Fundamental Properties of Stars

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2003 Michelson Summer School on Optical and Near-Infrared Interferometry

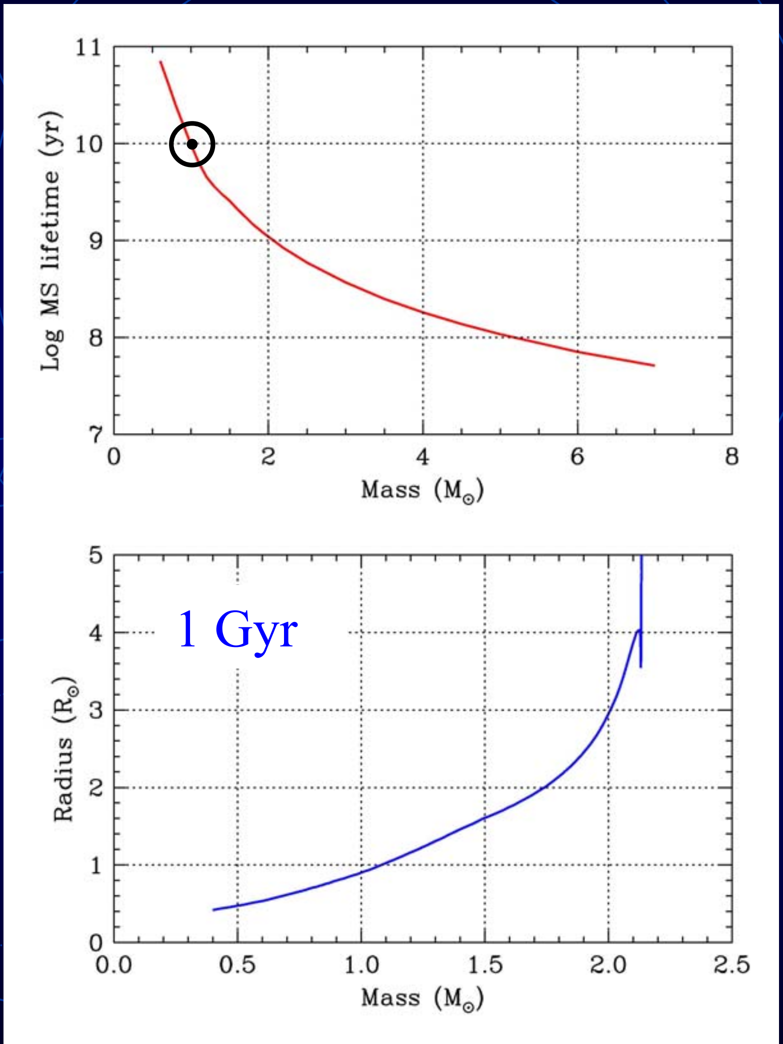
Fundamental Properties

- What are they? Which can we determine directly?
- Why do we care? What are they good for?
- Precision and accuracy: status
- How do we determine them?
- Examples: emphasis on comparison with theoretical models

What Are the Fundamental Parameters of Stars?

- ✓ Stellar mass (drives evolution): available only from binaries
- ✓ Stellar radius
- ✓ Effective temperature
- ✓ Chemical composition
- ✗ Age: only from models

Luminosity: $L \propto R^2\, T_{eff}^4$

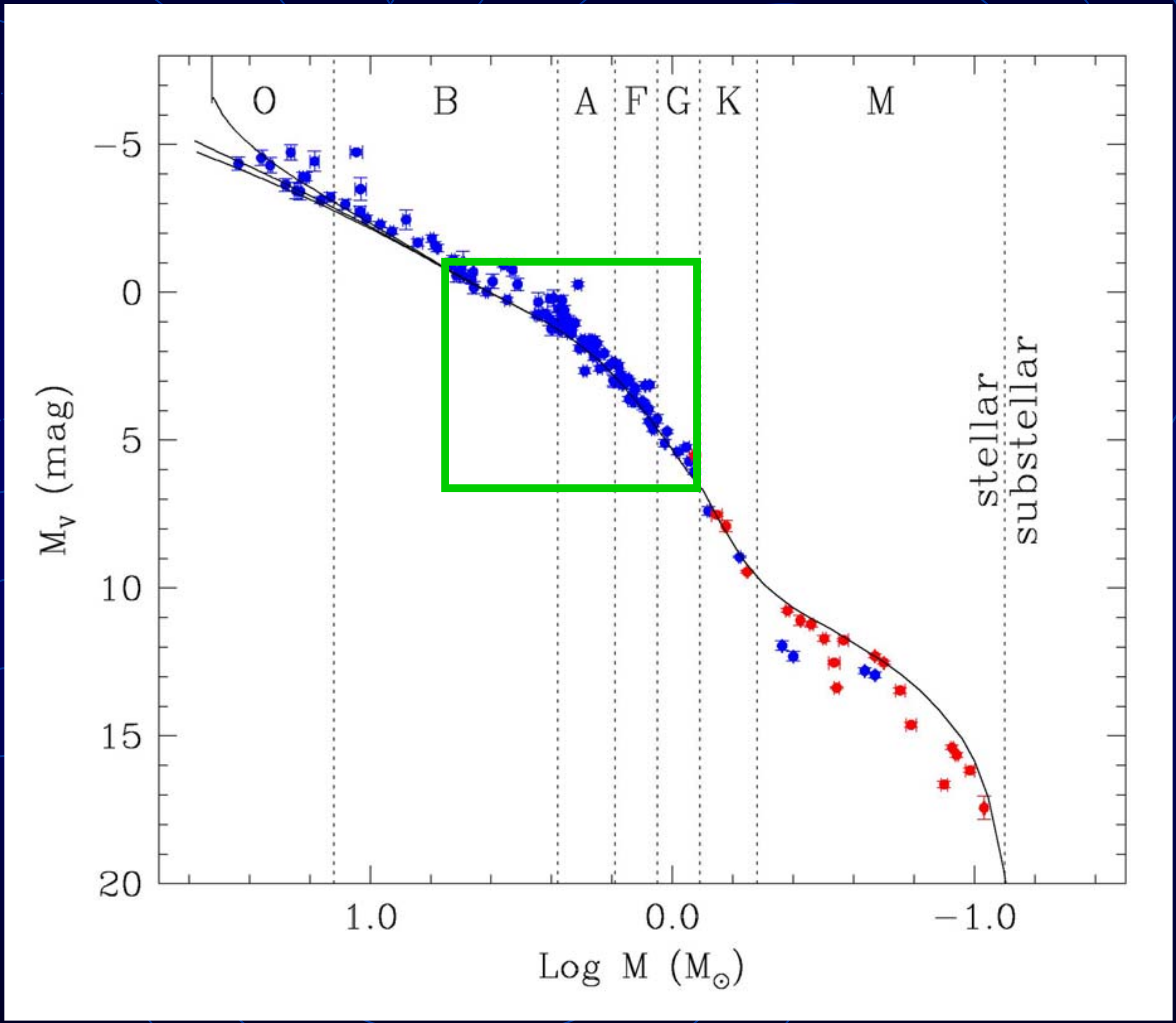


Why Do We Care About the Physical Properties of Stars?

- Improve our understanding of the structure and evolution of single stars (cosmological implications: globular cluster ages, etc.)
- Estimate total mass in clusters, through the mass-luminosity relation
- Use binaries as distance indicators in the Milky Way and beyond

The Mass-Luminosity Relation

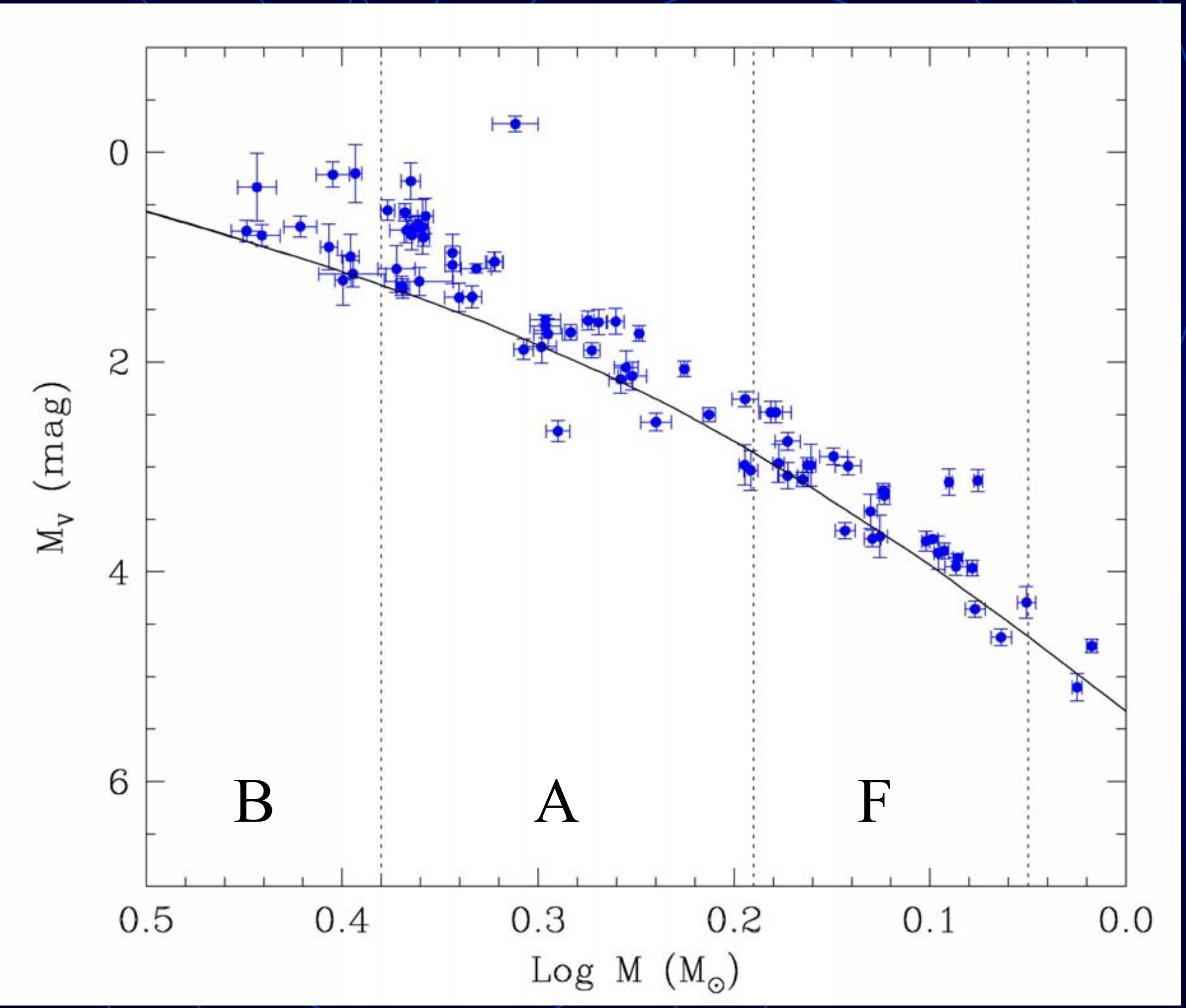
Eclipsing and **astrometric** binaries with mass errors ≤ than 5%



Cosmic Scatter

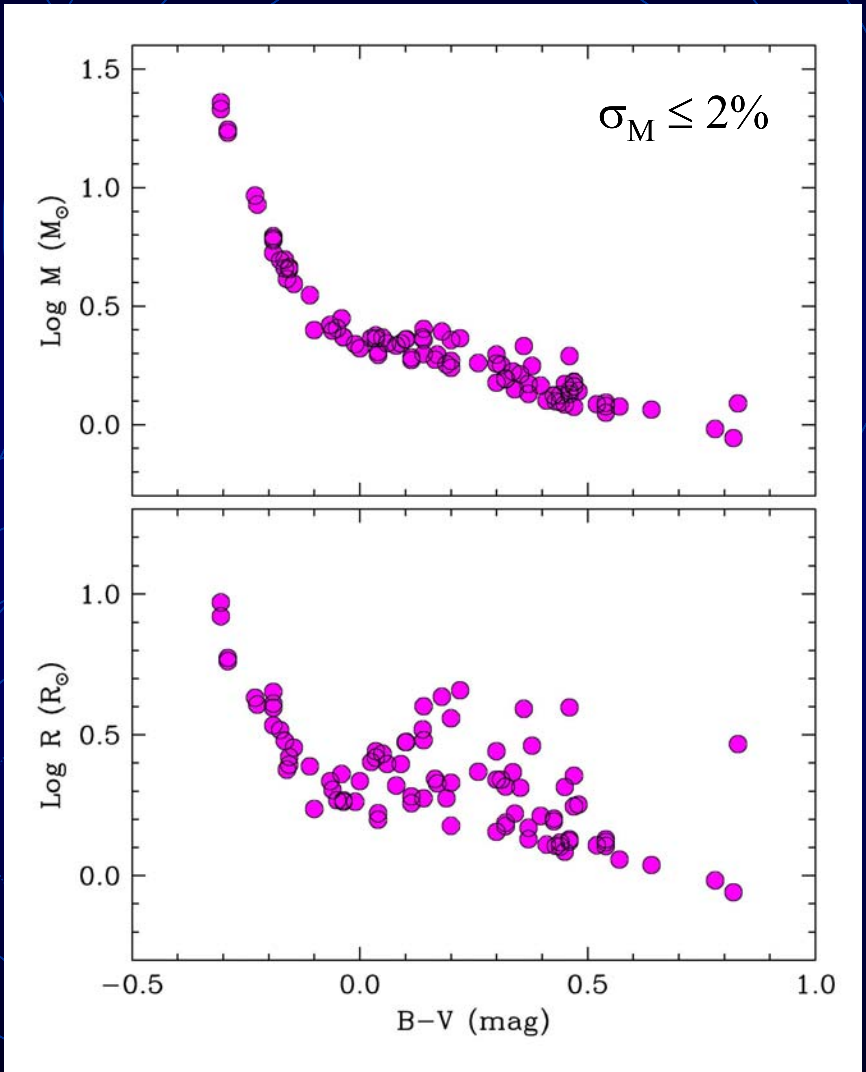
The scatter is produced by age and metallicity effects on the stellar luminosity.

