Data modelling and interpretation

2006 Michelson Summer Workshop

Frontiers of interferometry: stars, disks, and terrestrial planets

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Guy Perrin

l'Observatoire de Paris — LESIA

Laboratoire d'Études Spatiales et d'Instrumentation en Astrophysique

Outline

- Simple geometrical models
- Model fitting
- Miscalibration and data selection
- V^2 estimators
- Correlated noise
- Examples of data modelling and interpretation

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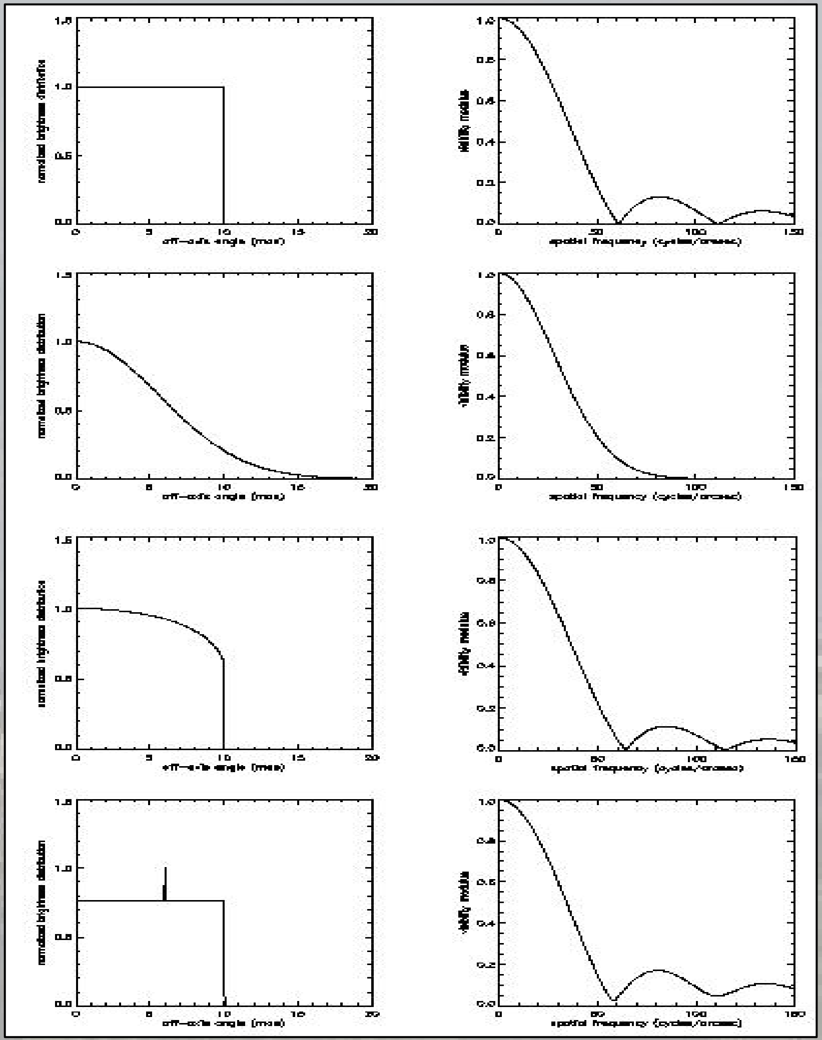
Simple geometrical stellar models

- uniform disk
- gaussian disk
- disk with limb-darkening
- stellar disk + spot



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Model fitting

A *very* classical technique that consists in searching for *the* model that best reproduces the data and in constraining its parameters.

The most common method is the χ^2 **method** (maximum likelyhood for gaussian random variables).

Example in the case of the uniform disk model:

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(\frac{V_i^2 - M(\text{Ø}_{UD}; S_i)}{\sigma_i} \right)^2$$

where *M* is the model with a single parameter, the star diameter Ø_{UD}. The V_i^2 are the measured squared visibilities and the σ_i the associated errors. The S_i are the spatial frequencies.

Model fitting

The method consists in searching for the minimum χ^2 to find the most probable parameters.

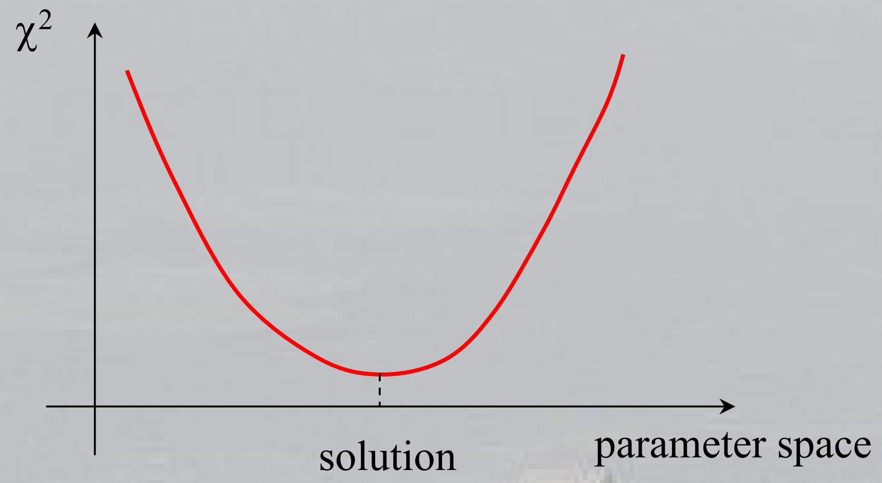
$$\chi^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{V_i^2 - M(\O_{UD};S_i)}{\sigma_i}\right)^2$$

Number of parameters

If the model is an exact representation of the object and if the error bars are correctly estimated then the ***average*** value of the χ^2 at minimum is 1.

N-p (p=1 here) is the number of degrees of freedom of the χ^2.

A unique solution ?



If the χ^2 is convex then the solution is unique. It is the case if the model is linear wrt to the parameters.

Most often the model is not linear ➲ local minima and multiple possible solutions

Caveat: the χ^2 surface is a realization of a random variable (the χ^2 !). If several solutions have close χ^2 values then *the true* solution may not have the smaller χ^2 and may appear as a secondary minimum ➲ ***statistics with small numbers.***

Model fitting

The χ^2 statistics allow to determine confidence intervals for the model parameters.

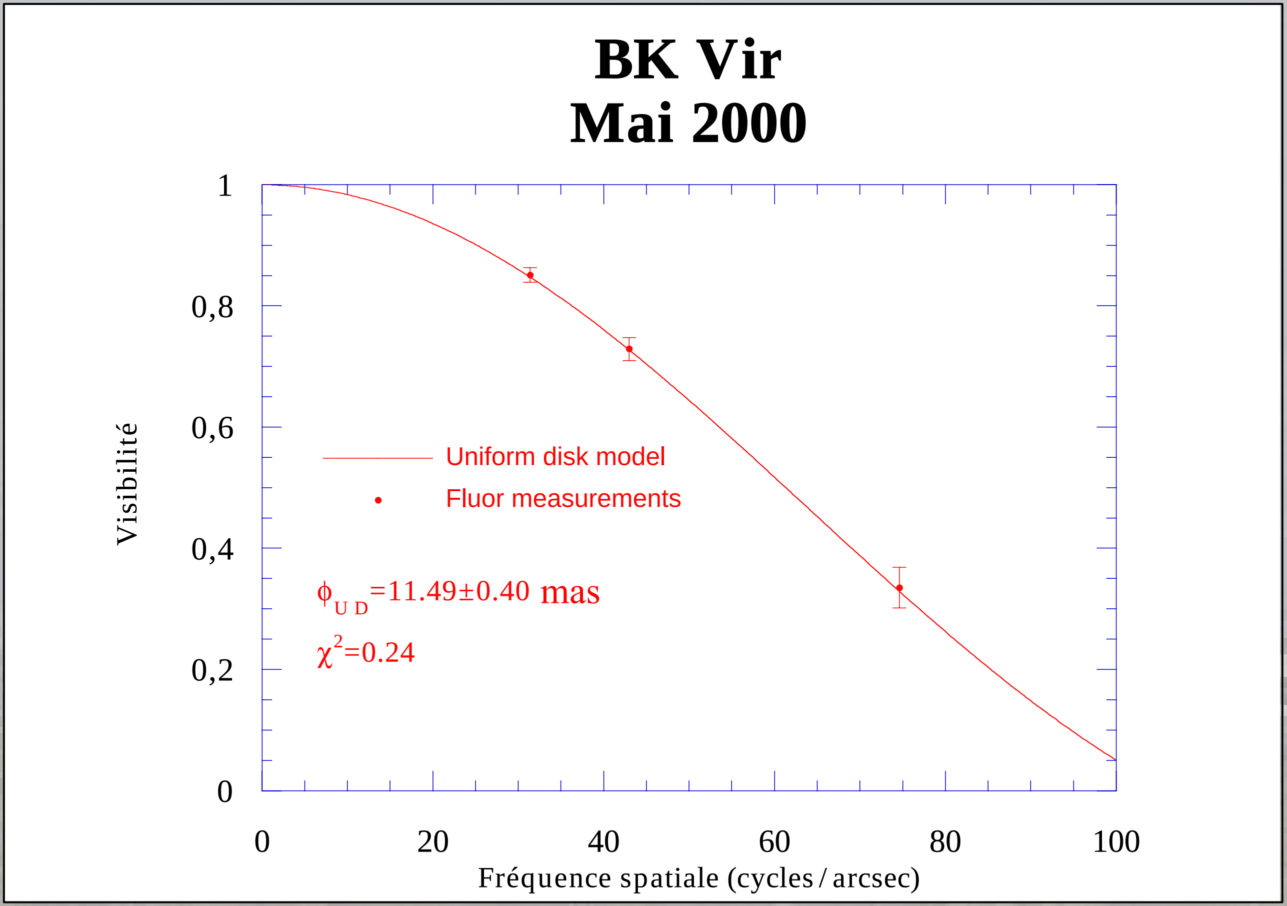
The error bars are computed by varying the χ^2 by $1/(N\text{-}p)$ around its minimum value. This yields the equivalent of the « 1σ » error for the gaussian distribution.

(the value of the confidence interval is not 63.7%, it depends on the number of measurements and on the number of parameters).

In the case of the one parameter model:

$$\chi^2\left(\text{Ø}_{UD} + \sigma_{\text{Ø}_{UD}}\right) = \chi^2_{\min} + 1/(N - p)$$

Model fitting examples



Use of closure phases

When available, closure phases can be included in the χ^2

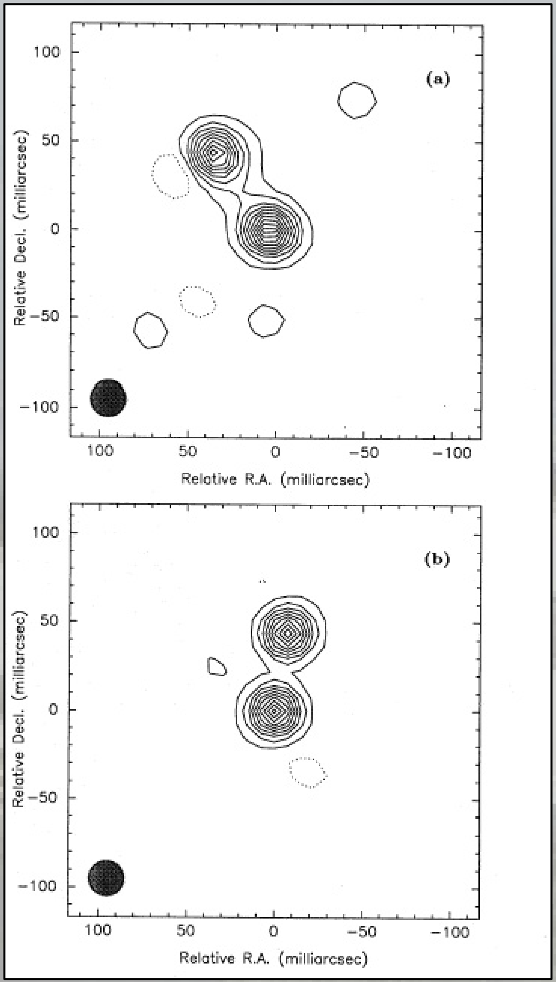
$$\chi^2 = \frac{1}{N-p}\left(\sum_{V_i^2}\left(\frac{V_i^2 - M_V(S_i)}{\sigma_i}\right)^2 + \sum_{\text{closure phases}}\left(\frac{\phi_i^{123} - M_\phi(S_i^{123})}{\sigma_i}\right)^2\right)$$

Minimizing this χ^2 is called *parametric imaging*.

