Atmospheric Limits to Interferometry

Michelson Summer Workshop 2006

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Acknowledgements:

Falkovich & Sreenivasan 2006, Lessons from Hydrodynamic Turbulence, Phys. Today, April, 43.

Quirrenbach 2003, Observing Through the Turbulent Atmosphere, Michelson Workshop Publication, 71.

Outline

- Description of limits
- What causes seeing
- Turbulence
- Descriptions for phase fluctuations
- Sky Background
- An example
- Omitted - scintillation, dispersion

Atmospheric Limitations Summary

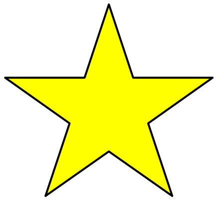
- Reductions in coherence of the electric field
  - Space, time, frequency
  - “phase screen” concept
- Space dimension implies coherence destroying processes are themselves incoherent
  - Compensation thus limited in space, time, frequency, and angle.
- Extra photons from sky thermal and line emission
  - Angular, temporal, frequency

Consequences

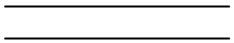
- Instrumental
  - Aperture size ☐limitation
  - Integration time limitation
  - Co-phasing limitation
- Observational
  - Magnitude limits
  - precision limitation (observable uncertainty)

What is seeing?

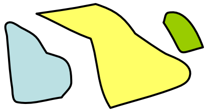
- Wave front for stellar light getting messed up by our atmosphere!



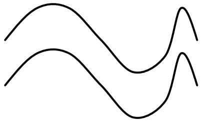
- Wave front is transformed from uniform horizontal phase to ***non-uniform*** horizontal phase.



- Caused by ***horizontal*** differences in the refractive index (“phase screen”).

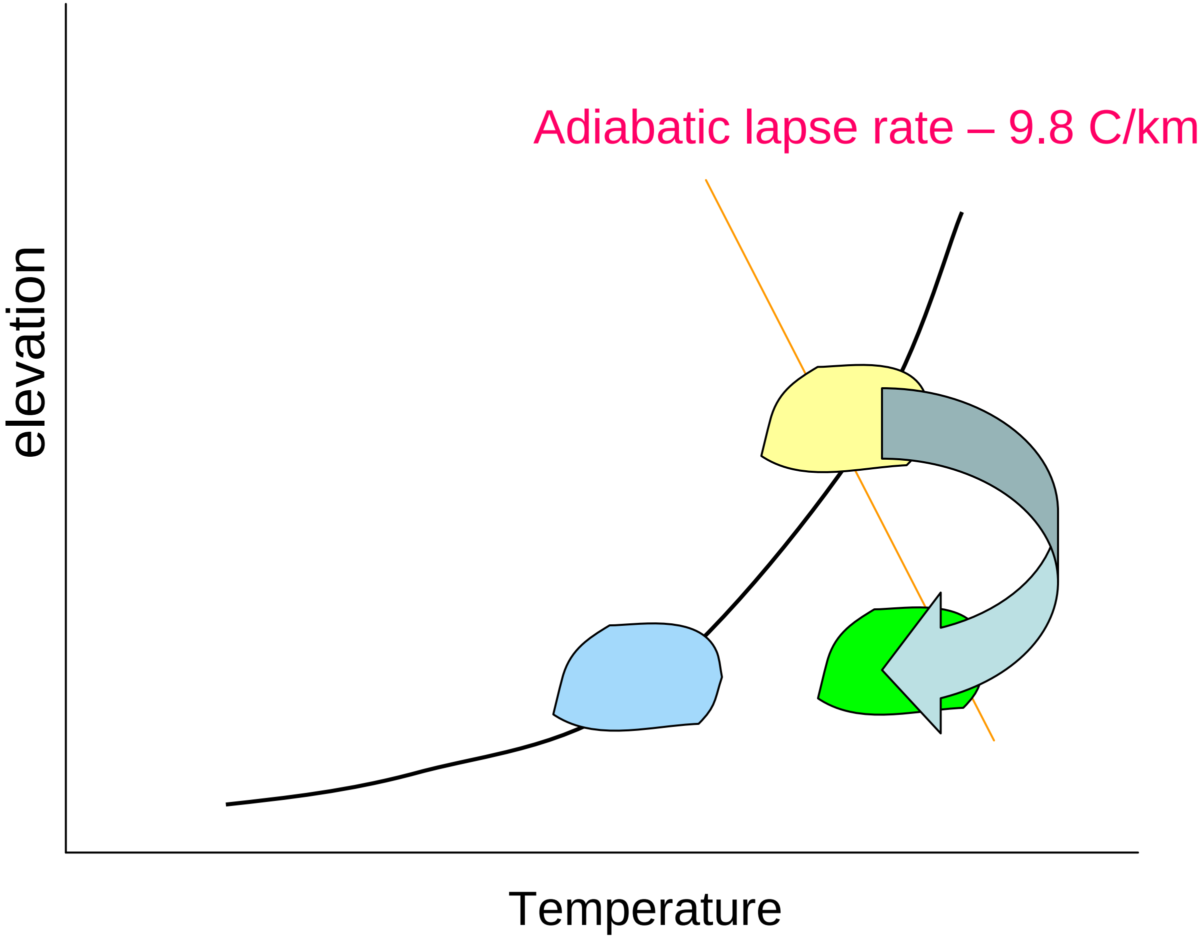


- Refractive index changes dominated by temperature fluctuations.



What makes seeing?

in Antarctic boundary layer



- ***Non-adiabatic temperature profile***
- ***Vertical mixing***
- Turbulence provides vertical transport of air parcel
- If atmosphere non-adiabatic, air parcel will not be in thermal equilibrium with surrounding air.
- This process produces horizontal refractive index structure.

What is omitted in this picture?

