CALIFORNIA INSTITUTE OF TECHNOLOGY
1891

Polarization with Interferometry

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Outline

- The basics: how does the picture of interference change when the E field is a vector quantity.
- What kind of observables result from combining polarimetry with long-baseline interferometry?
- How can we turn a “normal interferometer” into one capable of polarimetric observations? (+examples)
- How well should OIP calibrate?
- What science areas can most benefit from OIP?

Recap: E-field as a scalar

- Fringes are formed by splitting the electric field in two, phase-shifting one component and interferring:

$$E_d = E_A + E_B \exp(i\delta)$$

$$I_d = \langle E_d E_d^* \rangle = |E_A|^2 + |E_B|^2 + 2|E_A||E_B|\cos(\delta)$$

- This enables a ‘visibility’ to be defined as a ratio:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Interference with Vectors

- All photons are polarized. Unpolarized light consists of a statistical distribution of photons in definite polarization states (incoherent sum, mixed quantum state...).
- The polarization state of light is a complex vector:

$$\mathbf{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \exp(i[\mathbf{k}.\mathbf{x} - \omega t]) = \begin{bmatrix} E_R \\ E_L \end{bmatrix} \exp(i[\mathbf{k}.\mathbf{x} - \omega t])$$

- The electric field can be split into 2 or more parts, and recombined. This is vector addition: interference!

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} \exp(i[\mathbf{k}.\mathbf{x} - \omega t]) + \begin{bmatrix} E_x^{'} \\ E_y^{'} \end{bmatrix} \exp(i[\mathbf{k}.\mathbf{x} - \omega t + \phi]) = \begin{bmatrix} E_x + e^{i\phi} E_x^{'} \\ E_y + e^{i\phi} E_y^{'} \end{bmatrix} \exp(i[\mathbf{k}.\mathbf{x} - \omega t])$$

Jones Matrices

- Optics operate on the electric field vector one at a time by matrix multiplication:

$$\mathbf{E}^{'} = \mathbf{J}_1\mathbf{J}_2 \ldots \mathbf{J}_n\mathbf{E}$$

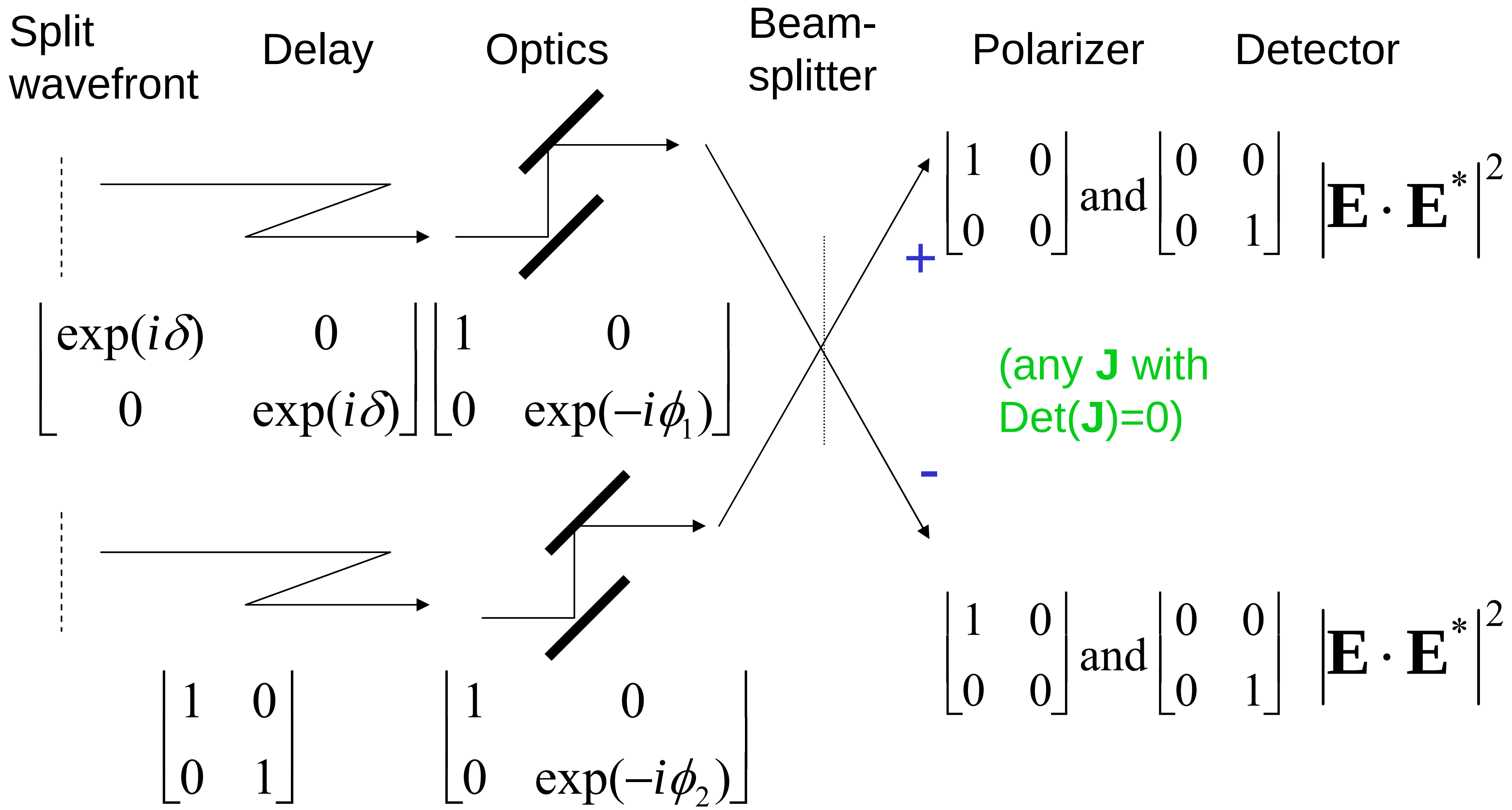
- Example Matrices:

Polarizer $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Retarder (Mirror) $\begin{bmatrix} 1 & 0 \\ 0 & \exp(-i\phi) \end{bmatrix}$

Image Rotator $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

Jones Matrices: Example.



