = \int \frac{(I - \sum_i a_i C_i)^2}{\sigma^2}\, dx\, dy \qquad C_i = K_i \otimes R$$

- solve linear equation

$$\mathbf{M}\,\mathbf{a} = \mathbf{V}$$

- scalar products
  of image vectors

$$M_{ij} = \int \frac{C_i(x,y)\, C_j(x,y)}{\sigma(x,y)^2}\, dx\, dy$$

$$V_i = \int \frac{I(x,y)\, C_i(x,y)}{\sigma(x,y)^2}\, dx\, dy$$

Kernel basis

- n~3 fixed width Gaussians with polynomial warps

$$K(u,v) = \sum_n e^{-(u^2+v^2)/2\sigma_n^2} \sum_{0 \le p+q \le D_n} a_{pq}\, u^p\, v^q$$

- count components (flatten index)

$$\{n, pq\} \to i \qquad 0 \le p, q \le D_n$$

$$i = 1, \ldots, \sum_n (D_n + 1)(D_n + 2)/2$$

- single "kernelet"

$$K_i(u,v) = u^{p_i}\, v^{q_i} e^{-(u^2+v^2)/2\sigma_{n_i}^2}$$

Smooth background

$I \models B + K \otimes R$

- introduce more image level vectors

$$B(x,y) = \sum_i a_i B_i(x,y) = \sum_{p,q} a_{pq}\, x^p\, y^q \qquad 0 \le p+q \le D_B \qquad 0 \le p,q \le D_B$$

$$i = 1, \ldots, (D_B+1)(D_B+2)/2$$

$$I(x,y) = \sum_i a_i C_i(x,y)$$

$$C_i(x,y) = \begin{cases} B_i \equiv x^{p_i}\, y^{q_i} & i = 1 \quad \ldots \; N_B \\ R \otimes K_i & i = 1 + N_B \; \ldots \; N_K + N_B \end{cases}$$

$$N_K = \sum_n (D_n+1)(D_n+2)/2$$

$$N_B = (D_B+1)(D_B+2)/2$$

Variable kernel: brute force

- expand low-frequency component (~ Karhunen-Loeve decomposition)

$$a_i \rightarrow a_i(x,y) = \sum_{r,s} a_i^{rs}\, x^r y^s \qquad K(u,v,x,y) = \sum_i a_i(x,y) K_i(u,v)$$

- reformulate least squares fit $\qquad \{^{rs}_{i}\} \rightarrow n$

$$\tilde{M}_{nm} = \int \frac{\tilde{C}_n \tilde{C}_m}{\sigma^2}\, dx\, dy$$

$$\tilde{M}_{nm} = \int \frac{C_n P_n\, C_m P_m}{\sigma^2}\, dx\, dy = \int \frac{C_n C_m\, P_n P_m}{\sigma^2}\, dx\, dy$$

- separate high and low frequency parts

$$C_n = K_{i_n} \otimes R = C_{i_n} \qquad P_n = x^{r_n} y^{s_n}$$

Variable kernel: speed optimized

- consider small sub-domains

$$\int_\Omega dx\,dy \to \sum_k \int_{\Omega_k} dx\,dy$$

$$\Omega_k \subset \Omega \quad k = 1, \ldots, N_\Omega$$

- ignore kernel changes over a single domain

$$P_n(x, y) \simeq P_n(x_k, y_k)$$

$$\tilde{M}_{nm} = \sum_k P_n(x_k, y_k) P_m(x_k, y_k) \int_{\Omega_k} \frac{C_n C_m}{\sigma^2}\, dx\,dy$$

$$\tilde{M}_{nm} = \sum_k P_n(x_k, y_k) P_m(x_k, y_k)\, Q^k_{nm}$$

- recover matrix elements for constant kernel

$$\left\{{rs \atop i}\right\} \to n$$

$$Q^k_{nm} = \int_{\Omega_k} \frac{C_{i_n}(x, y) C_{j_m}(x, y)}{\sigma^2}\, dx\,dy = M^k_{i_n j_m}$$

Variable kernel: final result

- compute local least squares matrix and vector for each domain (constant kernel)
- compute global problem by accumulating local contributions taken with position-dependent weights (variable kernel)

$$\left\{ {rs \atop i} \right\} \rightarrow n \qquad P_n = x^{r_n} y^{s_n}$$

$$\tilde{M}_{nm} = \sum_k P_n(x_k, y_k) P_m(x_k, y_k)\, M^k_{i_n j_m}$$

$$\tilde{V}_n = \sum_k P_n(x_k, y_k)\, V^k_{i_n}$$

Flux conservation

$$a_i \rightarrow a_i(x,y) = \sum_{r,s} a_i^{rs}\, x^r\, y^s$$

- require constant kernel norm

$$\int K(u,v,x,y)\, du\, dv = const$$

- assume normalized basis

$$\int K_i(u,v)\, du\, dv = 1$$

- rearrange basis vectors

$$\tilde{K}_i \equiv \begin{cases} K_0 & i = 0 \\ K_i - K_0 & i > 0 \end{cases}$$