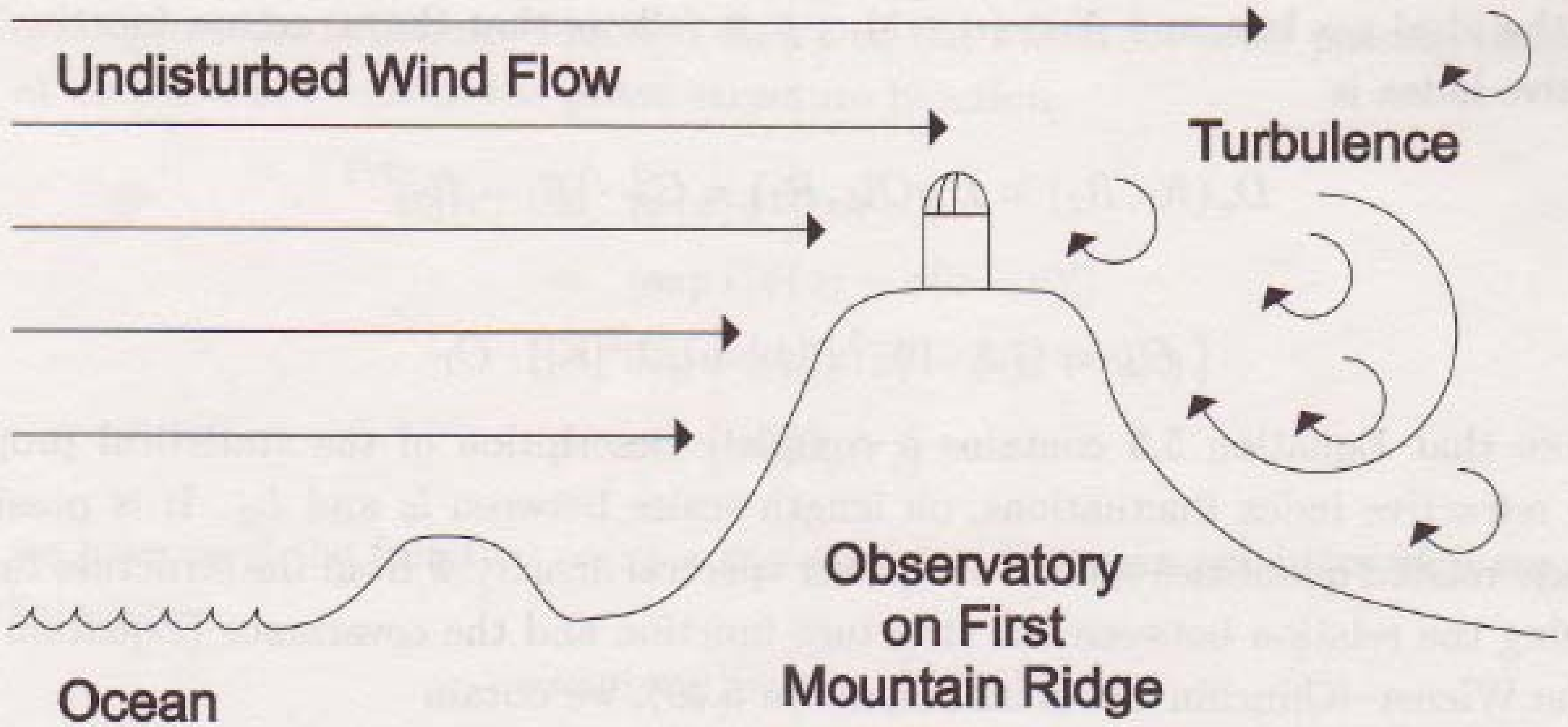


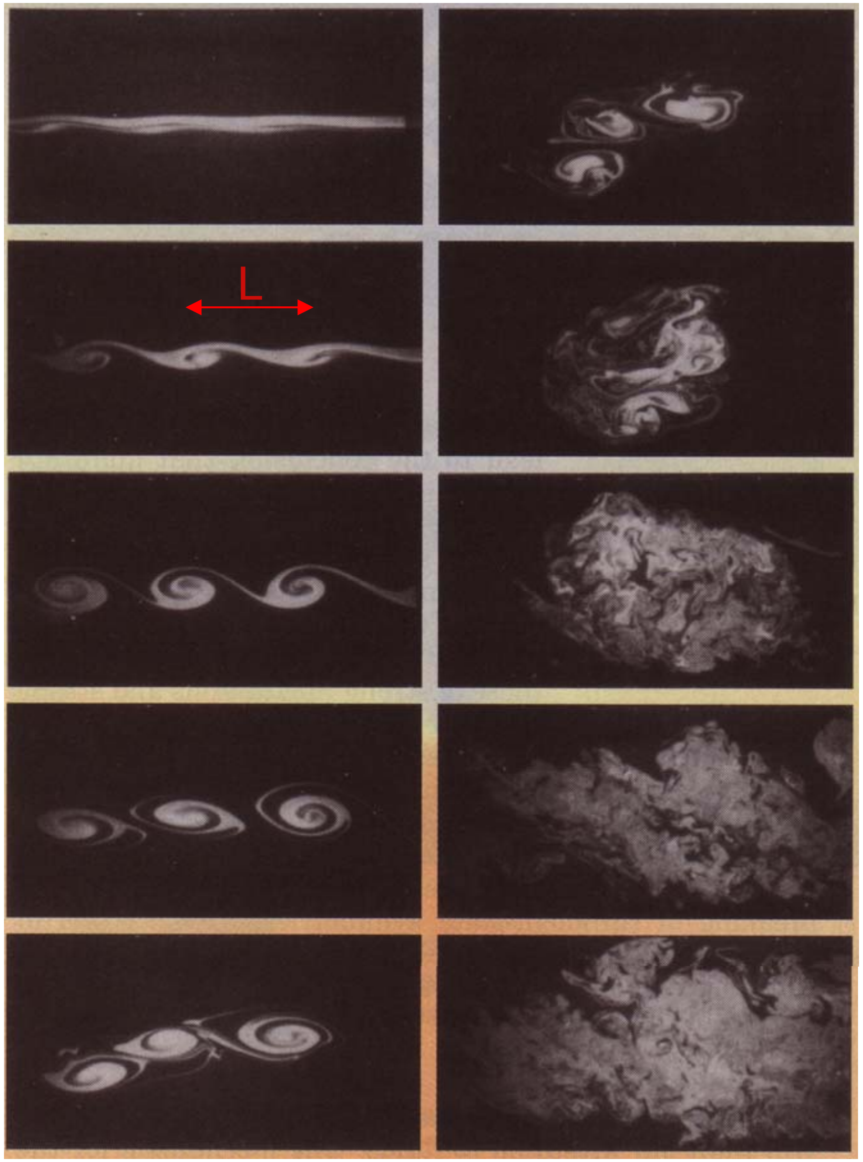
Figure 5.1: Schematic of turbulence generation in the wake of obstacles. Most world-class observatories are located on the first mountain ridge near the coast (or on mountains on islands), with prevailing winds from the ocean.

