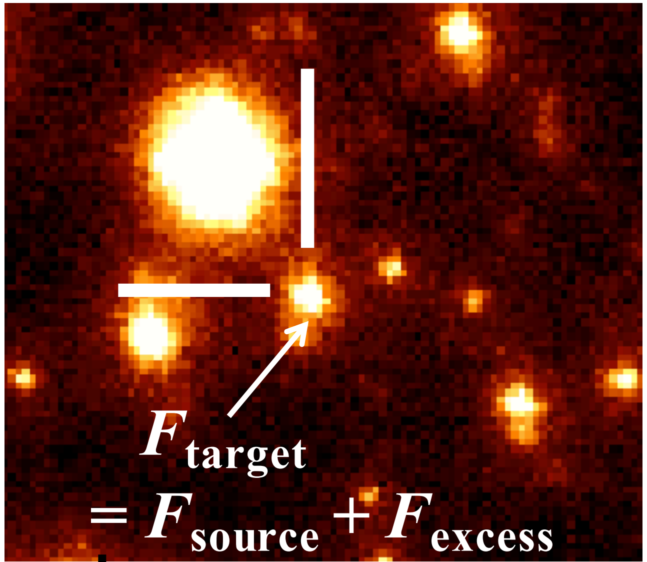
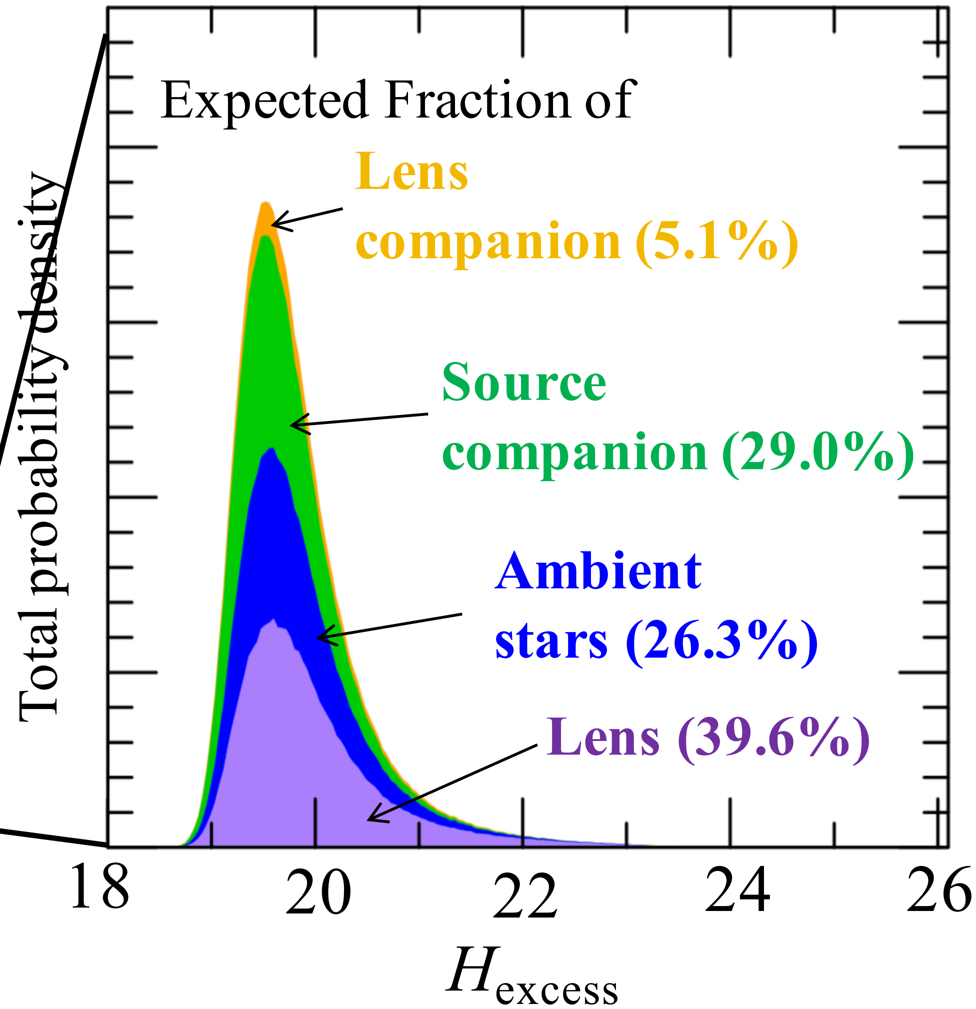
MOA-2016-BLG-227Lb: Possible Most Massive Planet around K-Dwarf in the Galactic Bulge

Keck Image of MOA-2016-BLG-227 provided by Y. Shvartzvald et al.



$\underline{H_{excess} = 19.50 \pm 0.36}$



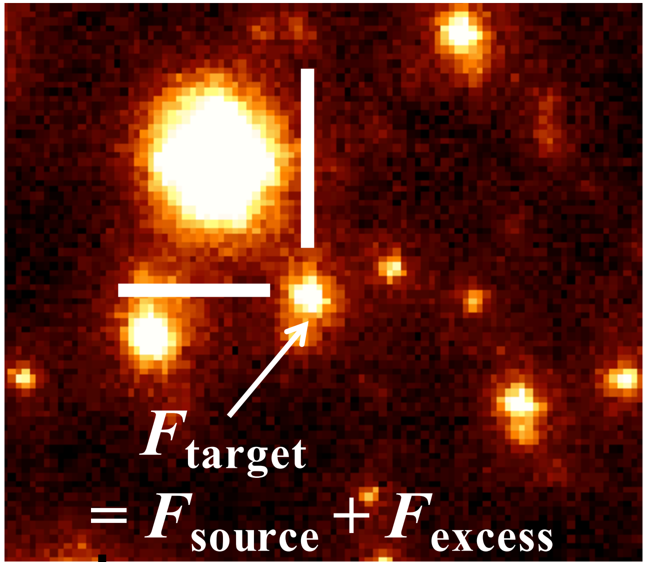
Naoki Koshimoto

Osaka University/NASA GSFC

MOA-2016-BLG-227Lb: Possible Most Massive Planet around K-Dwarf in the Galactic Bulge or Not

-A New Approach to Evaluate Contamination Probabilities-

Keck Image of MOA-2016-BLG-227
provided by Y. Shvartzvald et al.



