Abstract

The rapidly growing number of transiting exoplanets (TEPs) confirmed with RV observations enables in-depth statistical analysis of the correlations between planetary and host star parameters. Understanding these correlations - especially those involving host star metallicity - is crucial for justifying the models of planetary structures and formation. We therefore present a rigorous analysis using multiple statistical techniques, and investigate how the correlations change for different ranges of parameters, in particular for planetary mass. We compare the results to previous studies from the literature. We discuss observational biases and limitations, compare the parameter space of different surveys, and show what influence they have on the distribution and correlations of parameters of discovered TEPs. We also investigate outliers, and seek explanation for their nature.



period in days

The photometric presicion of a transiting exoplanet survey determines the relative transit depth sensitivity limit, while the temporal observing pattern sets the sensitivity to different periods. As ground-based surveys with more sites but otherwise identical observing strategies can take photometric measurements in a larger fraction of the day, one would expect them to have increased sensitivity towards long periods. To verify this, this figure presents the period and relative transit depth of confirmed transiting exoplanets by WASP (single site on each hemisphere) – solid purple trianles; other single site ground based surveys (MEarth, Lupus, OGLE, Qatar and XO) – empty purple triangles; HATNet (two sites 45° apart in longitude, Bakos et al. 2011) – solid green disks; TrES (three sites covering 94° in longitude on the Western hemisphere, Alonso et al. 2004) – empty green disks; and space based surveys (CoRoT, Kepler and SWEEPS) – cyan diamonds.

Indeed, one can see that HATNet with two sites does slightly better at longer periods than single site surveys. However, TrES with even more daily coverage does not provide us with longer period planets than other ground-based surveys. One can also see that space based surveys not only are able to discover planets with up to an order of magnitude longer periods due to their continuous observing baseline, but also have significantly higher photometric precision.

References

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Statistical methods to quantify the significance of a correlation

The null hypothesis is that the investigated parameters – e.g., planetary radius and host star metallicity – are independent. The alternative hypothesis is that they are related by some underlying phenomenon. A false positive is the rejection of the null hypothesis in spite of it being true. We implement three independent statistical methods to estimate the false positive probability: t-test (see e.g. Press et al. 1992, p. 640), bootstrap technique with 10 000 000 random sample sets (see e.g. Efron & Tibshirani 2003), and F-test (see e.g. Lupton 1993, p. 100). We denote the probability estimates by p_1 , p_2 and p_3 , respectively. In each case below, the probabilities agree to a high precision. These probabilities measure the correlation significance: the lower the probability, the more confidently the null hypothesis – i.e., no correlation – can be rejected. From the numerical results, we conclude that the t-test and F-test are always in good agreement, whereas the bootstrap method has some scatter due to finite number effects, especially for low values of p, but still gives a comparable result. See Béky et al. (2011) for more explanation and restrictions on the scope of these statistical methods.

Correlation of planetary parameters with host star metallicity

sample investigated by Enoch et al. 2010, due to the larger sample size.

restriction on planets	n	planetary parameter	r	p_1	p_2	<i>p</i> 3	
$0 < M_{\rm core}$	7	M _{core}	0.78	3.9%	na	na	reported by Guillot et al. 200
$M_{ m p}~<$ 0.6 $M_{ m J}$	18	$R_{ m p}$	-0.53	2.4%	na	na	reported by Enoch et al. 201
0.30 $M_{ m J}\leqslant M_{ m p}\leqslant$ 0.8 $M_{ m J}$:	33	$\dot{M_{ m p}}$	0.249	16.18%	16.17%	16.18%	
0.35 $M_{\rm J} \leqslant M_{\rm p} \leqslant 0.8 M_{\rm J}$	32	$\dot{M_{ m p}}$	0.094	60.78%	60.70%	60.79%	
0.40 $M_{\rm J} \leqslant M_{\rm D} \leqslant 0.8 M_{\rm J}$	31	$M_{ m p}$	0.222	22.99%	22.93%	22.99%	
0.30 $M_{\rm J} \leqslant M_{\rm p} \leqslant 0.8 M_{\rm J}$	33	$R_{ m p}$	-0.476	0.526%	0.485%	0.526%	
0.35 $M_{\rm J} \leqslant M_{\rm D} \leqslant 0.8 M_{\rm J}$		$R_{ m D}$	-0.580	0.0507%	0.0386%	0.00507%	
0.40 $M_{\rm J} \leqslant M_{\rm D} \leqslant 0.8 M_{\rm J}$		$R_{ m D}^{'}$	-0.535	0.193%	0.159%	0.193%	
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Correlation of planetary radius with host star metallicity for different mass ranges

