The low-redshift distance-redshift relation beyond FLRW

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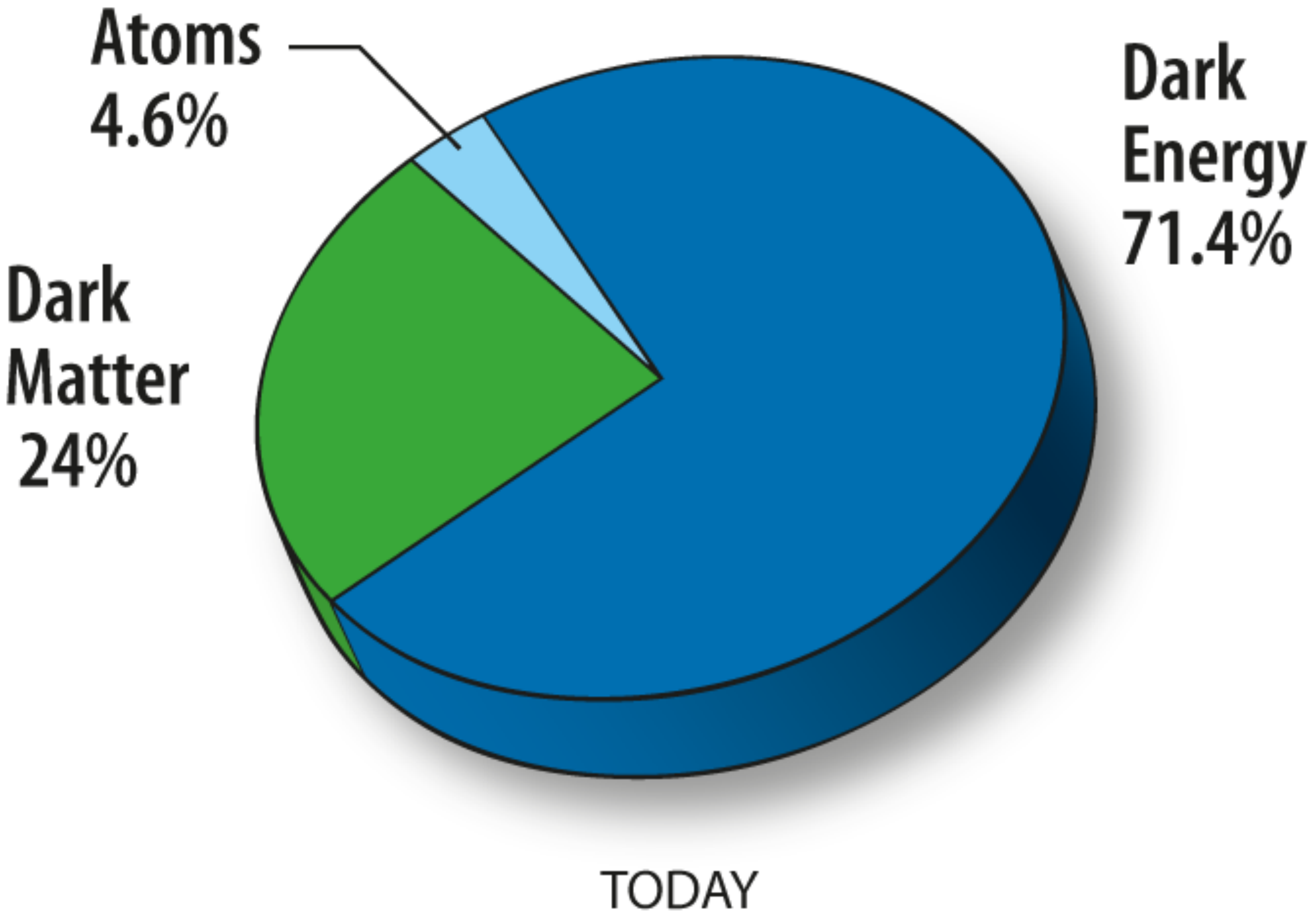
The standard model

- Flat, Lambda Cold Dark Matter - LCDM
- Based on General Relativity
- Successful in explaining most cosmological observations in a surprisingly simple framework
    - But, there are some interesting tensions



Hinshaw et. al (2013)

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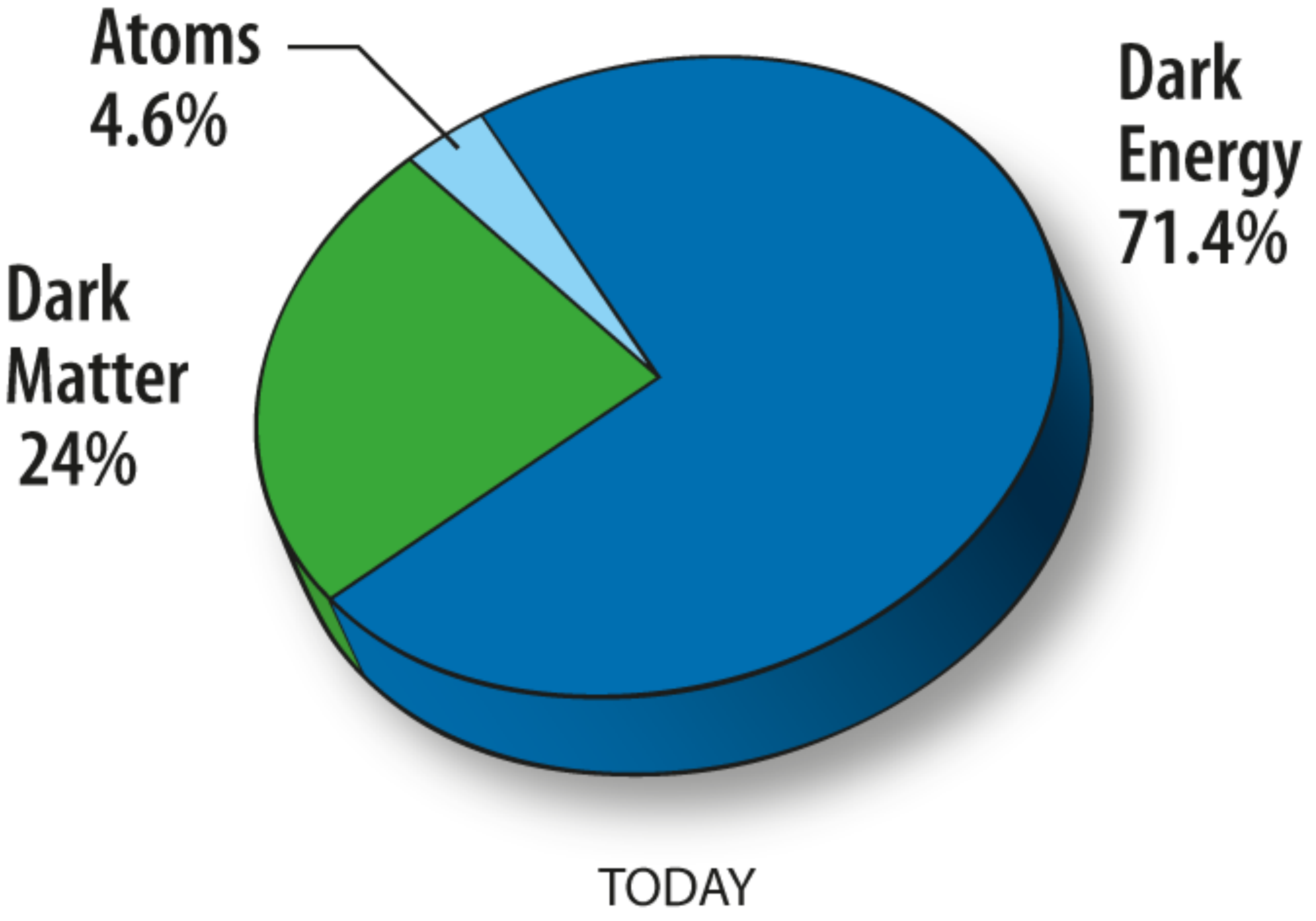
e.g. the Hubble tension

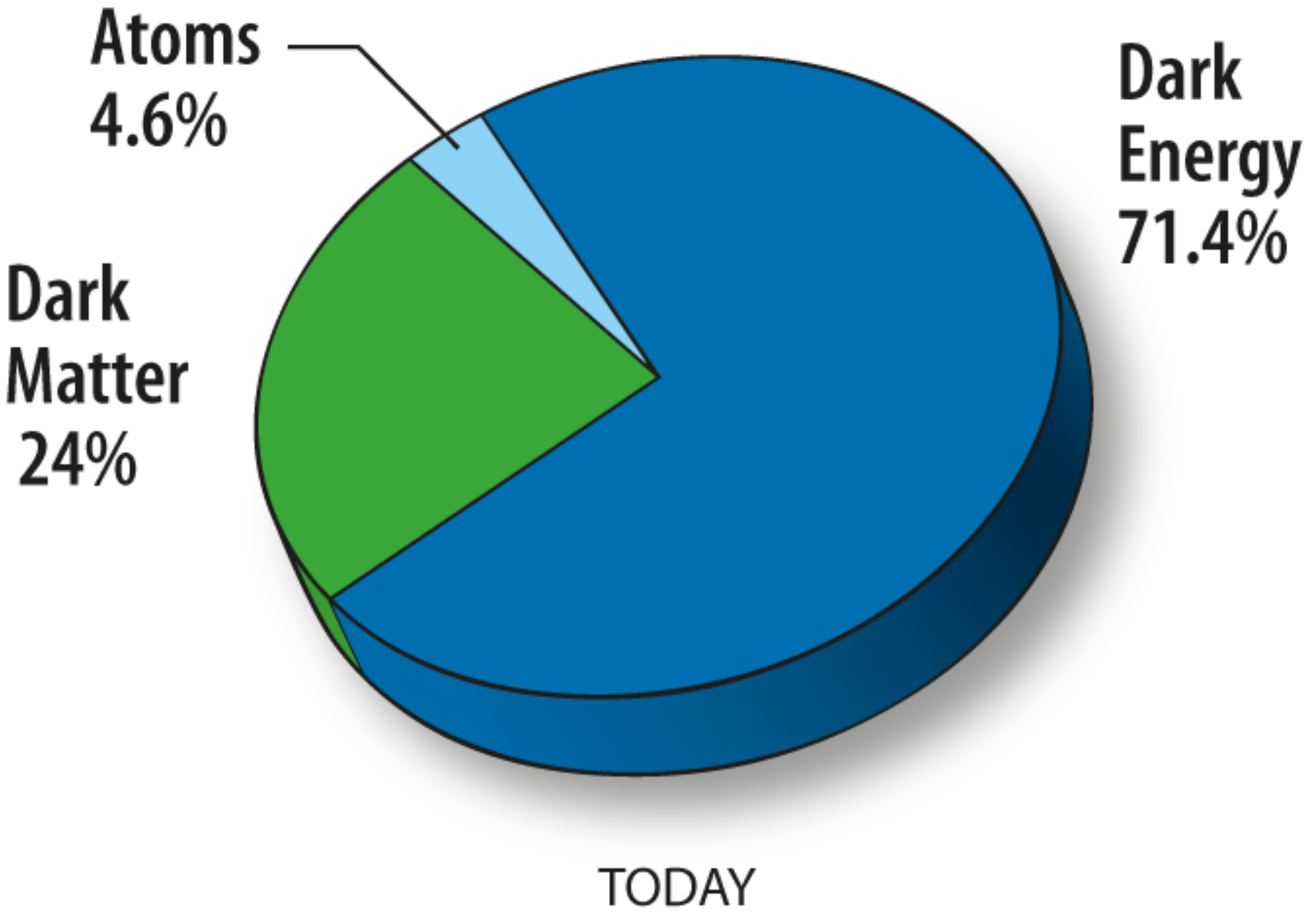


Hinshaw et. al (2013)


