UCLA GALACTIC CENTER GROUP

IMPROVING ORBITAL ESTIMATES FOR INCOMPLETE ORBITS WITH A NEW APPROACH TO PRIORS —

APPLICATIONS TO HR 8799

KELLY KOSMO O'NEIL, UCLA
G.D MARTINEZ, A. HEES, A. M. GHEZ, T. DO, G. WITZEL, **Q. KONOPACKY**,
E. E. BECKLIN, D. S CHU, J. LU, K. MATHEWS, S. SAKAI

EXSOCAL 2018
17 SEPTEMBER 2018

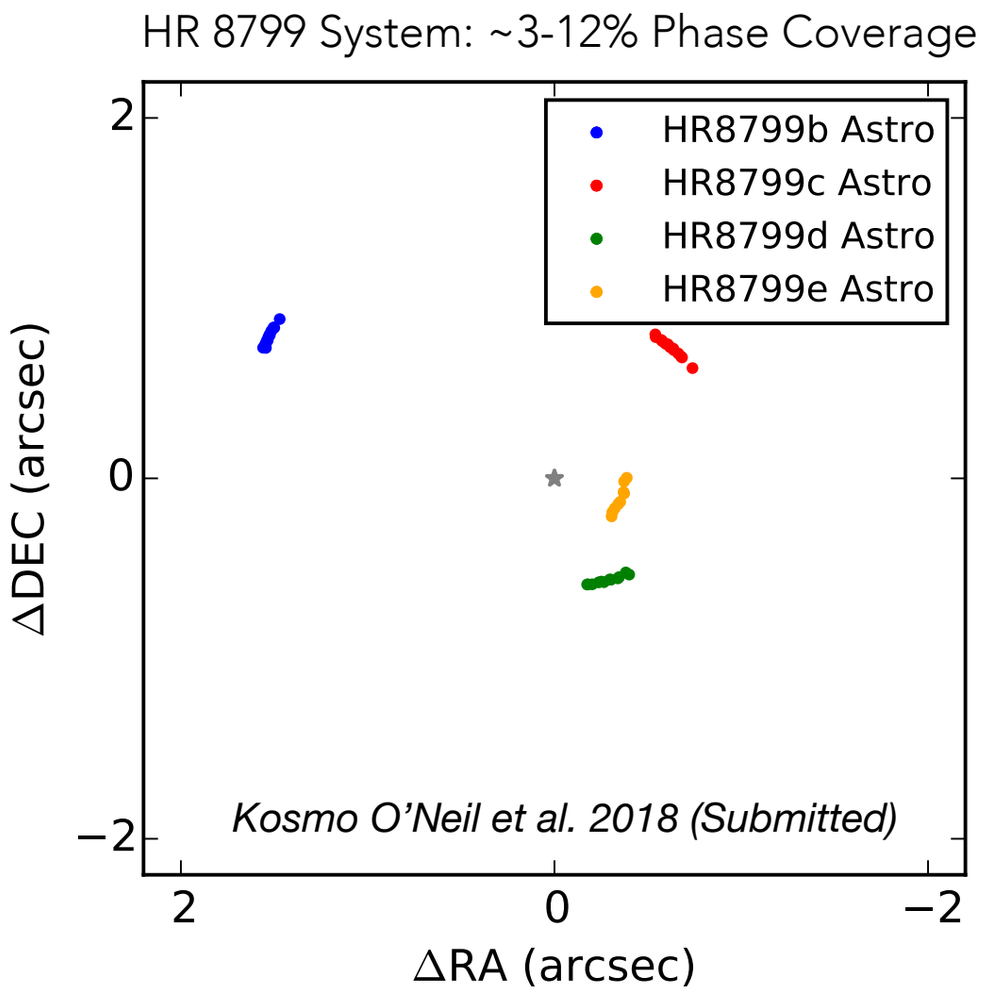
ExSoCal

See paper posted today! https://arxiv.org/labs/1809.05490

MANY DIRECTLY IMAGED PLANETS HAVE LOW ORBITAL PHASE COVERAGE

→ DATA CAN HAVE LOW CONSTRAINING POWER

→ PRIOR ASSUMPTIONS CAN BIAS STATISTICAL RESULTS



e.g. Marois+2008,2010; Chauvin+2012, Currie+2012, 2016; Esposito+2013; Maire+2015; Pueyo+2015; Rameau+2016; Konopacky+2016; Zurlo+2016; Wertz+2017; Wang+2018

BAYESIAN INFERENCE — WHEN DATA ARE NOT RIGOROUSLY CONSTRAINING, PRIORS HAVE MORE INFLUENCE

But we want data to contribute more information

Likelihood: encodes information from data

Prior: encodes knowledge of prior information

Priors add information!

$$P(\text{model} \mid \text{data}) = \frac{P(\text{data} \mid \text{model}) \times P(\text{model})}{P(\text{data})}$$

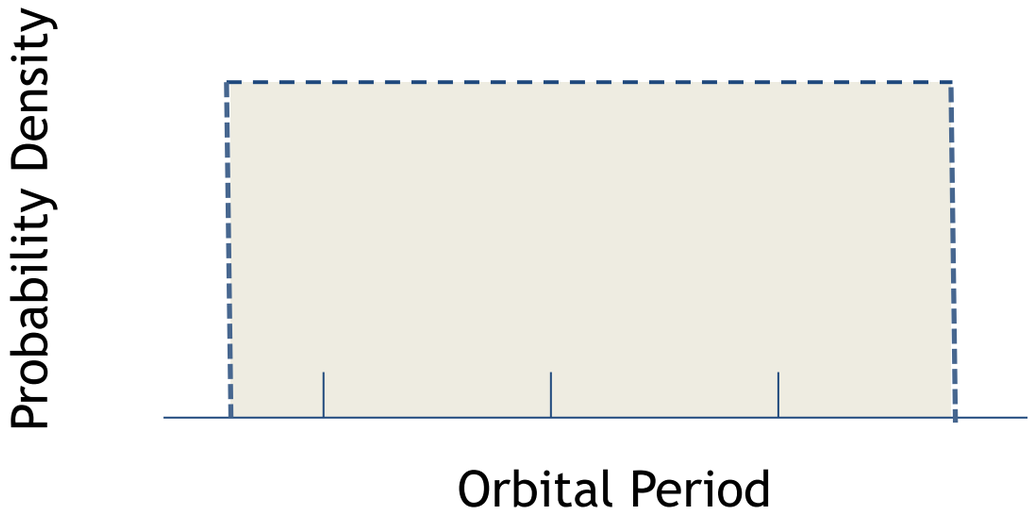
Posterior probability of model given data

Evidence: Probability of the data regardless of model

Want to add least subjective prior information possible!

