Introduction to Control Systems

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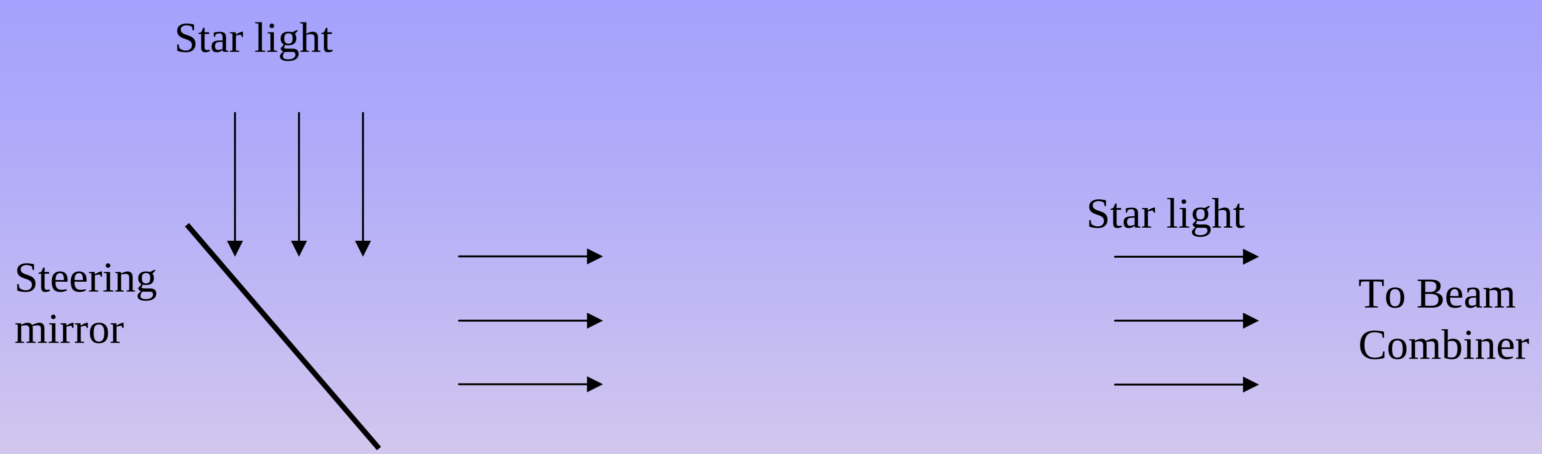
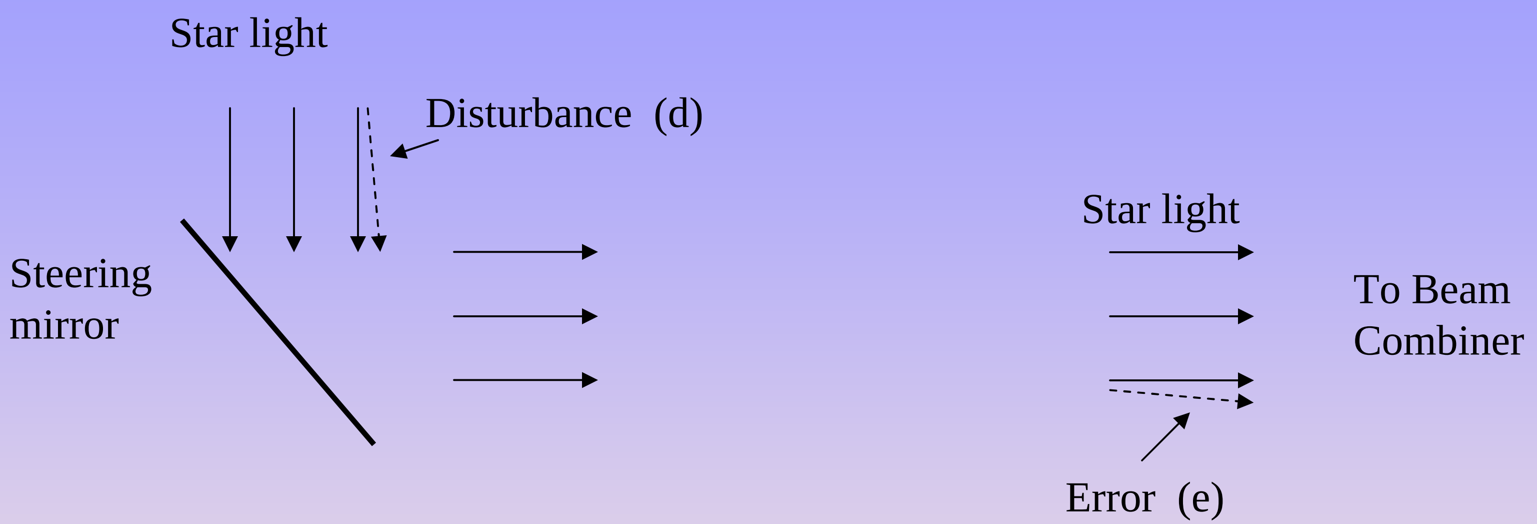
Sample Application: Star Tracker



Star light is delivered to beam combiner by steering mirror.

Want to maintain alignment of direction (wave fronts) of star light beam for good fringe visibility.

Star Tracker

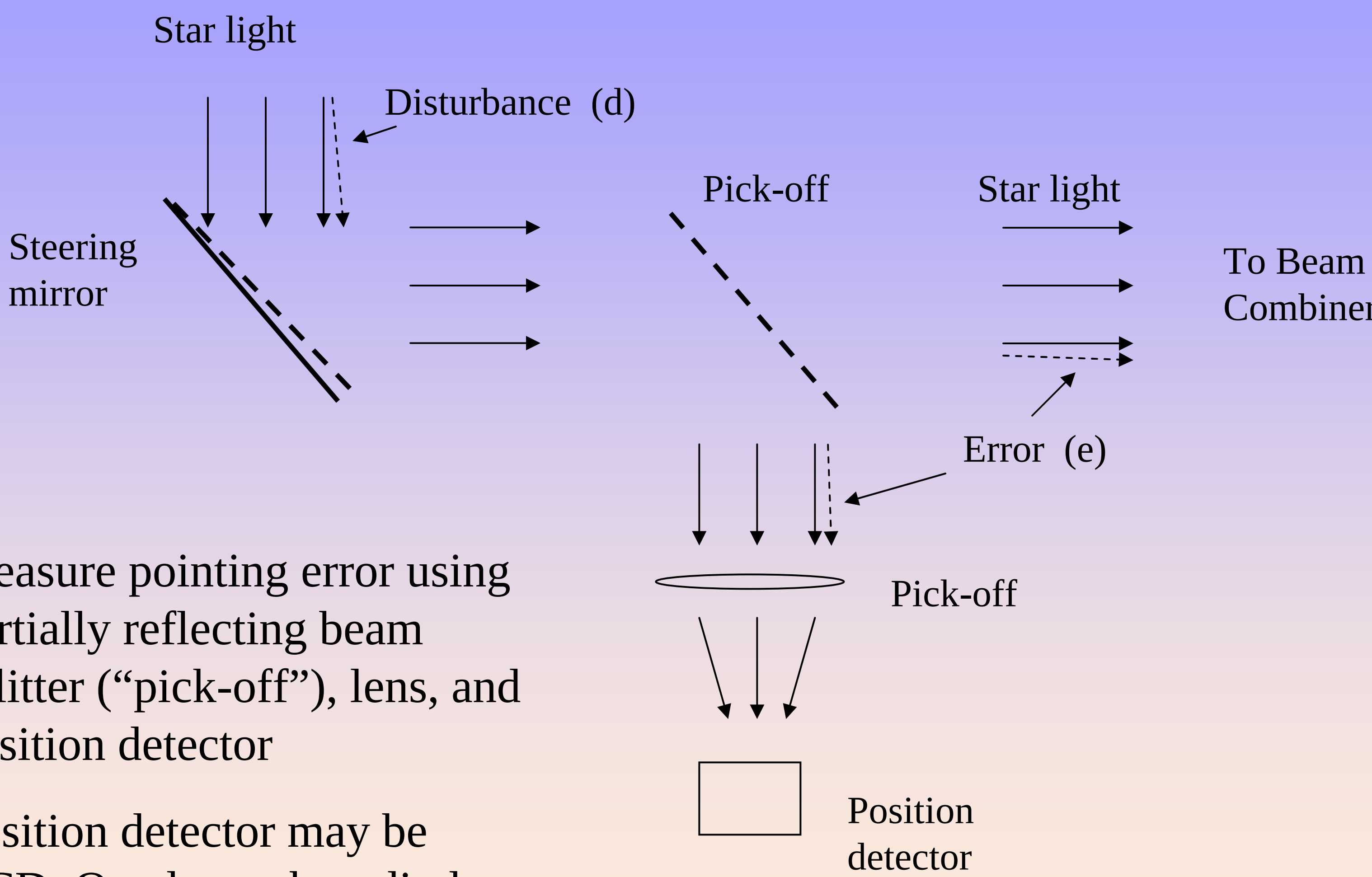


Imperfection changes direction of incoming beam of star light.