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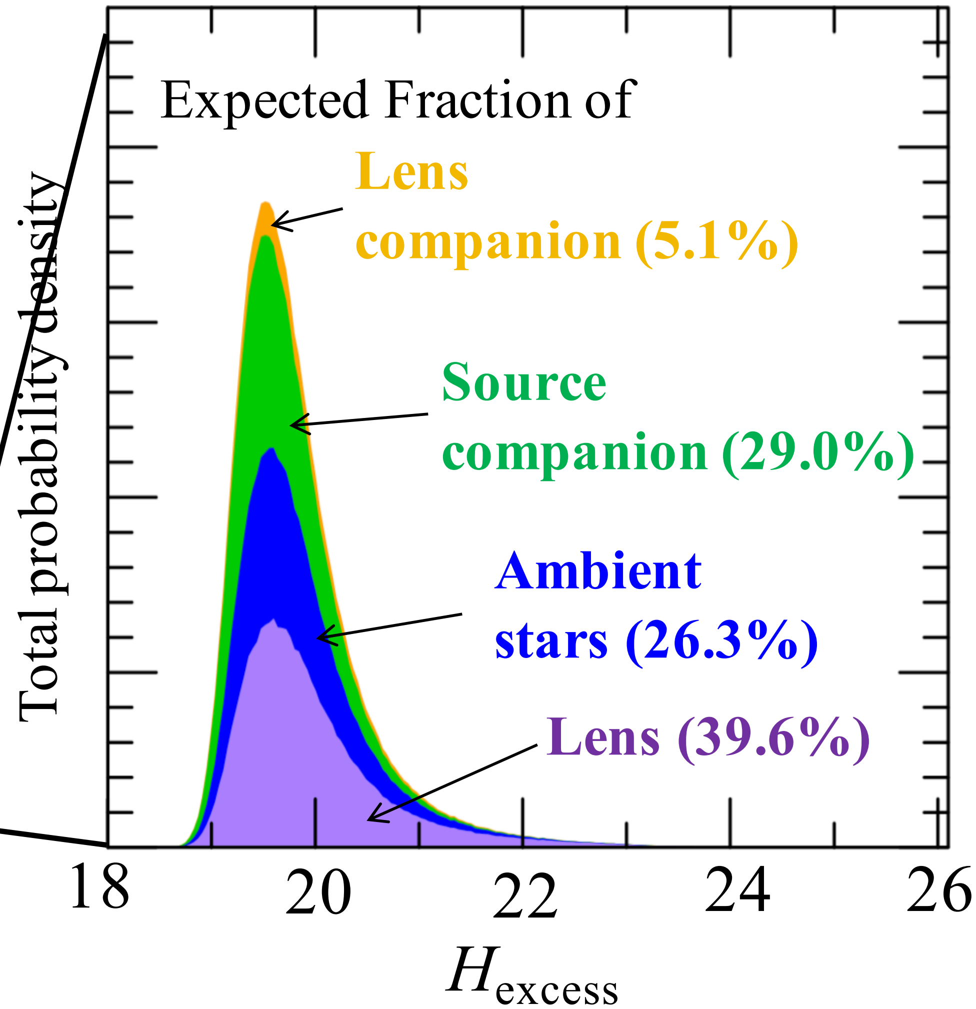


Naoki Koshimoto
Osaka University/NASA GSFC

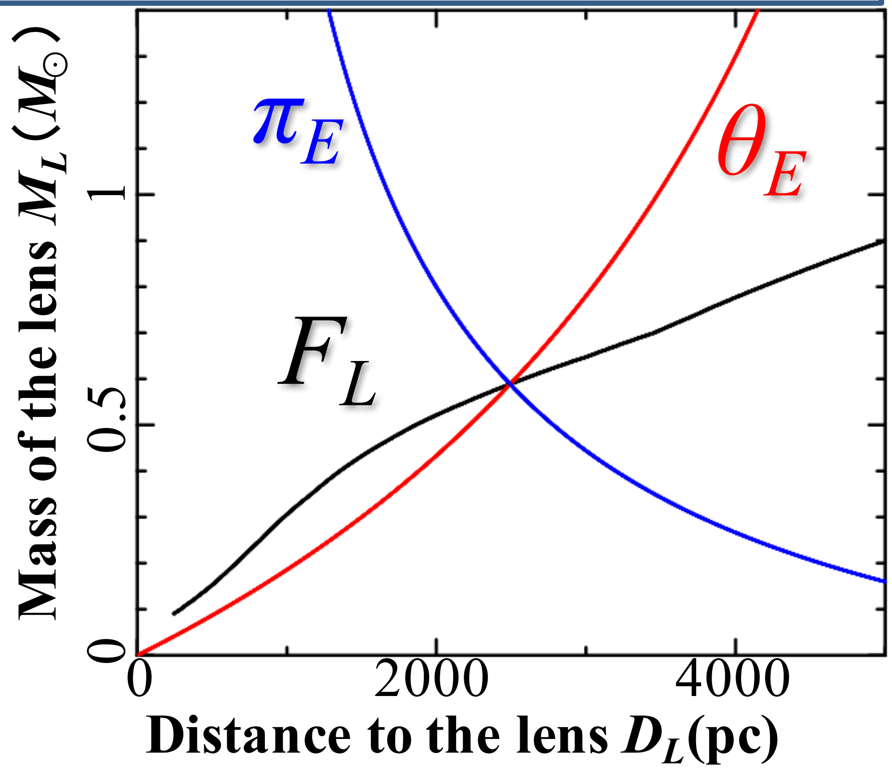
Lens Flux Is Useful

Mass-Distance relations

Angular Einstein radius	$\theta_E(M_L, D_L)$	←	Finite source effects
Microlensing Parallax	$\pi_E(M_L, D_L)$	←	Parallax effects
Lens Flux	$F_L(M_L, D_L)$	←	Observations with high angular resolution

We can measure the lens mass if two of the three observables are obtained.

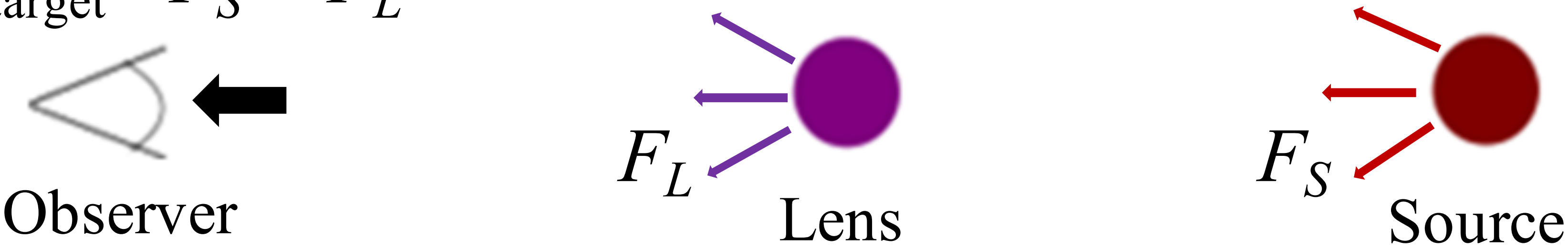
The lens flux can be obtained even after the end of the event!



