An introduction to closure phases

Michelson Summer Workshop

Frontiers of Interferometry: Stars, disks, terrestrial planets

Pasadena, USA,
July 24th-28th 2006

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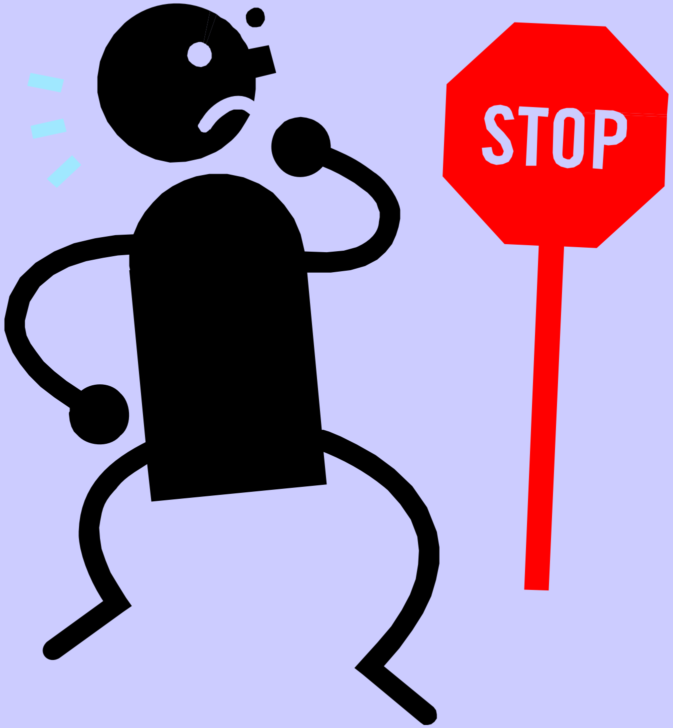
25th July 2006

Crawl, walk, run



Outline

- Why closure phases:
  - The effect of the atmosphere.
  - Telescope-based errors.
- Closure phases:
  - Definition.
  - Importance.
- Uses:
  - Heuristic interpretation.
  - Toy examples.
  - Imaging.

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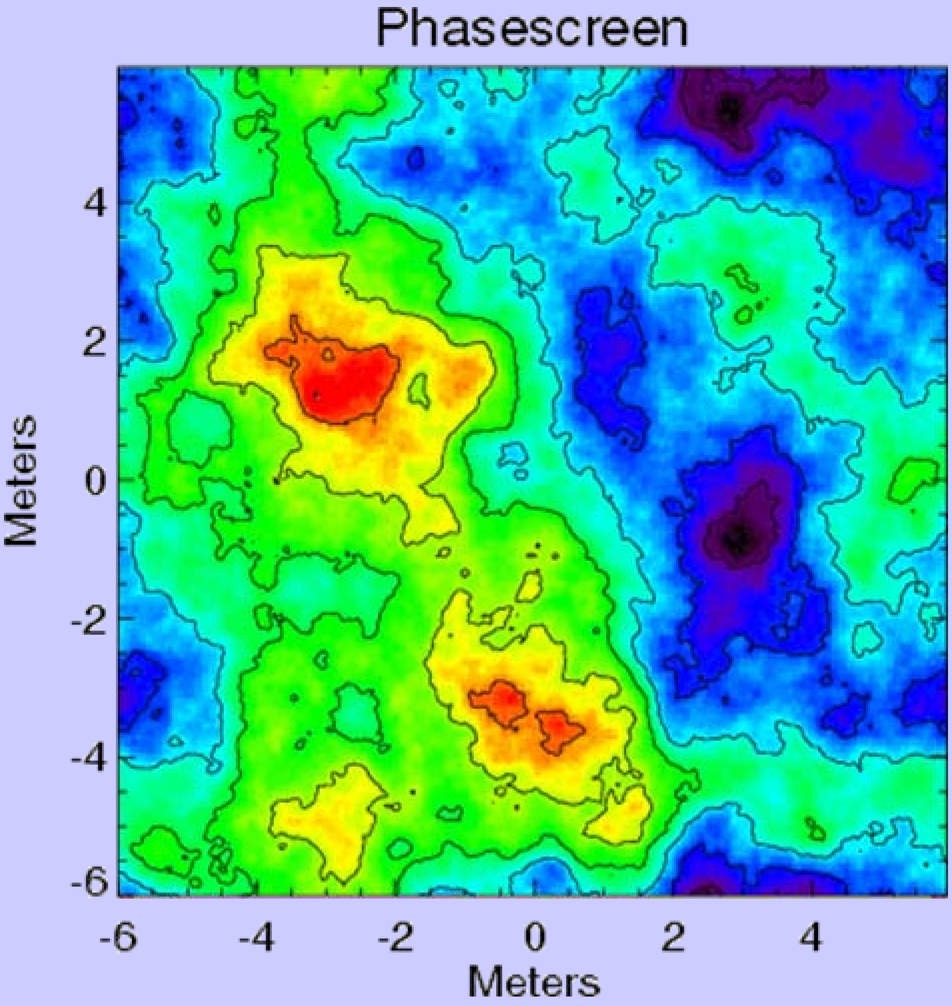
Starting points

- Interferometers measure the Fourier components of the sky brightness distribution.
- These are complex quantities, and so we would like, in principle, to measure both the amplitude and phase of these Fourier components (i.e. the amplitude and phase of the visibility function).
- The problem is that the atmosphere perturbs the electric fields sampled by the interferometric collectors and this impacts the measurements of the visbility function.

Note that, under all circumstances, the visibility phase can be measured – the problem is that its value no longer depends only on the source structure, but on lots else!

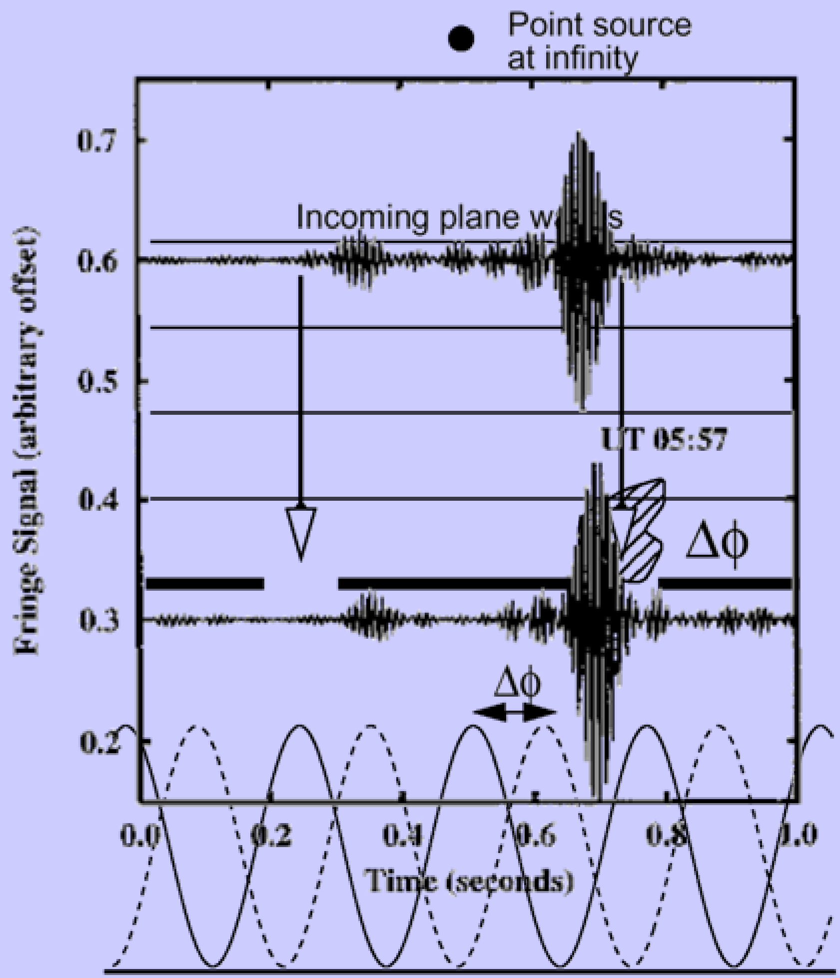
The atmosphere

- A 12m × 12m patch of good atmosphere.
- Each contour represents one radian of phase delay for light at a wavelength of 2 microns.



What this does

- So the effect of the atmosphere is to introduce an unknown phase delay above each interferometric collector.
- In the cartoon on the right, we depict what happens when only one aperture is affected.
- This gives a shift in the output of the interferometer away from the expected white-light position.



Dyck et al. 1995 (IOTA).

Modelling this antenna-based corruption

- Key idea is that electric field is altered in amplitude and phase:

$$\begin{aligned}\tilde{E}_i^{\mathrm{measured}} &= \tilde{G}_i \tilde{E}_i^{\mathrm{true}} \\ &= |G_i| e^{i\Phi_i^G} \tilde{E}_i^{\mathrm{true}}.\end{aligned}$$

Telescope Gain, e.g., coupling efficiency into single-mode fiber or reflectivity of mirror.