If the model is a set of independent pixels then this is how images can be reconstructed.

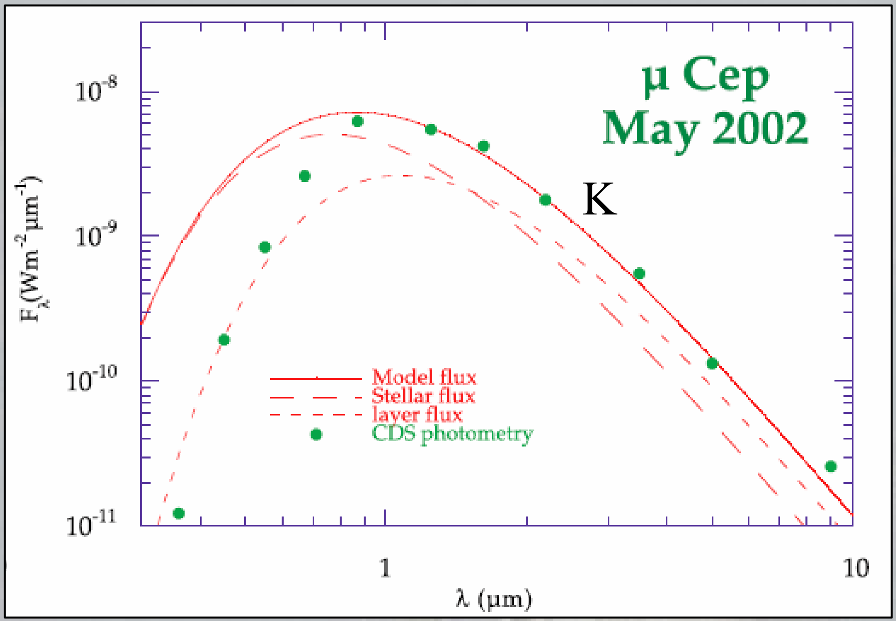
The relative weight between closure phases and visibilities can be adjusted (not a real χ^2 anymore)

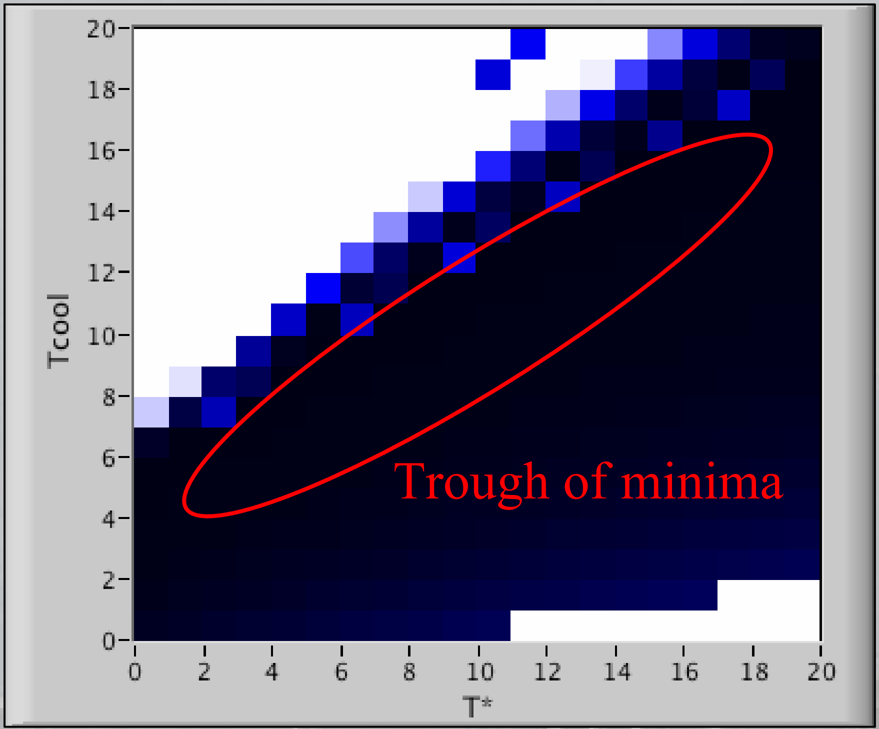
First image in optical interferometry
The binary star Capella seen by COAST



(Baldwin et al. 1996)

Parameter degeneracy





Visibilities are sensitive to the *relative* ratios of intensity between the photosphere and the layer which is a combination of optical depth and temperature.

The degeneracy is broken by forcing the model to comply with the flux of the source

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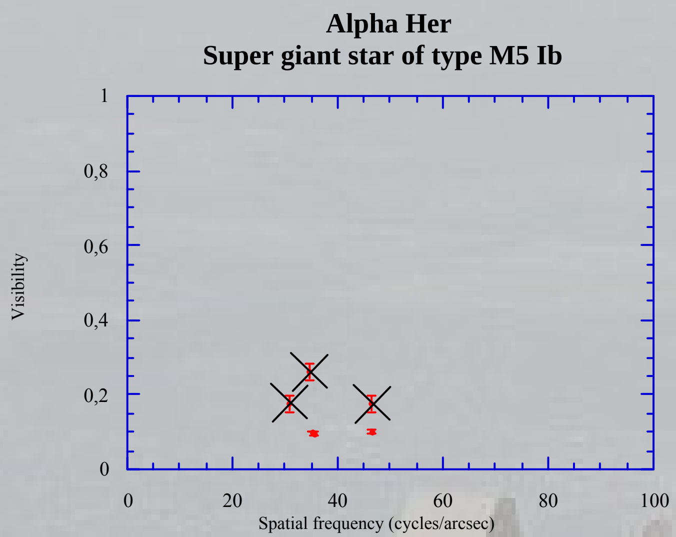
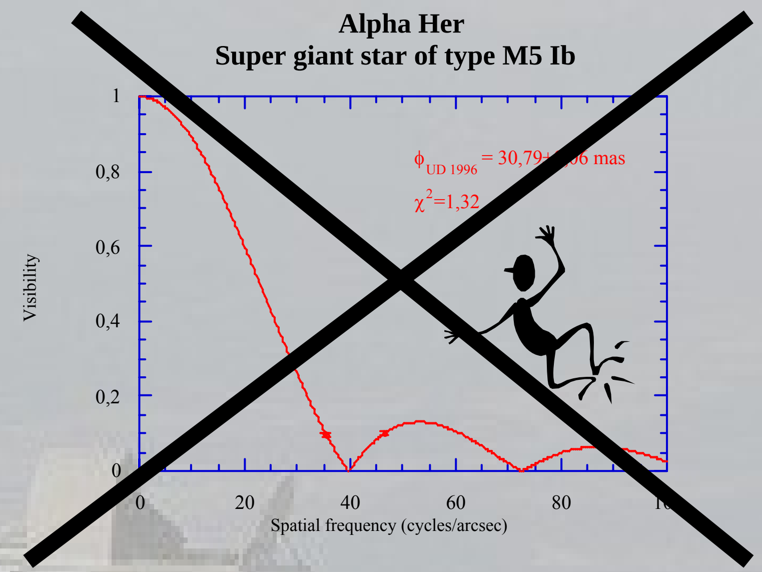
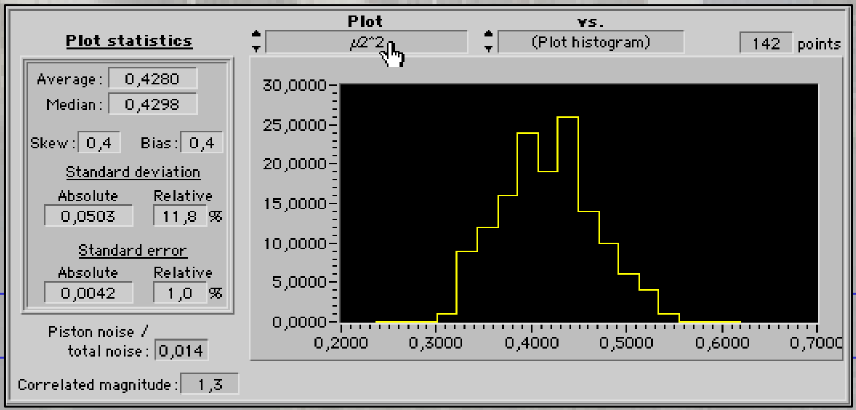
Rejecting bad data



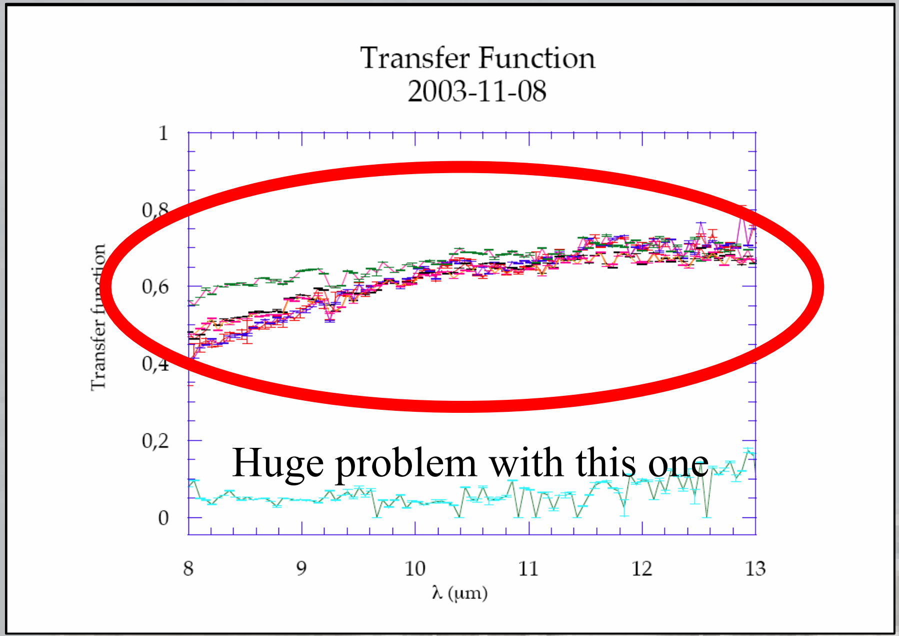


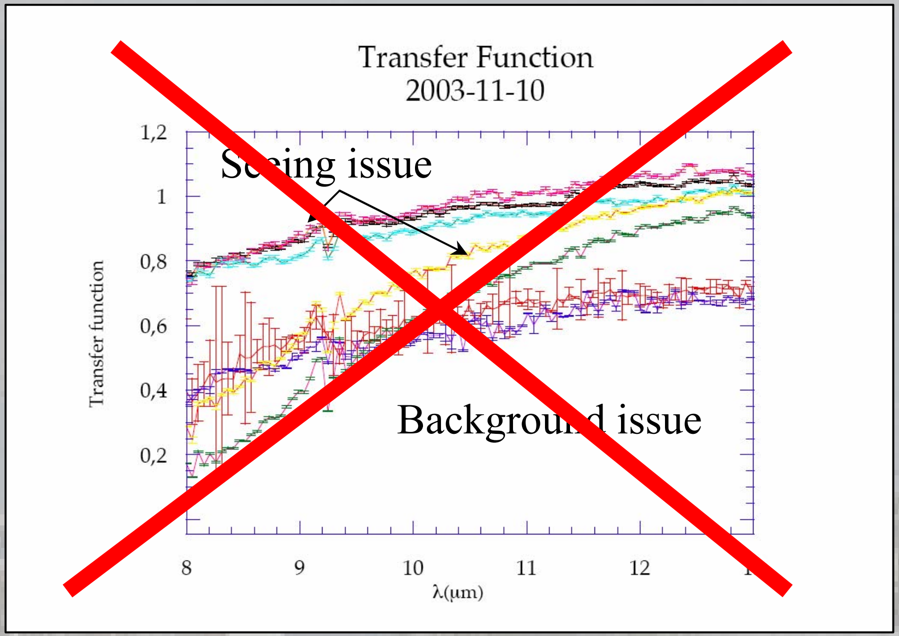
Examples of selection criteria:

- reject data for which the instrument was not stable (varying *transfer function*)
- reject data for which statistical distributions of *uncalibated* V^2 are not gaussian



Example of MIDI data: Betelgeuse





Same selection applied to the star data

Probable issue with error bar estimates

What to do if error bars are not well estimated

Error bars are first estimated for each series of scan (histogram method and propagation of errors).

