Universität Rostock

Traditio et Innovatio

INSTITUT FÜR PHYSIK

Constraining planetary interiors with the Love number k_2

Abstract

The recent ground- and space-based transit surveys have brought to light an overwhelming number of exoplanets. The variety of these planets cover giant Hot Jupiters to rocky Super-Earths. However, even with the combined data of transit and RV measurements the interior structure of exoplanets remains ambiguous, see e.g. GJ436b [1] or GJ1214b [2, 3]. A new planetary parameter sensitive to the planet's density distribution could help constraining this large amount of possible models further. We investigate whether the planet's tidal Love number k_2 can be such a constraining parameter, if known. We find that, while k_2 only depends on the density distribution of the planet, the inverse deduction of the density distribution from a given k_2 is non-unique. There is a degeneracy of k_2 with respect to a density discontinuity in the planet's envelope [4]. As a consequence, a precise value for the core mass cannot be derived. However, a maximum possible core mass can be infered which equals the core mass predicted by homogeneous zero metallicity envelope models. Using the example of the extrasolar transiting planet HAT-P-13b we show to what extend planetary models can be constrained by taking into account the tidal Love number k_2.

Love numbers

- Planetary parameters that quantify the deformation of the gravity field of a planet in response to an external perturbing body.
  - Tide raising potential: $W(s)=\sum_{n=2}^{\infty}W_n=(GM/a)\sum_{n=2}^{\infty}(s/a)^n P_n(\cos\theta')$ (M: mass of perturbing body, a: distance between planet and perturber, s: radial coordinate of the point under consideration inside the planet, θ': angle between the planetary mass element at s and the center of mass of M at a, P_n: Legendre polynomials) [5]
  - Induced change of the planet's potential at the surface: $V_n^{\mathrm{ind}}(R_\mathrm{p})=k_n W_n(R_\mathrm{p})$ (k_n: Love numbers)
- The only input needed for the calculation of a Love number is the radial density distribution of the planet.
- k_2 is, to first order, equivalent to J_2 for the solar system planets $\longrightarrow$ additional constraint.
- k_2 is a potentially observable parameter which could be obtained from tidally induced apsidal precession of close-in planets [6] or from the orbital parameters of specific two-planet systems in apsidal alignment [7, 8].

Degeneracy in three-layer models

To investigate the degeneracy with respect to a density discontinuity in the envelope, we considered three-layer models with a constant density core and two polytropic envelopes.
x_m: position of the envelope discontinuity in planet radii.
$\Delta\rho=(\rho_2-\rho_1)/\rho_1|_{x=x_m}$: relative density jump between inner (ρ_2) and outer (ρ_1) envelope.

