Mass-Radius Relationships for Solid Exoplanets

Sara Seager

Massachusetts Institute of Technology

Michelson Summer Workshop 2007

Image credit: NASA/JPL-Caltech/R. Hurt (SSC)

See Seager, Kuchner, Hier-Majumder, Militzer 2007

M-R for Solid Exoplanets

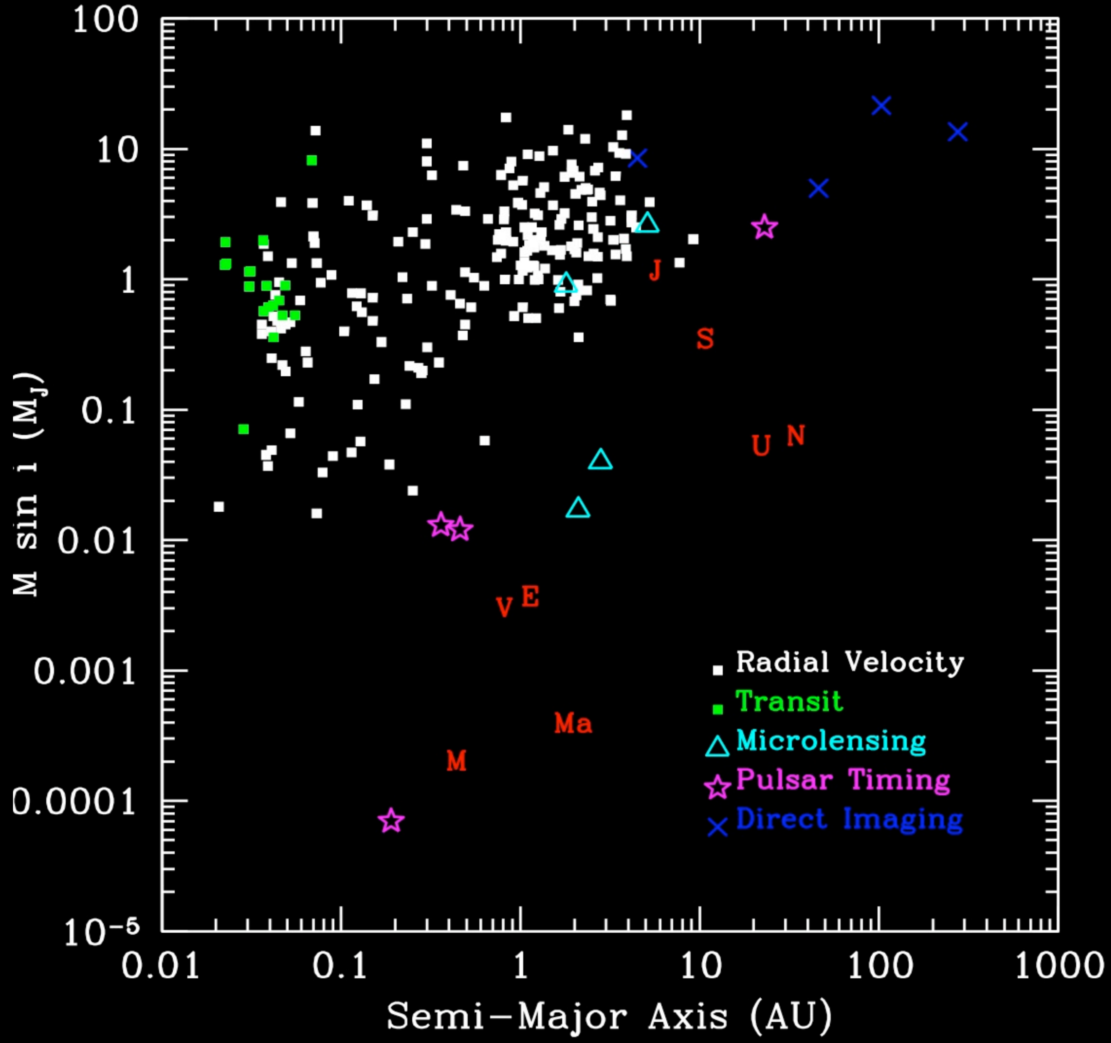
Overview

Equations of State

Approximations

Limitations and Degeneracies

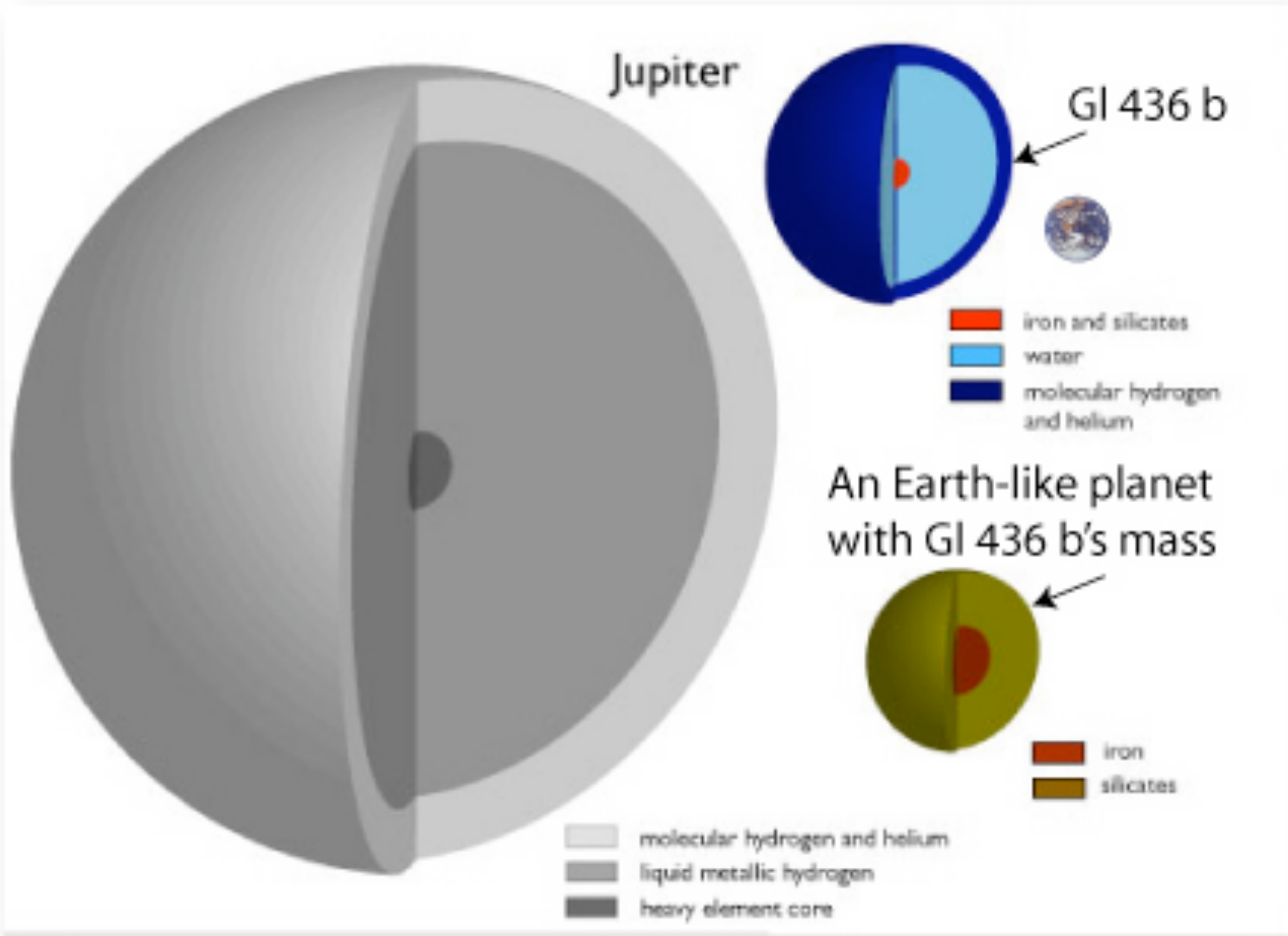
Known Planets



A huge diversity!