Turbulence Summary

- Unrepeatable in detail, irregular in time and space
- Involves constant dissipation and is thus sustained only by addition of energy
- Characterized by strongly interacting degrees of freedom and are far from equilibrium
- Flows characterized by the Reynolds number Re = LV/v
  - Re small -> smooth flow due to viscous damping
  - Re big -> motion dominated by nonlinear effects
  - Consequence of the nonlinear effects is the creation of smaller scales, which in turn create smaller scales
- Turbulence is steady state condition of maintaining the multi-scaled motion generation

Idealized Turbulence

- Energy injected at large scales (L) by bulk motion at speed (V). "cascades" (without loss) to ever smaller scales.
- The "cascade" is terminated by energy dissipation through viscous (*v*) loss on scales ~ η.
- Scales between L and η are termed the "inertial range".
- Properties of the flow can be described statistically in the inertial range.



Turbulence Described

- Structure function $S_n = \left\langle \{[\mathbf{v}(\mathbf{r},t) - \mathbf{v}(0,t)] \bullet \mathbf{r}/r\}^n \right\rangle$
- Kolmogorov showed $S_3 = -{}^4\!/_5 \langle \varepsilon \rangle r$

  in inertial range, where <ε> = average rate of energy dissipation.
- Closed form expression for higher moments unknown: Kolmogorov inferred. $S_n \propto r^{n/3}$
- This is the (assumed) scale invariance for homogenous, isotropic, 3-d turbulence.
- Equilibrium system equipartition of energy holds.
- Turbulence, non-equalibrium system, energy conservation in inertial range means energy flux across all scales is constant.

The Kolmogorov Approximation

- Restatement of structure function $D_v(\mathbf{r}_1,\mathbf{r}_2) = C_v^2 \left|\mathbf{r}_1 - \mathbf{r}_2\right|^{2/3}$
- The assumption of exact scale invariance is not correct (anomalous scaling).
- Relative growth of high moments means strong fluctuations become more probable as scales become smaller.
- Also, some observations show seeing observables depart slightly from what is expected using a Kolmogorov model.
- However, Kolmogorov is a useful model.
- There is a no evidence I know of suggesting astronomers are making significant errors in predicting atmospheric properties due to the non-Kolmogorov behavior of the atmosphere.

Refractive index fluctuations

- Note: the second moment of the structure function is the energy contained in the Fourier modes with wavenumbers larger than 1/r.
- Assume temperature and refractive index fluctuations (turbulence) follow velocity.

$$D_N(\mathbf{r}_{1,},\mathbf{r}_2) = C_N^2 \left|\mathbf{r}_1 - \mathbf{r}_2\right|^{2/3}, \quad C_N^2\ r^{2/3} = \int_{-\infty}^{\infty} d\kappa (1 - \exp(2\pi i \kappa r))\,\Phi(\kappa)$$

$$\Phi(\kappa) = 0.0365\ C_N^2\ \kappa^{-5/3}$$

- The refractive index fluctuation power spectrum, Φ(κ), of Kolmorgov turbulence ~ $\kappa^{-5/3}$.

The phase structure function

- Phase structure function for a layer of Kolmogorov turbulence of thickness δh

$$D_\phi(r) = 2.914\; k\; \delta h\; C_N^2\; r^{5/3}, \quad k = 2\pi/\lambda$$

- The phase coherence function☐ (integrating over the atmospheric column and accounting for zenith angle):

$$B(r) = \exp\left[-\frac{1}{2}\left(2.914\; k^2 (\sec z)\; r^{5/3} \int dh\; C_N^2(h)\right)\right], \quad B(r) = \exp\left[-3.44\left(\frac{r}{r_0}\right)^{5/3}\right]$$

- Atmospheric phase structure function defined in terms of the Fried parameter, r_0.

$$D_\phi(r) = 6.88\left(\frac{r}{r_0}\right)^{5/3}, \quad r_0 = \left[0.423 k^2 (\sec z) \int dh\; C_N^2(h)\right]$$

Seeing and the Fried parameter r_0

- r_0 is the length scale on which the phase variance is 1 radian², $\sigma_\phi^2 = 1\,\text{radian}^2$
- Isoplanatic patch scaling $r_0 \propto \lambda^{6/5}$
- Seeing limited resolution is $seeing_{rad^2} = \frac{\pi}{4}\left(\frac{r_0}{\lambda}\right)^2$
- r_0 scaling implies better image quality at longer wavelengths
- Strehl $S = e^{-\sigma_\phi^2}, \quad \sigma_\phi^2 = 1.03\left(\frac{D}{r_0}\right)^{5/3}$
- Useful interferometer telescope size ~2r_0 with active tip-tilt correction.