TRADITIONAL METHOD: "UNIFORM PRIORS"

Sample from distribution uniform in model parameters

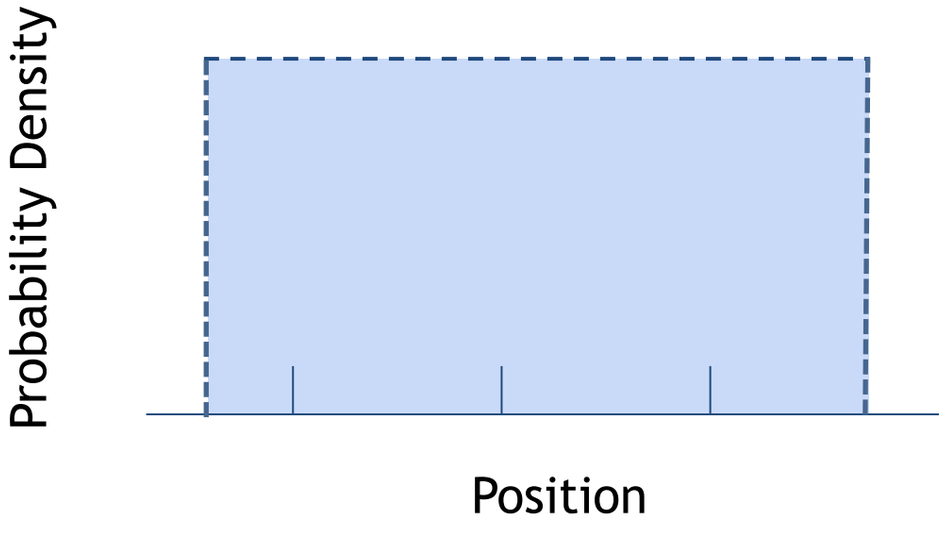


- Depends on parameterization!

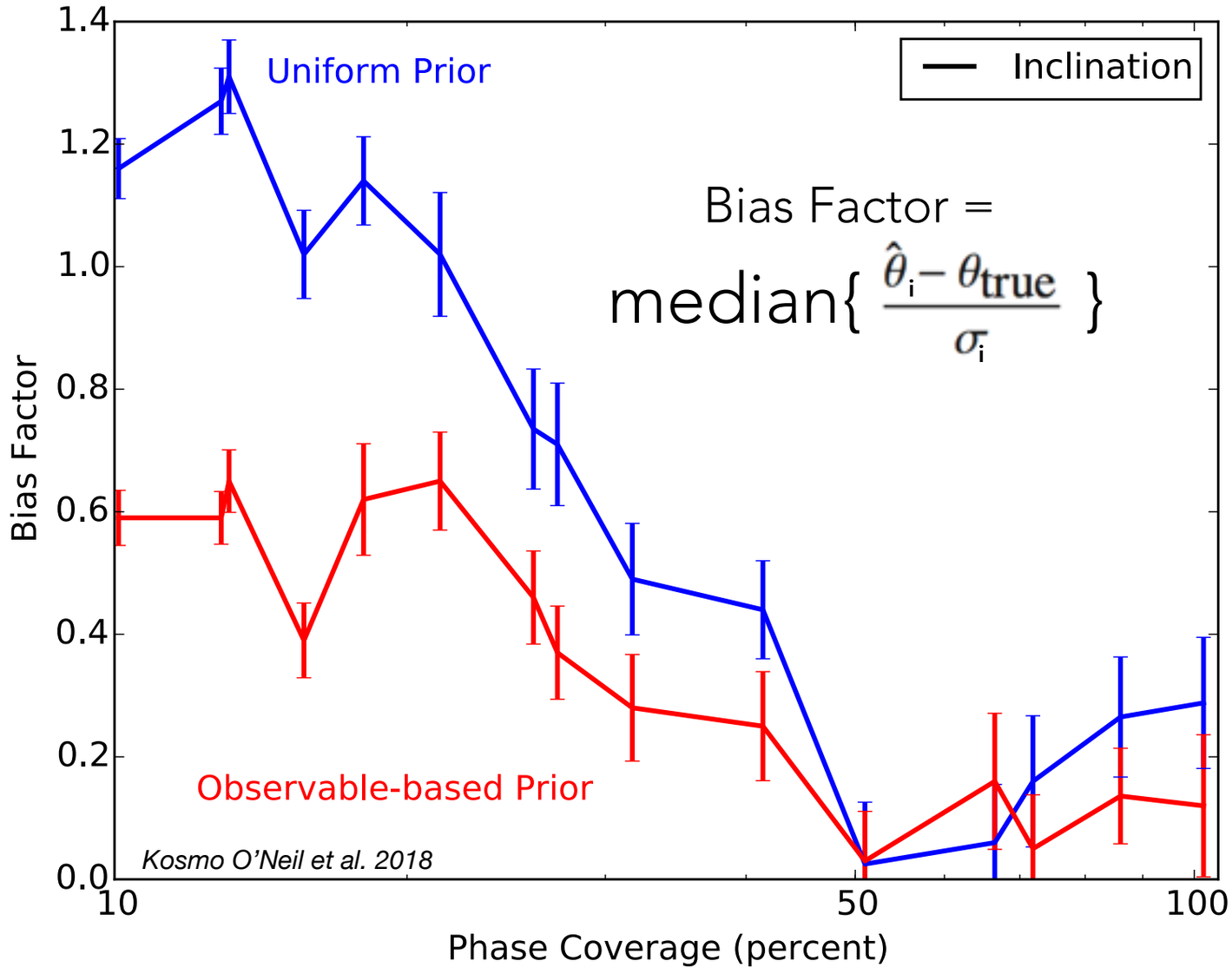
NEW METHOD: OBSERVABLE-BASED PRIORS:

Sample from distribution uniform in observables (sky positions)

- Allows all measurements to be equally likely before observation
- Less subjective
- **See paper posted today! https://arxiv.org/labs/1809.05490**