- "isolate" kernel norm
  in a single constant vector

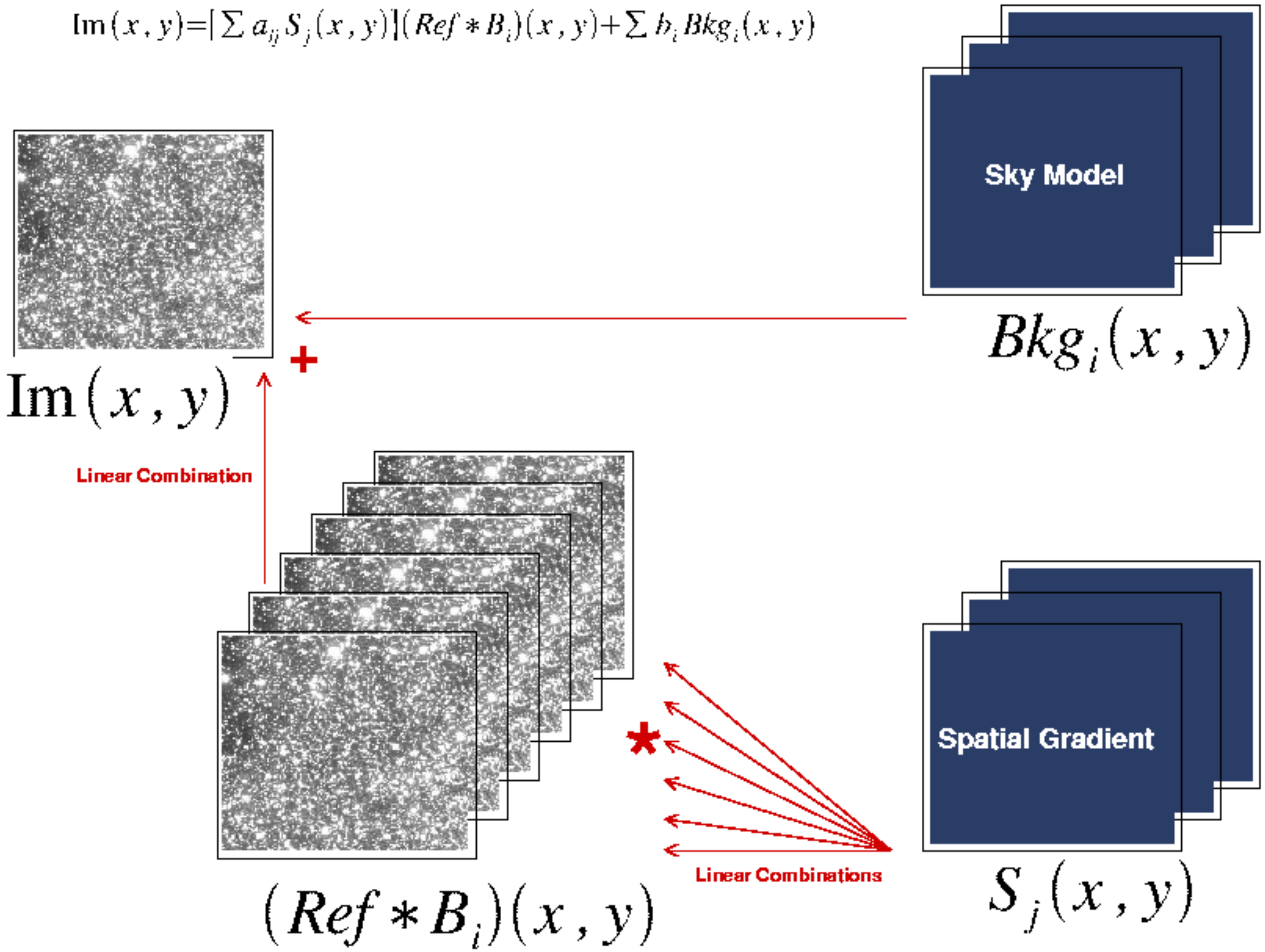
$$\int \tilde{K}_i(u,v)\, du\, dv = \begin{cases} 1, & i = 0 \\ 0, & i > 0 \end{cases}$$