E.g., warping of mirror mount due to a change in temperature.

No longer have good alignment at beam combiner; fringe visibility is degraded.

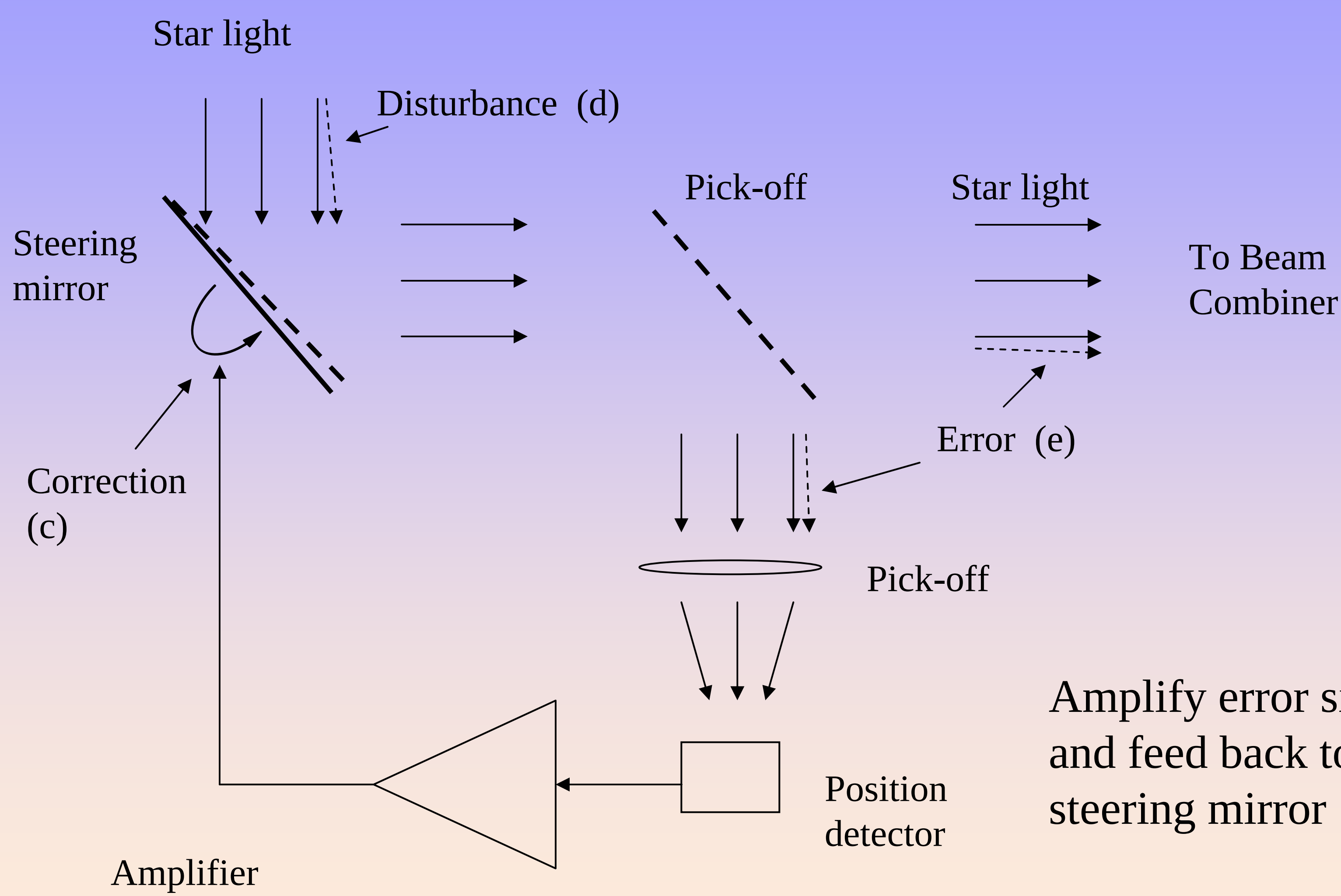
Star Tracker



Measure pointing error using partially reflecting beam splitter ("pick-off"), lens, and position detector

Position detector may be CCD, Quadrant photodiode, etc.

Star Tracker



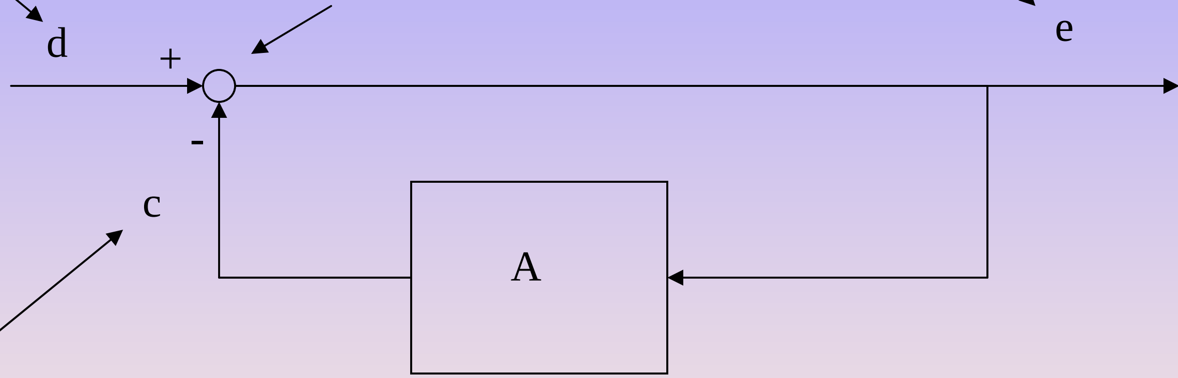
Amplify error signal and feed back to steering mirror

Block Diagram Representation

Misalignment of incoming starlight

Correction subtracted from misalignment at steering mirror

Residual alignment error in light reaching beam combiner

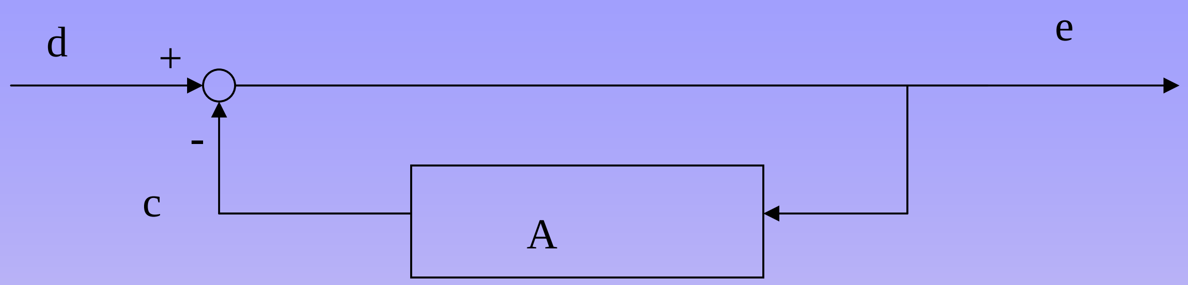


Correction applied to steering mirror

$c = A\,e$

A is loop gain or “open loop gain.” Response of mirror, lens, position detector, and amplifier (angle in, angle out).

Gain Setting



Suppose amplifier gain is adjustable.