Observational errors in mass and M_V are typically small on the scale of this diagram.



Predicting properties of single stars

- For a given spectral type or color index, errors can be $\sigma_M \sim 15\%$ and $\sigma_R \sim 50\%$ or more
- Abundance and age effects in M and R are significant
- Bi-parametric fits including a luminosity indicator do somewhat better: $\sigma_M \sim 3\%$, $\sigma_R \sim 5\%$
- Averaging any number of accurate masses and radii will not help

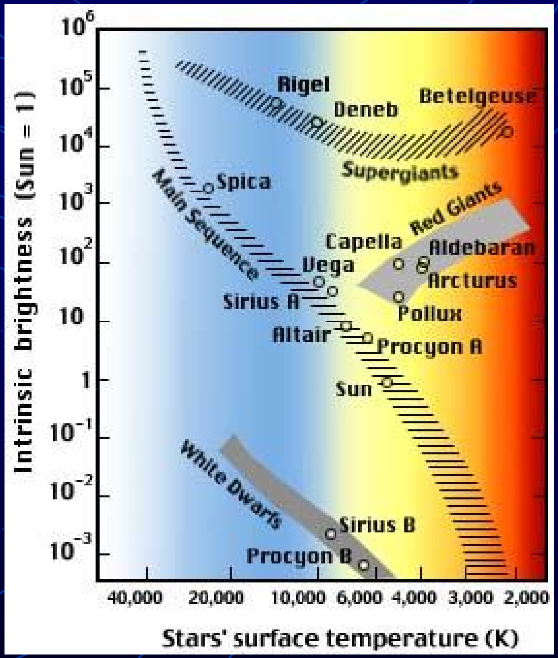


Precision Required for the Measurements to Be Most Useful

- Mass and radius determinations: 1–2%
- Effective temperature determinations: ~2%
- Metallicity determinations: ~25% (0.1 dex)
- Mass and radius measurements with errors larger than about 5% provide essentially no useful information for testing stellar evolution models

Status

- Stars with the best determined properties (detached eclipsing binaries): only a few dozen systems
- Areas of the H-R diagram that are well covered: 1–10 $M_\odot$ main sequence stars
- Areas that need more work:
  - Low-mass stars
  - High-mass stars
  - Evolved stars (giants)
  - Pre-main sequence stars
  - Metal-poor stars



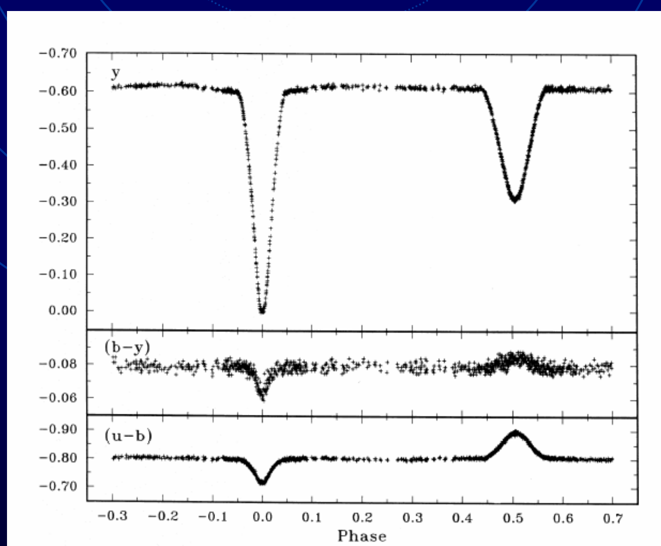
Stellar Mass Determinations

- Double-lined eclipsing binaries

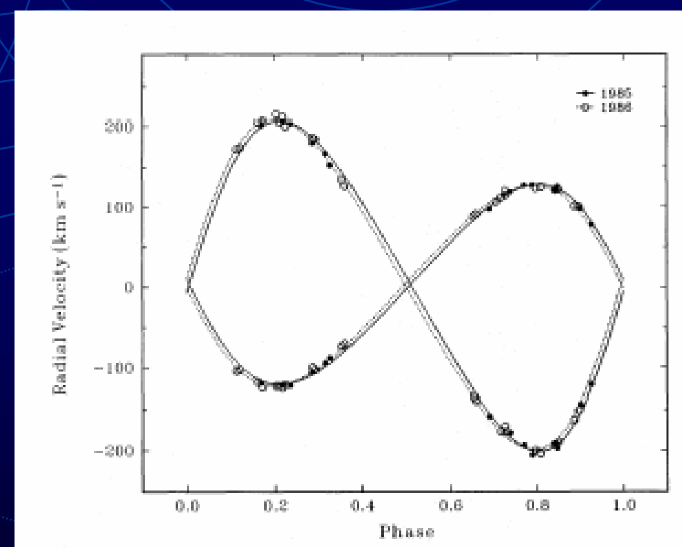
$$M_1 \sin^3 i = P(1-e^2)^{3/2}(K_1+K_2)^2 K_2$$
$$M_2 \sin^3 i = P(1-e^2)^{3/2}(K_1+K_2)^2 K_1$$

GG Lup

Andersen, Clausen & Giménez 1993, A&A, 277, 439



Light curves

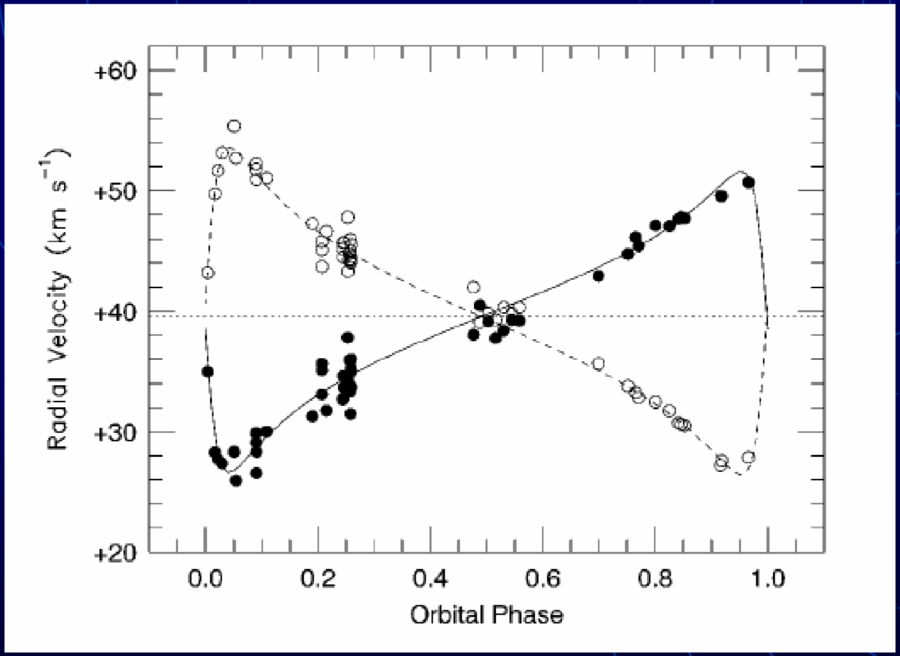


Radial velocity curves

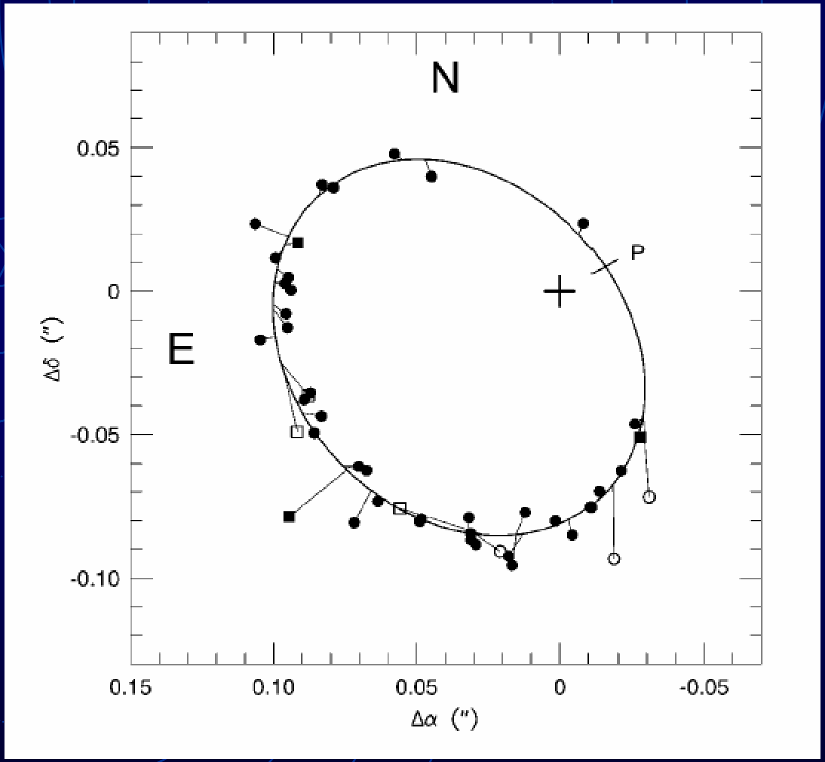
- Double-lined astrometric-spectroscopic binaries

Torres, Stefanik & Latham 1997, ApJ, 479, 268

70 Tau



Radial velocity curves



Astrometric orbit

- Absolute astrometry (orbits measured for both stars)