How to Get The Lens Flux

Optimistically

$$F_{\mathrm{target}} = F_S + F_L$$

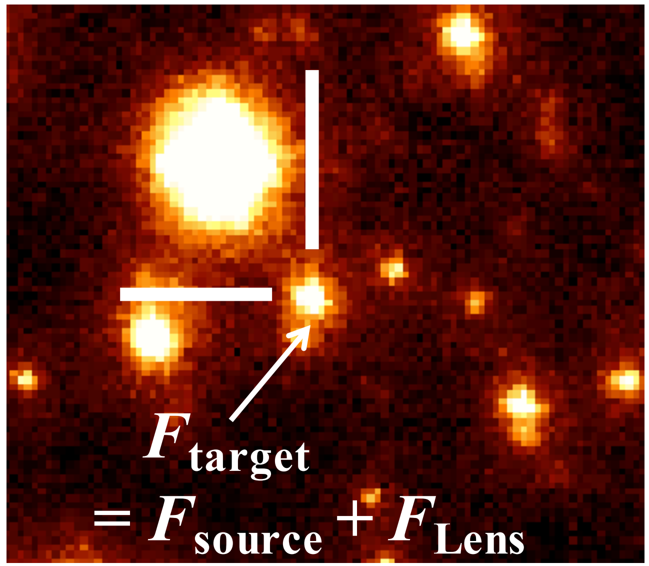


Lens Flux

$$F_L = F_{\mathrm{target}} - F_S$$

↑ from fitting

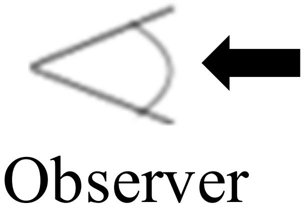
Keck Image of MOA-2016-BLG-227 provided by Y. Shvartzvald et al.

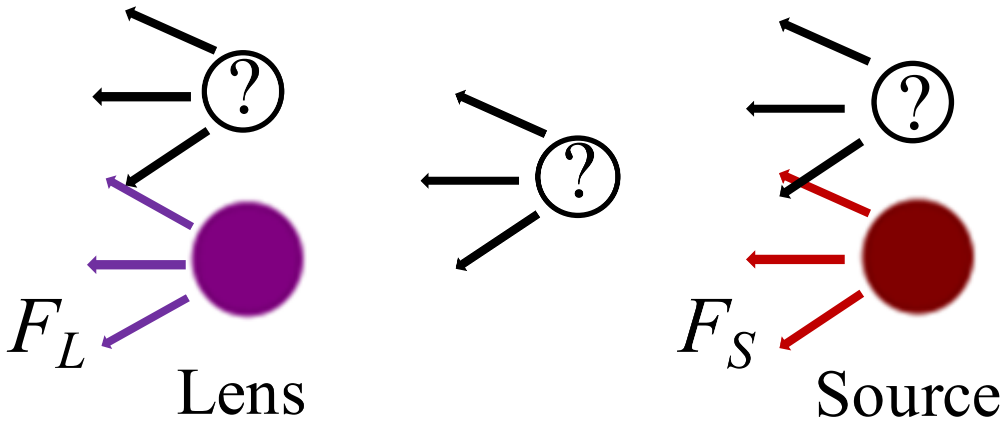


How to Get The Lens Flux

w/o wishful thinking

$$F_{\text{target}} = F_S + F_{\text{excess}}$$





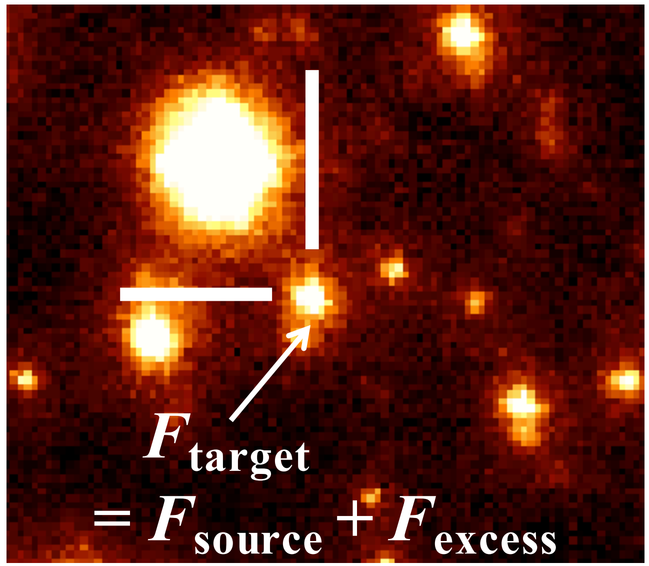
Excess Flux

$$F_{\text{excess}} = F_{\text{target}} - F_S$$

↑ from fitting

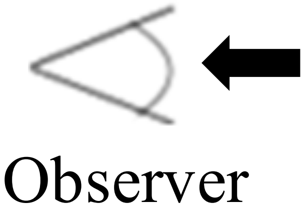
→ **We need to evaluate the probability of $F_{\text{excess}} = F_L$!!**

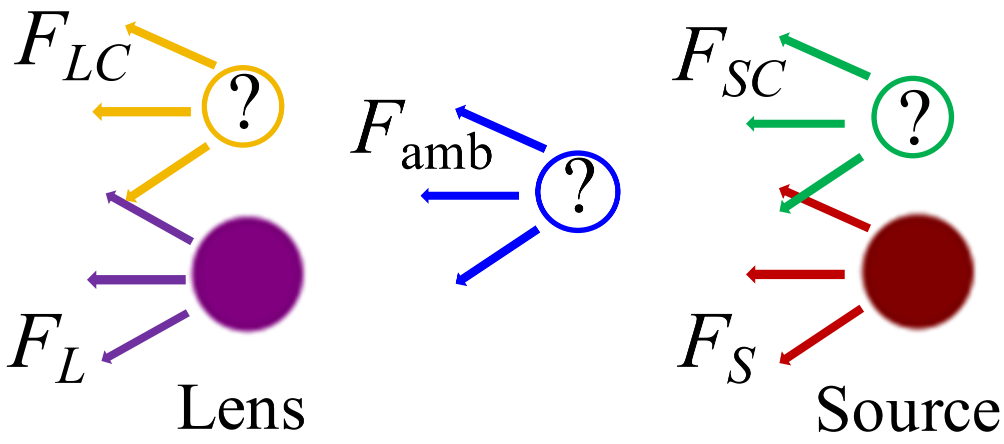
Keck Image of MOA-2016-BLG-227 provided by Y. Shvartzvald et al.



