Apodized Coronagraphs Designed for Wavefront Control

Carlotti A., Kasdin N. J., Groff T. – Princeton University

Abstract: All coronagraphs achieve high contrast by removing the diffracted starlight from the discovery zone. This is done in one of two ways: the first uses pupil masks (apodization) to change the PSF and transfer energy outside the discovery zone directly in the image plane. The second uses focal plane masks to transfer energy outside the exit pupil so it never gets focused into the final image plane. Examples of the first approach include all apodized and pupil mapping coronagraphs. Examples of the second include all Lyot type (bandlimited and otherwise), phase masks, and vector vortex coronagraphs. Combinations of pupil and focal masks are also possible; the leading example is the APLC. We show in this paper that for every such coronagraph the energy amplitude distribution in the exit pupil is changed. This makes the ultimate performance extremely sensitive to amplitude errors; although one deformable mirror can be used to correct for phase errors, it cannot correct for these amplitude errors everywhere in the image plane. As a result, a coronagraphic system cannot be designed without an amplitude correcting device; the most likely such device employs two deformable mirrors in series. This also implies that the coronagraph need only produce contrast to the point where amplitude errors dominate. Thus we focus our work on hybrid systems that include a pupil mask and two deformable mirrors. We optimize the transmission of the pupil mask so that it achieves contrast to the level at which amplitude errors dominate, and the remaining contrast is then achieved via the amplitude correcting device. We consider several shaped pupil designs. Each of them is associated to a different mirror profile, and we discuss the design tradeoffs of this hybrid concept.

1 – All coronagraphs are sensitive to amplitude errors…

Let's consider the one-dimension case:

The classic diffraction pattern is the square modulus of the Fourier transform of the entrance electric field (assumed to be unity) limited by the aperture shape $A_e(x)$:

$$P_0 = \left|\mathcal{F}\{A_e(x)\}\right|^2$$

The coronagraph acts as a linear operator that transforms the incoming electric field: $E_1 = \mathcal{C}\{A_e(x)\}$

The new electric field can be expressed as: $E_1 = A_o(x)e^{i\phi(x)}$, where $A_o(x)$ is the change in amplitude, and $\phi(x)$ is the phase change.

The PSF of the coronagraph is then: $P_1 = \left|\mathcal{F}\{A_o(x)e^{i\phi(x)}\}\right|^2$

There are multiple definitions of the **contrast**, and we choose to define it as:

$$C = \frac{\int_{\Delta_\Omega} P_1(\omega)d\omega}{T_\Omega \Omega P_0(0)}$$

For that we integrate the coronagraphic PSF over the discovery space Δ_Ω and normalize by the original maximum intensity $P_0(0)$ corrected by the local transmission T_Ω of the coronagraph.

We rewrite the contrast by introducing the region Δ_C complimentary to Δ_Ω:

$$C = \frac{1}{2T_\Omega \Omega D^2}\left[\int_{-\infty}^{\infty} P_1(\omega)d\omega - \int_{\Delta_C} P_1(\omega)d\omega\right]$$

And using Parseval's theorem, we obtain a new expression for the contrast:

$$C = \frac{\int_{-D/2}^{D/2} |A_o(x)|^2 dx}{2T_\Omega \Omega D^2}\left[1 - \frac{\int_{\Delta_C} P_1(\omega)d\omega}{\int_{-\infty}^{\infty} P_1(\omega)d\omega}\right]$$

Conclusion: All coronagraphs may achieve the detection of faint objects in either two ways:

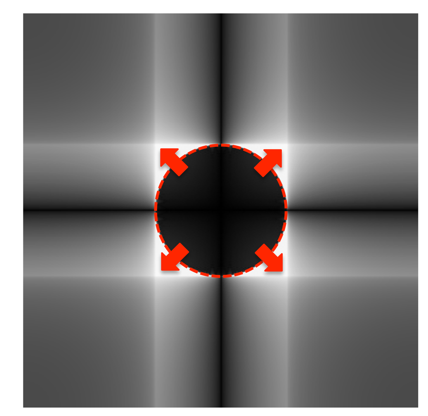
Minimize $\int_{-D/2}^{D/2} |A_o(x)|^2 dx$ or Equalize $\int_{\Delta_C} P_1(\omega)d\omega$ and $\int_{-\infty}^{\infty} P_1(\omega)d\omega$

‣ These are energy transfers in the pupil plane and image plane respectively.