Coherence time & isoplanatic angle

- Angular phase variance

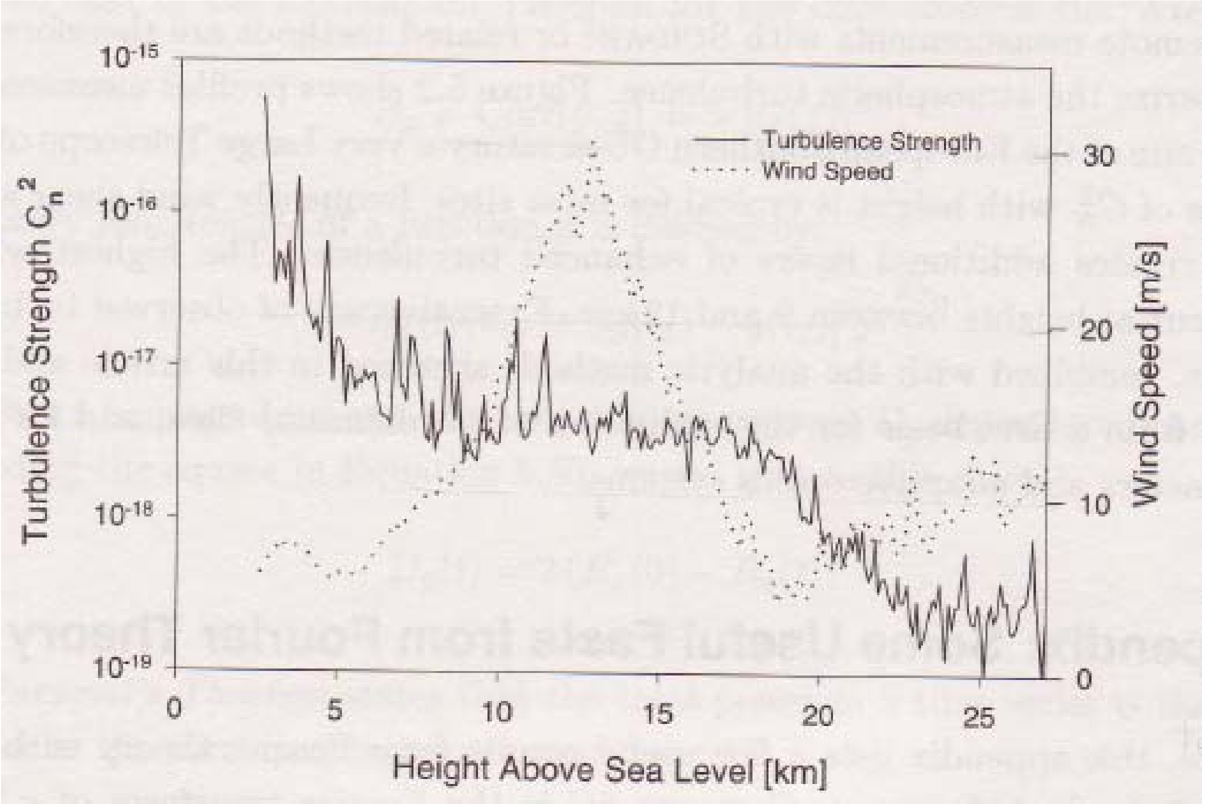
$$\langle \sigma_\theta^2 \rangle = \left(\frac{\theta}{\theta_0} \right)^{5/3}, \qquad \theta_0 = \left[2.914\, k^2 (\sec z)^{8/3} \int dh\, C_N^2(h)\, h^{5/3} \right]^{-3/5}$$

- Coherence time, and turbulence effective height

$$\tau_0 = \frac{1}{4} \frac{r_0}{v}, \qquad H = \left(\frac{\int dh\, C_N^2(h)\, h^{5/3}}{\int dh\, C_N^2(h)} \right)^{3/5}$$