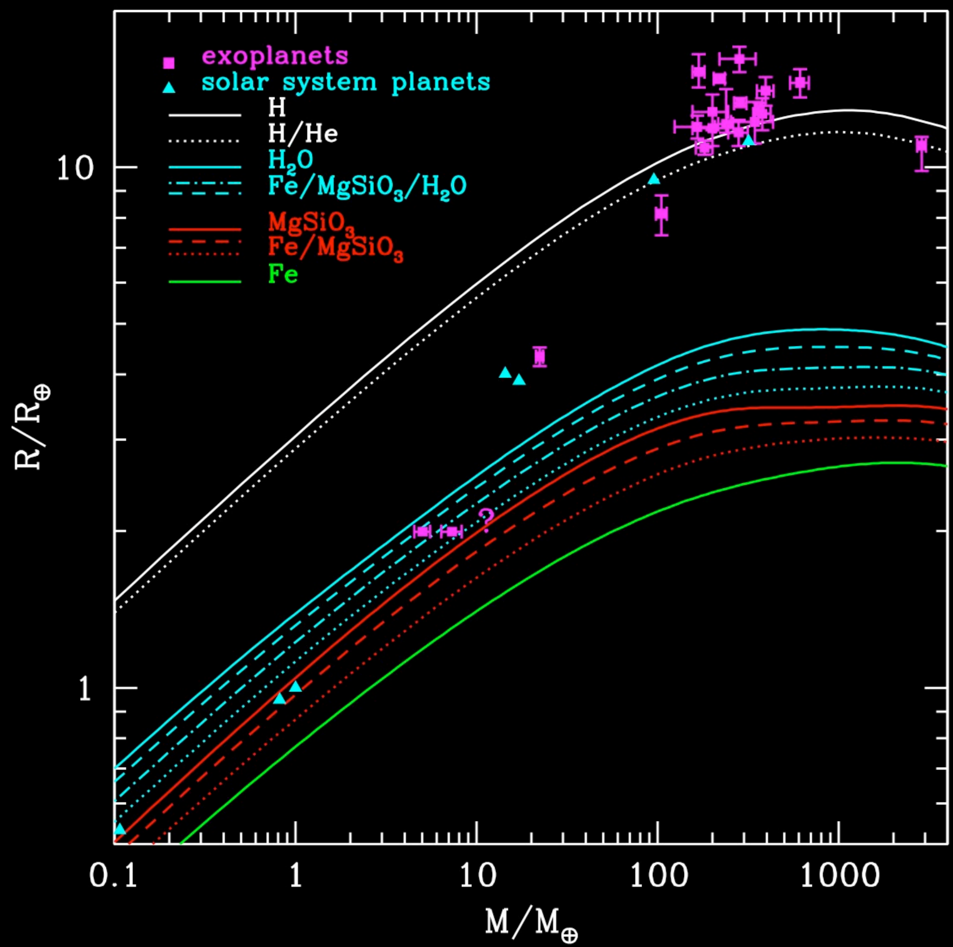
Based on data compiled by J. Schneider

Interior Composition



Courtesy G. Laughlin

Exoplanet Mass-Radius Relations



$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2}$$

$$\rho(r) = F(P(r))$$

Seager, Kuchner, Hier-Majumder, Militzer ApJ, 2007

Zapolsky and Salpeter 1969
Stevenson, 1982
See also: Valencia et al. 2006ab, 2007; Fortney et al. 2007; Sotin et al. 2007

We infer an exoplanet's bulk composition from its M and R

Mass-Radius Equations

Mass of a spherical shell

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equiilbrium

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2}$$

Equation of state

$$\rho(r) = F(P(r))$$

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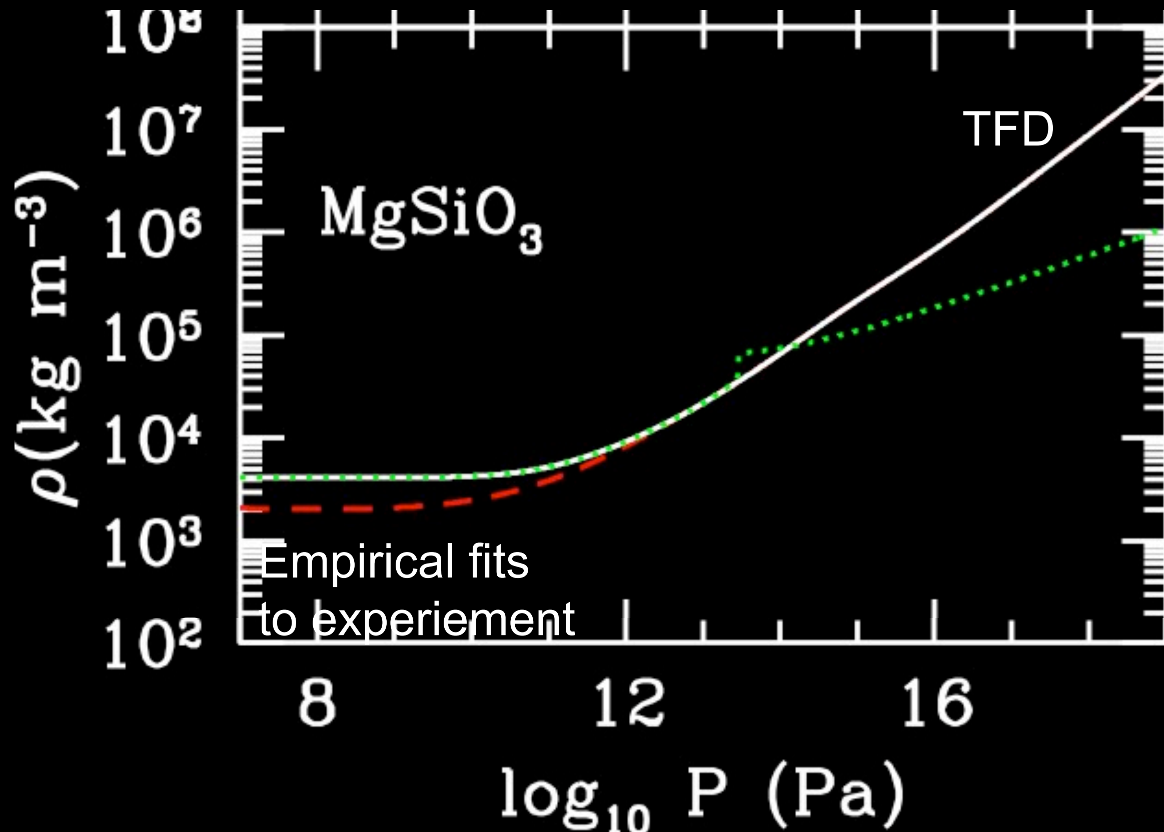
Equations of State

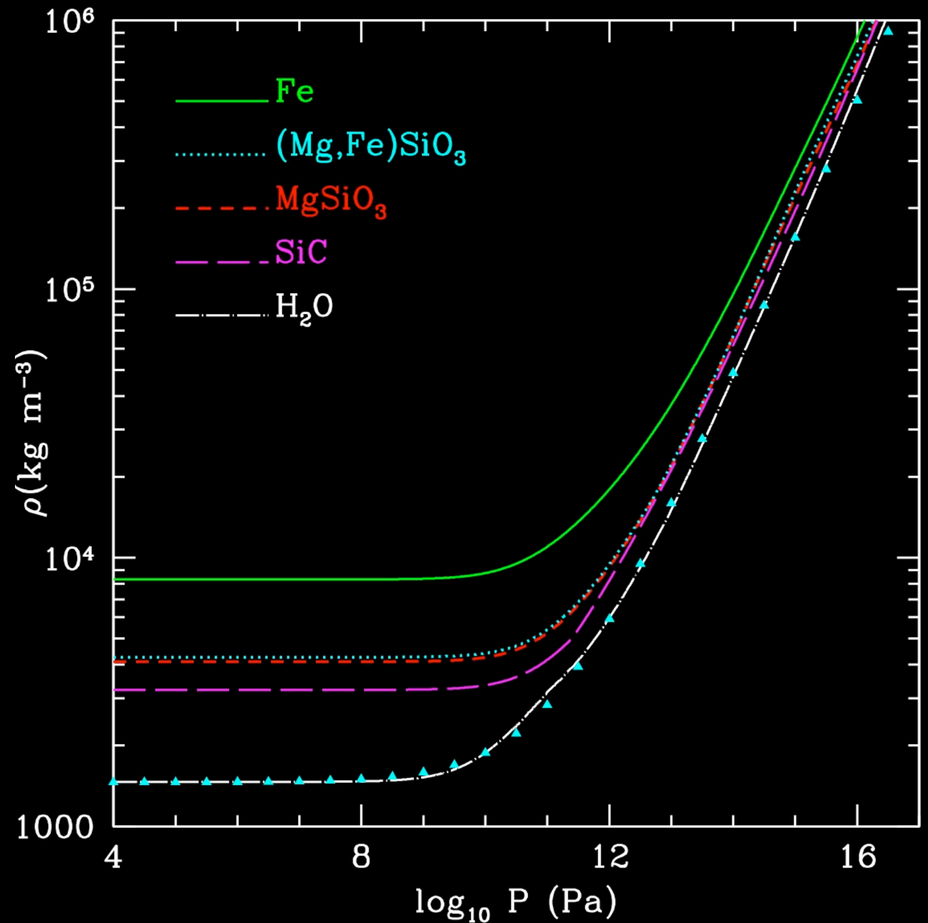


An EOS is a relationship between *P*, ρ, and *T*.