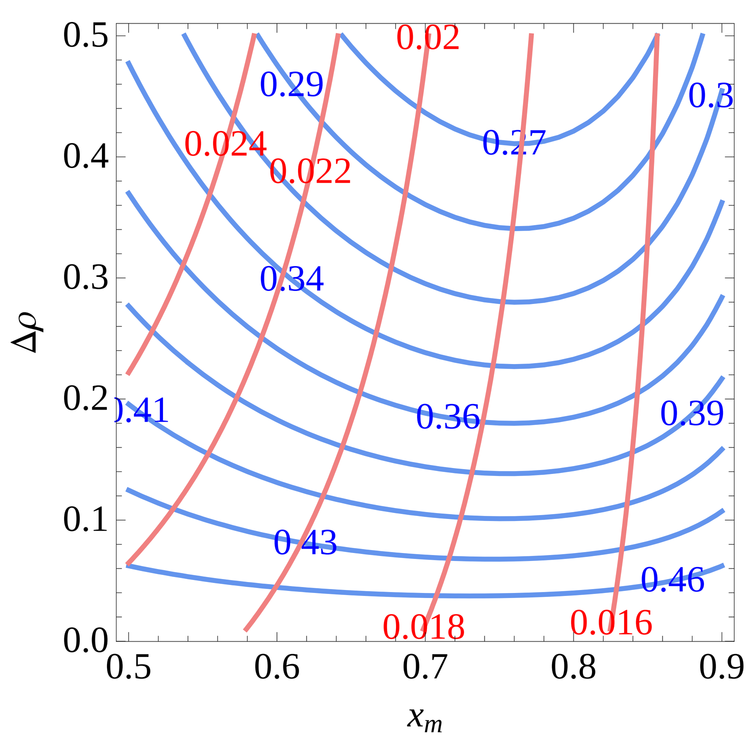

FIGURE 1: Lines of equal k_2 (blue) and equal $M_\mathrm{core}/M_\mathrm{total}$ (red) in $\Delta\rho$-x_m parameter space for a theoretical polytropic three-layer model. Numbers give the corresponding values of k_2 and $M_\mathrm{core}/M_\mathrm{total}$, respectively. The equi-k_2-lines demonstrate the degeneracy of k_2 with respect to the position (x_m) and size ($\Delta\rho$) of a density discontinuity in the envelope and the ratio of core mass to total mass (a blue line intersects several red lines). For the same k_2 value planetary models with different envelope characteristics and mass ratios are possible [4].

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HAT-P-13b

The system HAT-P-13 [9] consists of two planets which are assumed to be in apsidal alignment so that an allowed interval for the Love number k_2 can be determined [8].
We constructed models for HAT-P-13b with the following characteristics:

- $M_\mathrm{p}=0.853\,\mathrm{M_J}$, $R_\mathrm{p}=1.281\,\mathrm{R_J}$
- Two-layer models with a rocky core and a H/He/metal-envelope.
- Varying temperatures in the outer atmosphere which was assumed to be fully adiabatic or isothermal to 1 kbar (*solid* and *dashed colored* lines in Fig. 2).
- Varying pressure P_ad at the transition from isothermal to adiabatic layer for the models consistent with model atmospheres [10] (*black* lines in Fig. 2). These models have $T_{1\,\mathrm{bar}}=2080$ K and $P_\mathrm{ad}\leq 72$ bar.
- Varying metallicity in the envelope.

Results for the core mass and Love numbers k_2 are displayed in Fig. 2 and Fig. 3:

- The k_2-interval given in [8] gives a maximum possible core mass of $\approx 87\,\mathrm{M_\oplus}$.
- We construct a *new interval* based on the analysis of [8], the new eccentricity measurement by [11], and our limiting interior model that gives the most homogeneous interior of HAT-P-13b. Our new interval constrains the core mass further to be $M_\mathrm{core}\lesssim 27\,\mathrm{M_\oplus}$.
- The use of a model atmosphere favors models with $T_{1\,\mathrm{bar}}=2080$ K, a thin isothermal layer down to ~ 10 bar, and ~ 4 times stellar metallicity (see *black dashed* line in Fig. 2).