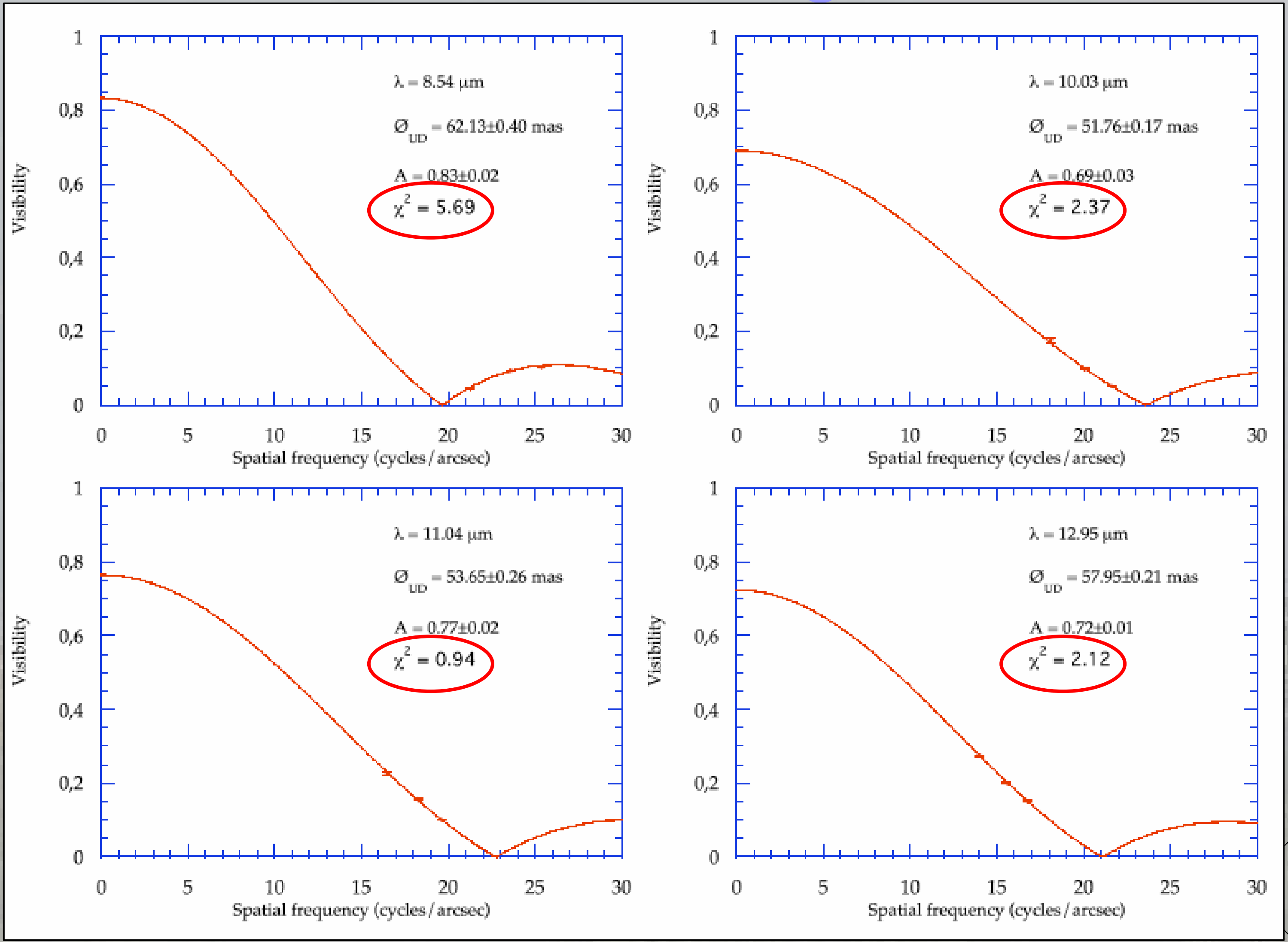
Visibilities are then binned by spatial frequencies -> several visibility estimates per bin.

The consistency of visibility sets per bin is checked:

$$\chi^2 = \frac{1}{N-1} \sum_{i=0}^{N-1} \frac{(M - X_i)^2}{\sigma_i^2}$$

If $\chi^2 > 1$ then the variance of the estimated average is multiplied by χ^2 to make the scattered visibility estimates consistent (probably one of the best among the worst methods !).

Result for Betelgeuse



Perrin et al. (≥2006)

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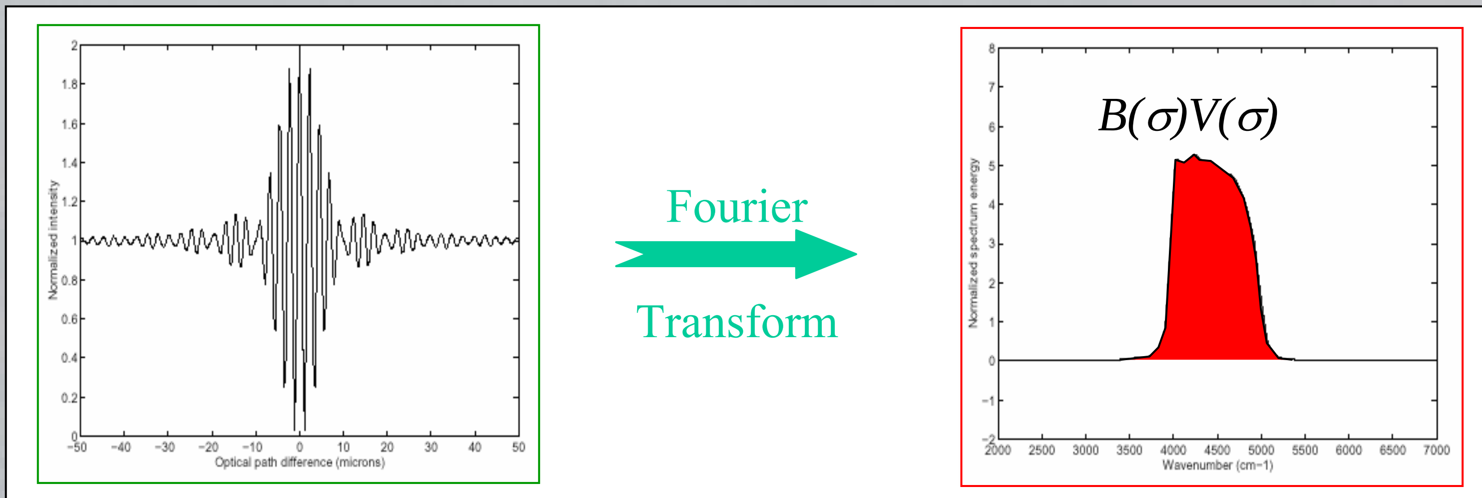
Caveat: model and visibility estimator (wide band)



Example: wide-band visibility estimators

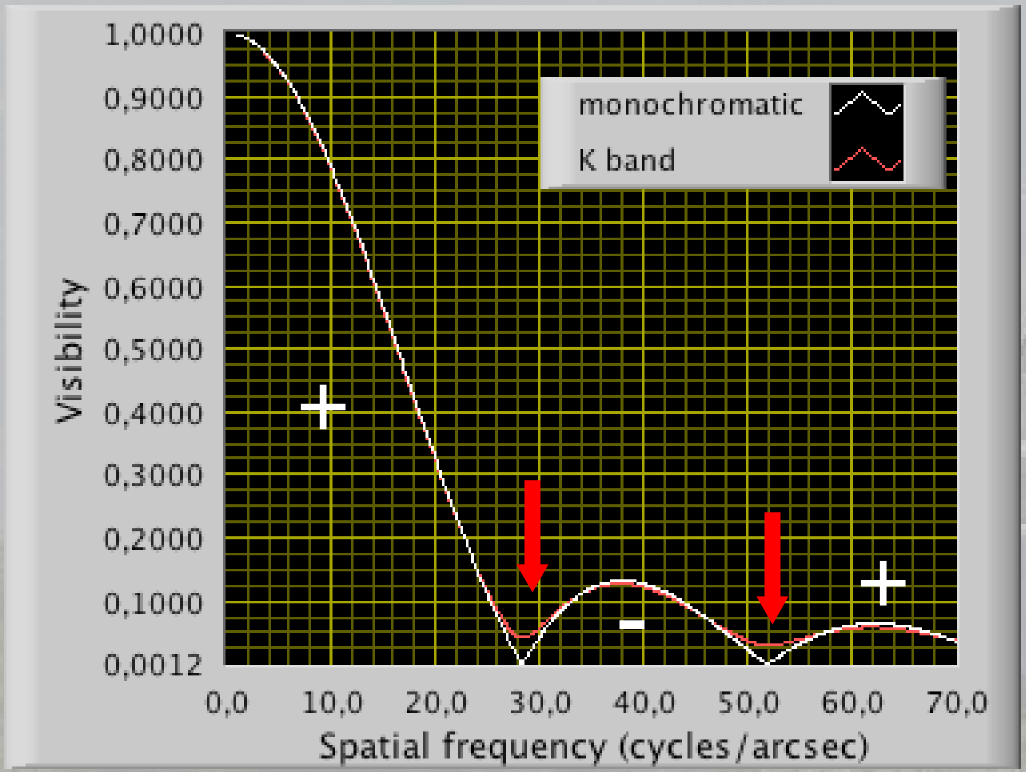
Estimator # 1 $$\tilde{V}^2 \propto \int_{band} |V|^2(\sigma) B^2(\sigma) d\sigma$$

Estimator # 2 $$\tilde{V}^2 \propto \left| \int_{band} V(\sigma) B(\sigma) d\sigma \right|^2$$