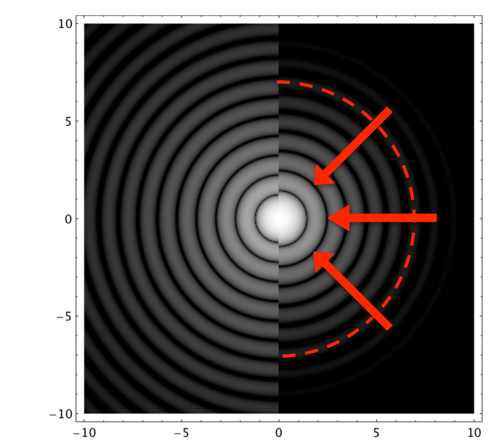
Coronagraphs in the First case:
- Lyot & Bandlimited Lyot
- 4 Quadrants & Vortex
- AIC & other nuller

Coronagraphs in the second case:
- All apodizers (smooth, binary, PIAA…)

Both techniques can be combined, as it is the case in the PALC.

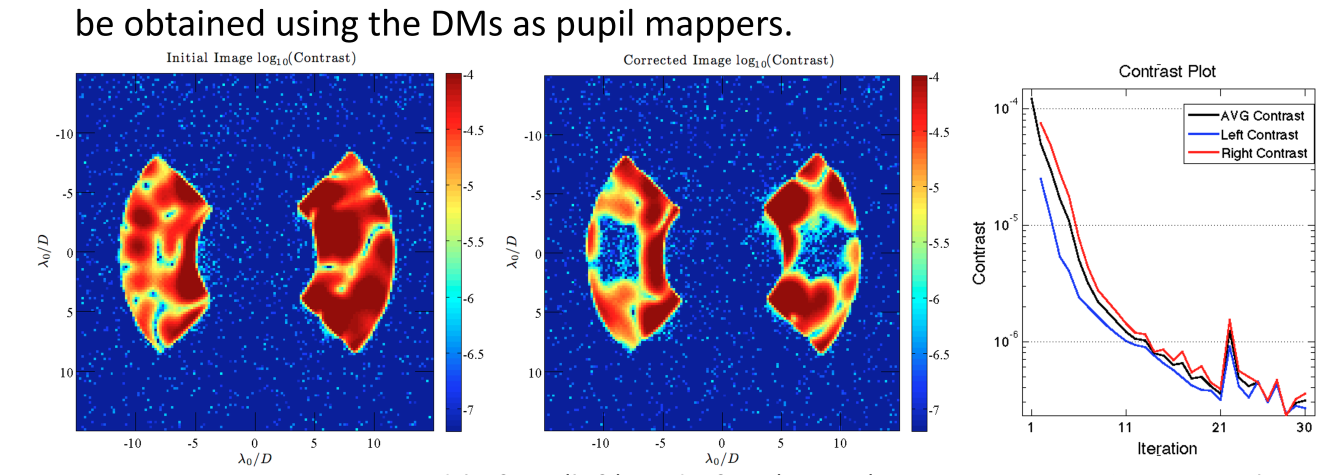


Example of $|A_o(x)|^2$ in the case of the 4 Quadrants phase mask. The energy is removed from the aperture limits and distributed outside.



Left: PSF of a uniform circular aperture. Right: PSF $P_1(\omega)$ of an apodized aperture. The energy is concentrated in the closest diffraction rings.

This underlines the necessity for the amplitude (and the phase) to be actively corrected. This can be achieved using 2 deformable mirrors (DMs) in series, in non-conjugate planes. Using current correction algorithm, the area in which the contrast can be increased is limited, thus it is not necessary for the apodizer to increase the contrast outside of this region. Moreover we believe that it is not necessary for the apodizer to achieve a theoretical contrast below the limit fixed by the quality of the instrument's optics before correction (typically 10^{-6} to 10^{-7}). The remaining contrast will be obtained using the DMs as pupil mappers.