GJ 1005

Hershey & Taff 1998, AJ, 116, 1440

$$M_1 + M_2 = \frac{(a_1 + a_2)^3}{\pi^3 P^2}$$

$$M_2/M_1 = a_1/a_2$$

Period = 4.566 yr

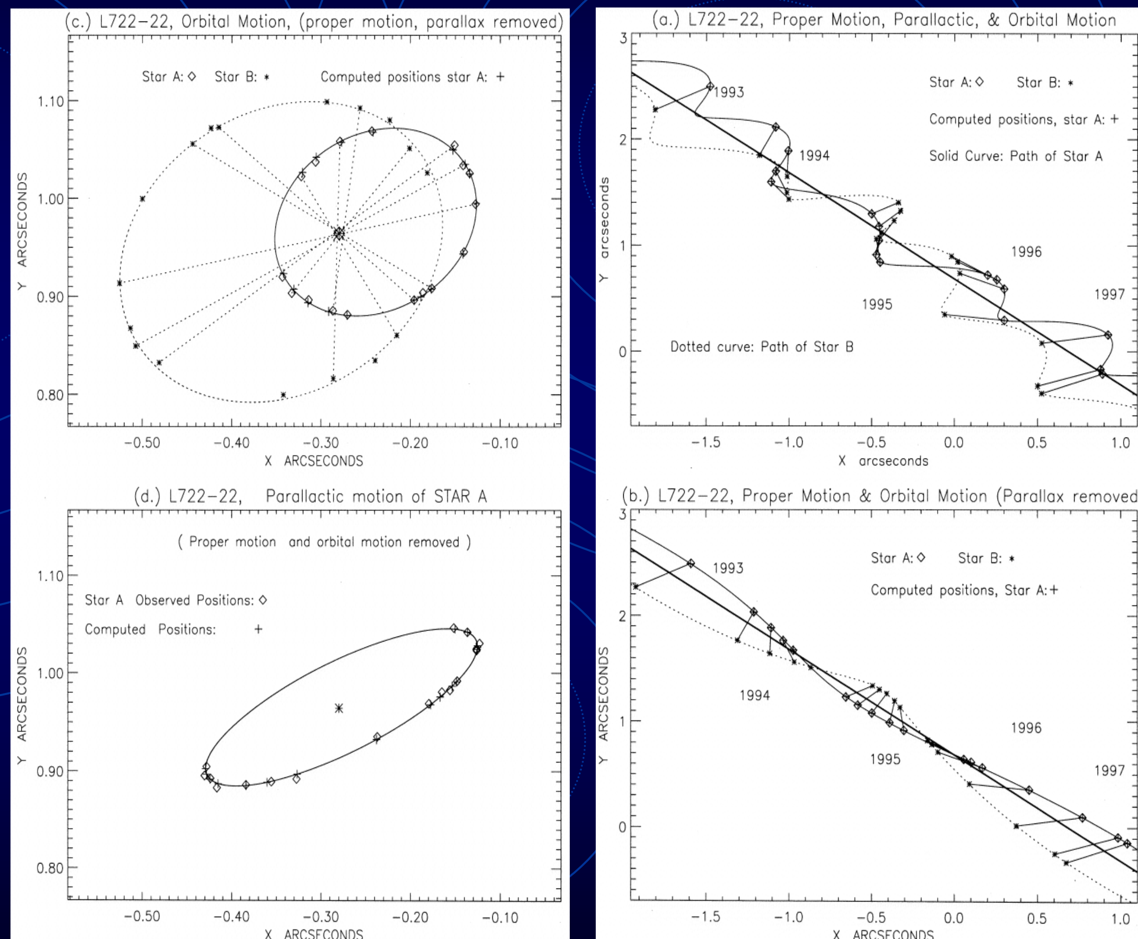
Distance = 6 pc

V mag = 11.5 and 14.4

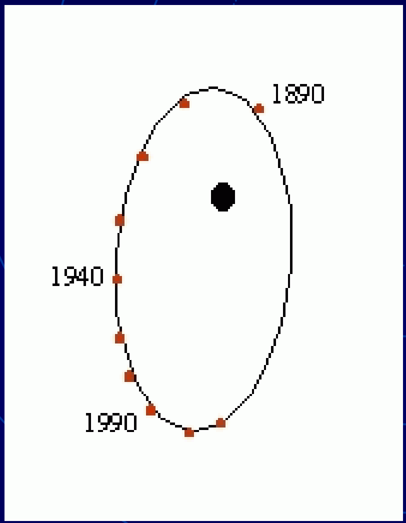
Spectral type = dM4.5

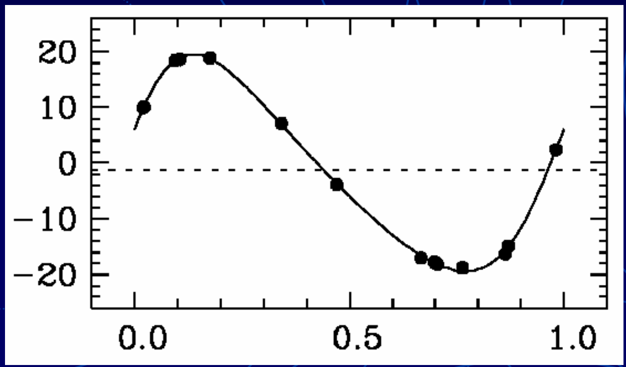
$M_A = 0.179 \pm 0.003\ M_\odot$

$M_B = 0.112 \pm 0.002\ M_\odot$



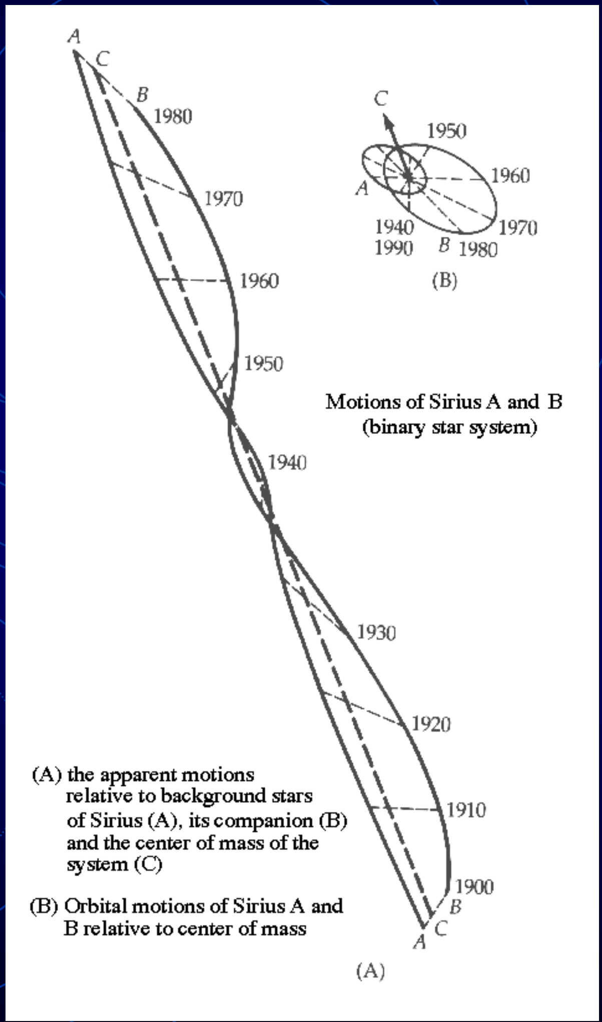
- Single-lined spectroscopic binary + relative orbit + parallax





$$M_1 + M_2 = a^3/\pi^3 P^2$$

$$M_2 = \frac{a^2\sqrt{1-e^2}K_1}{\pi^2 P \sin i}$$

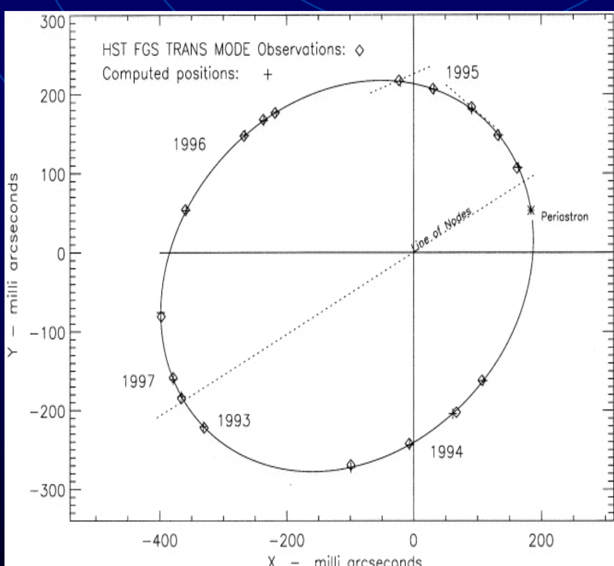


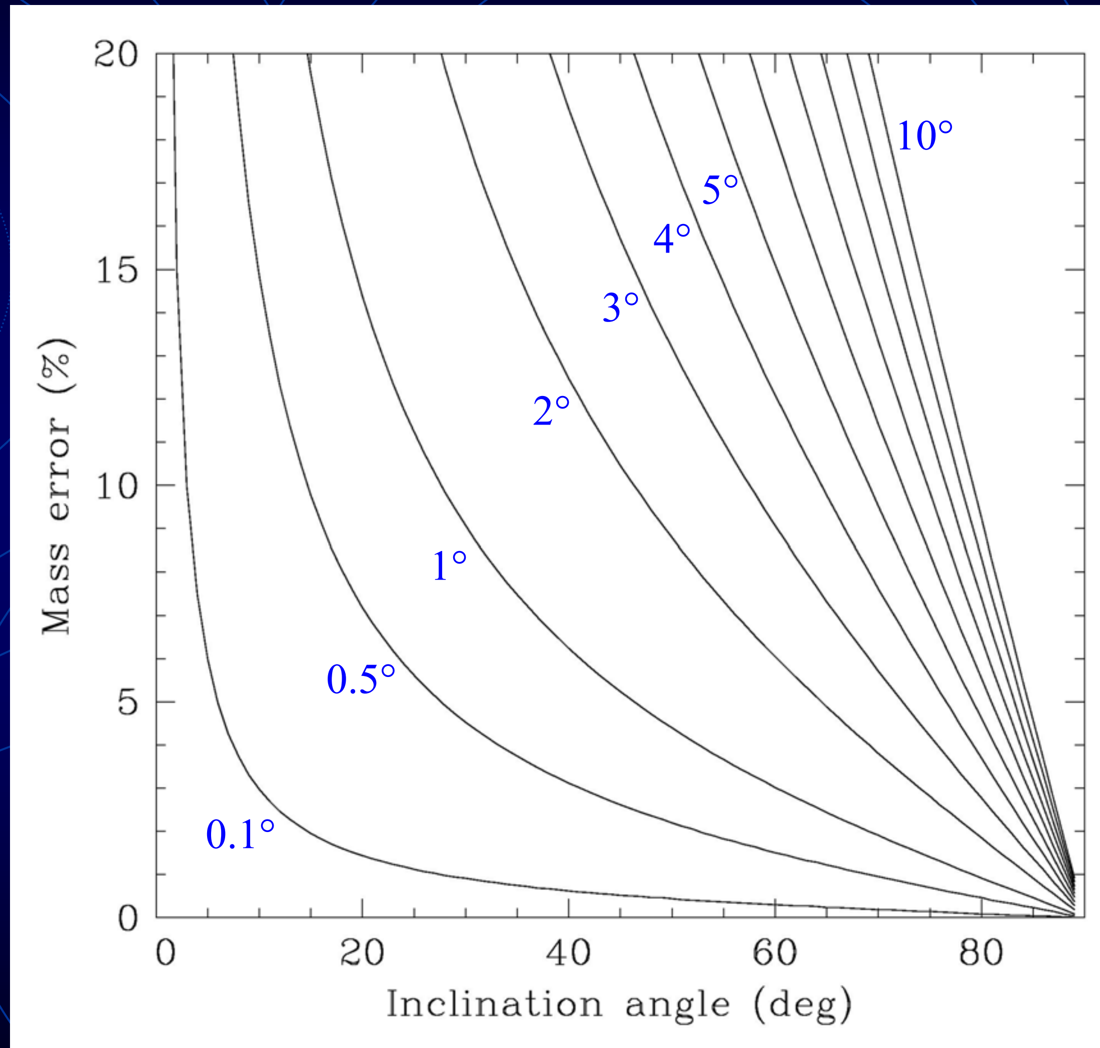
- Other combinations
  - Multiple systems →

Astrometric requirements on precision

Precision needed in the inclination angle of astrometric binaries, in order for the mass determinations **not** to be limited by the astrometry.

$$M \sin^3 i = f(P, e, K_1, K_2)$$





Stellar Radius Determinations

- Double-lined spectroscopic eclipsing binaries are the primary source of precise radius determinations: errors can be ≤1%, and the measurements are independent of distance
    - $R_1/a\ ,\ R_2/a\quad ,\quad a\sin i = P(K_1 + K_2)\sqrt{1-e^2}$
- Angular diameters (θ) + parallaxes
    - Few angular diameters for dwarfs
    - Difficult in binaries

Status of Angular Diameter Measurements in the Lower Main Sequence

- Lane, Boden & Kulkarni 2001, ApJ, 551, L81 **[PTI]**
- Ségransan, Kervella, Forveille & Queloz 2003, A&A, 397, L5 **[VLTI]**
- Pijpers et al. 2003, A&A, in press (internal σ_θ < 10μas, or 0.5%!) **[VLTI]**
- Kervella et al. 2003, astro-ph/0303634 **[VLTI]**