Static compression ~ P = 150 Gpa, T = 2000 K.

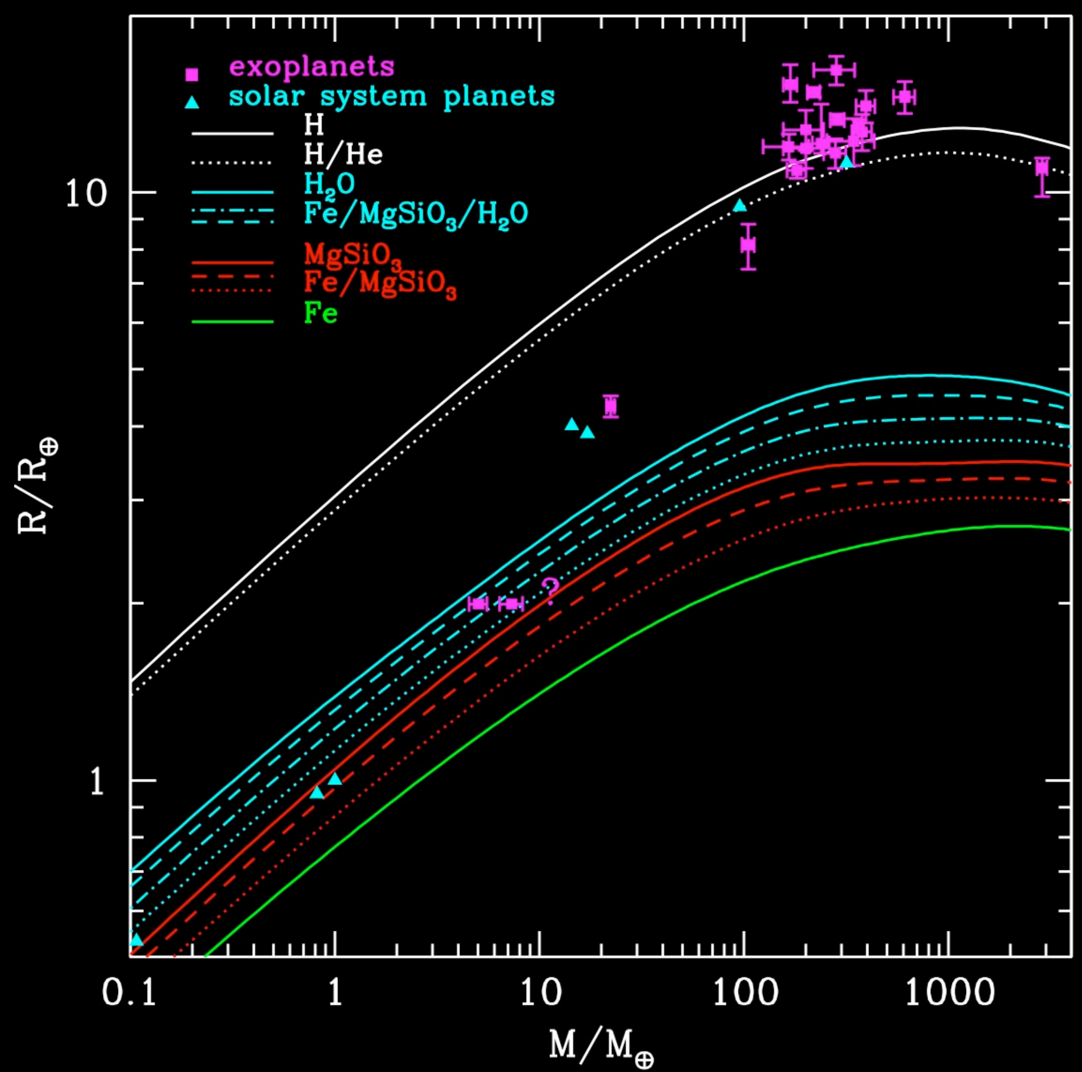
Shock experiments up to P = 10^4 Gpa …but high T!

Equations of State

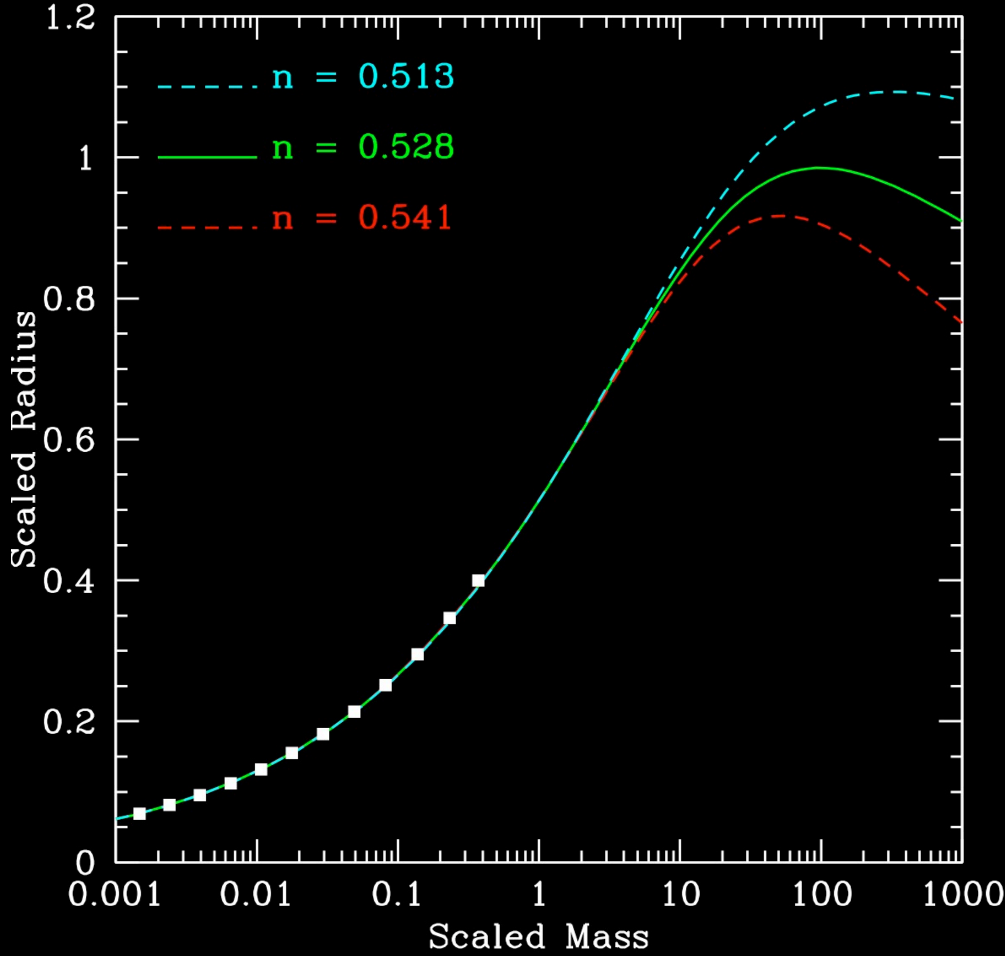


The EOSs for solid materials approximately follow $\rho = \rho_0 + cP^n$. A modified polytrope.