NEW PRIOR REDUCES BIAS IN FITTED PARAMETERS AT LOW PHASE COVERAGE

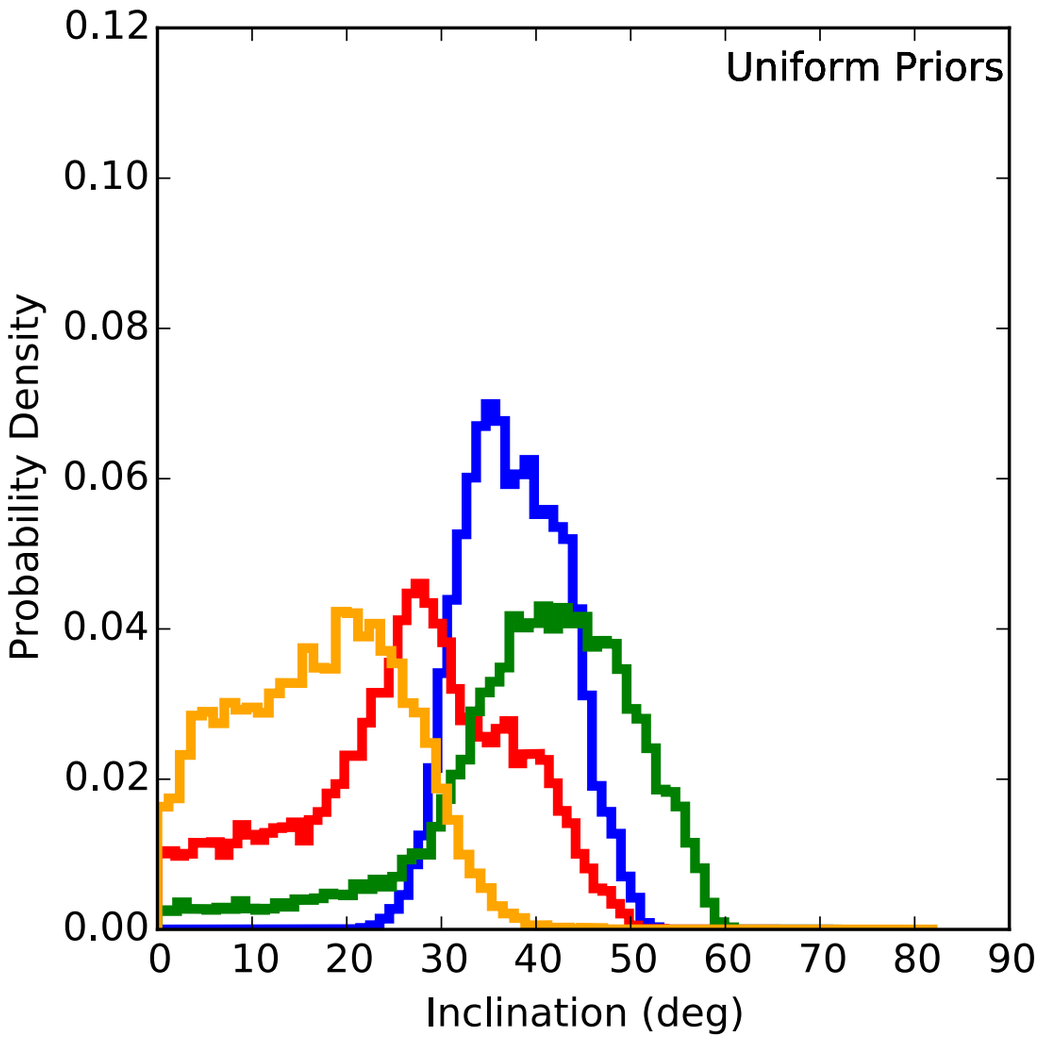


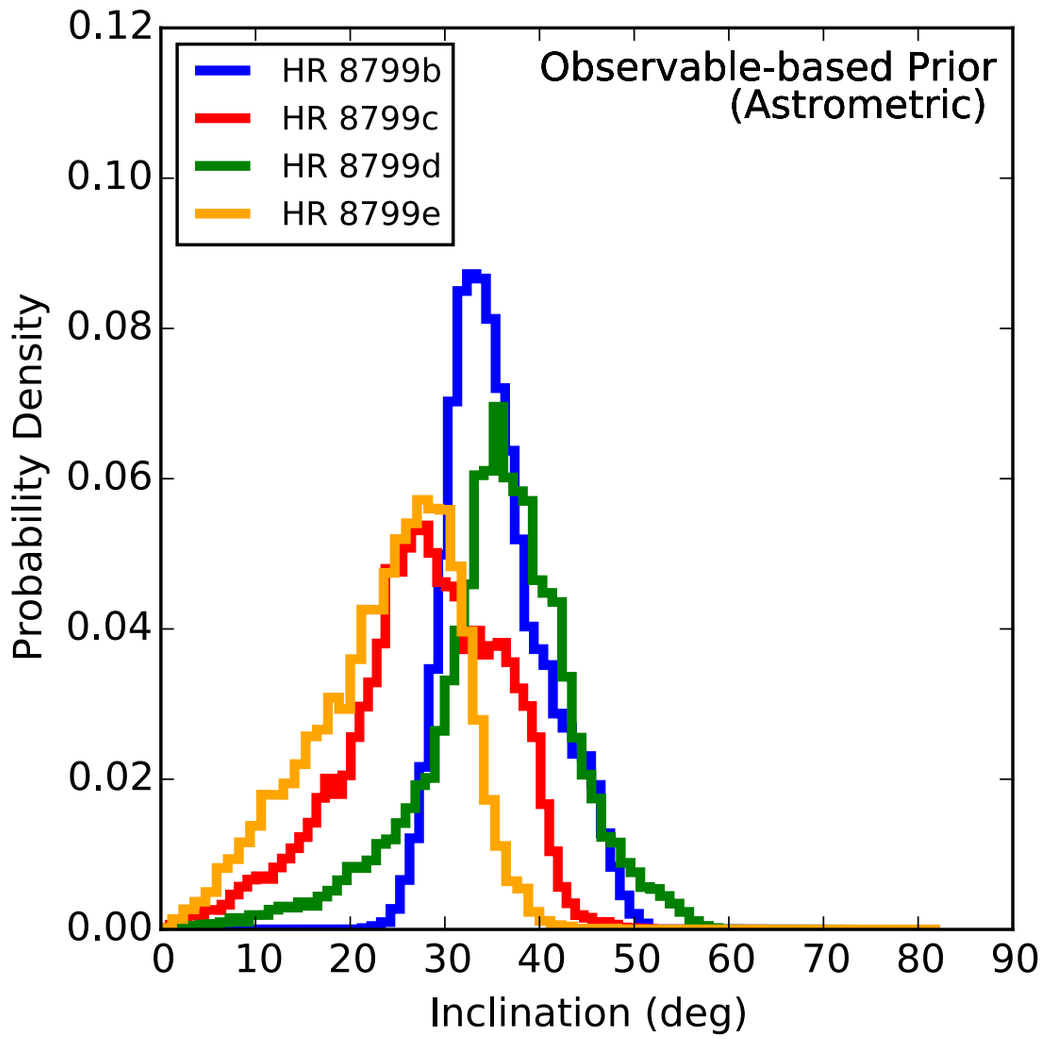
In addition, we look at *Information gained:*

Data contribute 25 — 35% more information over the prior for each of the HR 8799 planets