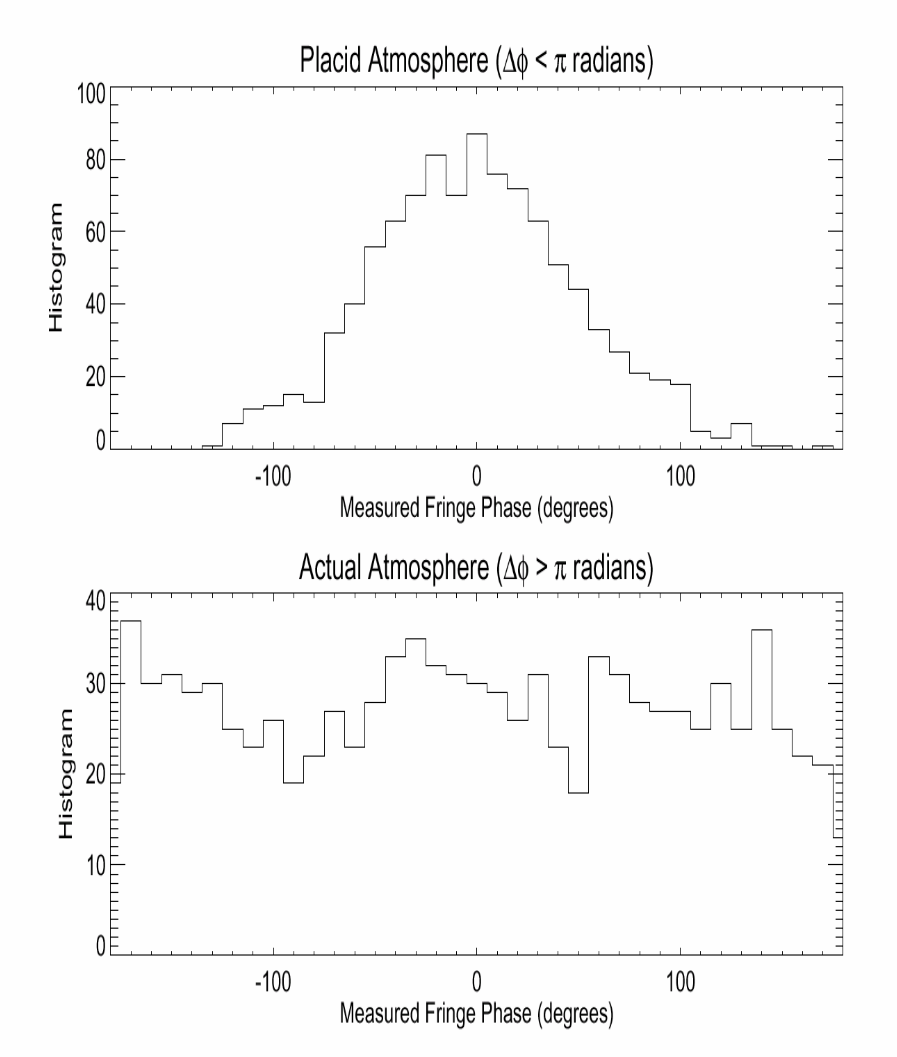
Telescope phase shift, e.g., atmospheric piston, thermal drifts.

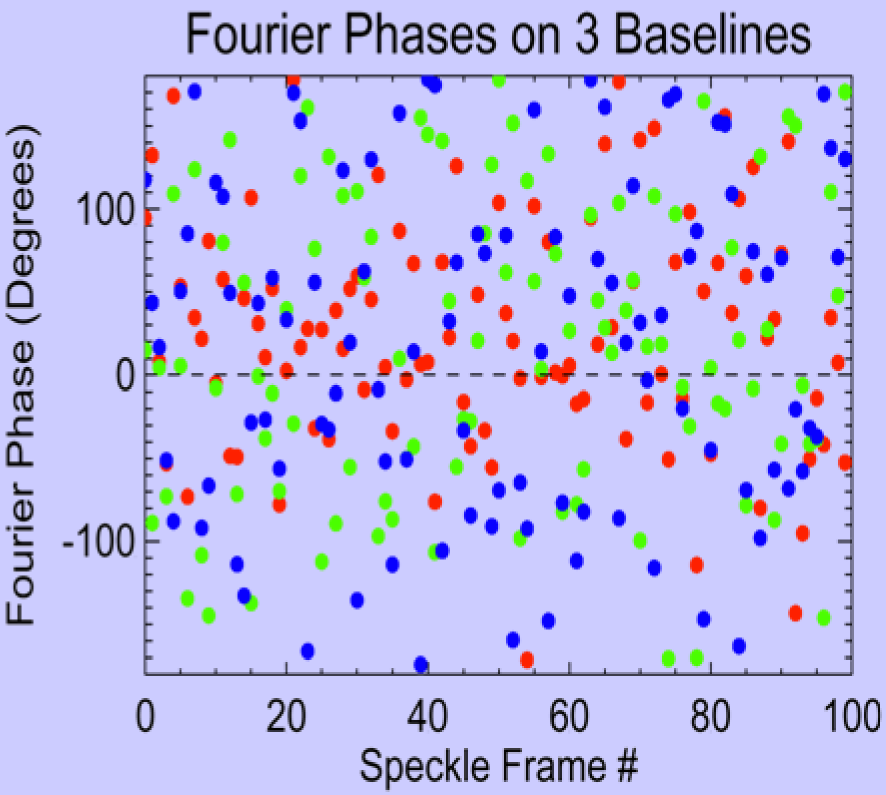
$$\begin{aligned}\text{Since} \qquad \tilde{\mathcal{V}}_{ij} &\propto \tilde{E}_i \cdot \tilde{E}_j^*, \\ \tilde{\mathcal{V}}_{ij}^{\mathrm{measured}} &= \tilde{G}_i \tilde{G}_j^* \tilde{\mathcal{V}}_{ij}^{\mathrm{true}} \\ &= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\mathrm{true}}\end{aligned}$$

Phase shift of detected fringe

- Amplitude of complex visibility is scaled: $|V_{ij}^{\mathrm{true}}| \rightarrow |G_i||G_j|\, |V_{ij}^{\mathrm{true}}|$
- Phase of complex visibility is altered: $\mathrm{Arg}(V_{ij}^{\mathrm{true}}) \rightarrow \mathrm{Arg}(V_{ij}^{\mathrm{true}}) + \Phi_i^G - \Phi_j^G$

What would pt. source visibility phases look like?





- Different colors are different baselines.
- x-axis direction represents time.

Key ideas 1

- The atmosphere effects the phase of the radiation collected by the interferometer.
- This in turn alters the phase of the measured complex visibility.

- Measured optical/near-infrared complex visibilities have random phases that are unrelated to the source structure:
  - These cannot be easily made use of.

Quiz 1

1. What does the model we have used to describe the effects of the atmosphere imply regarding the size of the collectors used in an interferometric array? *(Look at slide 6 – "The atmosphere")*
2. What would happen if we averaged the measured visibility phase on a given interferometer baseline for a long time, e.g. for 10 seconds?

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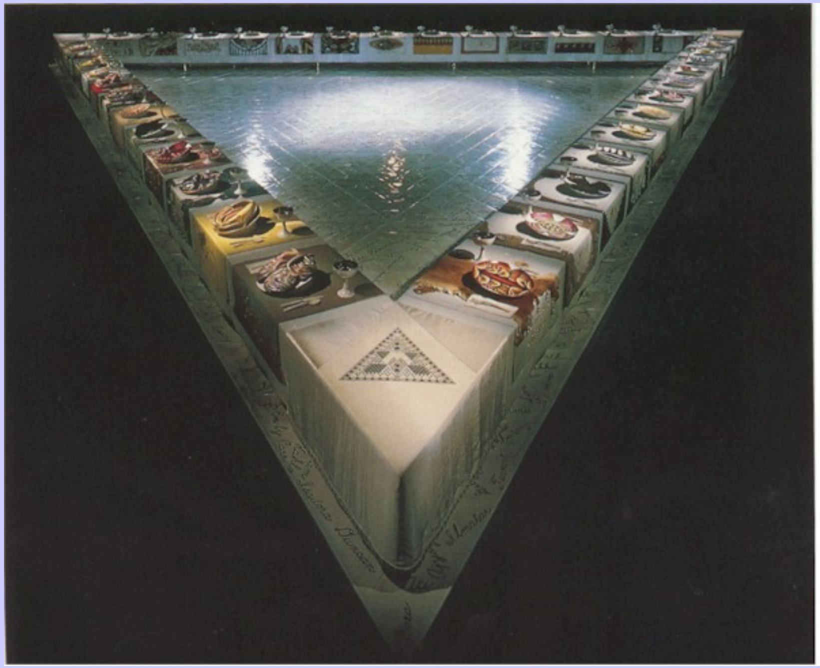
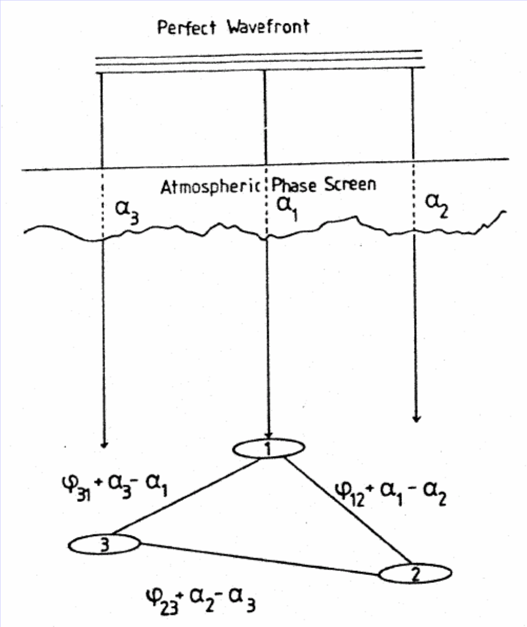