Freedman (2017)
80
75
70
65
H_0 [km s^{-1} Mpc^{-1}]
KP
SH$_0$ES
SH$_0$ES
CHP
SH$_0$ES
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15+BAO
Distance Ladder
ΛCDM
2000
2005
2010
2015
Publication Year
Late Universe
SH0ES (Riess+2022)
P20
Early Universe (CMB)
2021

The standard model

- Flat, Lambda Cold Dark Matter - LCDM
- Based on General Relativity
- Successful in explaining most cosmological observations in a surprisingly simple framework
  - But, there are some interesting tensions
  - …and 95% of the Universe remains unexplained
  - ***New physics beyond LCDM?***
  - ***Modifications to GR?***
  - ***Existing physics that's neglected?***



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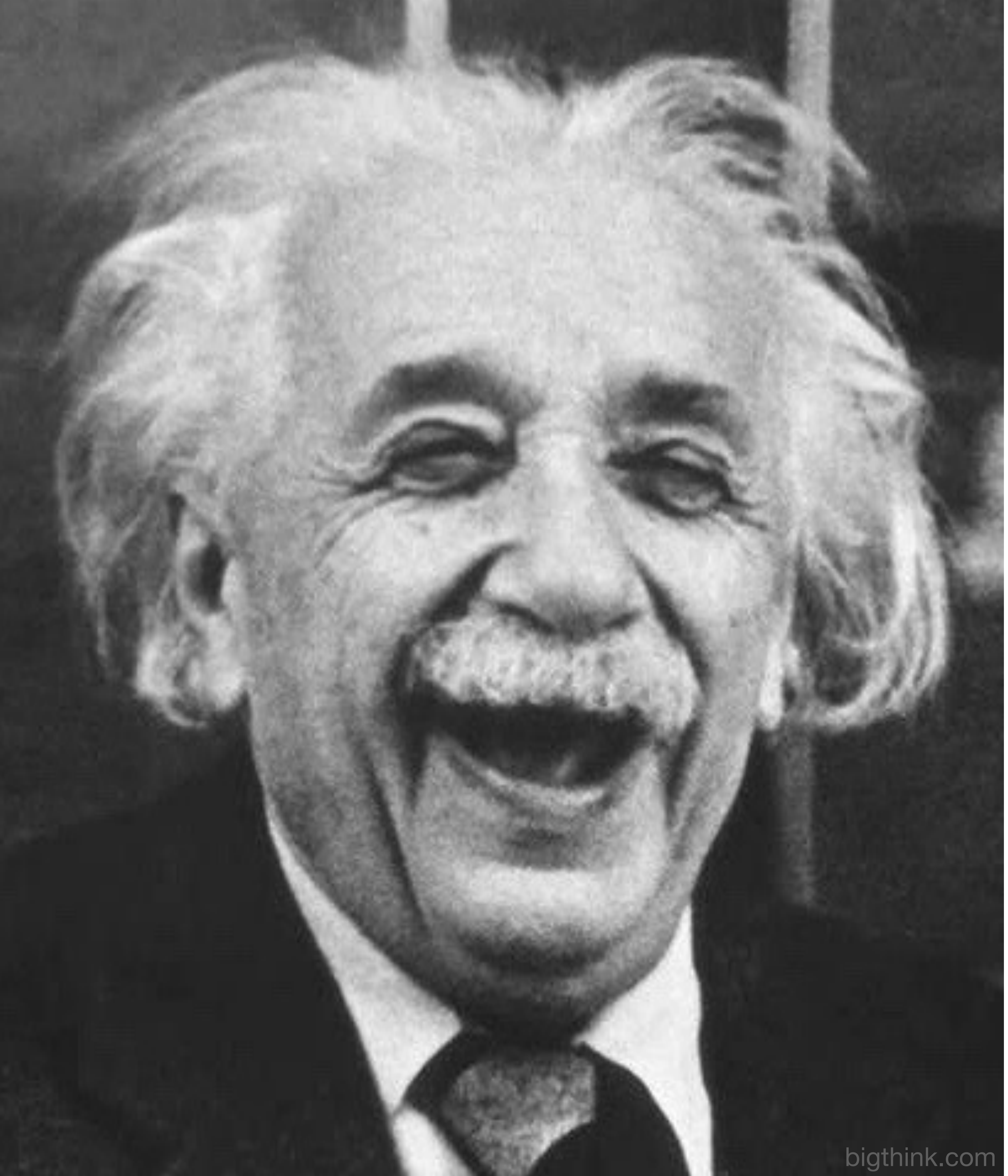
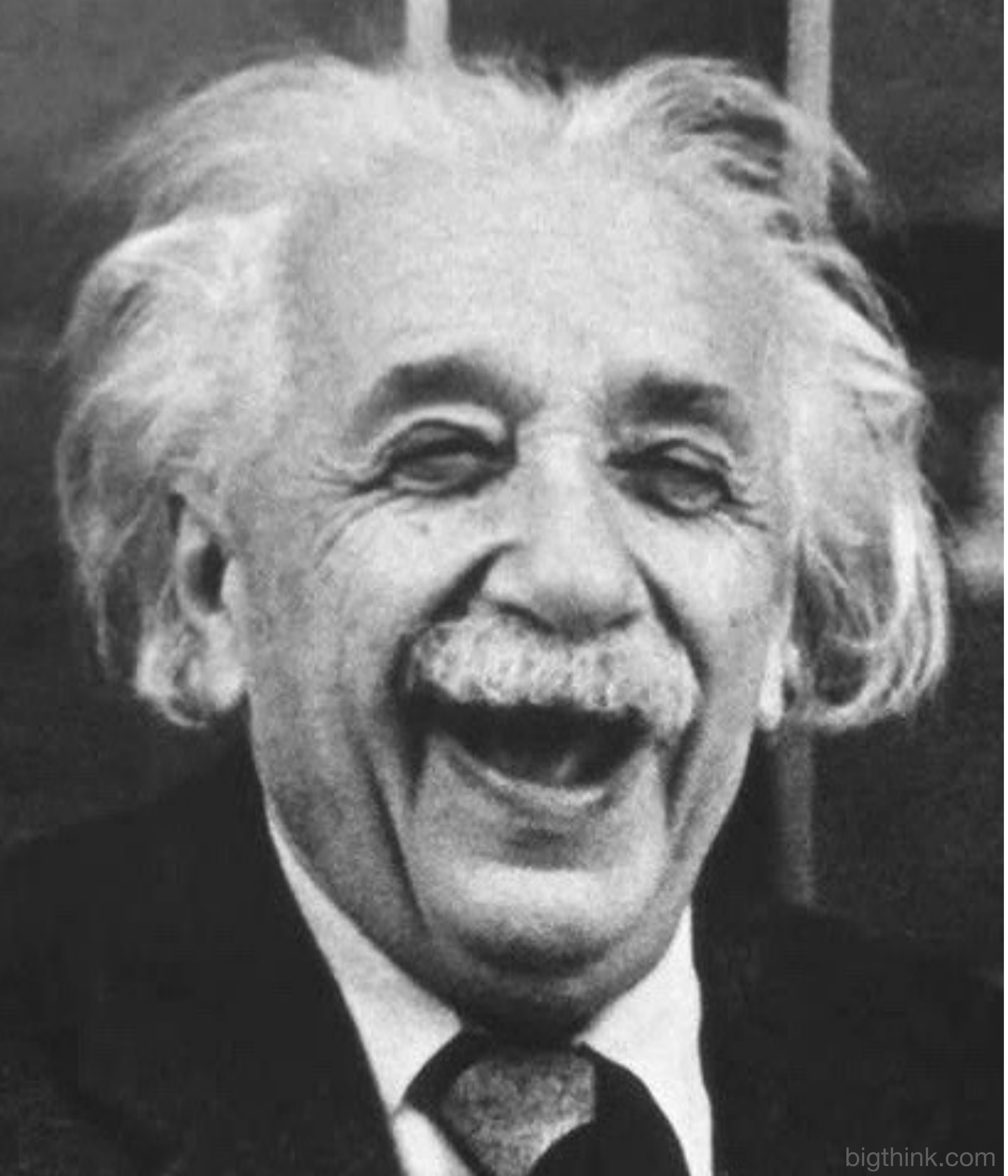


Modern cosmology is based on general relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



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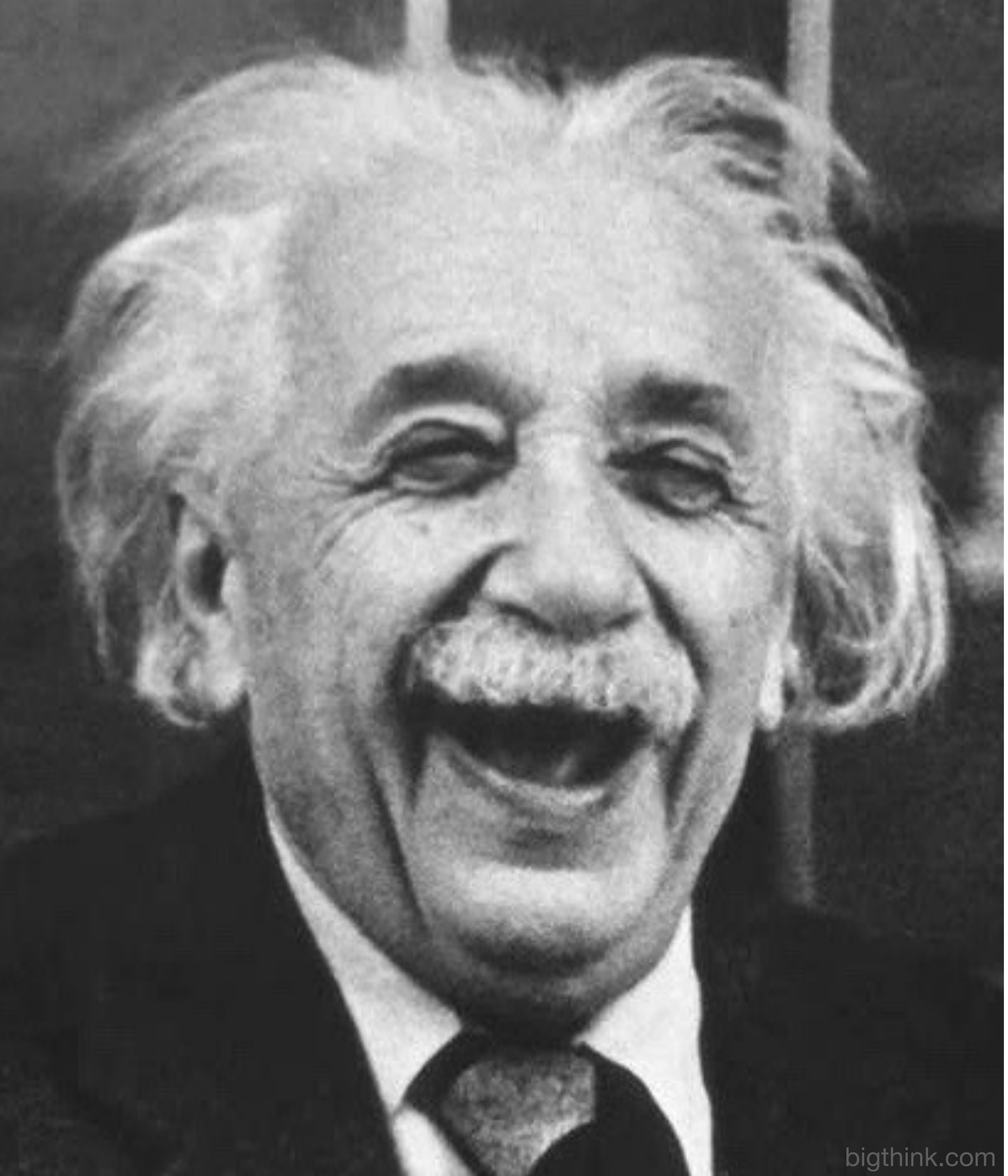
SPACETIME = MATTER