NEW PRIOR SHOWS STRONGER EVIDENCE FOR CONSISTENT INCLINATIONS OF THE HR 8799 PLANETS

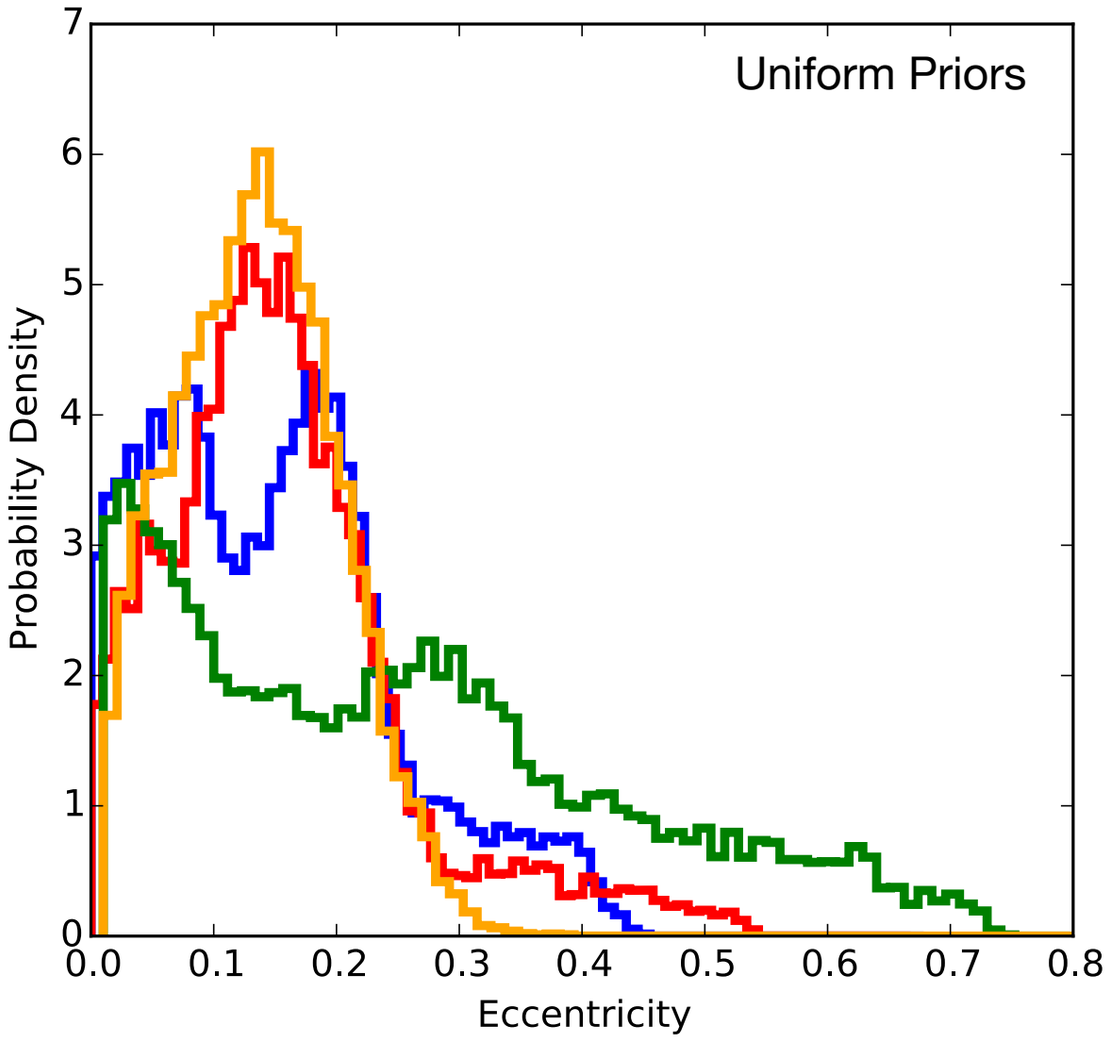
~30 deg to within 1-sigma

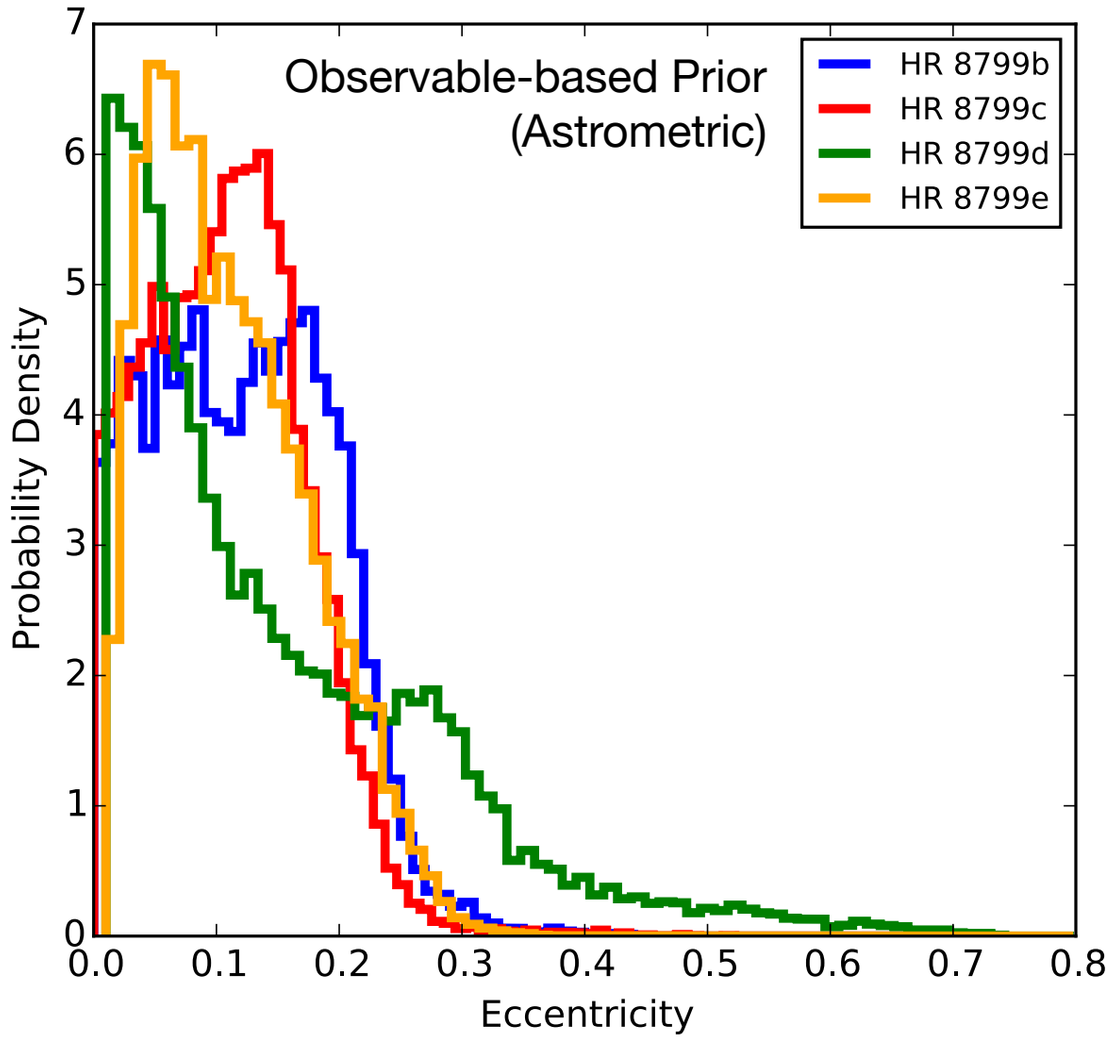




Kosmo O'Neil et al. 2018

NEW PRIOR SHOWS STRONGER EVIDENCE FOR POSSIBILITY OF NEARLY CIRCULAR ORBITS FOR THE HR 8799 PLANETS





Kosmo O'Neil et al. 2018

SUMMARY

- If data are not rigorously constraining (e.g. low phase coverage), prior assumptions can influence results and bias statistical quantities

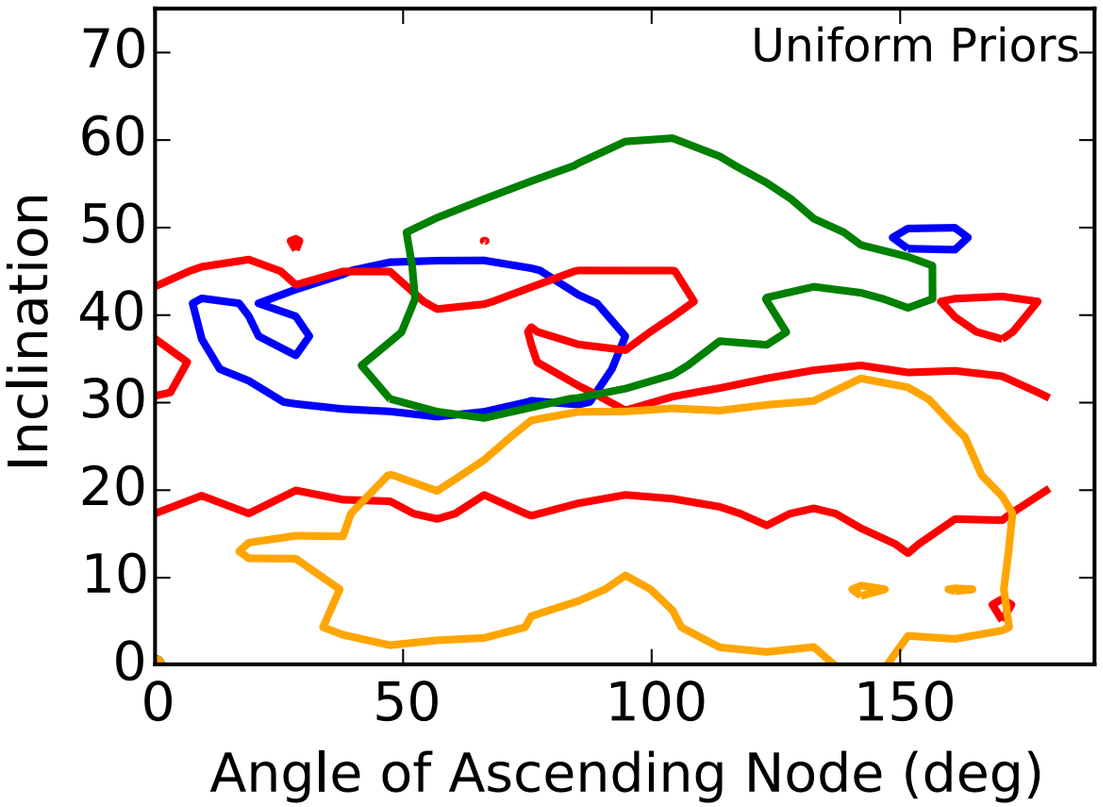
- Proposed solution: Assume uniformity in the observables rather than in inferred model parameters (less subjective prior)
    - Can help limit biases and increase data influence in regions of prior dominance