Split wavefront

Delay

Optics

Beam-splitter

Polarizer

Detector

$$\begin{bmatrix}\exp(i\delta) & 0\\ 0 & \exp(i\delta)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \exp(-i\phi_1)\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 0\\ 0 & \exp(-i\phi_2)\end{bmatrix}$$

+

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} \text{ and } \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} \qquad \left|\mathbf{E}\cdot\mathbf{E}^*\right|^2$$

-

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} \text{ and } \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} \qquad \left|\mathbf{E}\cdot\mathbf{E}^*\right|^2$$

(any **J** with Det(**J**)=0)

Jones Matrices: Example.

- The intensities as a function of delay δ are:

$$I_{1,x} = \left|E_x\right|^2(1+\cos(\delta))$$

$$I_{1,y} = \left|E_y\right|^2(1+\cos(\delta-\phi_1+\phi_2))$$

$$I_{2,x} = \left|E_x\right|^2(1-\cos(\delta))$$

$$I_{2,y} = \left|E_y\right|^2(1-\cos(\delta-\phi_1+\phi_2))$$

⇒Phase Shifted Visibilities

- For polarizers at 45 degrees to x and y:

$$I_{1,x+y} = I_{1,x-y} = \frac{1}{2}(\left|E_x\right|^2+\left|E_y\right|^2)(1+\cos(\frac{\phi_1-\phi_2}{2})\cos(\delta-(\frac{\phi_1-\phi_2}{2})))$$

$$I_{1,x+y} = I_{1,x-y} = \frac{1}{2}(\left|E_x\right|^2+\left|E_y\right|^2)(1-\cos(\frac{\phi_1-\phi_2}{2})\cos(\delta-(\frac{\phi_1-\phi_2}{2})))$$

⇒ Visibility amplitude reduction

Mueller Matrices and Stokes Parameters

- In optical interferometry, we measure the response to partially-polarized sources, and measure intensity not electric fields.
- From the intensities in different polarization states, we get the Stokes parameters:

$$I = I_x + I_y = \left\langle \left| E_x \right|^2 \right\rangle + \left\langle \left| E_y \right|^2 \right\rangle$$

$$Q = I_x - I_y = \left\langle \left| E_x \right|^2 \right\rangle - \left\langle \left| E_y \right|^2 \right\rangle$$

$$U = I_{45} - I_{-45} = \left\langle E_x E_y^* + E_x^* E_y \right\rangle$$

$$= \left\langle 2 E_x E_y \cos(\delta) \right\rangle, \text{with } \delta \text{ the } y - x \text{ phase shift.}$$

$$V = I_L - I_R = i \left\langle E_x E_y^* - E_x^* E_y \right\rangle = \left\langle 2 E_x E_y \sin(\delta) \right\rangle$$

Mueller Matrices and Stokes Parameters

- A Mueller Matrix **M** transforms a stokes vector [I,Q,U,V] into a vector [I’,Q’,U’,V’]
- Any Jones matrix can be written as a Mueller Matrix, but Mueller matrices also allow depolarization of the electric field.
- We can also form visibility Mueller matrices: e.g. calculation of $\mathbf{M}_{11}$ for a “fringe signal” (visibility numerator $E_A E_B^*$):

$$\mathbf{M}_{11} = [0,1,0,0]\mathbf{M}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\left([1,1,0,0]\mathbf{M}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + [1,-1,0,0]\mathbf{M}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} - [1,1,0,0]\mathbf{M}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} - [1,-1,0,0]\mathbf{M}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \frac{1}{4}(([1,0]_{Ins}\mathbf{J}_A\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{Sky} \cdot ([1,0]_{Ins}\mathbf{J}_B\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{Sky})^* + \ldots)$$

Recap so far...

- Fringes are formed by adding (complex) electric fields and squaring.
- But, the E field is best represented as a vector. Fringes are formed by vector addition, and squaring the components.
- The action of optics on the E field is represented by multiplication by 2x2 complex Jones matrices.
- To consider intensities, we have to move from electric field vectors to Stokes parameters. These are intensity differences and sums of pure polarization states.
- The action of optics on Stokes parameters is represented by multiplication by 4x4 Mueller matrices. These are real for incoherent intensities, and complex for fringe signals.