$B(\sigma)$ = source brightness spectral distribution

Example with the uniform disk visibility function in the K band

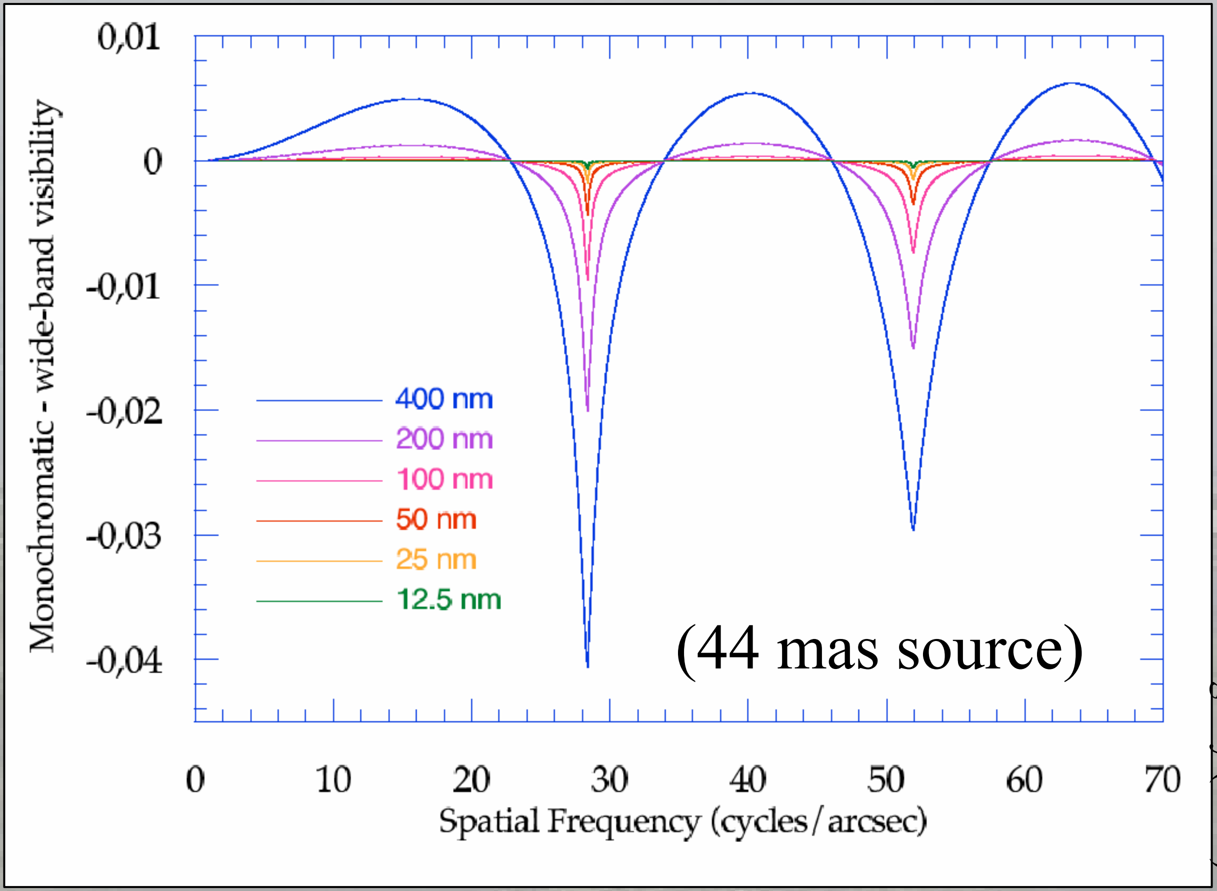
Wide band vs. Monochromatic estimator



$$\tilde{V}^2 \propto \int_{band} |V|^2(\sigma) B^2(\sigma) d\sigma$$

Example with the uniform disk visibility function in the K band

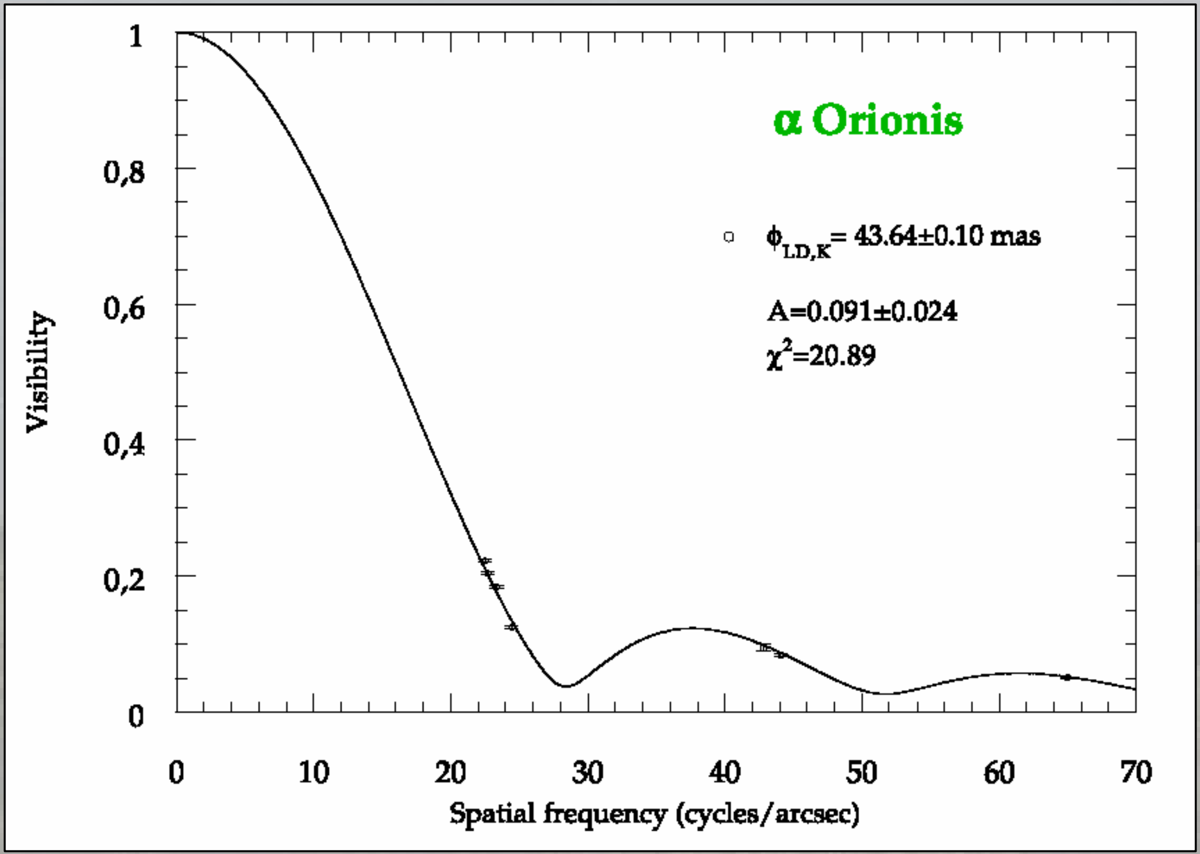
Wide band vs. Monochromatic estimator



Perrin & Ridgway (2005)

Errors and biases on fringe contrasts measurements

Wide band vs. Monochromatic estimator



Perrin et al. (2004)

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Correlated noise and relative interferometry

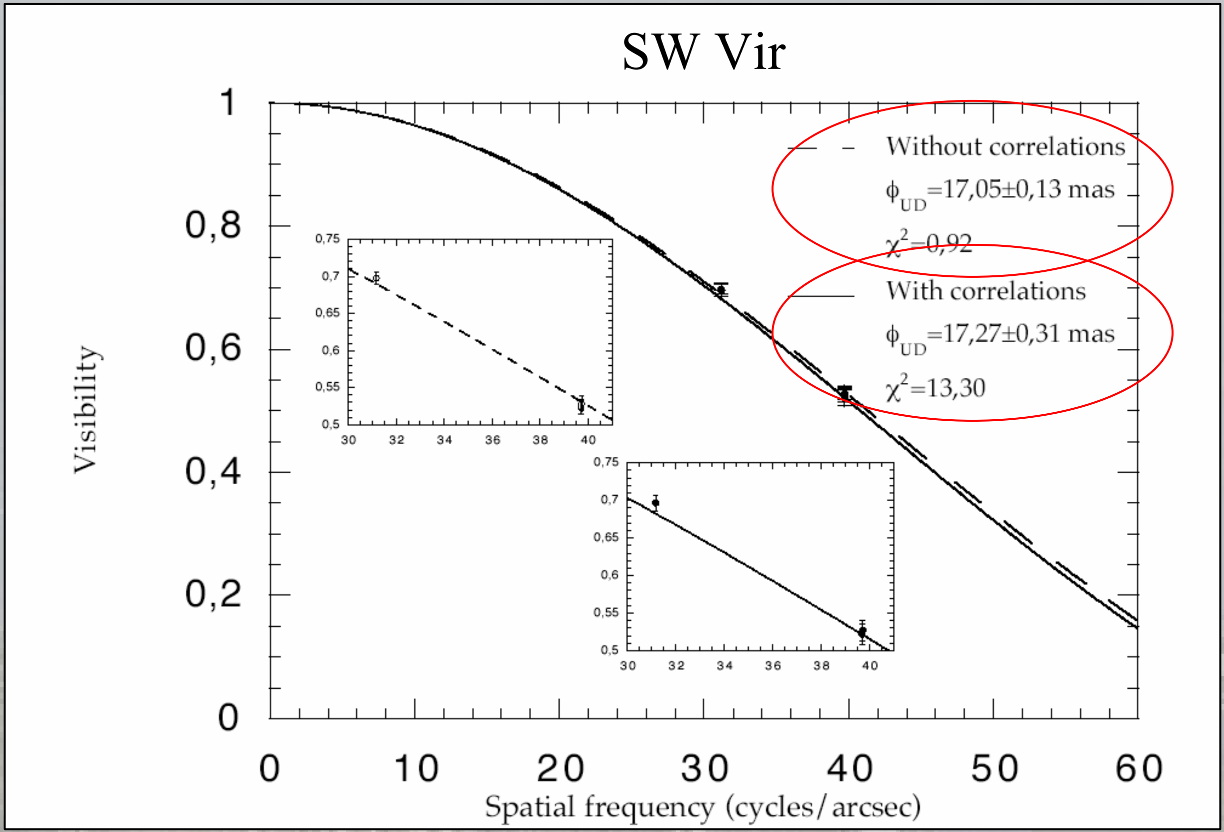
If different sets of visibilities have ***calibrators in common*** then different measurements have ***errors in common***

When fitting data, measurements cannot be assumed independent

➲ Lower accuracy on fitted parameters (correlated errors do not average down to zero)

However, systematic errors can be disentangled from statistical errors to improve accuracy on some relative parameters

Correlated noise



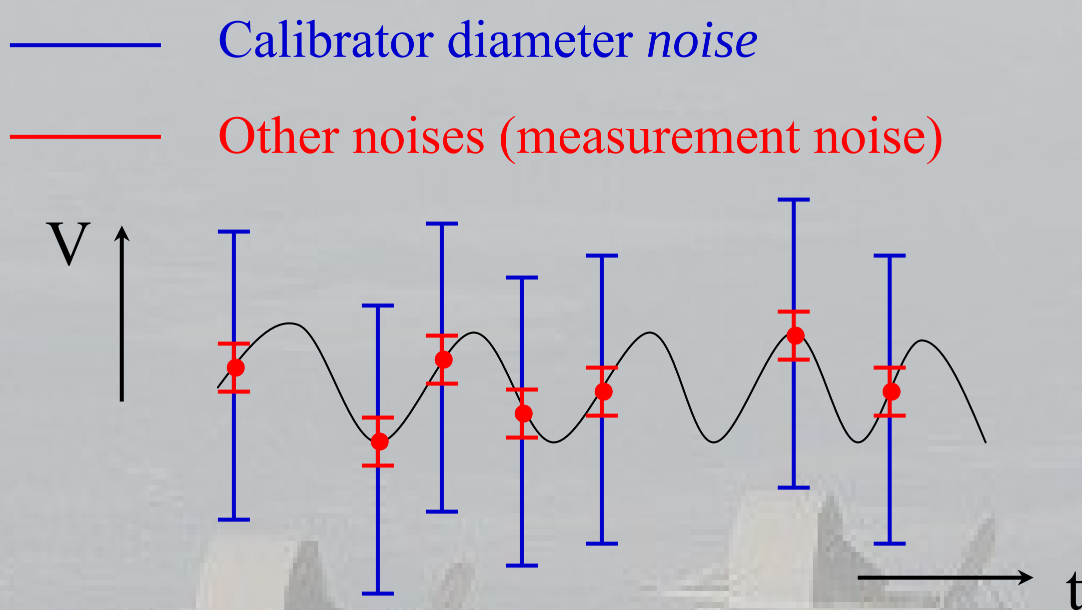
Perrin et al. (2003)

$$C = \begin{bmatrix} 1 & 0.96 & 0.96 \\ 0.96 & 1 & 0.97 \\ 0.96 & 0.97 & 1 \end{bmatrix}.$$

A single calibrator was used

Only 4% of the noise is uncorrelated

Correlated noise and relative interferometry



Absolute visibilities are consistent with a constant value:

- the absolute diameter (e.g.) can be determined whose accuracy is limited by that of the calibrator

The periodic modulation is compatible with relative visibilities

- relative diameter (e.g.) variation can be determined

Rather than using several calibrators, use of a single stable calibrator may be a good strategy to detect tiny variations

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Data modelling: how to choose a model ?