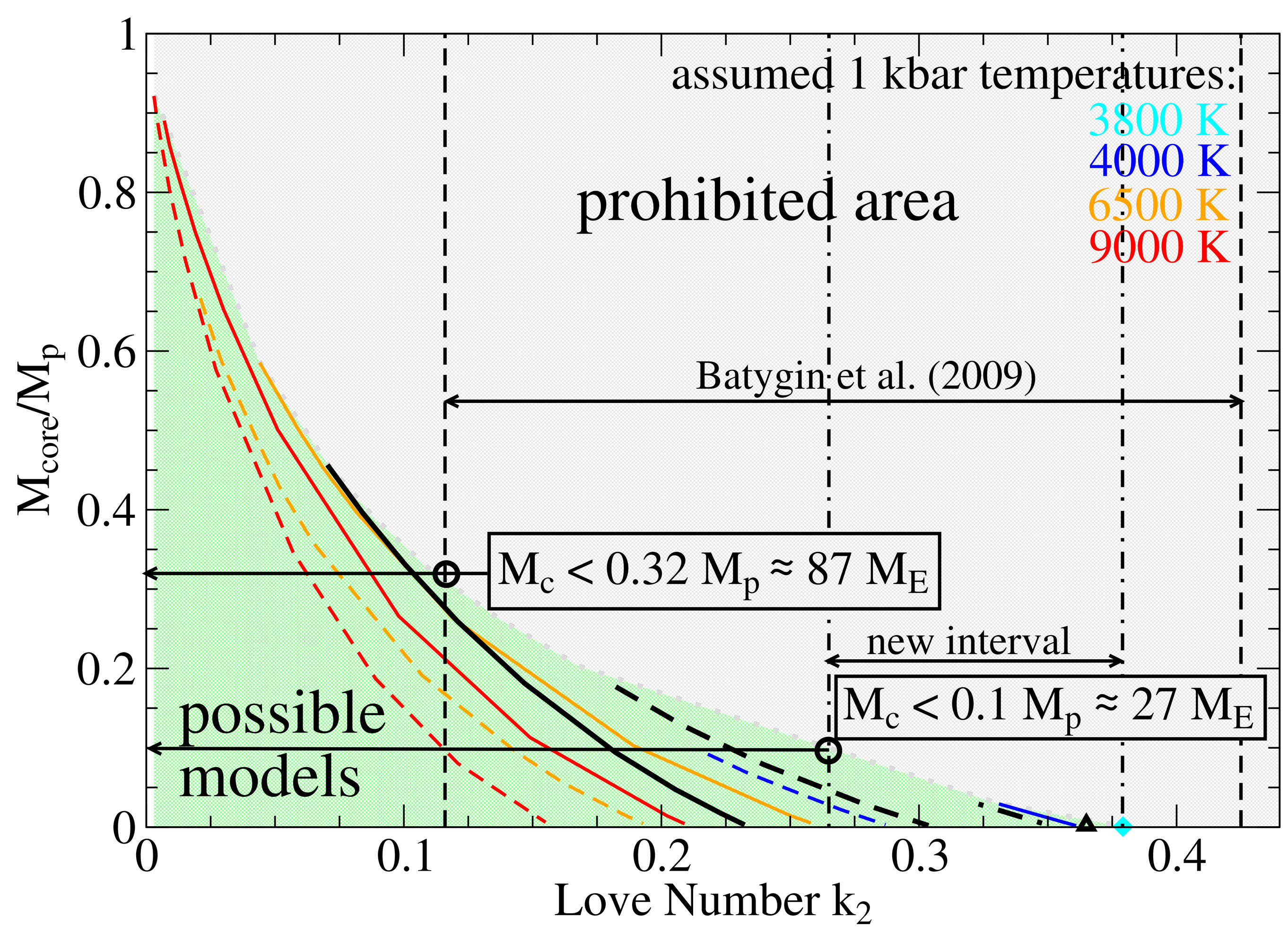

FIGURE 2: Core masses and Love numbers for several two-layer models of HAT-P-13b. Shown are models with a fully adiabatic (*colored solid* lines) or isothermal to 1 kbar (*colored dashed* lines) envelope for different 1 kbar temperatures (color coded) and models consistent with a model atmosphere with $T_{1\,\mathrm{bar}}=2080$ K (*black* lines) and a thin isothermal layer down to 10 bar (*black dashed*), 50 bar (*black dot-dashed*), and 72 bar (*black triangle*). The grey dotted line consists of models with a fully adiabatic, zero-metallicity envelope.

