Space-Based Imaging Astrometry

Jay Anderson
STScI
July 27, 2011

Overview of the Talk

- **Brief history of astrometry**
- **Astrometry with HST**
  - Issues
    - Undersampling
    - Distortion
    - Differential nature
  - Science
    - General
    - Microlensing

History

- **B.C.:** astronomy ***was*** astrometry
- **1718:** Halley noticed proper motions in stars
- **1838:** first parallax (0.3", by Bessel)
- **1844:** Sirius wobble (Sirius B) (Bessel)
- **1920s:** PM of M31 (van Maanen; wrong!)
- **1920s-1980s:** Photographic plates
  - Large baselines, large fields of view
  - Methods: “Blinking” “Measuring engines” → 10 mas
  - Limitations
    - Quality of the first epoch (darned technology…)
    - Different telescopes, cameras and set-ups, distortions
- **1980s CCDs**
  - Limited FOV: hard to compare with plates
  - Long-term gain
  - Advantages: linearity, dynamic range, digital signal
- **1990s HST**…

HST vs GROUND

	HST	GROUND
FOV:	$1'$	1°
FWHM:	$0.1''$	$1''$
Sampling:	undersampled	oversampled
Stars:	V>17	V<18
Crowding:	$d \sim 0.3''$	$d \sim 5''$
Distortion:	large, but static breathing (small)	atmosphere, optics gravity flexure
Reference:	differential	can be absolute
Detector:	better pixels	more pixels
Baseline:	a few years	a few decades

… Very different niches

Astrometry with HST

- **One of the original selling points**
  - FGS: always planned
  - Also imaging astrometry

- **Several challenges**
  1) Undersampling → PSFs
  2) Distortion
  3) Differential astrometry → Transformations

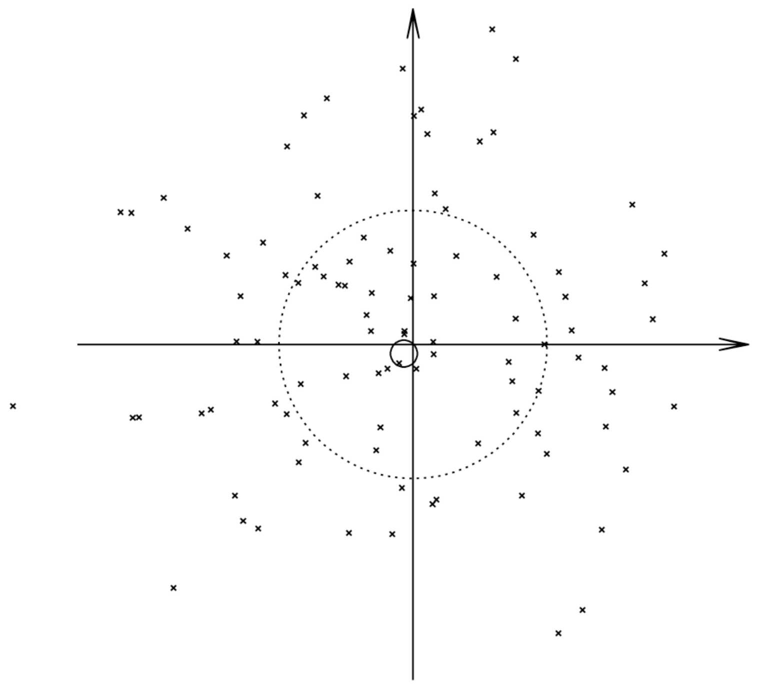
  …. took several years to address these issues.

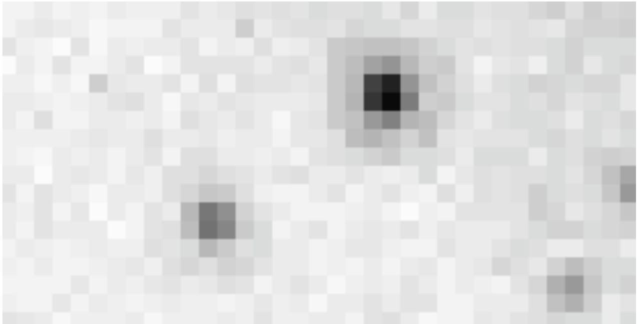
Goal of talk: an appreciation of the issues

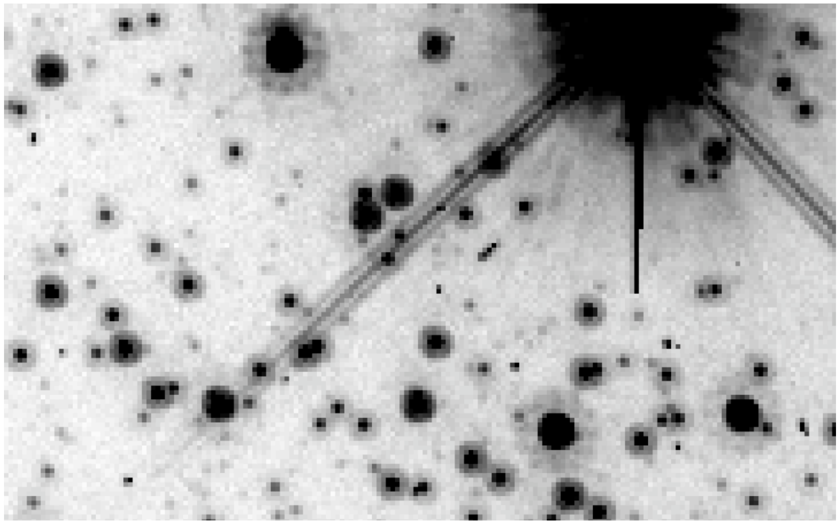
Astrometry:

Fundamental limitations

- **Poisson statistics**
  - Gaussian source
    $\delta x \sim \sigma_x / \sqrt{N}$
  - Best position
- **Pixelization**
  - Complication: loses information
  - Requires good PSF (Point-Spread Function)

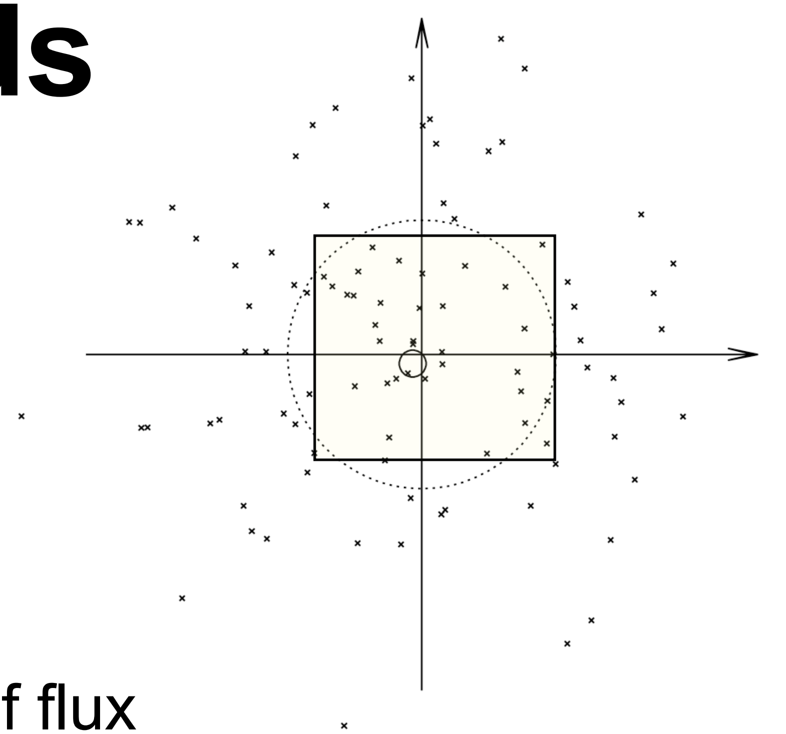






WFC3/UVIS SWEEPS FIELD

Simple Centroids



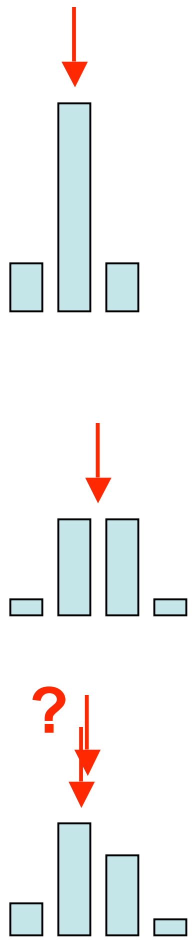
- **Literal centroids:**

$$\vec{x}_* = \sum F(\vec{x})\,\vec{x}$$

  - Finite window
  - Assumption: constant shuffle of flux
- **Undersampling Bias**
  - Hard to see the bias
    - Need to dither!

Illustration of Undersampling

Where is the center?

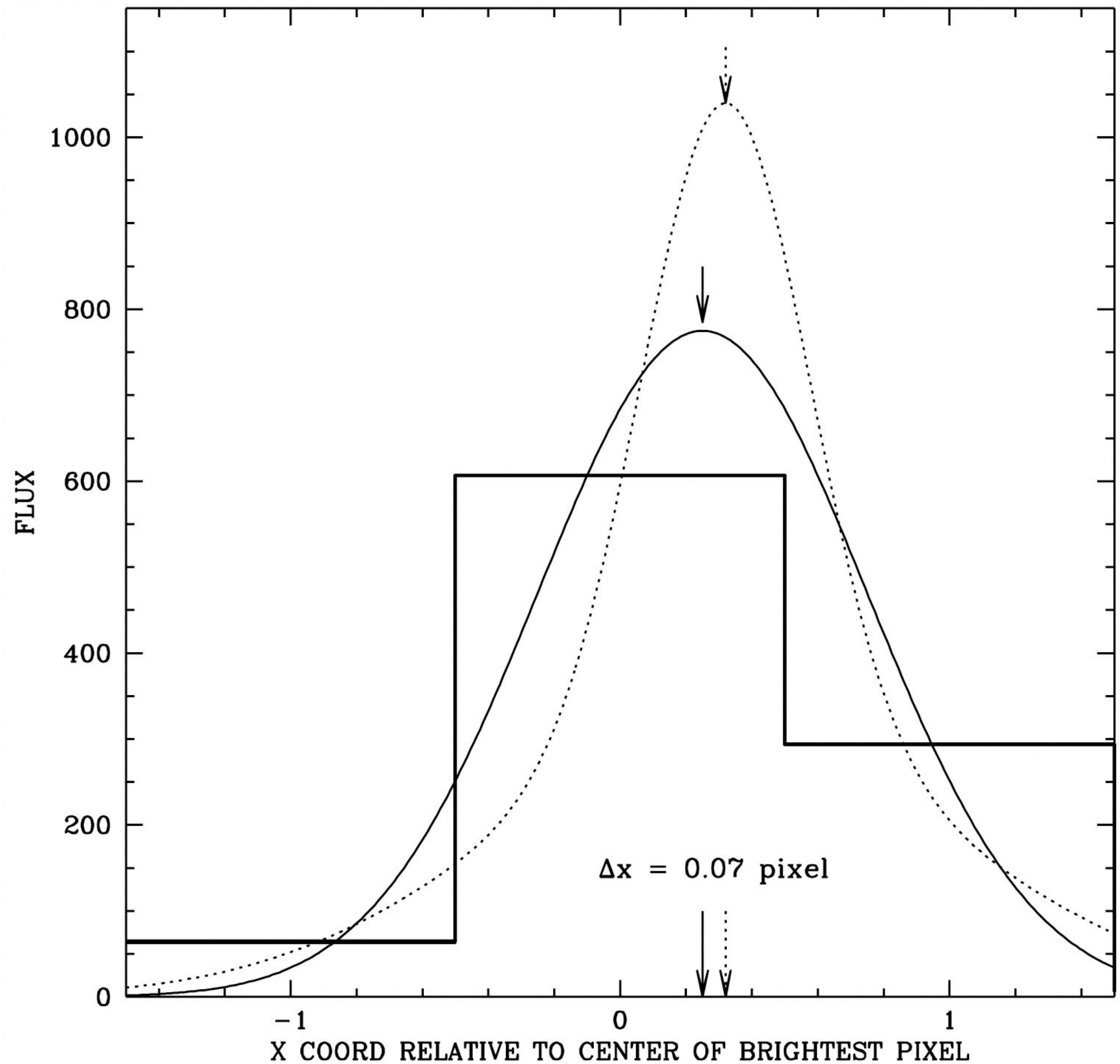


Easy

Easy

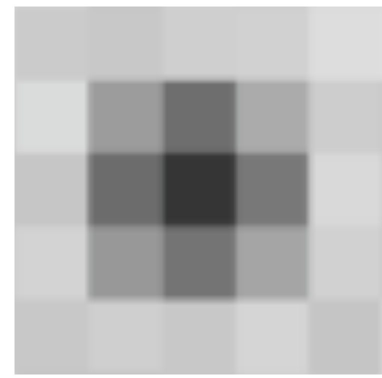
Harder

We need an accurate PSF!



Undersampling and Astrometry

- **Impossible?**
  - A point source has "no hair"
    - 3 parameters (x,y,f), ~9 pixels
  - Minimal requirements: "slosh"
    - The ideal PSF for astrometry
    - Pathological case: FWHM < 1 pixel
- **What is possible?**
  - 0.005-0.01 pixel possible ~ $(S/N)^{-1}$
  - Need good PSF model
  - Need good dithering
- **Limitations**
  - Individual images; no stacks
  - Hard in crowded fields
    - Neighbor finding/subtraction
  - Ideal in "semi-crowded" regime



PSFs: Photometry -vs- Astrometry

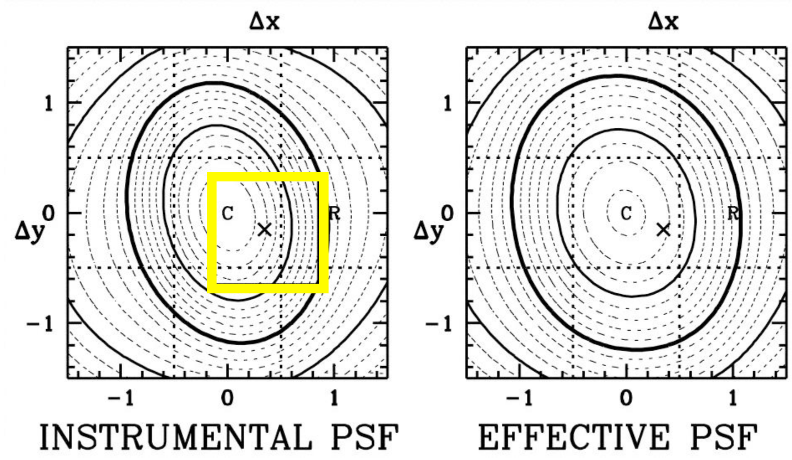
- **Photometry: how much flux is there? (SUMS)**
- **Astrometry: where is the flux? (DIFFERENCES)**
  - Both require good PSF, but they make different demands
- **PSF Modeling**
  - Ground
    - Variable-seeing dominated
    - Simple centroids, Gaussian-fitting models, DAOPhot
  - HST
    - Stable, undersampled, new regime
    - Sophisticated models possible

What do we mean by the PSF?

