The baroclinic instability in the context of layered accretion

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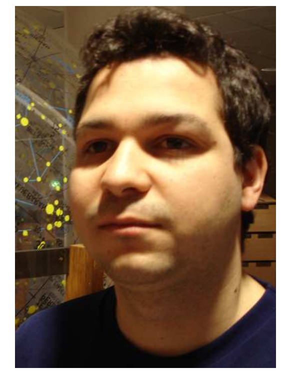


The problem

Turbulence and angular momentum transport in accretion disks remain a topic of debate. With the increasingly disturbing realization that dead zones are robust features of protoplanetary disks, the search for hydrodynamical sources of turbulence continues. A possible source is the baroclinic instability (BI, Klahr & Bodenheimer 2003), which has been shown to exist in unmagnetized nonbarotropic disks (Petersen 2007, Lesur & Papaloizou 2010). We aim to verify the existence of the baroclinic instability in 3D magnetized disks, as well as its interplay with other instabilities, namely the magneto-rotational instability (MRI, Balbus & Hawley 1991) and the magneto-elliptical instability (Mizerski & Bajer 2009).

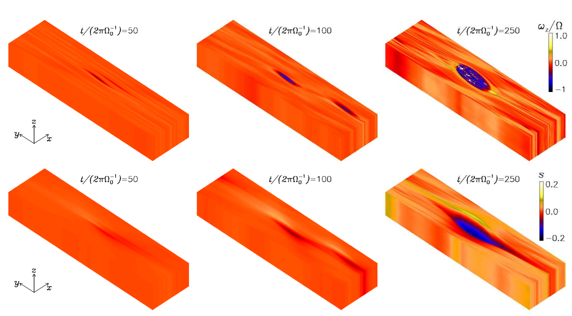
Problem setup

We perform local simulations of non-isothermal accretion disks with the Pencil code. The entropy gradient that generates the baroclinic instability is linearized and included in the momentum and energy equations in the shearing box approximation. The model is fully compressible, so excitation of spiral density waves is allowed. The disk is modeled with thermal relaxation and as specified below.

- 3D local unstratified Cartesian box,
- Physical domain: $4{\times}16{\times}2\ H$,
- Resolution: $256{\times}256{\times}128$,

Baroclinic instability

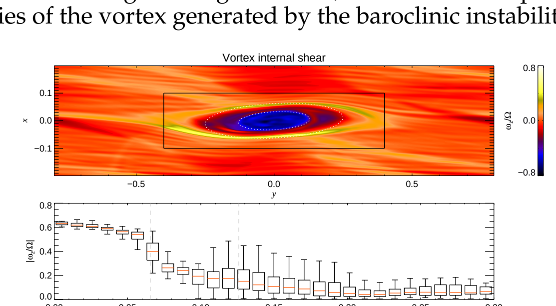
We seed the disk with finite amplitude perturbations. The non-linear instability grows as shown in the time series below. Upper panels show vorticity, lower panels show entropy.



When the thermal time is comparable to the eddy turnover time, the eddy is able to establish an entropy gradient around itself that compensates the large scale entropy gradient that created it. This entropy gradient back reacts on the eddy, generating more vorticity via buoyancy. This in turn reinforces the gradient. A positive feedback has been established, and the vortex grows. After 200 orbits the instability begins to saturate as vortices merge and the remaining giant vortex grows to the sonic scale.

Self-sustained vortex

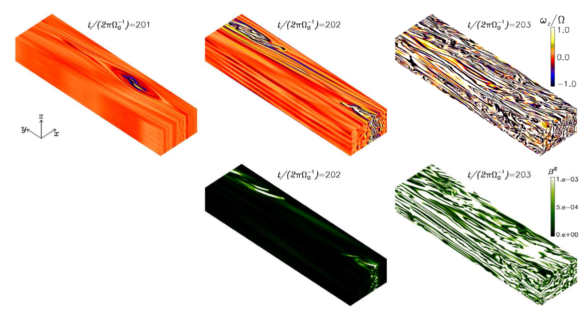
Before inserting the magnetic field, we measure the properties of the vortex generated by the baroclinic instability.