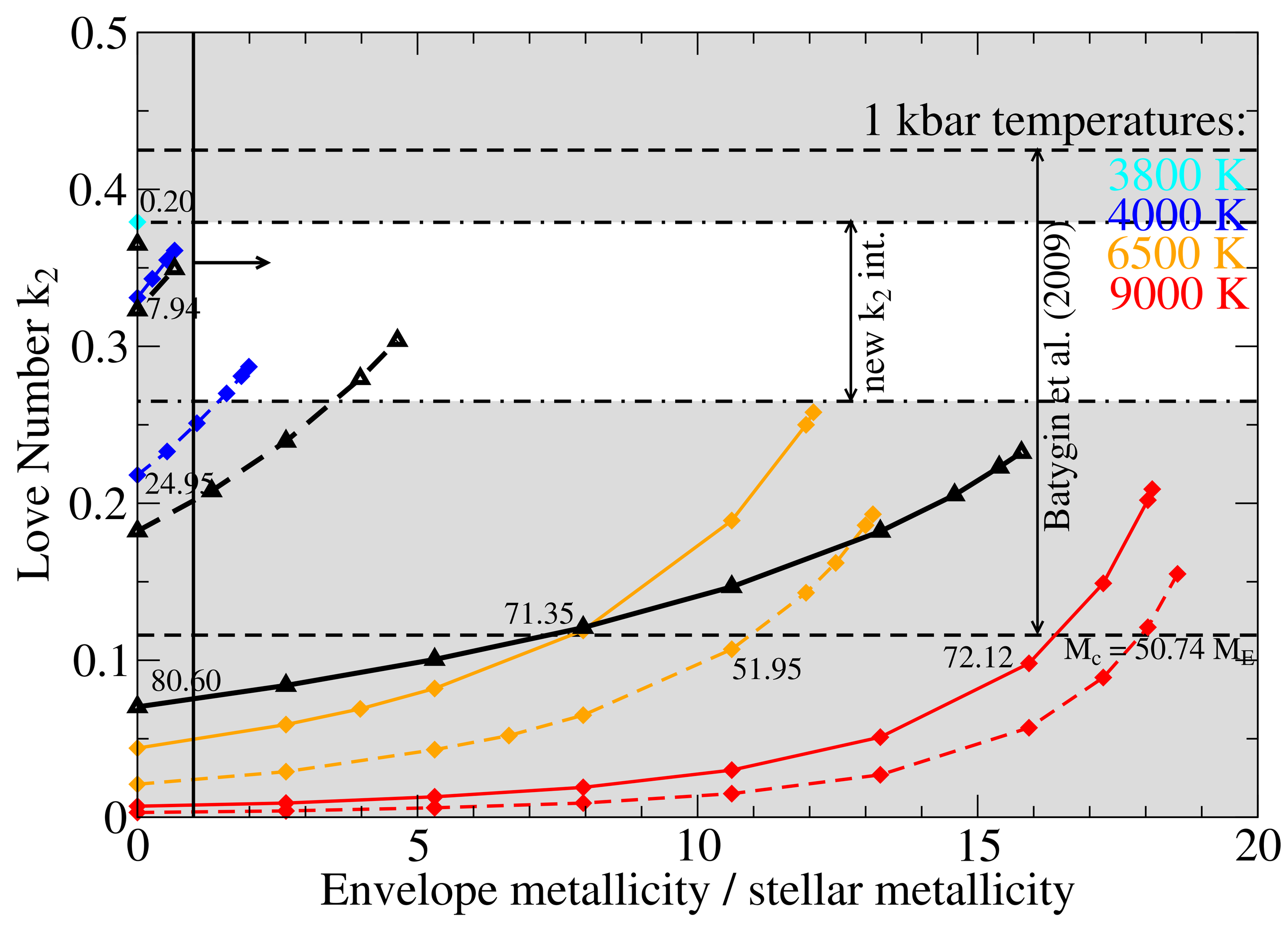

FIGURE 3: Love numbers over the envelope metallicity of HAT-P-13b. Models are the same as in Fig. 2. The shaded areas show which models can be ruled out based on the infered k_2 intervals and the assumption that the envelope metallicity should be at least the stellar metallicity.

We also calculated Love numbers for GJ 436b [1, 4], GJ 1214b [3] and Saturn [4].

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