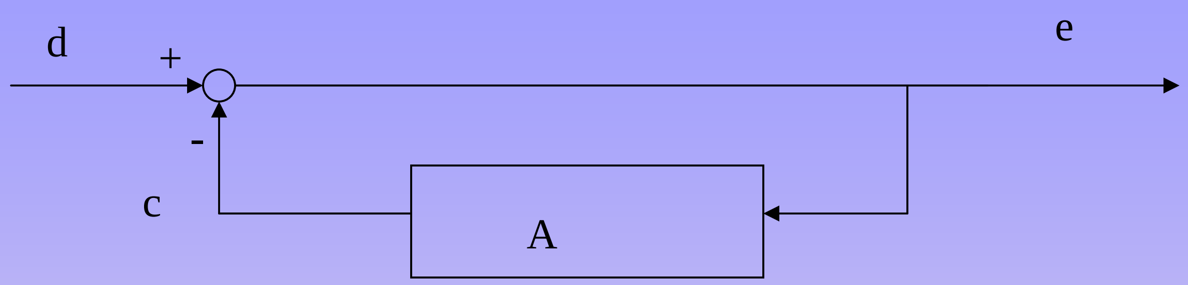
What is appropriate / optimum setting of amplifier gain?

$$e = d - Ae$$

$$e = \frac{d}{1+A}$$

Note: this holds for any frequency, if we allow the quantities involved (A, e, d) to be complex

Gain Setting



$$e = d - Ae$$

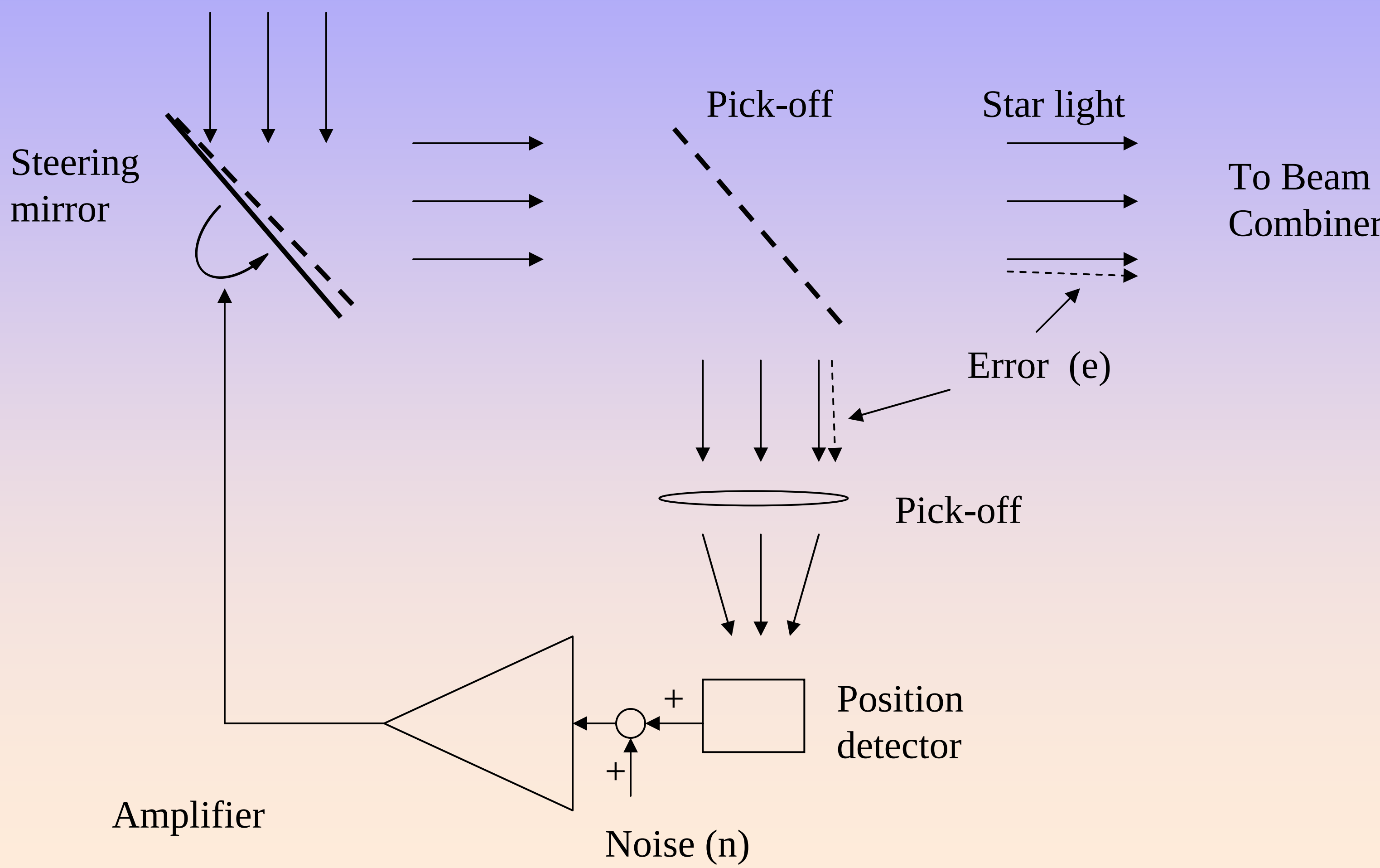
$$e = \frac{d}{1+A}$$

The higher the gain, the smaller the error. But…

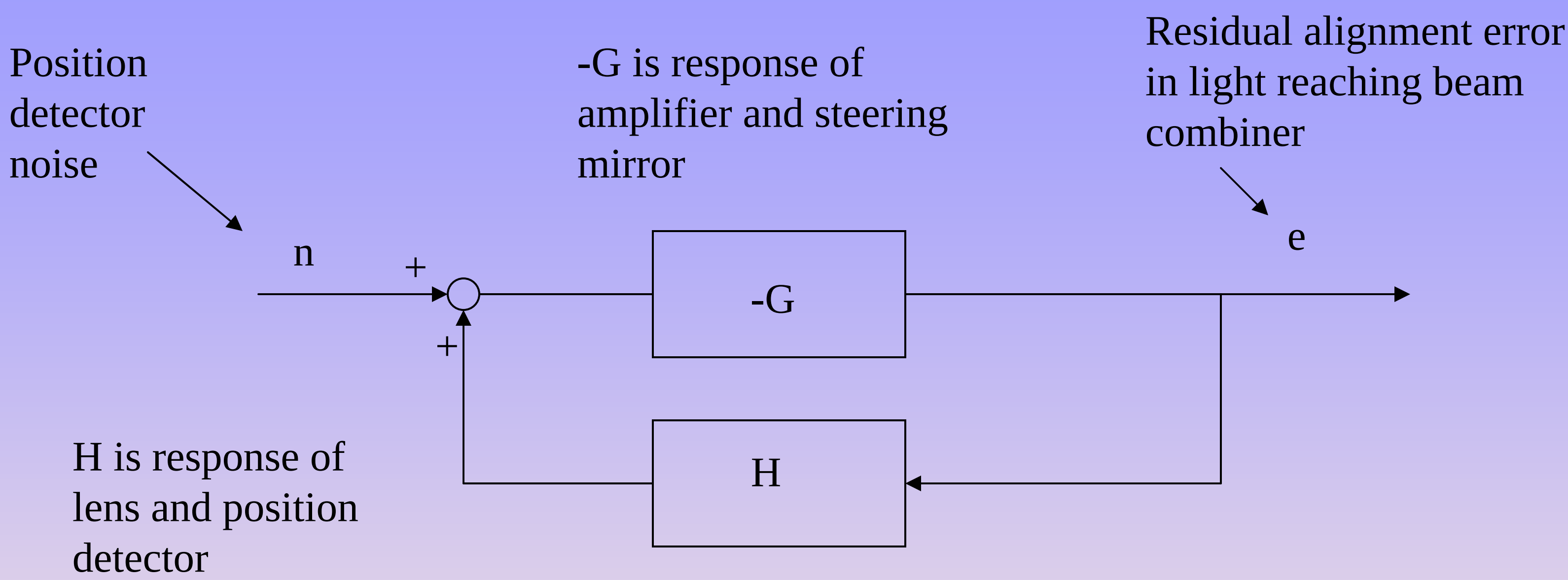
- Stability
- Other sources of error may not be suppressed

Other Sources of Error

E.G. noise in position detector



Other Sources of Error



•Use –G to denote gain of amplifier because we know there is an inversion in the loop, and we like to keep the values of the gains positive