$$K(u,v,x,y) = a_0 K_0(u,v) + \sum_{i>0} a_i(x,y)[K_i(u,v) - K_0(u,v)]$$

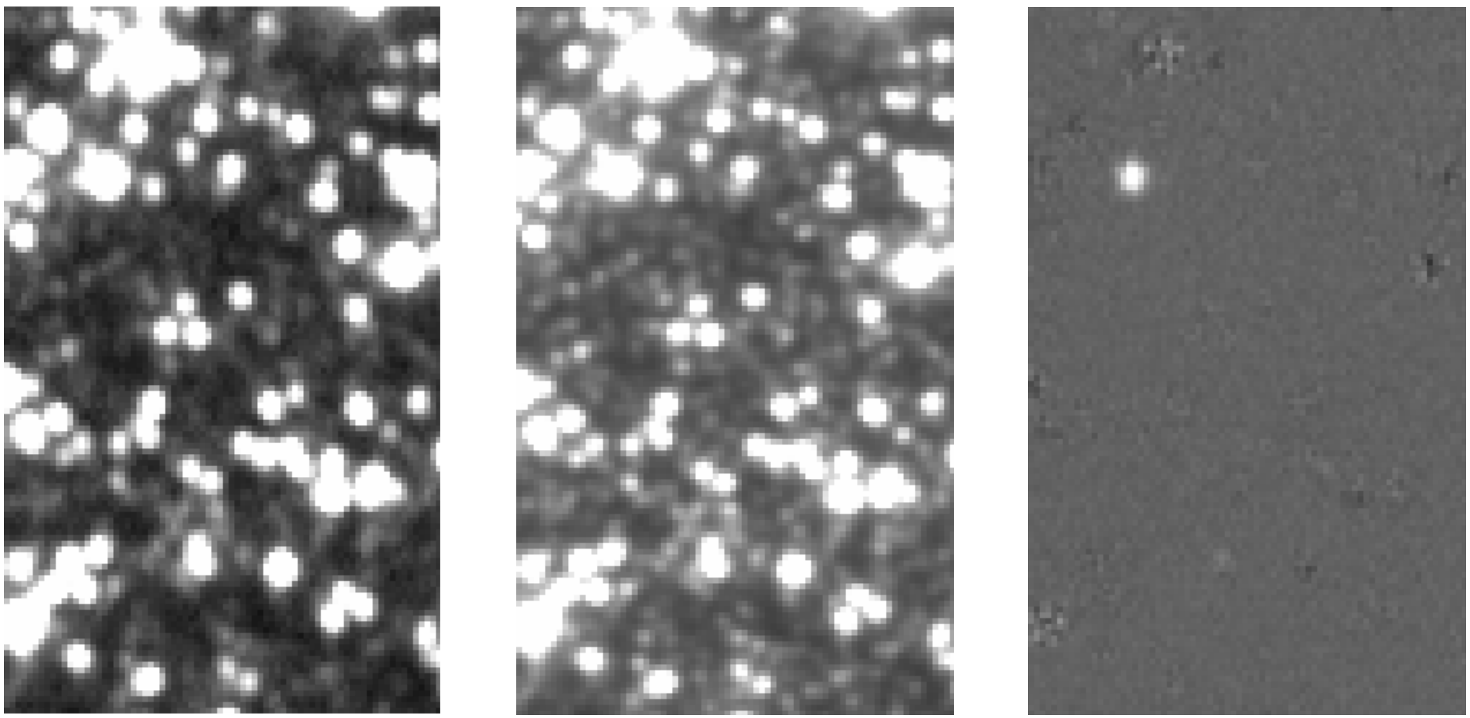
$$\int K(u,v,x,y)\, du\, dv = \int \sum_i a_i(x,y) \tilde{K}_i(u,v)\, du\, dv = a_0$$

Factoring out specific choice of functions

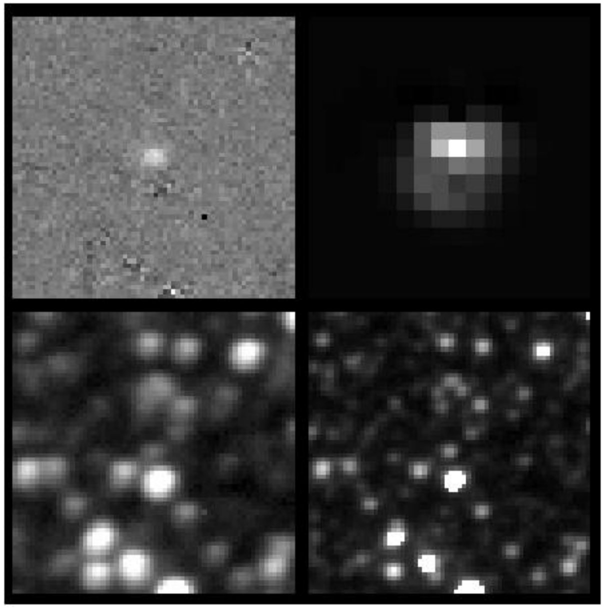
$$\mathrm{Im}(x,y)=[\sum a_{ij} S_j(x,y)](Ref * B_i)(x,y)+\sum b_i Bkg_i(x,y)$$



It works!