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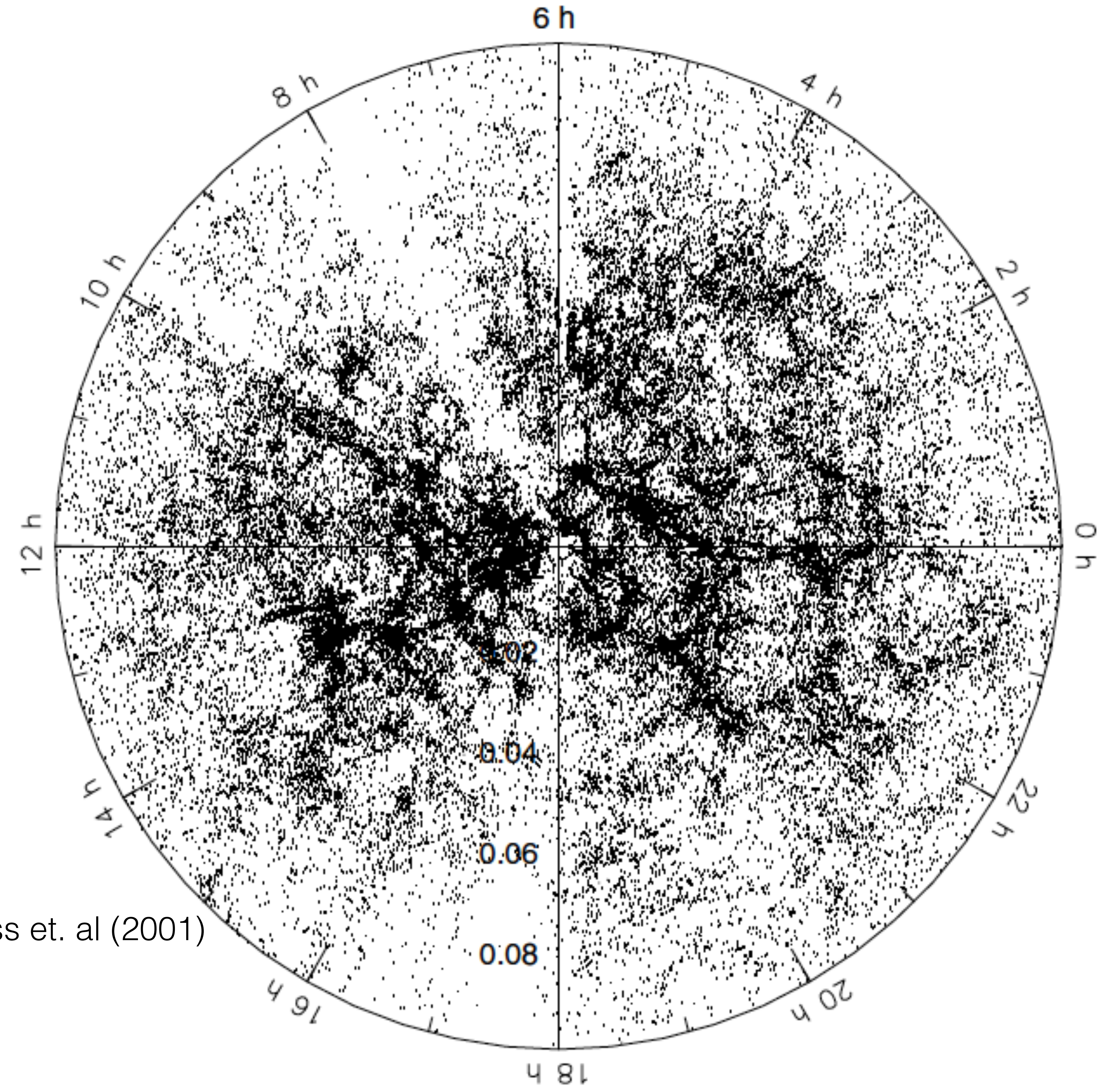
Homogeneity & isotropy

Yield FLRW models

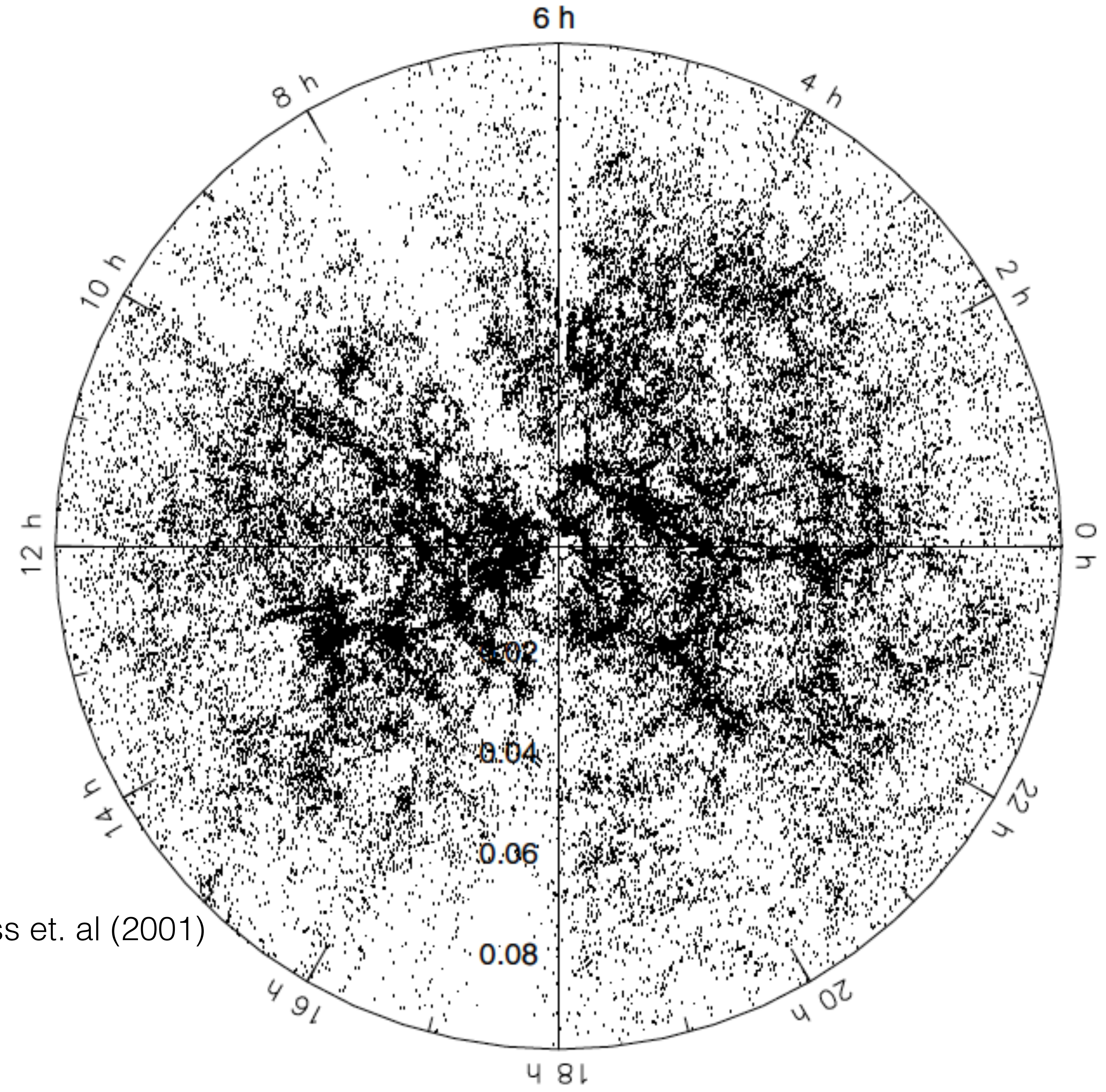
Homogeneity and isotropy in cosmology



Colless et. al (2001)

- The Universe is *homogeneous* and *isotropic*

Homogeneity and isotropy in cosmology



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  this occurs at ~ 100-300 Mpc

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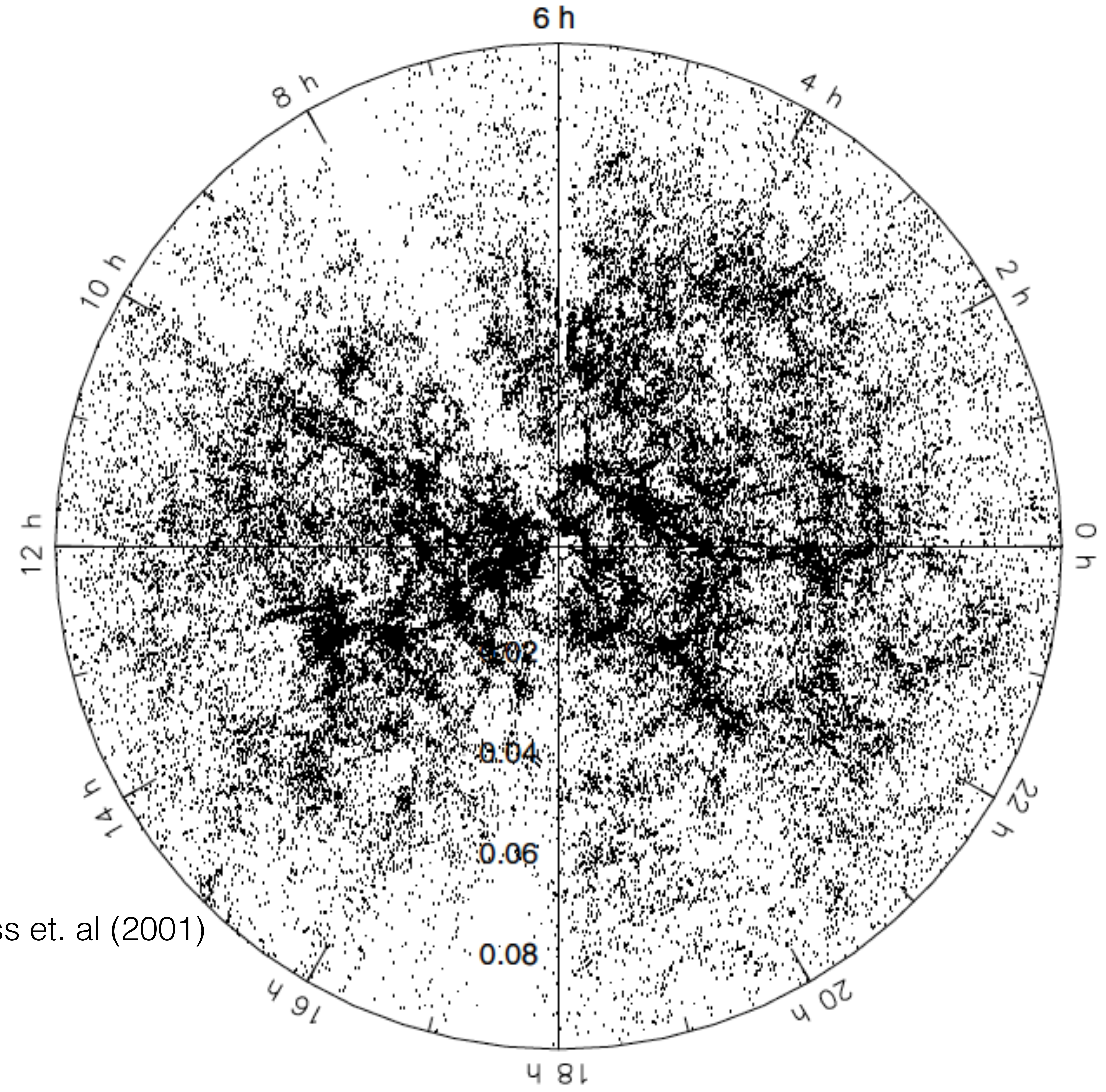