Four Possibilities for The Origin of The Excess

$$F_{\text{target}} = F_S + F_{\text{excess}}$$



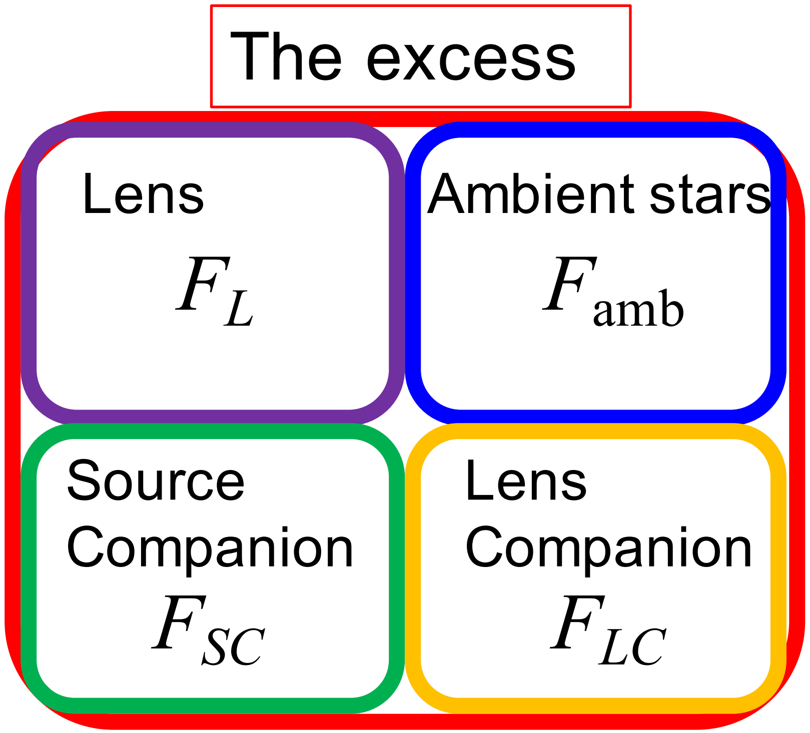


Excess Flux

$$F_{\text{excess}} = F_{\text{target}} - F_S$$

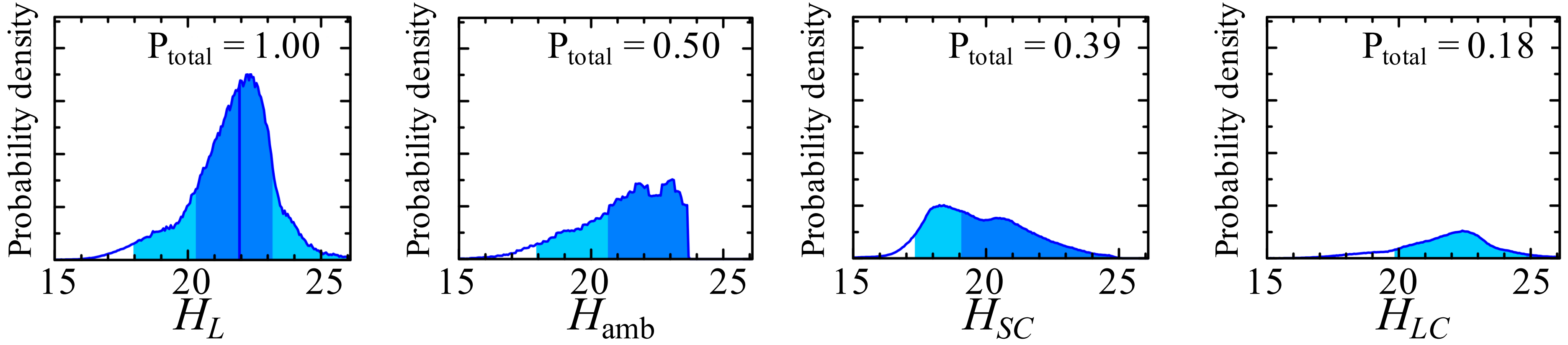
↑ from fitting

→ **We need to evaluate the probability of $F_{\text{excess}} = F_L$!!**



How to Evaluate The Possible Contaminations -New approach through Bayesian analysis-

1. Calculate prior probability distributions of the four possibilities



from
Bayesian analysis with t_E, θ_E and the Galactic model

from
Number density,
Luminosity function,
FWHM of the image.

from
Binary distribution for nearby stars,
θ_E (for closer limit)
FWHM (for wider limit)

from
Binary distribution for nearby stars,
θ_E, u_0 (for closer limit)
FWHM (for wider limit)

2. Pick a number of combinations of H_L, H_{amb}, H_{SC} and H_{LC} out of the prior distributions (if you have the excess flux in H-band).

3. Calculate $F_{excess} = F_L + F_{amb} + F_{SC} + F_{LC}$ for each combination and extract combinations which is consistent with the observed excess flux.

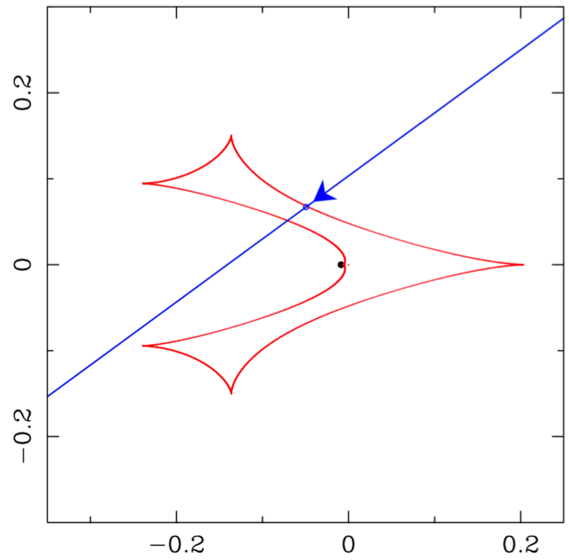
MOA-2016-BLG-227

- In K2C9 footprint (not superstamp)
- No parallax effect is detected
- Finite source effect is detected