- **$\psi_{INST}(\Delta x,\Delta y)$: the "Instrumental" PSF:**
  - The PSF as it hits the detector
  - Good theoretical motivations: Gaussians, Moffat
  - See only *indirectly* in images
  - Solve for: deconvolve the PSF from the pixels
    - Saving grace: often solve for limited set of parameters

- **$\psi_{EFF}(\Delta x,\Delta y)$: the "Effective" PSF:**
  - The PSF after pixelization: $\psi_{EFF} = \psi_{INST} \otimes \Pi$
  - Empirical: no natural basis function to describe
  - Tod Lauer's 1999 tutorial in PASP on image reconstruction
    - OLD: Pixels as light buckets
    - NEW: Pixels as point-samplings of a continuous scene
  - We never deal with anything BUT the effective PSF
    - See *directly* in images
    - Can measure directly from images

The "Effective" PSF



- **What it represents:**
  – Fraction of light that falls in a pixel, relative to the center of the star
- **Modeling images:**

OLD: $P_{ij} = S + F_* \times \int\int_{x,y\in(i,j)} \psi_{INST}(x-x_*,y-y_*)\,dx\,dy$

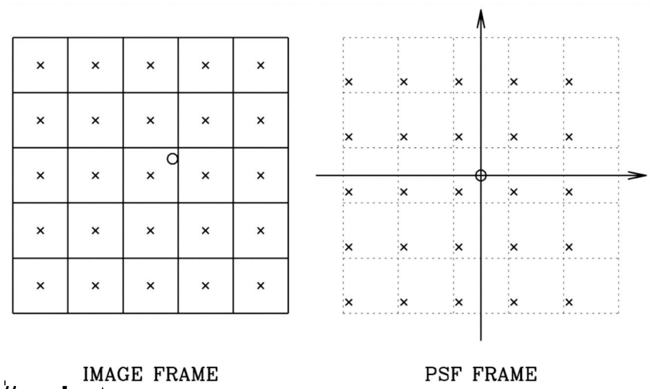
NEW: $P_{ij} = S + F_* \times \psi_{EFF}(i-x_*,j-y_*)$

- **How to "see" it:**

$\psi_{EFF}(\Delta x,\Delta y) = (P_{ij} - S)/F_*$

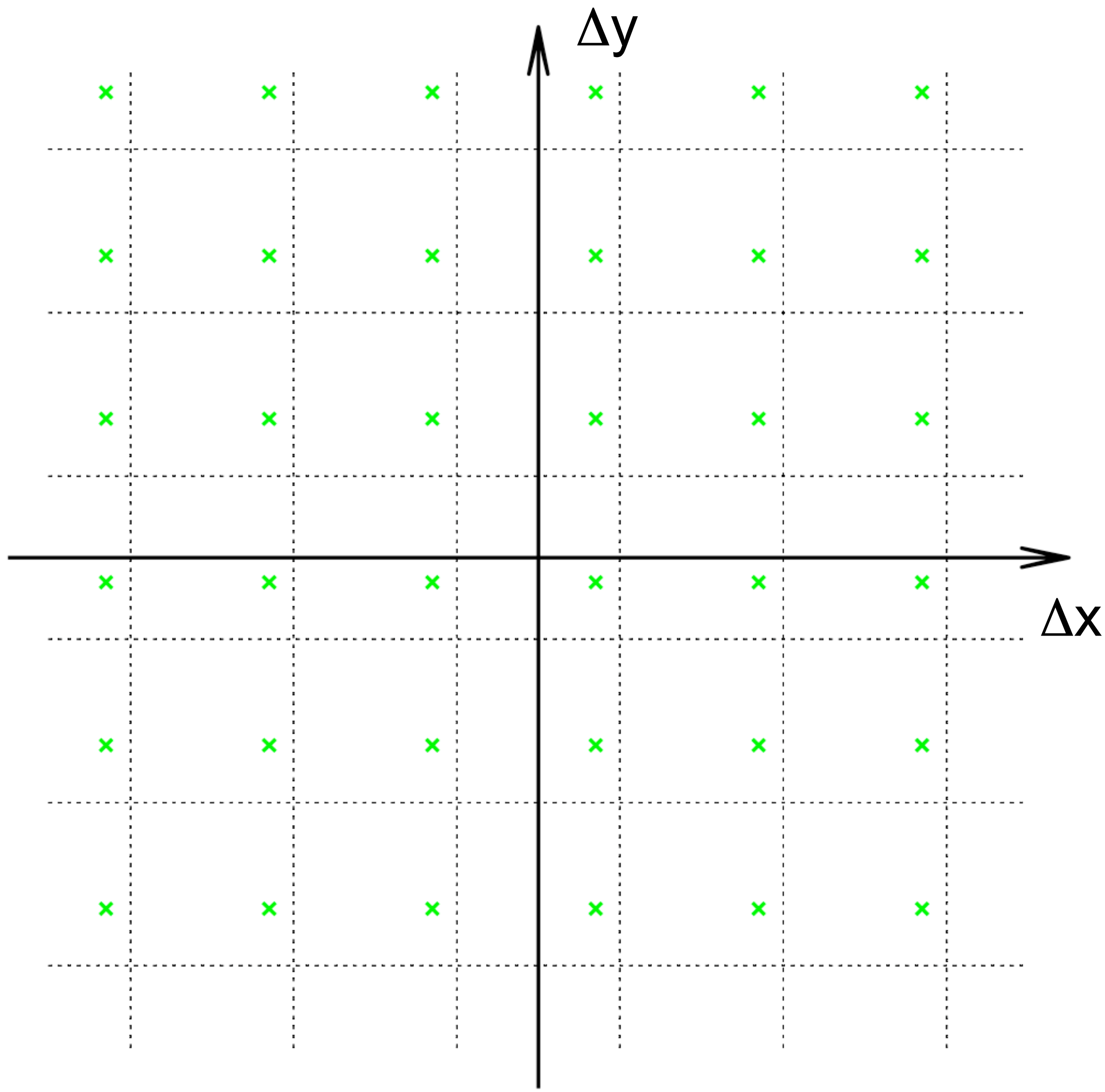
  – Where: $\Delta x = i - x_*$, etc
  – We have to know (x_*,y_*) and F_*
  – Each pixel represents a "point sampling" of ψ_{EFF}
  – Many many pixels in many many stars

Circular!



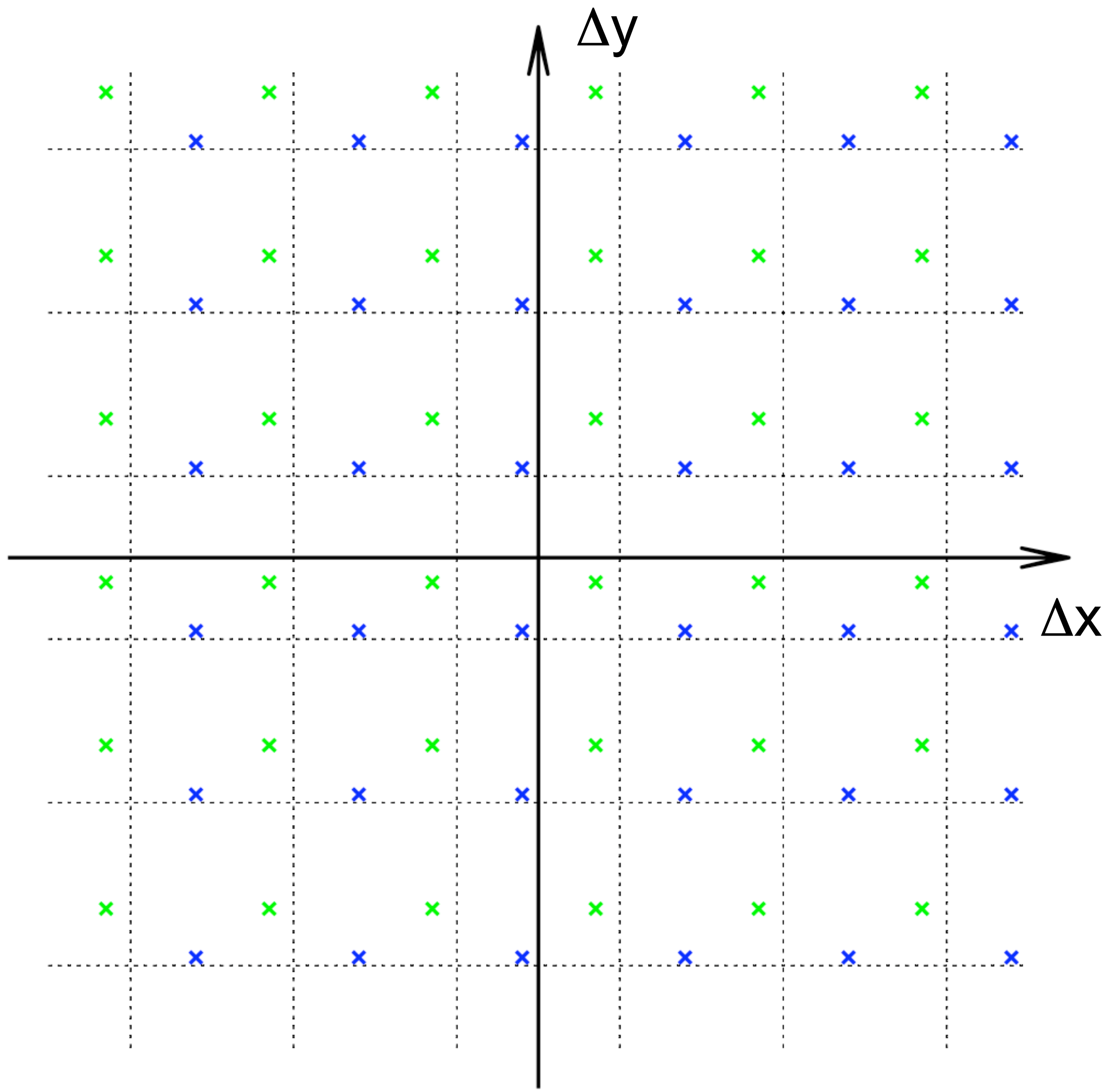
How a single star samples $\psi_E(\Delta x, \Delta y)$

- A single star has an array of pixels about its center.
- Each pixel contains a fraction of its flux.
- Each pixel reports ψ_E at one point in ψ_E's domain.



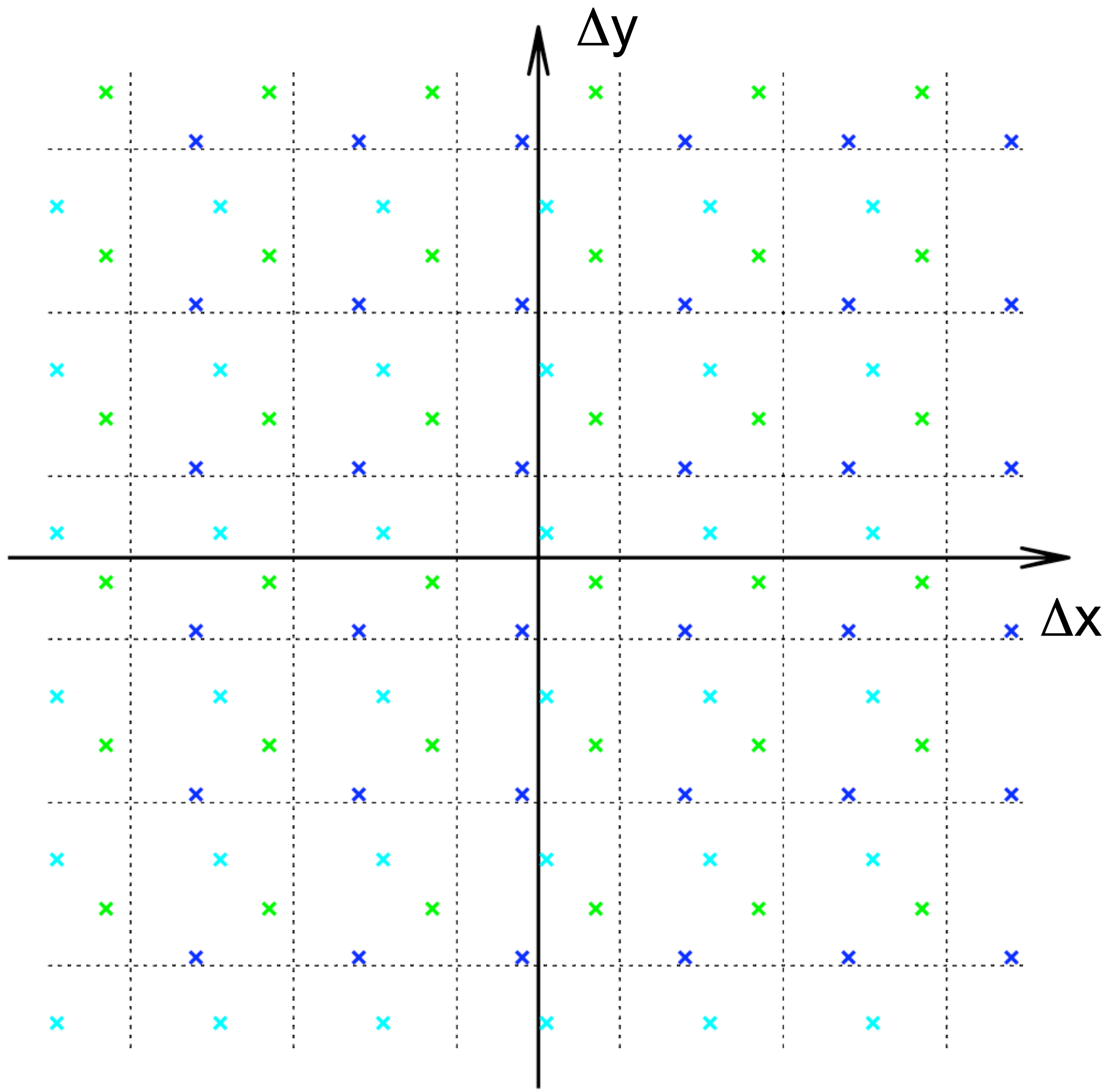
How two stars sample $\psi_E\,(\Delta x, \Delta y)$

- In general, the two stars will be at different pixel phases.
- This gives us a different array of samples of ψ_E



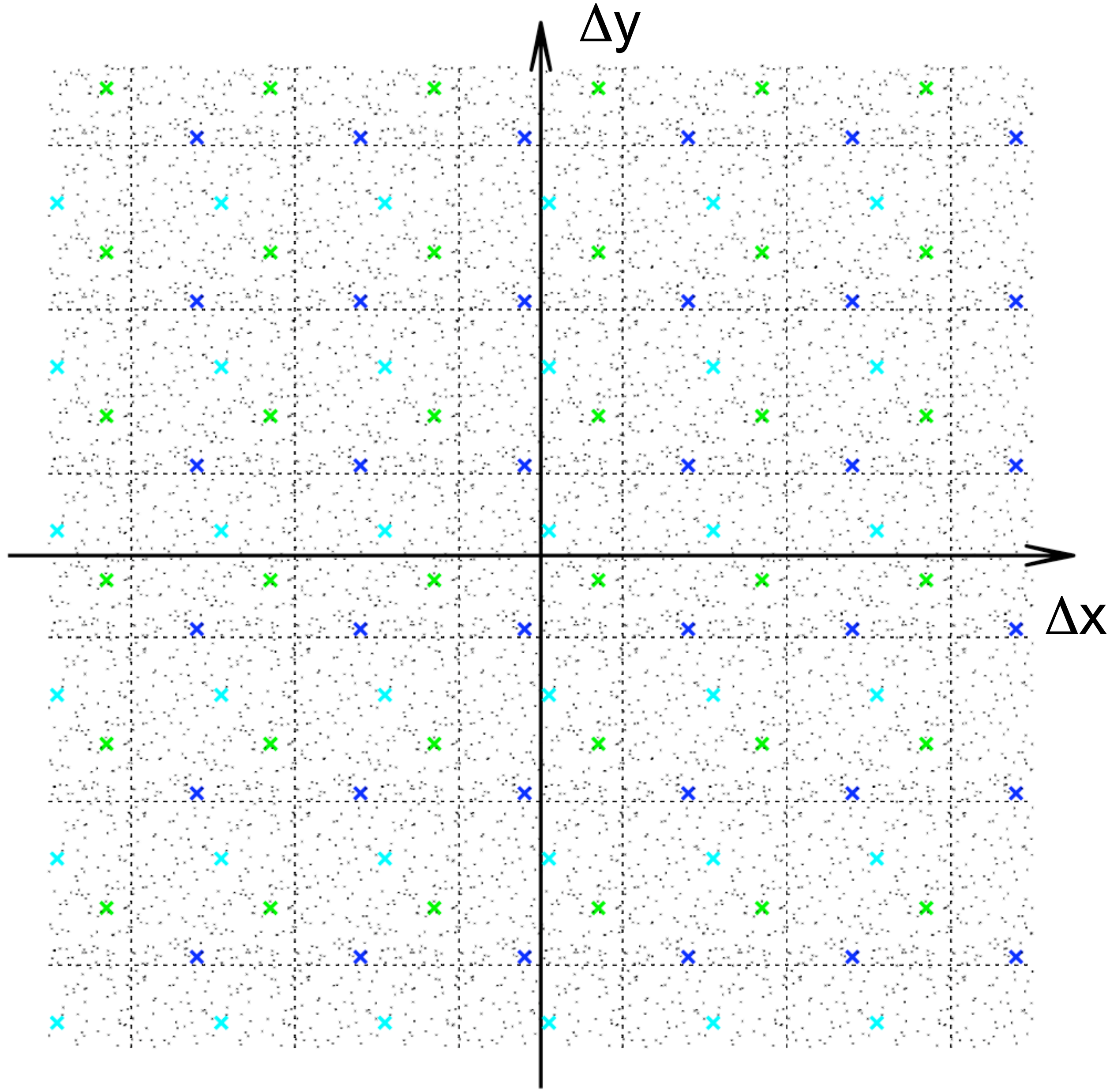
How three stars sample $\psi_E (\Delta x, \Delta y)$