Definition of the closure phase (i)

- Consider measurements of the three complex visibilities associated with a 3-element interferometer.
- Let each complex visibility be corrupted by the atmosphere in the way we have described earlier:



$$
\begin{aligned}
\tilde{\mathcal{V}}_{ij}^{\mathrm{measured}} &= \tilde{G}_i \tilde{G}_j^* \tilde{\mathcal{V}}_{ij}^{\mathrm{true}} \\
&= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\mathrm{true}}
\end{aligned}
$$

- Let's examine the product of the three complex visibilities:

$$
\begin{aligned}
\tilde{B}_{ijk} &= \tilde{\mathcal{V}}_{ij}^{\mathrm{measured}}\, \tilde{\mathcal{V}}_{jk}^{\mathrm{measured}}\, \tilde{\mathcal{V}}_{ki}^{\mathrm{measured}} \\
&= |G_i||G_j|\, e^{i(\Phi_i^G - \Phi_j^G)}\, \tilde{\mathcal{V}}_{ij}^{\mathrm{true}} \cdot |G_j||G_k|\, e^{i(\Phi_j^G - \Phi_k^G)}\, \tilde{\mathcal{V}}_{jk}^{\mathrm{true}} \cdot |G_k||G_i|\, e^{i(\Phi_k^G - \Phi_i^G)}\, \tilde{\mathcal{V}}_{ki}^{\mathrm{true}} \\
&= |G_i|^2\, |G_j|^2\, |G_k|^2\, \tilde{\mathcal{V}}_{ij}^{\mathrm{true}} \cdot \tilde{\mathcal{V}}_{jk}^{\mathrm{true}} \cdot \tilde{\mathcal{V}}_{ki}^{\mathrm{true}}.
\end{aligned}
$$

Definition of closure phase (ii)

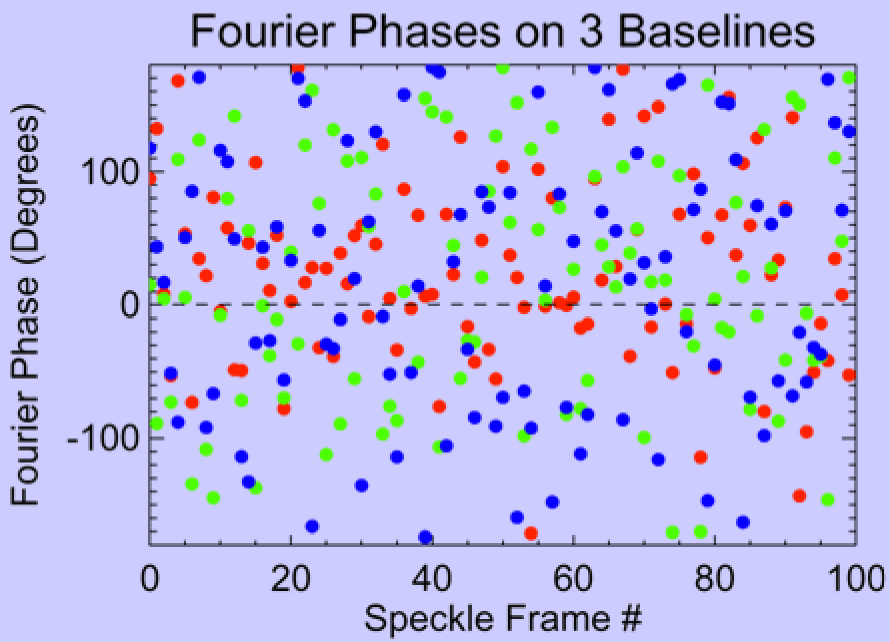
$$
\begin{aligned}
\tilde{B}_{ijk} &= \tilde{\mathcal{V}}_{ij}^{\text{measured}}\,\tilde{\mathcal{V}}_{jk}^{\text{measured}}\,\tilde{\mathcal{V}}_{ki}^{\text{measured}} \\
&= |G_i||G_j|\,e^{i(\Phi_i^G-\Phi_j^G)}\,\tilde{\mathcal{V}}_{ij}^{\text{true}}\cdot|G_j||G_k|\,e^{i(\Phi_j^G-\Phi_k^G)}\,\tilde{\mathcal{V}}_{jk}^{\text{true}}\cdot|G_k||G_i|\,e^{i(\Phi_k^G-\Phi_i^G)}\,\tilde{\mathcal{V}}_{ki}^{\text{true}} \\
&= |G_i|^2\,|G_j|^2\,|G_k|^2\,\tilde{\mathcal{V}}_{ij}^{\text{true}}\cdot\tilde{\mathcal{V}}_{jk}^{\text{true}}\cdot\tilde{\mathcal{V}}_{ki}^{\text{true}}.
\end{aligned}
$$

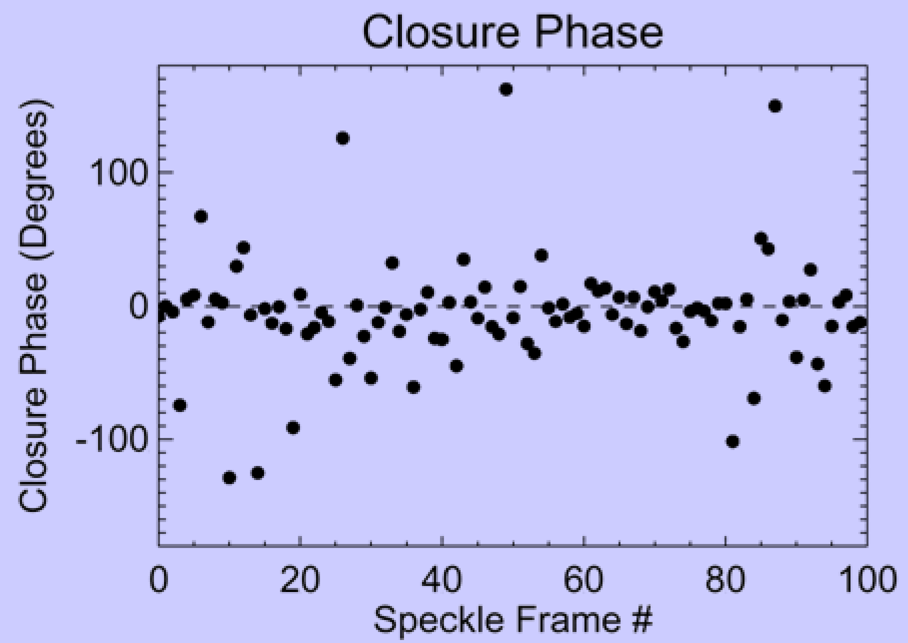
- So, the argument of this "triple product":
  - Doesn't contain any of the atmospheric error terms like Φ_i^G, Φ_j^G and Φ_k^G.
  - Has a value that is determined by the true values of the visibility function phases on the baselines that form this closed loop: $\mathrm{Arg}(B_{ijk}) = \mathrm{Arg}(V_{ij}^{true}) + \mathrm{Arg}(V_{jk}^{true}) + \mathrm{Arg}(V_{ki}^{true})$.
  - Is hence a "good" observable.

- This sum of visibility (Fourier) phases round a closed loop of baselines is called a "closure phase".
- It is perhaps more useful to refer to it as the argument of the triple product or of the "bispectrum".

Example of closure phases

- The resistance of the closure phase to atmospheric perturbations can be visualized in many ways:

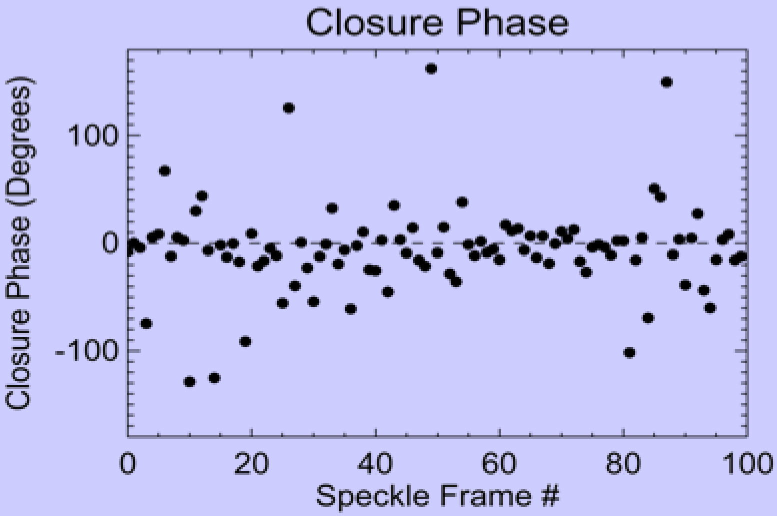


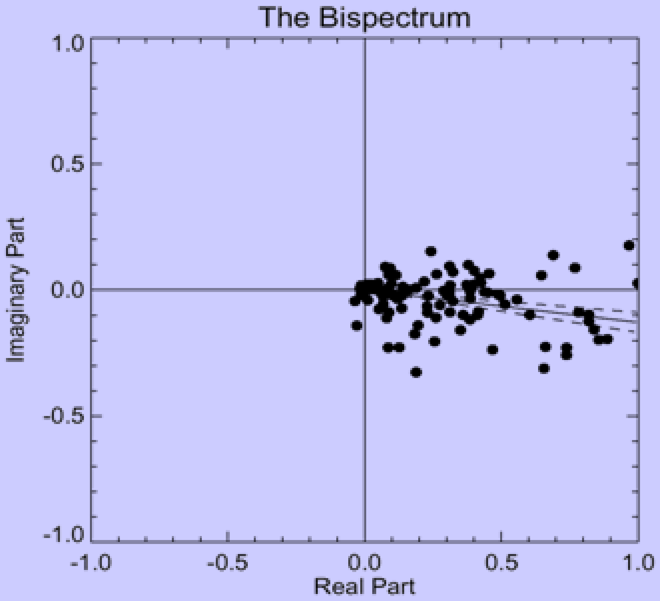


- Although the visibility phases on any given baseline are random, the closure phase for this point source is always close to the expected value of 0°.
  - Deviations away from zero here are due to measurement noise.