Generic Mass-Radius Relations



Generic Mass-Radius Relation



$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2}$$

$$\rho(r) = \rho_0 + cP(r)^n$$

$M_s << 1$

$$M_s = \frac{4}{3}\pi R_s^3 \left[1 + \left(1 - \frac{3}{5}n\right)\left(\frac{2}{3}\pi R_s^2\right)^n\right]$$

$M_s <\sim 4$

$$\log_{10} R_s = -0.209 + 1/3 \log_{10} M_s - 0.0804 \times M_s^{0.394}$$

Solid planets have a similar M-R relation because EOSs of planetary materials are similar.

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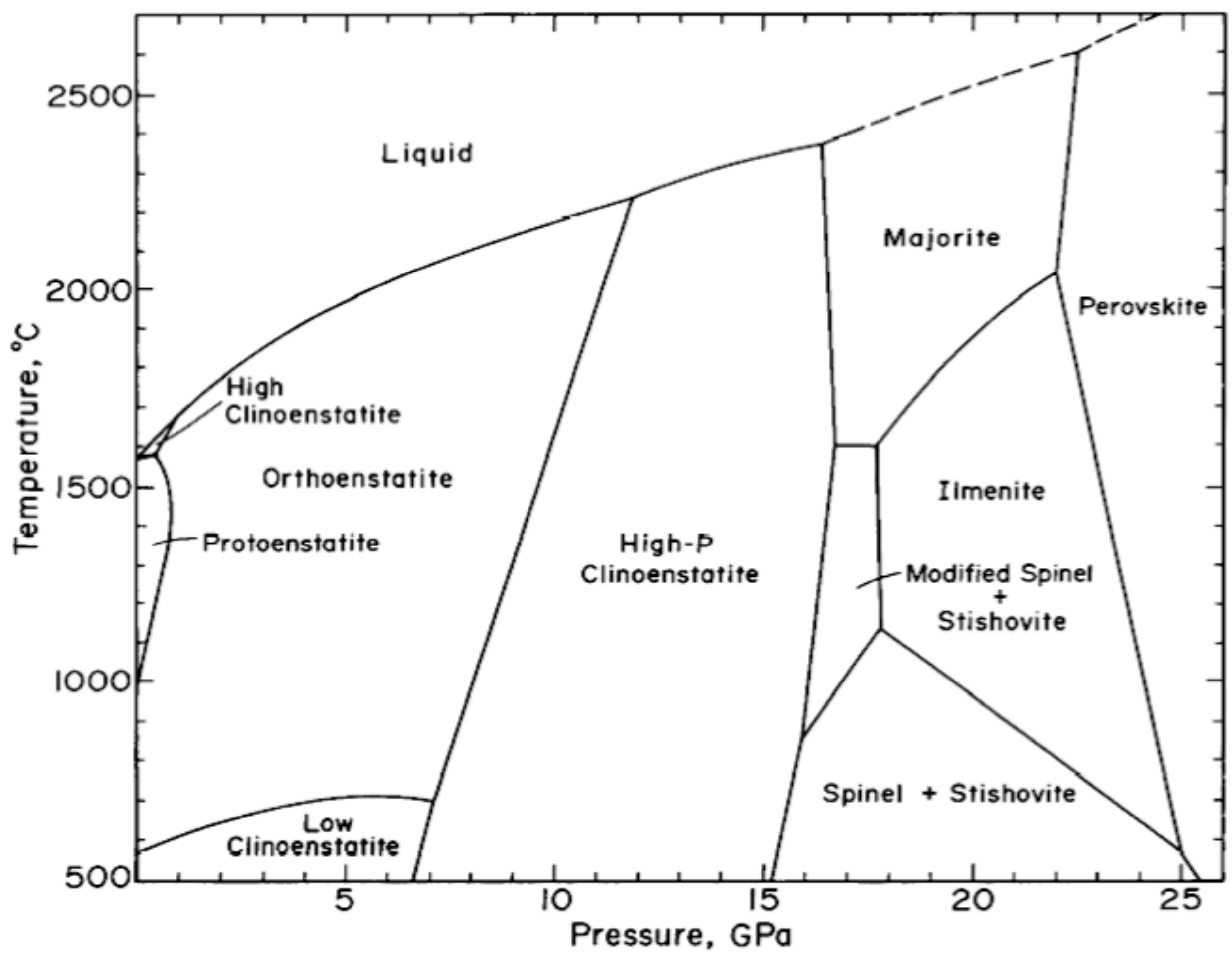
$$\rho(r) = F(P(r))$$