•With this convention, A = GH

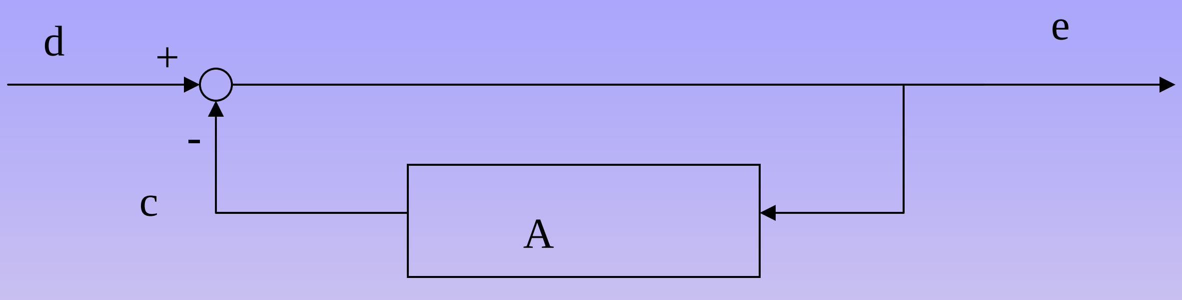
$$e = -Gn - GHe$$

$$e = \frac{-Gn}{1+GH}$$

If GH ™1, increasing G doesn't help

Stability

Recall:



$$e = \frac{d}{1+A}$$

Don't want A ≅ -1 at
any frequency.
(Necessary condition)

Stability

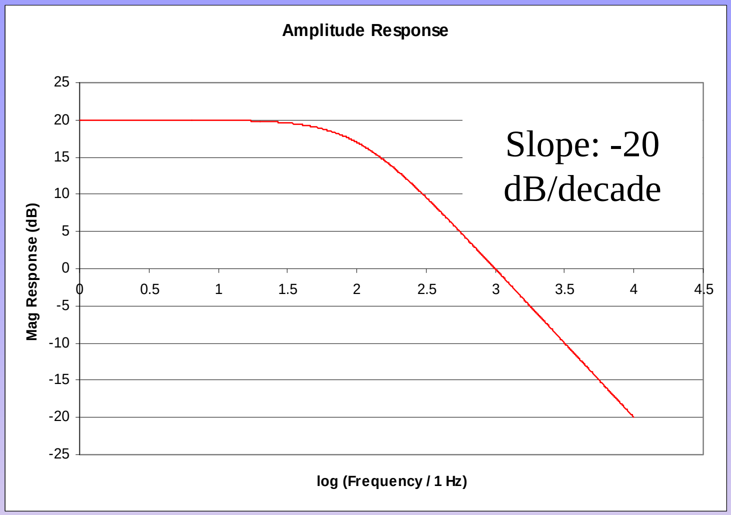
Typical frequency response for A

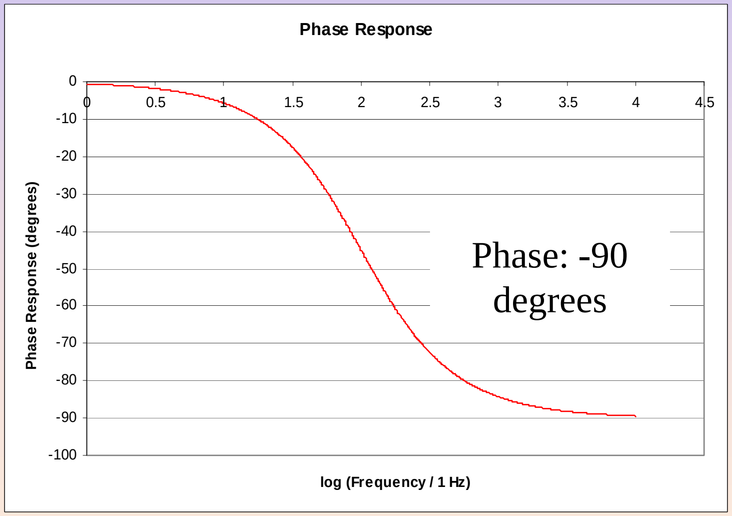
$$A = \frac{A_0}{1 + \frac{j\omega}{\omega_c}}$$

“Single pole response”

left half-plane pole at

$s = \sigma + j\omega = -\omega_c$





Stability

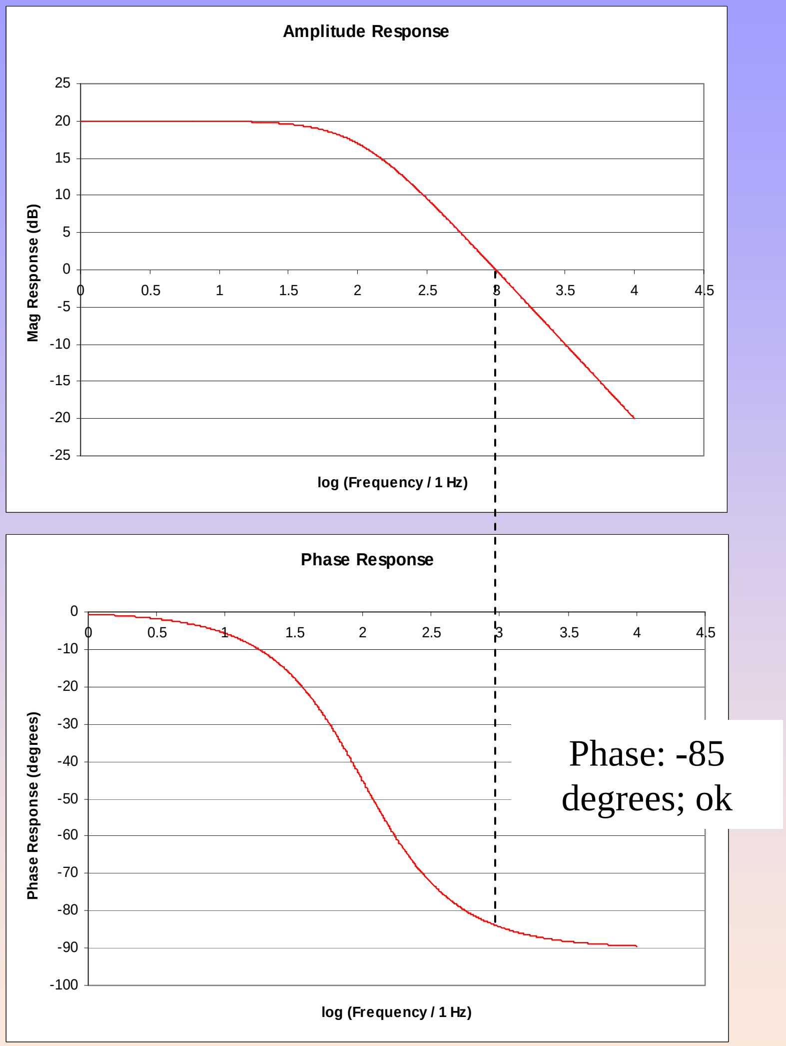
Bode stability criterion:

Phase at unity-gain frequency

> -180 degrees

Okay to turn up gain?