A better representation

- A more valuable way to present instantaneous measurements of the triple product argument, is to display the triple product measurements on an Argand diagram:

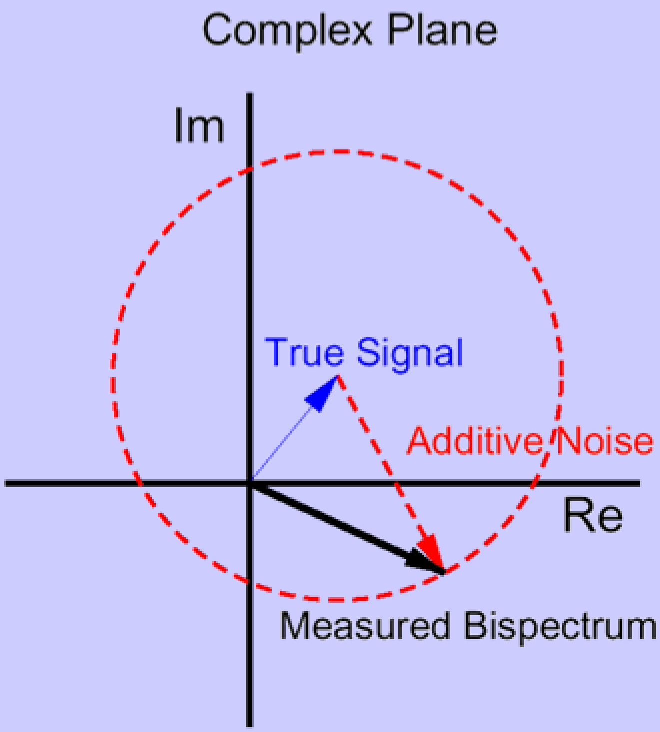


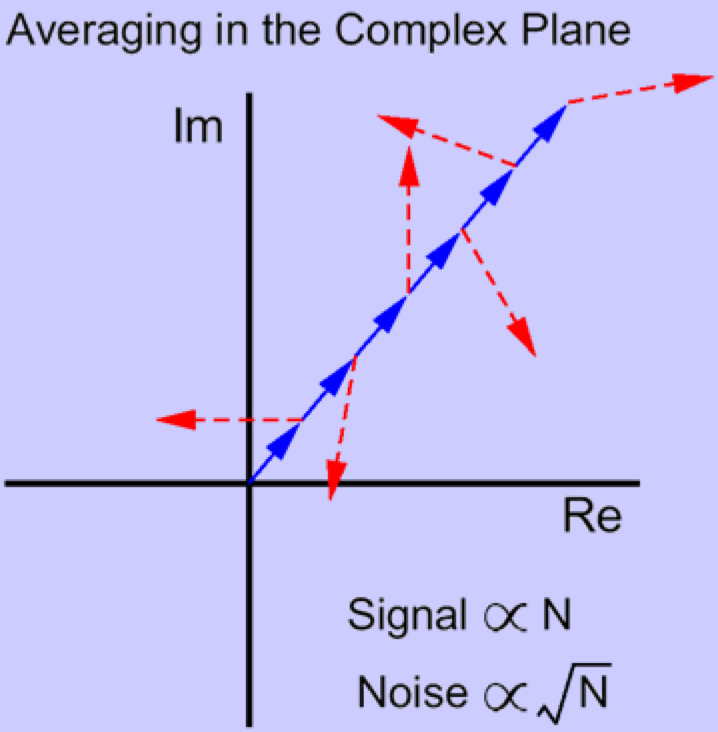


  - When the amplitude of the triple product is small, its argument can be very different from its expected value (= zero in this example).
  - When the amplitude of the triple product is large, its phase is always close to the expected value.

What to do with multiple measurements?

- A key feature of the triple product (bispectrum) is that it can be averaged coherently over multiple measurements:
  - This is in direct contrast to measurements of complex visibilities.
- So, individual estimates of a given triple product can have very low S/N.





What's the big deal?

- The triple product retains information about the source Fourier phases, albeit in a scrambled form:
  - More precisely, we can measure linear combinations of the visibility phases.
- The amount of phase information retained by the closure phase is a monotonically increasing function of the number of telescopes in an interferometric array:

Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

- The triple product phase is not usually biased by atmospheric turbulence:
  - This is in contrast to measurements of the squared visibility, which are scaled by factors like $|G_i|^2 \times |G_j|^2$

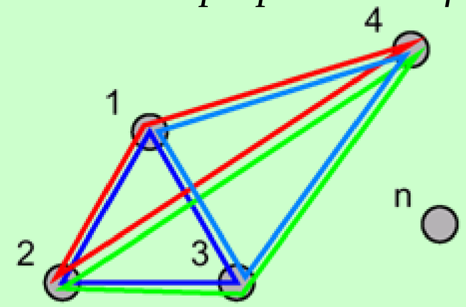
Key ideas 2

- The closure phase is the argument of the triple product of complex visibilities around a closed loop of baselines.
- The measurements of these complex visibilities must all be made at the same time for the atmospheric (or any telescope-based) errors to cancel.
- It must also be the case that the atmospheric (or telescope-based) phase errors can be characterized by a single number at each telescope.

- The triple product can be averaged coherently over multiple measurements.
- The triple product phase is, generally, not biased by the atmosphere.
- The closure phase retains a large fraction of the Fourier phase information that we are interested in determining.

Quiz 2

1. Why do you think the triple product is sometimes called the bispectrum? *(Consider how the spatial frequencies of the three measured complex visibilities are related to each other).*
2. Review the table on slide 18 that enumerates the number of Fourier phases, closure triangles and "independent" closure triangles for an n-element array. Can you generate the numbers presented in the table? *(You may wish to consider the following diagram, and ask yourself how any given closure triangle can be decomposed into superpositions of other triangles.)*



3. The triple product is just one of a hierarchy of multi-products that are resistant to antenna-based phase errors. Can you guess why these higher order multi-products are not widely used in astronomical observations?

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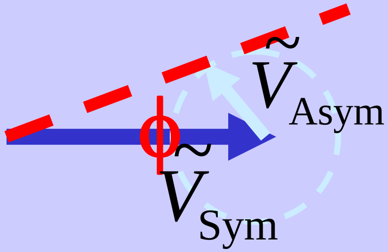
What does a closure phase measure?

1. The closure phase is insensitive to the source position:
    - It tells us nothing about where the target is – c.f. Fourier phases.

2. The triple product is real if the source has point-symmetry:
    - This implies that for these types of sources the closure phase is either 0° or 180°.

3. The closure phase measures the amount of "asymmetric" flux in the target.
    - In an order of magnitude sense:

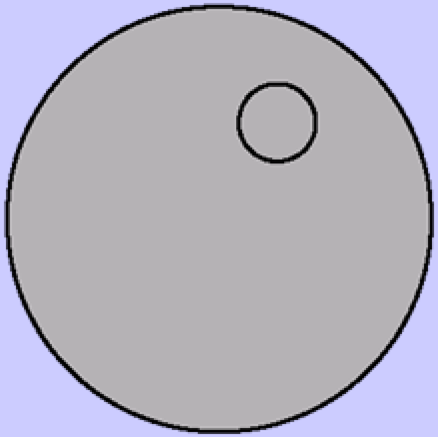
$$|Closure\,Phase(radians)| \approx \frac{Asymmetric\,Flux}{Symmetric\,Flux}$$



- The amount of "asymmetric" flux should be based on the resolution of the baselines (Nothing is asymmetric if its unresolved!)
- E.g., for an unequal resolved binary system, the closure phase will typically be roughly the brightness ratio.

A simple example (i): star and spot

- Can consider the case of a resolved stellar disc with a superposed hotspot.