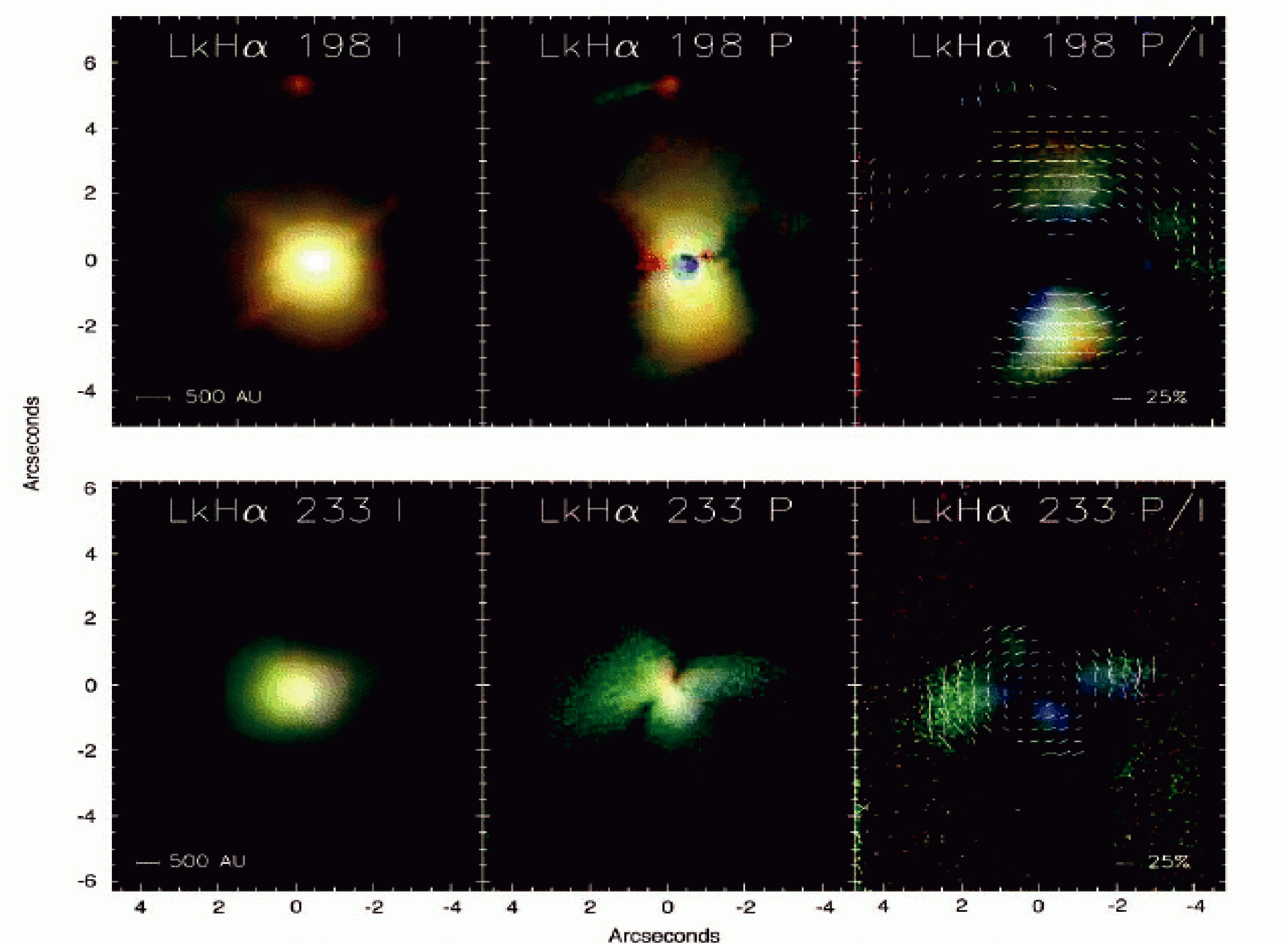
Visibilities in Stokes Parameters

- Visibilities are always a ratio. The numerator (fringe signal) and denominator (incoherent flux) can both be expressed in terms of Mueller Matrices.
- Define the Stokes intensity vector $\underline{I}$ and the Stokes fringe vector $\underline{F}$. Then a polarizaton state $\underline{P}$ (e.g. x:[1,1,0,0]) has one of 2 obvious visibility definitions. Only one of these has a non-zero denominator in general.

$$V_P = \frac{\underline{P}\mathbf{M}_F\underline{F}}{\underline{P}\mathbf{M}_I\underline{I}} \text{ or } \frac{\underline{P}\mathbf{M}_F\underline{F}}{I_0} \quad ??$$

NB Elias (2004) (25 page, 83 Eqn paper) uses the second, and calls $\underline{I}$: I_0 and $\underline{F}$: I_{12})