Accessing atmospheric quality

- The common metrics r_0, τ_0, θ_0 used.
- All depend profiles of wind velocity and $C_N^2(h)$.
- Interferometer sensitivity scales as coherence volume $\approx r_0^2 \tau_0$.



Cerro Paranal wind and turbulence profiles.

Sky background

- Becomes a significant limitation by ~ 3 μm.
- Chopping used to subtract background.
- 10 μm (N band) observations strongly effected.

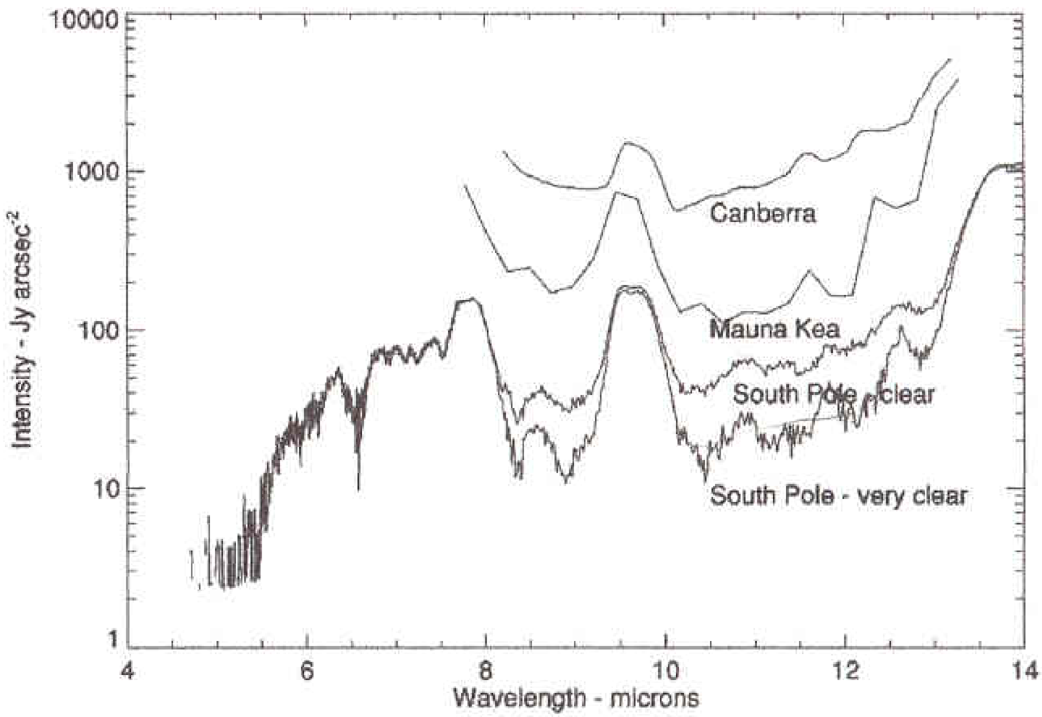
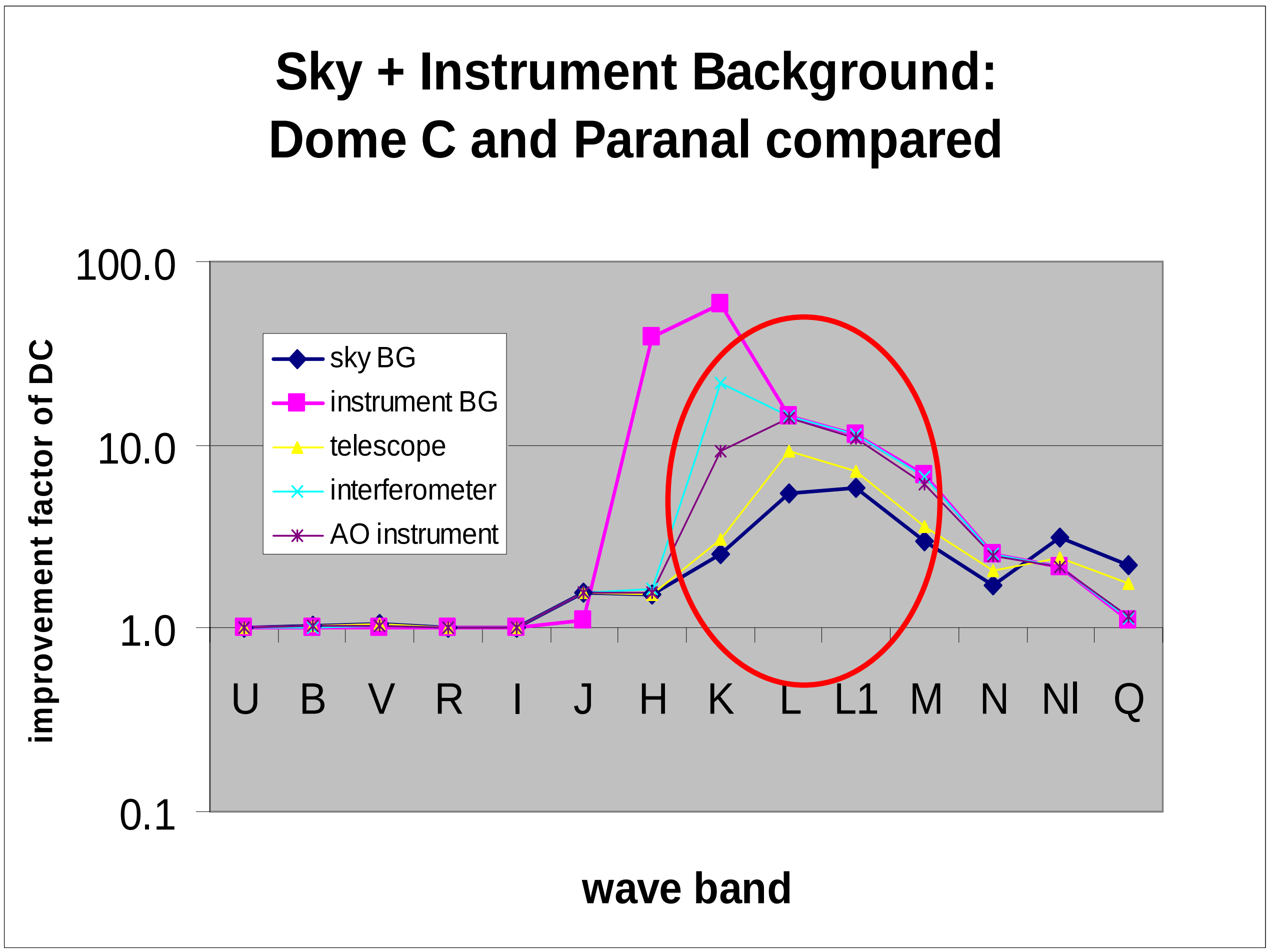