$MgSiO_3$ Phase Diagram

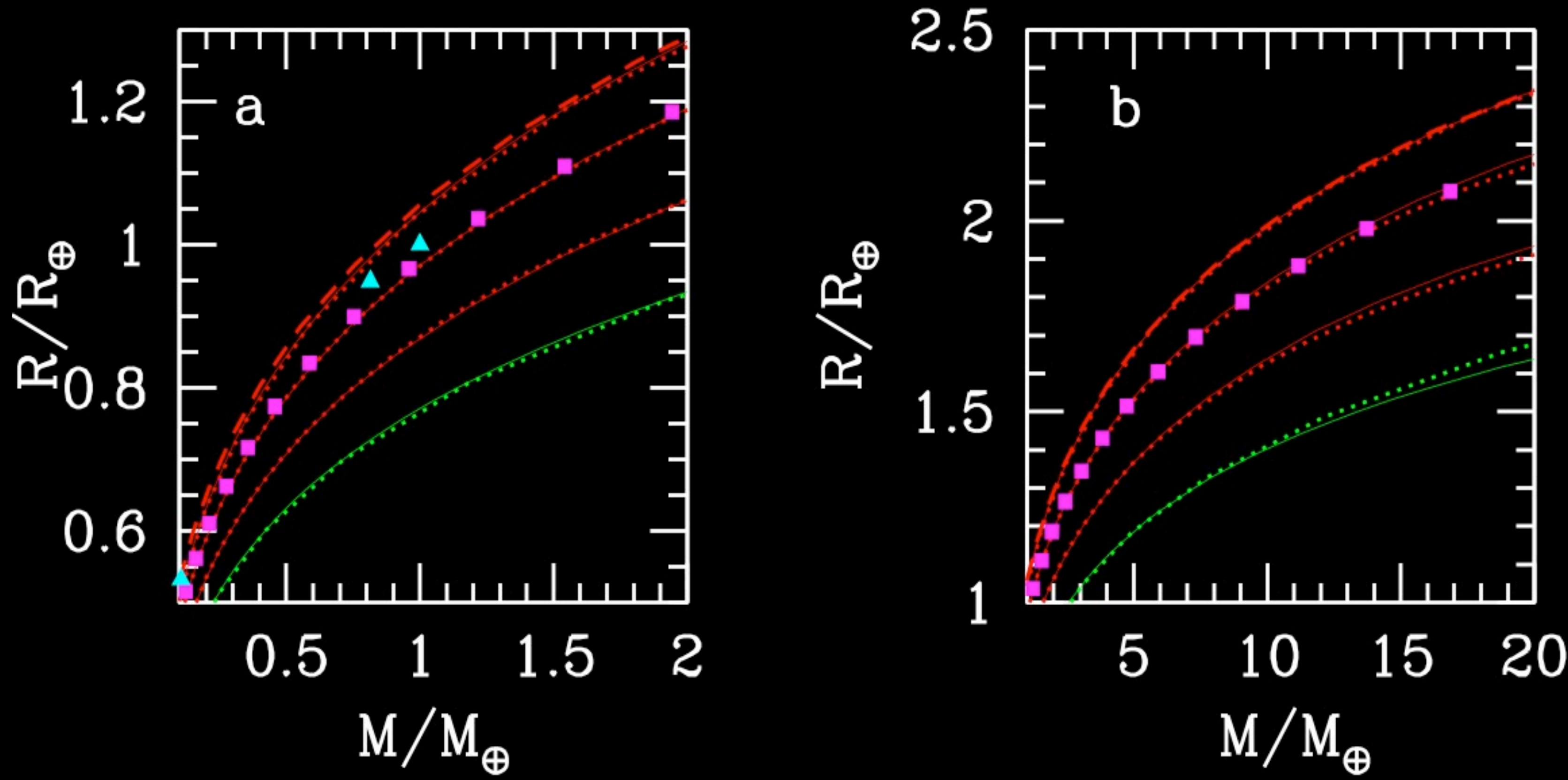


Post-Perovskite >125 Gpa at 2500K

Presnall, 1995

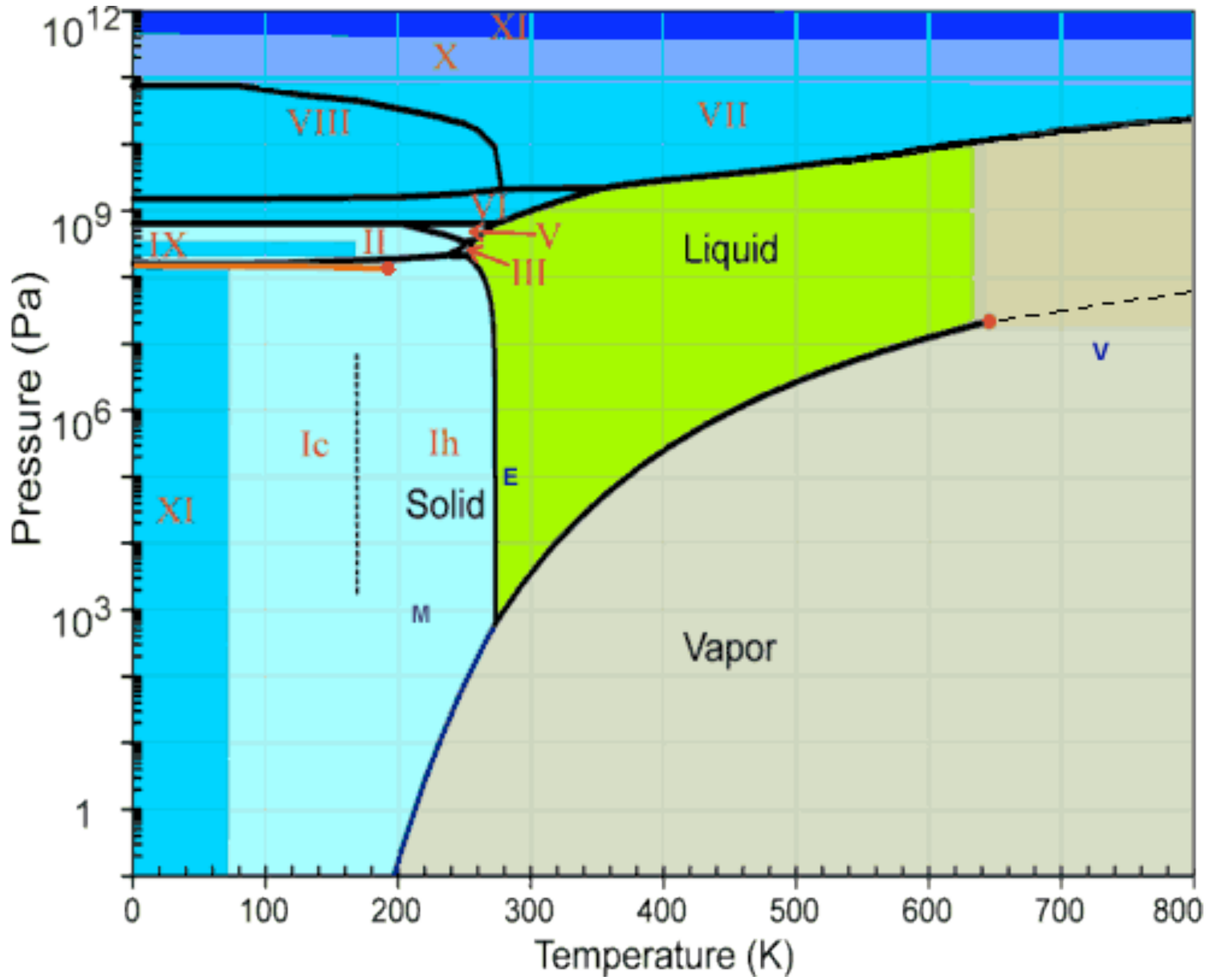
Solid-solid phase changes, e.g. graphite and diamond

Phase Changes

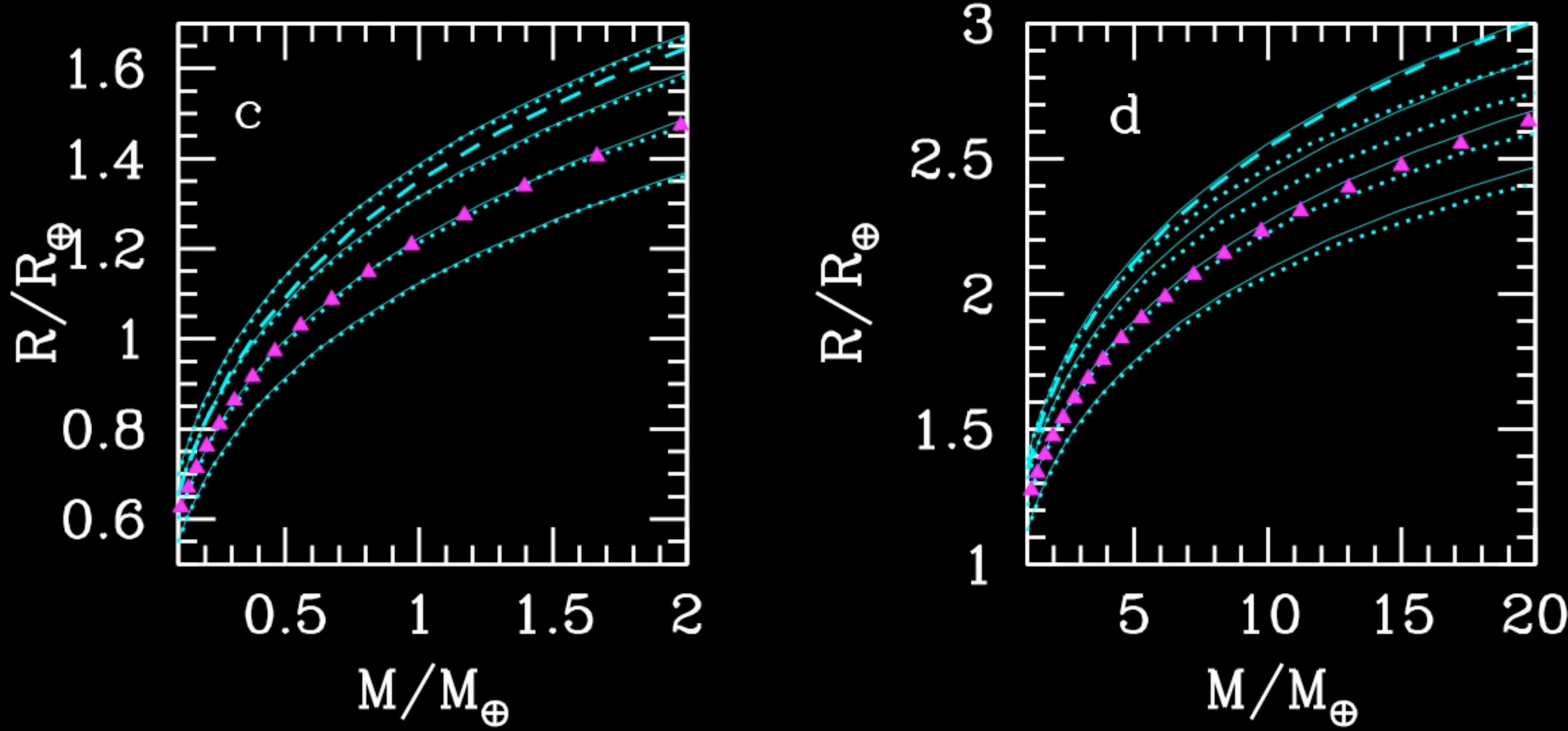


Most of a planet's mass is at high pressure
Small change in R for model with enstatite < 10 GPa

Liquid Water Phase Diagram



Phase Changes



Phase Changes Summary

- Low Pressures (< 10 GPa)
  Most of a planet's mass is at high pressure.

  Low-pressure phase changes therefore have a small effect on the planet's radius.