Yes. Phase asymptotes at –90 degrees



Stability

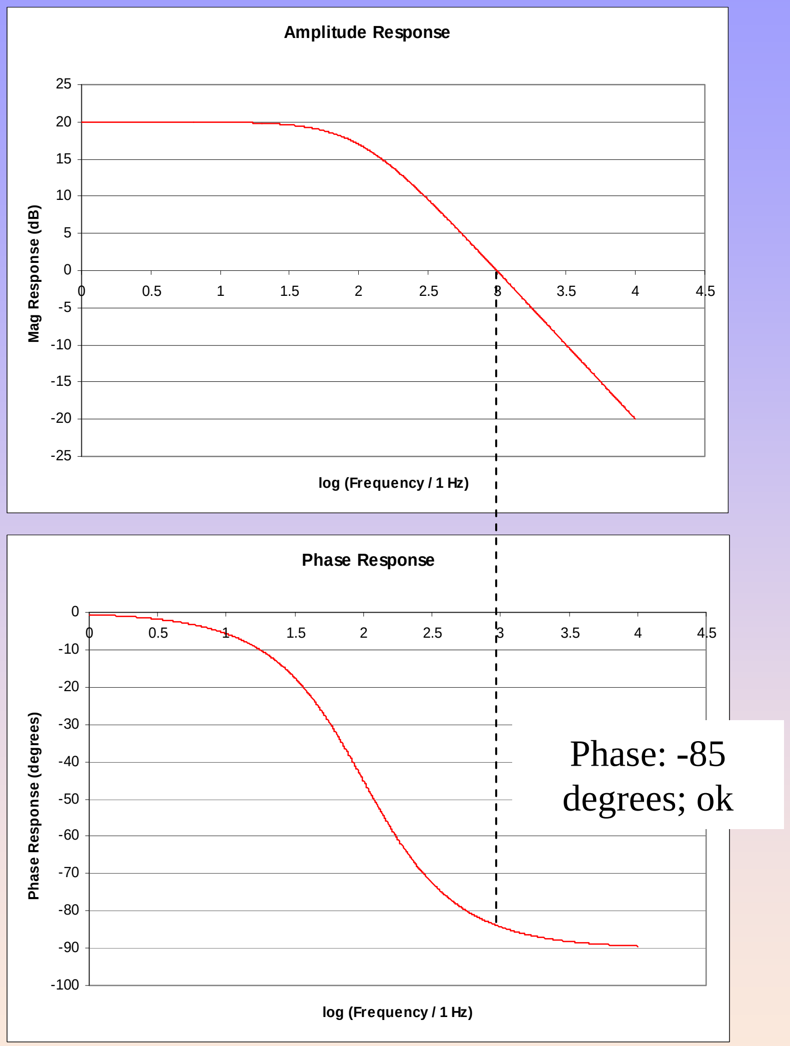
Bode stability criterion:

Phase at unity-gain frequency

> -180 degrees

Rule of thumb: provide at least 30 degrees of phase margin and 6 dB of gain margin.

In this case, phase margin is 96 degrees and gain margin is infinite.

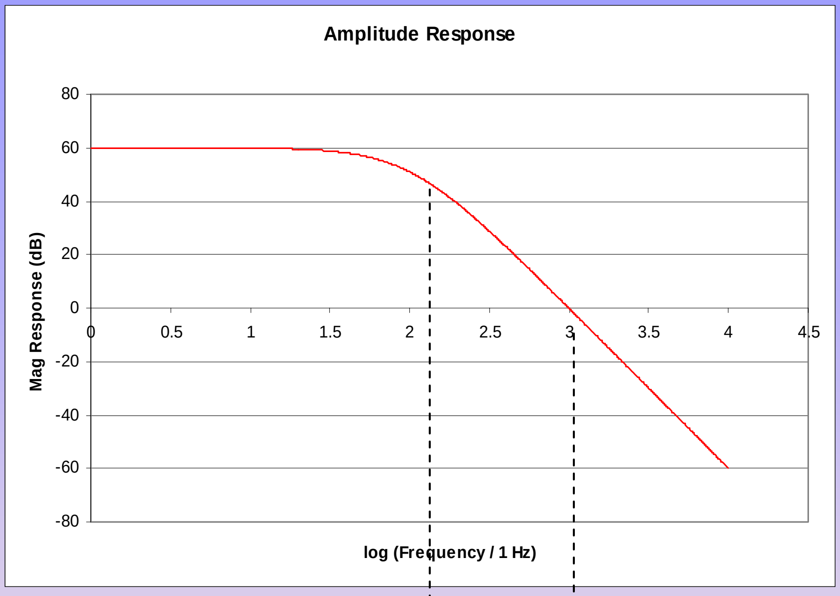


What if gain isn't adjustable?

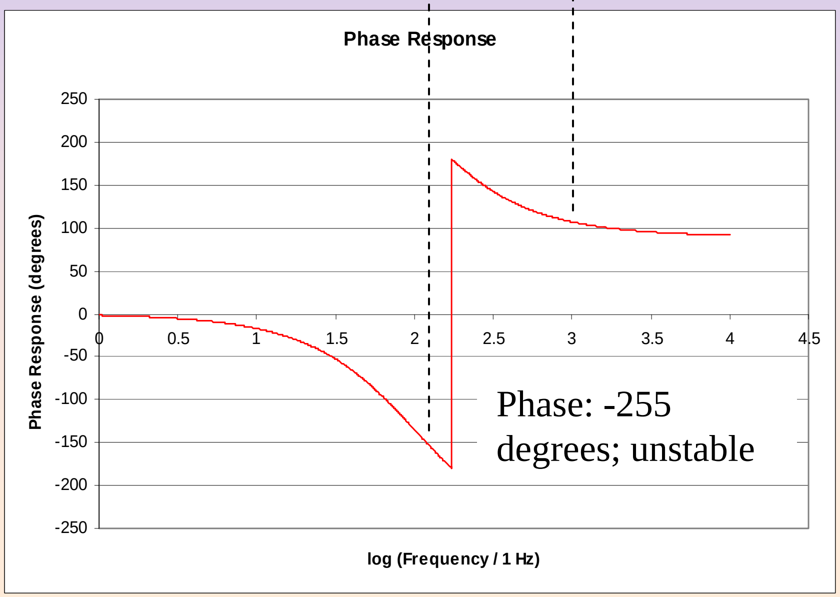
Stability

Cascade three amplifiers: too many poles

$$A = \frac{A_0^{\;3}}{\left(1+\dfrac{j\omega}{\omega_c}\right)^3}$$



Insert attenuator to reduce gain for stability?



Need to reduce gain by over 40 dB; using one amplifier is better.

Two amplifiers might be better still.

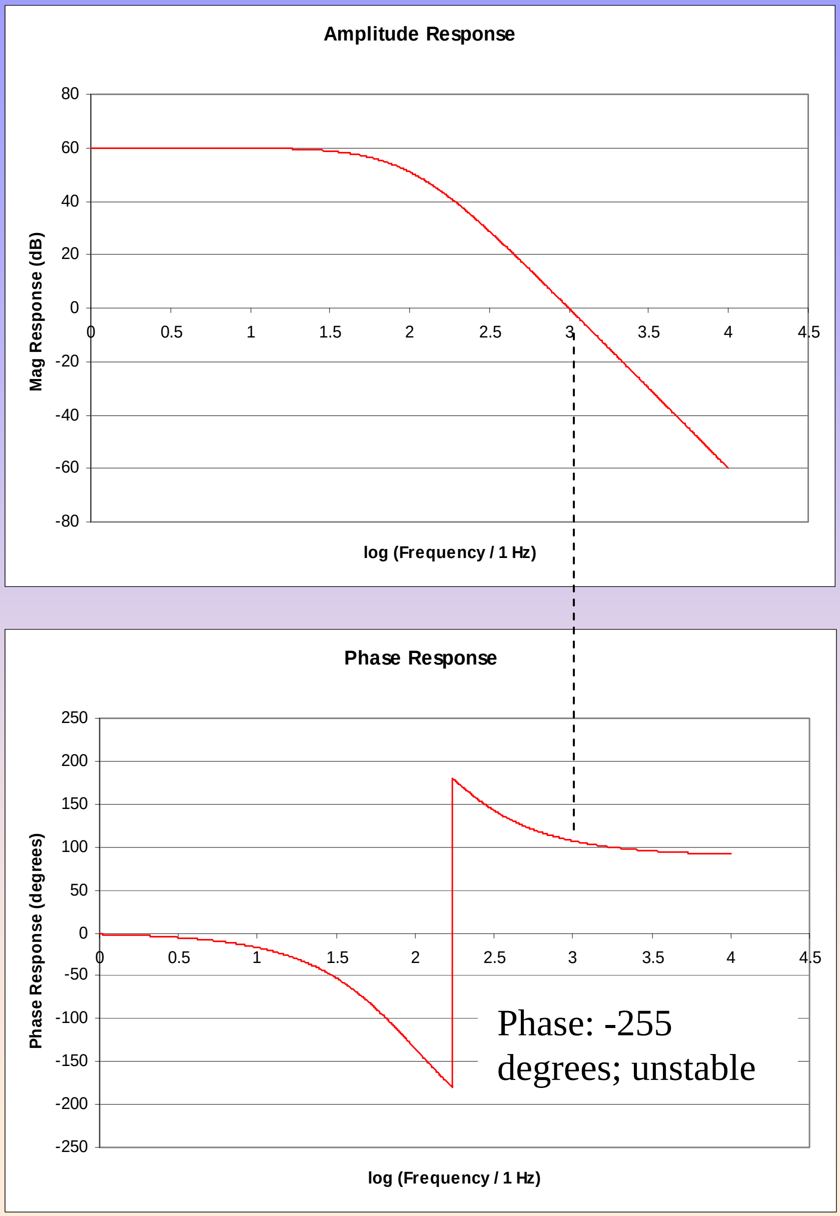
Stability

Why is this –255 degrees and not +105 degrees?