Current contrasts measured before (left) and after (center) a 2 DMs correction using the stroke minimization algorithm. The correction process evolution is displayed on the right.

… all coronagraphs need to be combined to an amplitude correcting device.

2 – Designs of shaped pupils to create dark holes

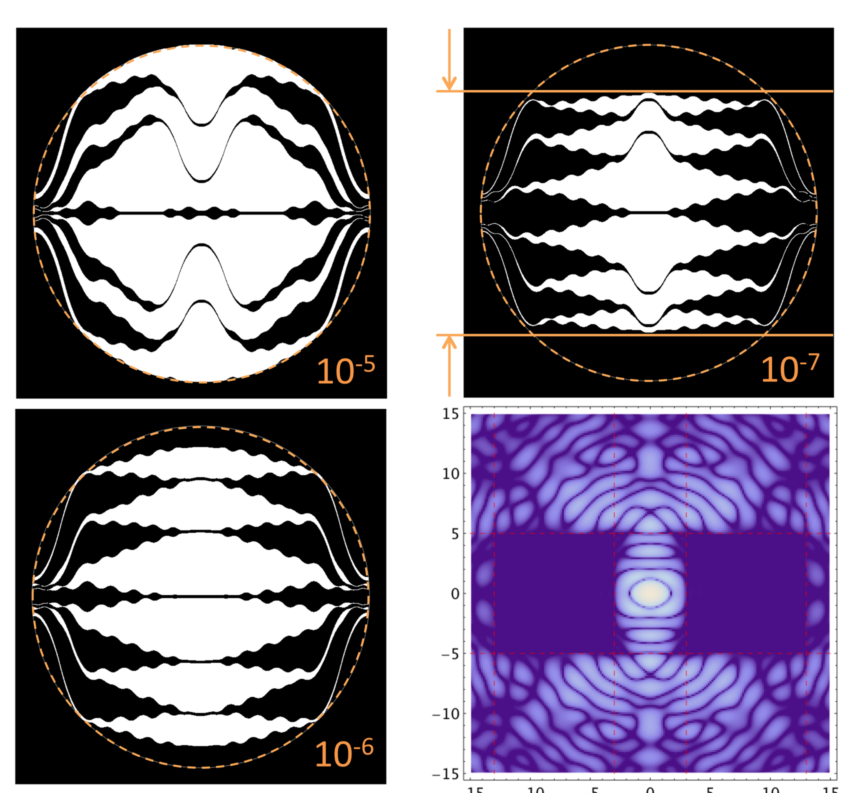
We present here how ripple masks and checkerboard masks can be optimized to create dark holes of various sizes, angular proximity to the star and contrast. Ripple masks and checkerboard masks have been selected among all the shaped pupils already proposed (see Spergel 2001, Vanderbei 2003, Kasdin 2003) because of their ability to create localized high contrast regions suitable for further corrections using a phase and amplitude correcting device.

Ripple masks

Transmission vs. Contrast

The transmission of ripple masks decreases as the contrast for which they are designed goes down. The table below gives the transmission for 2 different inner working angles (IWA) and contrasts going from 10^{-5} to 10^{-7}. In some cases (too small IWA, too small contrast) optimizations cannot be found. The width of the dark holes was fixed to 10 λ/D and their height to ± 5 λ/D.