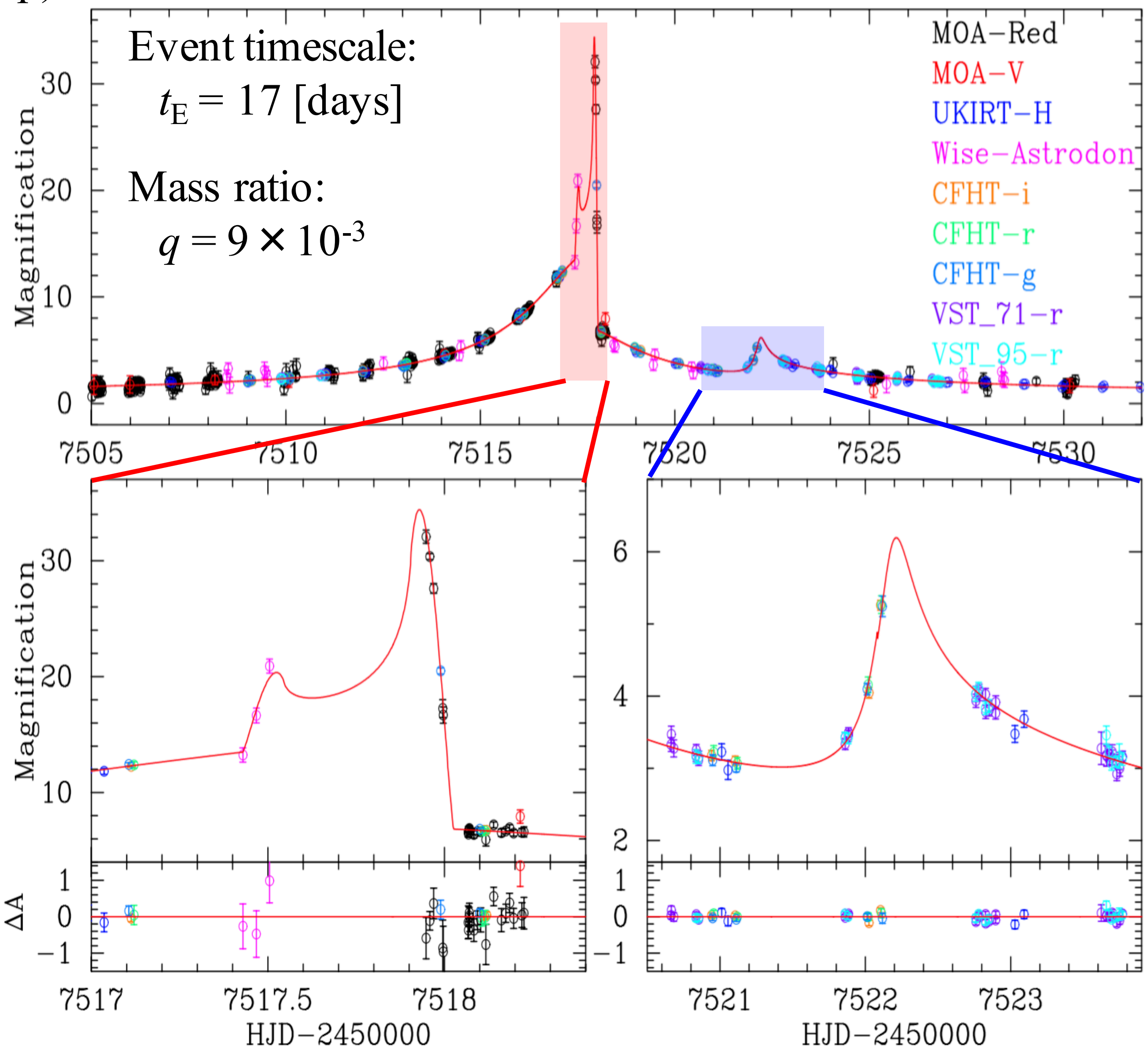
Angular Einstein radius:
$\theta_E = 0.23$ [mas]

- Observed by Keck with AO

Magnitude of excess flux :
$H_{excess} = 19.50 \pm 0.36$



Binary fit with finite source effect



Assumptions for The Prior Probability Distributions

We used similar prior probability distributions to those of the previous papers (e.g., Janczak+10, Batista+14, Fukui+15, Koshimoto+16),

Except that we used the prior for the lens flux as well.

Table 1. Assumptions and undetectable limits used for the prior distributions

Prior probability for	Assumption	Closer limit	Wider limit	Used observed value	Paper for the assumption
H_L (Lens)	Galactic model	–	–	t_E, θ_E	Han & Gould (2003)
H_{amb} (Ambient stars)	Luminosity function	–	0.8 FWHM	FWHM, Number density	Zoccali et al. (2003)
H_{SC} (Source companions)	Binary distribution	$\theta_E/4$	0.8 FWHM	FWHM, θ_E, H_S	Duchêne & Kraus(2013)
H_{LC} (Lens companions)	Binary distribution	w_c[1] $< u_0$	0.8 FWHM	FWHM, θ_E, H_L, u_0	Duchêne & Kraus(2013)

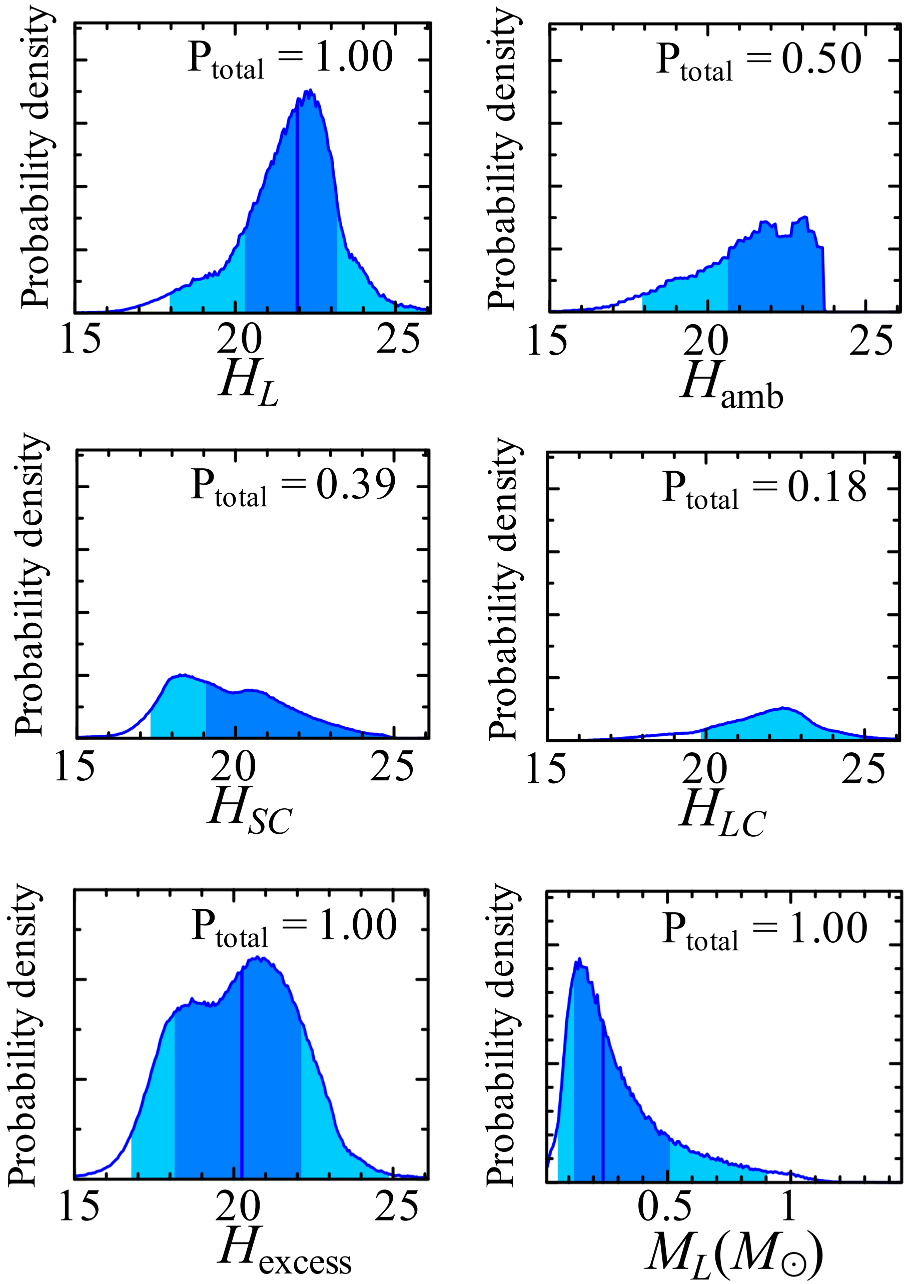
[1] The size of the caustic created by the hypothetical companion to the lens, $w_c = 4q_c/(s_c - s_c^{-1})^2$.