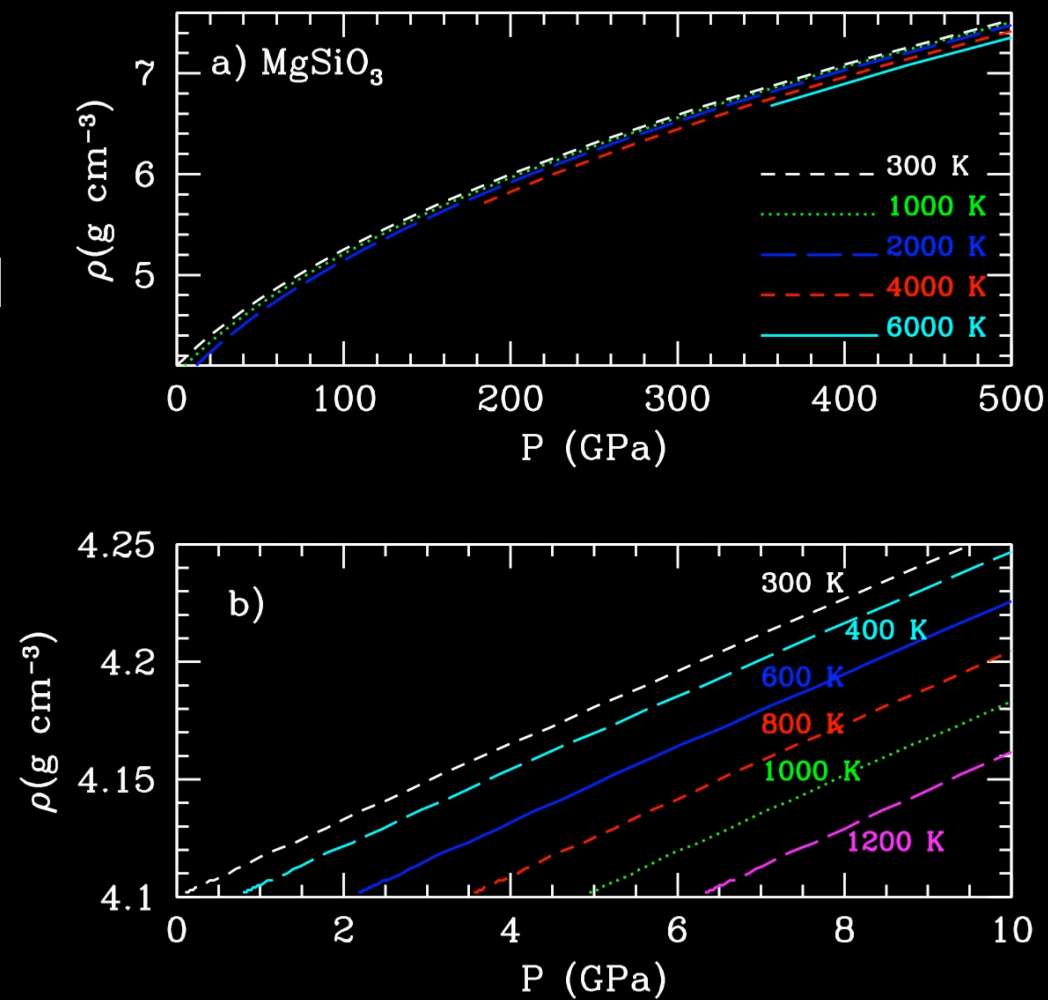
- High Pressures (> 10 GPa)
  At high pressure the atoms are close-packed resulting in little change to the EOS.

  Specifically $\rho(P)$, K and dK/dP are similar

Temperature

Density changes by up to ~4 percent due to temperatures expected in solid planet interiors.

Radius is hence affected up to ~1.5 percent, because $R \sim \rho^{-1/3}$.



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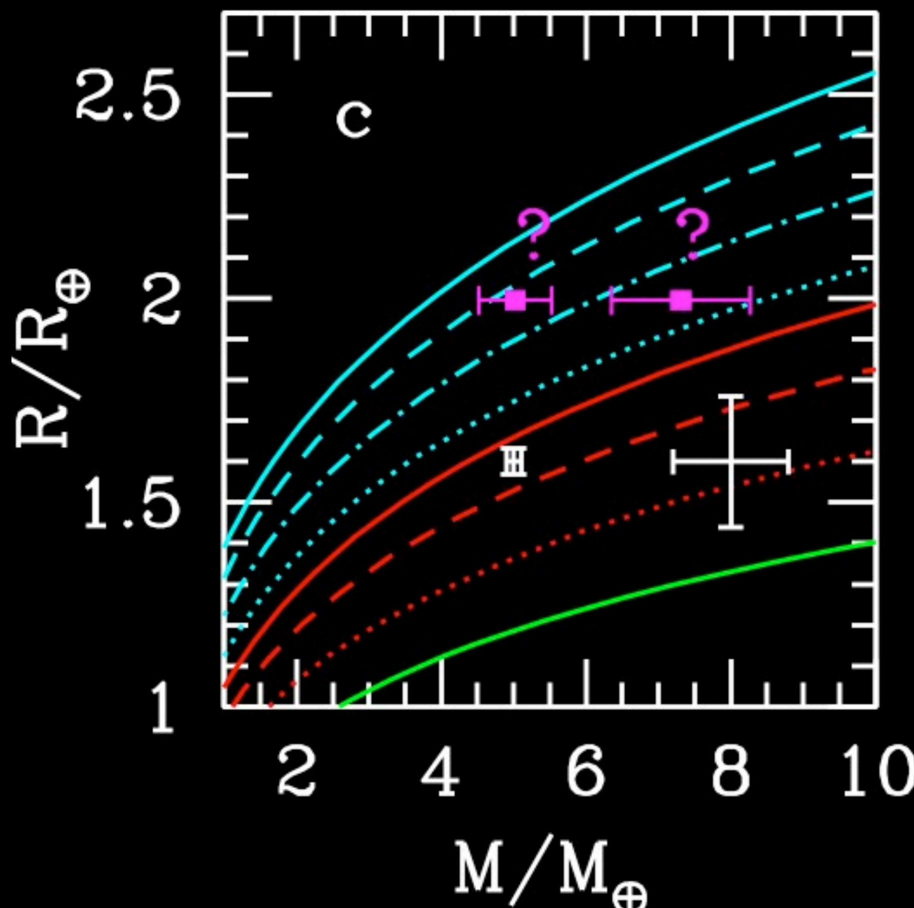
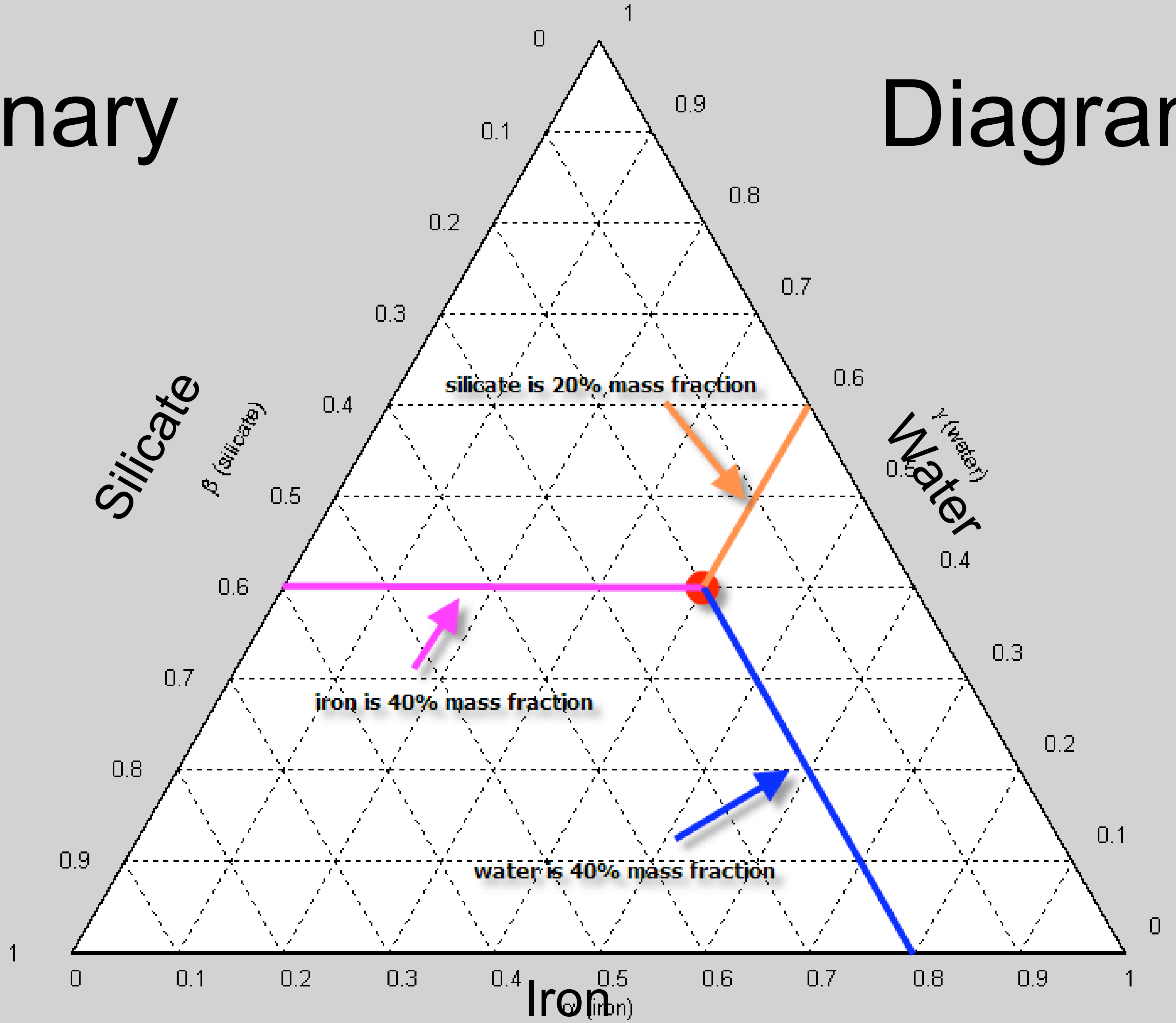
Super Earths