Relative errors in R as low as 2%

↓

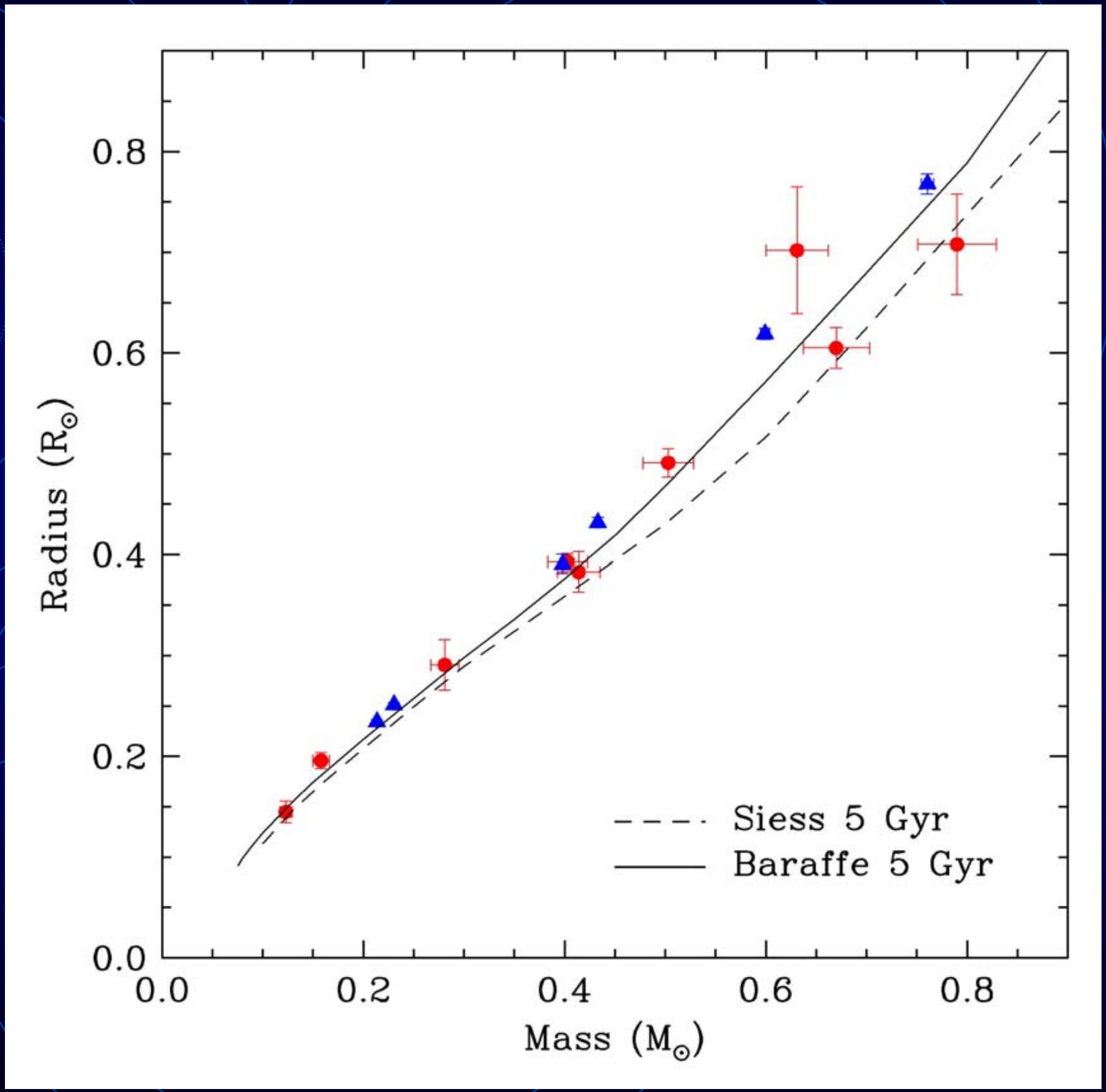
Object	Spectral Type	Photometry & Masses		Limb Darkening, K band (models)						Diameter [mas]			Radius [$R_\odot$]		Atm. prop.				
		M_K	$M/M_\odot$	g	$T_{\rm eff}$	a_1	a_2	a_3	a_4	$\theta_{\rm UD}$	$\theta_{\rm LD}$	σ_θ	R	σ_R	$T_{\rm eff}$	$\sigma_{T_{\rm eff}}$	g	σ_g	
GJ205	M1.5V	5.09	0.631 ± 0.031	4.70	3894	1.11	−1.11	0.92	−0.31	1.124	1.149	0.11	0.702	0.063	3520	170	4.54	0.06	→ 9.0%
GJ887	M0.5V	5.79	0.503 ± 0.025	4.80	3645	1.61	−2.35	2.00	−0.68	1.366	1.388	0.04	0.491	0.014	3626	56	4.76	0.03	→ 2.9%
GJ191	M1V	7.08	0.281 ± 0.014	4.98	3419	1.76	−2.72	2.39	−0.82	0.681	0.692	0.06	0.291	0.025	3570	156	4.96	0.13	→ 8.6%
GJ551	M5.5V	8.80	0.123 ± 0.006	5.19	3006	1.94	−2.80	2.39	−0.81	1.023	1.044	0.08	0.145	0.011	3042	117	5.20	0.23	→ 7.6%
GJ699	M4V	8.21	0.158 ± .008	5.11	3193	1.87	−2.88	2.54	−0.88	0.987	1.004	0.04	0.196	0.008	3163	65	5.05	0.09	→ 3.9%
GJ15A	M2V	6.27	0.414 ± .021	4.87	3541	1.66	−2.48	2.14	−0.73	0.984	1.000	0.05	0.383	0.02	3698	95	4.89	0.06	→ 4.9%
GJ411	M1.5V	6.33	0.403 ± .020	4.88	3533	1.67	−2.50	2.16	−0.73	1.413	1.436	0.03	0.393	0.008	3570	42	4.85	0.03	→ 2.0%
GJ380	K7V	4.77	0.670 ± .033*	4.65	4106	1.09	−1.01	0.83	−0.28	1.268	1.155	0.04	0.605	0.02	–	–	4.70	0.03	→ 3.2%
GJ105A	K3V	4.17	0.790 ± .039*	4.56	4603	0.86	−0.53	0.38	−0.13	0.914	0.936	0.07	0.708	0.05	–	–	4.63	0.05	→ 7.4%

Ségransan, Kervella, Forveille & Queloz 2003, A&A, 397, L5

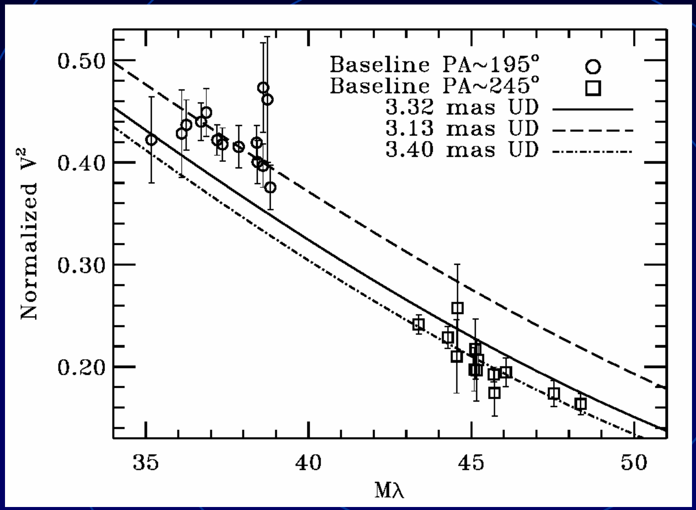
Status of Radius Determinations in the Lower Main Sequence

Mass-Radius relation

Radius determinations from angular diameter measurements and eclipsing binaries.

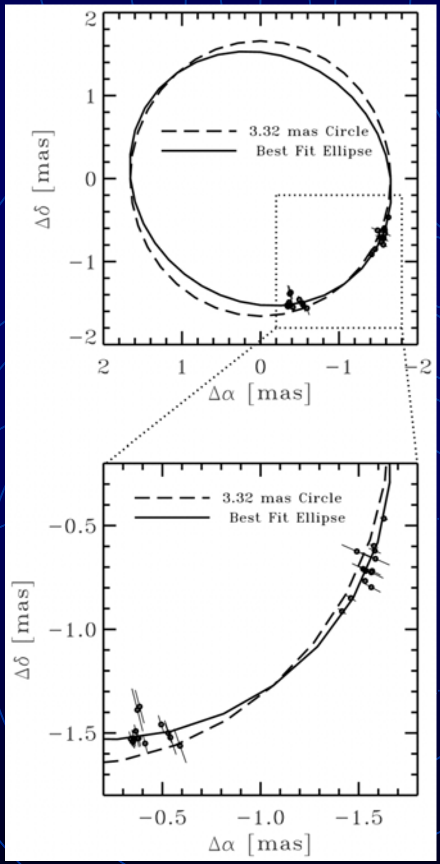


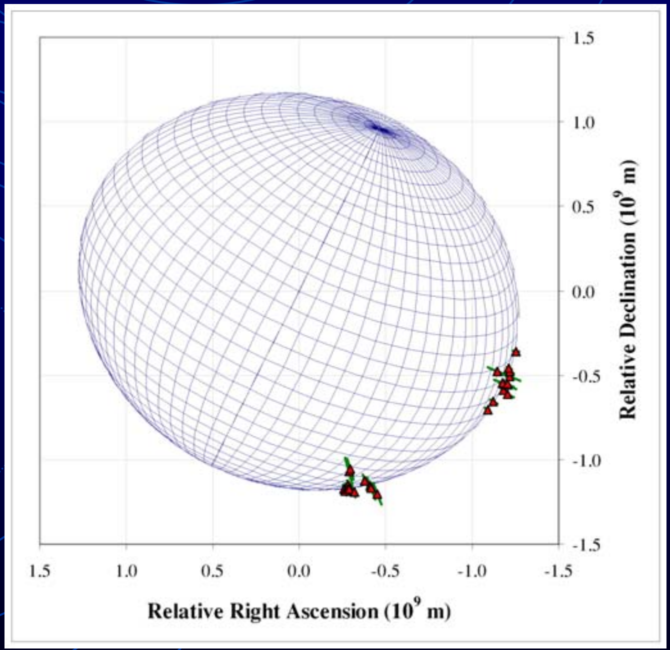
An interesting recent result



First direct measurement of an oblate photosphere in a main-sequence star!

van Belle et al. 2001, ApJ, 559, 1155





Altair (α Aql)

SpT = A7IV-V

$\pi = 194.45 \pm 0.94$ mas

$a = 3.461 \pm 0.038$ mas

$b = 3.037 \pm 0.069$ mas

$a/b = 1.140 \pm 0.029$

$\langle \boldsymbol{R} \rangle = 1.794 \pm 0.023\ R_\odot$

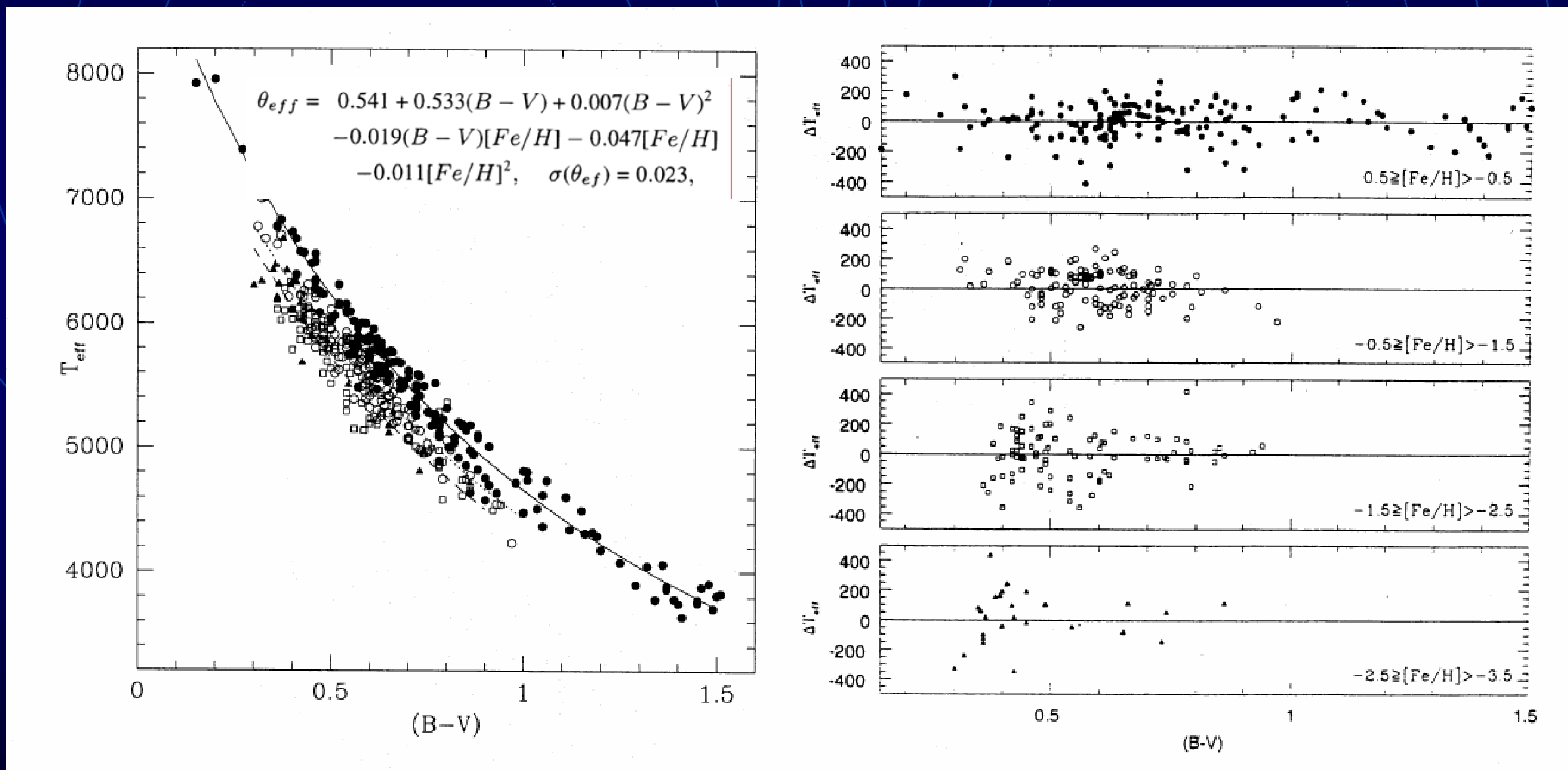
Effective Temperature Determinations

- Angular diameters and bolometric fluxes:

$$4\pi d^2 \int_0^\infty f_\lambda^E d\lambda = 4\pi R^2 \sigma T_{\rm eff}^4 \quad \rightarrow \quad f_{\rm bol}^E = \frac{\theta^2}{4}\sigma T_{\rm eff}^4 \quad , \ \sigma = 5.67 \times 10^{-5} \ {\rm erg\ s^{-1}\ cm^{-2}\ K^{-4}}$$