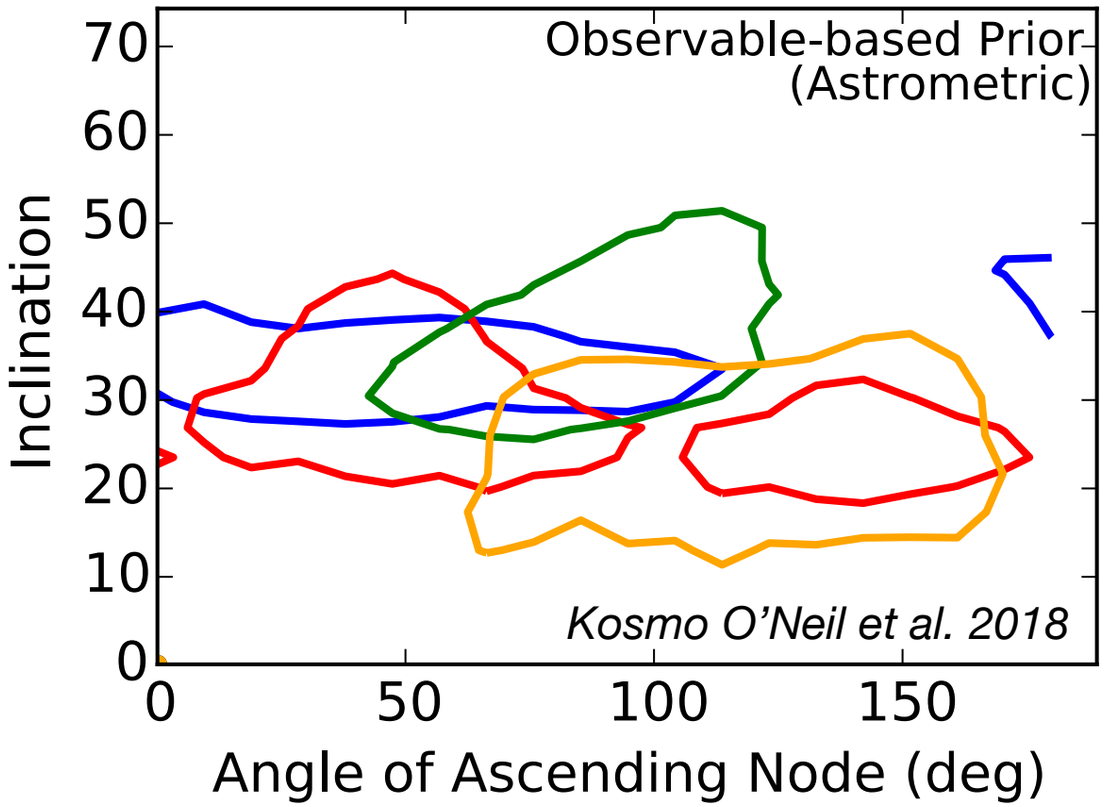
- Implications for HR 8799 orbit analysis:
    - Stronger evidence for consistent inclinations of the four planets
    - Stronger evidence for low eccentricity orbits

See paper posted today! https://arxiv.org/labs/1809.05490

EXTRAS

NEW PRIOR ALLOWS POSSIBILITY OF COPLANARITY





Future work: combine dynamical analyses of system stability (e.g. Wang+2018) with this prior analysis

EXPECTED INFORMATION GAINED:

AVERAGE RELATIVE ENTROPY BETWEEN POSTERIOR AND PRIOR = AVERAGE INFORMATION GAINED IN POSTERIOR OVER PRIOR

Relative Entropy: Kullback-Leibler Divergence (D_{KL})

$$\kappa \equiv \int d\mathscr{M}\, \mathscr{P}(\mathscr{M}|\mathscr{D}) \log\left[\frac{\mathscr{P}(\mathscr{M}|\mathscr{D})}{\mathscr{P}(\mathscr{M})}\right]$$

Average of relative entropy = Expected Information

→ Integrate over all possible data sets and maximize to find P(M):

$$\mathcal{I} \equiv \int d\mathscr{D}\, \mathscr{P}(\mathscr{D}) \int d\mathscr{M}\, \mathscr{P}(\mathscr{M}|\mathscr{D}) \log\left[\frac{\mathscr{P}(\mathscr{M}|\mathscr{D})}{\mathscr{P}(\mathscr{M})}\right]$$

$$= \int\int d\mathscr{D}\, d\mathscr{M}\, \mathscr{P}(\mathscr{M},\mathscr{D}) \log\left[\frac{\mathscr{P}(\mathscr{M},\mathscr{D})}{\mathscr{P}(\mathscr{D})\mathscr{P}(\mathscr{M})}\right]$$

$$= \int d\mathscr{M}\, \mathscr{P}(\mathscr{M}) \int d\mathscr{D}\, \mathscr{P}(\mathscr{D}|\mathscr{M}) \log\left[\frac{\mathscr{P}(\mathscr{D}|\mathscr{M})}{\mathscr{P}(\mathscr{D})}\right]$$

The sharper the prior, the less information gained in the posterior over the prior → Allow data to contribute maximally to posterior estimates

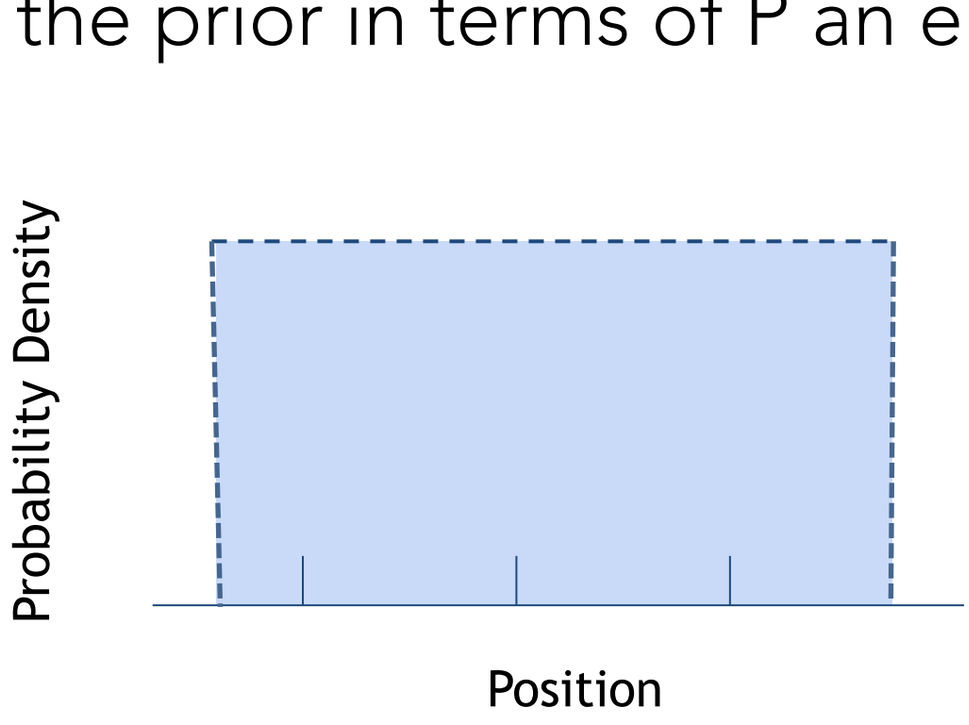
EXPECTED INFORMATION GAINED IN THE POSTERIOR OVER THE PRIOR FOR THE HR 8799 PLANETS.