FIG. 13.—Comparison of emission spectra taken in clear conditions at Canberra and Mauna Kea (Smith & Harper 1998) with data from the South Pole (this work).

Does Antarctica reduce the10 µm Noise Equivalent Power ?



- Sky and instrument background contributions differ.
- Net background controlled by emissivity and transmission.
- Instruments couple to background components differently.

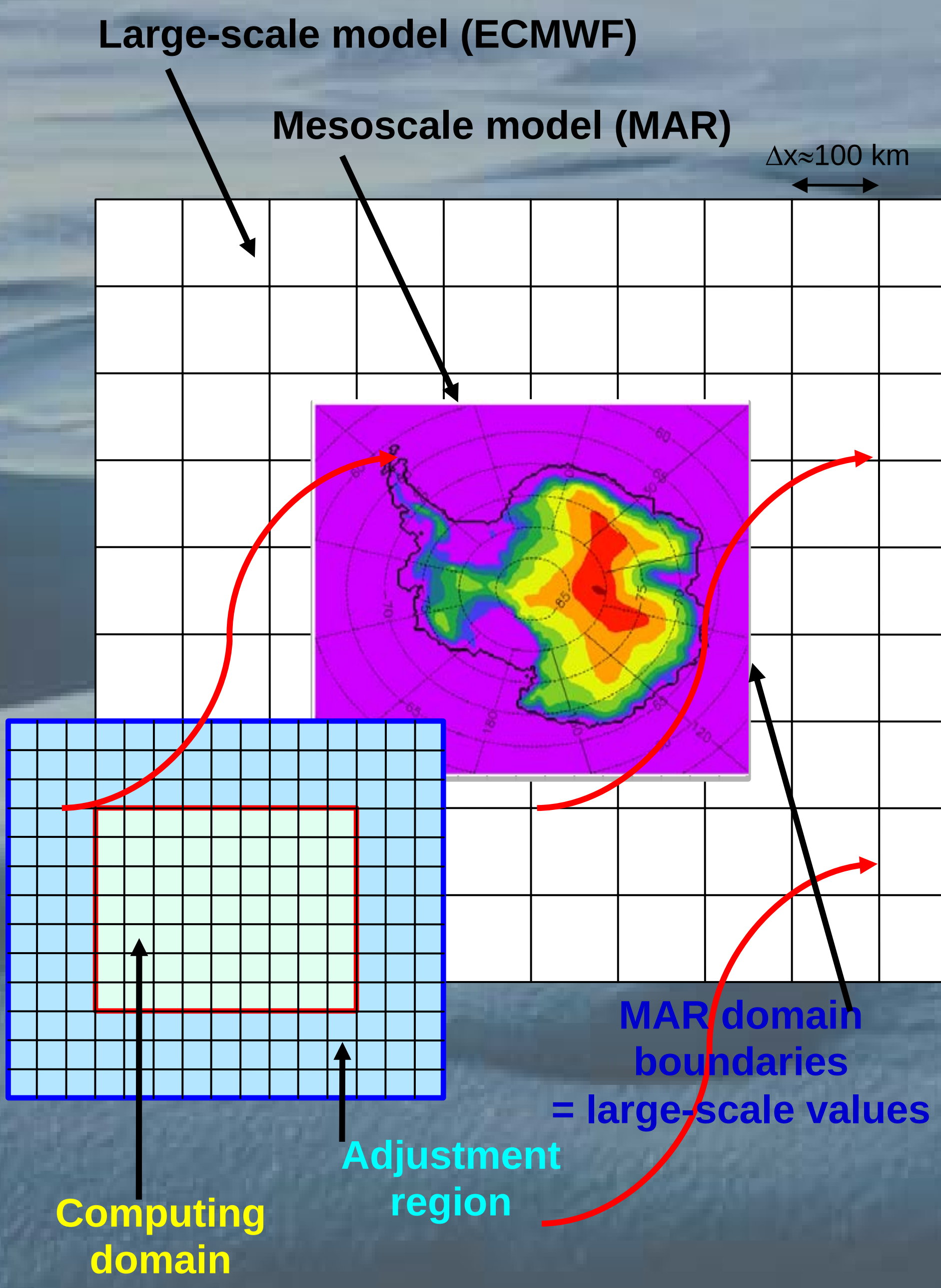
What to do with all this?