- Infrared Flux Method (Blackwell et al. 1980, 1990): combines $f_{\rm bol}^E$ and f_λ^E with model atmospheres to get $T_{\rm eff}$ (and θ)
- Color-temperature calibrations

Practical Determination of Effective Temperatures Through Color Indices



Alonso, Arribas & Martínez-Roger 1996, A&A, 313, 873

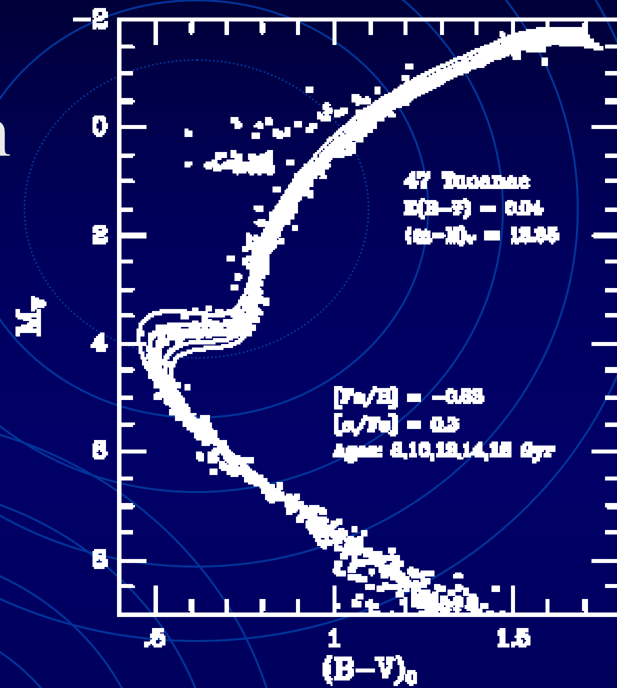
Luminosity Determinations

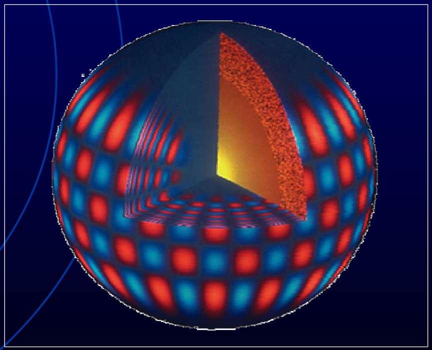
- From R and T_{eff} for eclipsing binaries (distance-independent, except for residual reddening effects on T_{eff}): $L \propto R^2\, T_{eff}^{\;4}$
- From parallaxes for nearby stars
  - Trigonometric parallaxes (e.g., HIPPARCOS)
  - Orbital parallaxes:

$$\pi_{\rm orb} = \frac{a \sin i}{P\sqrt{1-e^2}(K_1 + K_2)}$$

Comparing Observations With Theory

- Color-magnitude diagrams in open clusters and globular clusters
  - Constraints from large number of stars (same age, same metallicity)
- Binary systems with accurately determined properties
  - Simultaneous constraints from different stellar properties
- Single stars (asteroseismology)
  - Constraints on interior structure

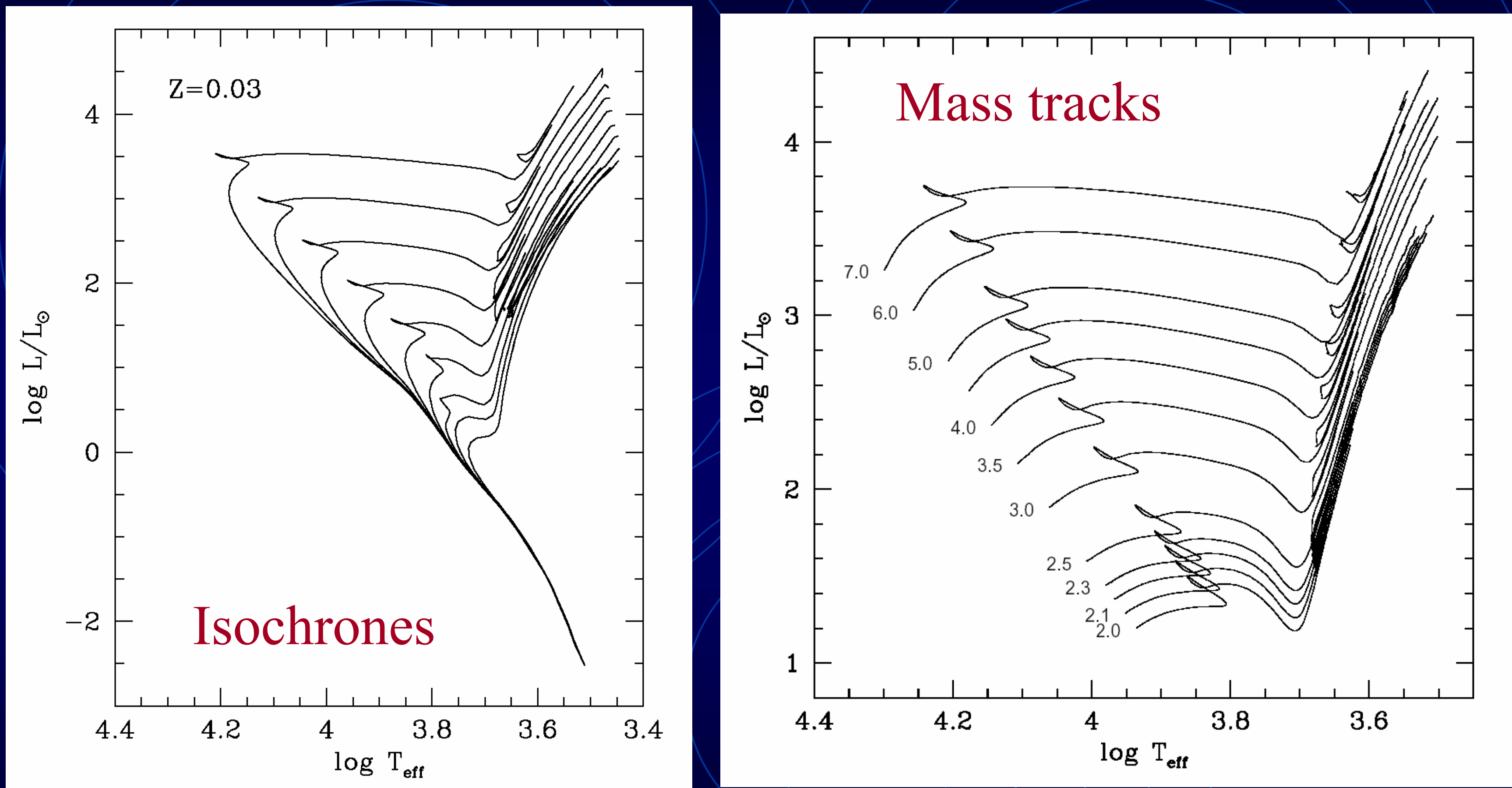




Aspects of Theory That Can Be Tested Using Binaries

- Stellar evolution (M–R diagram, T_{eff}–log g diagram, and others)
- Internal structure (apsidal motion in eccentric binary systems)
  - Contribution from General Relativity
- Tidal theories (rotational synchronization and orbital circularization)
  - Hydrodynamical currents (Tassoul 1987, 1988)
  - Turbulent dissipation and radiative damping (Zahn 1977, 1989)

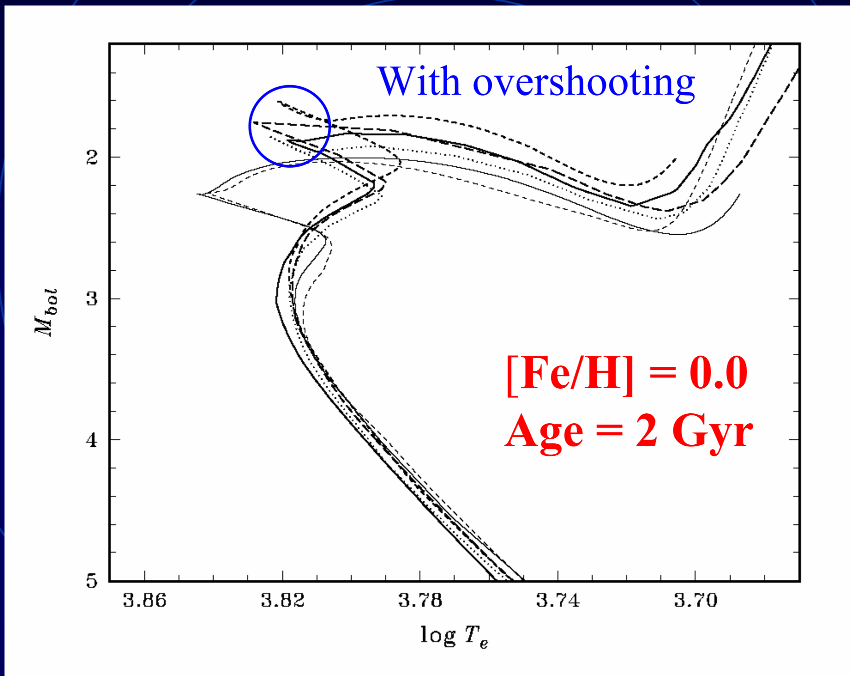
How Do We Use Stellar Evolution Theory?



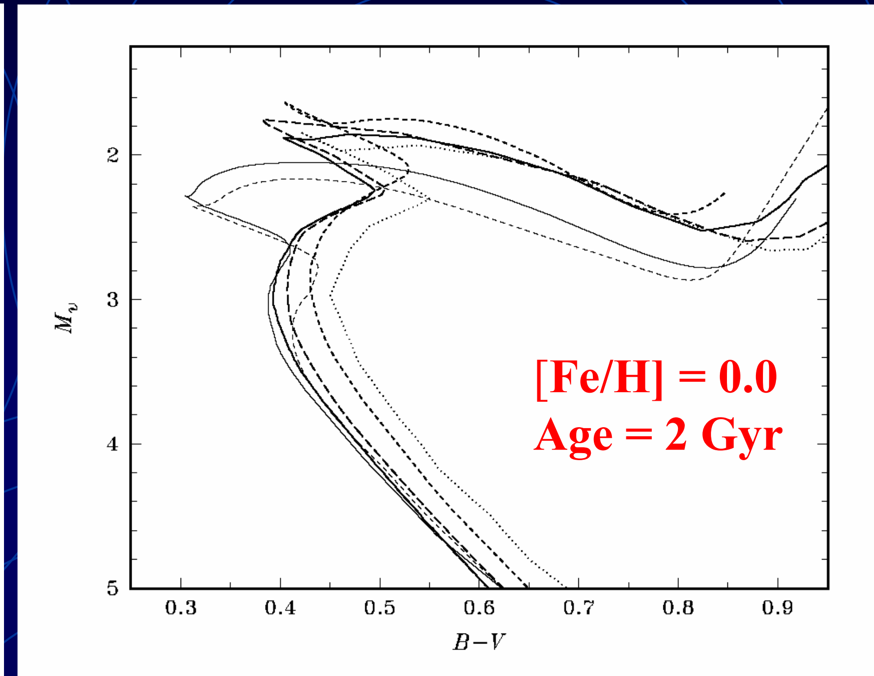
Girardi et al. 2000, A&AS, 141, 371

We compare isochrones and mass tracks with the observations.