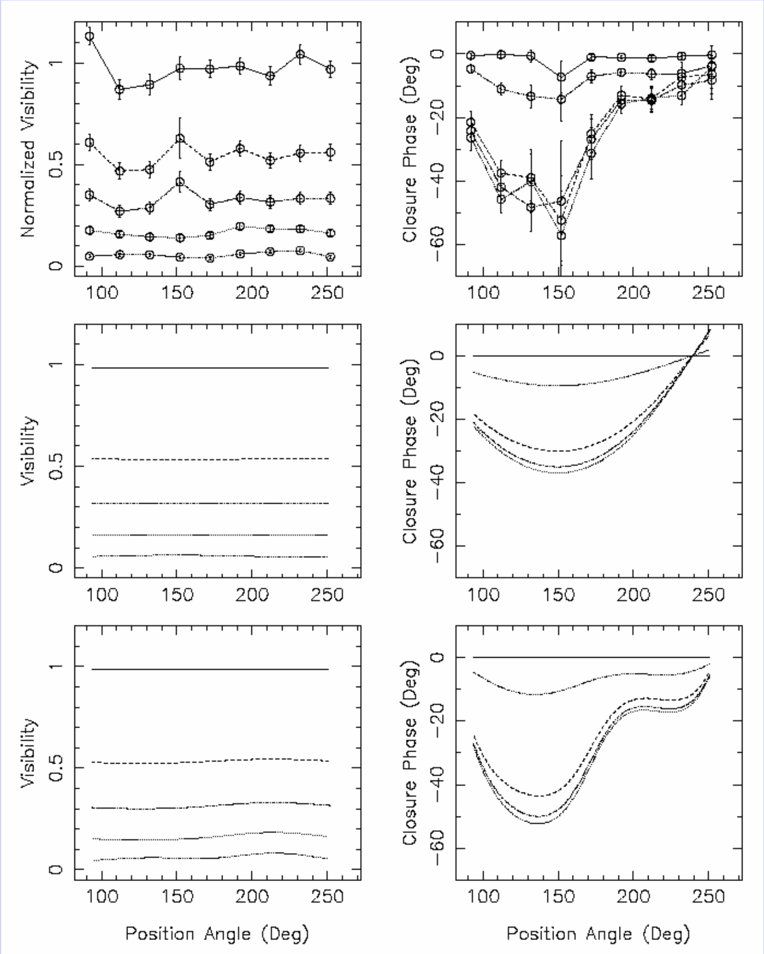


- At low angular resolution, the stellar disc dominates and the target looks centro-symmetric.
- Closure phases are small.

- At high angular resolution, the stellar disc is resolved out, and the off-center spot dominates.
- Closure phases are large.

Actual data for α Her.

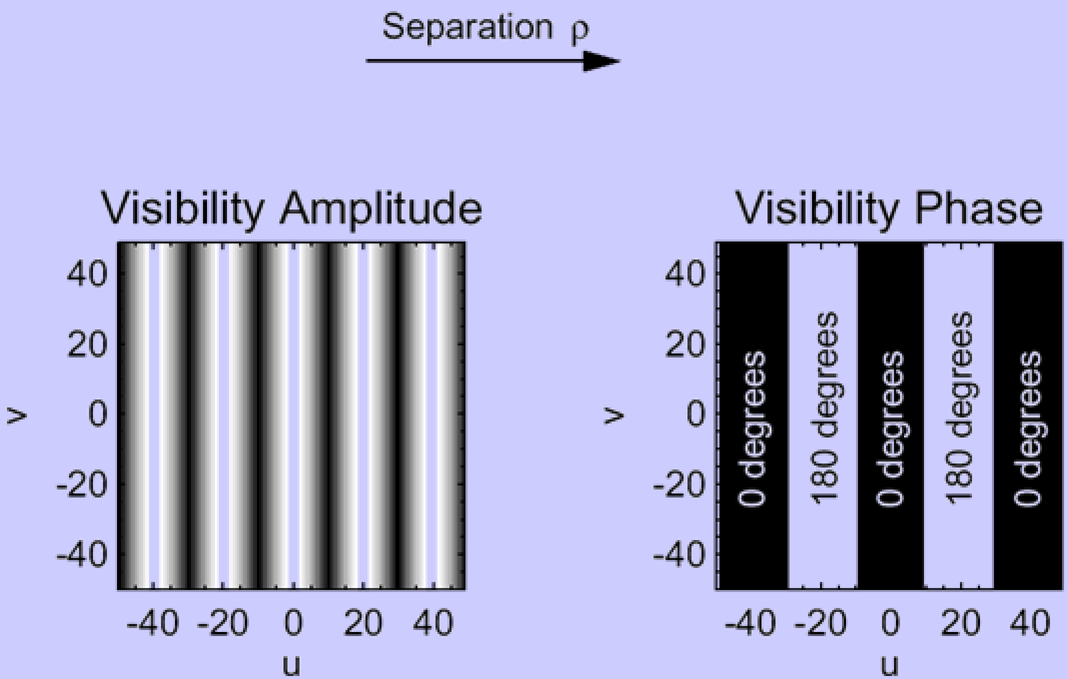
- Interferometric data from Tuthill (1994) – 5-element linear array, baselines up to 4.2m.



- Top panel is data:
  - LHS visibility amplitudes.
  - RHS closure phases.
  - x-axis is orientation of array with respect to cardinal points.
- Middle panel shows predictions for disc + single spot model.
- Bottom panel shows predictions for disk + 2 spot model.
- Model-fits confirmed by model-independent image reconstruction.

A simple example (ii): binary star

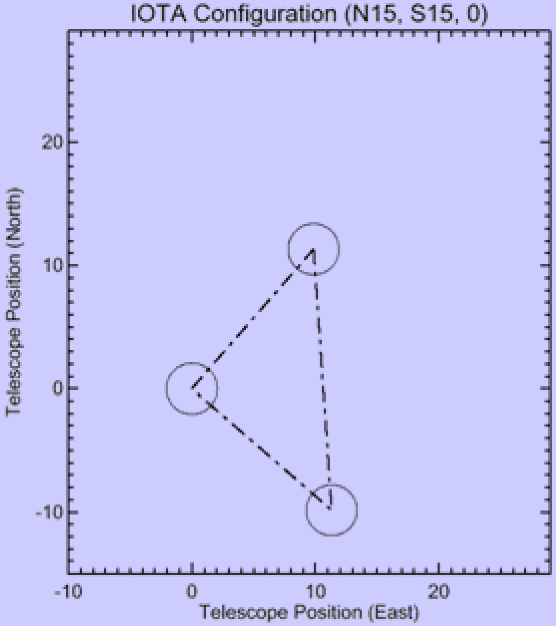
- Consider the case of an equal binary star:

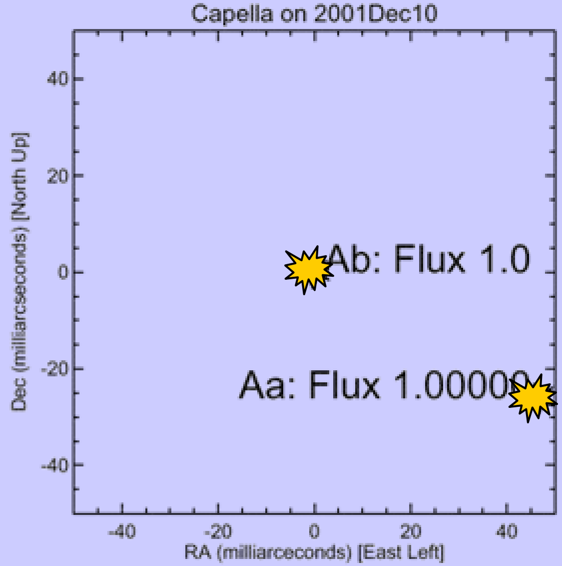


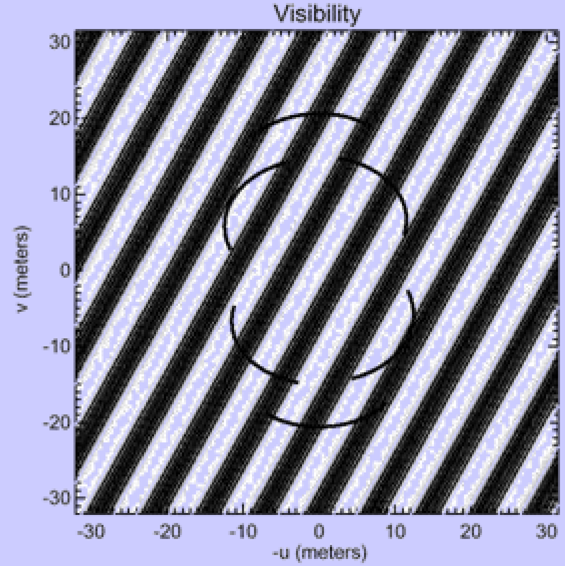
- Visibility amplitude is a set of sinusoidal stripes oriented orthogonal to the binary separation.
- Source is centro-symmetric:
  - Visibility phase is either zero or 180 degrees.