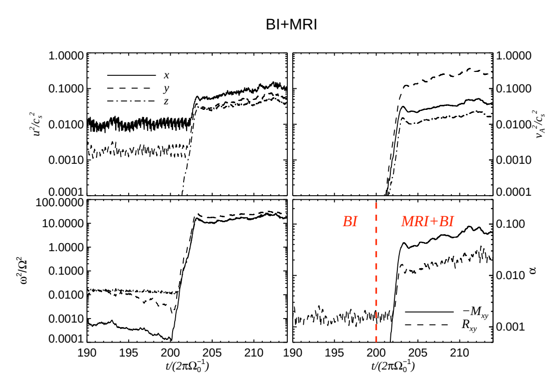
The vortex core is well enclosed by an ellipse of aspect ratio χ=4. We measure the vertical vorticity in the midplane of the simulation against the elliptical radius, in the grid points boxed by the thin black line. The conclusion is that the vortex core has a Gaussian angular velocity profile, thus close to uniform rotation at the center.

Magnetic disruption

We plot vorticity at three consecutive orbits after insertion of the magnetic field. Magnetic energy is also shown. The vortex, that in a non-magnetic run retains its coherence indefinitely, is destroyed when magnetic fields are included.



The growth of the MRI in the box is evident at the third snapshot, and led to complete disruption of the vortex. The end state is no different than a MRI only scenario, as shown below.



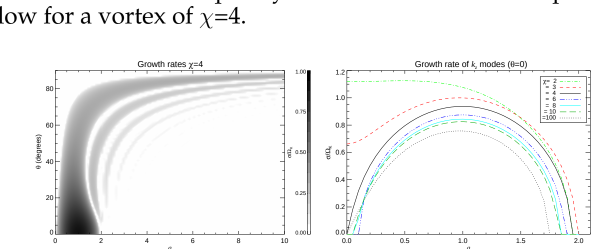
Yet, examining the vorticity in the second snapshot, we notice that the core also went unstable. This is visible in the magnetic energy snapshot, as well. This is curious since the core rotates close to uniform. This seems to be a signature of the magneto-elliptic instability.

Magneto-elliptic instability

The magneto-elliptic instability is in fact two different instabilities. One for k_z modes, which exists for anti-cyclonic vortices when

$$0 < k/k_{\rm BH} < 2\,|{\rm Ro}|^{1/2}$$

where Ro is the Rossby number. Its physical mechanism is that the magnetic field resists the strain of the motion in elliptical streamlines, as the MRI has its source on the magnetic field resisting shear. The other de-stabilizing mechanism is through resonances between MHD waves and the vortex turn-over frequency. Both are seen in the left plot below for a vortex of χ=4.



The figure in the left shows the growth rates as a function of q=$k/k_{\rm BH}$ and the angle θ between the wavevector of disturbances and the vertical axis. The strongest de-stabilization is for pure k_z modes. The resonances correspond to the weaker de-stabilization at intermediate θ, and exist for smaller wavelengths.
The right panel shows growth rates of the k_z modes for different aspect ratios. For χ<4 the purely hydro elliptical instability is seen as finite growth rates at q=0. For χ = 4 onwards the instability is magnetic and has a most unstable wavelength near q=1. Notice that for large χ, corresponding to pure shear flow, the curve approaches the MRI curve.