For Bode stability criterion, 0 degrees is defined as the low-frequency limit of the phase, where the slope of the amplitude response vanishes

If low-frequency limit of amplitude response slope is n * 20 dB/ decade, then low frequency phase is n * 90 degrees

See Thaler and Brown for details



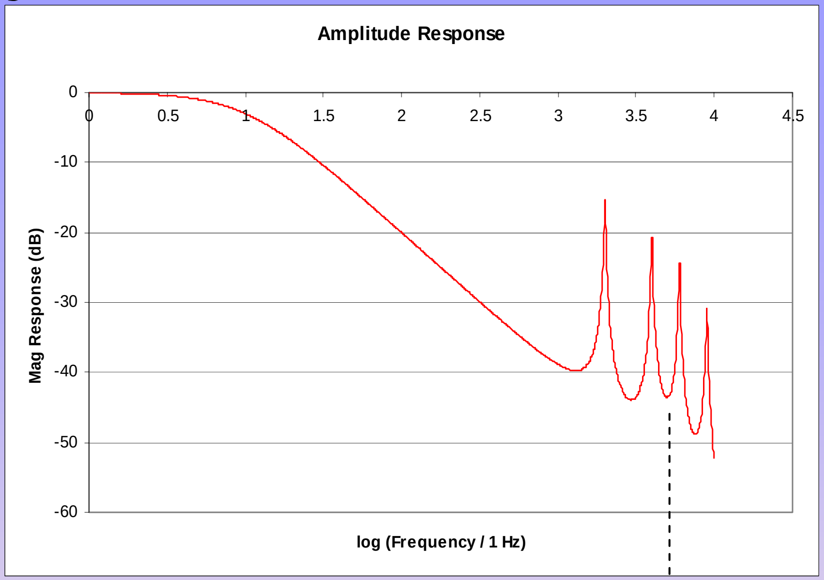
Stability

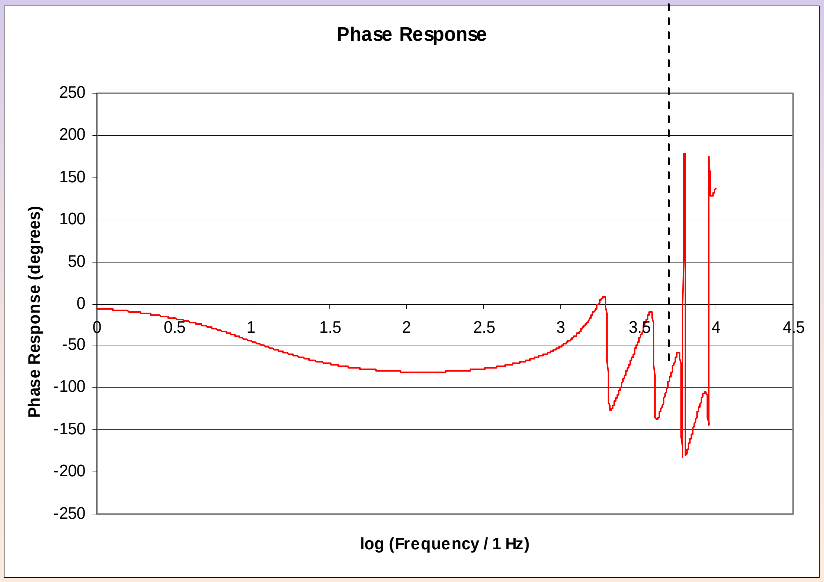
More typical gain limitation: mechanical resonances

Often have “forest” of resonances at frequencies above first resonance

Phase response depends on mechanism. Often design loop so that all resonances are below unity gain.

Note phase roughly proportional to slope of amplitude. Bode integral theorem.



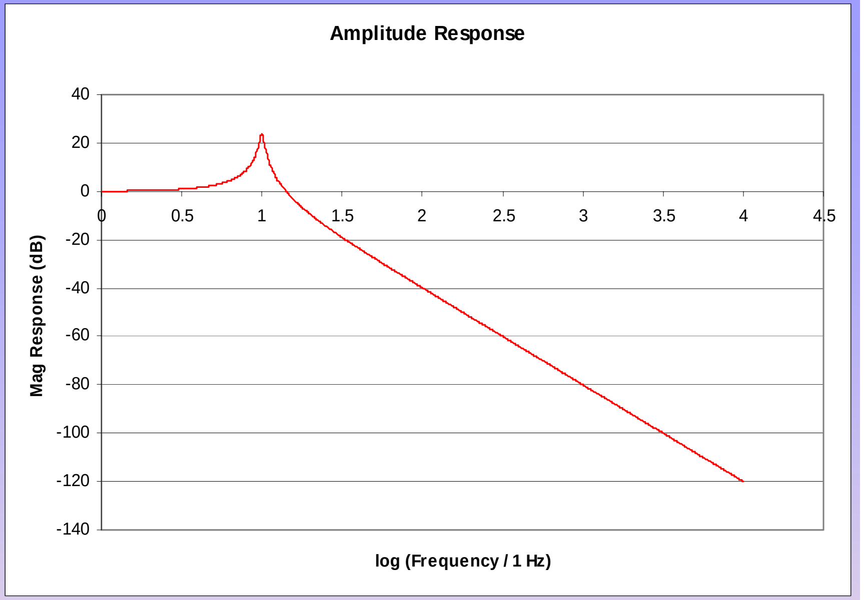


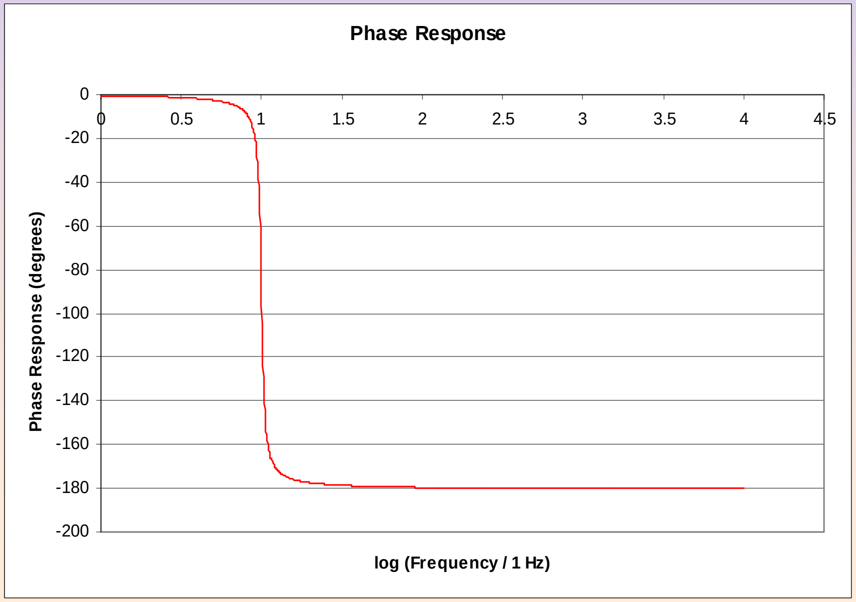
Example of Benign Resonance: Delay line with voice coil

OPD actuator consisting of retro-reflector mounted on soft flexures, with magnet driven by coil (simple harmonic osc.).

Used to control optical path delay. First resonance typically at very low frequency; next resonance much higher in frequency.