A simple example (ii): binary star

- Assume some given configuration of the interferometer and ask how does the measured closure phase change during the night:

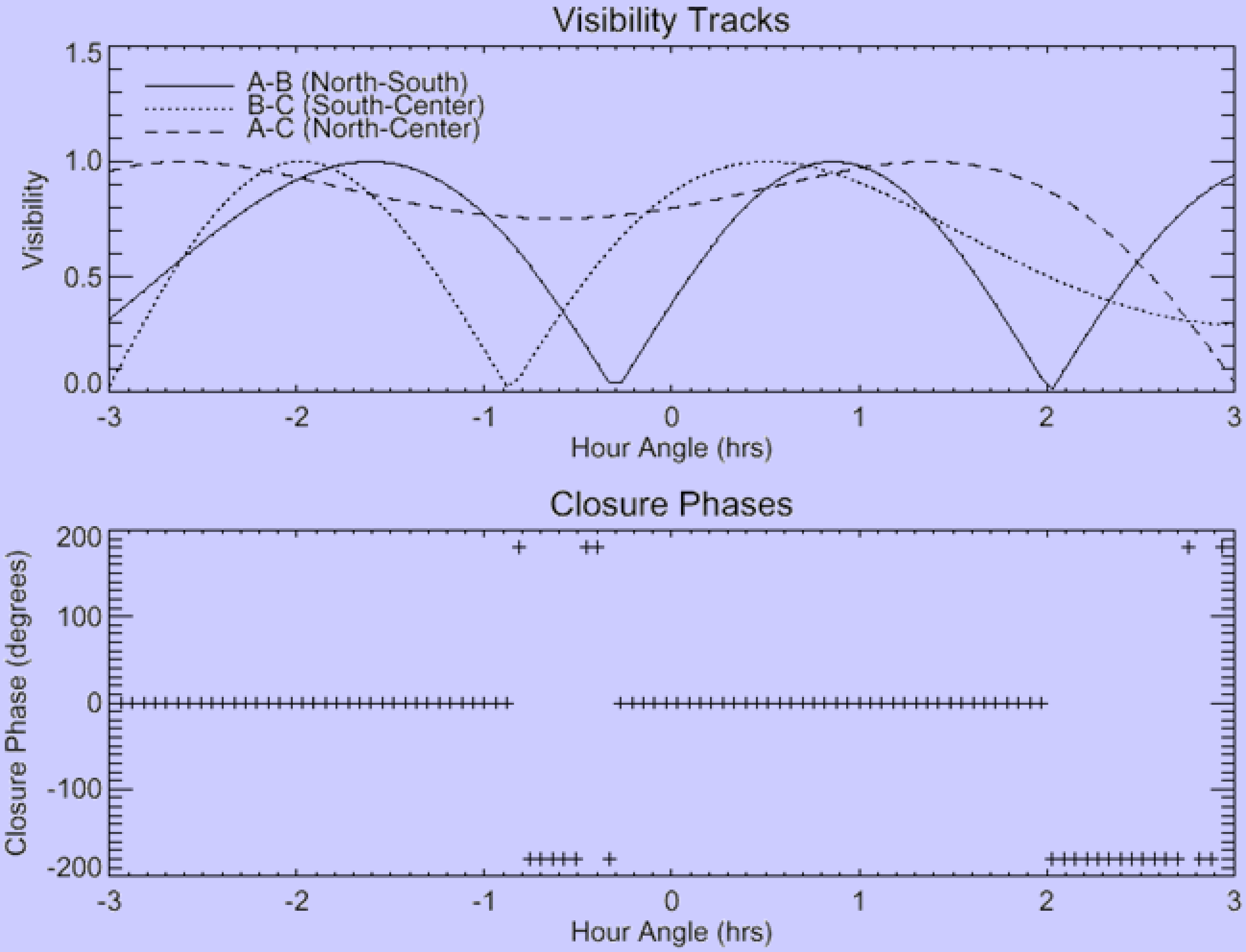




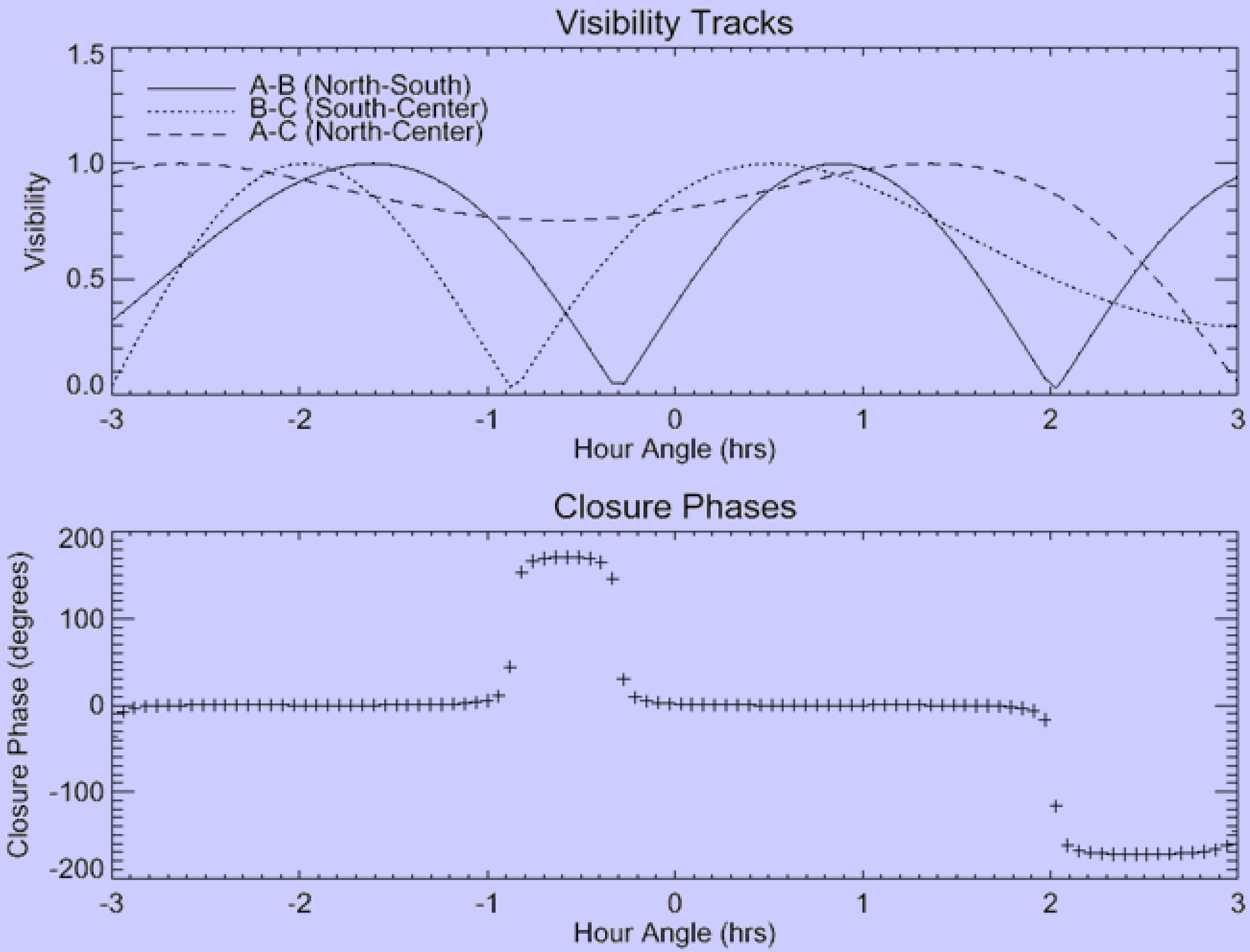


  - Note how two of the baseline tracks cross the nulls in the visibility function, while the other doesn't.