References

Balbus, S. & Hawley J. 1991, ApJ, 376, 214
Klahr, H. H. & Bodenheimer, P. 2003, ApJ, 582, 869
Lesur, G. & Papaloizou, J.C.B. 2010, A&A, 513, 60
Mizerski, K. A. & Bajer, K. 2009, J. Fluid Mech., 632, 401
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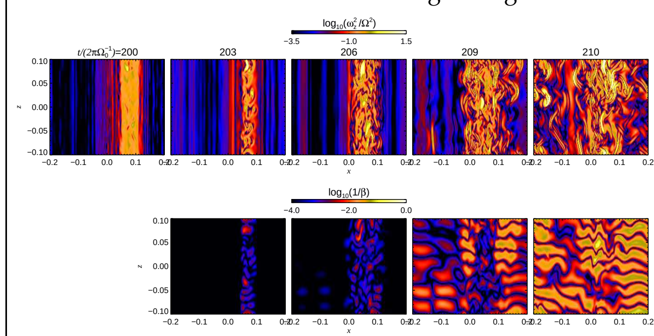
Isolating the magneto-elliptic instability

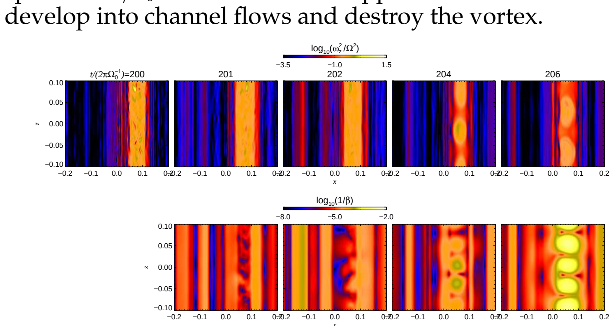
The maximum growth rate of the magneto-elliptic instability for χ=4 is $\sigma{\approx}0.95\Omega_K$. While the MRI is amplified a million-fold in three orbits, the magneto-elliptic instability is amplified by more than a billion-fold in the same time interval. We explore in the figure below the time window when the later is saturated whereas the former is still growing.



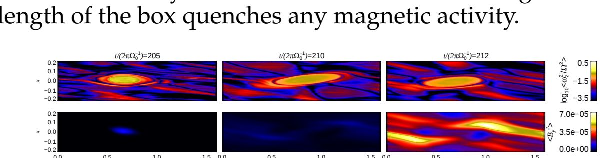
The vortex is magneto-elliptic unstable, yet it does not seem to lose its spatial coherence. The instability is violent, making the vortex bulge. During this period, however, the kinetic energy and enstrophy were nearly constant, so it is not clear if this magneto-elliptic turbulence would have lead to vortex destruction, or if it would have reached a steady state. One orbit later, the MRI started to develop in the surrounding Keplerian flow.

Resistivity

In this simulation, we set the Elsässer number to $\Lambda \equiv 2\pi v_{\rm A}^2/\Omega_K\eta = 1$, in order to quench the most unstable wavelength. Due to excitation of slower growing modes, radial and azimuthal fields grow inside the vortex core, and a conspicuous k/k_0=2 vertical mode appears. These eventually develop into channel flows and destroy the vortex.



In the xy-plane, shown below, the field gets looped around the vortex, which initially appears magnetized. Yet, due to the high resistivity, the field diffuses away. The radial field gets sheared into azimuthal by the Keplerian flow. After a few orbits, strong magnetic fields are seen in the vortex spiral waves. A Reynolds number near 1 for the largest wavelength of the box quenches any magnetic activity.



Summary

The results are summarized as follows.

- We find that the vortices show a core of nearly uniform angular momentum, yet they are unstable to the magneto-elliptic instability.
- The stability criterion and growth rates for the magneto-elliptic instability, when taken in the limit of infinite aspect ratio (no vortex) and with shear, coincide with those of the MRI. The MRI is a limiting case of the more general magneto-elliptic instability.
- After the vortex is destroyed, by channel flows, internal magneto-elliptic modes, or external MRI turbulence, the saturated state of the MRI+BI simulation in much resembles a MRI-only scenario.
- The baroclinic instability is important only when magnetic fields are too weakly coupled to the gas. This fits neatly in the layered accretion paradigm as the active layers are unmodified.