Isn't Theory Ok Already?



Theoretical plane



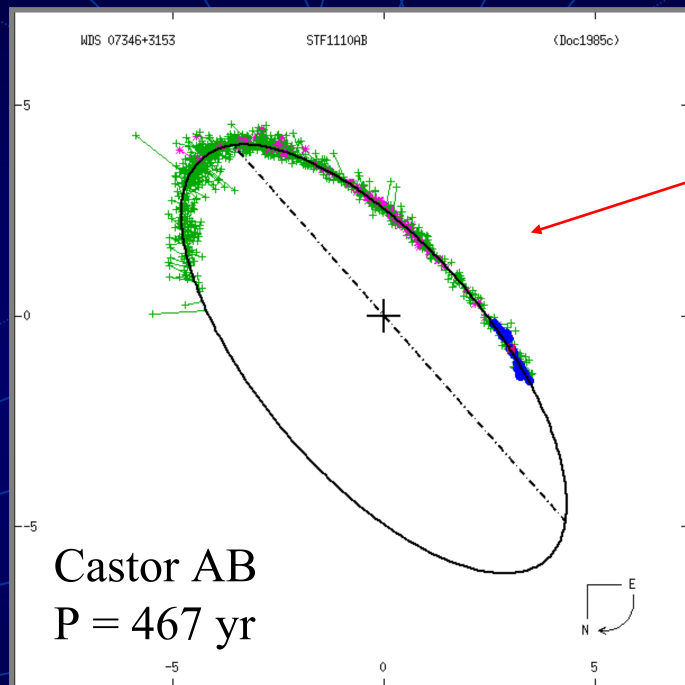
Observational plane

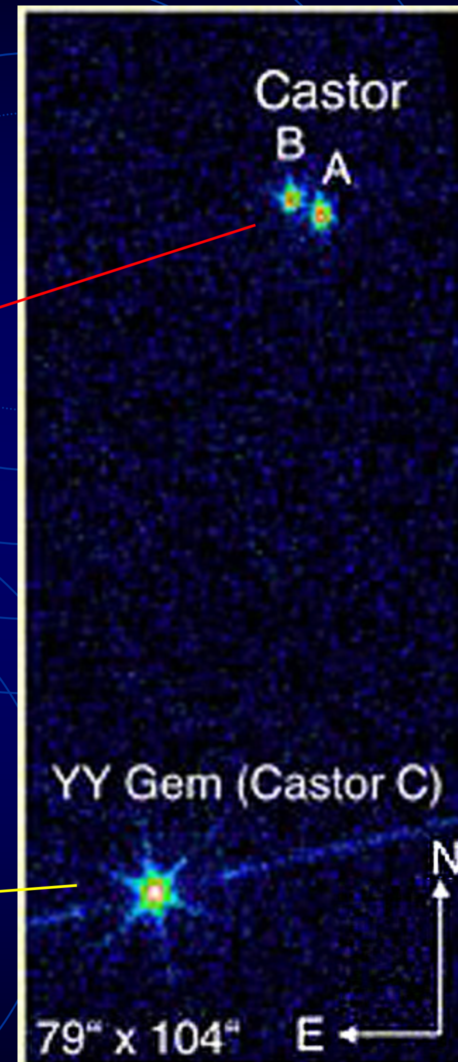
Nordström et al. 1997, A&A, 322, 460

There are still some disagreements between different models, because of the different physical assumptions and also the transformations to the observational plane.

Testing stellar evolution models in the lower main sequence: **YY Gem**

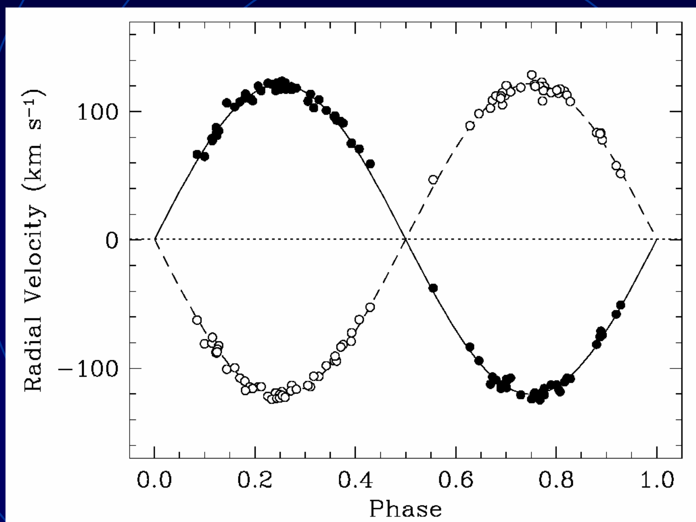
There are only 3 known double-lined eclipsing binaries with components that are M stars, with sufficiently accurate mass determinations.





The double-lined spectroscopic eclipsing binary YY Gem (Period = 0.814 days) is a member of the Castor sextuple system.

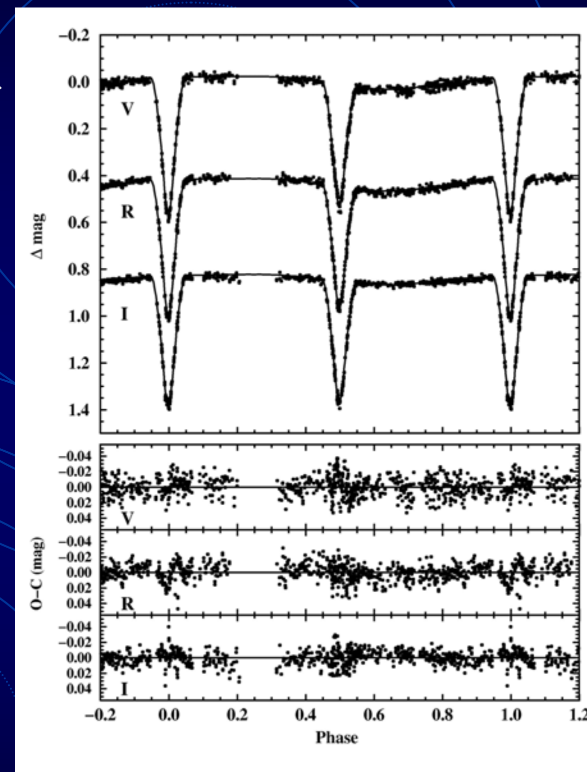
Determination of the Physical Properties of YY Gem from the Observations

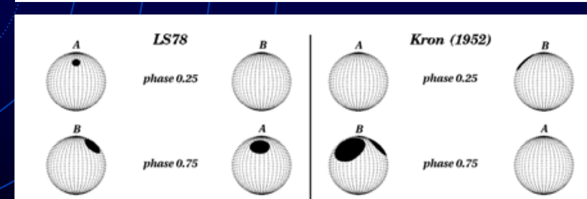


Torres & Ribas 2002, ApJ, 567, 1140

← RV curves

Light curves→



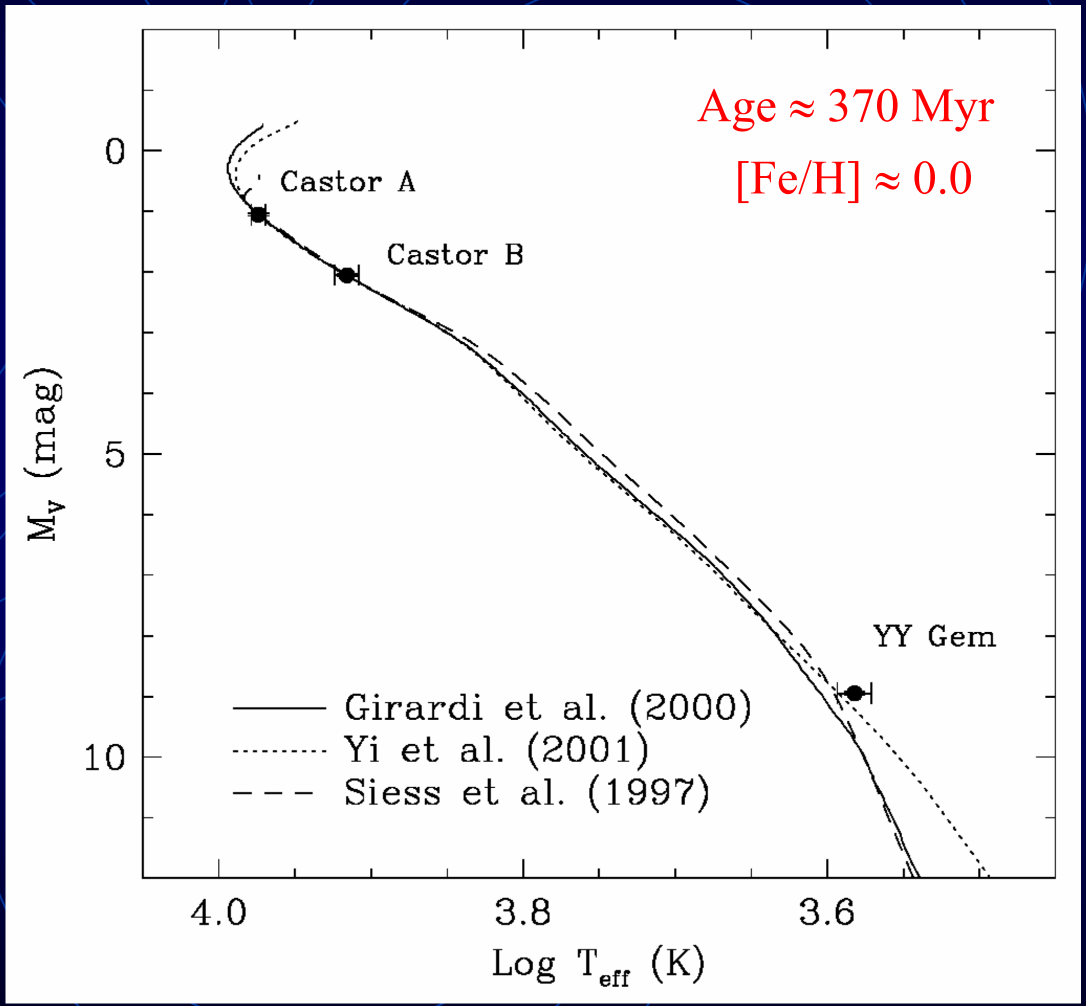


Parameter	Value	
Mass ($M_{\odot}$)	0.5992 ± 0.0047	← σ < 1%
Radius ($R_{\odot}$)	0.6191 ± 0.0057	← σ < 1%
Log g	4.6317 ± 0.0083	
Temperature (K)	3820 ± 100	← σ < 3%

The Age and Metallicity of YY Gem

Since YY Gem is a member of the Castor system, we may assume that all stars have the same age and chemical composition.

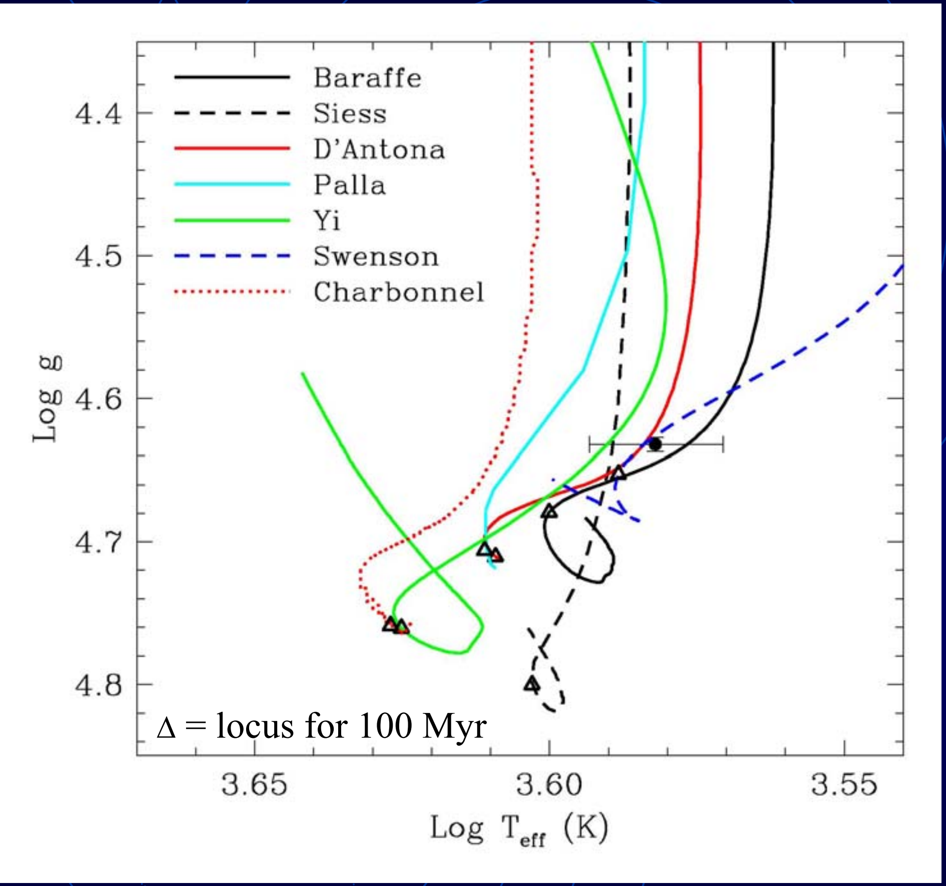
Isochrone fitting to Castor A+B gives an age of 370 Myr, and a metallicity very close to the solar value.