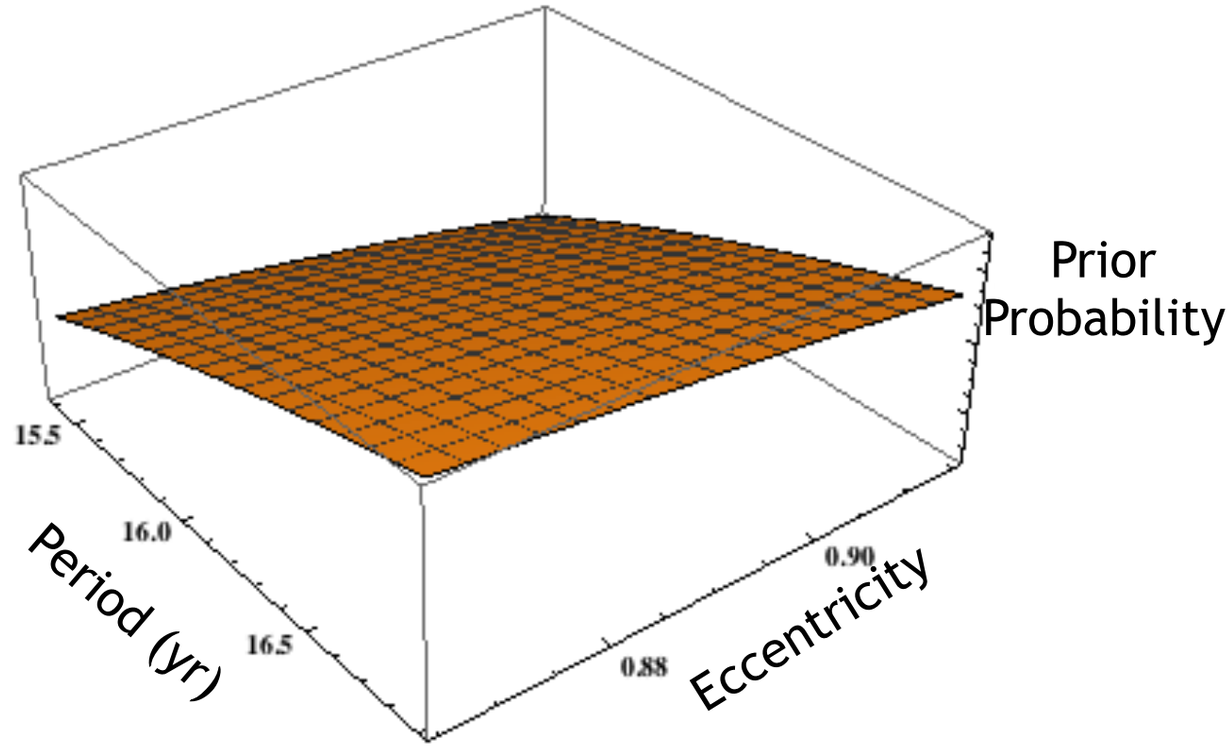
Planet	Uniform Prior (Commonly Assumed)	Observable-based Prior	Percent Increase
HR 8799b	16.4 ± 0.1	21.2 ± 0.1	29.3 ± 0.1 %
HR 8799c	16.7 ± 0.1	21.0 ± 0.1	25.7 ± 0.1 %
HR 8799d	16.1 ± 0.2	21.7 ± 0.3	34.8 ± 0.4 %
HR 8799e	14.0 ± 0.1	18.6 ± 0.4	32.9 ± 0.4 %

OBSERVABLES-BASED PRIOR –

SAMPLE FROM DISTRIBUTION UNIFORM IN OBSERVABLES

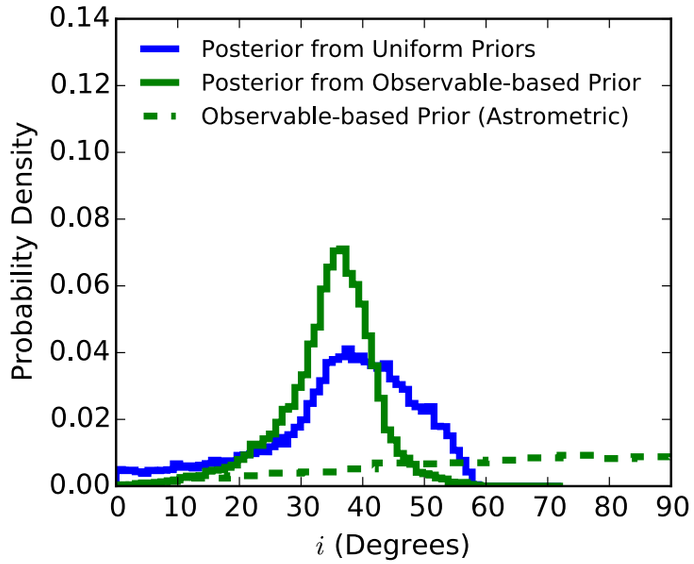
- Transform from observables space (sky positions + radial velocities) to model parameter space (Period + eccentricity)
- Sum Jacobian determinant over all epochs to produce form of the prior in terms of P an e