Little phase margin because at high frequencies, $- m \omega^2 x = F$

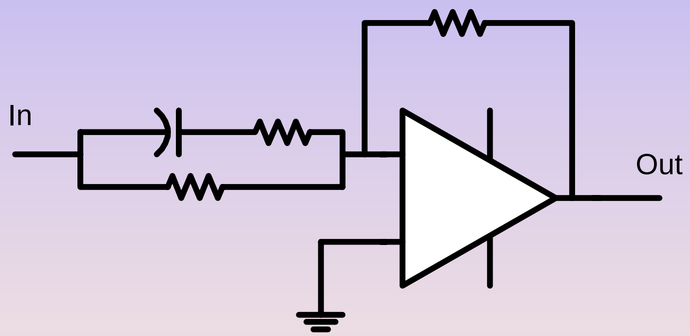


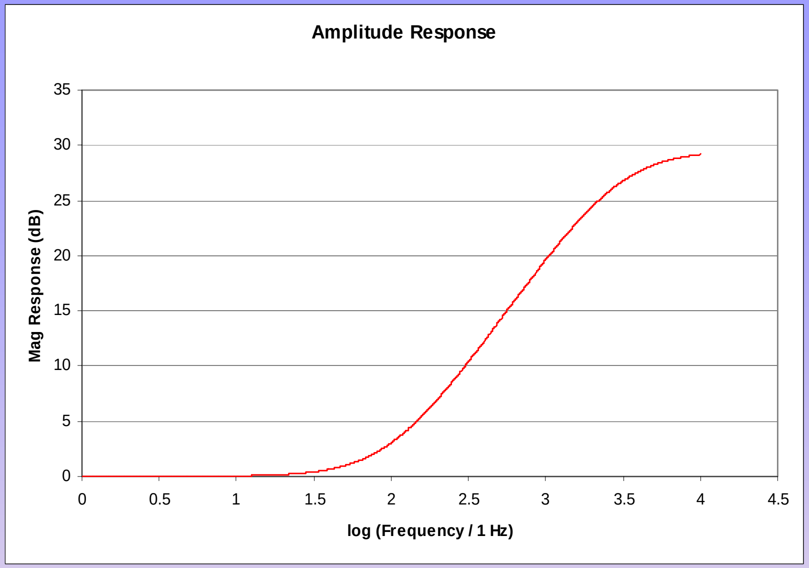


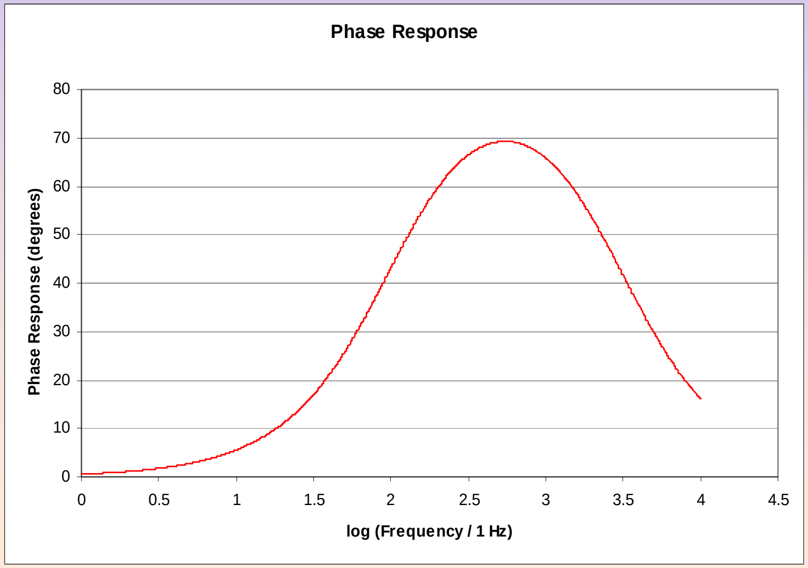
“Lead” Circuit

Construct using op-amp with a capacitive network.

Produces phase lead over a range of frequencies.

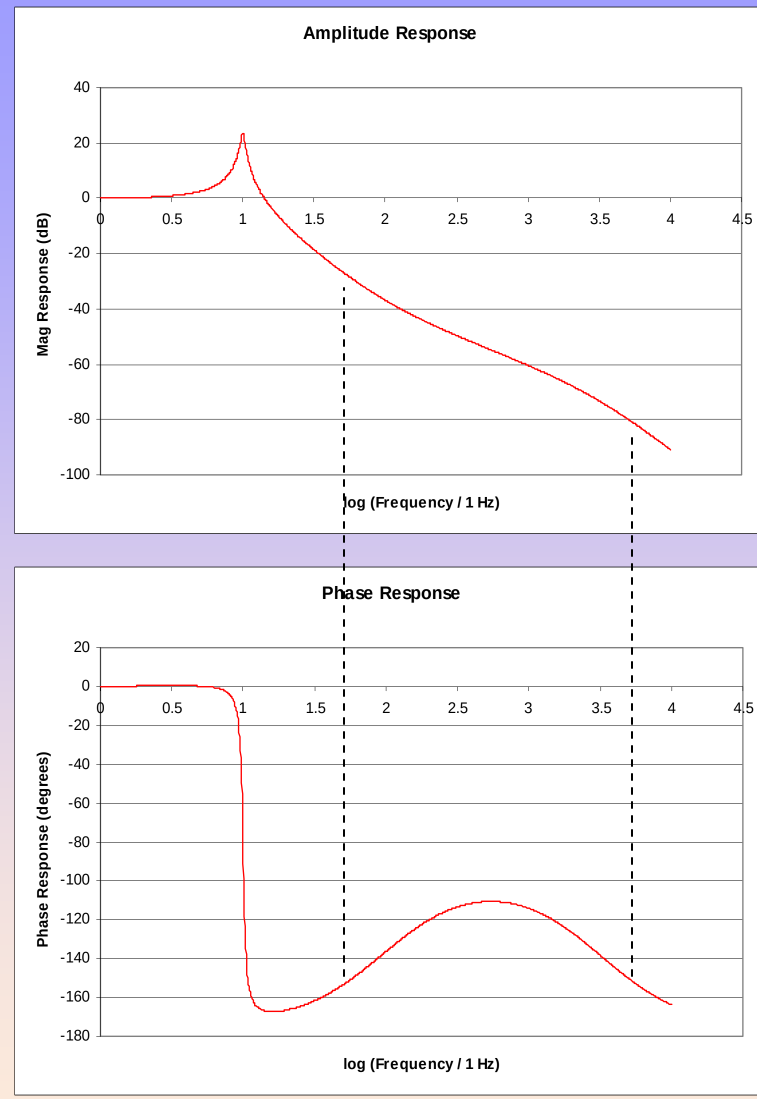


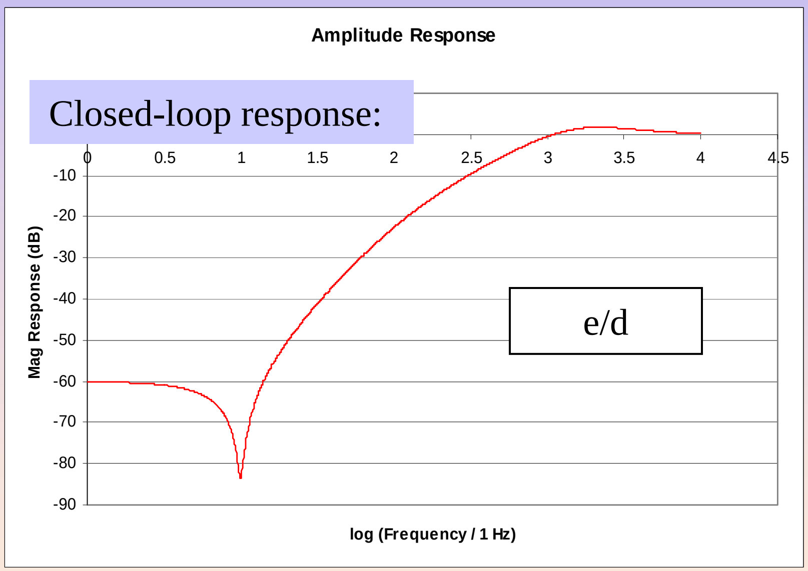




Cascade of Voice Coil and Lead

Range shown has good phase margin:





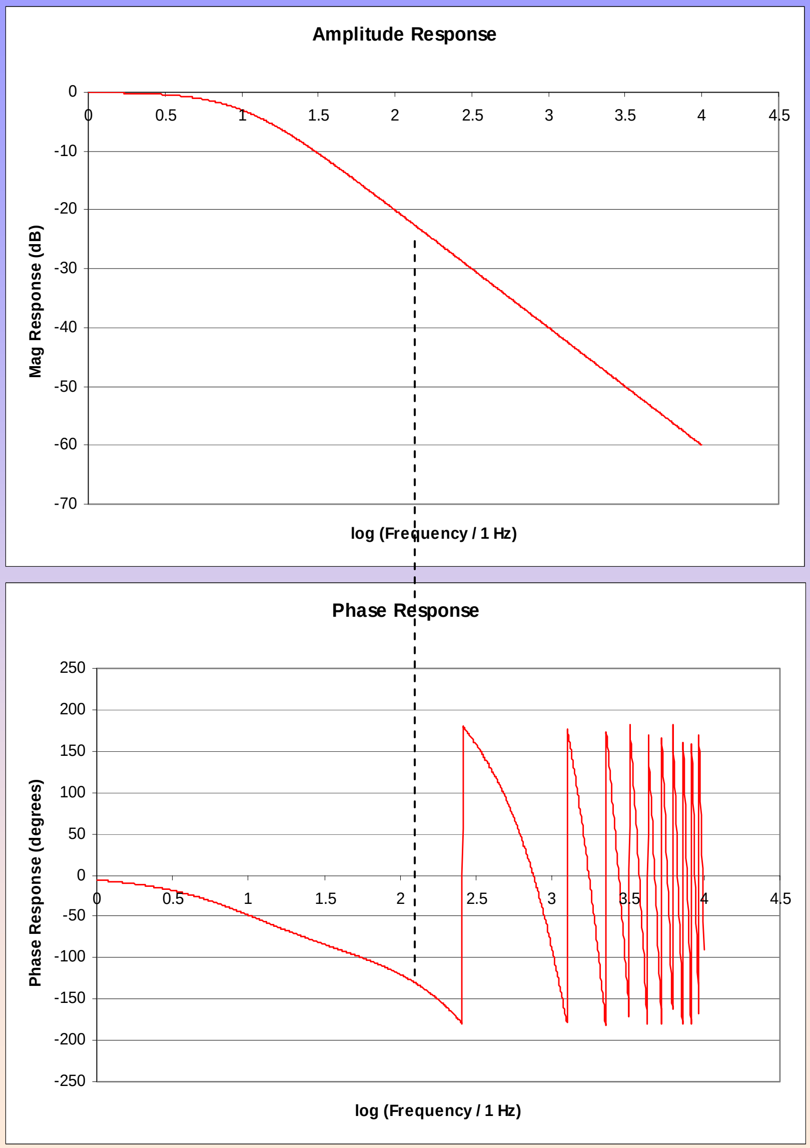
Delay

Pole at 10 Hz and 1 ms delay

E.g., sampled-data (digital) system with 1 kHz sample rate

Rule of thumb: in a sampled-data control system,

Unity-gain frequency = sampling rate / 10



Procedure for Simple Design

Make sure there is a broad range of frequencies around the desired unity-gain frequency over which the phase is > -150 degrees and the slope of the amplitude response is approximately –20 dB/decade. Provide a means to change the sign of the feedback.

Increase the gain gradually. If the error increases when the gain is increased, change the sign of the feedback.

Increase the gain until the system oscillates, then decrease it by a factor of 2.

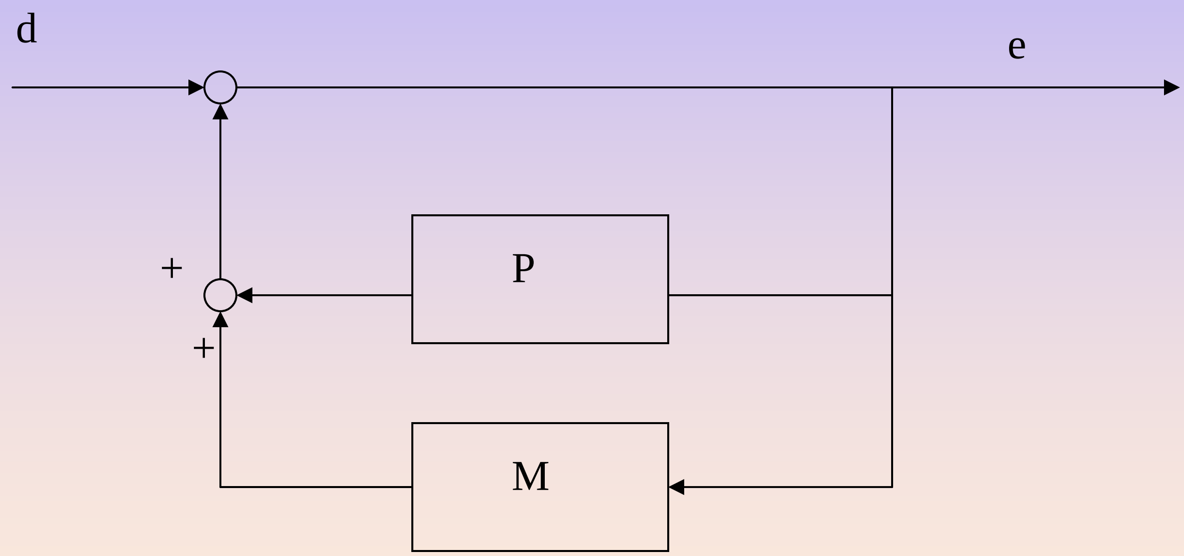
Multiple Loops

Back to star tracker example. Suppose range of steering mirror is too small. Add larger range, slower (e.g., motorized) actuator. Both actuators suppress disturbances.

Adjust gain and frequency response so that fast actuator suppresses high-frequency disturbances, and slow actuator suppresses low-frequency disturbances.

P is response of fast actuator

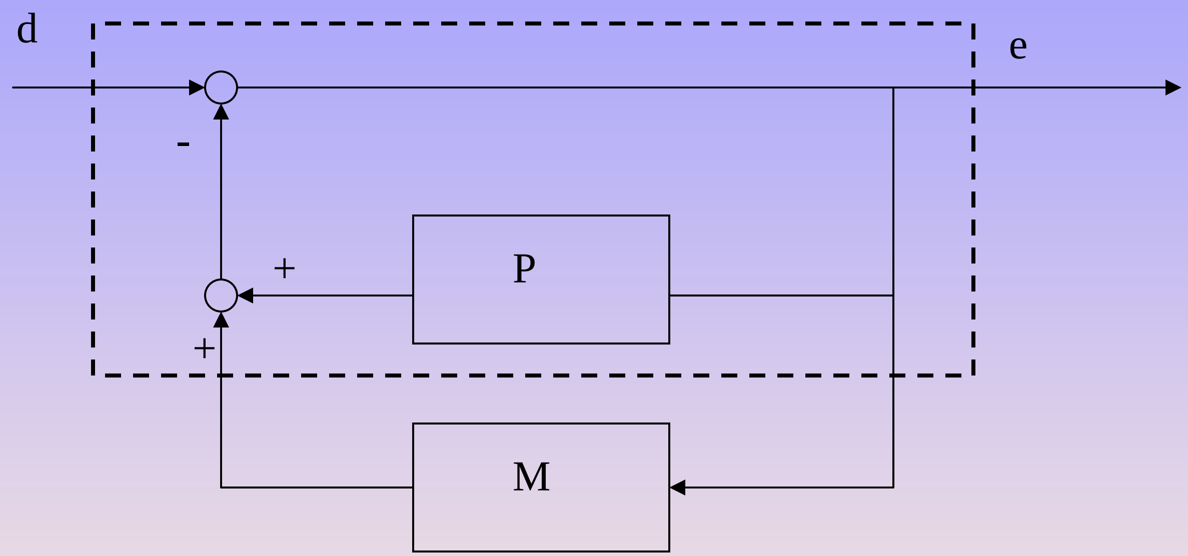
M is response of slow actuator



P+M must satisfy stability criterion. Suppose P already satisfies criterion; what are constraints on M?

Multiple Loops

P+M must satisfy stability criterion. Suppose P already satisfies criterion; what are constraints on M?



Open-loop gain, for $P >> 1$

Can be written as

$$A = M\frac{1}{1+P}$$

$$\cong \frac{M}{P}$$

This means that the ratio of M to P must satisfy the stability criterion.

At "cross-over," relative phase must be >-180 degrees