- A third star will give yet more variety in our sampling of ψ_E



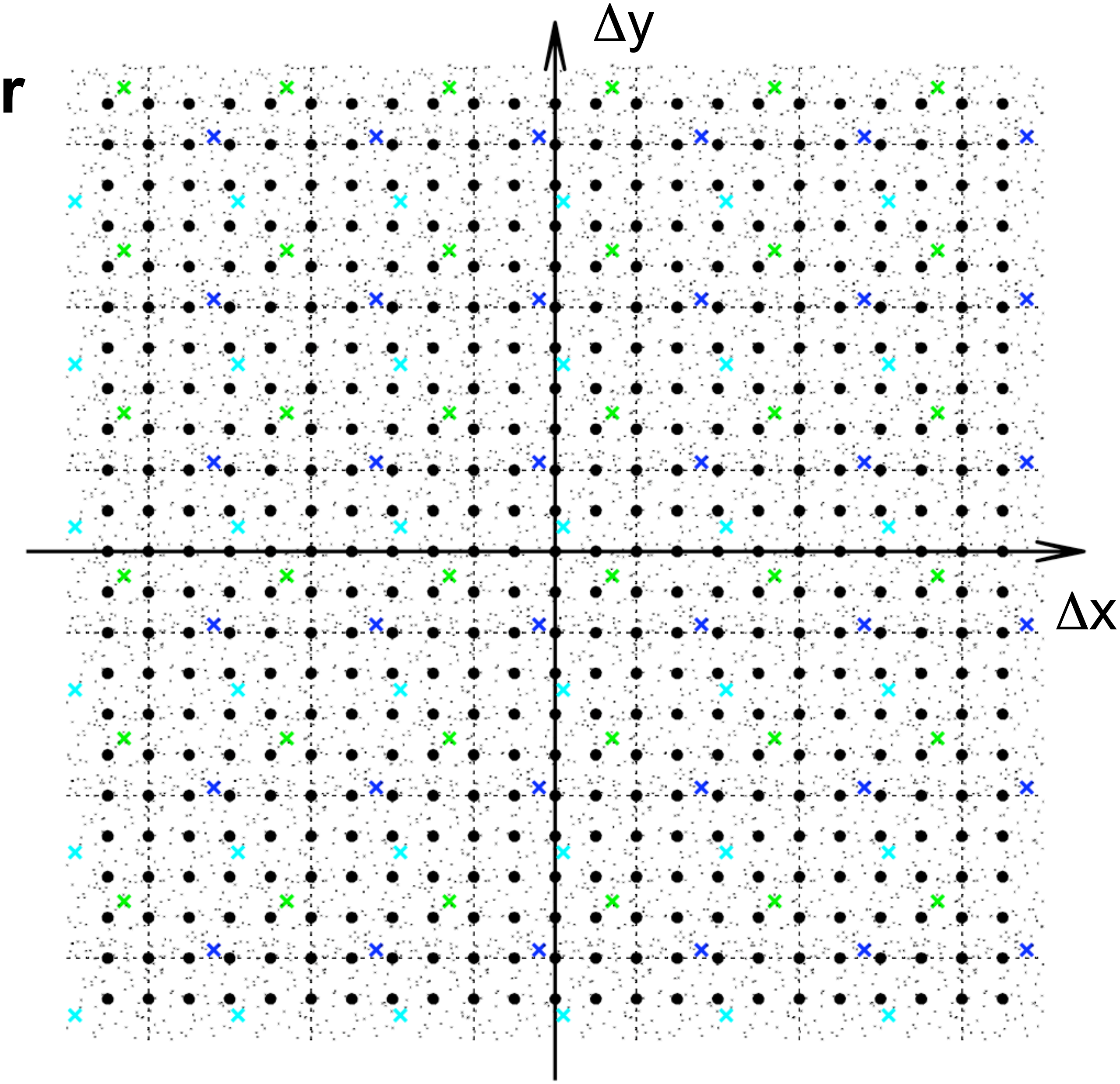
How 200 stars sample ψ_E (Δs, Δy)

- A large number of stars gives us an almost even coverage of ψ_E across its 2-D domain.

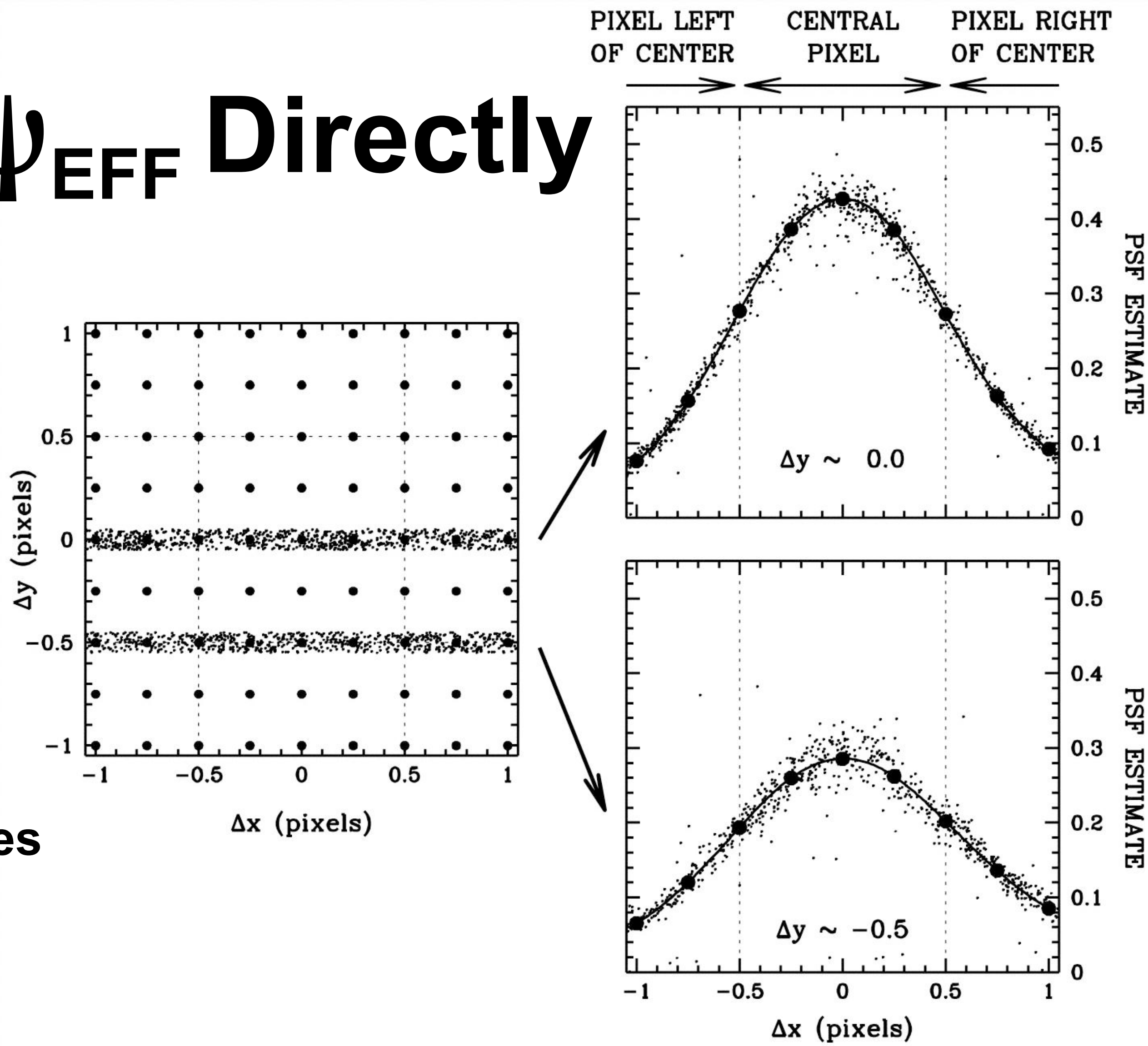


How to solve for $\psi_E(\Delta x, \Delta y)$

- A regularly-spaced array of grid-points
- Specify value of ψ_E at those points to best-fit the data.



“Seeing” ψ_{EFF} Directly

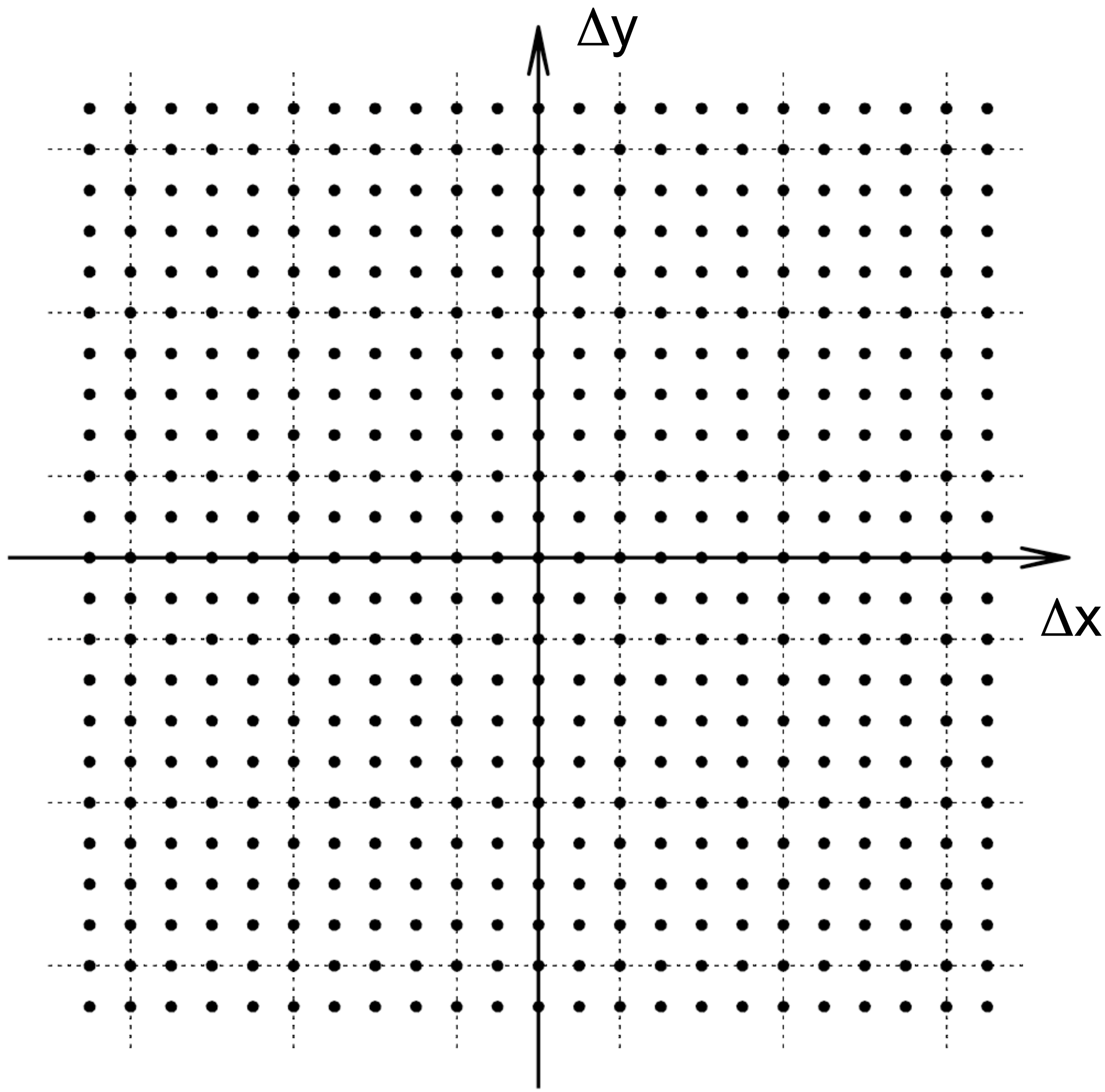


- **Examine samples**
- **Grid model**
  - “fiducial” spots
  - Interpolating

Anderson & King 2000 PASP

The model of $\psi_E(\Delta x, \Delta y)$

- Tabulated values of ψ_E at this array of points across its domain.



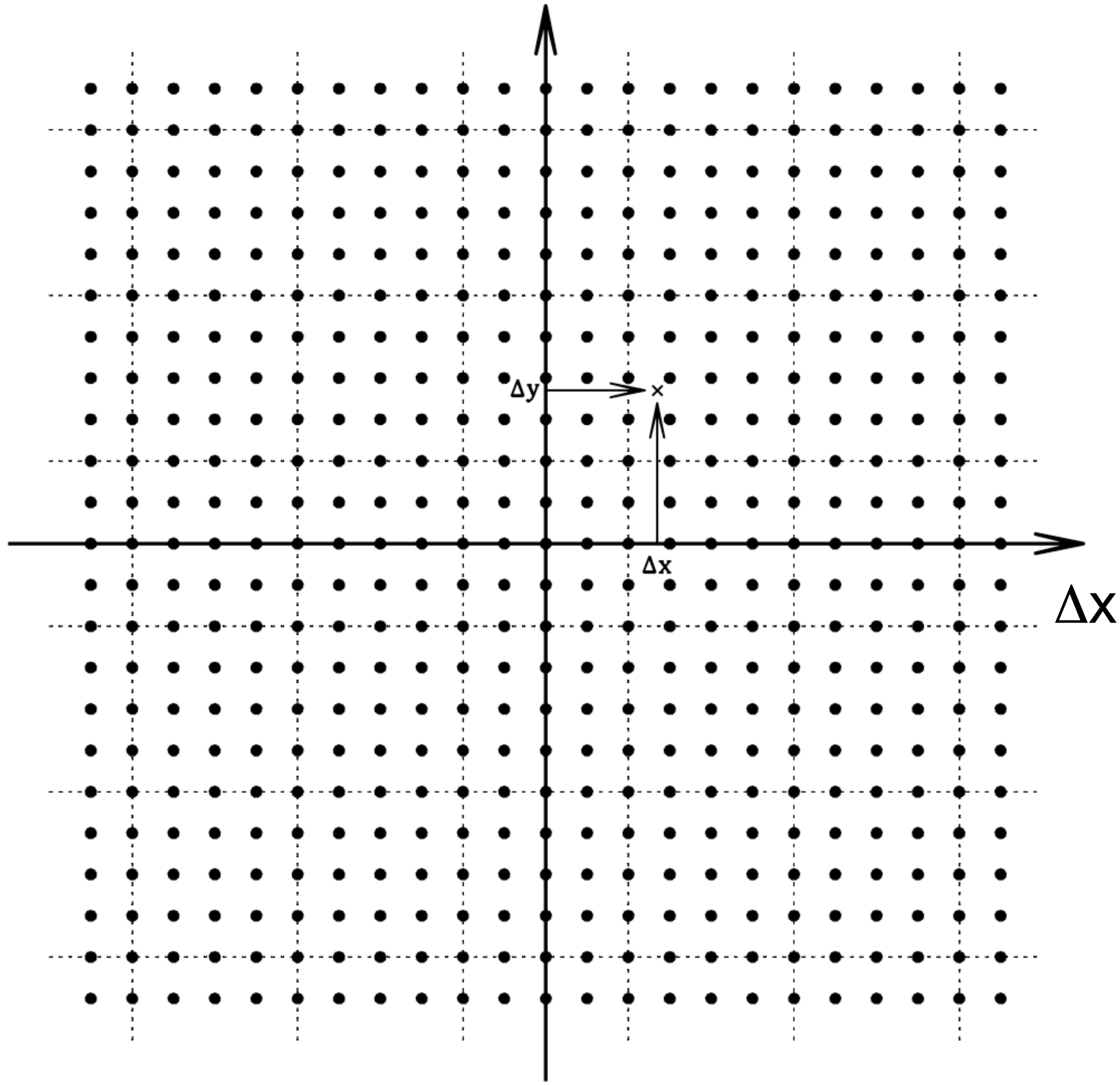
How to use $\psi_E(\Delta x, \Delta y)$

Need to know:

"What fraction of light should land in a pixel, if the pixel is centered at (Δx, Δy) relative to the point source?"