Comparing YY Gem with Stellar Evolution Models. I

When using the radius (or log *g*) as a measure of evolution, the models give ages for YY Gem in the range 30–85 Myr.

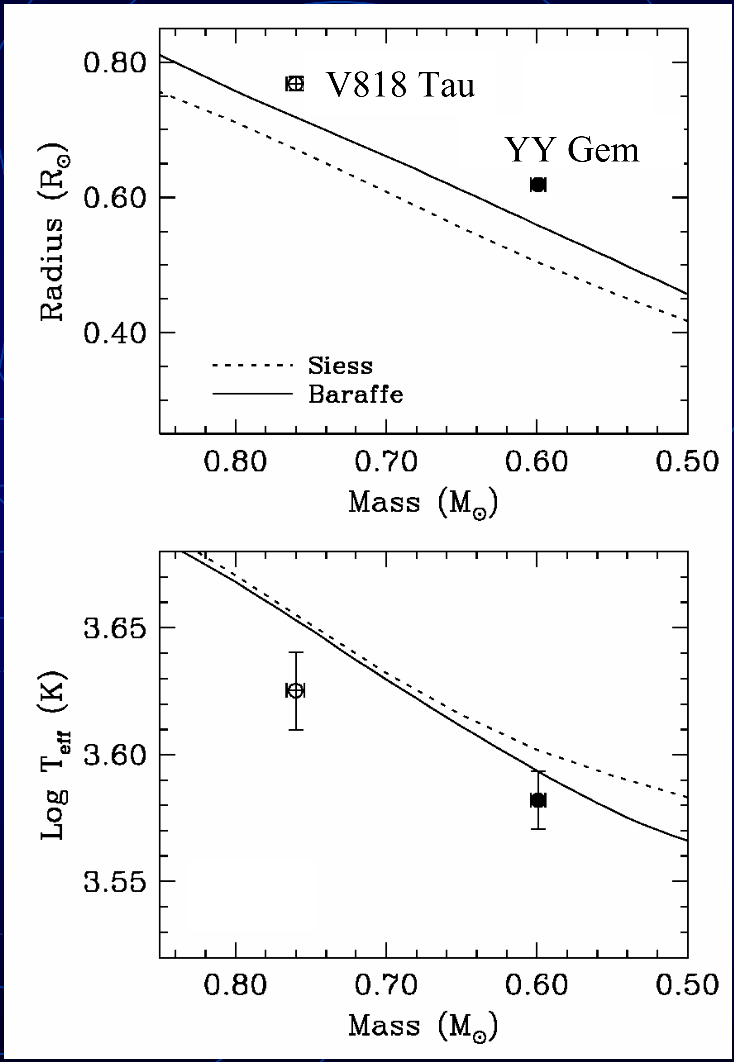
Therefore, the models seem to underestimate the age by factors of up to 10 in this mass regime (~0.6 $M_\odot$).



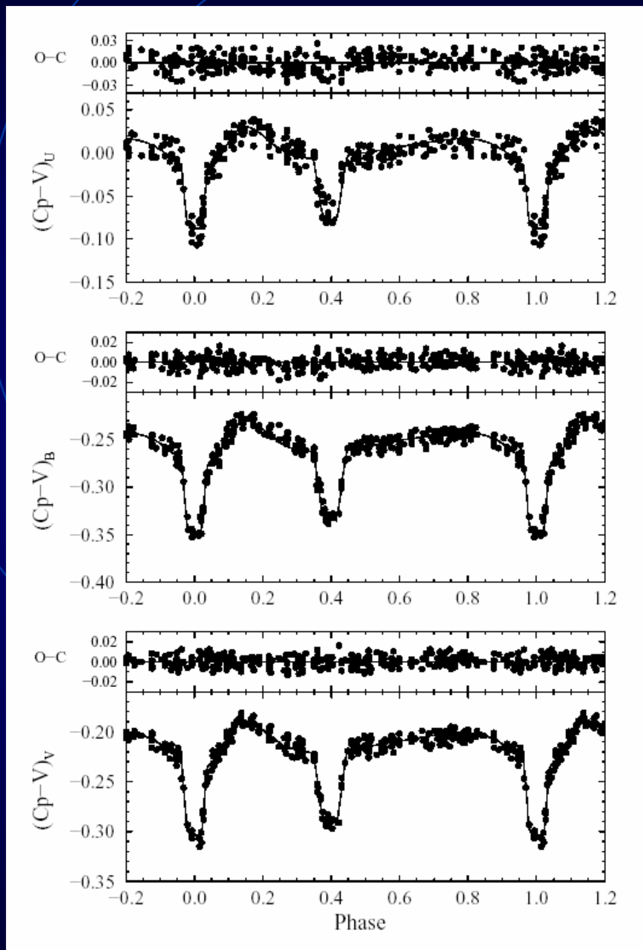
Evolutionary tracks for the mass and chemical composition of YY Gem.

Comparing YY Gem with Stellar Evolution Models. II

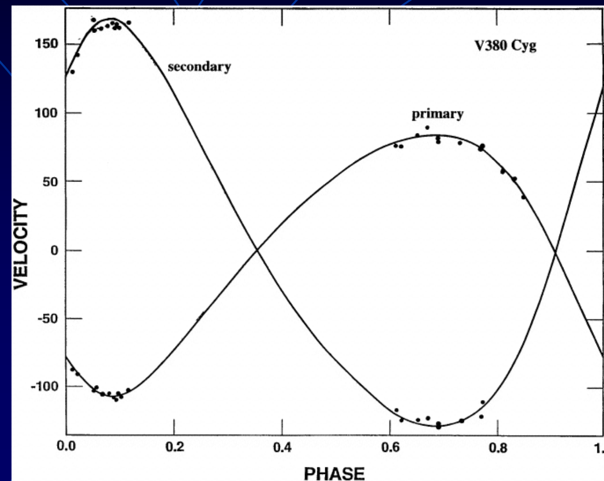
- The models underestimate the radius by up to 20%
- The models overestimate the temperature by ~150 K or more
- This can have important consequences for the estimate of ages for PMS stars
- But, radius and temperature discrepancies tend to compensate each other, so that the luminosities are not far off



Tests of Stellar Evolution for Massive Stars: **V380 Cyg**



Guinan et al. 2000, ApJ, 544, 409

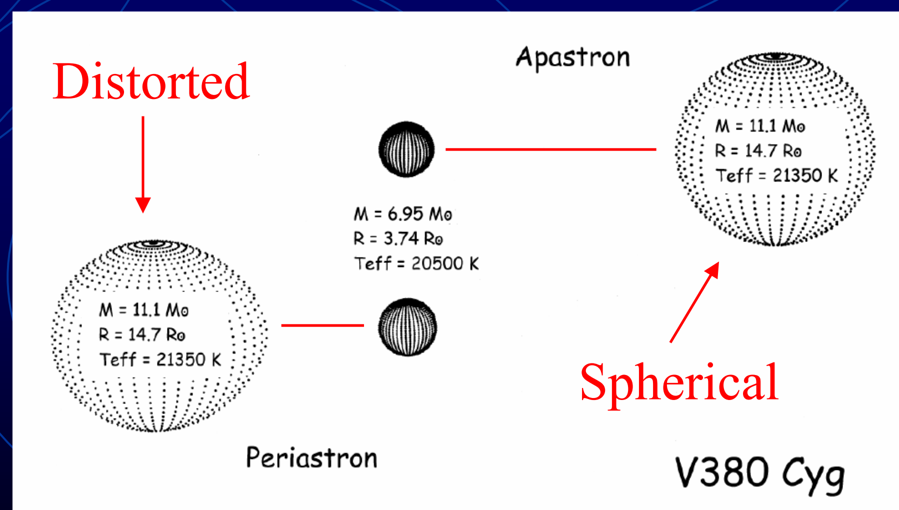


Popper & Guinan 1998, PASP, 100, 572

Spectral type:
B1.5 II-II + B2 V

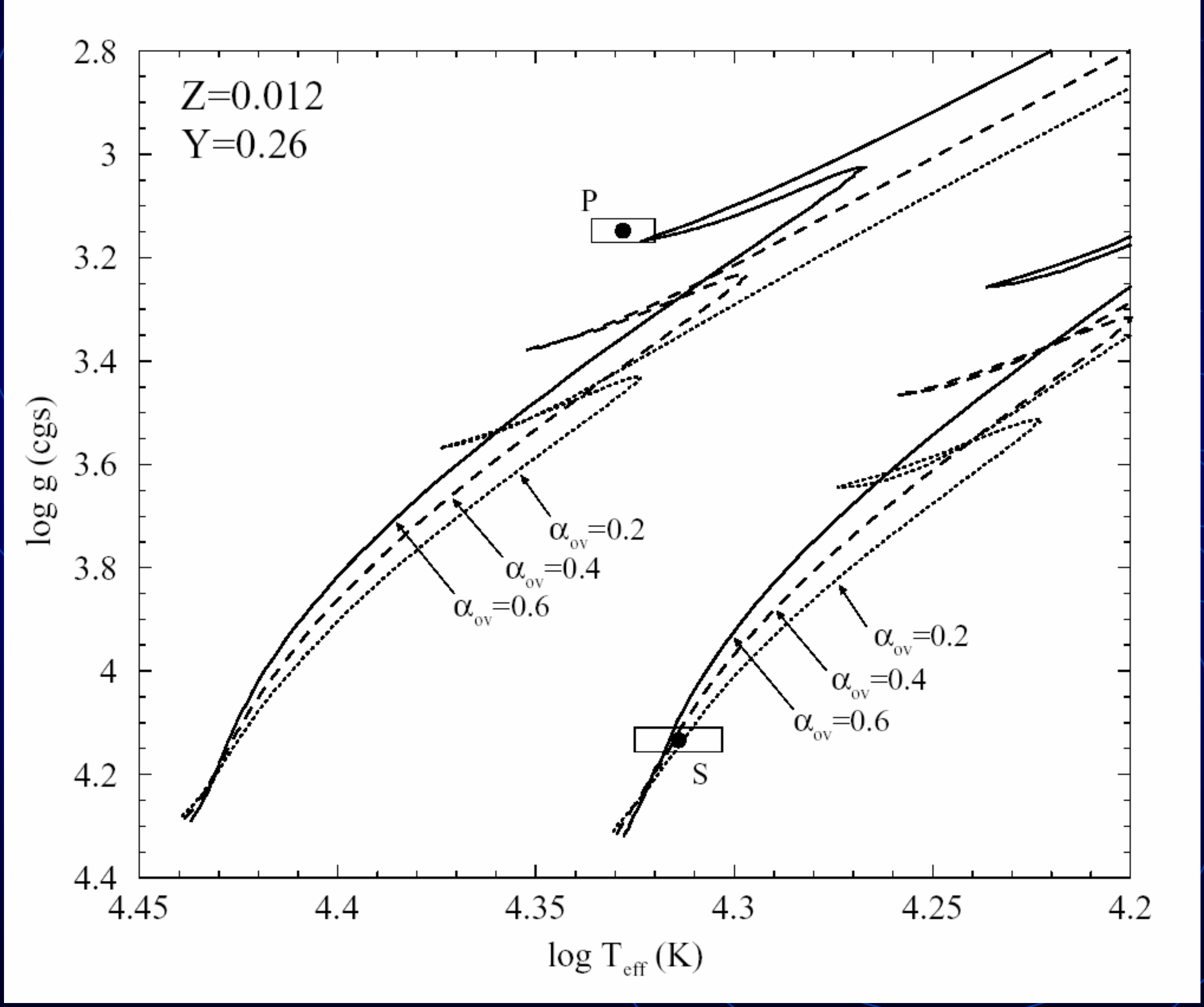
P = 12.426 days

Eccentric orbit:
e = 0.234



V380 Cyg and convective core overshooting

Guinan et al. 2000, ApJ, 544, 409

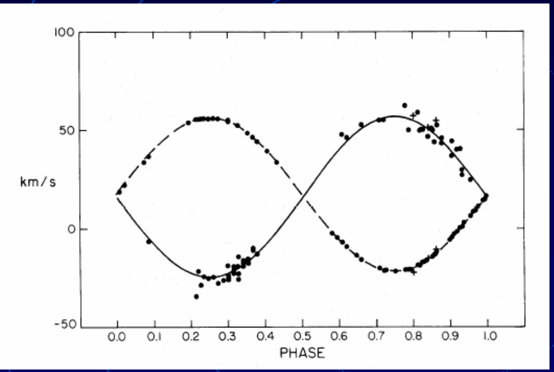


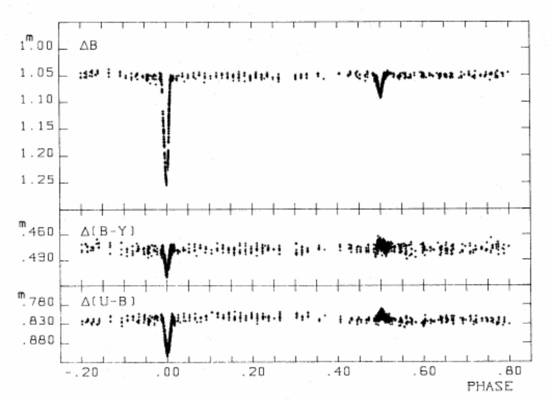
Masses and radii are good to 4% and 2%, respectively.