Seager, Kuchner, Hier-Majumder, Militzer, 2007

Ternary Diagrams



See Valencia et al. 2007. Figure from Zeng and Seager in prep.

$1.0\ M_\oplus \quad 1.0\ R_\oplus$

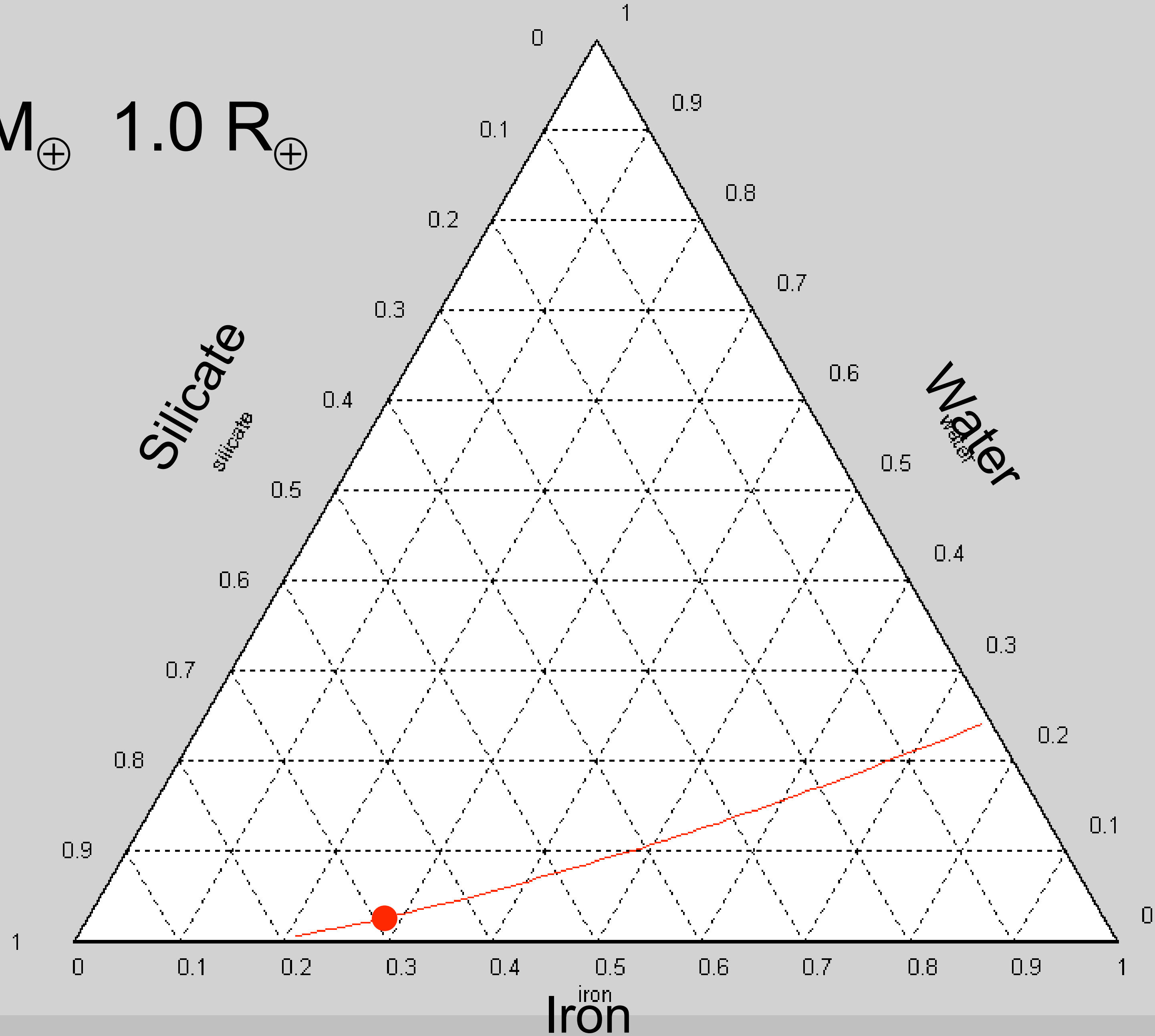
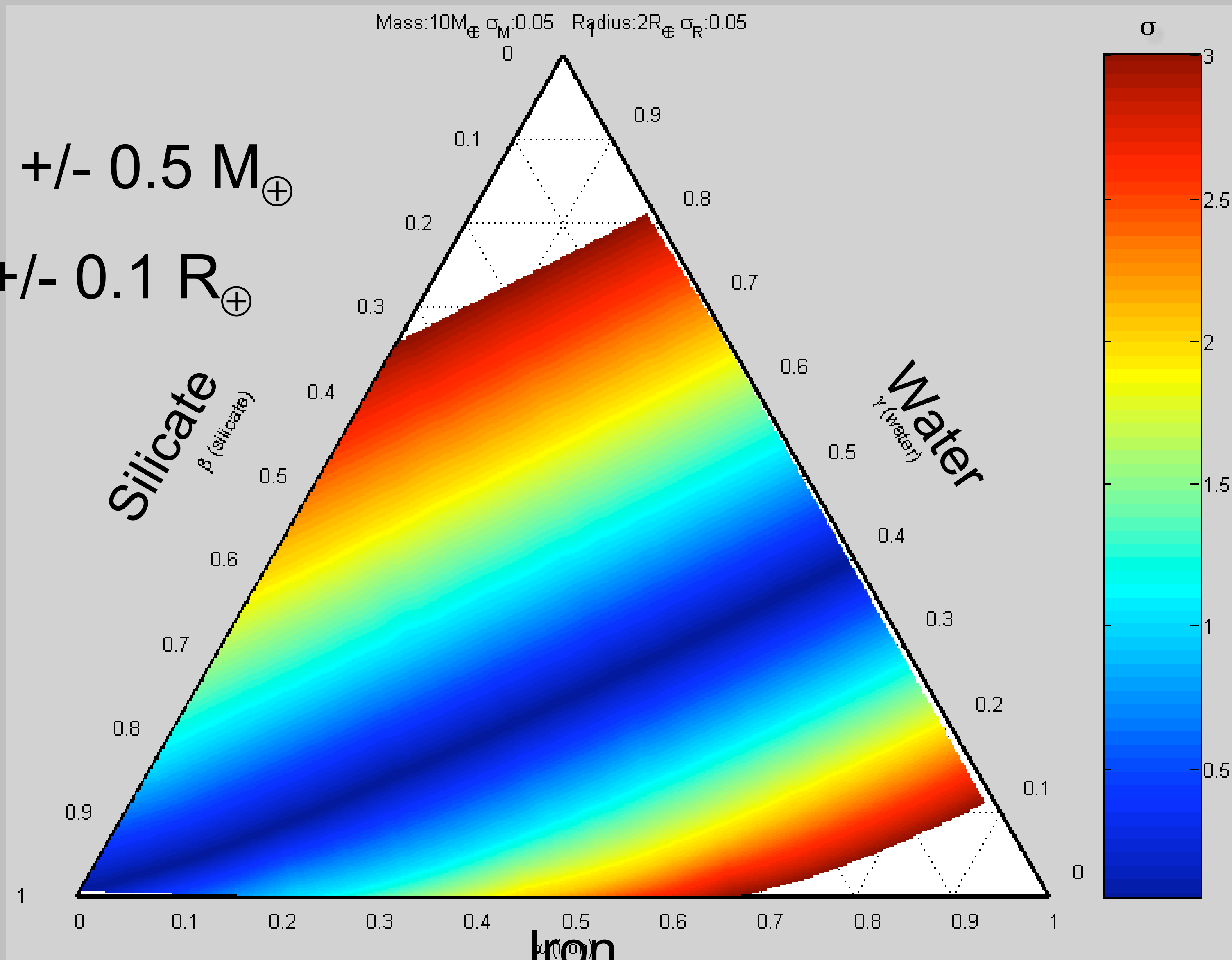


Figure from Zeng and Seager in prep.