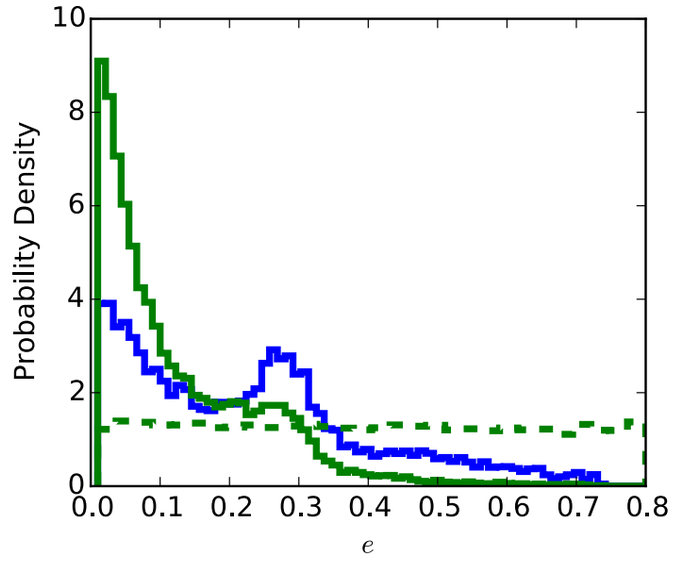


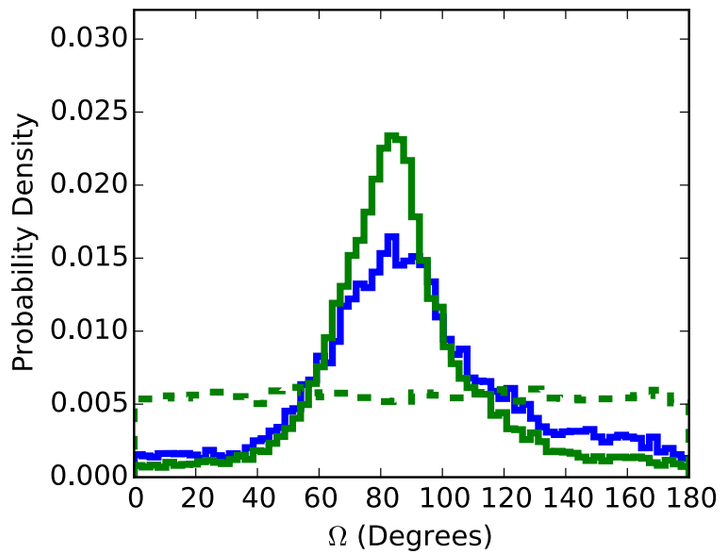


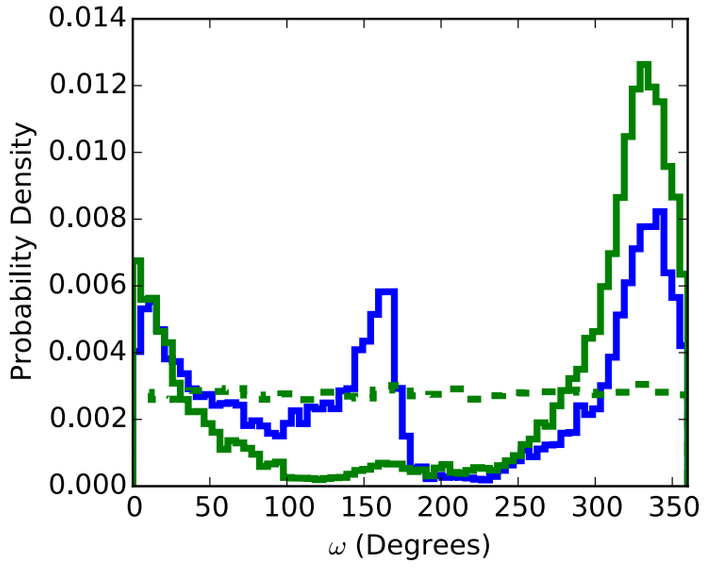
- Observables-based priors = less subjective

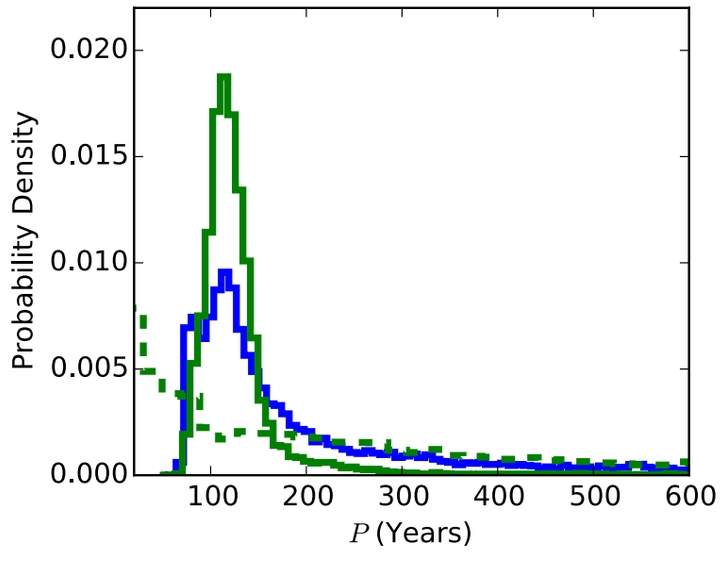
HR 8799d

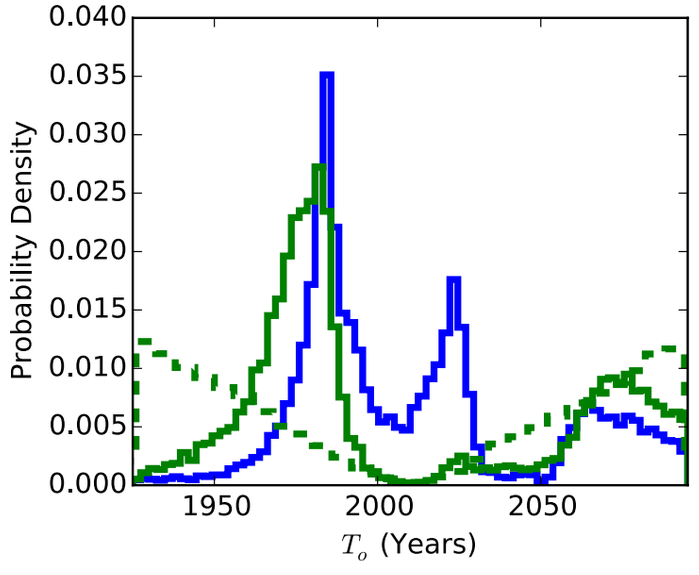

Posterior from Uniform Priors
Posterior from Observable-based Prior
Observable-based Prior (Astrometric)
Probability Density
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
70
80
90
i (Degrees)


Probability Density
10
8
6
4
2
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
e

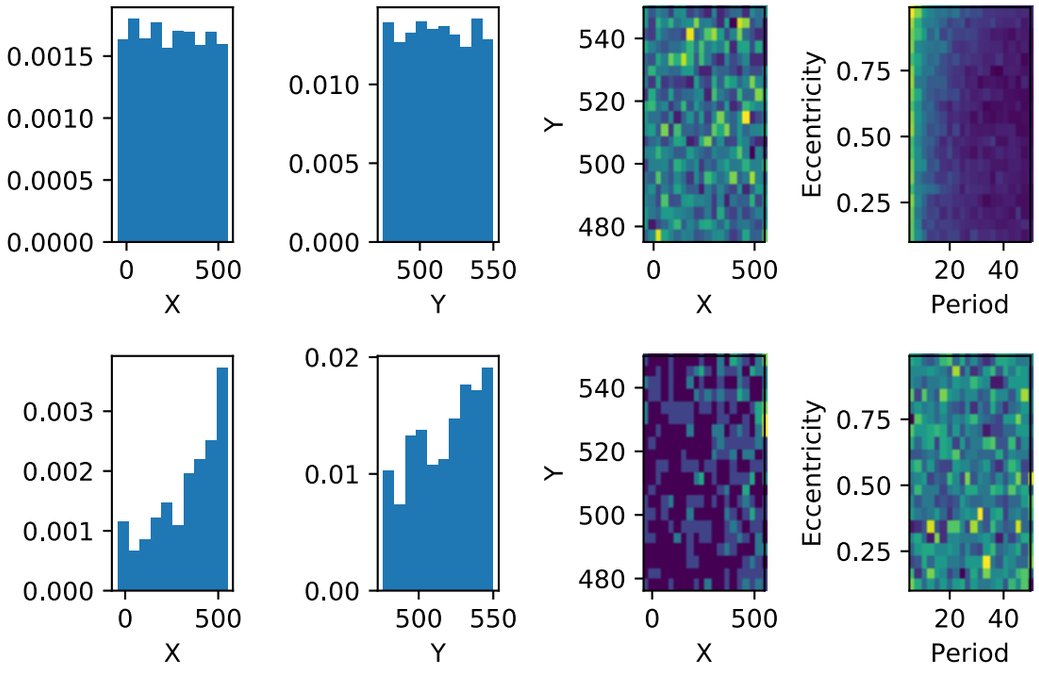

Probability Density
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0
20
40
60
80
100
120
140
160
180
Ω (Degrees)