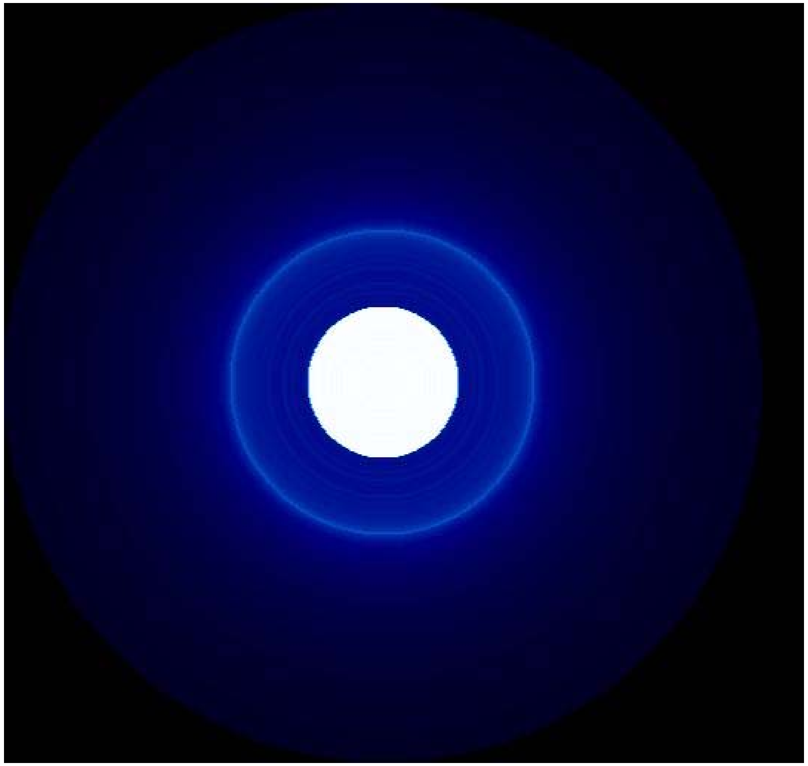
Astronomy with OIP



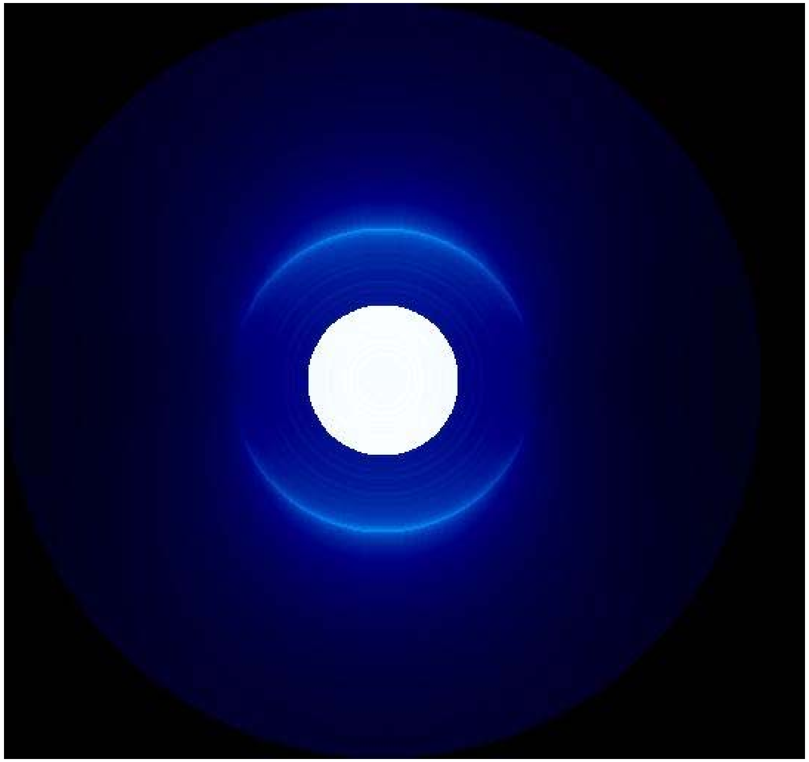
Herbig Ae/Be Stars from Perrin et al (2004)