- Ignore it. Take the site/instrument limits as a given, pick your targets, and be a happy astronomer. ☺
- Develop calibration methods (you will need more information than the material in this talk…).
- Model site quality and instrument performance.

An Example…

- Access potential of Antarctic sites for astronomy potential.
- Model atmospheric properties (since significant Antarctic astronomical site testing has only be done at two locations).
- Metrics for site quality.

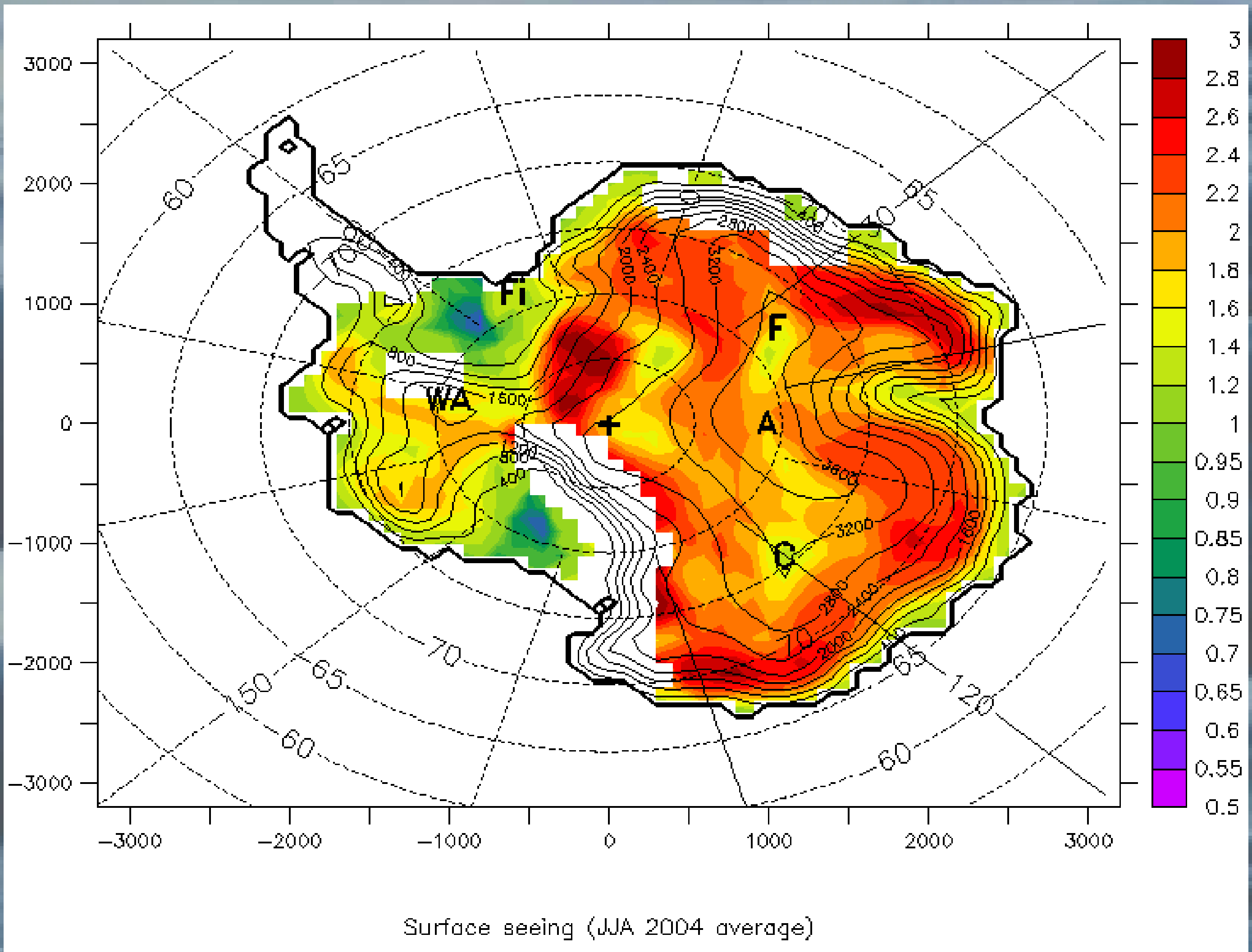
MAR and Lateral Forcing



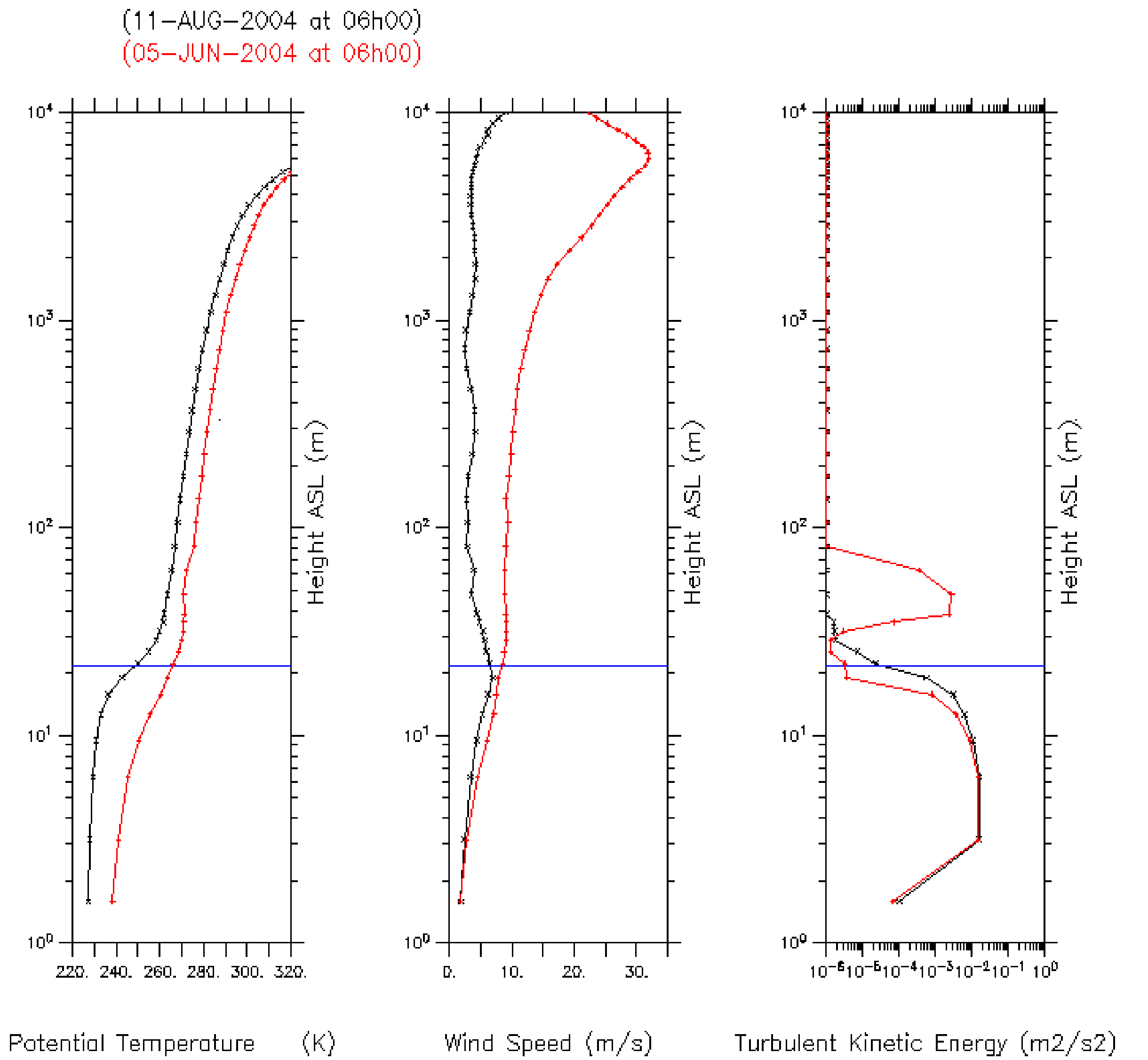
MAR Simulation of the Antarctic Atmosphere

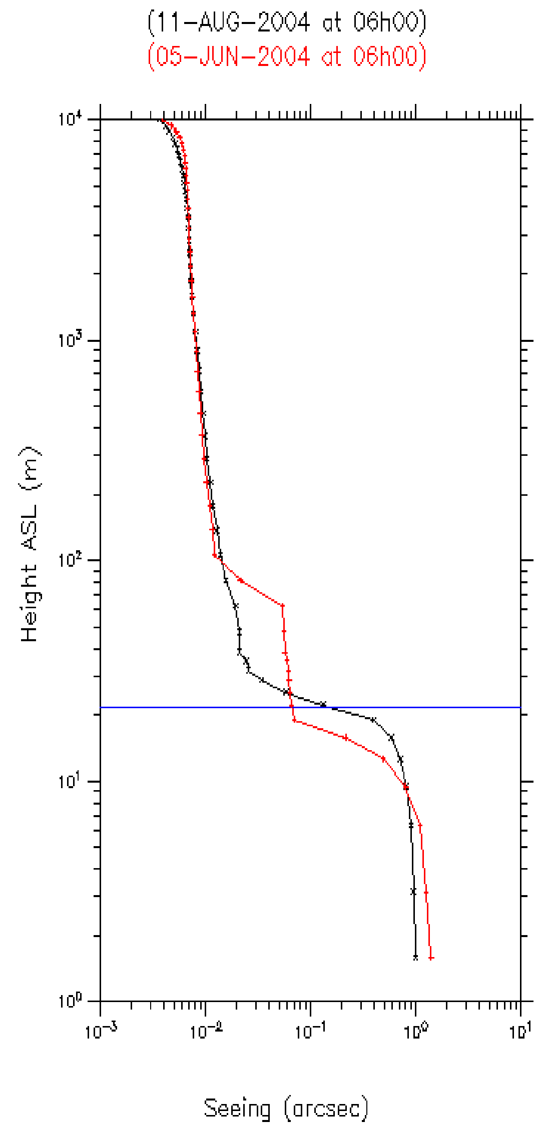
QuickTime™ and a
Video decompressor
are needed to see this picture.