Evolutionary tracks for V380 Cyg, computed for the exact mass of each component (11.1 $M_\odot$ and 6.95 $M_\odot$), and for different values of the overshooting parameter.

Age ≈ 25 Myr

Testing Stellar Evolution Theory in Evolved Stars





Andersen et al. 1991,
A&A, 246, 99

TZ For

F7 III + G8 III, P = 75.7 days, e = 0

Two very different components

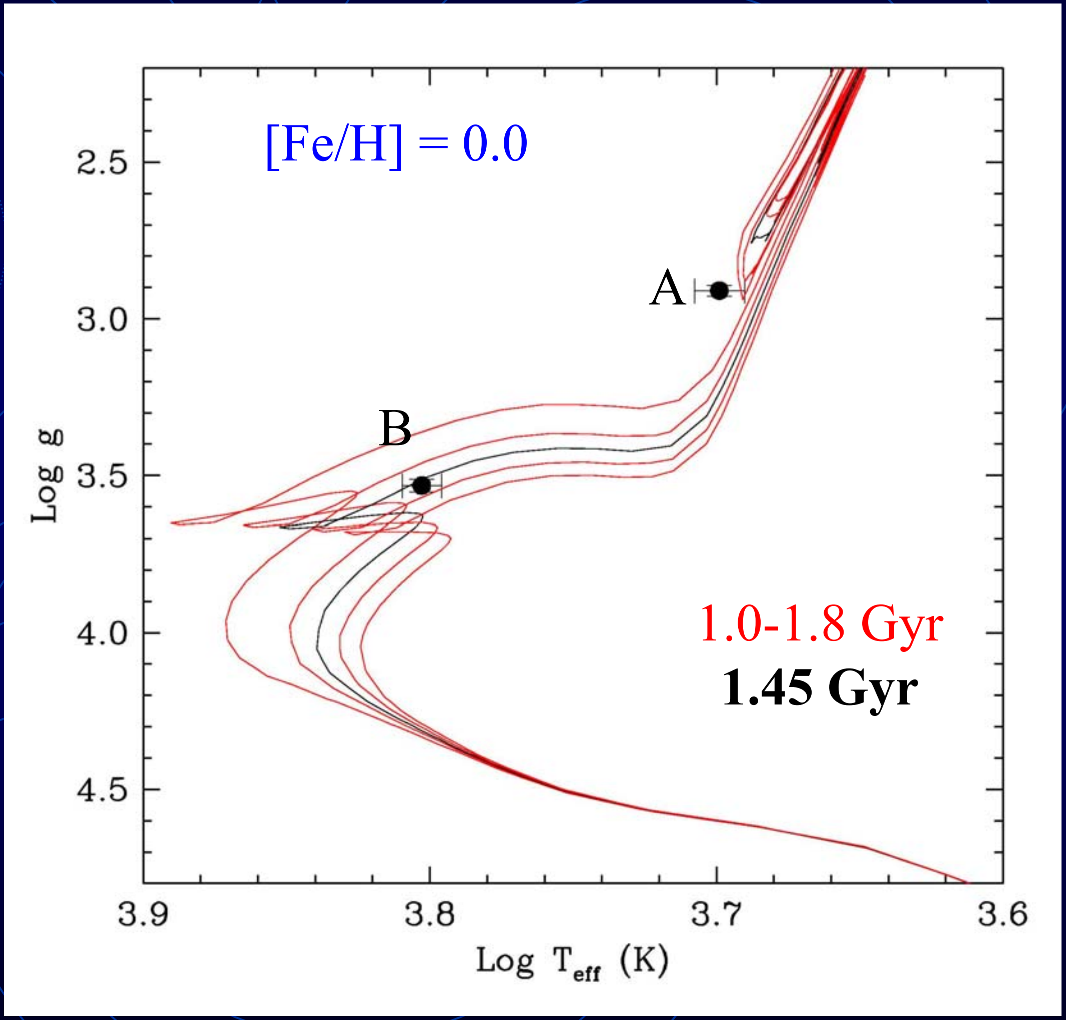
Parameter	Primary	Secondary
Mass ($M_\odot$)	**2.05 ± 0.06**	**1.95 ± 0.03**
Radius ($R_\odot$)	**8.32 ± 0.12**	**3.96 ± 0.09**
Log *g*	**2.910 ± 0.017**	**3.532 ± 0.020**
Temperature (K)	**5000 ± 100**	**6350 ± 100**
***v* sin *i* (km/s)**	**4 ± 1**	**42 ± 2**

Mass and radius errors < 3%

Isochrone Fits for TZ For

The components of TZ For are in very different evolutionary states: the primary (A) is an evolved star (clump giant).

The age is estimated to be 1.45 Gyr.



Isochrones from Girardi et al. 2000, A&AS, 141, 371

Tidal Theory (Zahn 1977, 1989)

Tidal forces tend to synchronize the rotation of the stars with the orbital motion, and to circularize the orbit. The timescales are:

Radiative envelopes

$$\frac{1}{(\tau_{sync})_{rad}} = -\frac{1}{\Omega_{\rm orb} - \Omega_{\rm rot}}\frac{d\Omega_{\rm rot}}{dt}, \quad (\tau_{sync})_{rad} = 2.03 {\beta_g}^2 M^{7/3} \frac{(1+q)^2}{q^2} E_2^{-1} \frac{P^{17/3}}{R^7}$$

$$\frac{1}{(\tau_{circ})_{rad}} = -\frac{1}{e}\frac{de}{dt}, \quad (\tau_{circ})_{rad} = 1.71 \times 10^1 M^3 \frac{(1+q)^{5/3}}{q} {E_2}^{-1} \frac{P^7}{R^9}$$

Convective envelopes

$$\frac{1}{(\tau_{sync})_{turb}} = -\frac{1}{\Omega_{\rm orb} - \Omega_{\rm rot}}\frac{d\Omega_{\rm rot}}{dt}, \quad (\tau_{sync})_{turb} = 3.95 \times 10^2 {\beta_g}^2 M^{7/3} \frac{(1+q)^2}{q^2} L^{-1/3} {\lambda_2}^{-1} \frac{P^4}{R^{16/3}}$$

$$\frac{1}{(\tau_{circ})_{turb}} = -\frac{1}{e}\frac{de}{dt}, \quad (\tau_{circ})_{turb} = 1.99 \times 10^3 M^3 \frac{(1+q)^{5/3}}{q} L^{-1/3} {\lambda_2}^{-1} \frac{P^{16/3}}{R^{22/3}}$$

Effective circularization time

$$-\frac{1}{e}\frac{de}{dt} = \frac{1}{\tau_{circ,A}} + \frac{1}{\tau_{circ,B}}$$

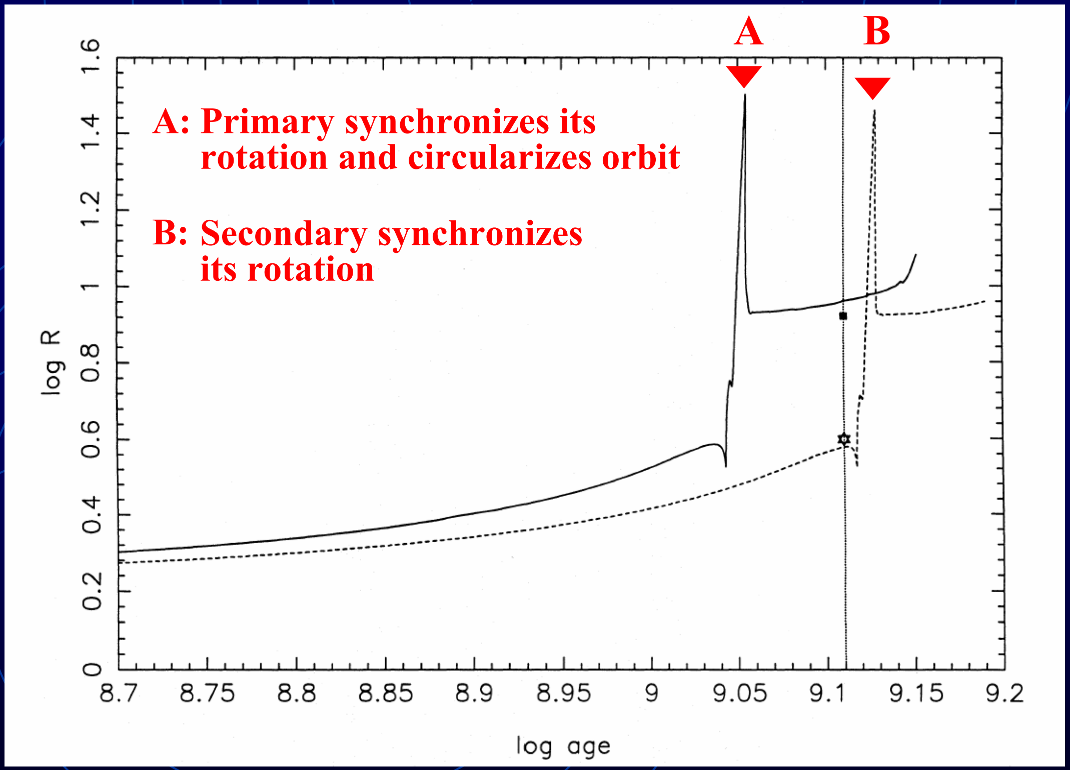
TZ For and Tidal Theory

Primary star has $v \sin i = 4$ km/s → synchronous rotation

Secondary star has $v \sin i = 42$ km/s → asynchronous rotation

Radius as a function of age.

Predictions for the time of circularization and synchronization agree with the observations.



Examples of Uncertainties Remaining in the Models

- Treatment of convection
  - Mixing length prescription (simplified treatment)
  - Overshooting
- Radiative opacities
- Nuclear reaction rates
- Helium and metal diffusion
- Equation of state
- Rotation (internal?)
- Mass loss
- Magnetic fields

Prospects

- Recent advances in observational techniques and analysis techniques are very promising
  - Spectroscopy: better instruments, new analysis tools
  - Interferometry: new instruments coming online
- Accurate fundamental parameters for stars are still needed, particularly for astrophysically important non-interacting binary systems
- Complete information is required in order to provide the tightest constraints on theory

Suggested Reading

- Popper, D. M. 1980, *Stellar masses*, ARA&A, 18, 115
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- Heintz, W. D. 1973, *Double stars*, Reidel Pub. Co.
- Henry, T. J. & McCarthy, D. W. 1993, *The mass-luminosity relation for stars of mass 1.0 to 0.08 solar mass*, AJ, 106, 773
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- Alonso, A. et al. 1996*, The empirical scale of temperatures of the low main sequence (F0V-K5V)*, A&A 313, 873
- Girardi, L. et al. 2000, *Evolutionary tracks and isochrones for low- and intermediate-mass stars: from 0.15 to 7* $M_{\odot}$*, and from Z=0.0004 to 0.03*, A&AS, 141, 371
- Yi, S. et al. 2003, *The* Y^2 *Stellar Evolutionary Tracks*, ApJS, 144, 259