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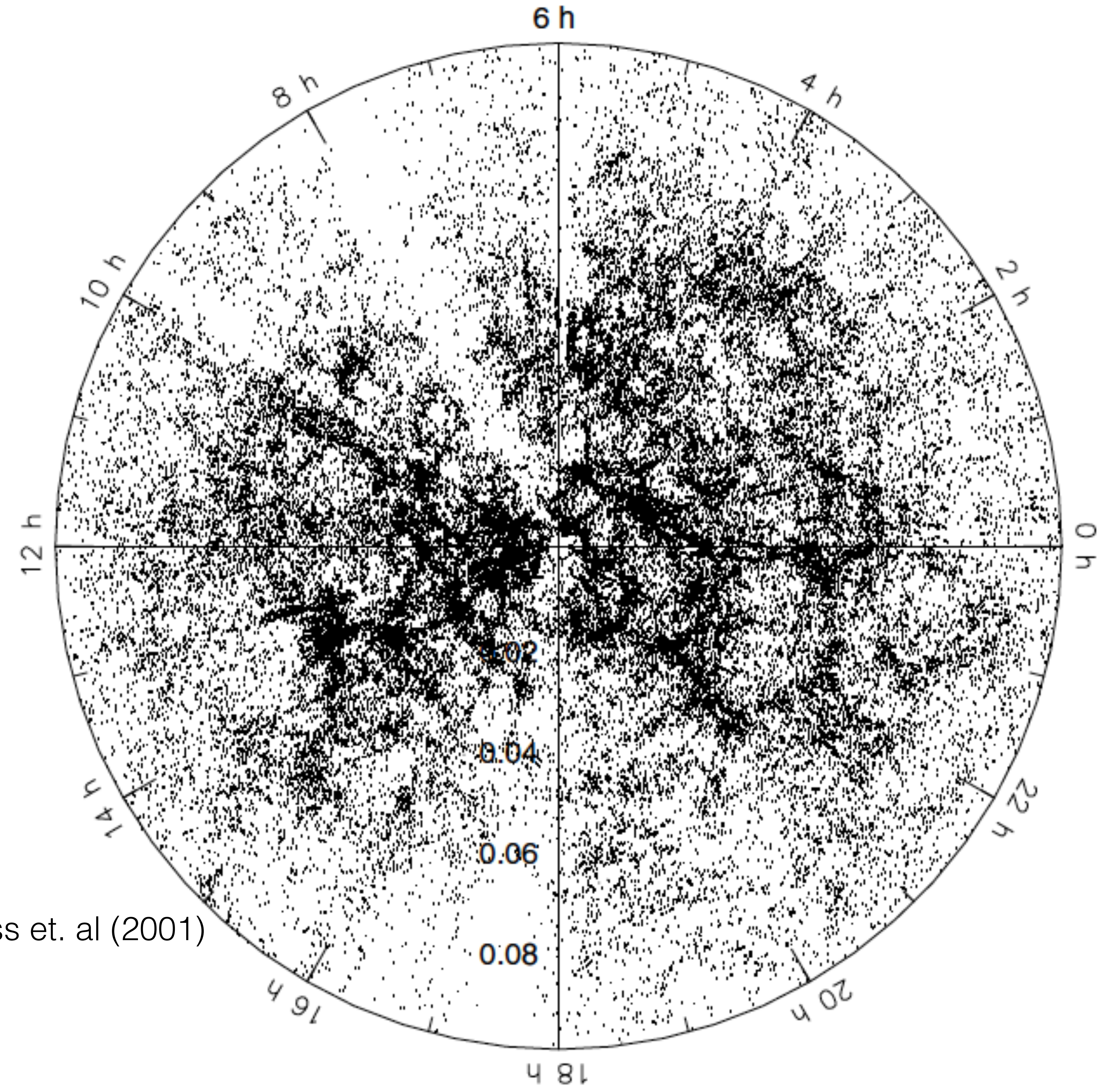

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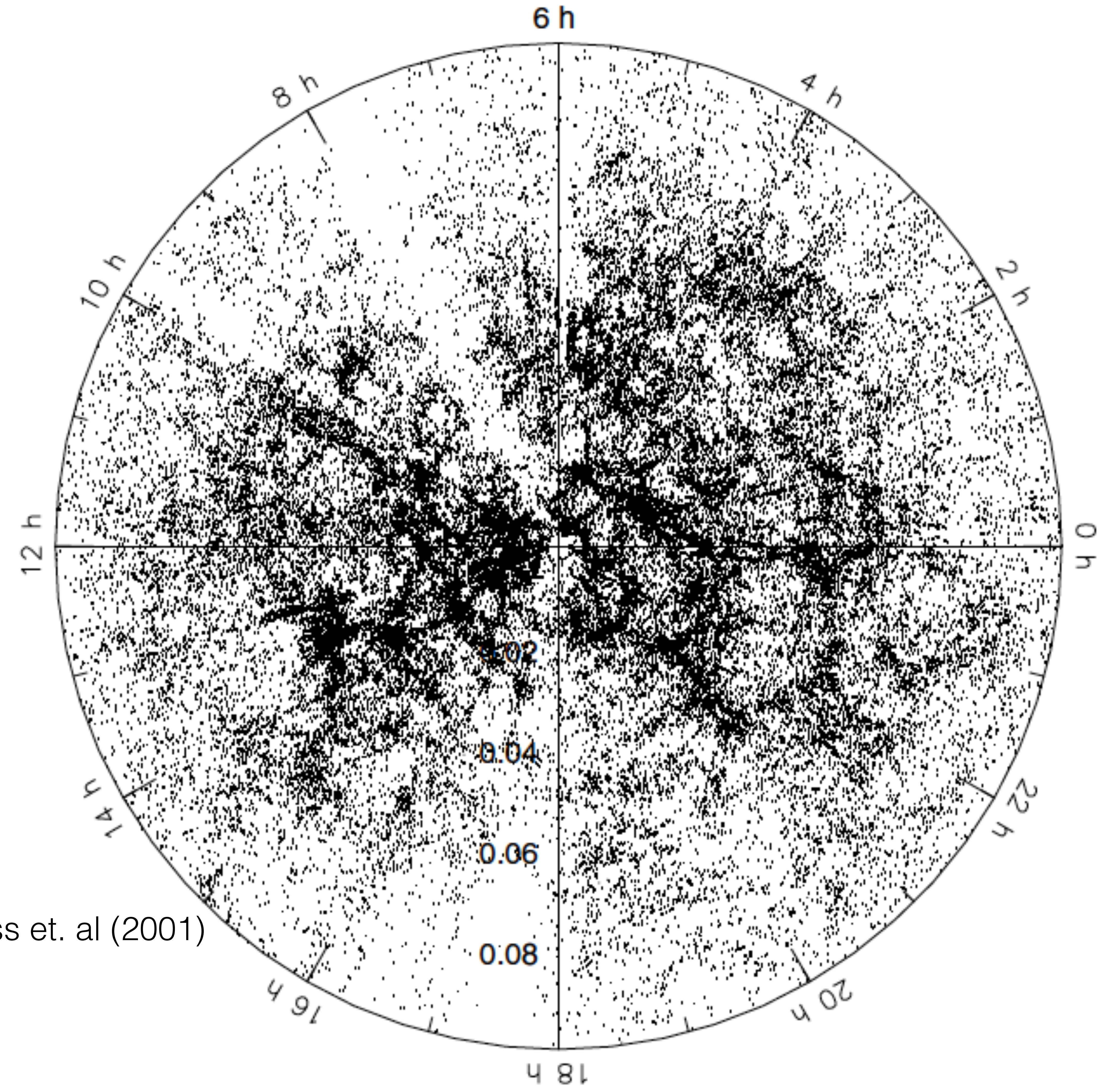

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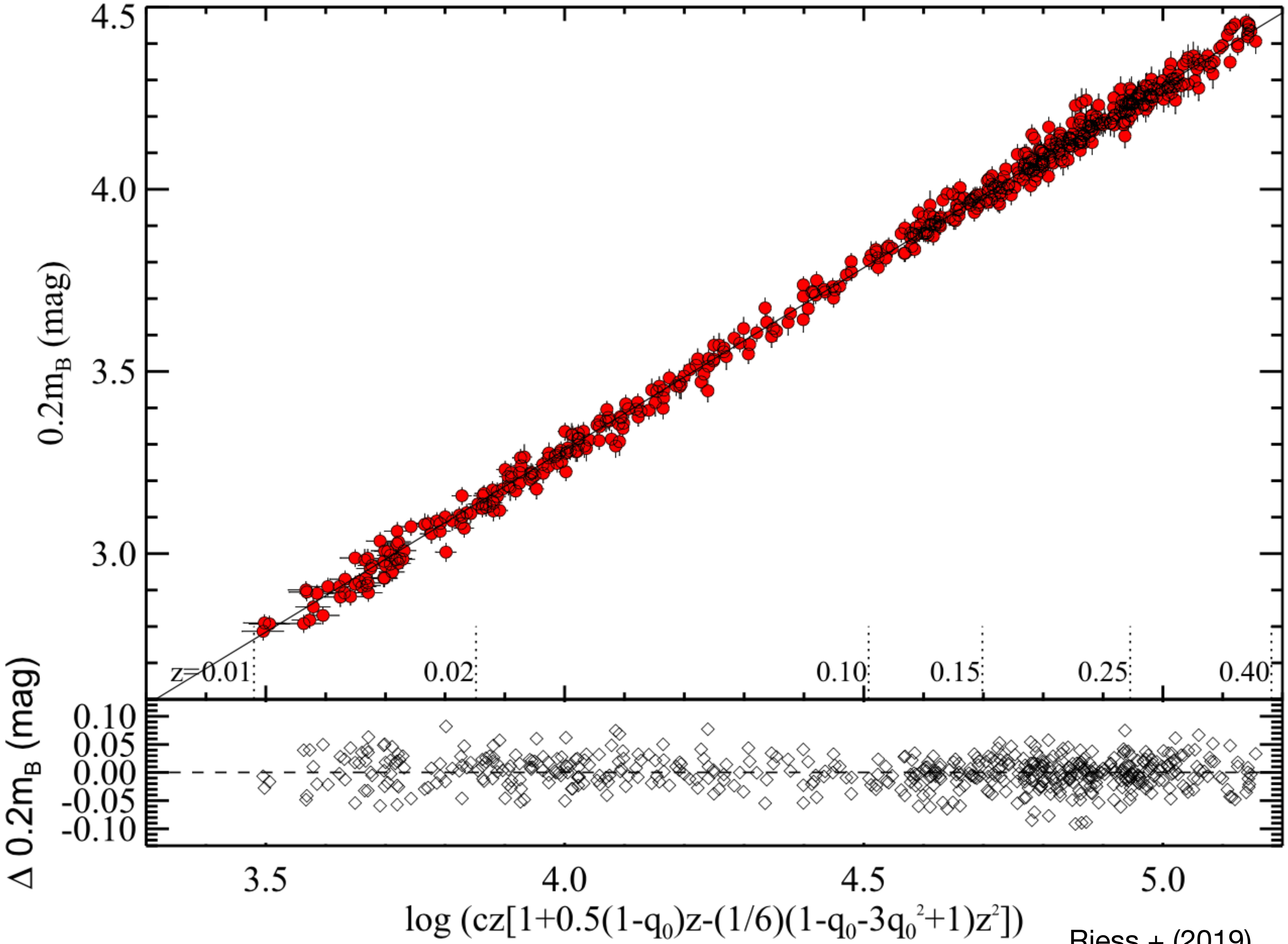
- We have cosmological observations that take place at or around this scale which impose FLRW
- *Is this okay??*



4.5
4.0
3.5
3.0
0.2m_B (mag)
z=0.01
0.02
0.10
0.15
0.25
0.40
Δ 0.2m_B (mag)
0.10
0.05
0.00
-0.05
-0.10
3.5
4.0
4.5
5.0
log (cz[1+0.5(1-q0)z-(1/6)(1-q0-3q0²+1)z²])

Riess + (2019)

Distance vs. redshift in FLRW

Visser (2004)

$$d_{L,\mathrm{FLRW}}(z) = d^{(1)}_{L,\mathrm{FLRW}} z + d^{(2)}_{L,\mathrm{FLRW}} z^2 + d^{(3)}_{L,\mathrm{FLRW}} z^3 + \mathcal{O}(z^4)\,.$$

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We've assumed an FLRW metric, but nothing about the equations that govern a(t)

"FLRW cosmography" is the framework used to, e.g., measure the local Hubble parameter

Valid for redshifts z<1

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Hubble

$$H \equiv \frac{\dot{a}}{a}\,,$$

deceleration

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jerk

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Distance vs. redshift in general

At low redshift, inhomogeneities distort the metric of space-time away from FLRW

Is this important for our current measurements? Or for future high-precision surveys?
Or is it totally negligible?