$10.0 \pm 0.5\ M_\oplus$

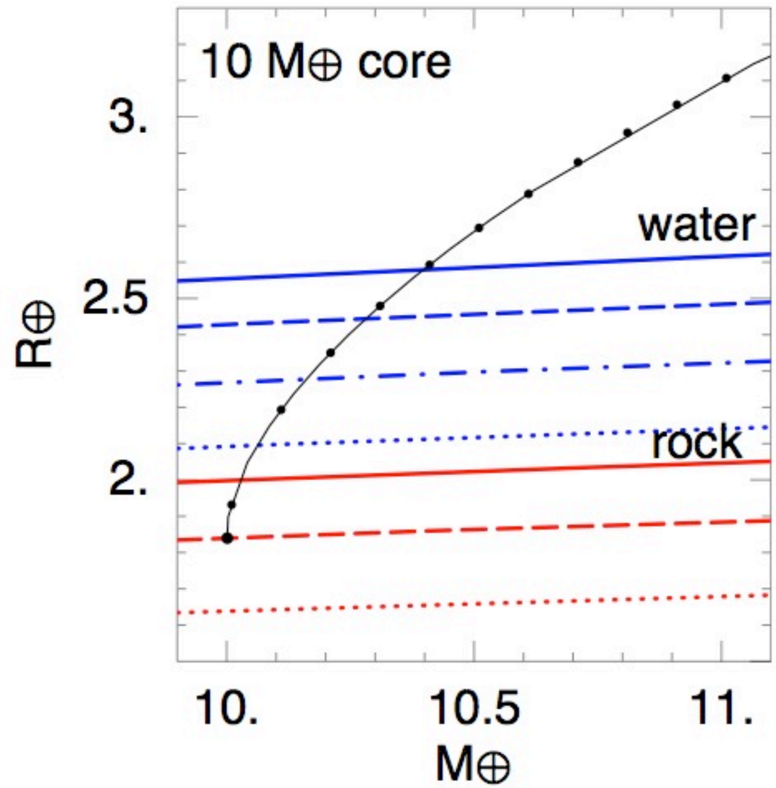
$2.0 \pm 0.1\ R_\oplus$



A 5% uncertainty in mass and radius can make it difficult to identify the bulk composition of a solid exoplanet

Figure from Zeng and Seager in prep.

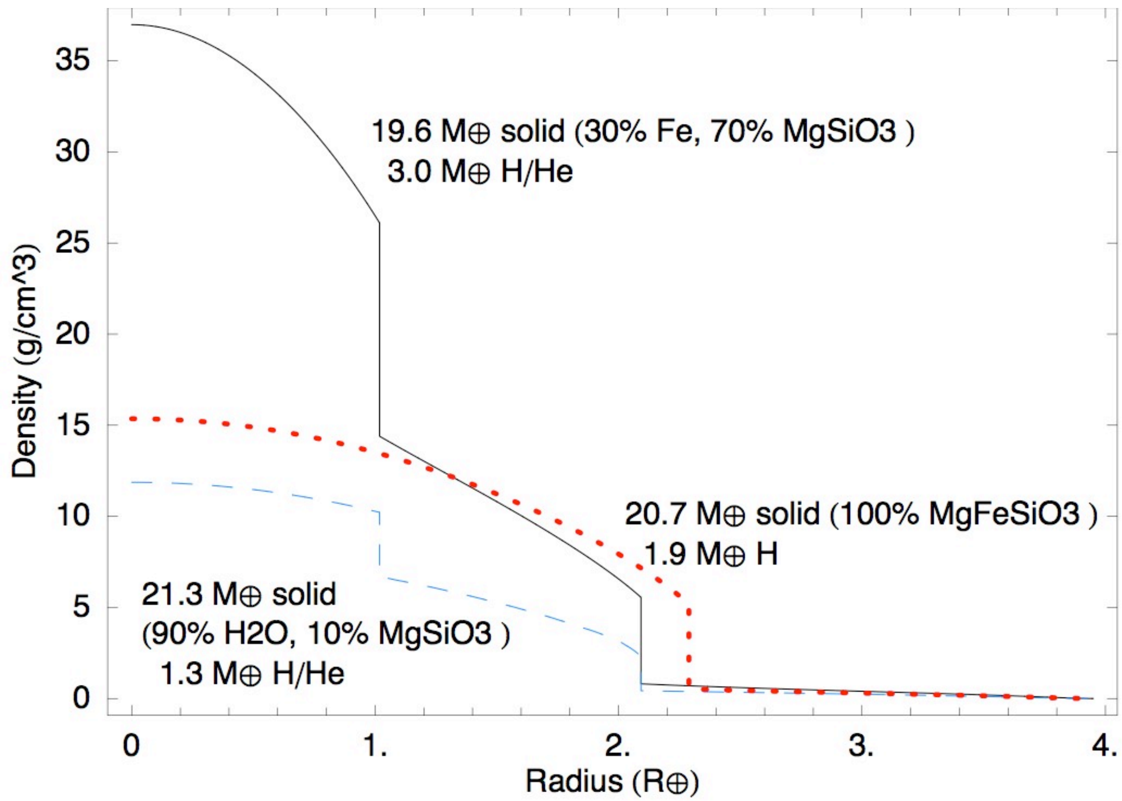
Ocean Planets or Hot Air?

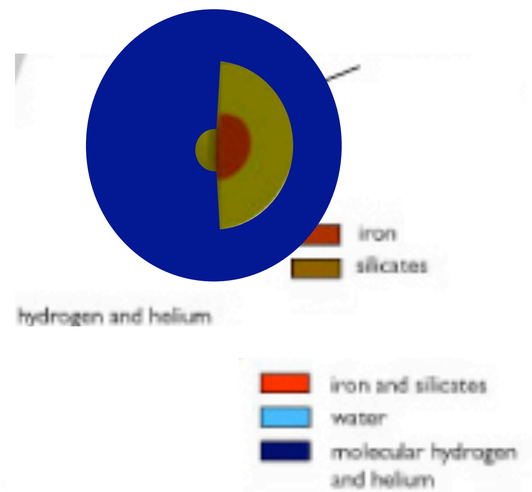


A more significant degeneracy: water planets cannot be uniquely identified unless a significant H/He atmosphere can be ruled out

Elisabeth Adams, Seager, Elkins-Tanton, submitted to ApJL

GJ 436b





GJ436b may be a water planet with 10% H/He, or it may be an iron/silicate planet with 15% H/He, or something else.

Elisabeth Adams, Seager, Elkins-Tanton, submitted to ApJL

JWST: 2013 Transit Planet Follow-up

26 m^2 collecting area

0.7 - 25 microns

Secondary eclipse spectra

Thermal phase curves

Transits: Kepler followup

See Beichman et al. 2006 PPV

Mass-Radius for Solid Exoplanets

Overview

- ✓We aim to derive the bulk composition from M and R
- ✓Aided by the very different *ρ* of water, silicates and iron.

Equations of State

- ✓Most of the physics is incorporated in the relationship between *ρ* and *P*
- ✓Experimental data at high *P* is limited

Approximations

- ✓OK to ignore phase changes and temperature, if you are only interested in the bulk composition

Habitable Worlds

- ✓To identify a habitable world we first need a transiting solid exoplanet