Table 2. Details of the binary distribution (Duchéne & Kraus 2013)

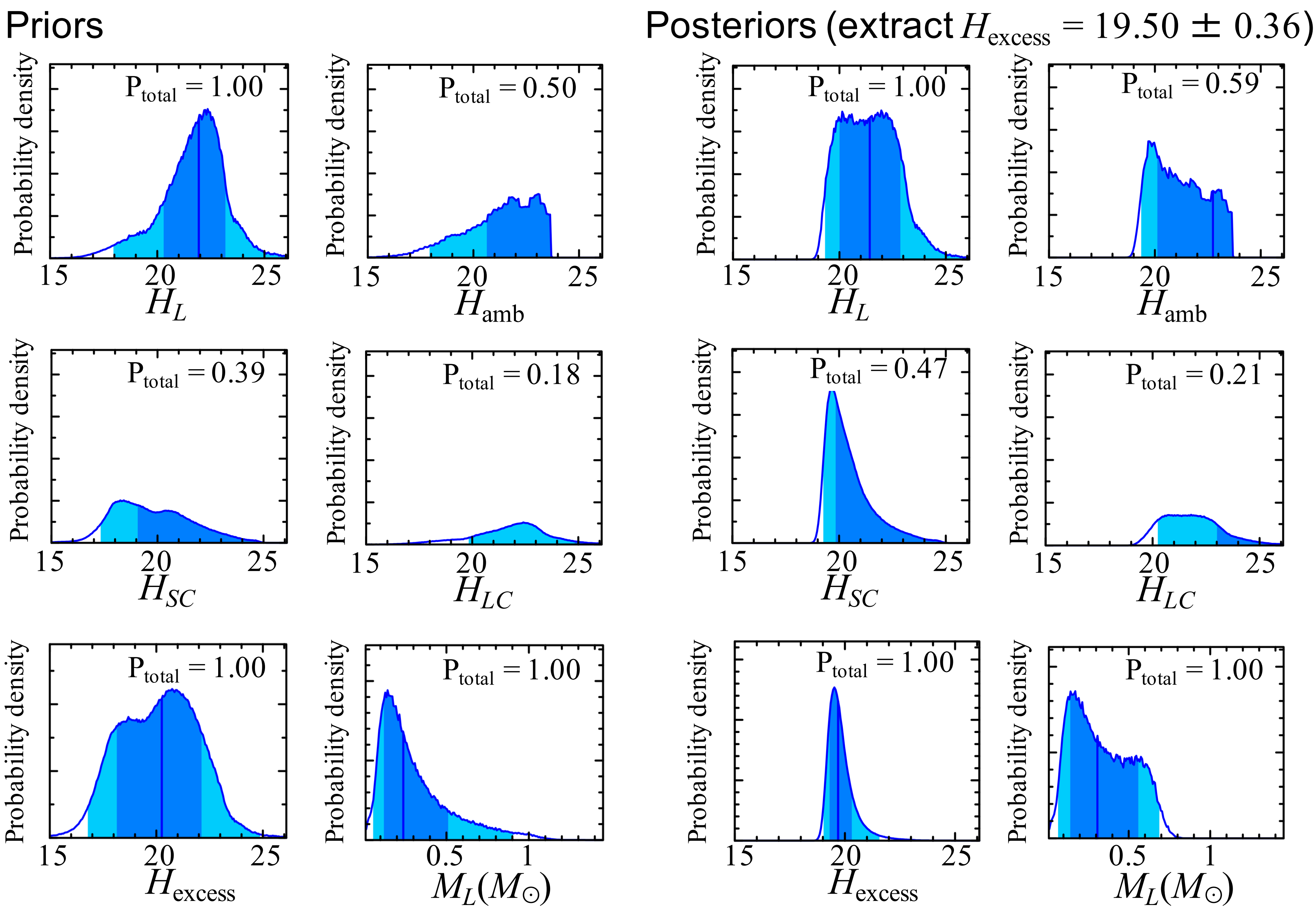
Parameter	Formula	Primary mass dependency
Multiplicity Fraction MF	-	$MF = 0.20 + 0.26 \times M$
Semi-major axis a	log normal, $N(\mu_{\log a}, \sigma^2_{\log a})$	$\mu_{\log a} = 0.57 + 1.01 \times M$
		$\sigma_{\log a} = 1.6 + 1.2 \times \log M$
Mass ratio q for $\log a < \mu_{\log a}$	Power law, $\propto q^{\gamma_c}$	$\gamma_c = 1.2 - 2.8 \times \log M$
Mass ratio q for $\log a > \mu_{\log a}$	Power law, $\propto q^{\gamma_w}$	$\gamma_w = 0$ (for $M > 0.34$)
		$\gamma_w = -3.1 - 6.7 \times \log M$ (for $M < 0.34$)