Need to interpolate:

→ Use bi-cubic interpolation



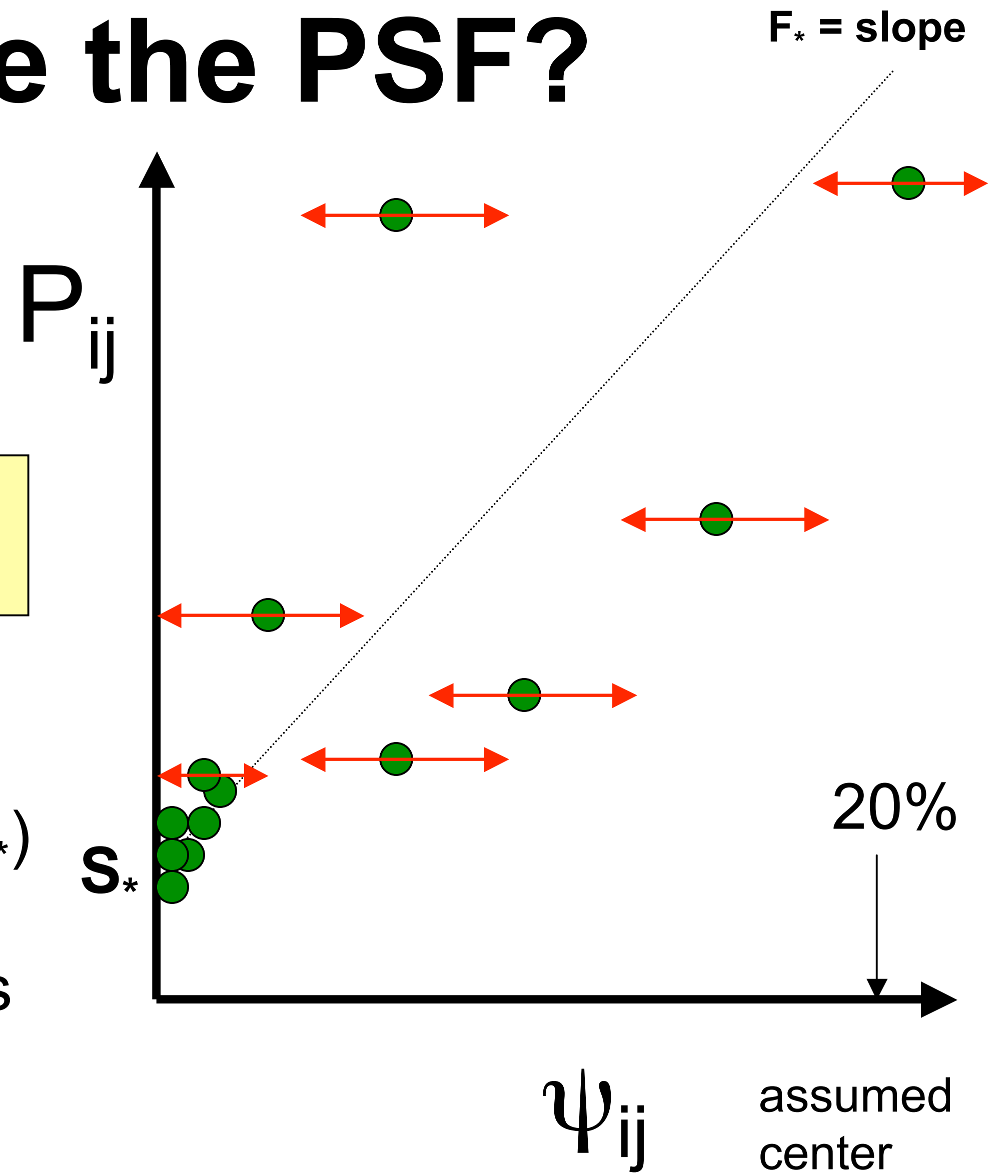
1) How to find the PSF?

2) How to use the PSF?

Fitting for Flux and position:

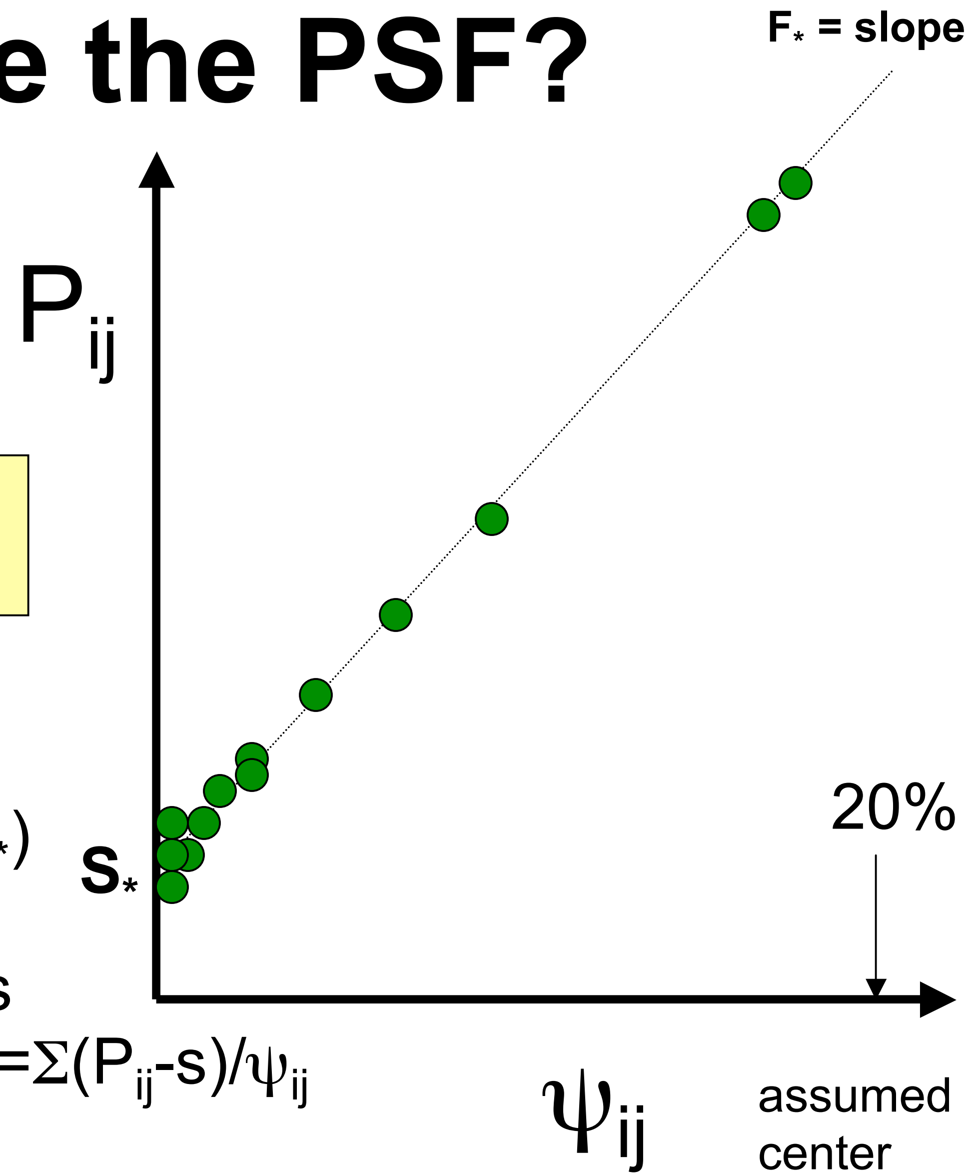
$$P_{ij} = S + F_* \times \psi_{EFF}(i - x_*, j - y_*)$$

- Nice, linear equation!
  - P in S_*, F_*, not (x_*, y_*)
- Which pixels to use?
- Sky from outer annulus
- For given (x_*, y_*), get F_*
- Find optimal (x_*, y_*)



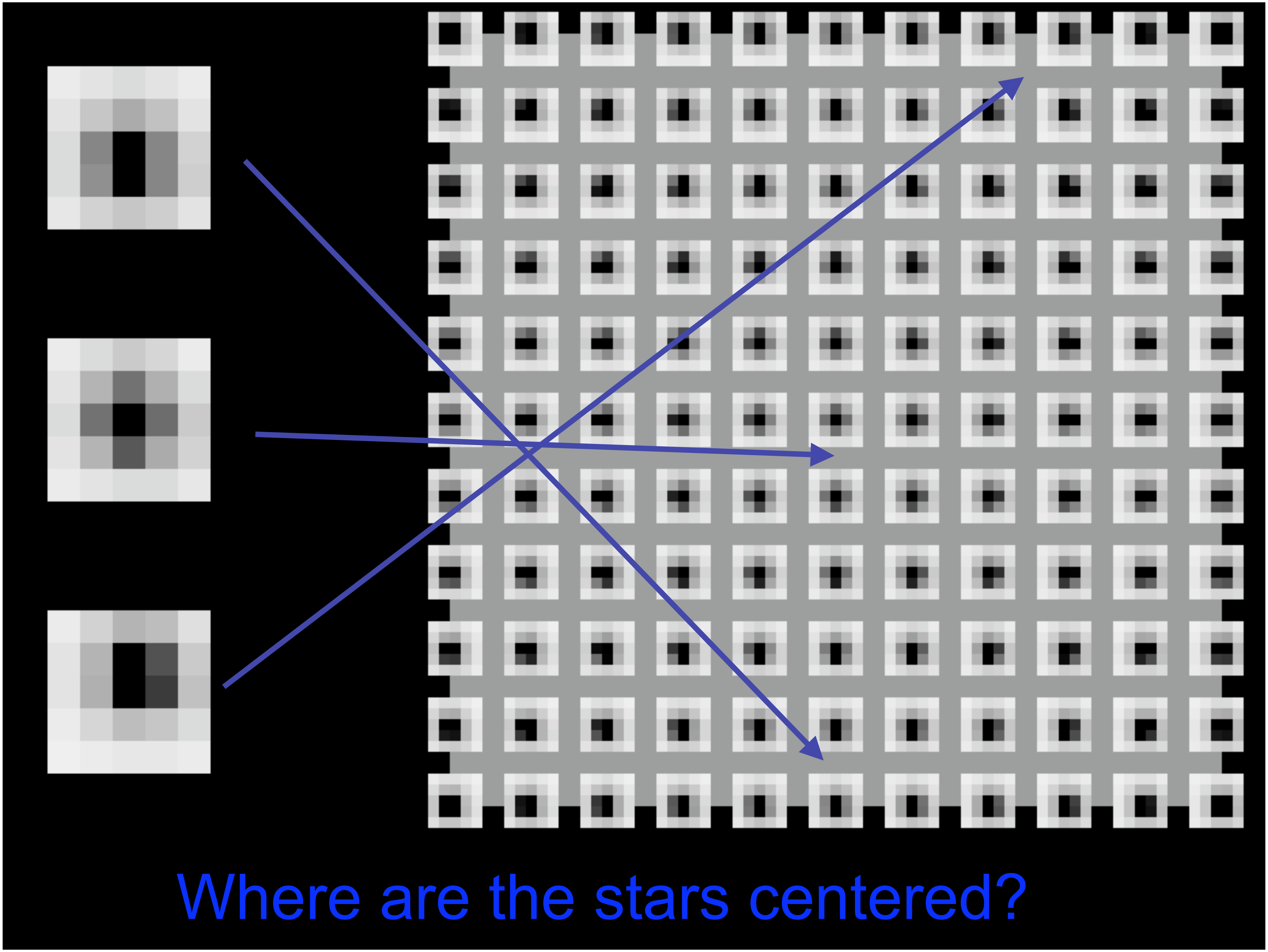
1) How to find the PSF?
2) How to use the PSF?

Fitting for Flux and position:



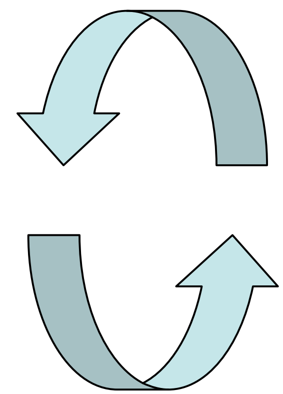
$$P_{ij} = S + F_* \times \psi_{EFF}(i-x_*, j-y_*)$$

- Nice, linear equation!
  - P in S_*, F_*, not (x_*, y_*)
- Which pixels to use?
- Sky from outer annulus
- For given (x_*, y_*), get $F_* = \Sigma(P_{ij} - s)/\psi_{ij}$
- Find optimal (x_*, y_*)


Where are the stars centered?

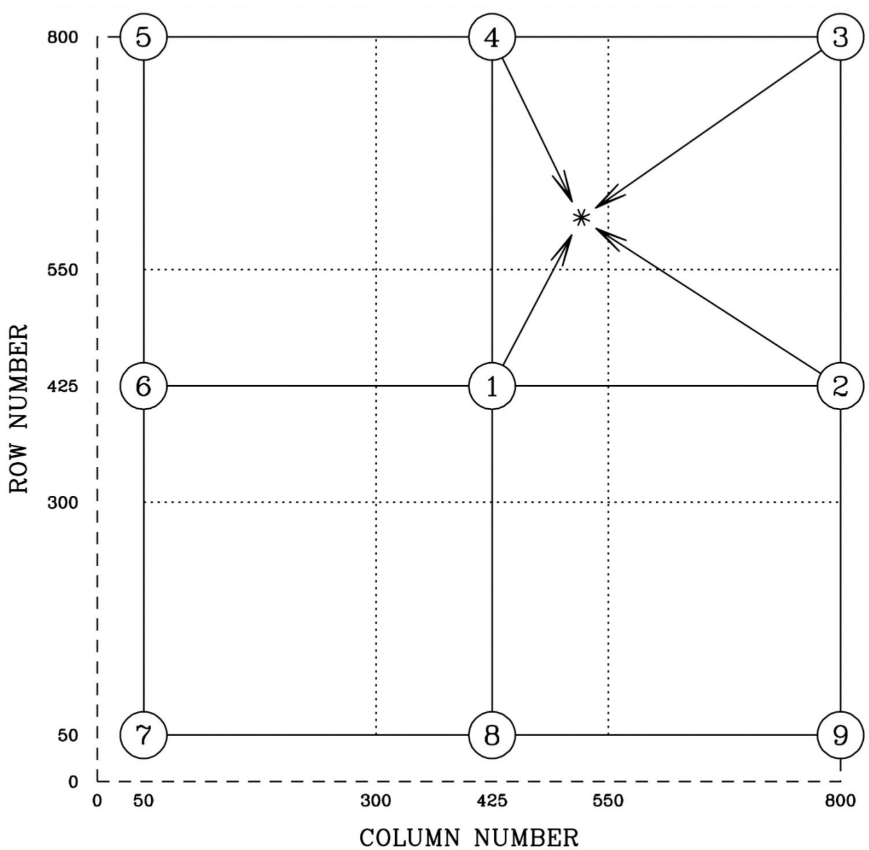
PSF: Finding -vs- Using

- **Degeneracy**:
  - Finding ψ_{EFF} requires (x,y,f)
  - Finding (x,y,f) requires ψ_{EFF}
- **Iteration**
  - Dithers break the degeneracy!