Circumstellar Scattering

Unpolarized light

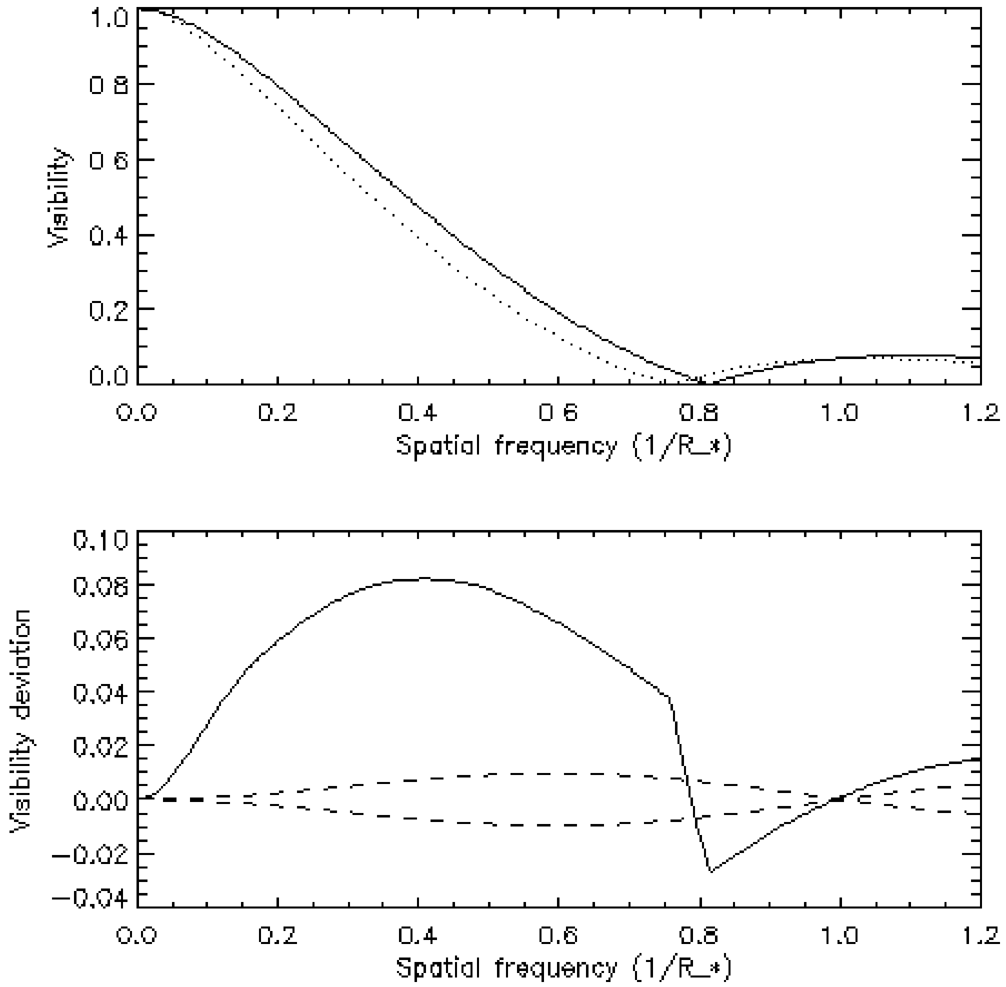


Polarized light



OIP Visibility Curves

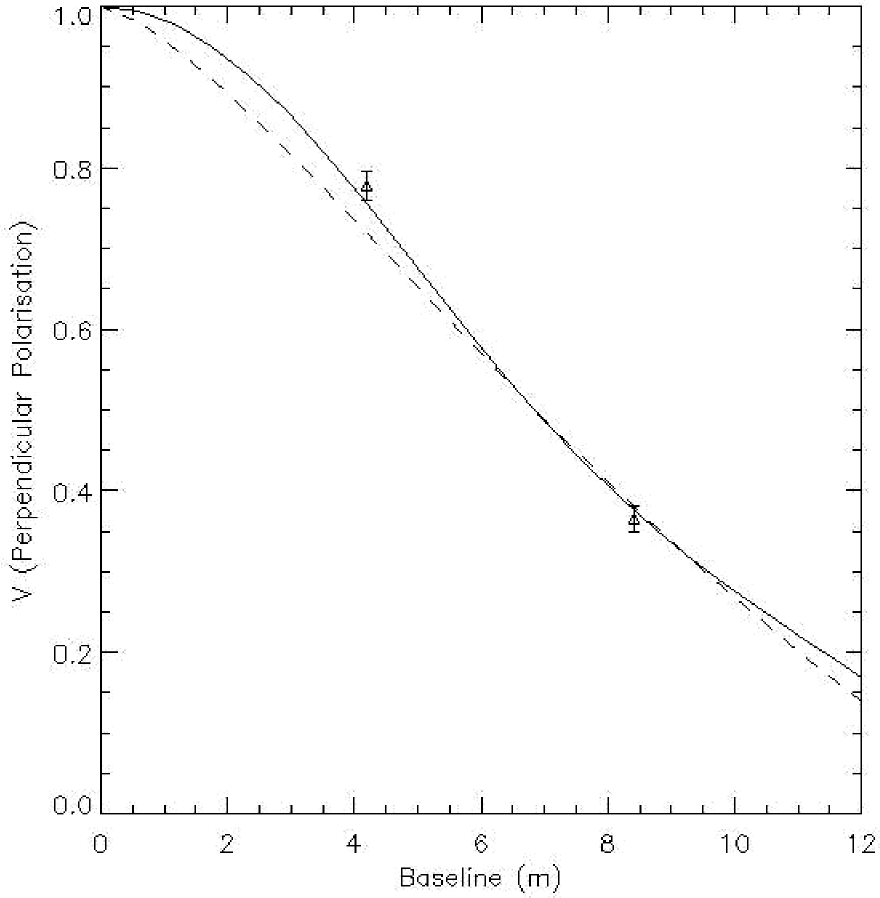
- In the case of spherical symmetry, we can predict visibility curves for linear polarization parallel and perpendicular to the baseline
- Obvious examples: winds from Hot stars, scattering around Mira variables...

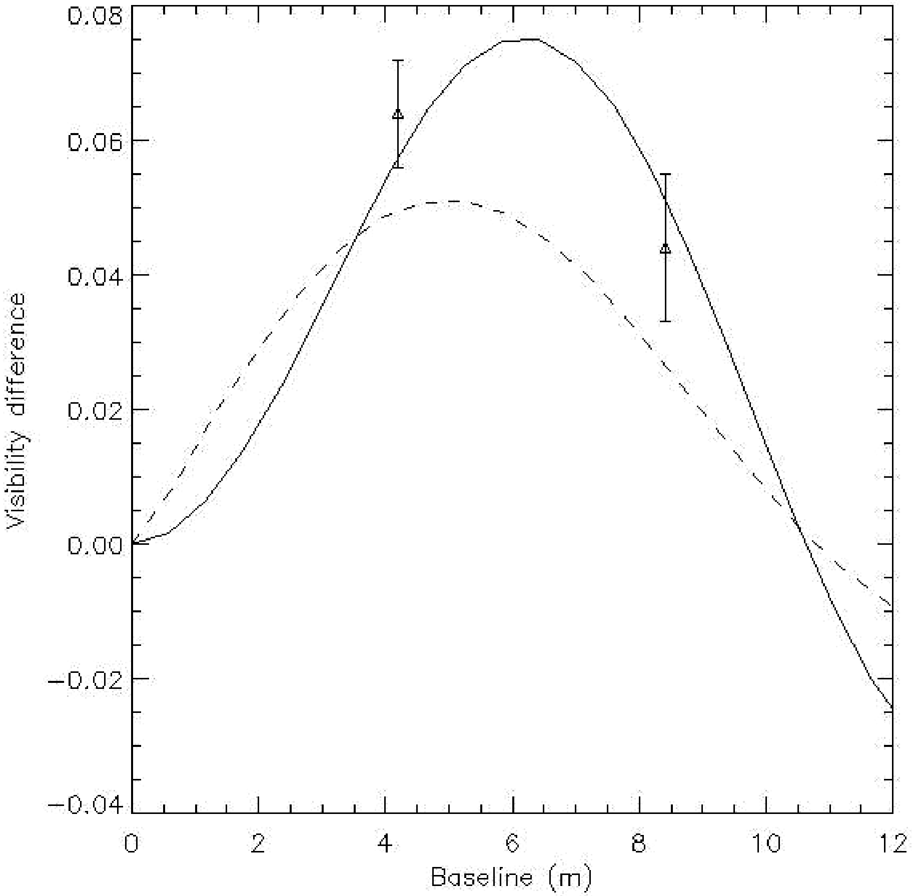


P Cyg Prediction (Chesneau 2003)