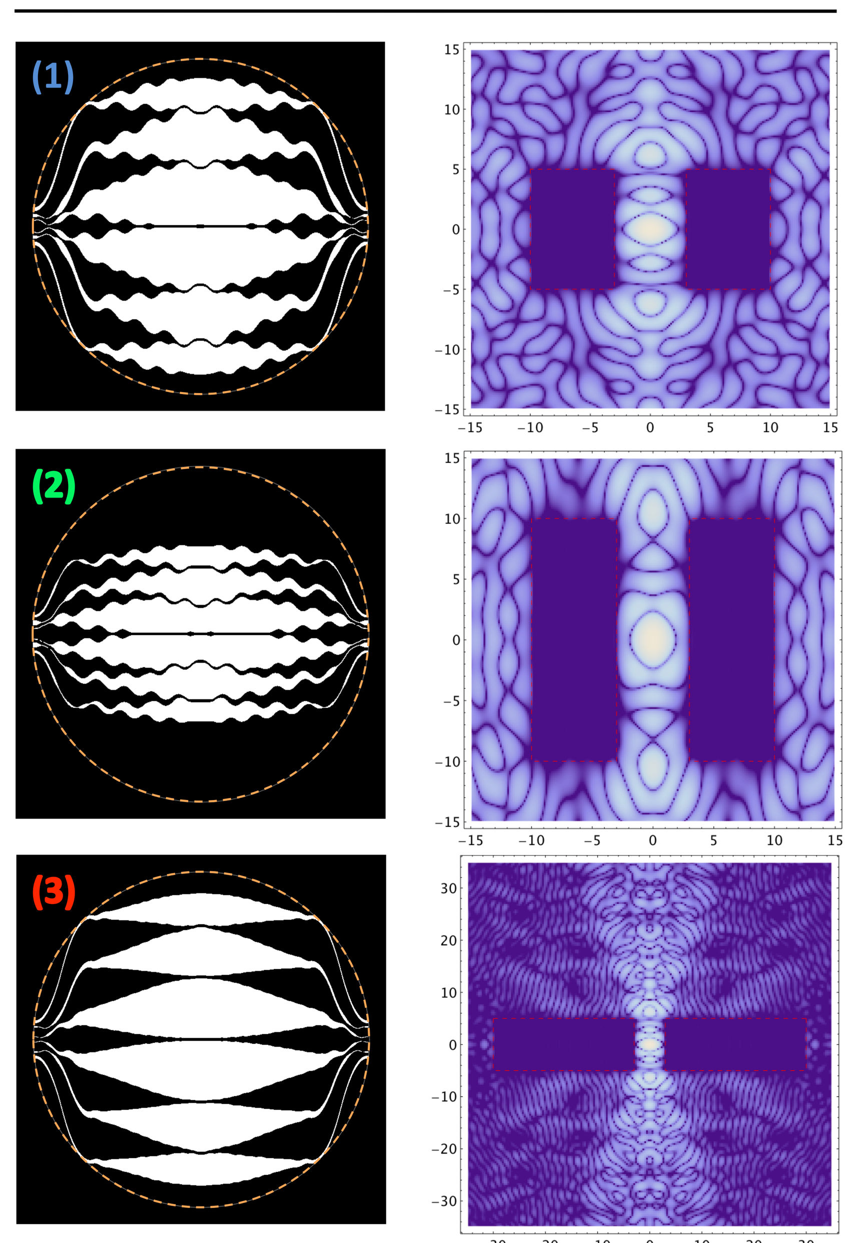
Contrast▶ / IWA▼	$10^{-5.0}$	$10^{-5.5}$	$10^{-6.0}$	$10^{-6.5}$	$10^{-7.0}$
2.5 λ/D	0.53	0.46	0.31	-	-
3.0 λ/D	0.68	0.61	0.49	0.42	0.35