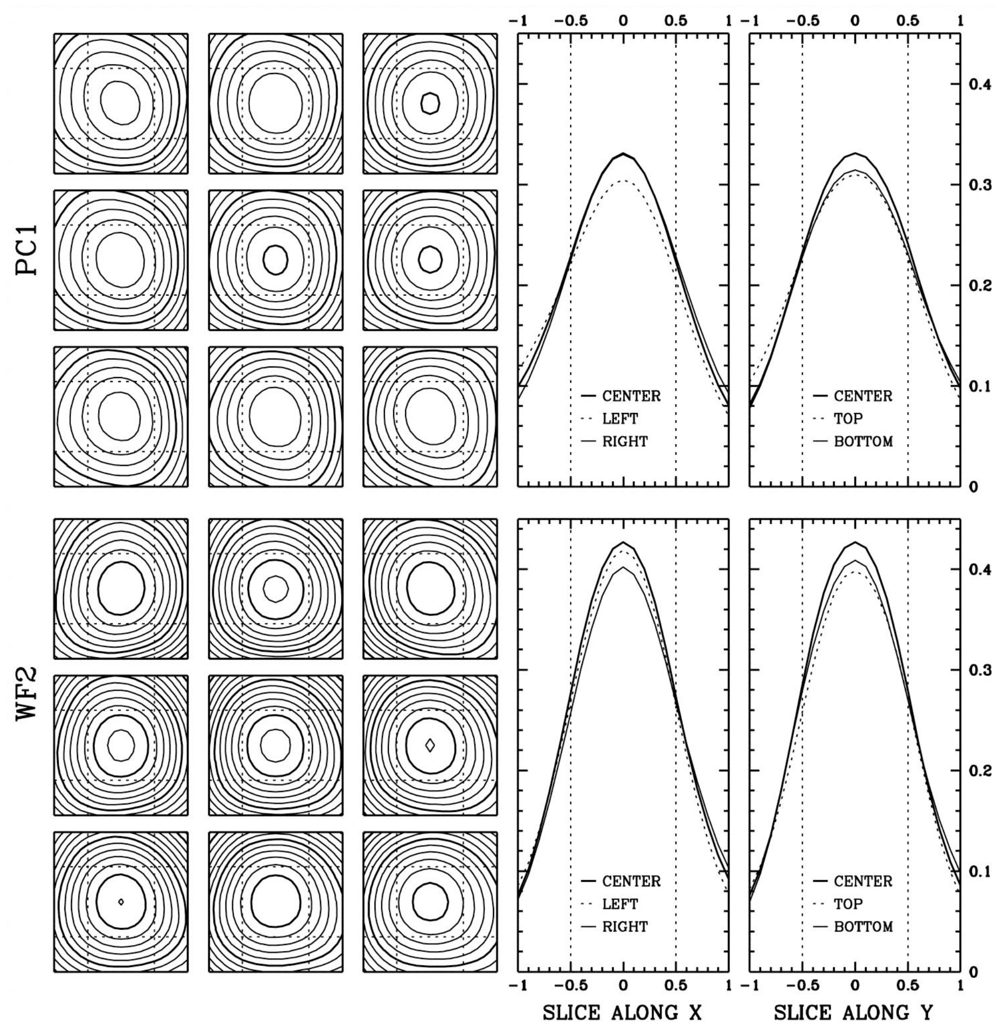


Higher-Level PSF Issues…

- **Spatial variability…**



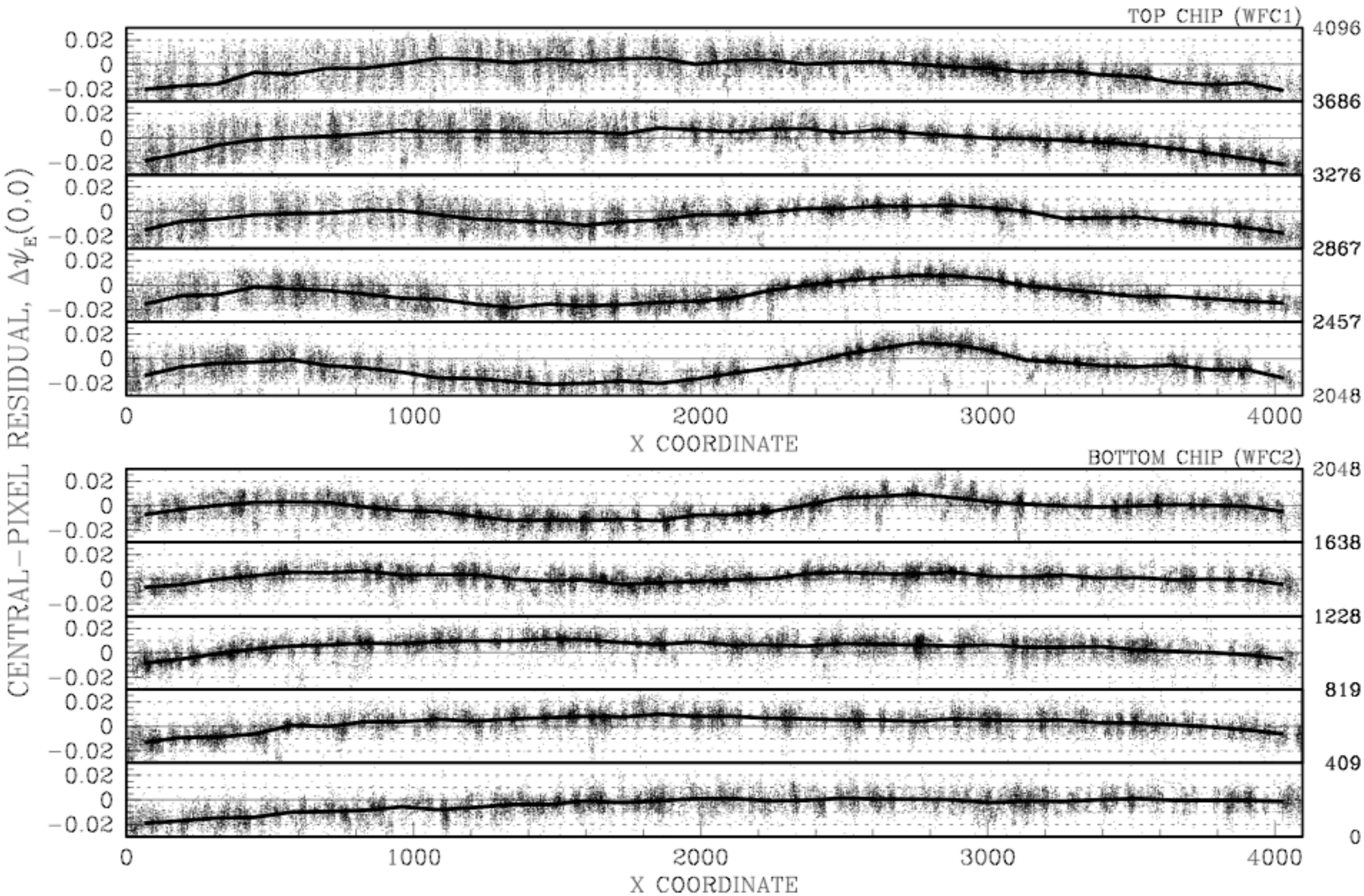
Array of Fiducial PSFs for WFPC2



Higher-Level PSF Issues…

- **Spatial variability…**

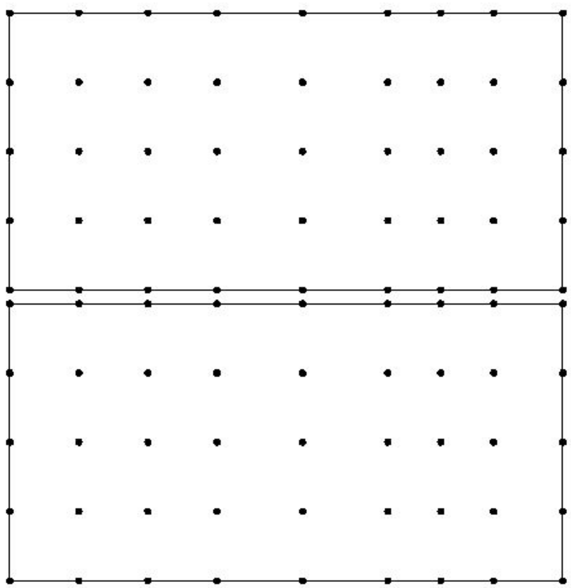
Central Pixel for F606W ACS PSF

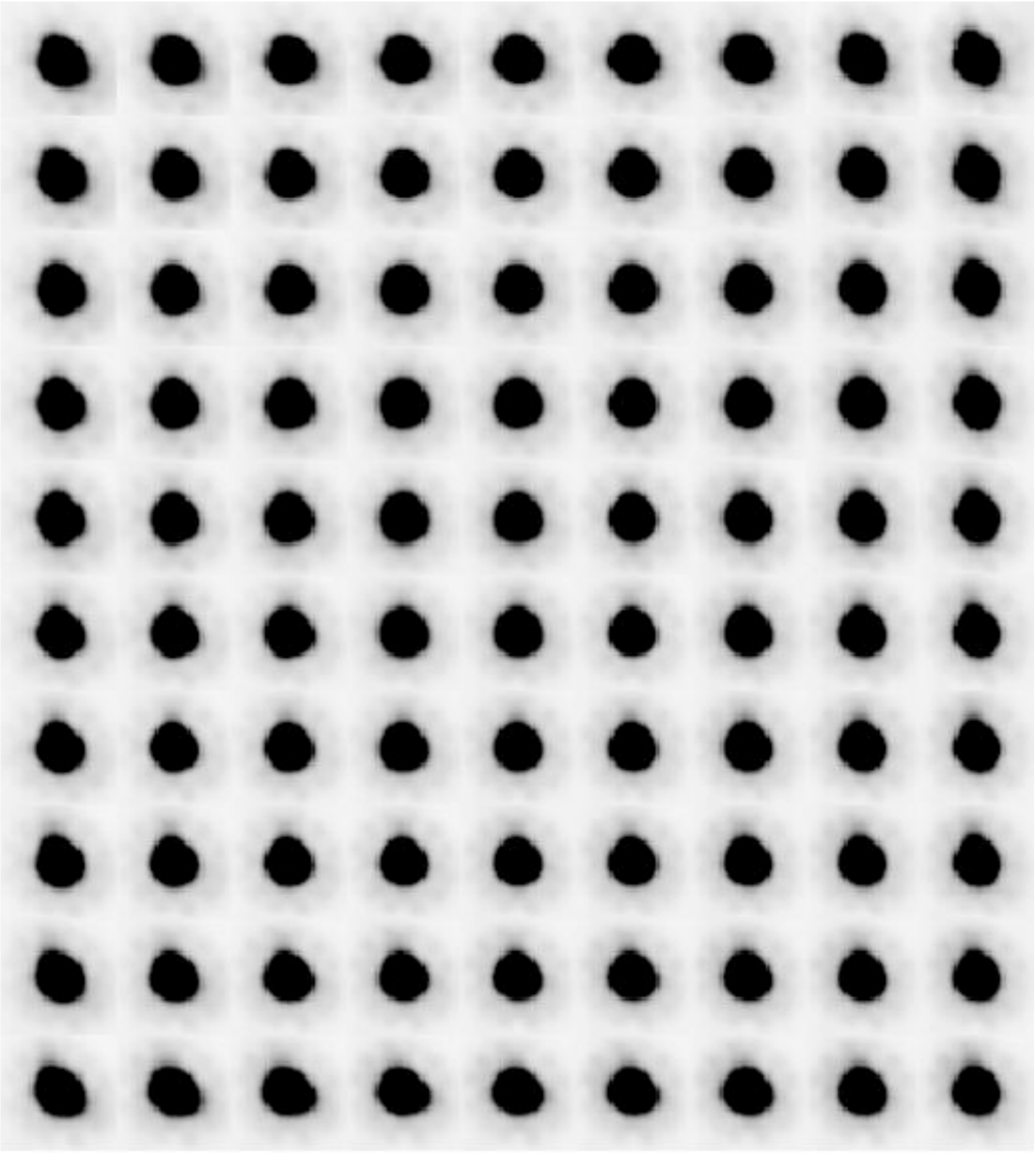


Higher-Level PSF Issues…

- Spatial variability…

Array of PSFs for F606W ACS

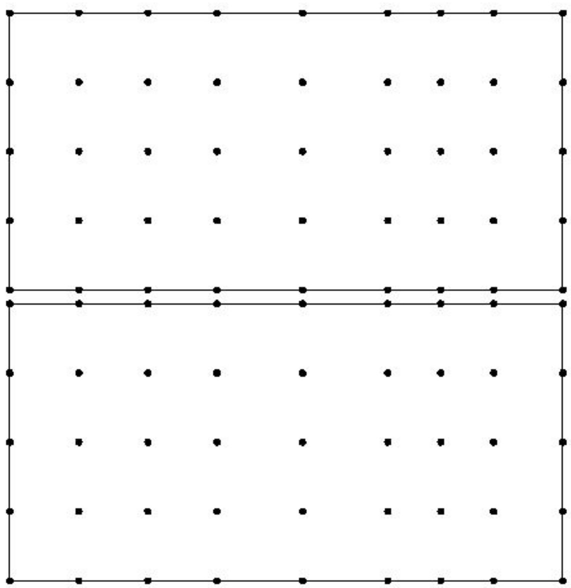


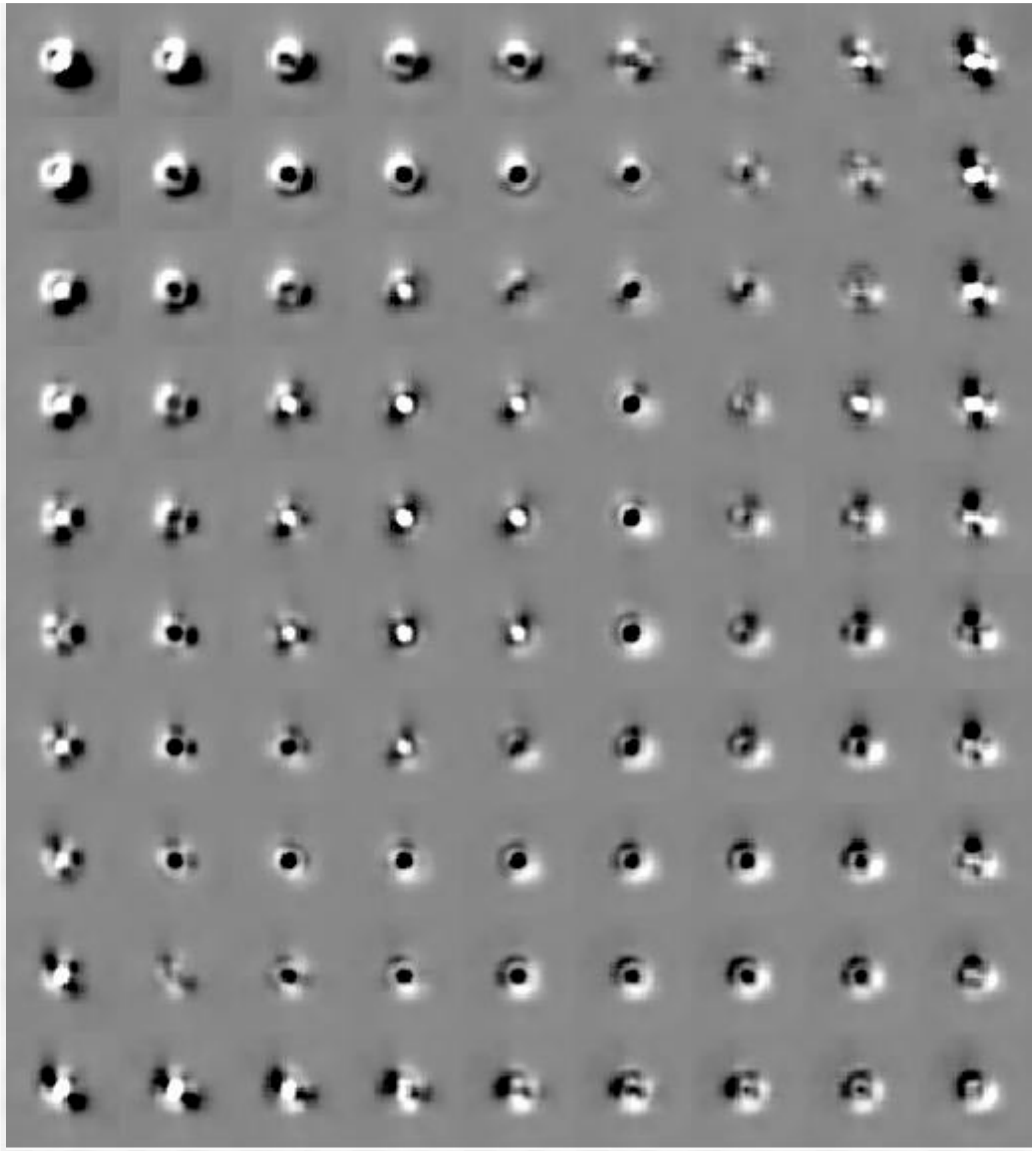


Higher-Level PSF Issues…

- Spatial variability…

Core intensity varies by ±10% over scales of ~500 pixels.

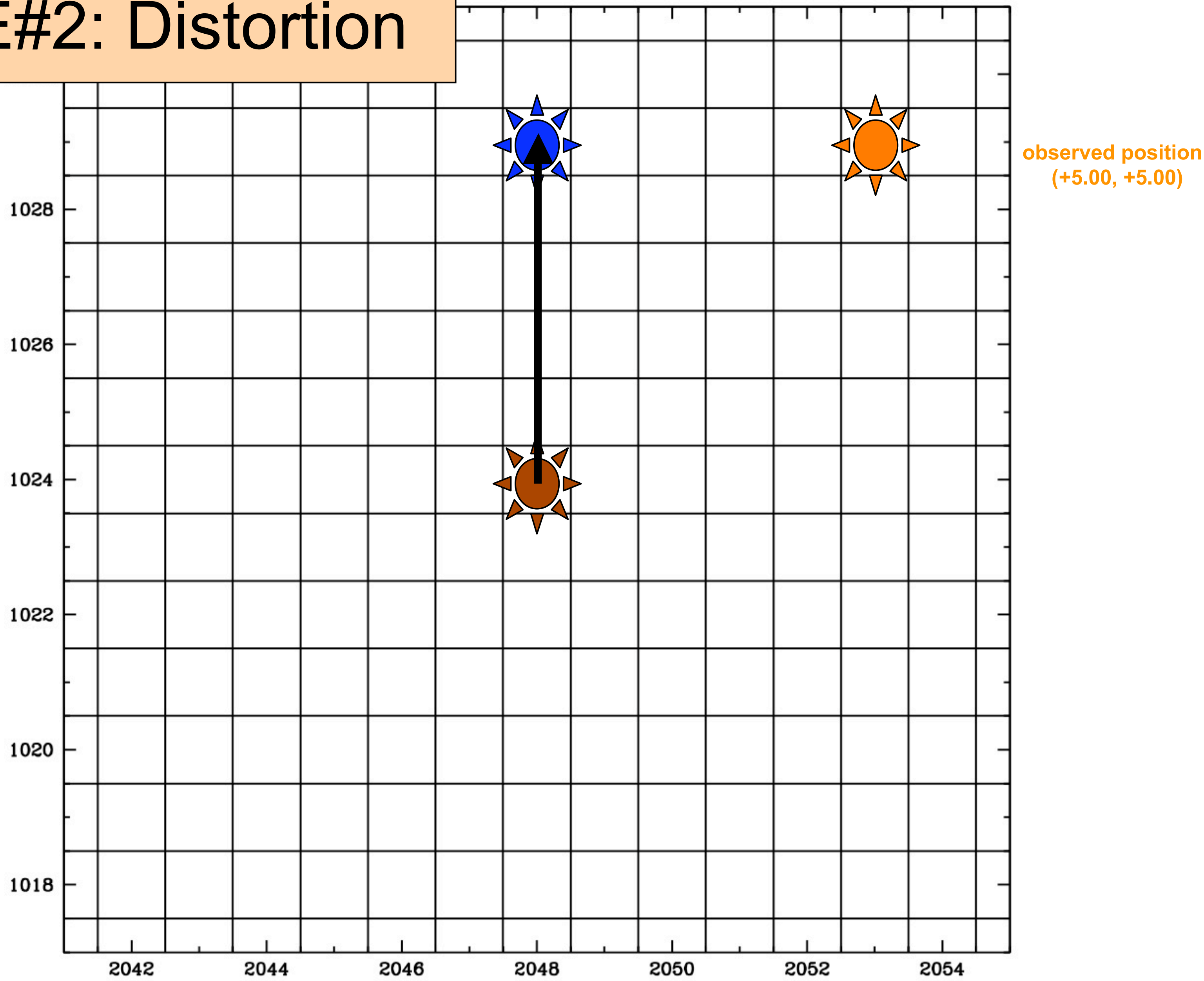


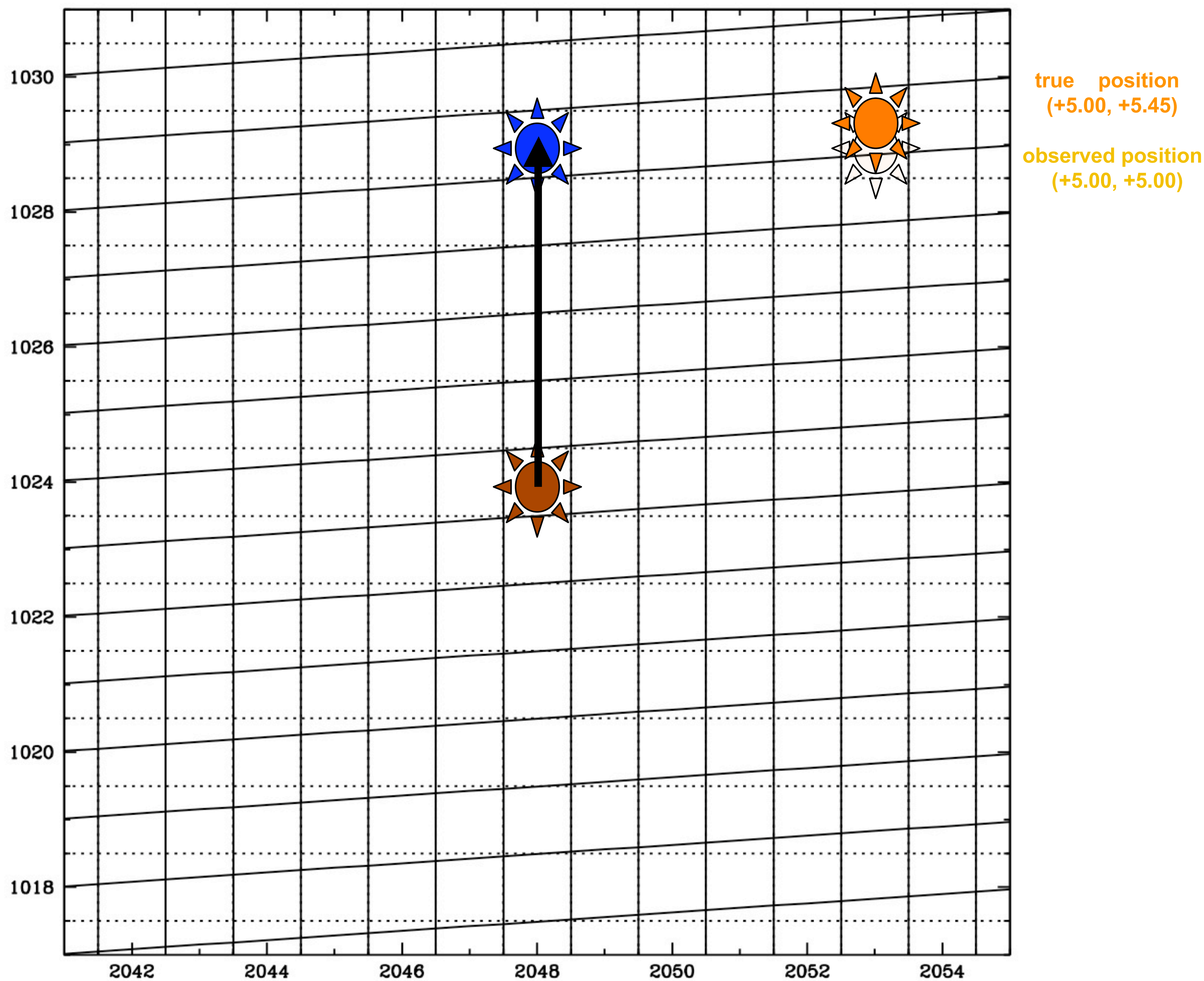


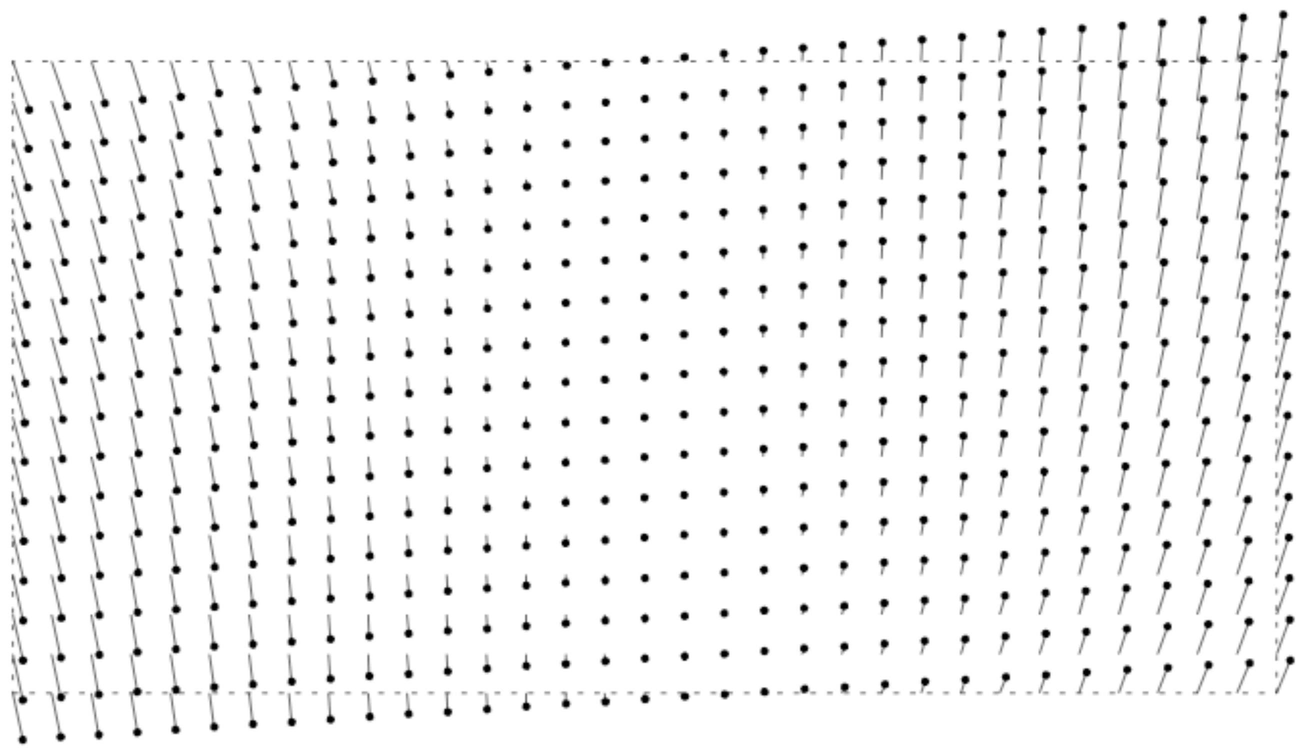
Higher-Level PSF Issues…

- **Spatial variability**
- **Time variability**
  - Breathing: +/- 2%
  - Not enough stars per exposure for full PSF extraction
  - Hybrid models:
    - PSF(x,y;t) = PSF(x,y) + PSF(t)
    - Good for ACS, not great for UVIS
  - Long-term variability (ACS)
- **Color variability:** ~0.002 pixel (extreme: 0.02 pixel)
- **How to define "center" ?**
  - Peak? Centroid? Point of Symmetry?
  - Cross-talk with distortion
- **Pixel-response function:** $\Pi(\Delta x,\Delta y)$
  - Total flux may depend on pixel phase
  - NICMOS, WFPC2, not others…
  - If flat, constraints on PSF…