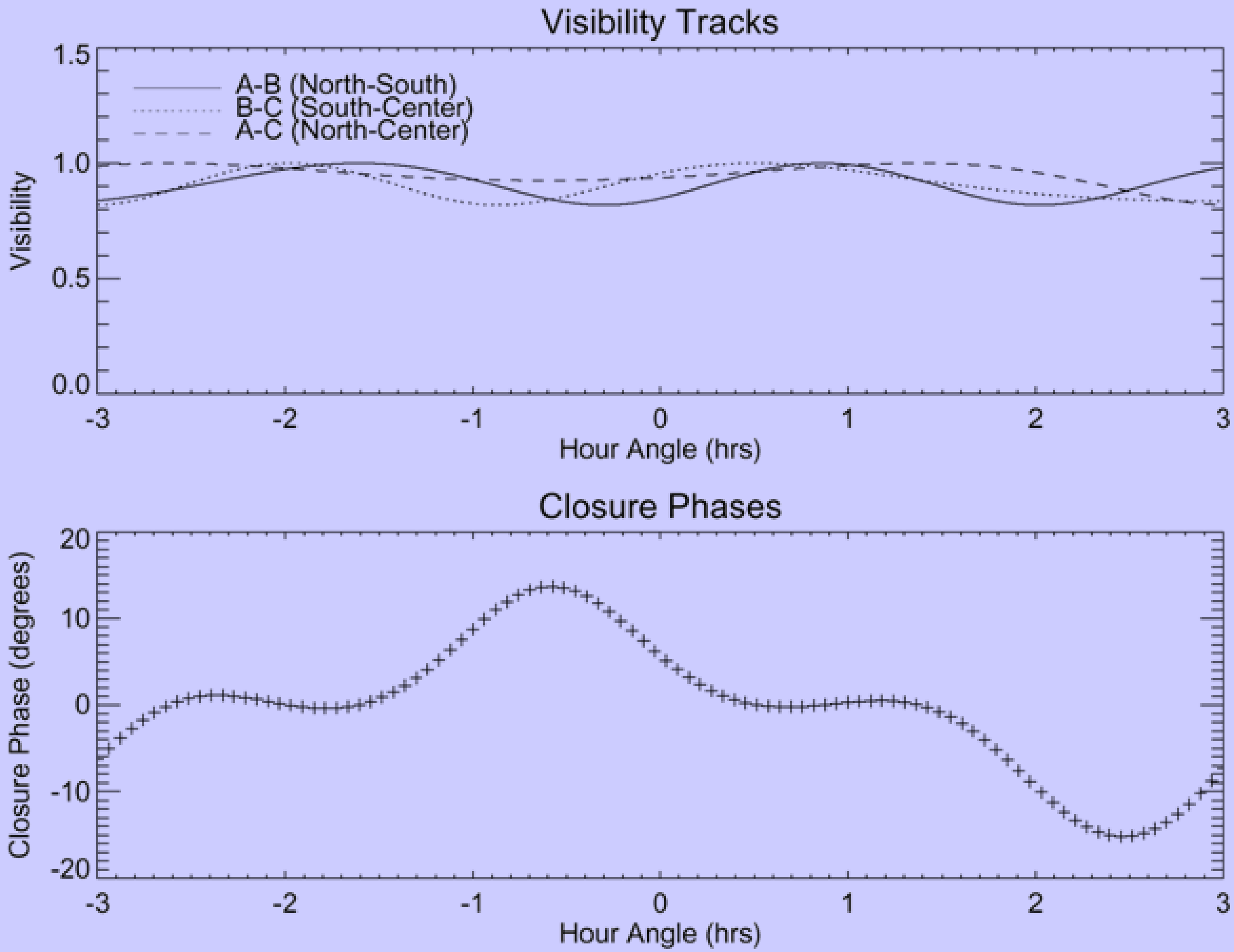
Equal binary – 1:1



Almost equal binary – 1.05:1



Unequal binary – 10:1

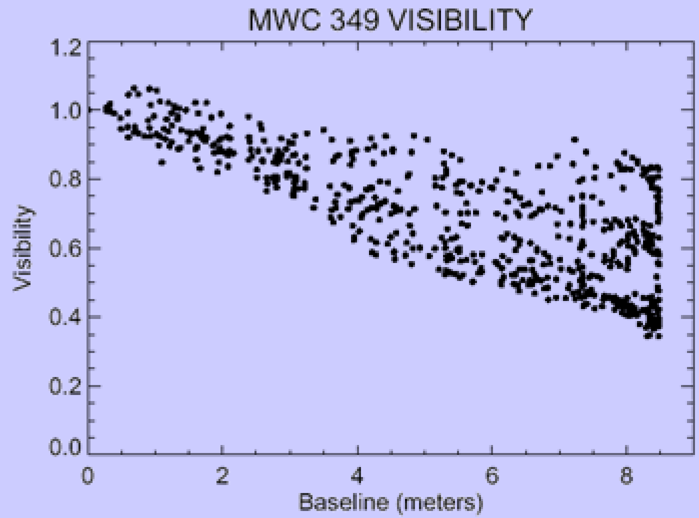


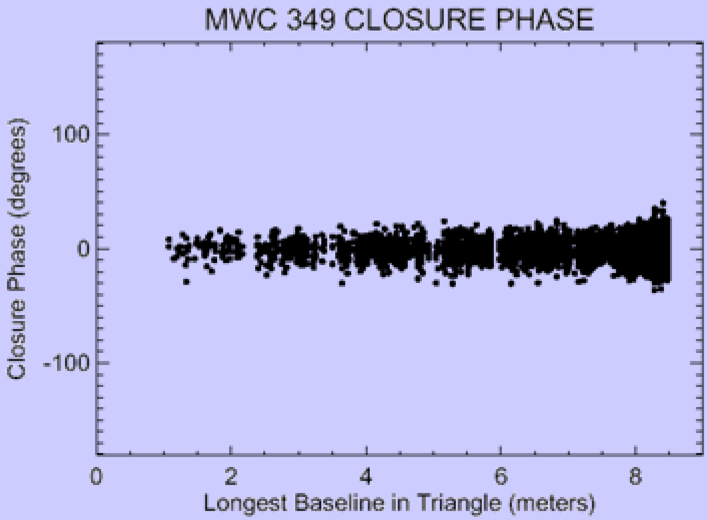
What can we expect for more complex targets?

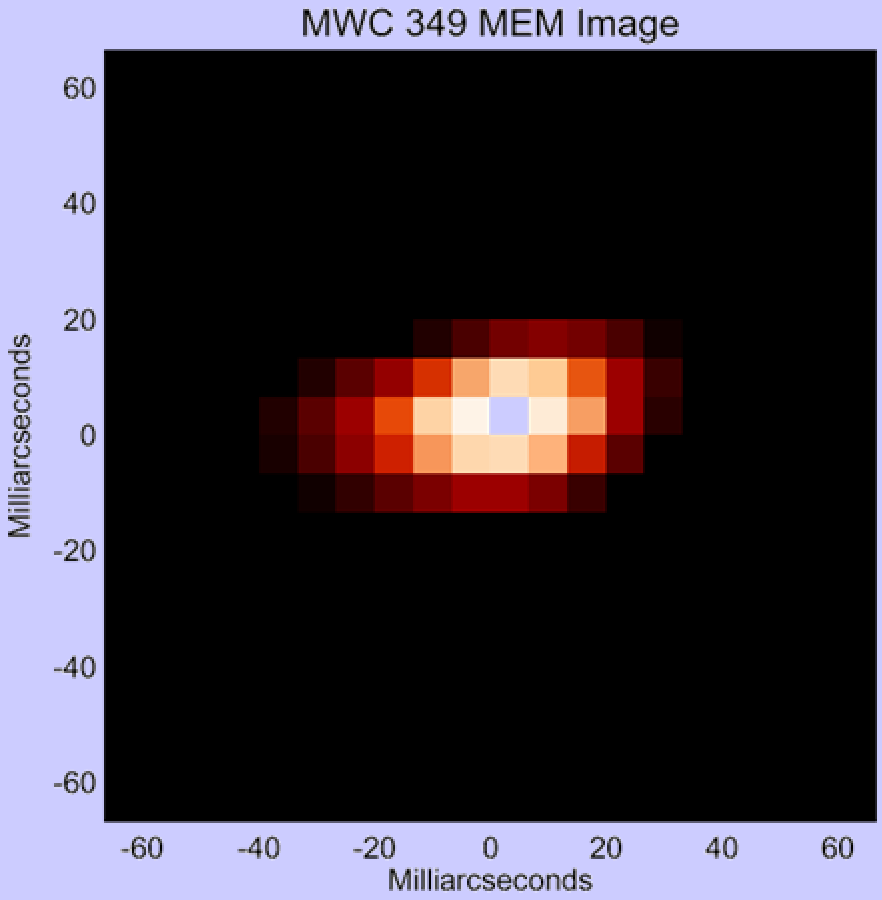
- Centro-symmetric targets:
  - Small or zero closure phases.
- Closure phases with short baseline triangles:
  - Small or zero closure phases – source is not resolved.
- Closure phases with long baseline triangles:
  - Larger closure phases – especially when symmetric flux is resolved out.

Following real examples are courtesy of John Monnier and Peter Tuthill.