Two cases

1. The shape of the source is known

select a model that represents most of the visibility contests and put some effort in modelling the remaining bits

<u>Example:</u> Betelgeuse

2. The source is not known at this resolution scale and there is no good a priori

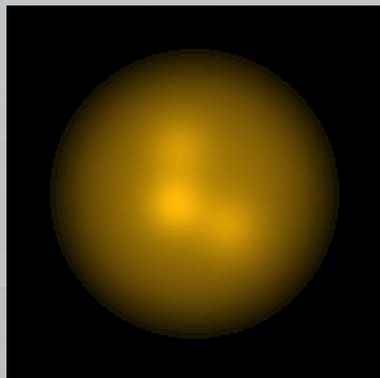
either it is possible to directly reconstruct a high fidelity image (radio case) or, try to analyse the visibilities with simple visibility models to find good hints on the nature of the object and then build a more complex model

<u>Example:</u> NGC 1068

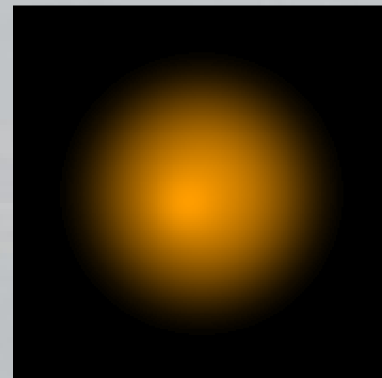
Facts about Betelgeuse

- Supergiant star of type M1.5
- Well-known diameter: from 50 mas in the UV to 44 mas in the NIR
- It has a dust shell (size ~ 1'' measured by the ISI interferometer at 11 μm)
- It is surrounded by a MOLsphere (NIR interferometry + ISI + spectroscopy)
- Spots have been detected at its surface in the visible and UV
- It has a limb-darkened disk

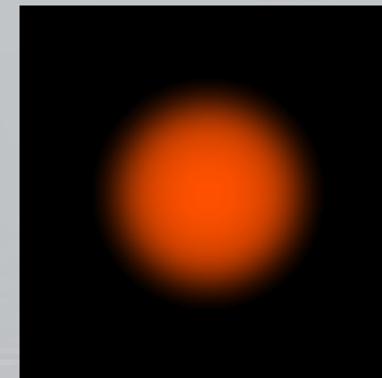
The asymmetries of Betelgeuse



700 nm

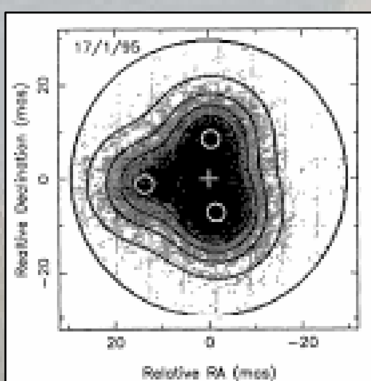


905 nm

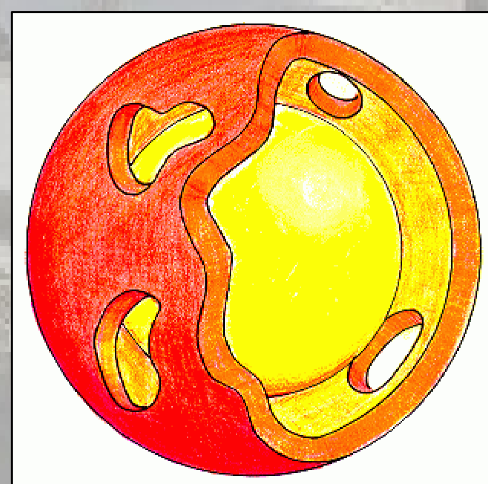


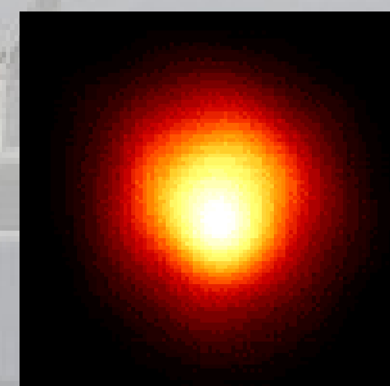
1290 nm

Young et al. (2000, COAST)



Wilson, Dhillon & Haniff (1997, 700 nm, WHT)

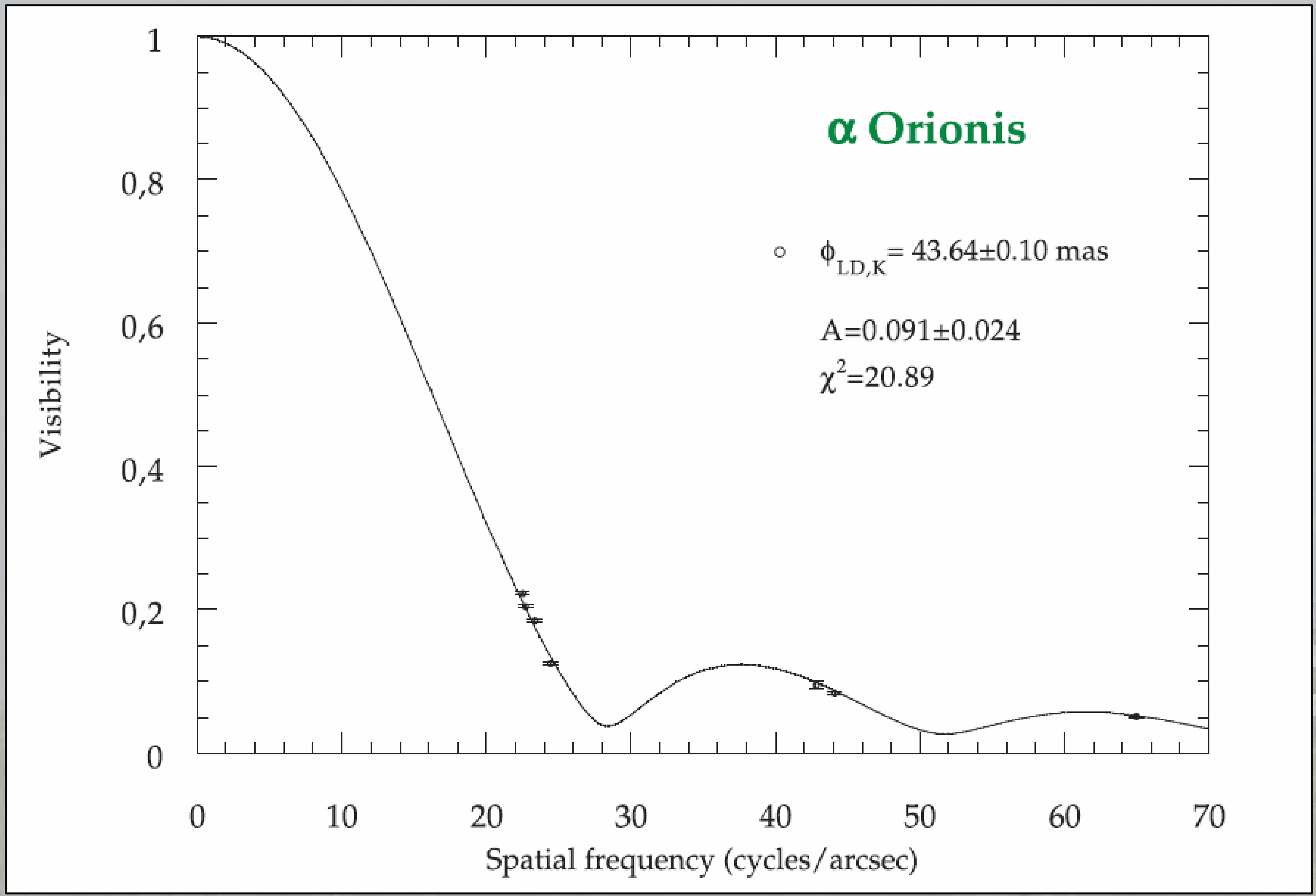




UV

Gilliland & Dupree (1995, HST)

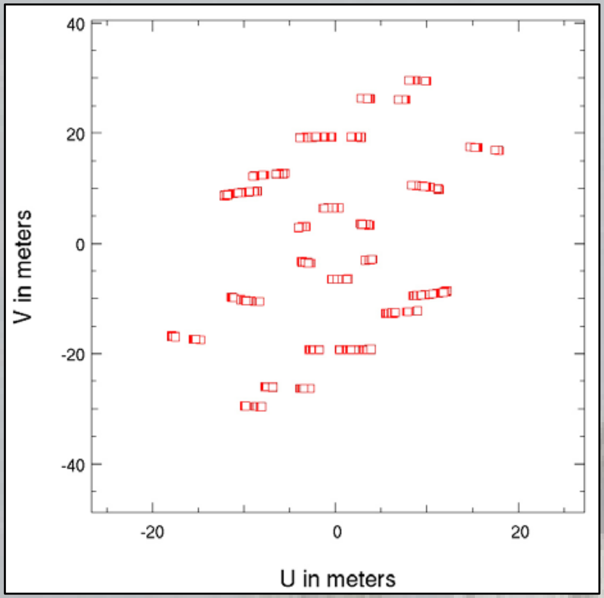
K band measurement

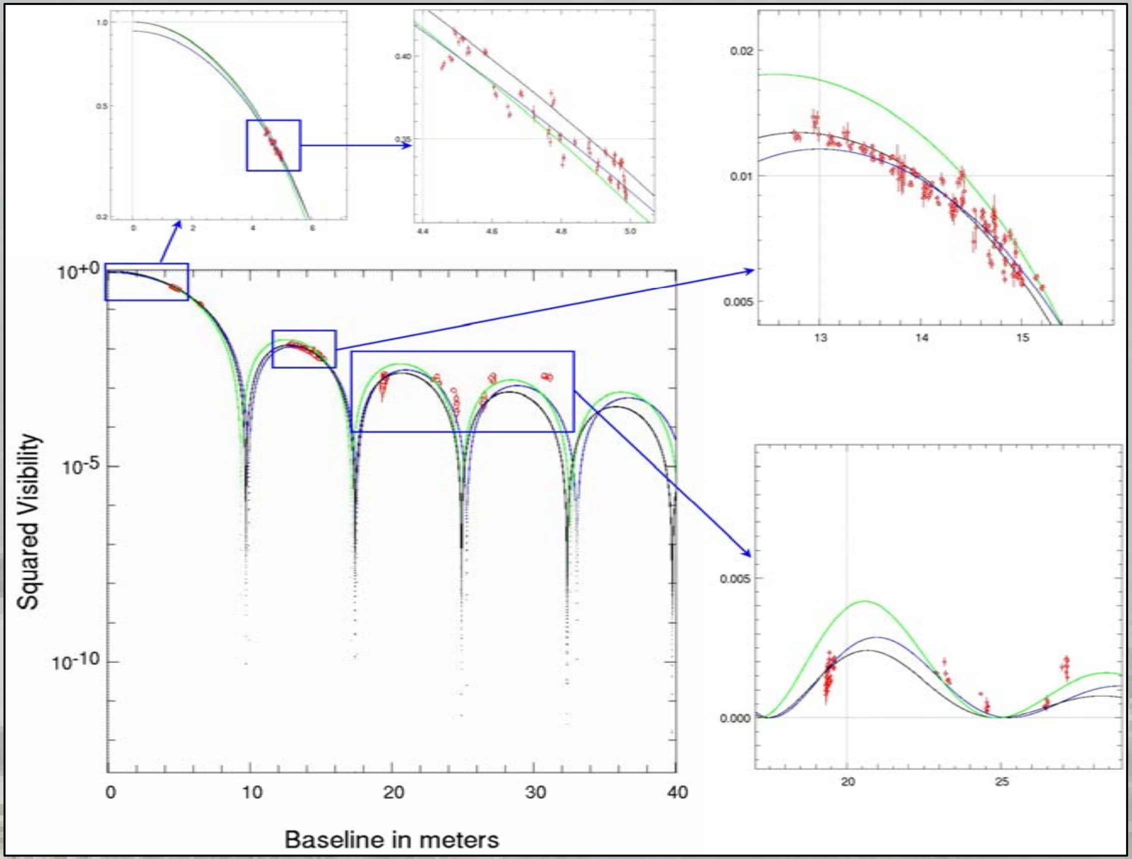


Perrin et al. (2004)

1.65 µm band data from IOTA

Haubois et al. (≥2006)