Results of The New Approach for MOA-2016-BLG-227

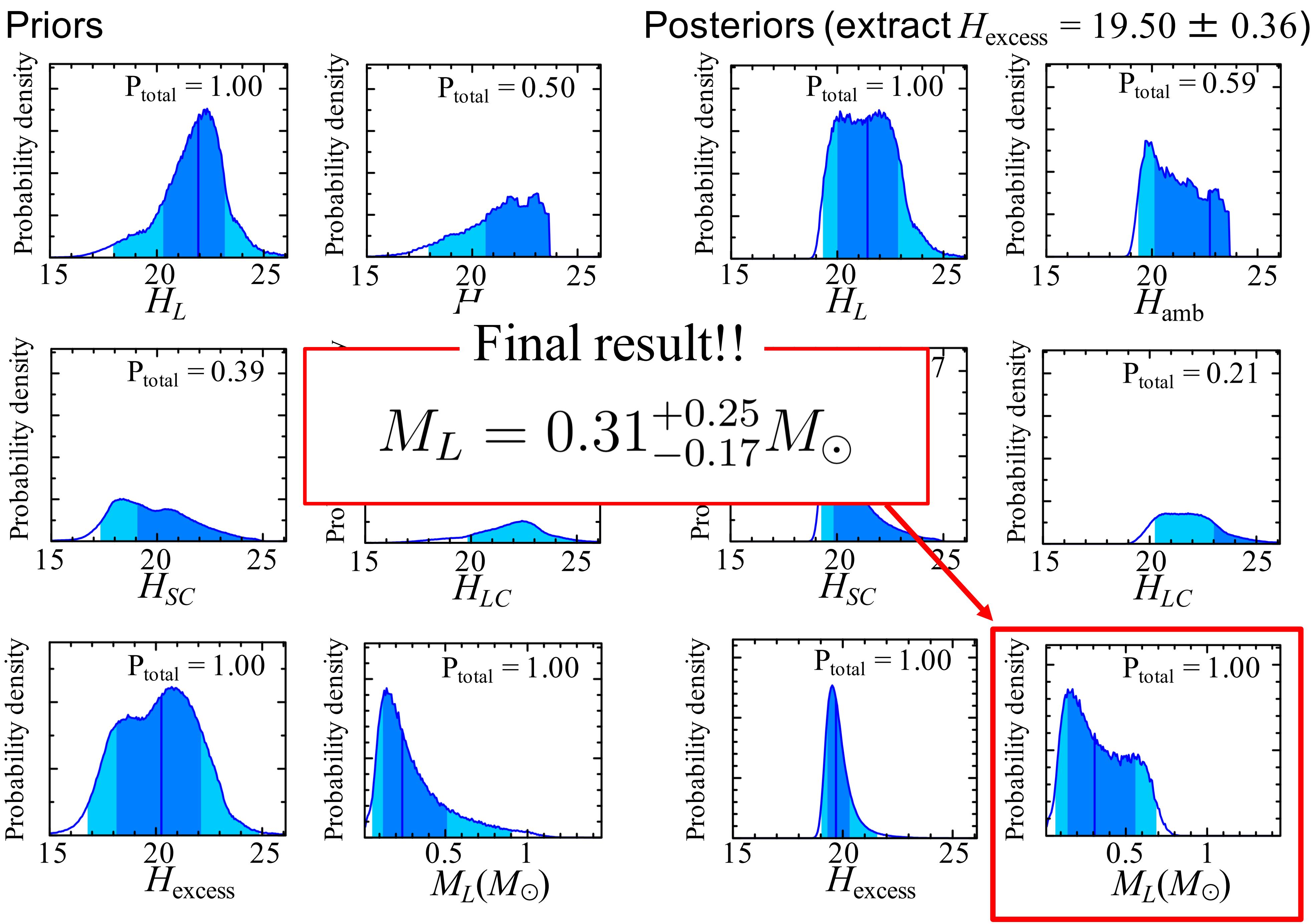
Priors



Results of The New Approach for MOA-2016-BLG-227

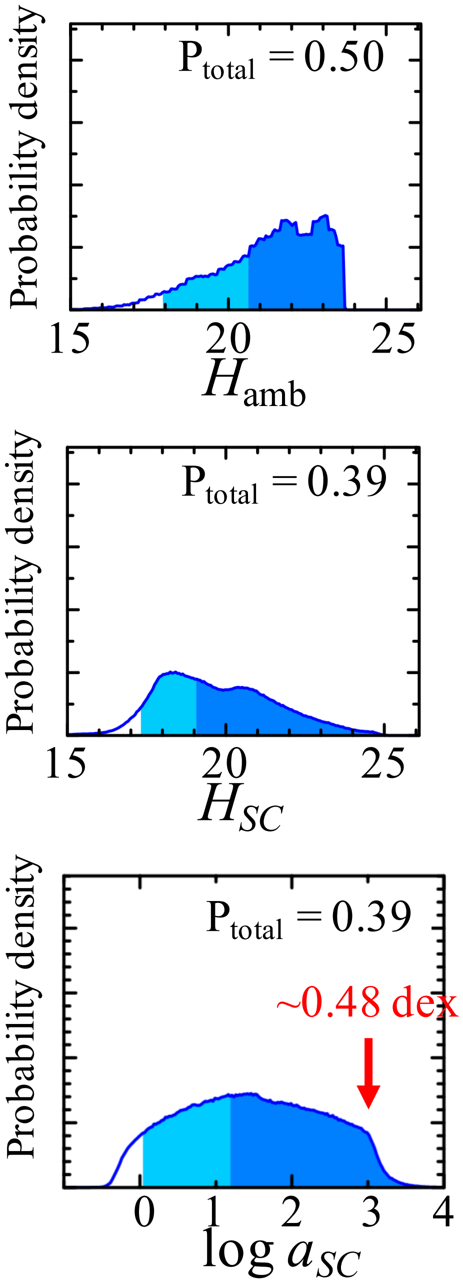


Results of The New Approach for MOA-2016-BLG-227



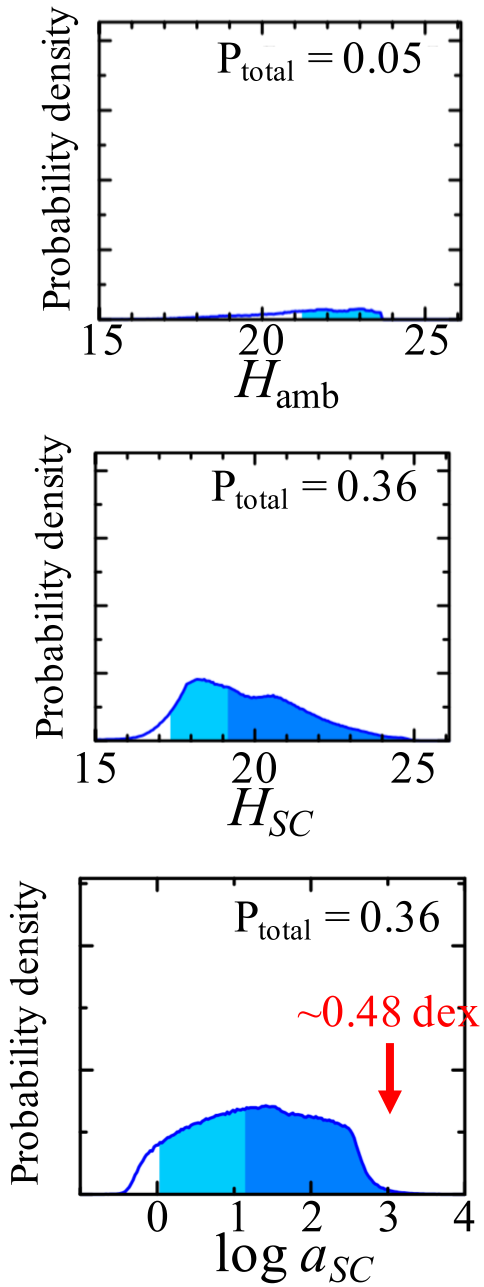
Comparison with Better FWHM Value

Priors (FWHM = 184 mas)



~ 1/3 FWHM

Priors (FWHM = 60 mas)



Excluded
($P_{amb} \propto \mathrm{FWHM}^2$)

Almost same!!
(1/3 FWHM
<=> ~0.48 dex)

The distribution of a is "log normal" !!