Average winter seeing at surface level

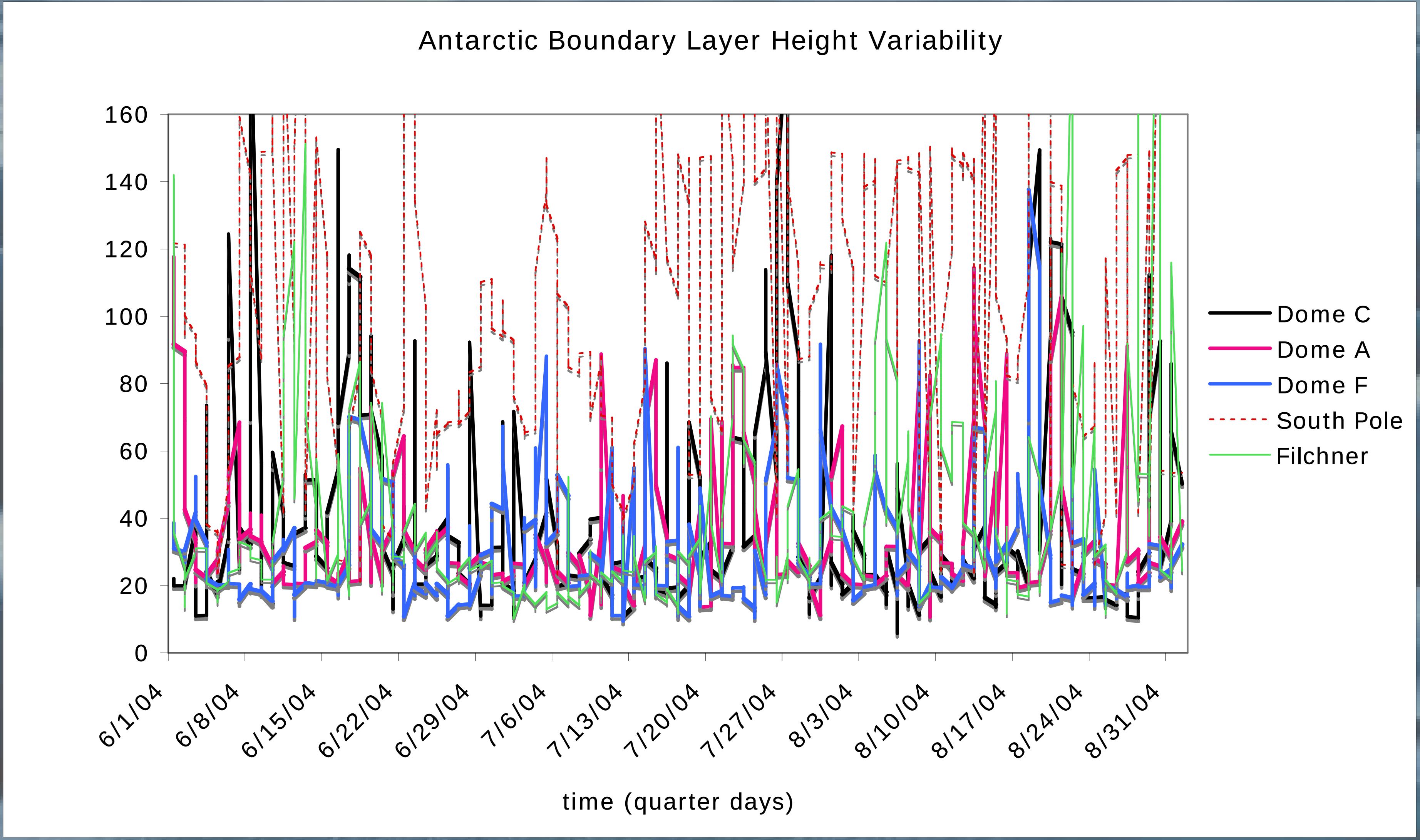


Vertical profiles for seeing and related parameters

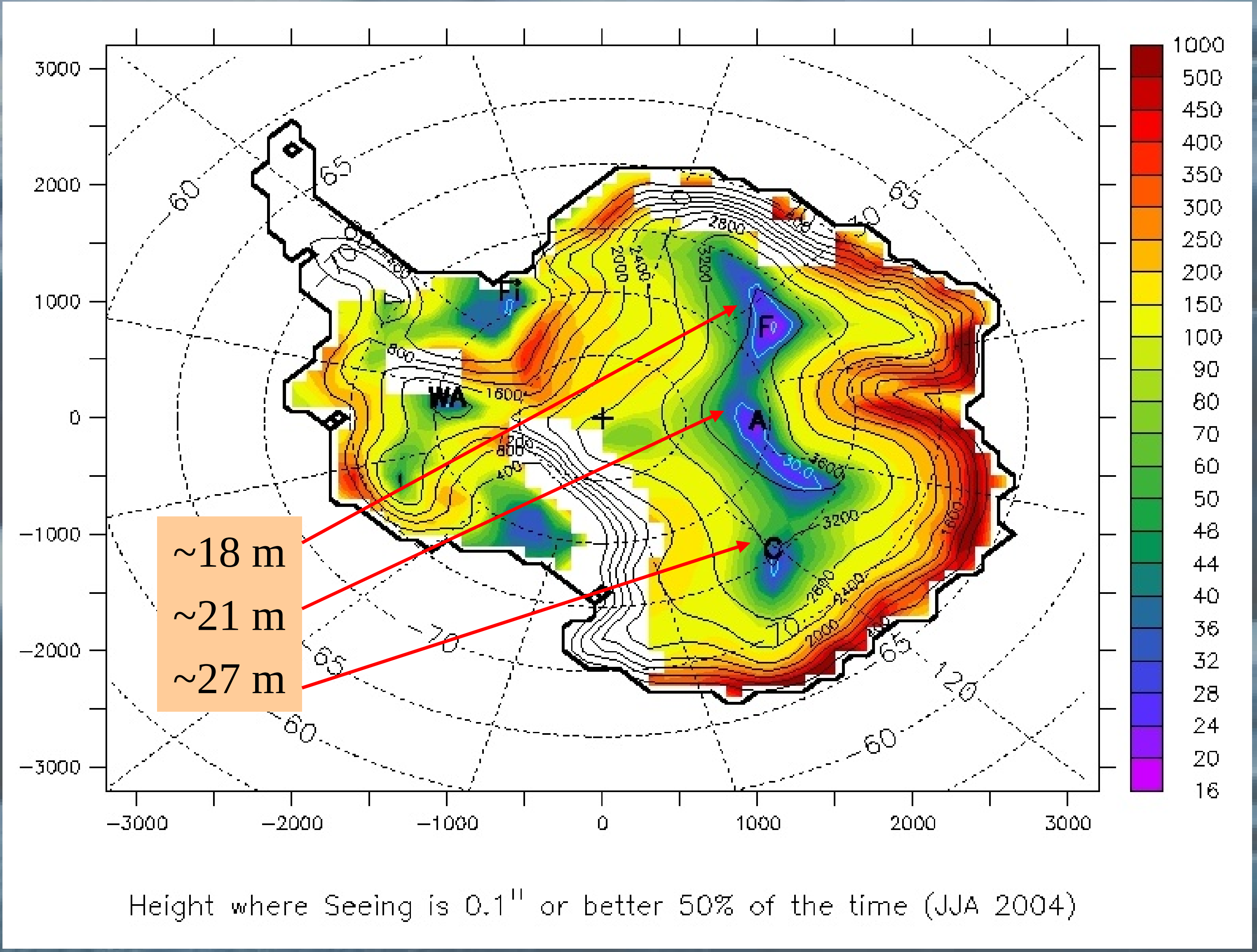




Boundary layer height variability



Elevated Telescopes



Boundary Layer: height and seeing

- Surface wind speed determines the boundary layer height and seeing.
- Seeing "saturates" above some wind speed
- Strong seeing everywhere because wind speed is high enough to put seeing in the "saturated" regime.
- The difference in the seeing/wind speed profile for Antarctic sites indicates the Dome A/F inversion is stronger.
- Strong inversion implies more "clear sky" time.

Dome A/F will have fewer clouds than Dome C.