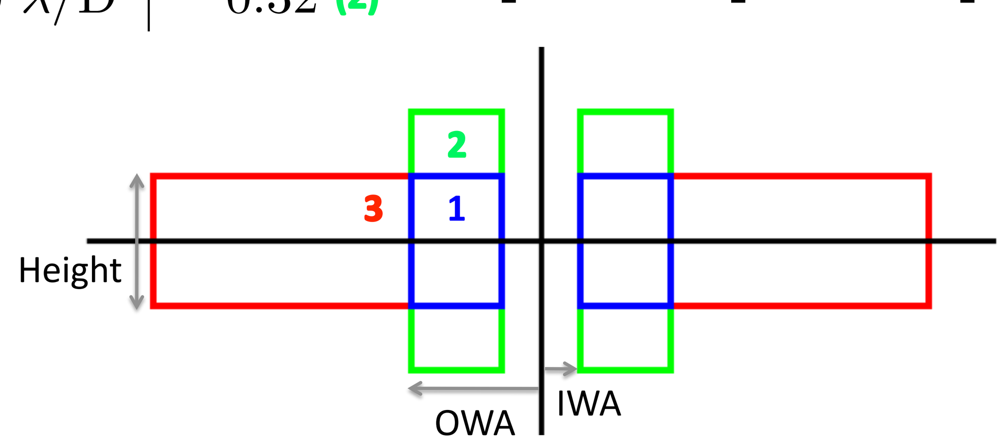


Transmission vs. Dark hole size

The width and height of the dark holes created in the image plane has a great influence on both the transmission and general aspect of the masks. The table below gives the transmission for different values of the outer working angle (OWA) and the height of the dark holes. In every case the targeted contrast was 10^{-6} and the IWA was 3 λ/D.

OWA▶ / Height▼	10 λ/D	15 λ/D	20 λ/D	30 λ/D
10 λ/D	0.51 (1)	0.48	0.49	0.48 (3)
15 λ/D	0.41	0.38	0.38	-
20 λ/D	0.32 (2)	-	-	-



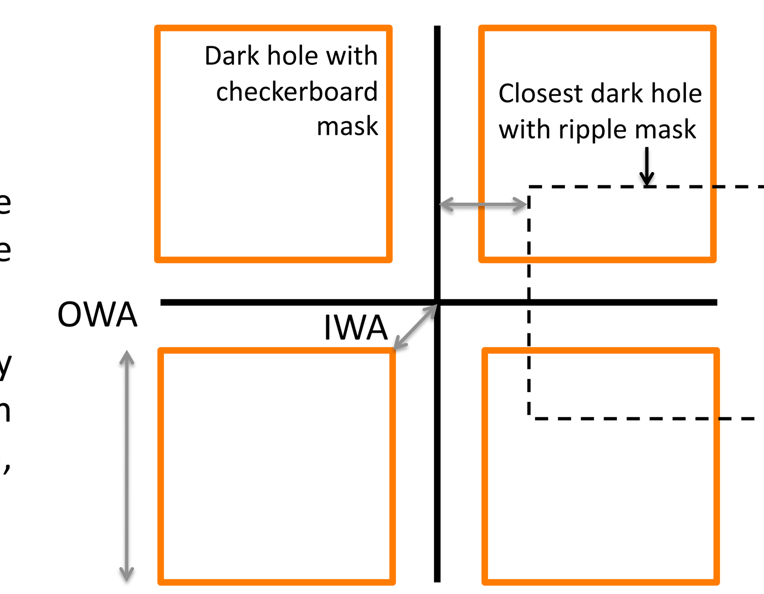


Schematic of the extensions of the dark holes created by the 3 masks displayed on the right. The transmission does not change with the OWA, although the structure of the mask changes: the lines that define the openings in the pupil plane are smoother with bigger OWA.
The transmission is however strongly dependent on the vertical extension of the dark hole. This height also limits the highest achievable OWA.

Checkerboard masks

Contrary to ripple masks, checkerboard masks cannot create rectangular dark holes along the horizontal axis, and instead concentrate the light along the horizontal and vertical axes, and create square dark holes in the four quadrants of the image plane.

The distance of their closest corner to the center defines the IWA of the coronagraph. The discovery spaces they provide is thus different and somehow complimentary from those that ripple masks can provide. They can also achieve smaller IWA than ripple masks (10^{-5}: 1.7 λ/D instead of 2.5 λ/D), although one should remember that the geometry of the pupils are different (square instead of circle).