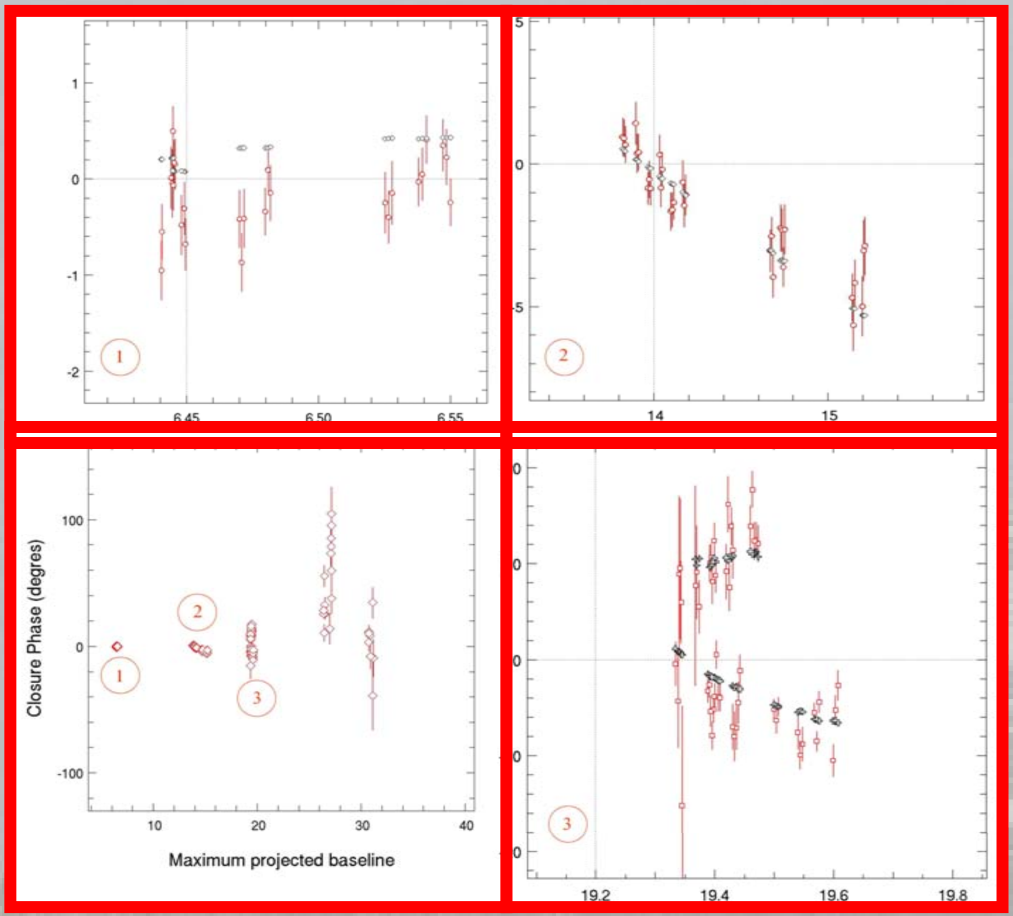


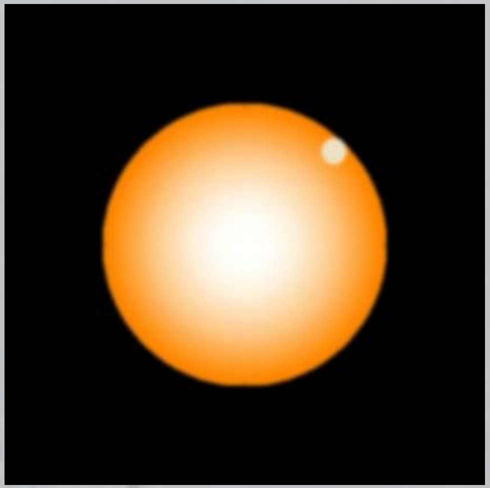


This is the baseline model

Green: UD model
Black: LD disk
Blue: LD disk + 5% total flux environment

Parametric imaging





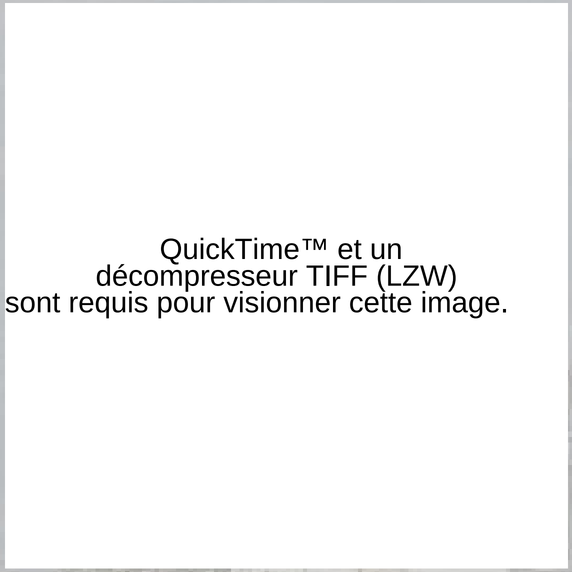
0.5% flux positive N-W unresolved spot

Or

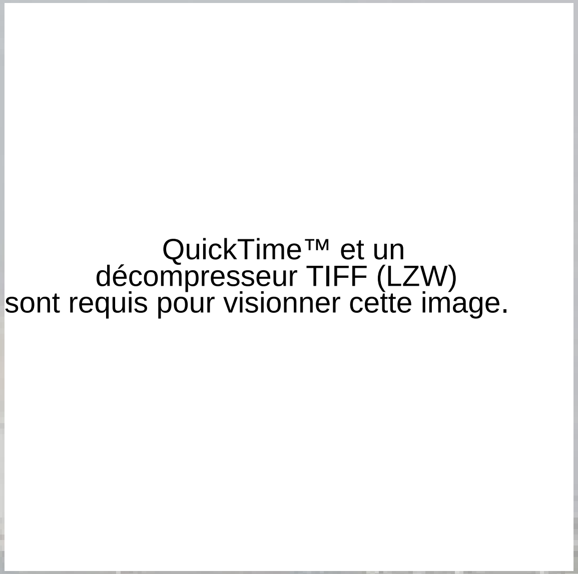
0.5% flux negative S-E unresolved spot

Direct image reconstruction

(S. Meimon)



Point source response



Reconstructed image

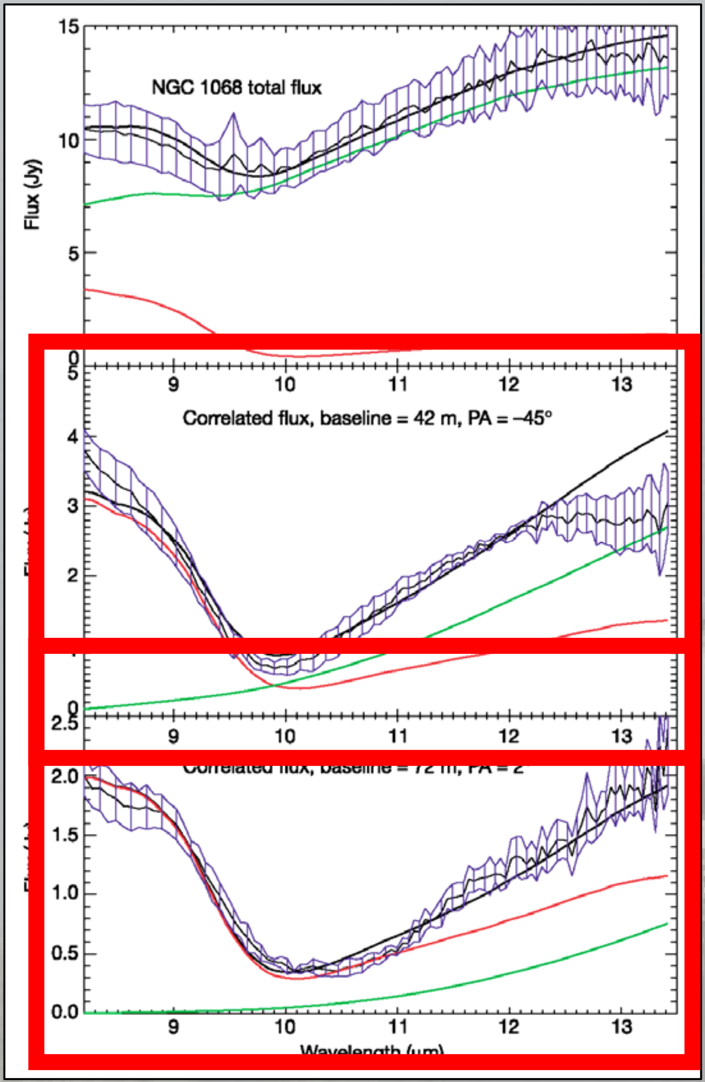
Parametric imaging and image reconstruction

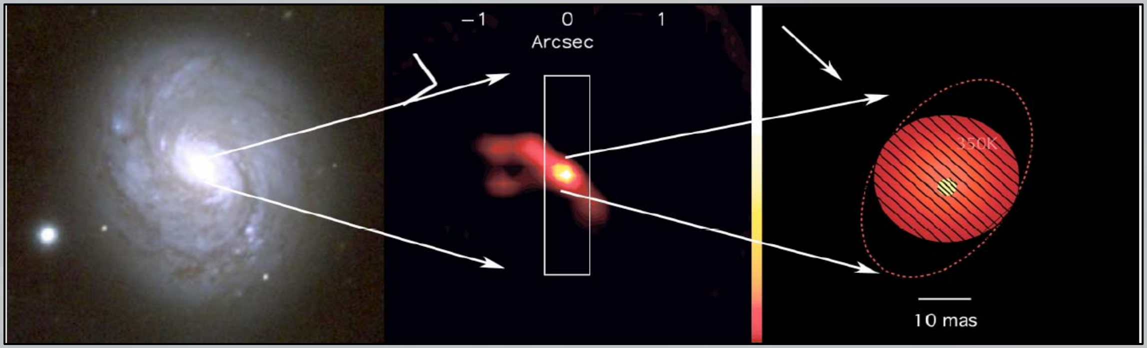
Betelgeuse is a simple example (although finding this low flux spot has been difficult)

Direct imaging will be useful for complex object to get a hint of what they look like (example: jets in YSOs, AGNs, …)

As long as we don't have very nice *uv*-plane coverages, parametric imaging will be a powerful tool to derive accurate quantities on the details of an image: spot flux, spot temperature, spot precise location …

Modelling NGC 1068 in the N band





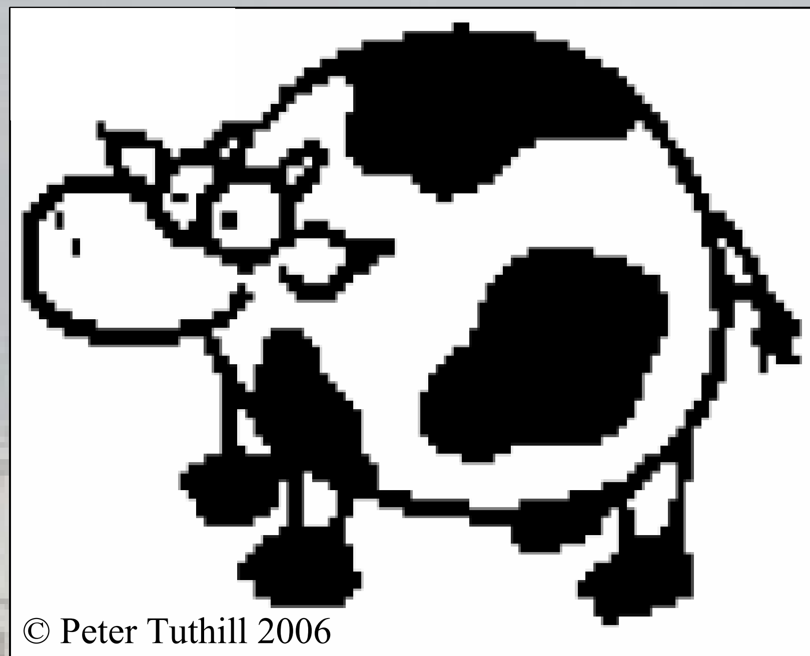
(Jaffe et al. 2004)

Choosen model:

2 elliptical gaussians (orientation provided by the larger scale jet direction) with the larger gaussian containing silicate dust grains (the core and the torus)