Example convolution kernel



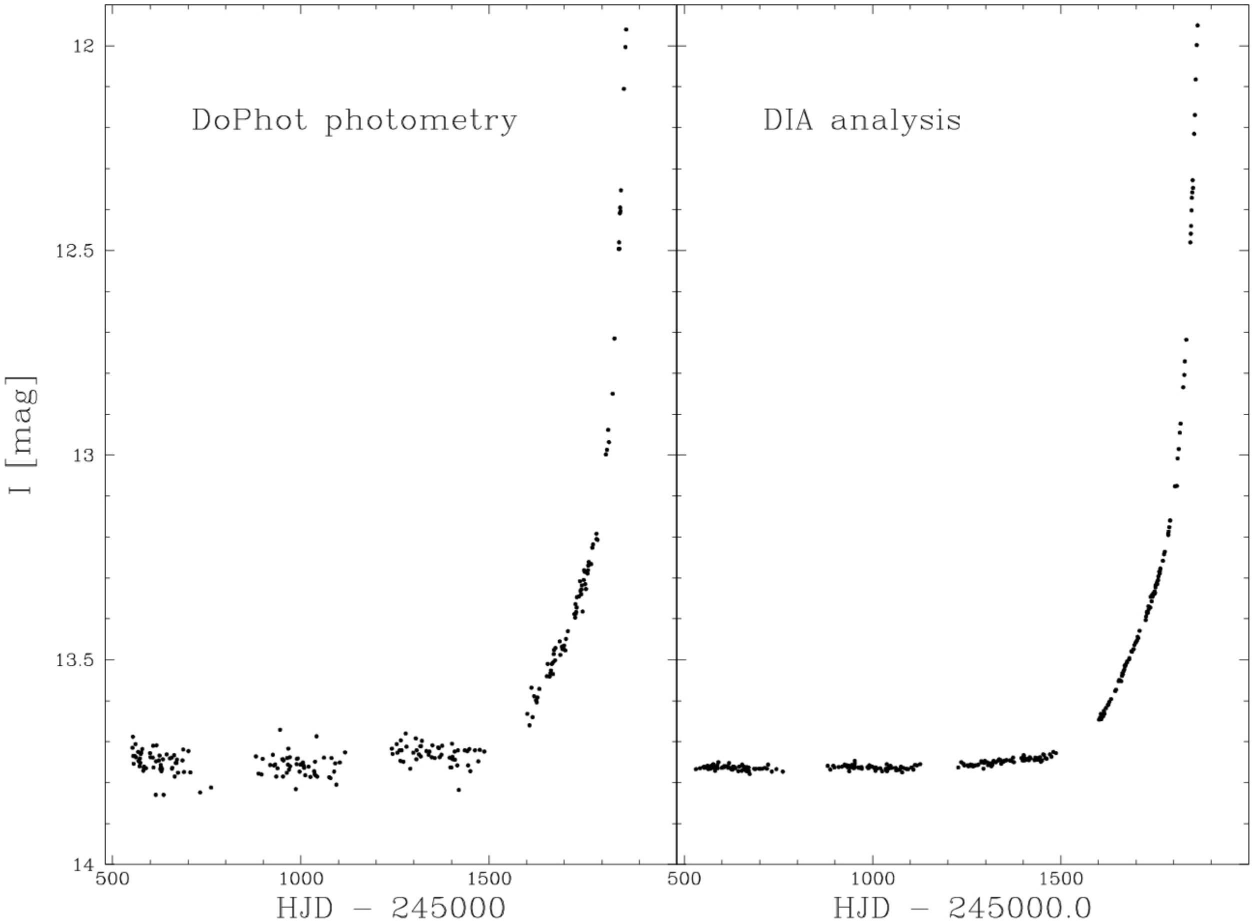
From images to light curves

1. Register images and resample to the same pixel grid
2. Construct photometric reference image
3. Run PSF matching and subtract the reference
4. Locate (variable) objects and perform photometry
5. Measure reference flux and convert to mag (AC/DC)

$$f(t_i) = f_{\rm ref} + f_{\rm DIA}(t_i) \qquad m_i = const - 2.5 \times \log_{10}(\Delta f_i + f_{\rm ref})$$

- run a conventional PSF package on the reference image
- compare entire DIA light curve to (noisier) PSF version
- use external info (e.g. HST photometry)
- for transients a suitable choice of reference image gives $f_{ref} = 0$

Light curve S/N improvement



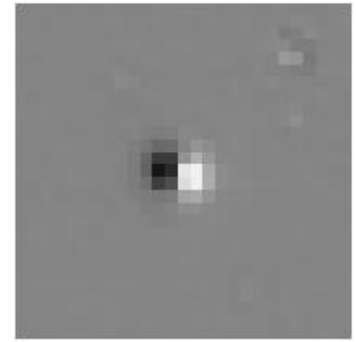
Implementation issues

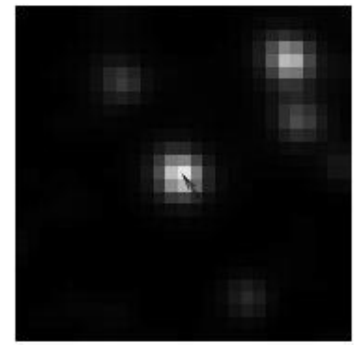
- Good reference frame (optimal image co-addition)
- Relative PSF weights of variable stars
- Interpolation techniques
- Clipping variable pixels
- Masking for background dominated regions and defects
- Flux conservation
- Cost functions
- Reference flux
- Variability detection vs. measurement
- Noise propagation: convolve with kernel squared
- Caching computation
- Separable kernels are fastest: $K(x,y) = f(x)g(y)$
- Choice of basis functions: shapelets, spherical harmonics, orthonormal polynomials, …
- For some problems can fit each kernel mesh pixel separately (delta function kernels)
- May solve each domain separately + PCA on coefficients to get spatial variability (LSST approach)