Heinesen (2020) derived the dL(z) cosmographic expansion to third order with **no assumptions of any underlying metric or field equations**

Building on: Kristian & Sachs (1966), Ellis & MacCallum (1970), Clarkson & Umeh (2011), Clarkson et. al (2012)

Distance vs. redshift in general

Heinesen (2020) arXiv:2010.06534

$$d_L(z) = d_L^{(1)} z + d_L^{(2)} z^2 + d_L^{(3)} z^3 + \mathcal{O}(z^4),$$

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Effective Hubble

$$\mathfrak{H} = -\frac{1}{E^2}\frac{\mathrm{d}E}{\mathrm{d}\lambda},$$

Effective deceleration

$$\mathfrak{Q} \equiv -1 - \frac{1}{E}\frac{\frac{\mathrm{d}\mathfrak{H}}{\mathrm{d}\lambda}}{\mathfrak{H}^2},$$

Effective curvature

$$\mathfrak{R} \equiv 1 + \mathfrak{Q} - \frac{1}{2E^2}\frac{k^\mu k^\nu R_{\mu\nu}}{\mathfrak{H}^2},$$

Effective jerk

$$\mathfrak{J} \equiv \frac{1}{E^2}\frac{\frac{\mathrm{d}^2\mathfrak{H}}{\mathrm{d}\lambda^2}}{\mathfrak{H}^3} - 4\mathfrak{Q} - 3.$$

"Effective" because interpretation of these parameters is not so simple outside the safety of FLRW...

(but they play the same roles in the general relation as they do in FLRW)

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e.g., ‘Q’ can be negative without any global acceleration of spacetime

$$q \equiv -\frac{\ddot{a}}{aH^2}$$

Distance vs. redshift in general

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All effective parameters can be written as *exact* multipole series expansions in the direction of observation

$$\mathfrak{H} = \frac{1}{3}\theta + e^{\mu}e^{\nu}\sigma_{\mu\nu}$$

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Reduces to Hubble parameter in FLRW limit

Depends on observer position - "*cosmic variance*"

Variations have been shown to be small* in:

- Perturbation theory and Newtonian sims (e.g. *Odderskov+2014,2016, Wojtak+2014, Ben-Dayan+ 2014, Wu & Huterer 2017*)
- Weak-field GR sims (*Adamek+ 2017*)
- NR sims (***HJM+, 2018***)

**on the scales of interest for the SH0ES measurement*

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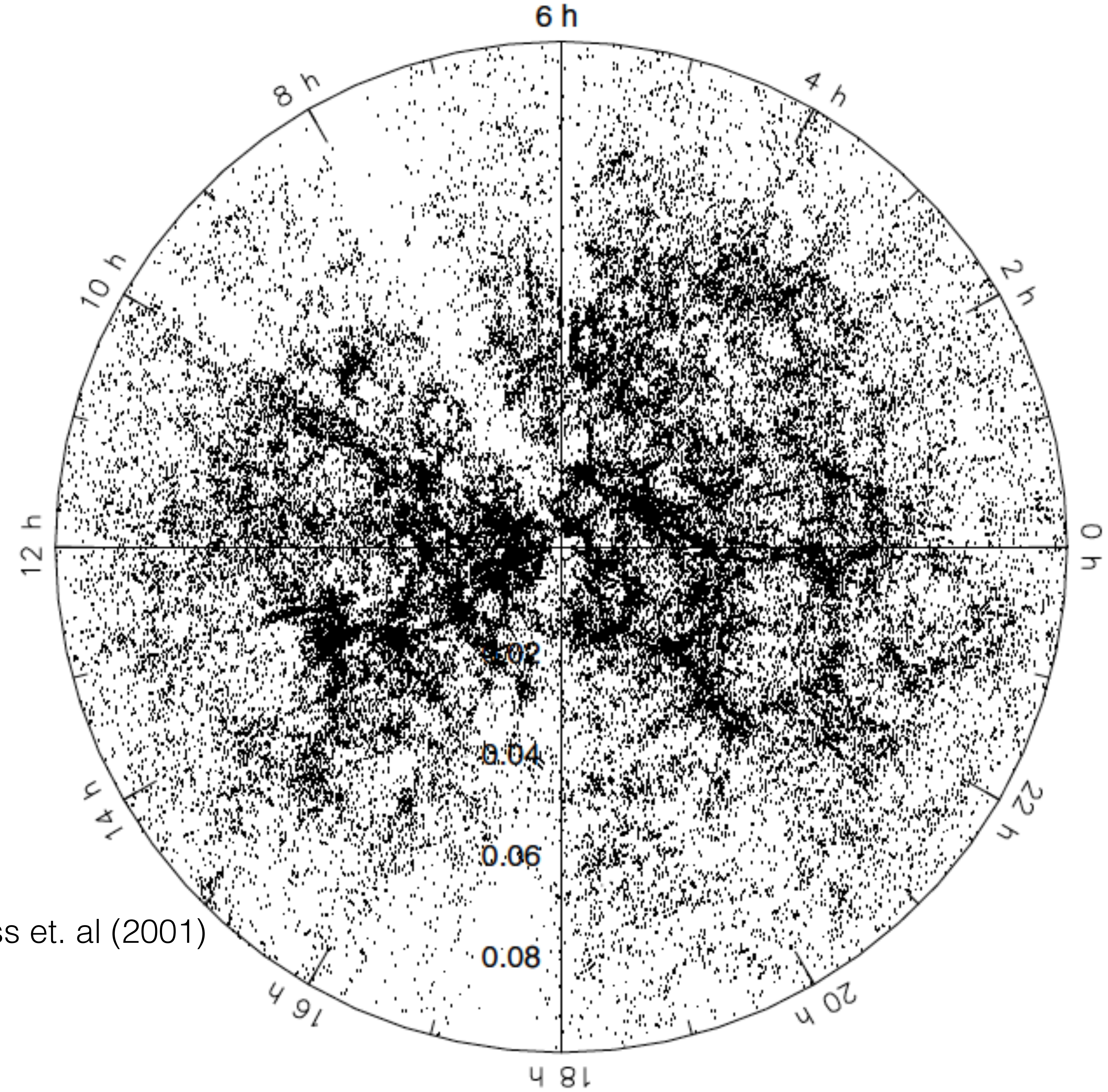
(Similar expansions can be done for the deceleration, curvature, and jerk parameters as well)

Low redshift anisotropies



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Does this impact low-redshift surveys which assume an FLRW expansion?

E.g., the Hubble tension, of course…

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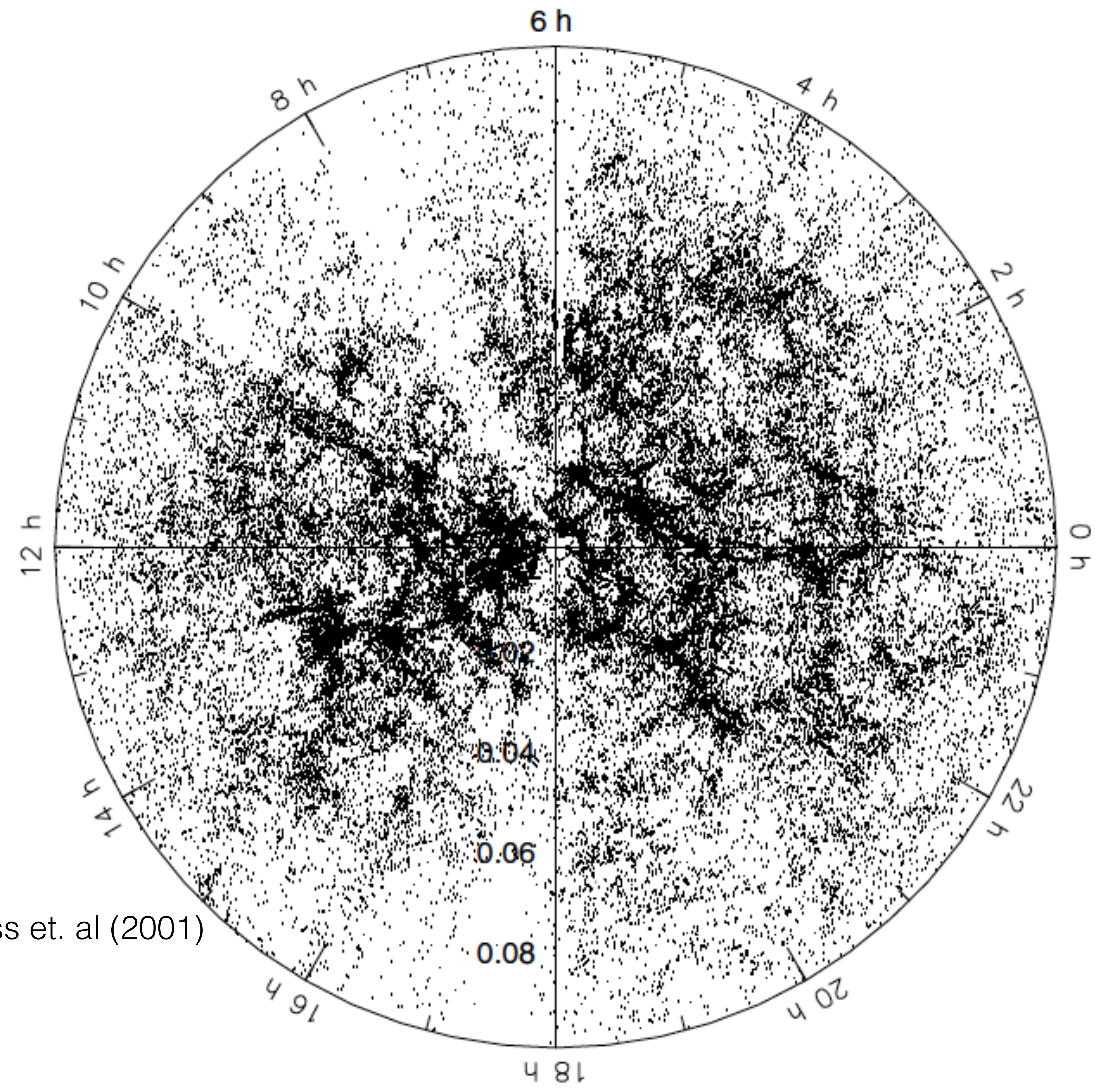