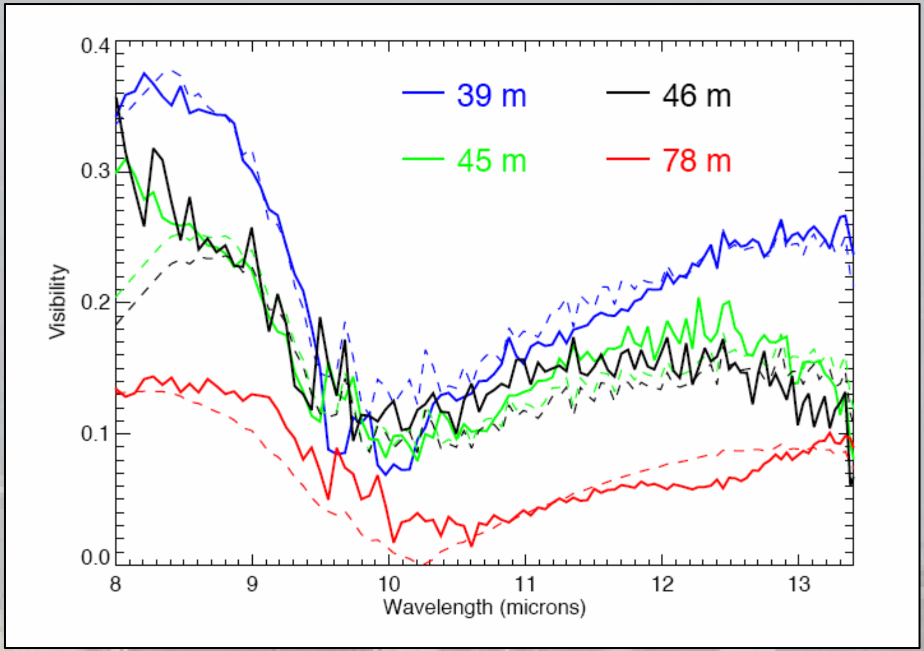
A lot of a priori information

A real *Spherical Cow* interferometric science example



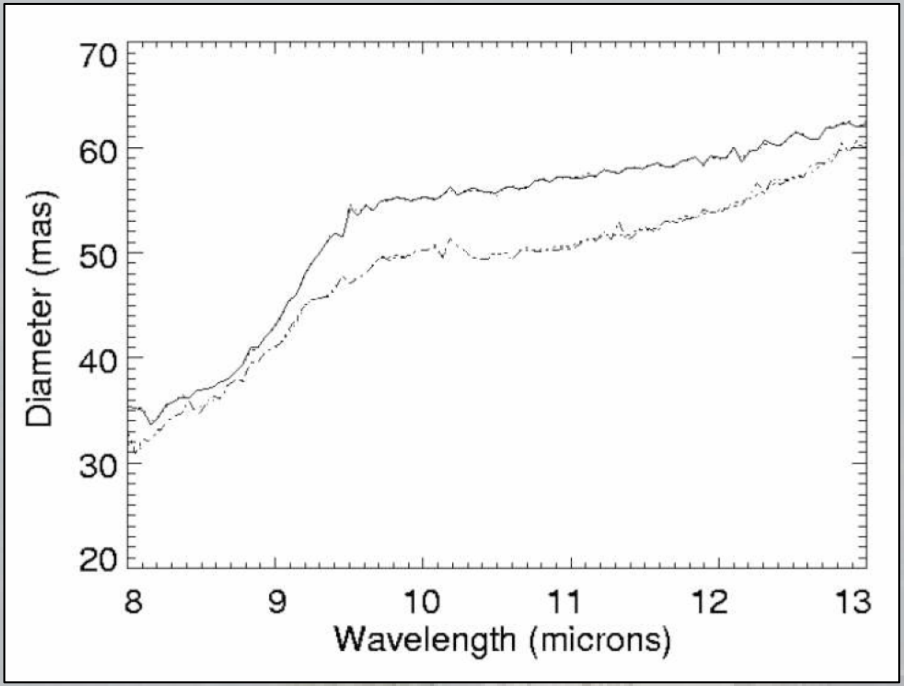
Spherical Cow modelling of NGC 1068

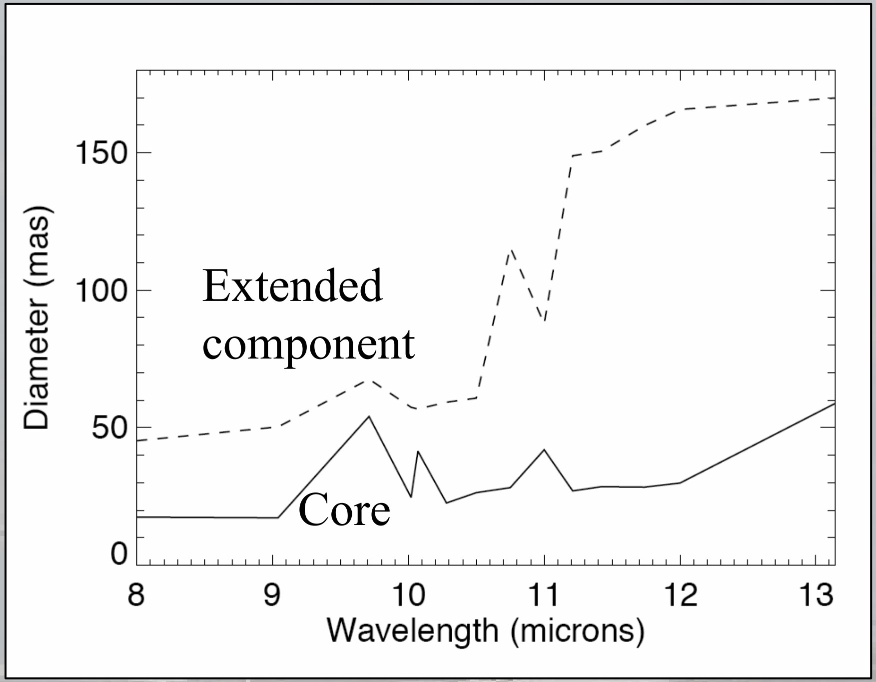
Poncelet, Perrin & Sol (2006)



Two more visibility spectra available compared to the Jaffe et al. (2004) paper

Spherical Cow modelling of NGC 1068





Fit of the visibilities with a λ-dependant UD model along each azimuth

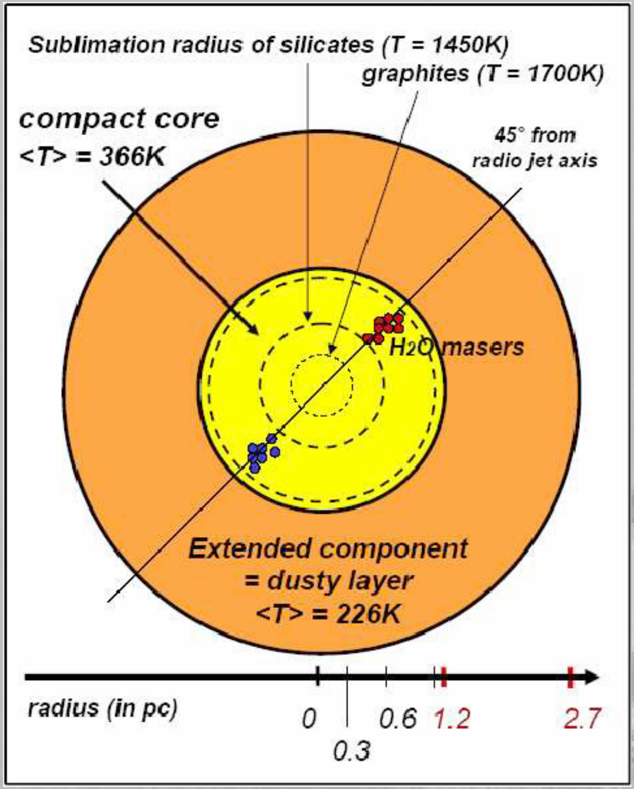
Elliptical shape not statistically significant

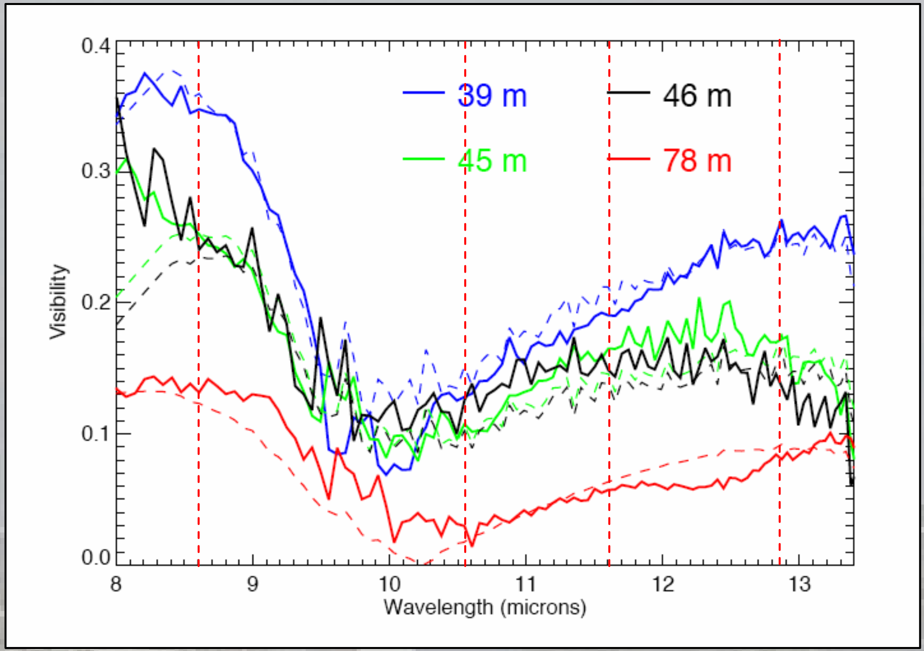
➲ Two scales are clearly visible: 30 mas (2 pc) and 60 mas (4 pc)

Fit of the visibilities with a 2 UD model + λ-dependant relative intensity

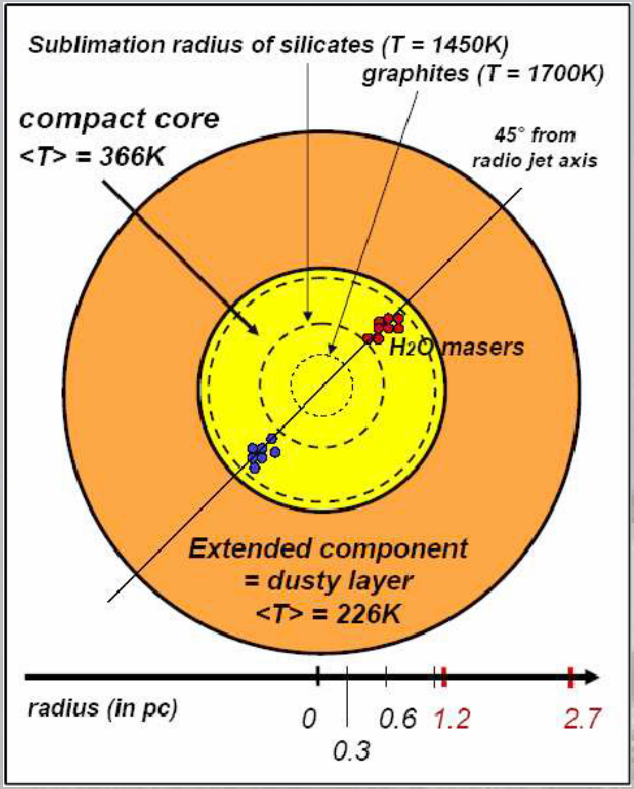
➲ Need for radiative transfer between the two components