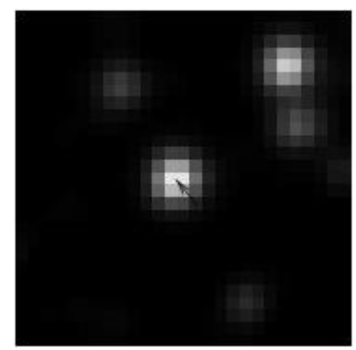
Good properties

- $PSF_{1,2}$ not required to find transition PSF_1 -> PSF_2 !
- Guarantees min. chi2 result
- Flexible and generic approach
- Can get down to photon noise
- Turns crowding into advantage
- Surprisingly good in sparse fields too
- Reasonably fast
- Works in 1-D (forrests of spectral lines)
- Sharpening kernels possible
- Slight under-sampling OK
- Unbiased centroid
- Removes residual image mis-registration or detects subtle motions

Astrometry, motions and registration

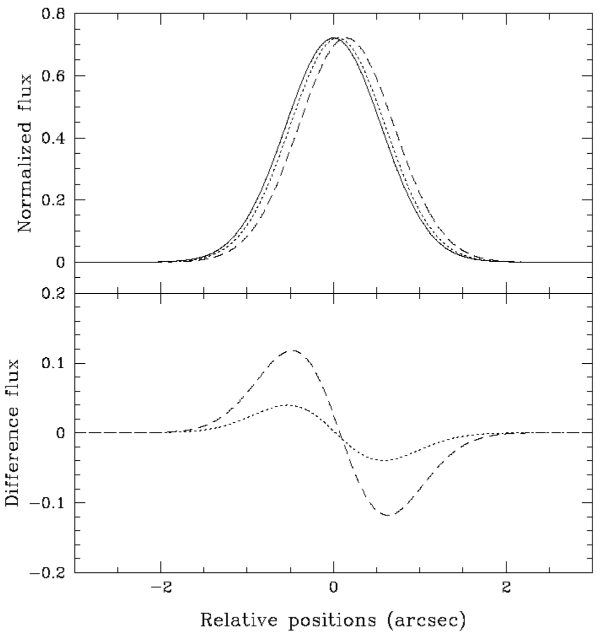




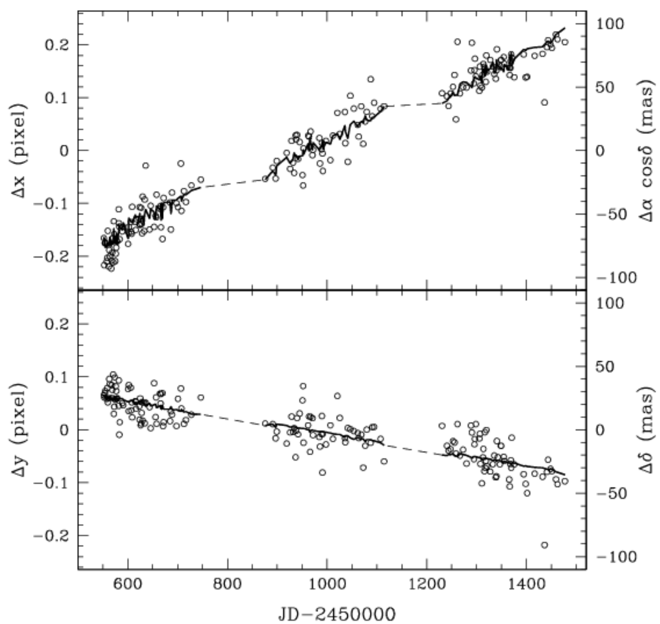


$$f(x) = \frac{F_{\mathrm{tot}}}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu t)^2}{2\sigma^2}} - \frac{F_{\mathrm{tot}}}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$\mu = \frac{\sqrt{2\pi}\sigma}{F_{\mathrm{tot}}} \gamma$$

