Introduction
Of the thousands of planets found by the Kepler Space Telescope, the most dynamically interesting and information rich systems are those with multiple known planets. However, it is very likely that many additional planets have not been discovered in these systems. Evidence of these unseen planets can be seen in the transit timing variations (TTVs) observed in many of the known planets.

We examined the unascrivable TTVs in 46 Kepler multi systems to see if the addition of an unseen planet could provide a solution. Using the photodynamical modeling tool PhoDyMM, we analyzed each system both with only their known planets and with an unseen planet added.

KOI-759.01 TTV Data
System has two known planets: 32.6 days (shown) and 91.8 days. Figure shows observed ttv data with error bars.

PhoDyMM
For our analysis, we used PhotoDynamical Multi-planet Model (PhoDyMM) which builds models using both photometric and dynamical aspects. This tool models the lightcurve of every planet in a system using the Bayesian Monte Carlo Markov Chain method to fit planet parameters. PhoDyMM is publicly available at: https://github.com/dragozzine/PhoDyMM.git

KOI-759.01 With Best Known Planet Model
Known planets in the system can not produce a good fit to the observed TTVs while maintaining reasonable physical and orbital parameters. Red lines are error bars from TTV data points.

Known Planets Model
Under the principle that the simplest solution is often the correct one, we first tried to fit the observed TTVs with the known planets in each system. We tried variations of the physical and orbital parameters of the primary perturbing planet. For most of the systems, we could not create a good fit for TTVs using the known planets.

Predicting an Unseen Planet
To predict possible unseen planets, we focused on the 6 strongest, small integer resonances with the planet experiencing the TTVs. We used these and the amplitude of the TTVs to predict several orbital periods. We also used the TTV amplitude to predict a mass for each planet. Using the mutual Hill radii distance between the unseen planet and the known planets, we made a stability estimate for the system. We then selected the unseen planet with the strongest resonance in a stable orbit to add to the system.

KOI-759.01 With Best Unseen Planet Model
Addition of an unseen planet produces a very accurate model to the observed TTVs.

Unseen Planet Model
Now with an unseen planet in the system, we performed the same analysis as with just the known planets. We adjusted the parameters of the added planet to create an improved model. Most of the systems we examined were vastly improved by the addition of an unseen planet.

Results and Future Work
Of the 46 systems we analyzed, 5 produced a reasonable fit to the TTVs with only the known planets, 38 systems created good fits with the addition of an unseen planet, and 3 of the systems will need more work to find a satisfactory answer.

We plan to run each system for a longer number of steps to better converge on the parameters of the unseen planet. Due to the degeneracy of the added planets, these planets are only planetary candidates. These new candidates can help us to learn more about the architecture and dynamics of these exoplanetary systems.