Probability Density
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
50
100
150
200
250
300
350
ω (Degrees)


Probability Density
0.020
0.015
0.010
0.005
0.000
100
200
300
400
500
600
P (Years)


Probability Density
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1950
2000
2050
T_o (Years)

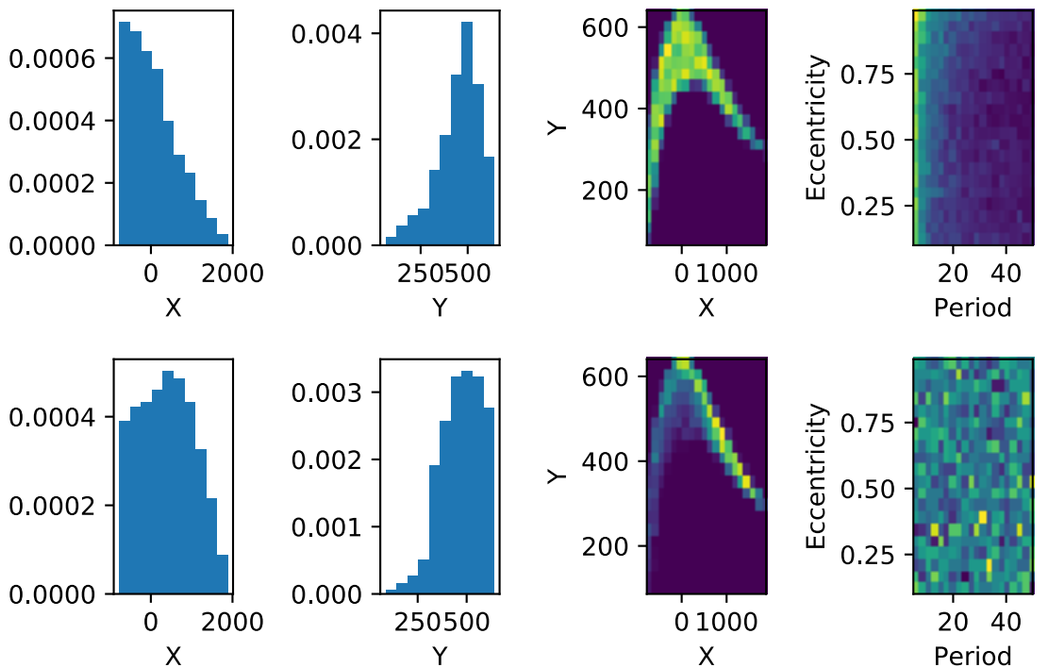
For a single date with T_o fixed



Observables-based prior (astrometric)

Uniform Priors

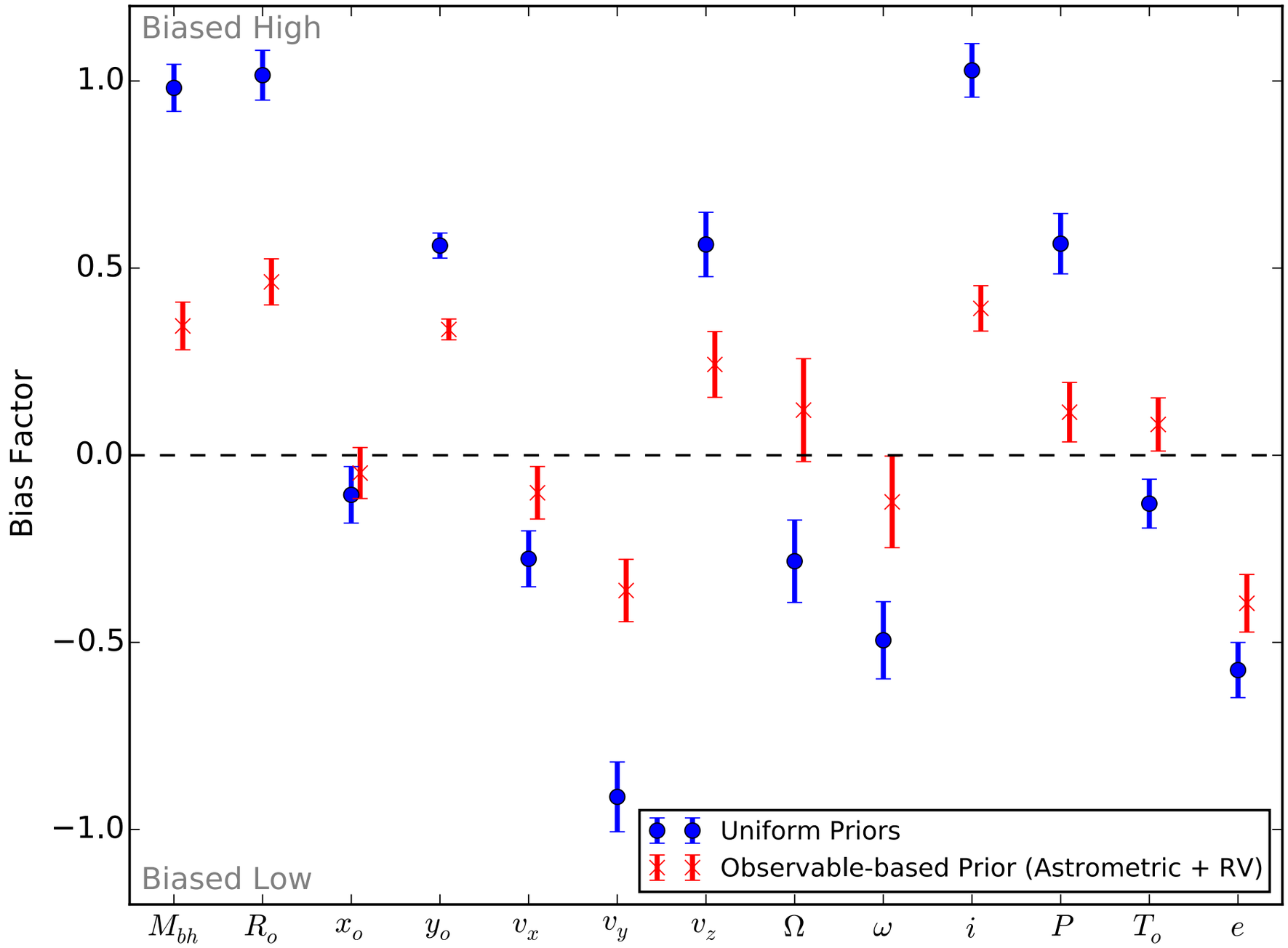
For a single date with T_o fixed, full parameter space



Observables-based prior (astrometric)

Uniform Priors

Simulation with 16% phase coverage



OUTLINE

- If data are not rigorously constraining (e.g. low phase coverage), prior assumptions can influence results and bias statistical quantities

- We propose a new observable-based prior that limits biases and increases data influence in regions of prior dominance

- Implications for HR 8799 orbit analysis:

    - Stronger evidence for consistent inclinations of the four planets

    - Stronger evidence for low eccentricity orbits

    - Allows possibility of coplanarity