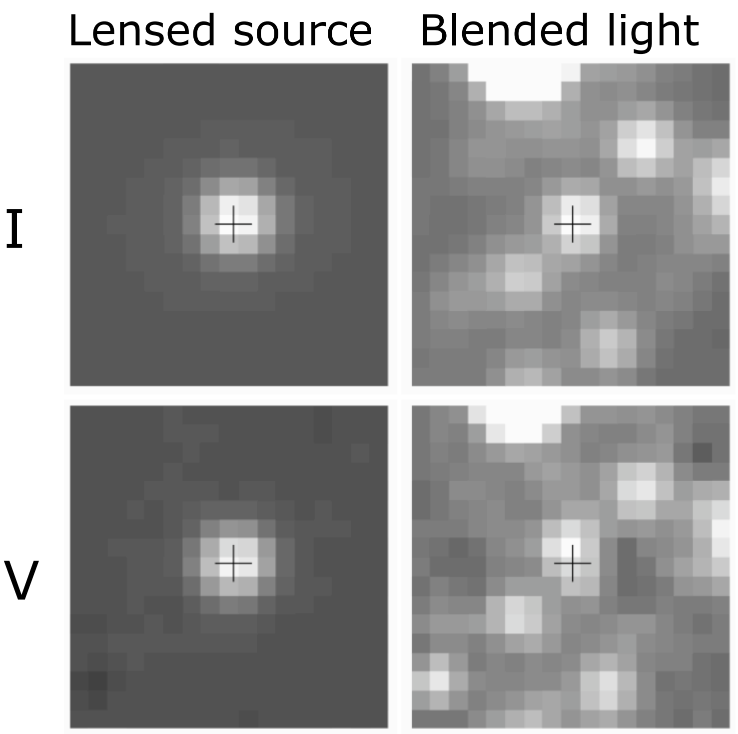


Eyer & Wozniak (2001)



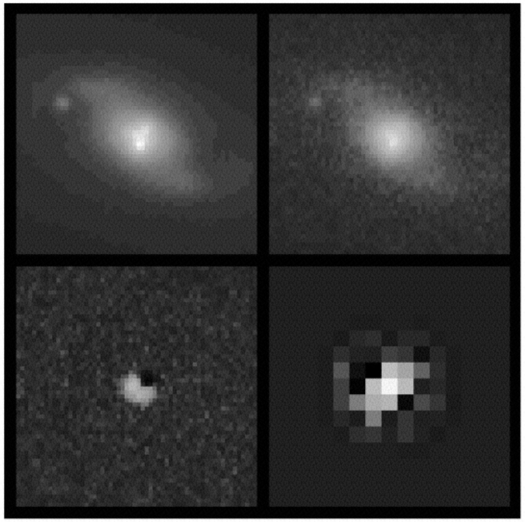
Tricks with PSF matched frames

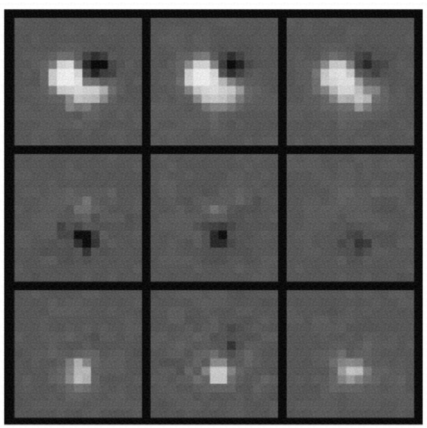
A prescription to separate the source flux from the blend using a linear combination of images with weights determined by the light curve:
Gould & An (2002)

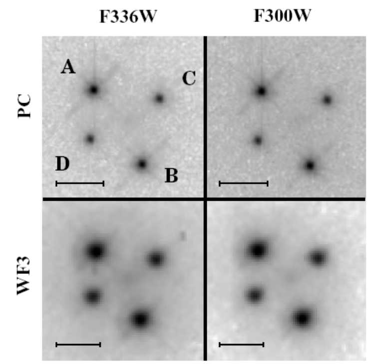


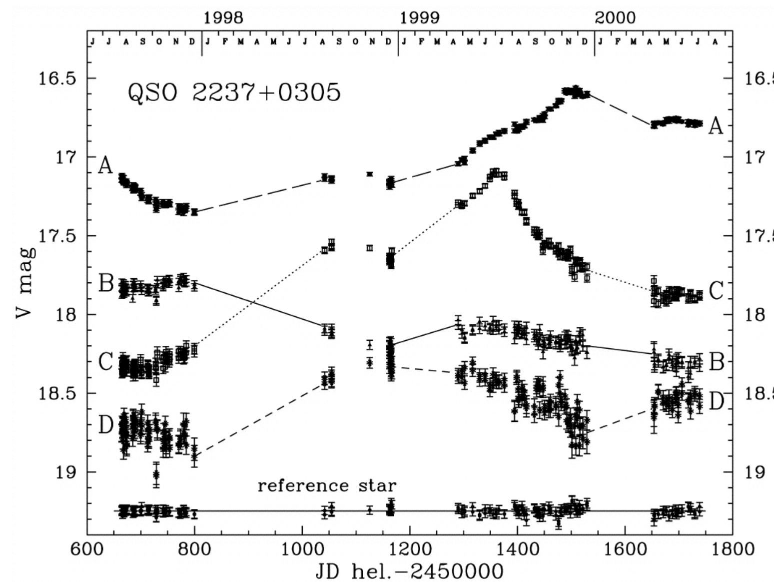
Smith et al. (2002)