ISSUE#2: Distortion




1030
1028
1026
1024
1022
1020
1018
2042
2044
2046
2048
2050
2052
2054
true position
(+5.00, +5.45)
observed position
(+5.00, +5.00)



WFC/ACS DISTORTION

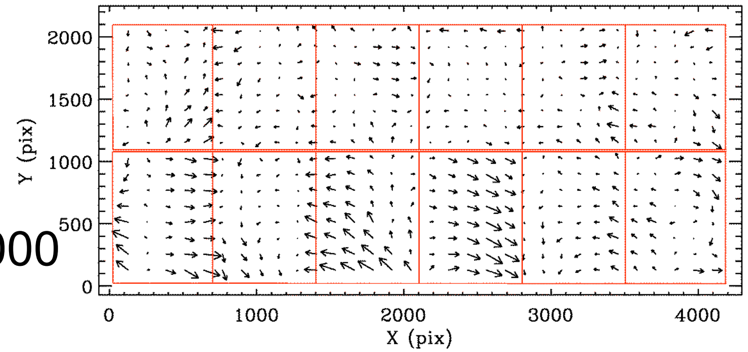
Dealing with Distortion

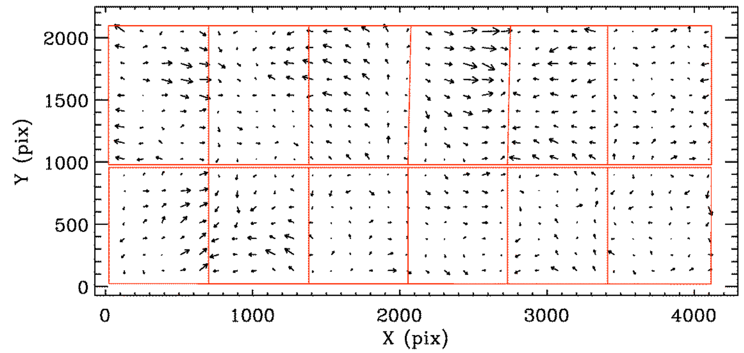
- **Why?** Fewer reflections, better throughput
- **How to solve for and remove?**
  - → **Easy way:** astrometric reference frame
    - → Instant solution
    - → Need: Depth, precision-match, PMs
      flat distribution, good S/N
      - UVIS: ω CEN: PMs
      - JWST preparation: LMC field
  - → **Hard way:** self-calibration
    - → Very often necessary
    - → Large dithers / multiple orients
      - Same stars, different places on detector
      - Simultaneous solution
- ***What* solution?**
  - → (x,y,S,θ) arbitrary ("conformal transformations")
  - → Must "choose" a frame → choose a convenient one

Sources of Distortion

1) Geometric optics:

- Linear-vs-higher order
  - Linear “skew”: 500 pixels over 2000
    → Parallelogram pixels
  - Non-lin: 50 pixels over 2000
- Why? Minimize reflections





UVIS

2) Filters introduce distortion

- Offsets, scale changes
- “Fingerprint” of ~0.05 pixel

3) Detector “stitching” defects

- WFPC2: every 34.1333th row 3% shorter
- ACS/WFC: pattern every 68.2666th column
- WFC3/UVIS: 2-D zones

4) CTE losses...