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We looked at this in:

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Cowell, Dhawan, ***HJM*** (2023, arXiv:2212.13569)

HJM (2024, arXiv:2402.09659)

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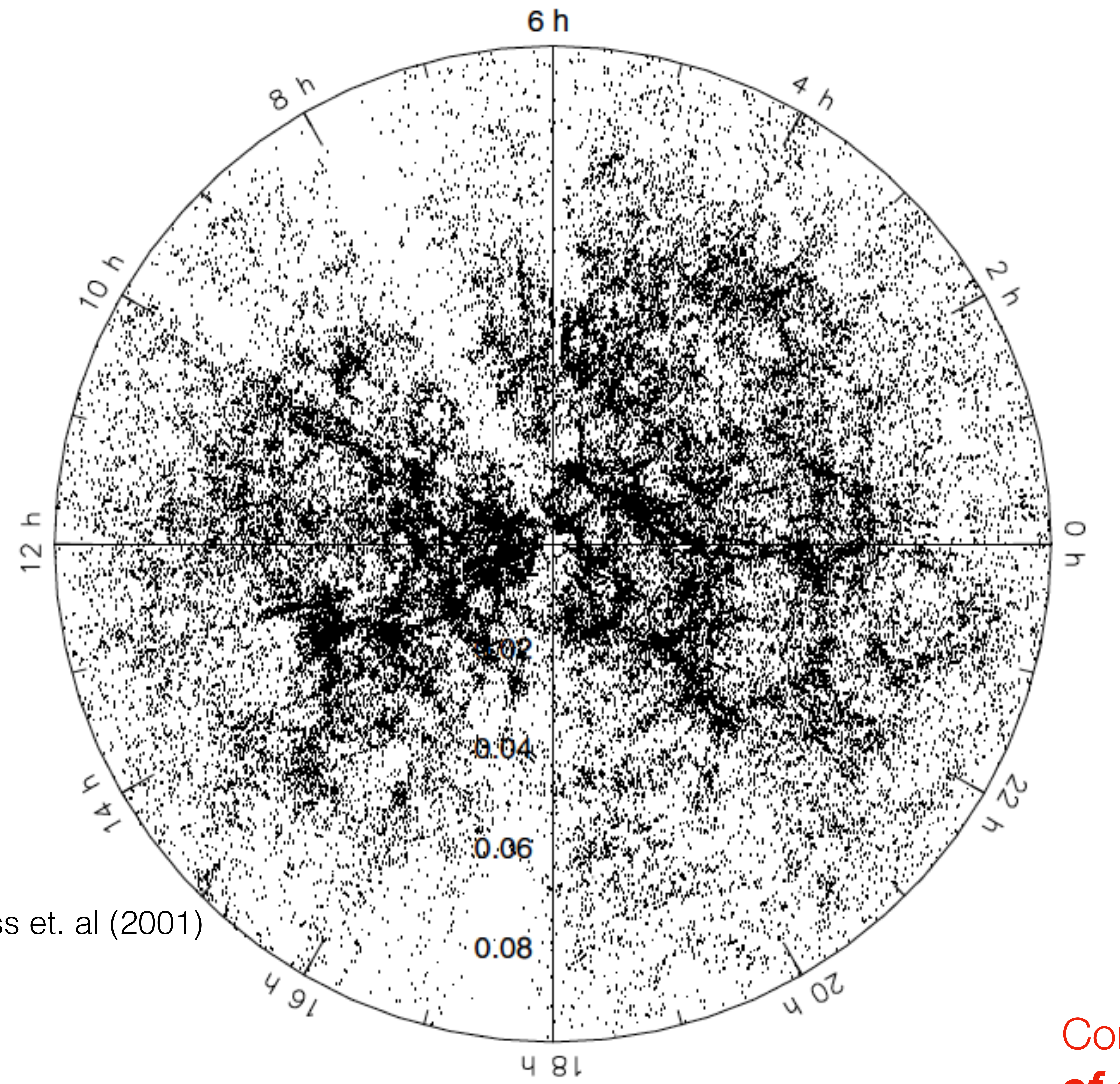

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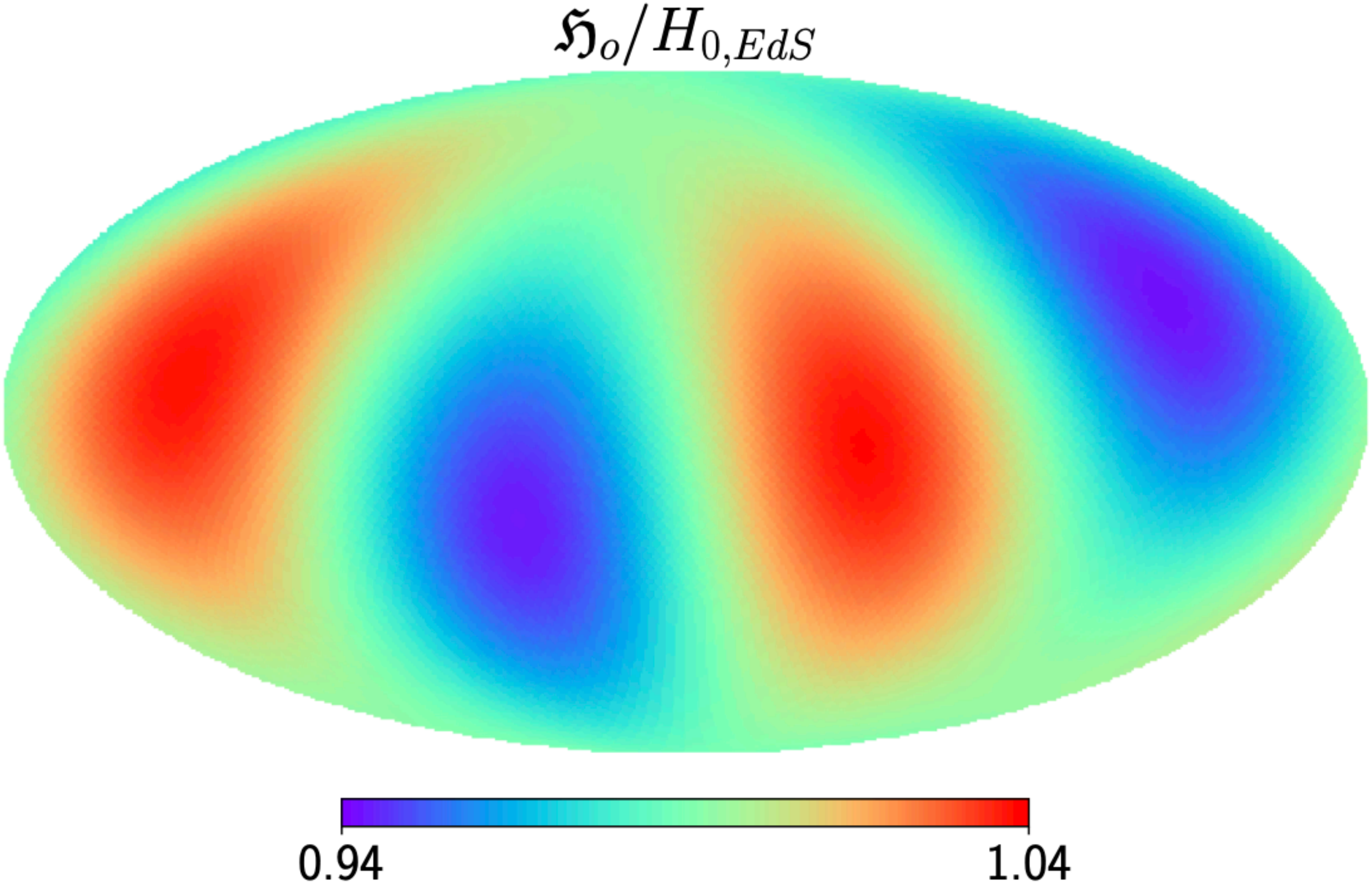
Cowell, Dhawan, ***HJM*** (2023, arXiv:2212.13569)

HJM (2024, arXiv:2402.09659)

Conclusion… probably not. ***But it could still impact the accuracy of our measurements!***

Cowell, Dhawan, ***HJM*** (2022)
See also ***HJM*** & Heinesen (2021)

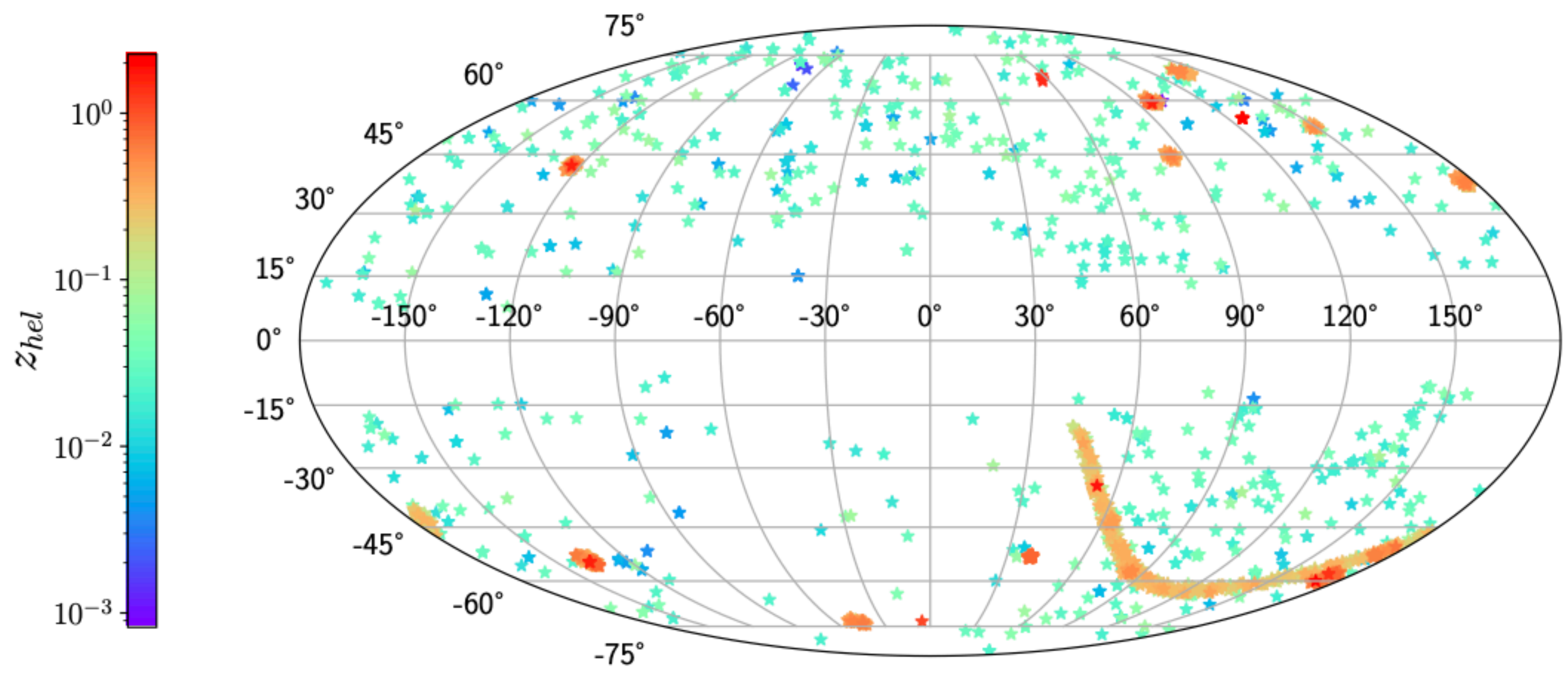


$$\mathfrak{H} = \frac{1}{3}\theta + e^{\mu}e^{\nu}\sigma_{\mu\nu}$$

Hubble expansion variance across the sky for one observer in an NR simulation

Dominated by a quadrupole, as expected in Heinesen (2021; 2010.06534)

Can we see this in data?