SUSI Results (R Car)

Solid lines: Thin shell model, Dashed lines: Outflow model

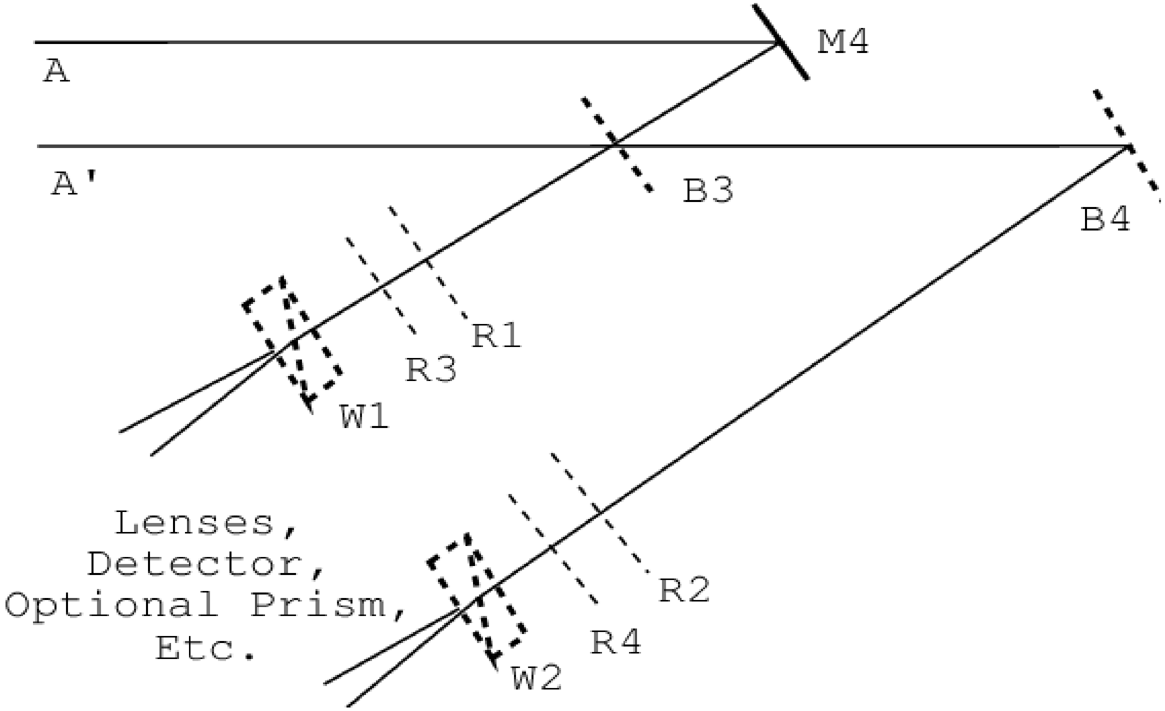




Ireland et al (2005)

Implementation at PTI

- R1, R4 : half-wave plates. R3 : LCVR. R2: QWP. W1, W2 : YVO_4 Wollastons.
- Enables Measurement of arbitrary polarization states.



Calibration Strategy 1: Raw Observables

- Complex visibility ratios are *un-affected by the atmosphere* or instrumental polarizations.

$$\mathbf{M}_F(\text{seeing}+\text{optics}) = f(\text{seeing})\mathbf{M}_F(\text{optics})$$

- Simple strategy: define back-end observables, and fit on-sky models for intensity and fringe signal directly to these observables. E.g. for a measurement of V'_x/V'_y

$$\frac{V_x^{'}}{V_y^{'}} = \frac{F_x^{'} I_y^{'}}{I_x^{'} F_y^{'}} = \frac{[1,1,0,0]\mathbf{M}_F \vec{F} \cdot [1,-1,0,0]\mathbf{M}_I \vec{I}}{[1,1,0,0]\mathbf{M}_I \vec{I} \cdot [1,-1,0,0]\mathbf{M}_F \vec{F}}$$

Calibration Strategy 2: ‘Intuitive’ Stokes Observables

- Assume that the polarimetric signal is small, i.e.

$$\frac{V_Q}{V_I} << 1, \frac{V_U}{V_I} << 1, \frac{V_V}{V_I} << 1$$

...and that as a good approximation $\underline{I}=[I_0,0,0,0]$

- Then:

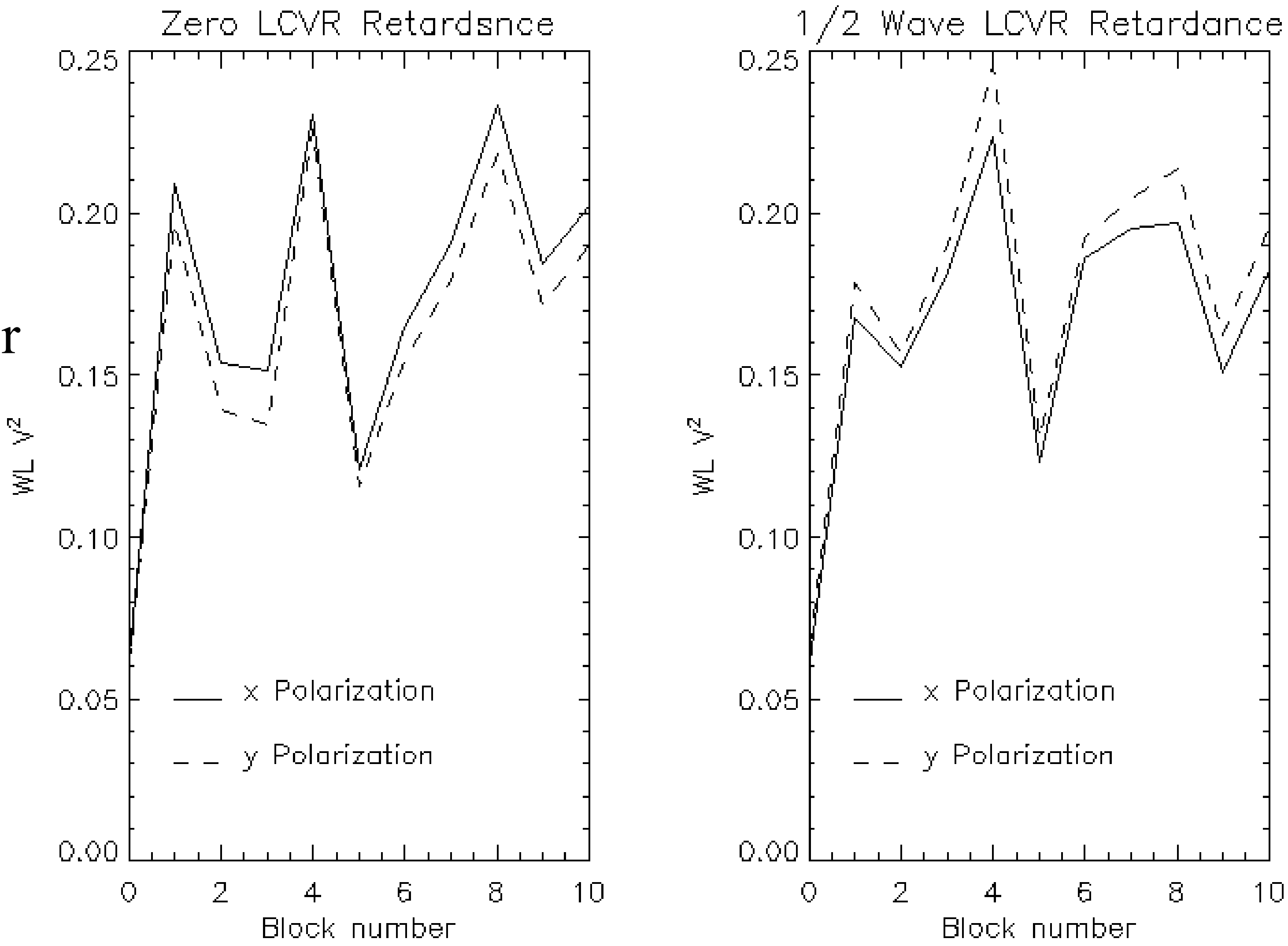
$$\frac{V_Q^{'}}{V_I^{'}} = \frac{1}{2}(1 - \frac{V_y^{'}}{V_x^{'}})$$

$$\frac{V_Q^{'} V_{I,C}^{'}}{V_I^{'} V_{Q,C}^{'}} = [a,b,c,d] \frac{\underline{F}}{I_0 F_0}$$

...where [*a,b,c,d*] is calculated from the system fringe Mueller matrix $\mathbf{M}_F$ (work out yourself or ask me later).