- ACS Solution now available

Need empirical approach…
Plot everything against everything else…

ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3…
Transformations

- **All HST astrometry is differential astrometry**
  - → Guide-star precision ~ 0.5″ (improved from 1.5″!)
  - → No reference stars in typical field
  - → We never know the true pointing

- **Always need to define a *local* reference frame**
  - → Pixels/positions have only relative meaning.
  - → Choosing a frame
    - → Base it on a population of objects (3+) in the frame
    - → Must know *a priori* something about them
      - → absolute $\mu = 0$ (galaxies)
      - → average μ = same (clusters)
      - → average μ = unchanging (field)
    - → Frame is specified by positions in it
    - → Often choose a convenient frame that is close to RA/Dec…

ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3…

Transformations

- **Least-squares linear transformations**
  - → Have a list of N "point" associations: $(X_n, Y_n ; U_n, V_n)$
  - → General: 6 parameter ; "conformal" 4-parameter
  - → Form: (X,Y) → (U,V)

$$\begin{pmatrix} U - U_0 \\ V - V_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \end{pmatrix}$$

  - → (X_0,Y_0) or (U_0,V_0) is arbitrary ; can be centroid $X_0 = \sum X_n/N$
  - → Solution: $A = \frac{\langle ux\rangle\langle y^2\rangle - \langle uy\rangle\langle xy\rangle}{\langle x^2\rangle\langle y^2\rangle - \langle xy\rangle^2}$, where $u = U - U_0$; similar for C,D

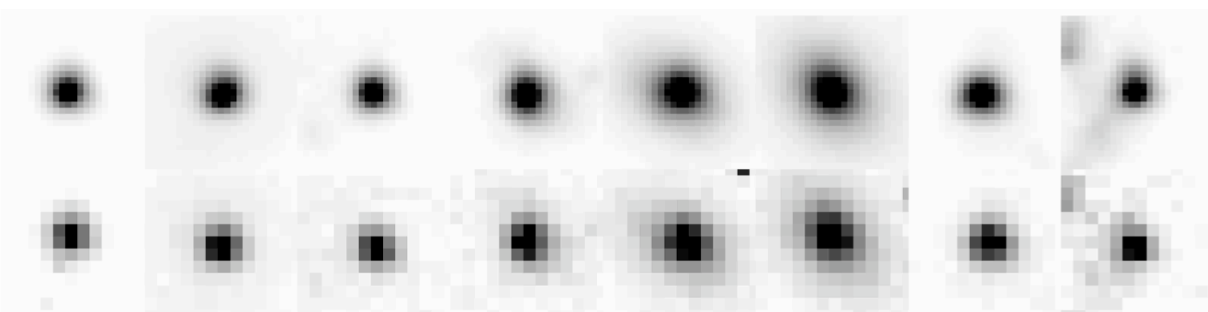
$$B = \frac{\langle uy\rangle\langle x^2\rangle - \langle ux\rangle\langle xy\rangle}{\langle x^2\rangle\langle y^2\rangle - \langle xy\rangle^2}$$

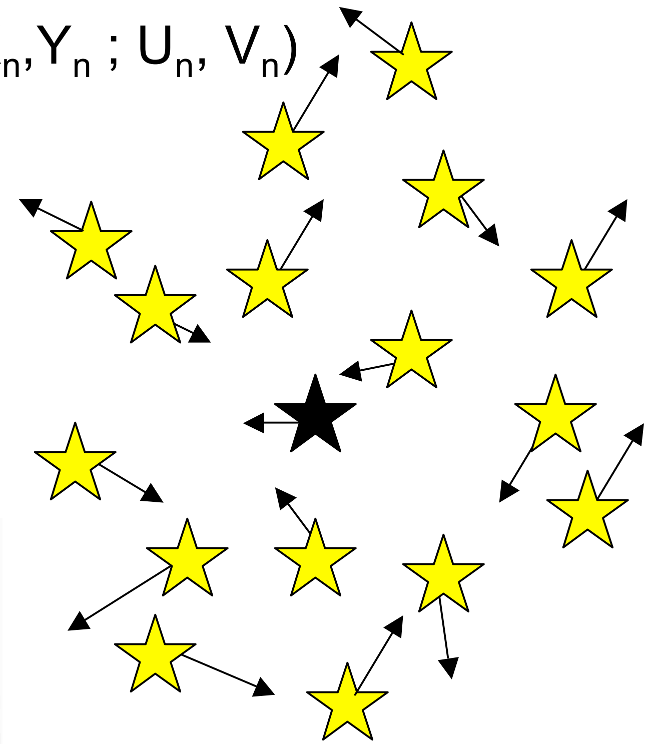
  - → Allows comparison of positions in different frames

ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3…

Transformations

- **Errors in the transformations**
  - → "Point" associations are not perfect: $(X_n, Y_n ; U_n, V_n)$
    - Measurement error
    - Proper motions (dispersion)
      - → random + systematic
    - "Fuzzy handles" for galaxies
  - → Distortion not perfectly removed
    - Make transformations more local





$V_{SYST} = \sigma/\sqrt{N}$

ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3: Transformations

Good News: All manageable issues

Undersampling/PSFs:

→ Ways to model accurately, get 0.01-pixel positions

→ Libraries available, usually sufficient

Distortion:

→ Stable, modelable, small variations, ~ 0.01 pixel

Transformations:

→ Can optimize for program

Bad news:

No one size-fits-all solutions…

Astrometric Science with HST…

1) **Bulk motions**

   Membership: WDs, CVs, binaries, unusual stars…

2) **Absolute motions**

   Clusters, field stars, rotations, even other galaxies

3) **Internal motions in clusters**

   IMBHs? Absolute distances, internal dynamics

4) **Individual stars**

   Parallax, SN-progenitor ID,

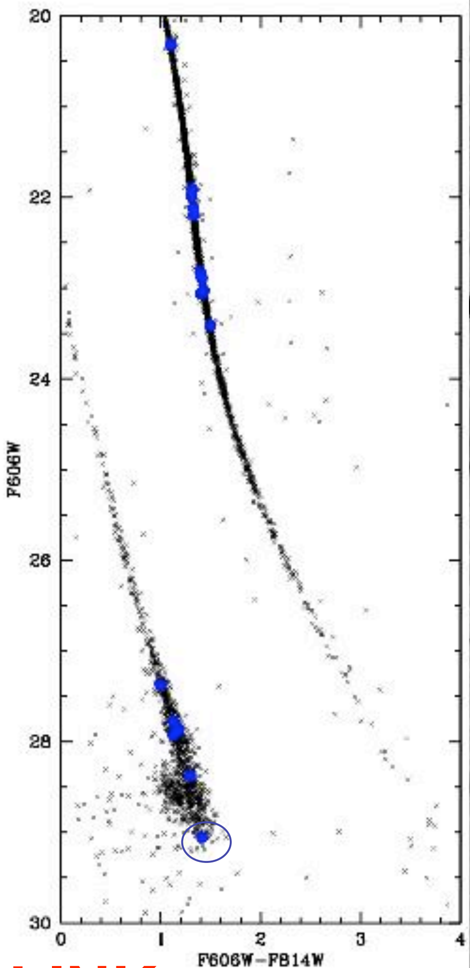
   microlensing applications

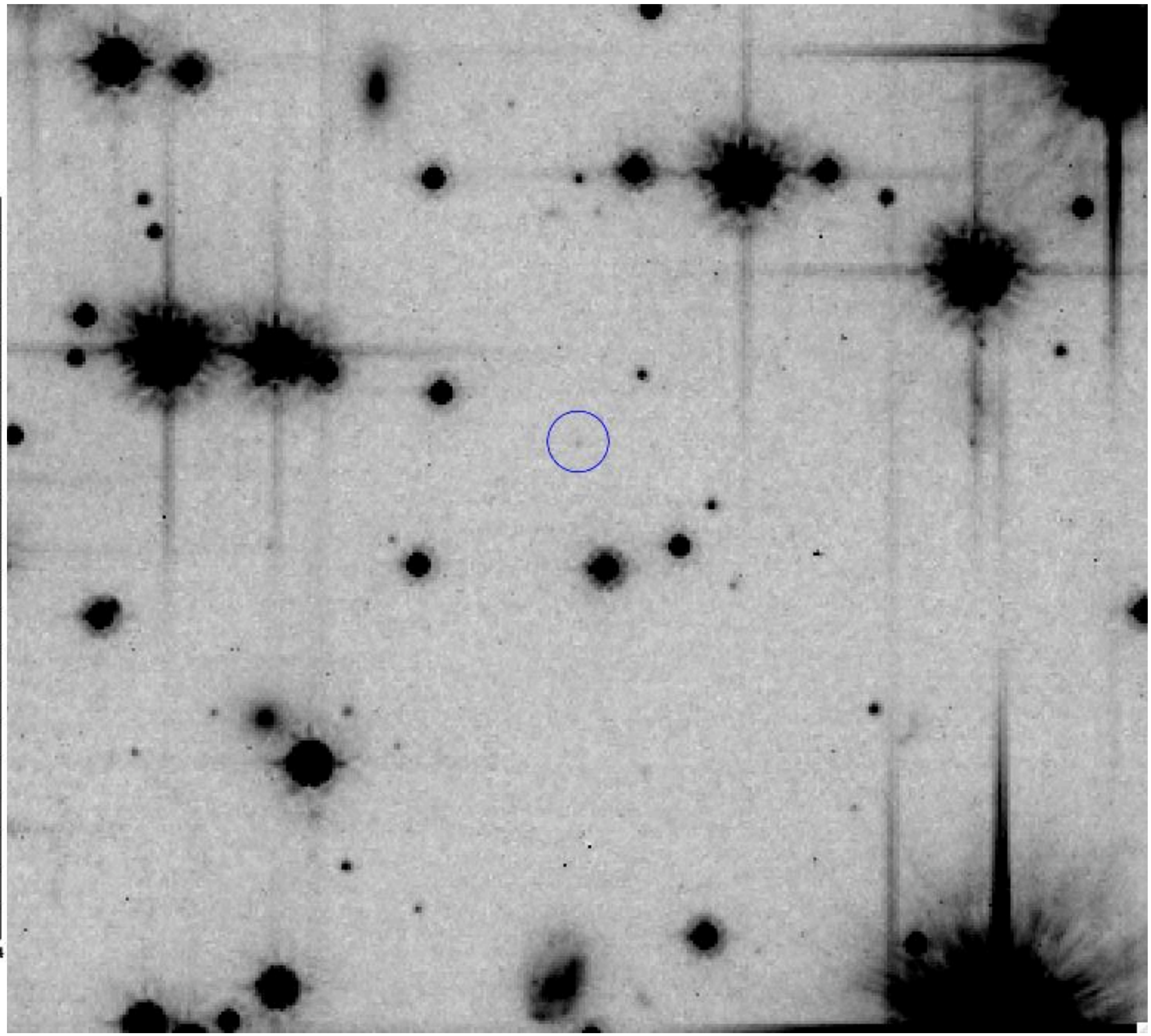
1) Bulk motions:

NGC6397

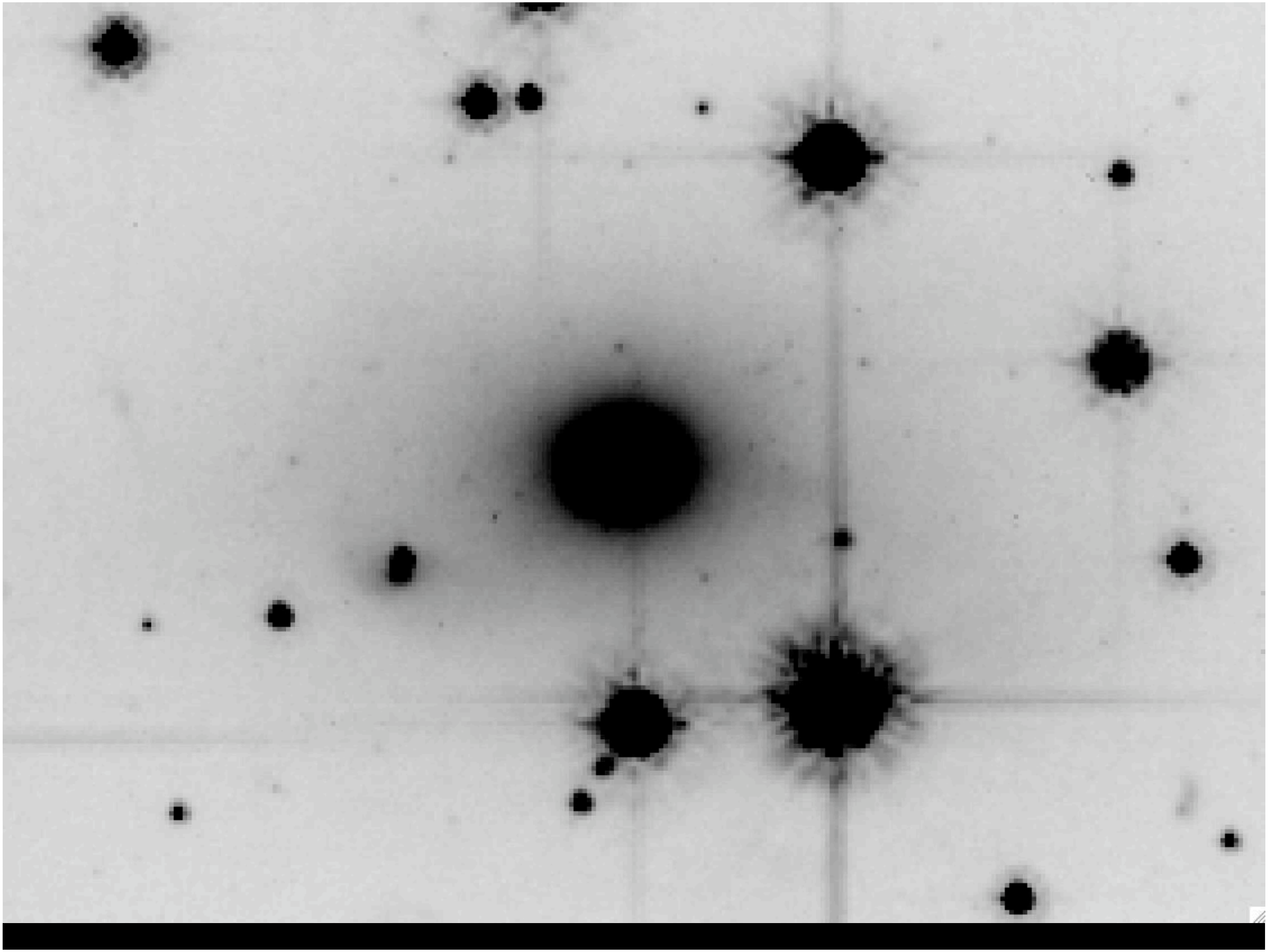
PI-Rich, UCLA

Proper-Motion Cleaning



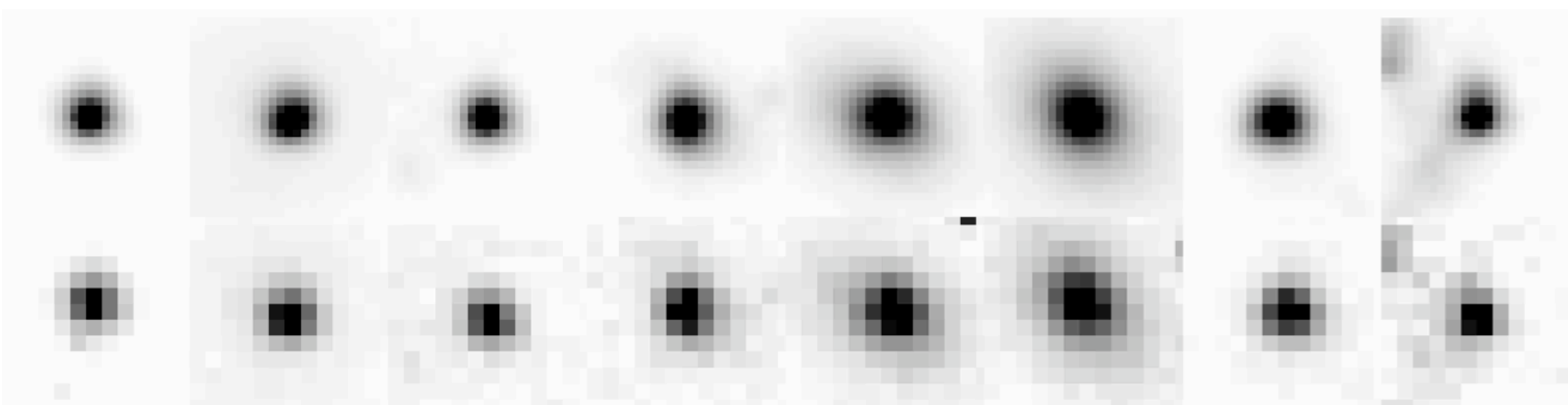


BLINK



2) Absolute Proper Motions

- **Challenge**: measure stars relative to galaxies
  - Regimes: HST -vs- ground
  - Challenge: Galaxies not PSFs → "GSFs"
- **Several projects in the works**
  - Hyper-velocity stars (Brown et al 2010, Gnedin-PI)
  - Dwarf spheroidals
  - M31…

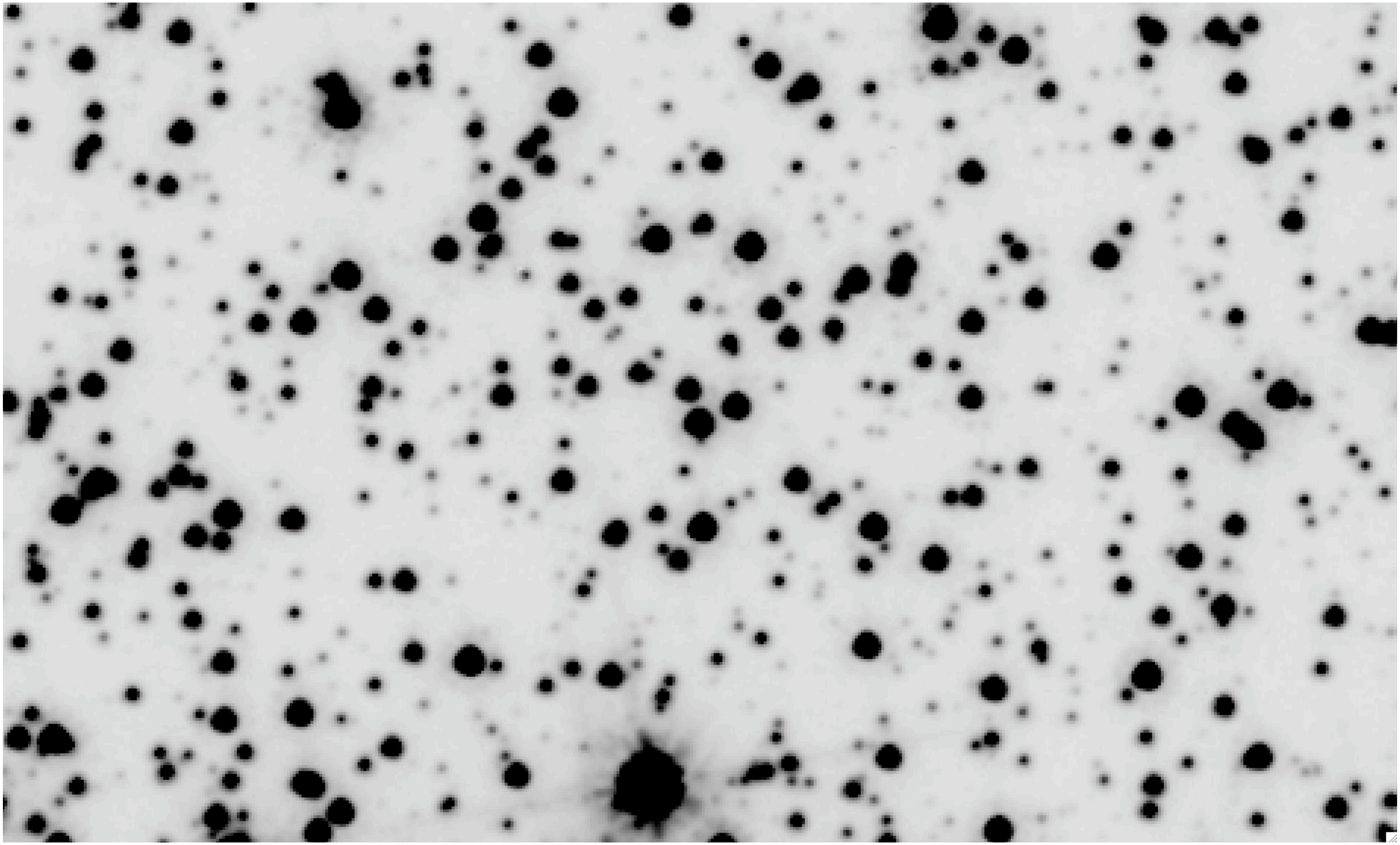


3) Internal cluster motions

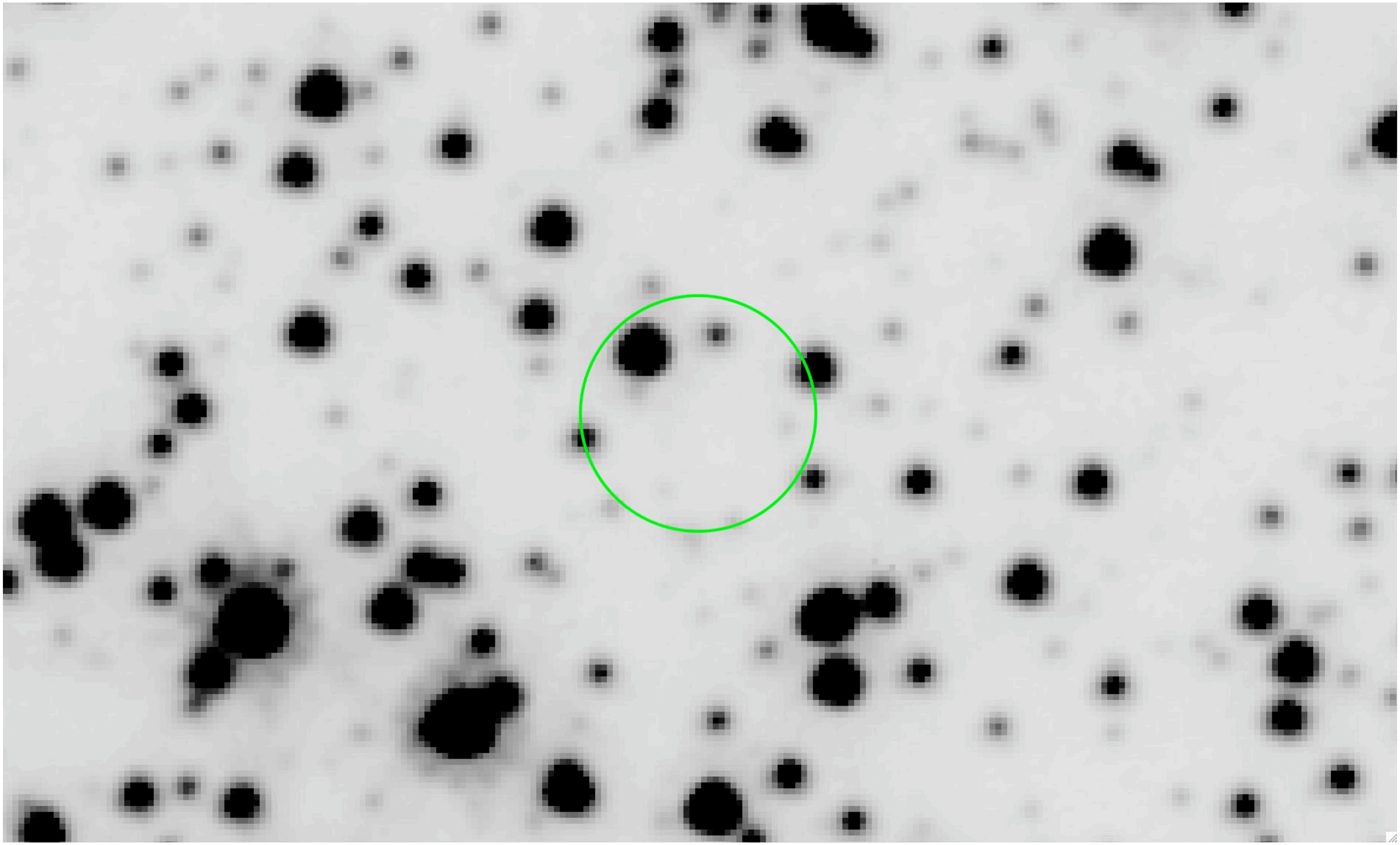
- Search for IMBHs
  - Are clusters little galaxies?
  - General rise, smoking gun
- Distances from dispersions: $D = k\ \sigma_V$ (km/s)/ σ_{PM} (mas/yr)
- Anisotropy
- Equipartition $\rightarrow \Psi(R)$
- General dynamical modeling
  - Higher moments of DF, etc.
  - Formation signatures?

FIRST CASE: Omega Cen

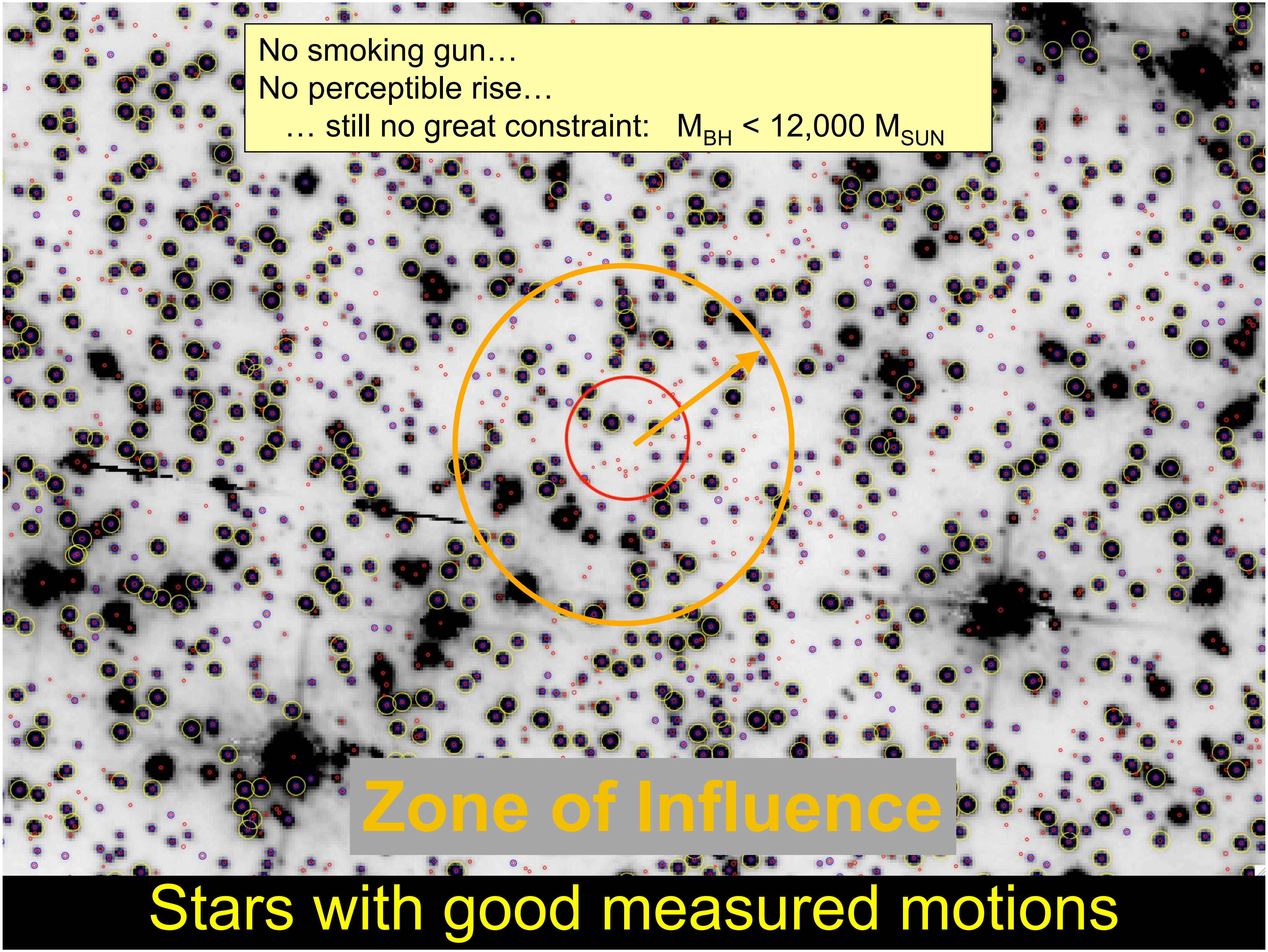
Omega Cen: a Ground-Based Image (Lehman)



A typical field star…



Motions at the center


No smoking gun…
No perceptible rise…
… still no great constraint: M_{BH} < 12,000 M_{SUN}
Zone of Influence
Stars with good measured motions

4) Individual stars

- **The Challenge:**
  - Defining the reference frame
  - Parallax/Motions/Positions
- **Microlensing applications**
  - Color-dependent centroid shift (1st moment)
    - Color difference between lens/source → μ
  - Deblending (measure 2nd moment)
    - Need exquisite PSF → μ
  - Astrometric signature of lensing (mass of lens)
    - Accepted 3-cycle proposal: BHs, NSs, etc (PI-Sahu)

Summary:
HST Imaging Astrometry

- **Technical aspects**
  - Positions good to 0.01 pixel = 0.5 mas per image
  - Differential astrometry
  - Attention to PSFs, Distortion, Transformations
  - Should extend to other, non-HST missions

- **Scientific possibilities**
  - Open/Globular Clusters, Field stars, absolute PMs/orbits
  - Microlensing: constraints to break degeneracy