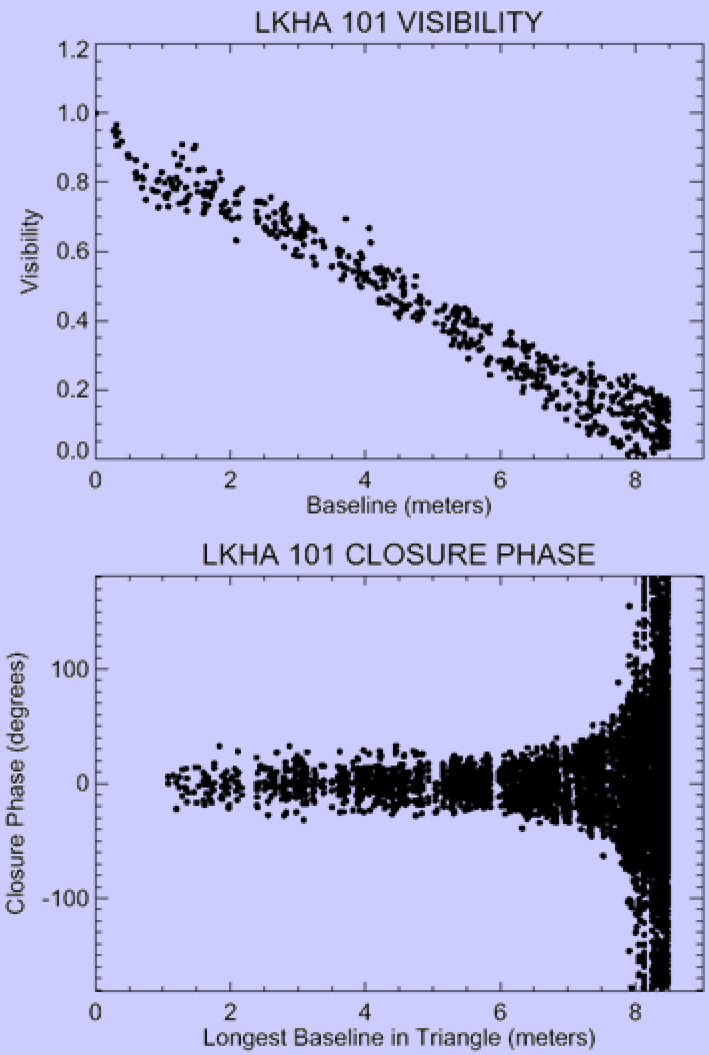
Example 1 – YSO MWC 349

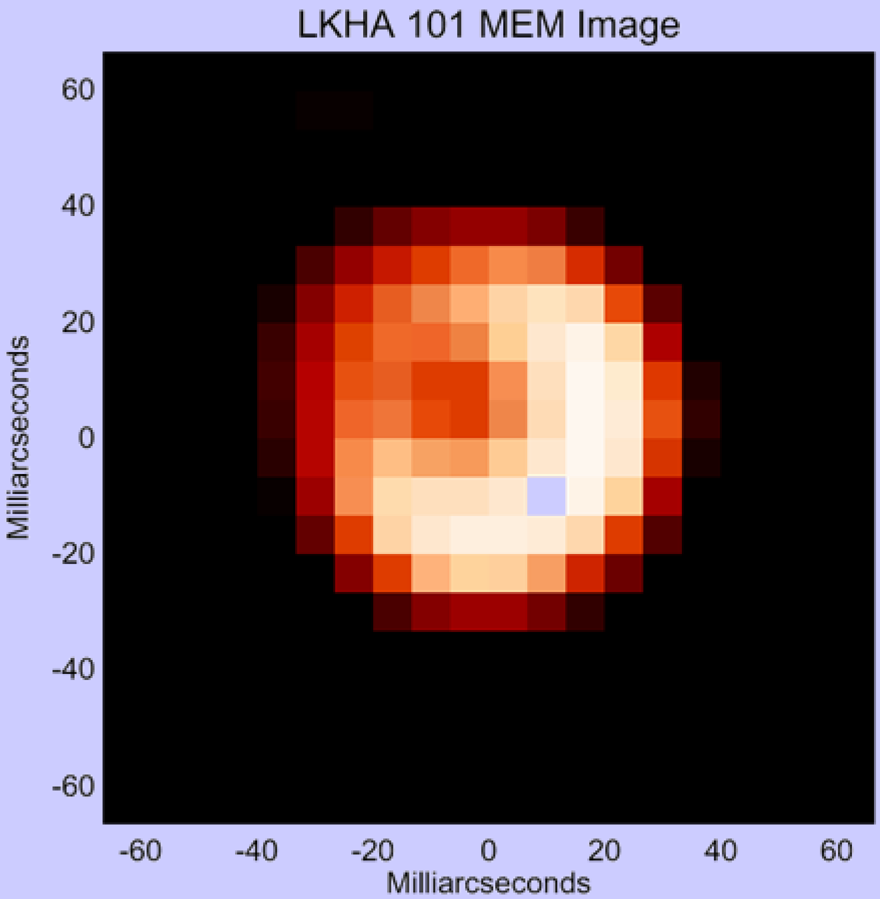






Example 2 - YSO LkHα101





Key ideas 3

- Closure phases are not sensitive to an overall translation of image.
- The bispectrum is real for sources with point symmetry. That is, the closure phases are all 0 or 180 degrees
- The closure phase is sensitive to asymmetric structure in the source.
- If non-zero closure phases are see from a point source this is indicative of either non-closing baselines or baseline-dependent phase errors.
- For large closure phase signals to be measured, the target being observed must be resolved.
- For well modeled targets, e.g. binary or multiple stellar systems, the closure phase can be a very sensitive discriminator for different models.

Quiz 3

1. Use the charts on slides 27-29 to assess whether closure phase measurements might be a useful detection strategy for hunting for extra-solar planets. Assume measurements made at a wavelength of 1 micron of a solar-system like system at 5pc, and consider the following:
   a) The baseline lengths needed.
   b) The expected closure phase signal.
   c) The level of precision required to claim a reliable detection.

Imaging with closure phases

- One of the most valuable uses of closure phases in interferometry is to permit model-independent imaging using visibility amplitude and closure phase data.

- Conventional wisdom for interferometric imaging:
  - Sequential one-loop process:
    - "Bundle" amplitudes and phases.
    - Grid, weight and perform inverse Fourier transform.
    - Correct for the sampling distribution (e.g. CLEAN, MEM etc)
    - $\Rightarrow$ map of the sky.

- But, we only have linear combinations of the Fourier phases:
  - Necessitates an iterative approach:
    - Strategy is to solve for the telescope-based gain errors that corrupted the Fourier phase data.
    - Take advantage of the small number of these as compared to the number of visibility data.
    - Leverage additional information on source positivity and extent.
    - Still include all steps like gridding, weighting, deconvolution, etc.

Self-calibration

$$\tilde{V}_{ij}^{measured} = \tilde{G}_i \tilde{G}_j^* \tilde{V}_{ij}^{true}$$

Generate a set of Fourier phases that are consistent with the closure phases and bundle these with the amplitudes.

Create an initial source model.

[1] Adjust the telescope gain errors so the least-squares misfit between the measured data and the model data is minimized.

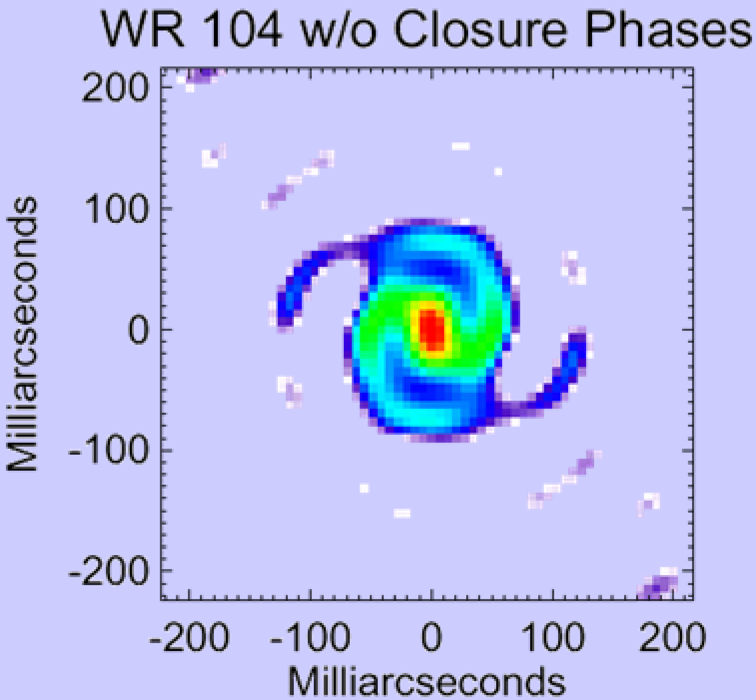
$$M = \sum_{ij} w_{ij} \left| \tilde{V}_{ij}^{measured} - \tilde{G}_i \tilde{G}_j^* \hat{V}_{ij} \right|^2$$

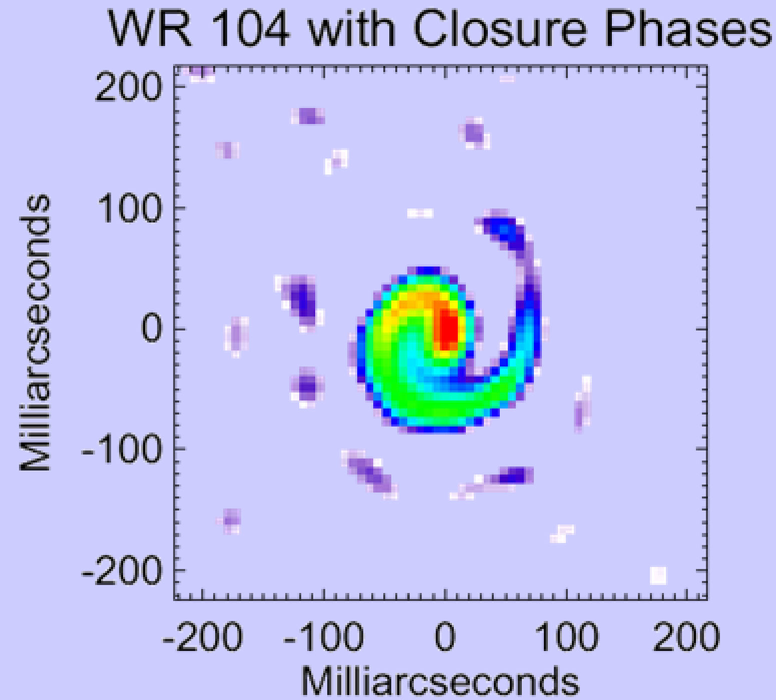
[2] Correct the measured data for the gain errors and map the data.

[3] Use this map as a new source model in step [1] unless converged.

- Crucial element is model for process by which Fourier phase data is corrupted.
- Although the process begins with an initial estimate of the source structure, the final result rarely depends on it.
- Convergence means that you have found a brightness distribution in the sky, and a set of telescope-based gain errors, that are fully consistent with the measured data <u>and</u> the a-priori information you input.

Impact of closure phase data





- Without closure phase data, map is necessarily centro-symmetric.
- With closure phase data, correct source brightness distribution is discovered.
- Phase data is key to model-independent image reconstruction.

Key ideas 4

- Most straightforward strategy for using closure phase data in image reconstruction is through use of self-calibration:
  - Other methods exist, but are beyond the scope of this lecture.
- Process is necessarily iterative because data is incomplete.
- Basic driving force is through use of additional constraints such as positivity and extent:
  - Without these an infinity of possible source brightness distributions can fit the visibility amplitudes and closure phases.

Summary

- Closure phase is simply the argument of the complex triple product.
- For particular types of atmospheric effects – notably telescope-dependent phase errors, the triple product phase is uncorrupted:
  - We have the possibility to retain Fourier phase information in the presence of the atmosphere.
  - The closure phase is less likely to be biased by the atmosphere than the visibility amplitude.
  - The Fourier phase data is retained in a scrambled form – linear combinations.
- Broadly speaking the closure phase is sensitive to asymmetric structural information:
  - Absolute source location is lost.
- Closure phases can be used in many different ways:
  - Precision diagnostics for well-modeled systems.
  - Powerful constraints in image reconstruction.