Transmission vs. Contrast

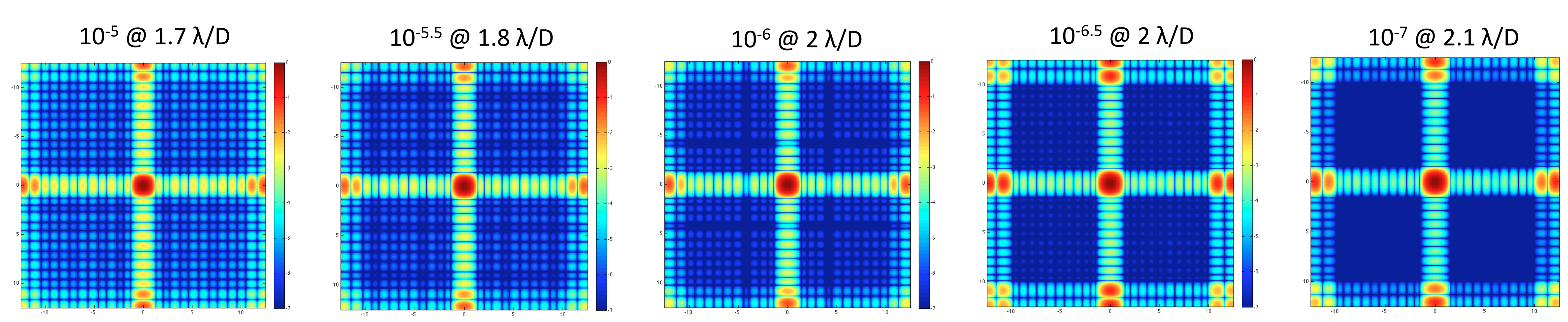
As in the precedent case, the transmission decreases with the contrast. In the case of a specific IWA (2.1 λ/D), and for a fixed OWA (10 λ/D), the table below gives the transmissions achieved for contrasts from 10^{-5} to 10^{-7}. The OWA is not measured along the diagonal but along the x-axis.

Contrast▶ / IWA▼	$10^{-5.0}$	$10^{-5.5}$	$10^{-6.0}$	$10^{-6.5}$	$10^{-7.0}$
2.1 λ/D	0.57	0.52	0.48	0.45	0.42

Minimum IWA vs. Contrast

For each contrast there is a minimum IWA. It can be noted that the transmission associated to these minimum IWA are not very different from those given in the precedent table.

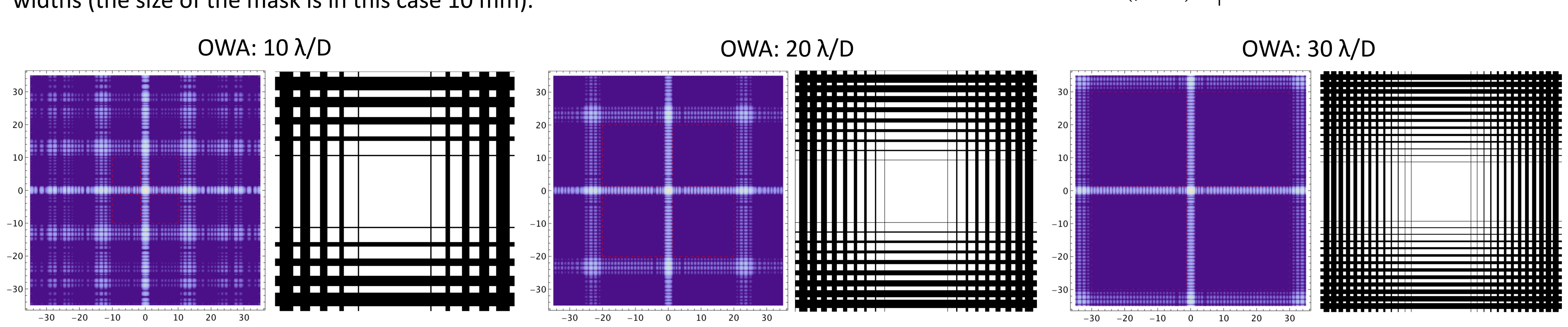
Contrast	$10^{-5.0}$	$10^{-5.5}$	$10^{-6.0}$	$10^{-6.5}$	$10^{-7.0}$
IWA_{min} (λ/D)	1.7	1.8	2	2	2.1



Minimum size of openings and gaps vs. OWA

While the transmission of the masks does not significantly change with the OWA, the minimum size of the openings and gaps between openings decreases with the OWA. In the case of a contrast of 10^{-6}, and for an IWA of 2 λ/D, this table displays these minimum widths (the size of the mask is in this case 10 mm).

OWA (λ/D)	10	15	20	30
Min. Hole (μm)	205	160	105	65
Min. Gap (μm)	55	25	15	10



This work is funded by NASA APRA grant #NNX09AB96G - Contact: acarlott@princeton.edu