Jess Cowell

Part III & summer @ IoA, Cambridge

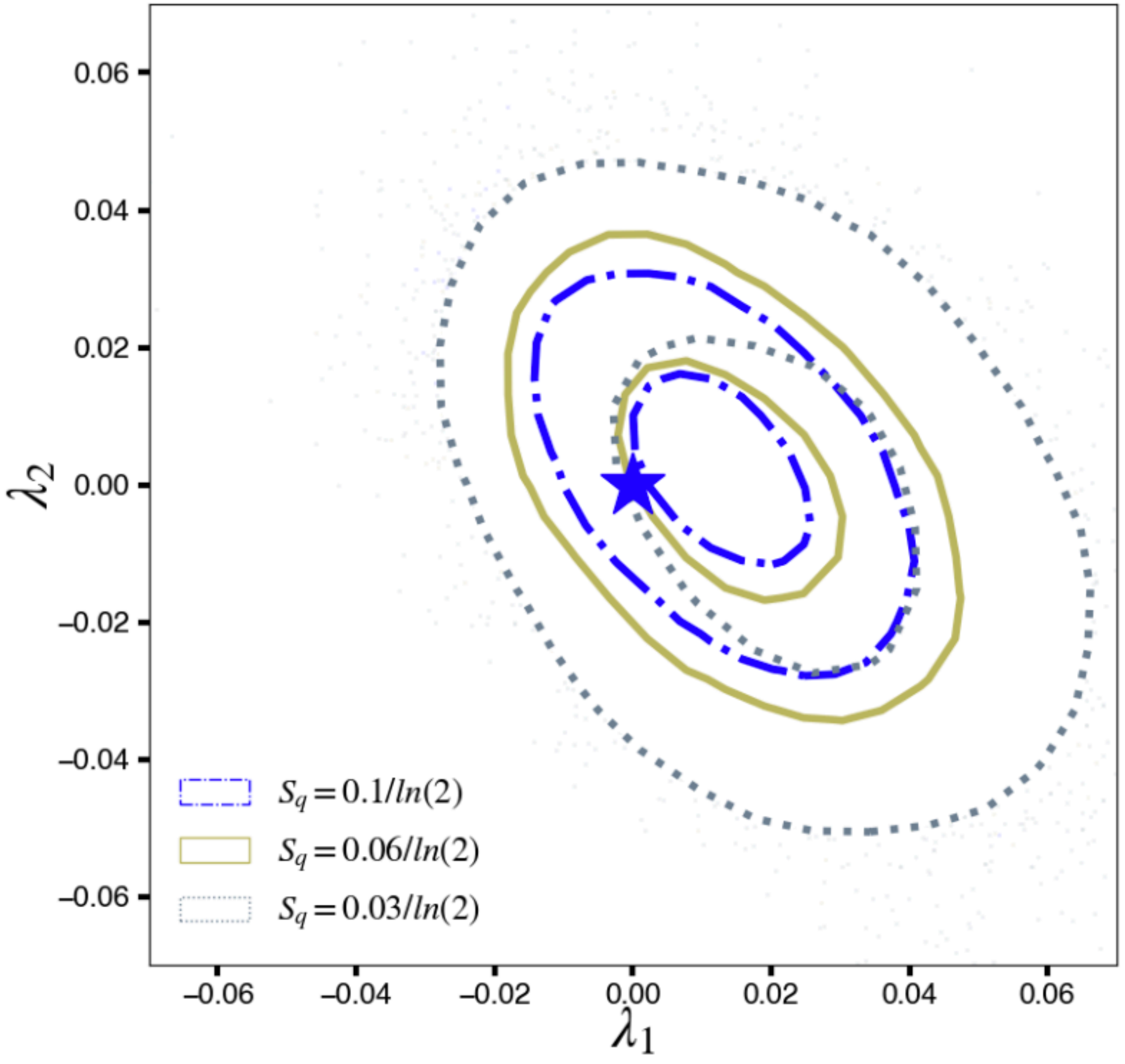
Now PhD @ Oxford/IPMU w/ Alonso & Liu

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae

Jessica A. Cowell,[1,2,3]★ Suhail Dhawan,[1] Hayley J. Macpherson,[4,5,6]
[1] *Institute of Astronomy and Kavli Institute for Cosmology, University of Cambridge, Madingley Road, Cambridge CB3 0HA, UK*
[2] *Department of Physics, University of Oxford, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK*
[3] *Kavli Institute for the Physics and Mathematics of the Universe (IMPU), the University of Tokyo, Kashiwa, Chiba, 277-8582, Japan*
[4] *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK*
[5] *Kavli Institute for Cosmological Physics, The University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637, USA*
[6] *NASA Einstein Fellow*

arXiv:2212.13569

Dhawan, Borderies, ***HJM***, Heinesen (2022)



Pantheon: No

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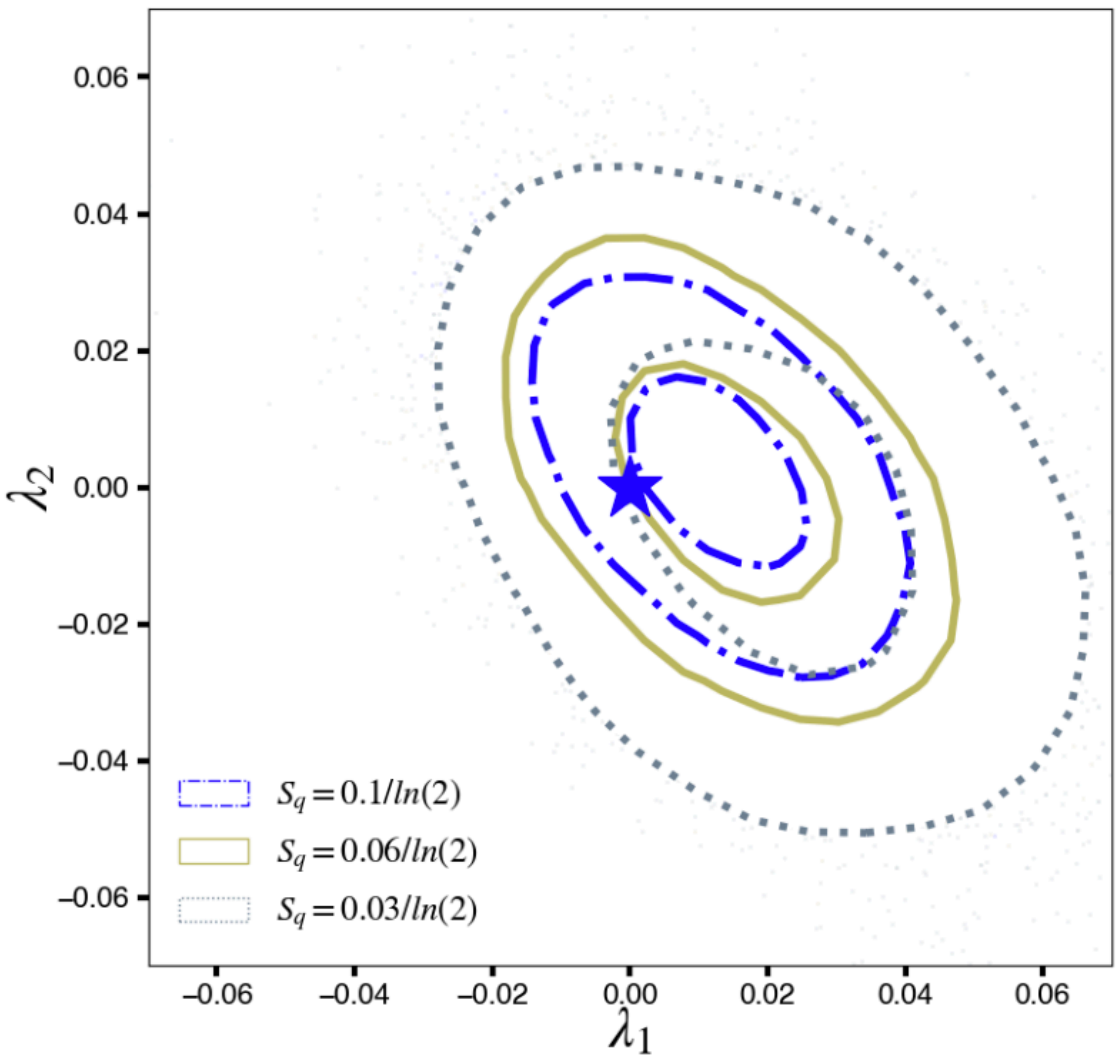

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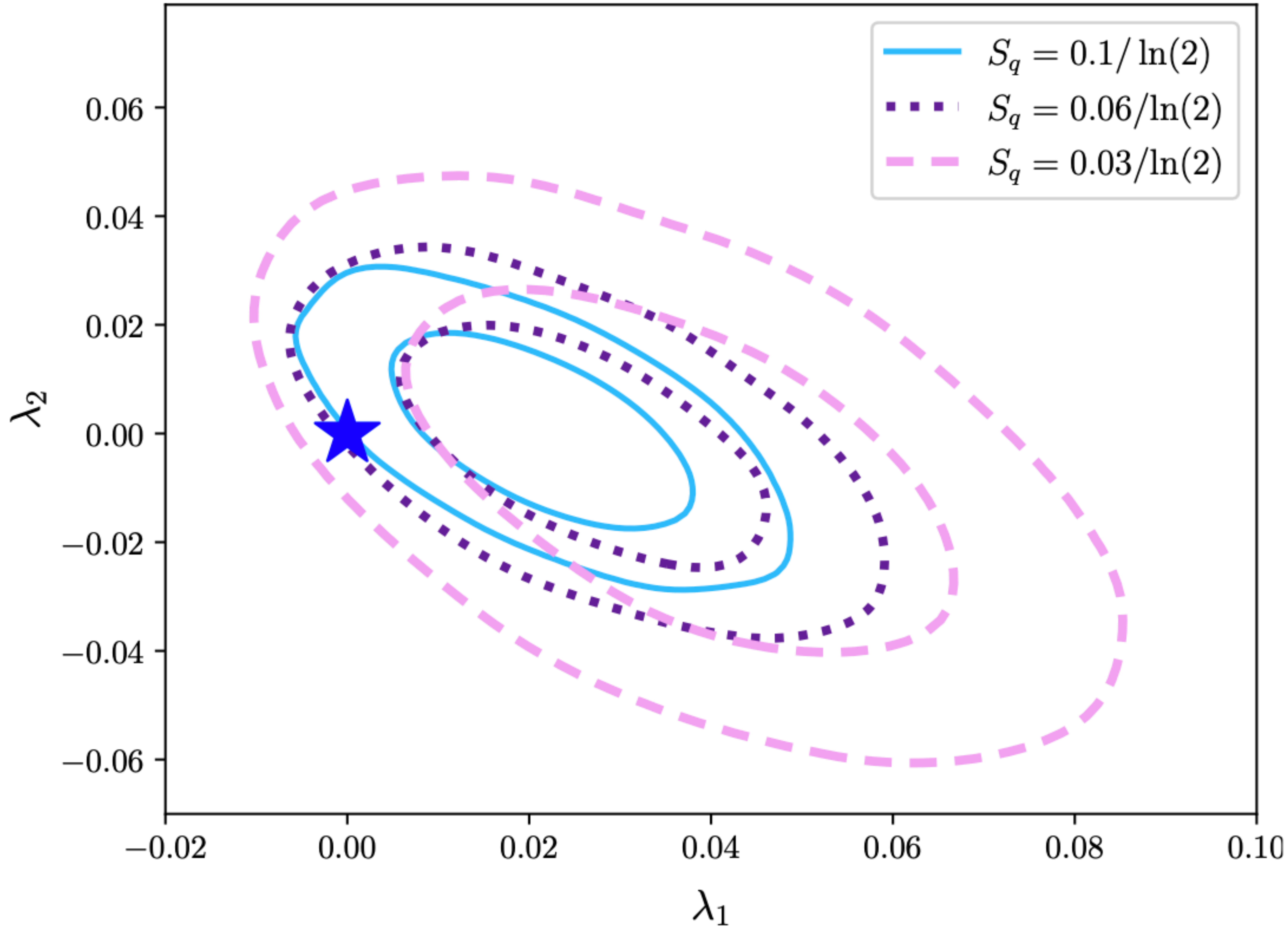