Enoch et al. 2010 reported that the strong $[Fe/H]-R_p$ correlation holds only for transiting exoplanets below 0.6 M_J , and is very weak for more massive planets. To quantify this phenomenon, we partition the known transiting exoplanets by mass into equal subsets, and run the same tests on these sample sets as above. Indeed we find that planetary radius correlates very strongly with host star metallicity for the range between 0.3 $M_{\rm I}$ and 0.6 $M_{\rm I}$ (where one finds the most dense planets), but is statistically insignificant for other ranges, even the sign of the correlation might change. For more massive planets, this can be explained by noticing that the theoretical planet models of Fortney et al. 2007, Bodenheimer et al. 2003 and Baraffe et al. 2008 all suggest that the radius of a planet is more sensitive to its composition for low mass planets than it is for more massive ones. For the mass range 0.3 $M_{\rm J}$ to 0.8 $M_{\rm J}$ with a high correlation, we further split the sample into two equal bins based on orbital period p. We find that the planets in closer orbits have radii that correlate stronger with host star metallicity, possibly indicating that the planet inflation mechanism depends on metallicity.

- 0.30 A 0.60 A
- 0.92 **A**
- 1.40 **A**
- 3.00 A
- 0.30 A
- 0.30 A

Outliers

WASP-21 is likely to be in the thick disk population, which would explain its being old and metal poor compared to typical transiting exoplanet hosts, according to Bouchy et al. (2010). HD 149026b, on the other hand, is likely to have a large core between 67 M_{\oplus} and 78 M_{\oplus} , and a structure more similar to that of Neptune than Saturn or Jupiter, see Sato et al. (2005).

The relation between host star metallicity [Fe/H] and planetary composition was studied by Guillot et al. 2006. A positive correlation was found between the inferred mass of the planetary core and stellar metallicity for the n = 7 transiting exoplanets known at that time with positive inferred core mass. Enoch et al. 2010 also reported a strong negative correlation between [Fe/H] and $R_{\rm p}$ for the n = 18 transiting exoplanets below 0.6 $M_{\rm J}$ known that time. The reason for these findings might be that planets have formed from the same cloud as their host stars, suggesting that their metal content correlates. However, it is not clear how stellar metallicity is connected to planetary metallicity, especially because a larger rocky core is likely to accrete more gas during the planet's formation.

We examine further the correlation of planetary mass and radius with host star metallicity, using the sample of up to n = 33 known transiting exoplanets with masses between 0.3 $M_{\rm J}$ and 0.8 $M_{\rm J}$. The limits are chosen to maximize the statistical significance of the correlations. One has to note that this significance depends strongly on the choice of the lower mass limit. The two least massive planets in this range are WASP-21b with a mass of 0.3 $M_{\rm J}$, very low host star metallicity of -0.4, and average radius of 1.07 $R_{\rm J}$; and the dense HD 149026b with a mass of 0.368 $M_{\rm I}$, very high host star metallicity of +0.36, and low radius of 0.813 $R_{\rm I}$: they influence the significance of the correlations strongly due to their extreme metallicities. To have an unbiased result, none of these two outliers can be excluded without a justified reason, therefore we need to compare the false positive probabilities of the three nested samples. For the planetary mass—host star metallicity correlation, they scatter between 16% and 61%, neither supporting, nor rejecting a $[Fe/H]_{\star}$ — $M_{\rm D}$ correlation. However, the planetary radius—host star metallicity correlation has a maximum false positive probability of 0.526% for the three cases, therefore this correlation is statistically significant for all our choices of lower mass limits. This is at least a fourfold improvement over the

The following table lists the correlation coefficient r and the false positive probability estimates p_1 , p_2 and p_3 of the correlation between host star metallicity and a given planetary parameter for the aforementioned sample sets.

restriction on planets	n	r	p_1	p_2	<i>p</i> 3
$M_{ m p} \leqslant$ 0.30 $M_{ m J}$	20	-0.134	57.268%	57.426%	57.268%
$M_{ m J}$ $<$ $M_{ m p}$ \leqslant 0.60 $M_{ m J}$	20	-0.625	0.321%	0.278%	0.321%
$M_{ m J}$ $<$ $M_{ m p}$ \leqslant 0.92 $M_{ m J}$	20	0.152	52.333%	52.314%	52.333%
$M_{ m J}$ $<$ $M_{ m p}$ \leqslant 1.40 $M_{ m J}$	20	-0.236	31.611%	31.617%	31.611%
$M_{ m J}$ $<$ $M_{ m p}$ \leqslant 3.00 $M_{ m J}$	20	0.164	49.064%	49.103%	49.064%
$M_{ m J} < M_{ m p}$	15	-0.385	15.582%	15.536%	15.582%
$M_{ m J} < M_{ m p} \leqslant$ 0.80 $M_{ m J}$, $p <$ 3.53 d	16	-0.661	0.533%	0.541%	0.533%
$M_{ m J} < M_{ m p} \leqslant$ 0.80 $M_{ m J}$, $p \geqslant$ 3.53 d	16	-0.511	4.326%	4.116%	4.326%