Some early results: caustics in Q2237+0305





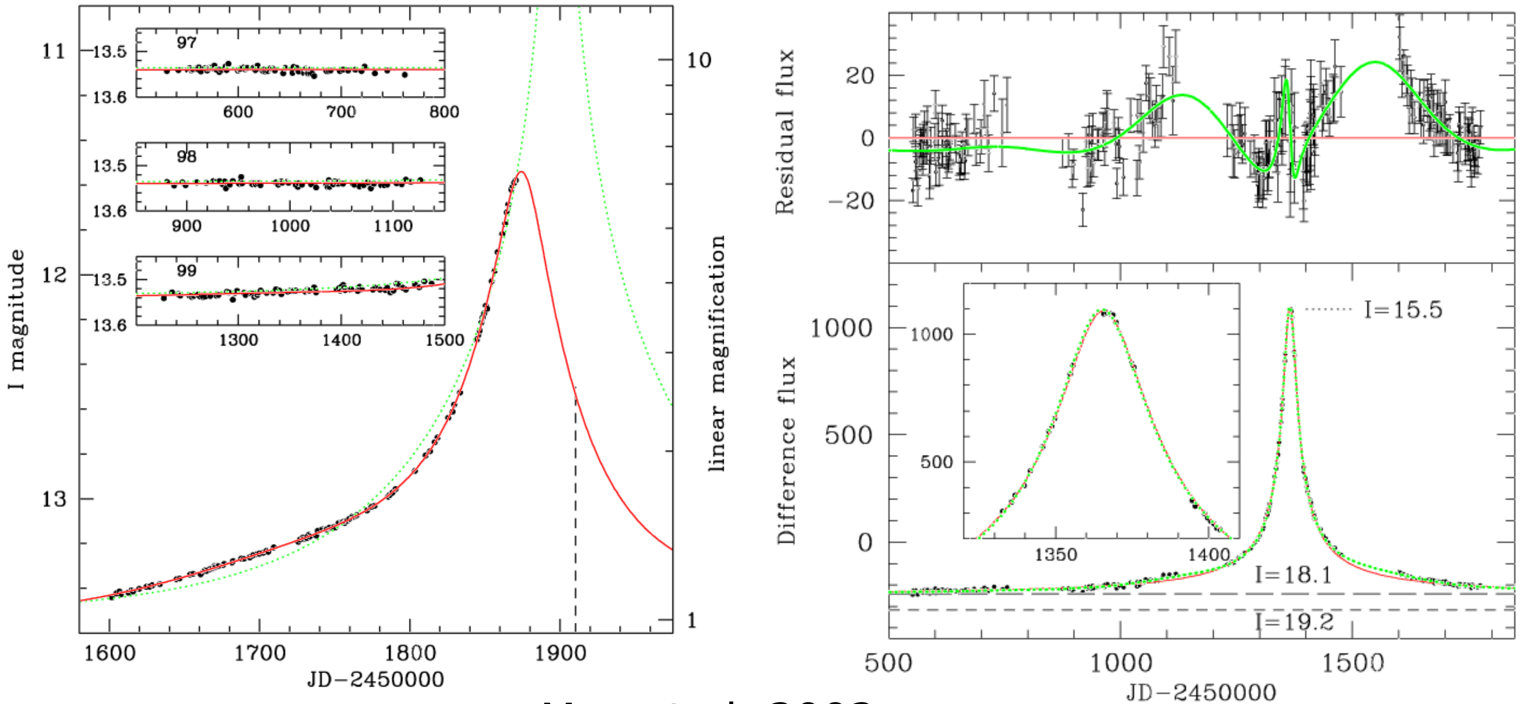




Wozniak et al. (2000ab)