Pantheon: No

Pantheon +: Yes! (ish)

arXiv:2205.12692

arXiv:2212.13569

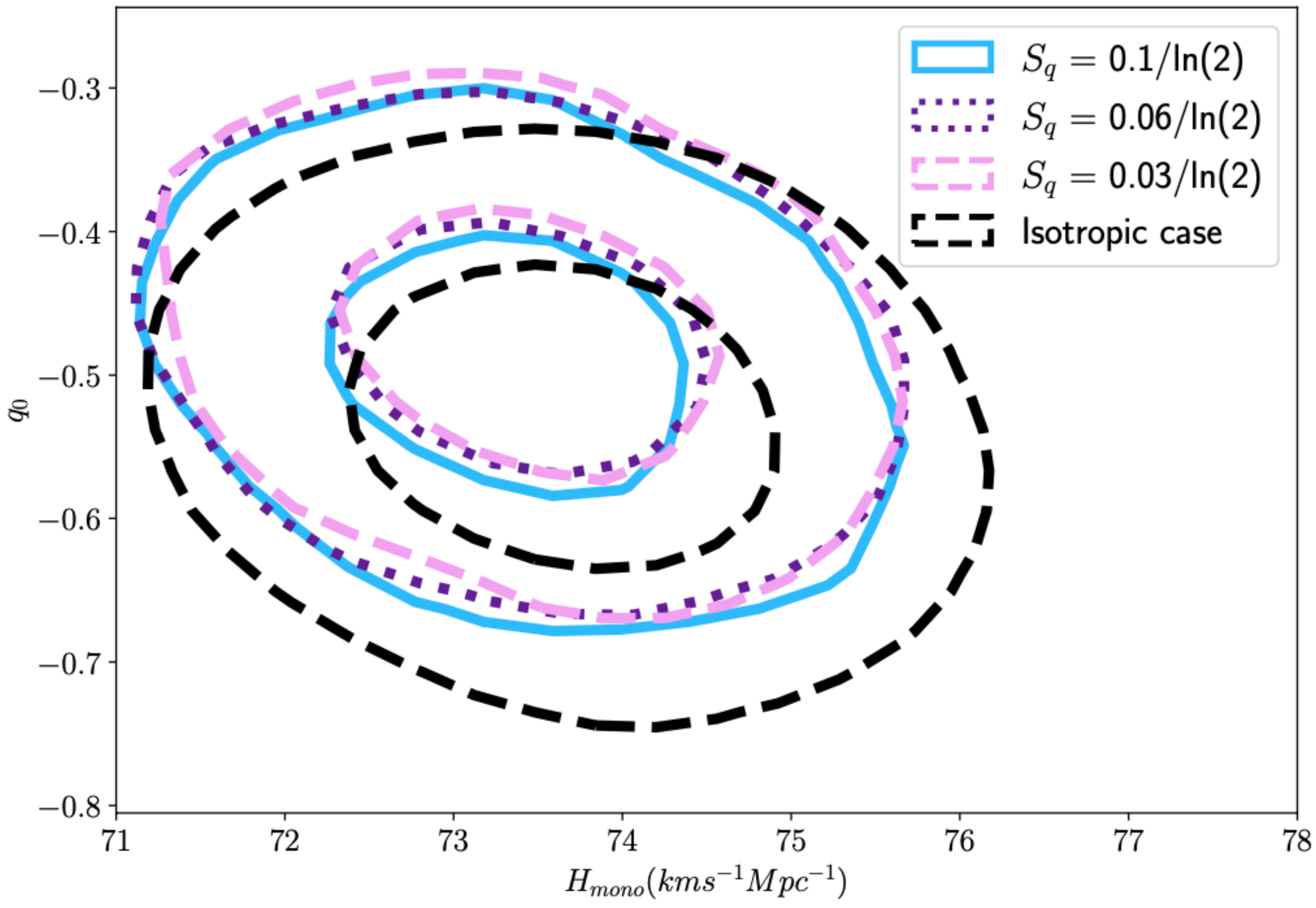
What about the Hubble tension?

Add calibrator SNe into the mix so we can also constrain the monopole

Try isotropic vs anisotropic fits to see if the monopole changes

… it doesn't, maximum shift is ~ 0.5 km/s/Mpc

Anisotropy ***of this kind*** is unlikely to explain the Hubble tension



Cowell, Dhawan, ***HJM*** (2023), arXiv:2212.13569

Conclusions

- The distance-redshift relation generalises in a nontrivial way in the presence of inhomogeneities
- We can directly constrain these anisotropic signatures in local expansion
- These don't seem to shift the monopole parameters enough to solve the Hubble tension
- Can this act as a secondary probe of the homogeneity scale?

hayleyjmacpherson@gmail.com

Extras

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae

arXiv:2212.13569

Use SNe distances from P+ to constrain the cosmographic expansion (as usual)

Model the distance using the 3rd order cosmography:

$$d_L(z) = \frac{1}{H_0} z + \frac{1 - q_0}{2H_0} z^2 + \frac{-1 + 3q_0^2 + q_0 - j_0 + \Omega_{k,0}}{6H_0} z^3 + \mathcal{O}(z^4)$$

Replace:

$$H_0 \to \mathfrak{H}(\boldsymbol{e})$$

$$\begin{aligned}\mathfrak{H}(\boldsymbol{e}) &= H_{\text{mono}} + H_{\text{quad}}(\boldsymbol{e})\,\mathcal{F}(z, S_q) \\ &= H_{\text{mono}}\left\{1 + \left[\lambda_1 \cdot \cos^2\theta_1 + \lambda_2 \cdot \cos^2\theta_2 \; - (\lambda_1 + \lambda_2) \cdot \cos^2\theta_3\right]\mathcal{F}(z, S_q)\right\}\end{aligned}$$

Without calibrators, we cannot constrain the monopole. *But we can still constrain the anisotropy in expansion*

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Replace:

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Separation between each SNe and quadrupole eigendirection

We fix this to be that found in RFGC by Parnovsky & Parnowski (2012)

$$\mathfrak{H}(\boldsymbol{e}) = H_{\text{mono}} + H_{\text{quad}}(\boldsymbol{e})\,\mathcal{F}(z, S_q)$$

$$= H_{\text{mono}}\left\{1 + \left[\lambda_1 \cdot \cos^2\theta_1 + \lambda_2 \cdot \cos^2\theta_2 - (\lambda_1 + \lambda_2) \cdot \cos^2\theta_3\right]\mathcal{F}(z, S_q)\right\}$$

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Eigenvalues of the quadrupole — constrain these!

$$= H_{\text{mono}}\left\{1 + \left[\boxed{\lambda_1}\cdot\cos^2\theta_1 + \boxed{\lambda_2}\cdot\cos^2\theta_2 - \boxed{(\lambda_1+\lambda_2)}\cdot\cos^2\theta_3\right]\mathcal{F}(z, S_q)\right\}$$

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Replace:

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Arbitrary decay function — choose exponential decay:

$$\mathcal{F}(z, S_q) = \exp\left(-\frac{z}{S_q}\right)$$

with 3 x different fixed decay scales

$$\mathfrak{H}(\boldsymbol{e}) = H_{\text{mono}} + H_{\text{quad}}(\boldsymbol{e})\, \mathcal{F}(z, S_q)$$

$$= H_{\text{mono}}\left\{1 + \left[\lambda_1 \cdot \cos^2\theta_1 + \lambda_2 \cdot \cos^2\theta_2 - (\lambda_1 + \lambda_2) \cdot \cos^2\theta_3\right] \boxed{\mathcal{F}(z, S_q)}\right\}$$

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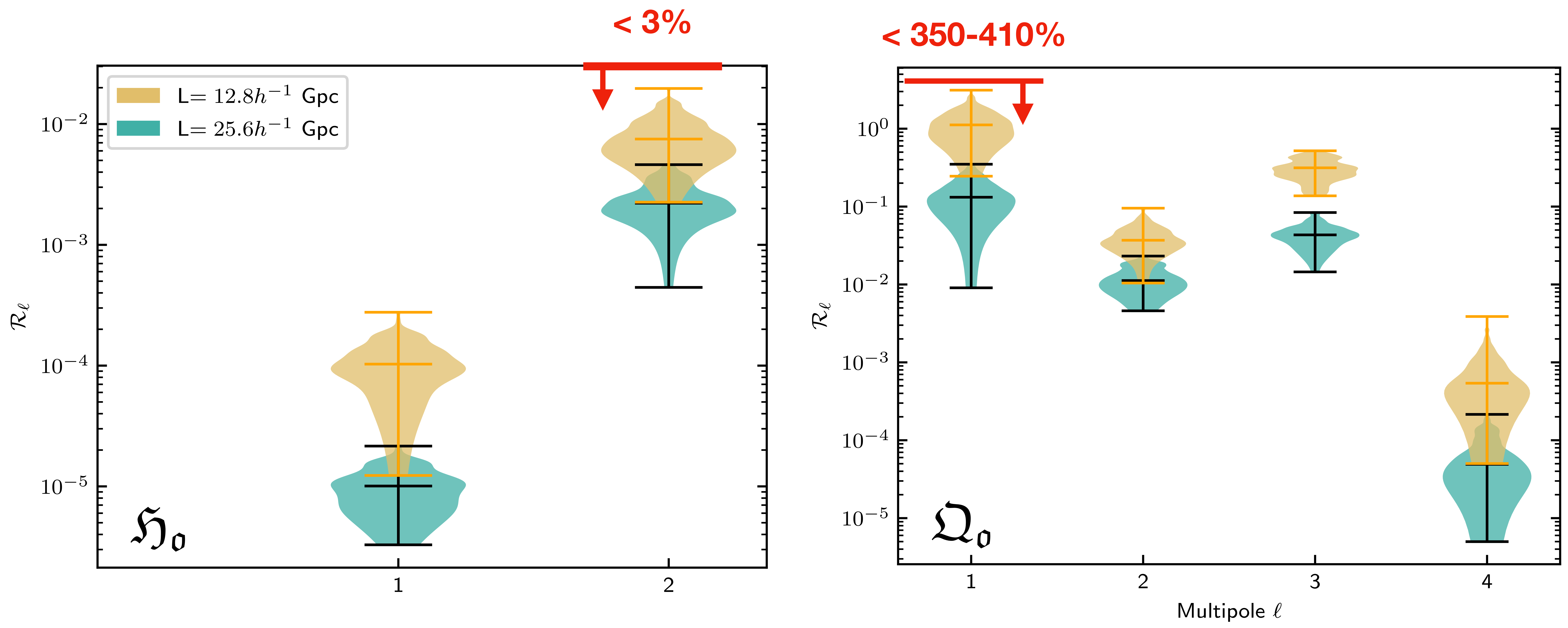
Observational limits match our expectation from simulations



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