Real PTI Example

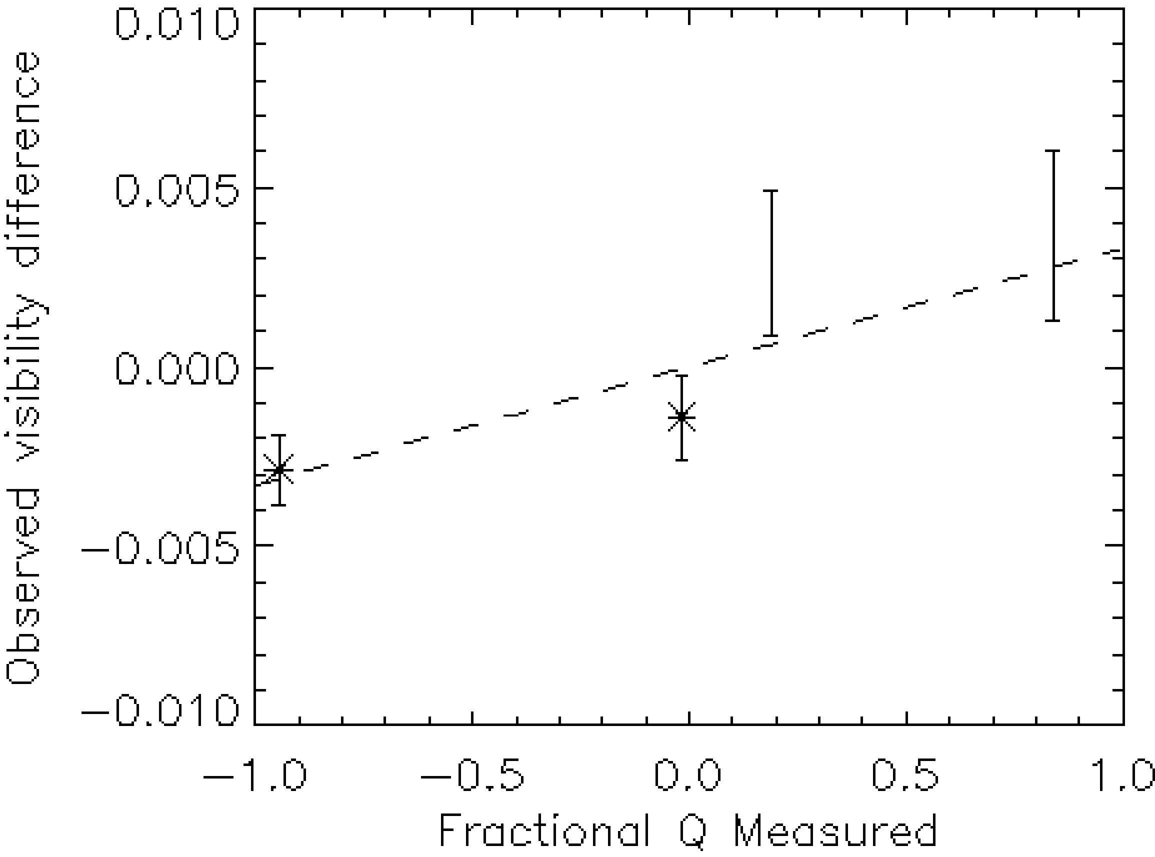
HD 215373 (calibrator) H-band, ~10s integration per Point.



Real PTI Example

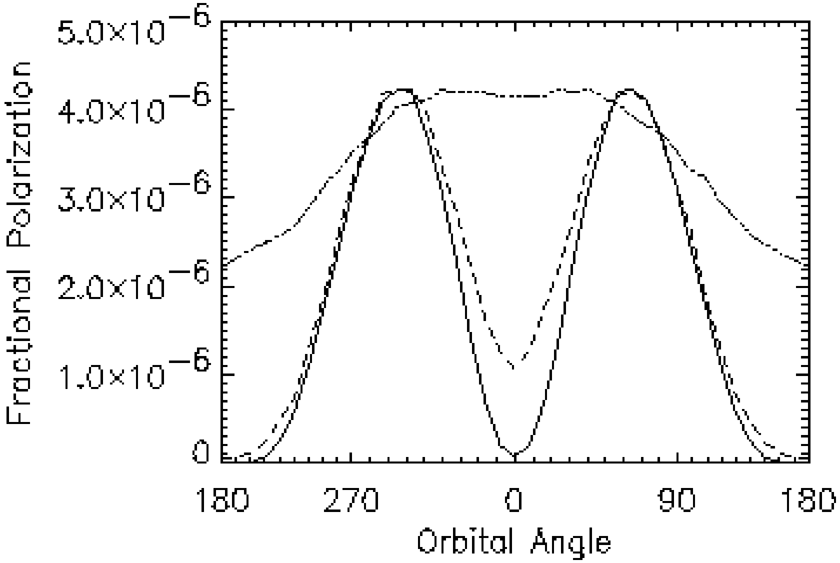
- When visibility ratios are converted to on-sky V_Q values.
- A signal of 0.003 with an error of 0.001!

S Lac “Q” parallel-perpendicular

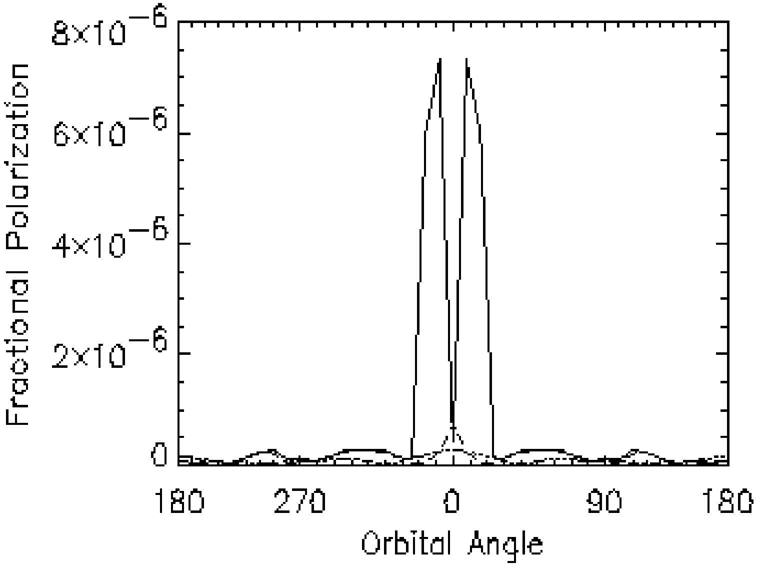


The Future: Planetary Signals?

- A detection of polarized light from a CEGP will give a clear determination of grain size in the dust clouds.



λ = 5 times grain radius



λ = 0.05 times grain radius

Seager et al. (2000)

Summary

- The E-field is a vector, which can be manipulated by multiplication by Jones matrices and vector addition.
- The interferometer response to intensity is best characterized using Mueller matrices.
- OIP is ideal for the science of circumstellar scattering.
- OIP is a differential technique, that cancels-out atmospheric effects (high precision).
- There are several ways that data can be calibrated: the final observables are functions of Stokes visibilities and not the visibilities themselves