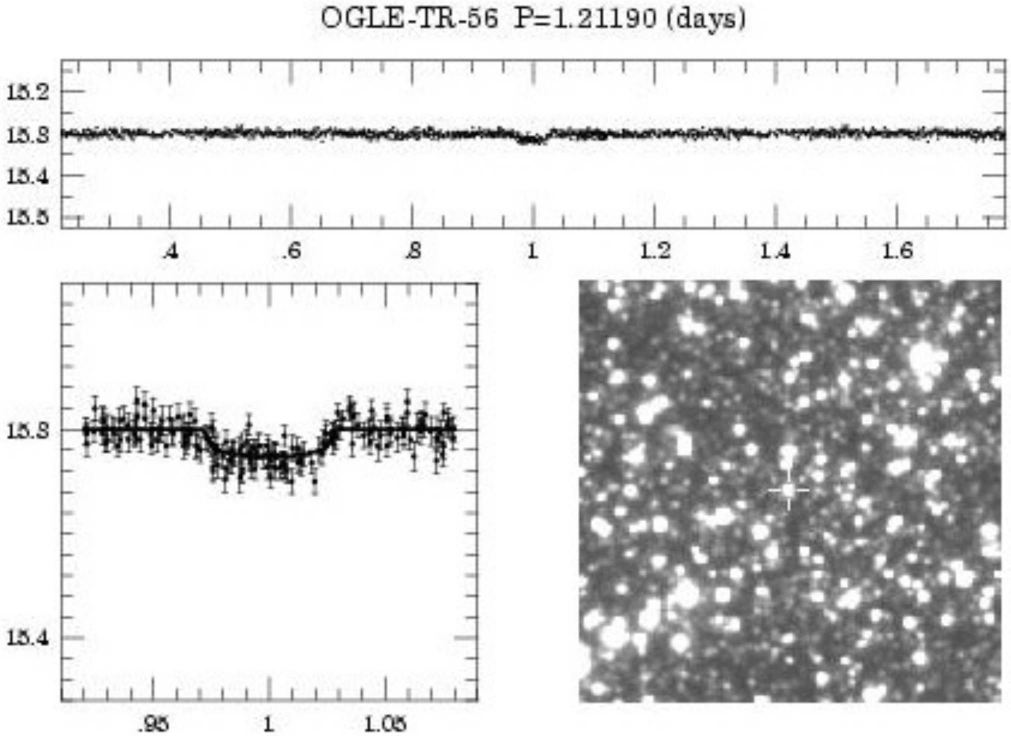
Exotic microlensing events

Possible black hole lens OGLE-1999-BUL-32



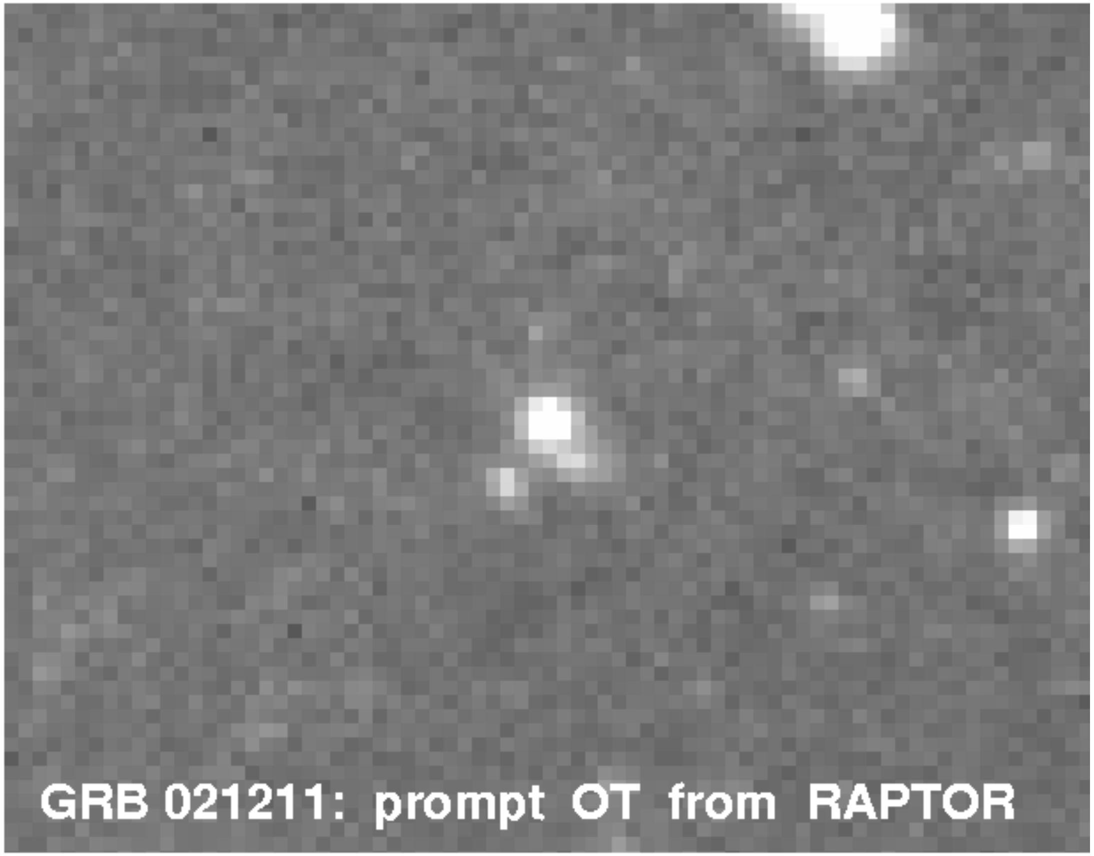
Mao et al. 2002

Planetary transits



Udalski et al. (2002)

OT from GRBs and other explosive transients



Summary

The Alard & Lupton algorithm for image subtraction and PSF matching based on convolution is the corner stone of photometric data pipelines in the current generation microlensing surveys. It enables high precision photometric monitoring of crowded fields on a massive scale. Conventional PSF photometric codes (DoPHOT) continue to provide reference flux measurements for DIA light curves.