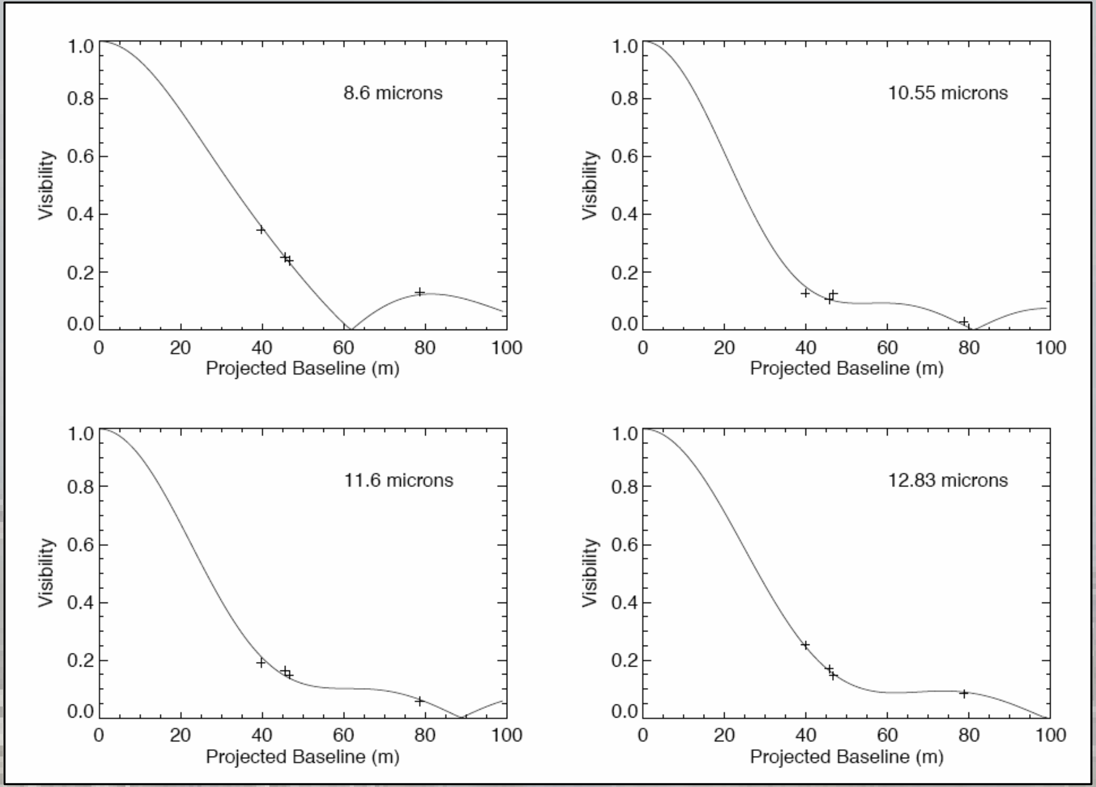
Spherical Cow modelling of NGC 1068



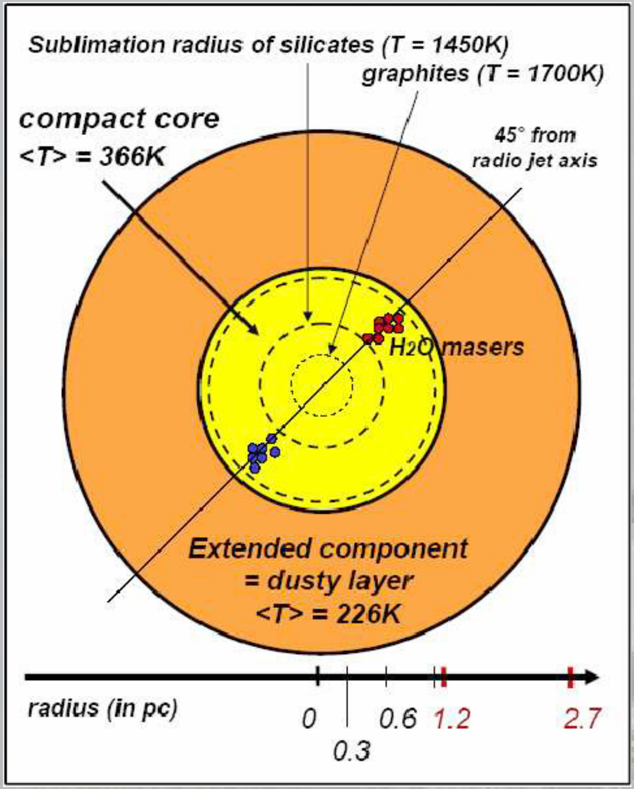


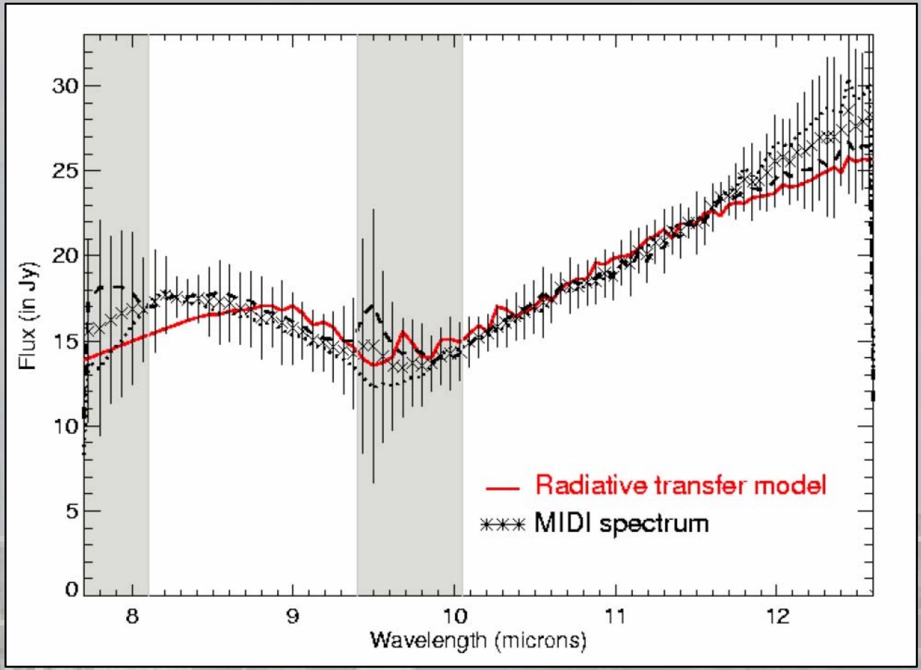
Spherical Cow modelling of NGC 1068





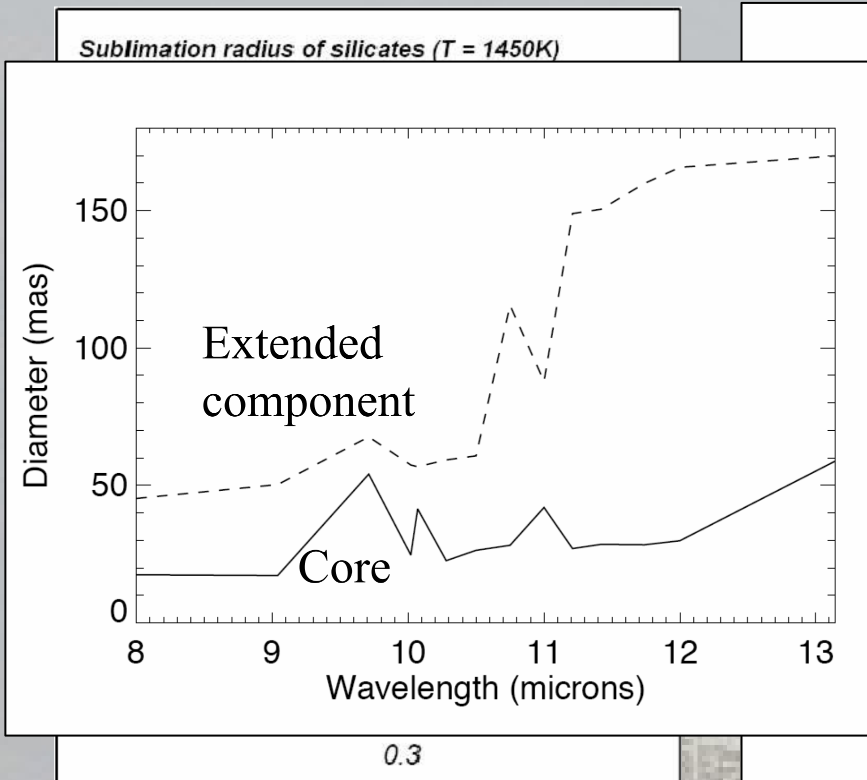
Spherical Cow modelling of NGC 1068

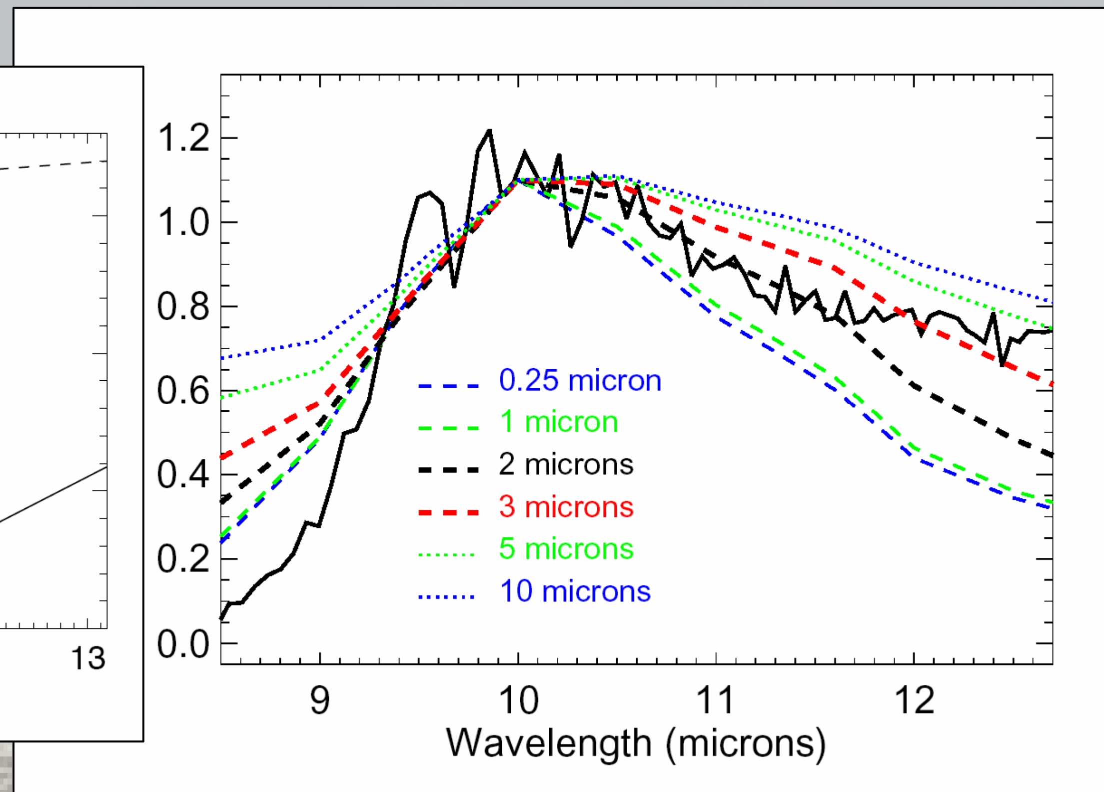




The observed spectrum was used to remove the degeneracy between the optical depth and the temperature

Spherical Cow modelling of NGC 1068





Spectrum of the optical depth of the extended component

Conclusion on the modelling of NGC 1068

We have detected a spherical cow at the center of NGC 1068 !

Very simple phenomenological model

But it is good to use as it requires less *a priori* to analyse the data.

A better *uv*-plane coverage (and closure phases) is needed to better assess (constraint) departure from spherical symmetry

A better *uv*-plane coverage is needed to better characterize the optical depth of the extended component.

Spatial frequency data close to zero are also required to measure the amount of uncoherent flux due to the larger scale structures

➲ observations with a single telescope

Some conclusions

Make sure data are well calibrated and have no bias

Beware of correlated noise (use of same calibrators), especially for high dynamic range modelling

Be careful (mostly in wide band) that your model and visibility data are based on the same estimator

Do not be afraid of object or data complexity.

Help us ask your funding agencies for more telescopes !