Comparison of the Results with Other Priors of P_{host}

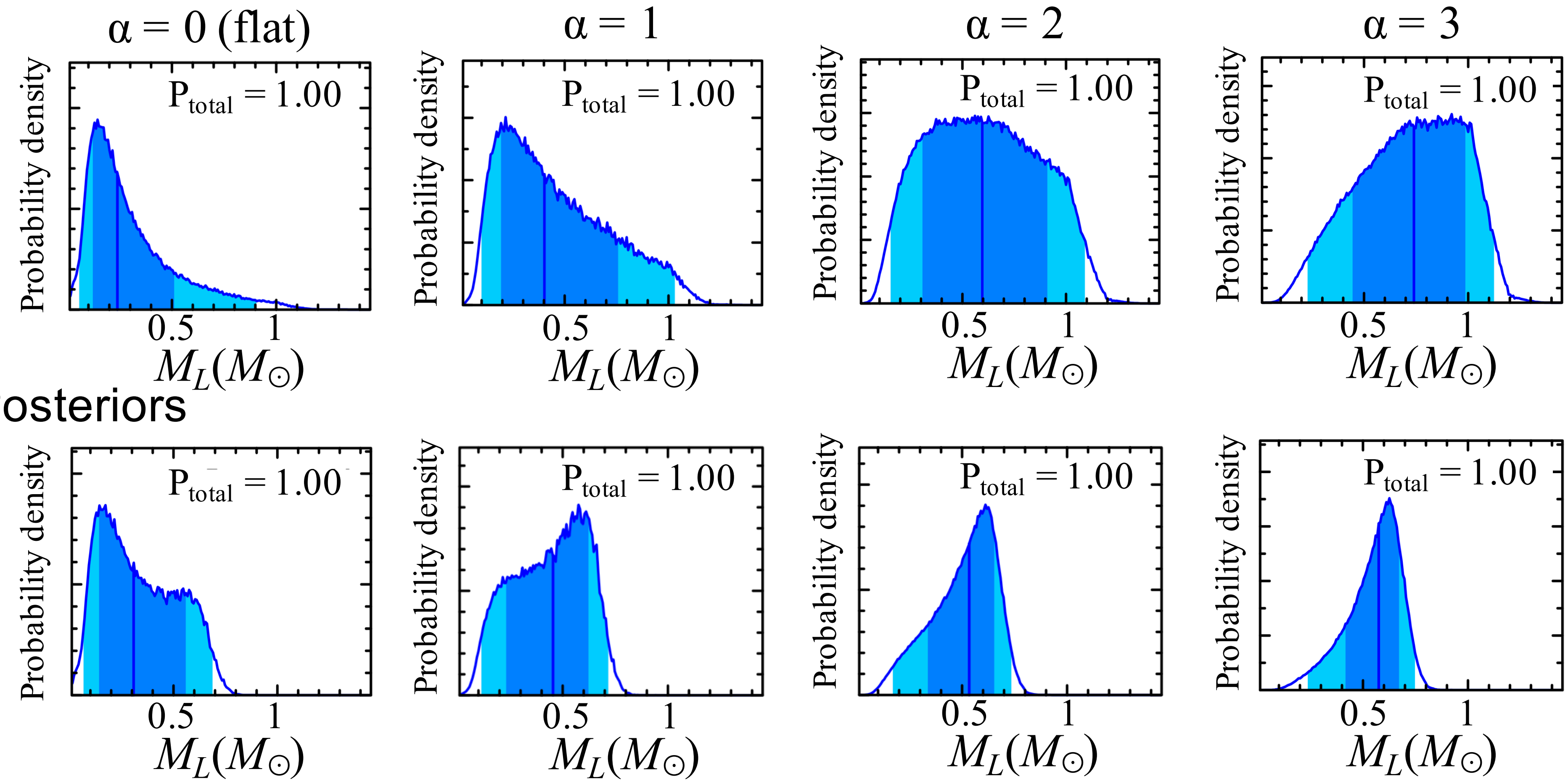
When we derive the prior distribution for M_L, we assume that the host mass dependency of the probability of hosting planet is flat.

→ Compare with the non-flat probabilities.

$$\text{Assuming } P_{\mathrm{host}} \propto M^{\alpha}$$

Priors

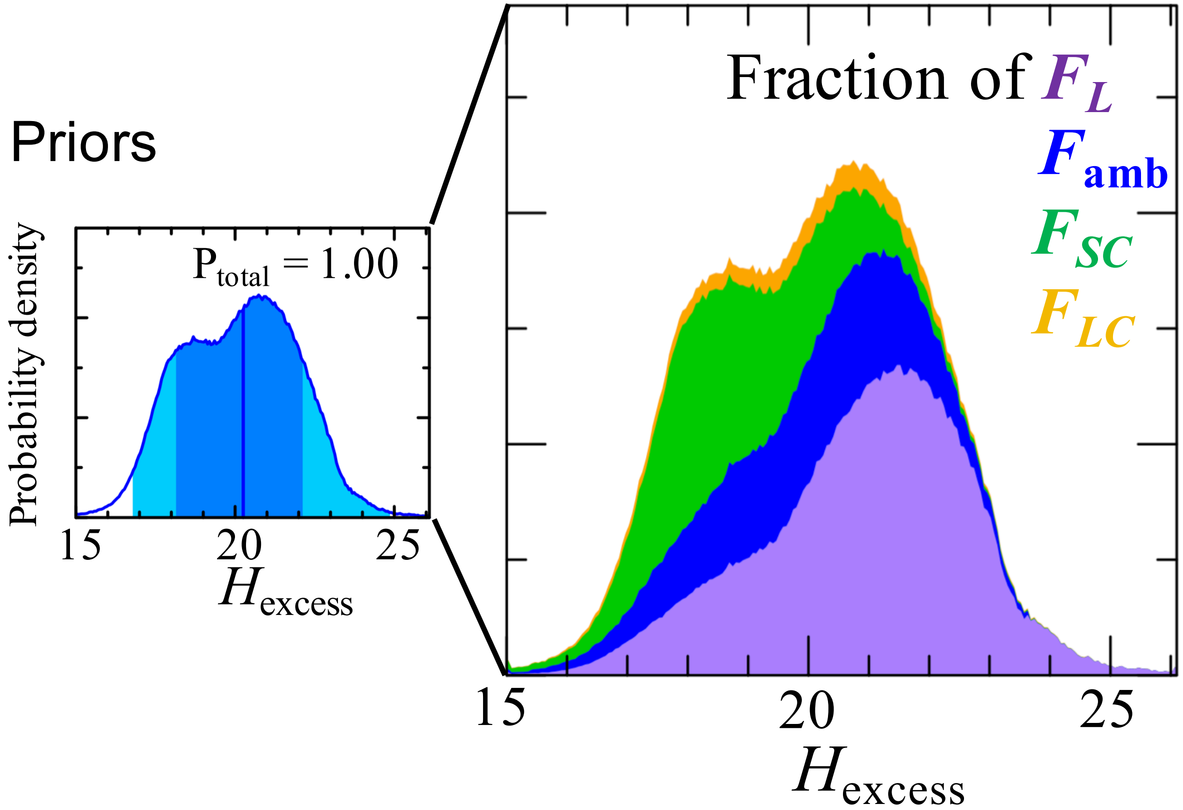
Posteriors



Contributions of Each Possibilities to the Excess
-What can we know before the follow-up observation?-

We can calculate the prior distributions before the follow up high-angular resolution imaging assuming a FWHM value.

Priors



We can determine the lens mass well if the H_{excess} > ~23

We can know, before any follow-up observations,

with which H_{excess} value, how much we can constrain the lens mass.

Contributions of Each Possibilities to the Excess
-What can we know before the follow-up observation?-

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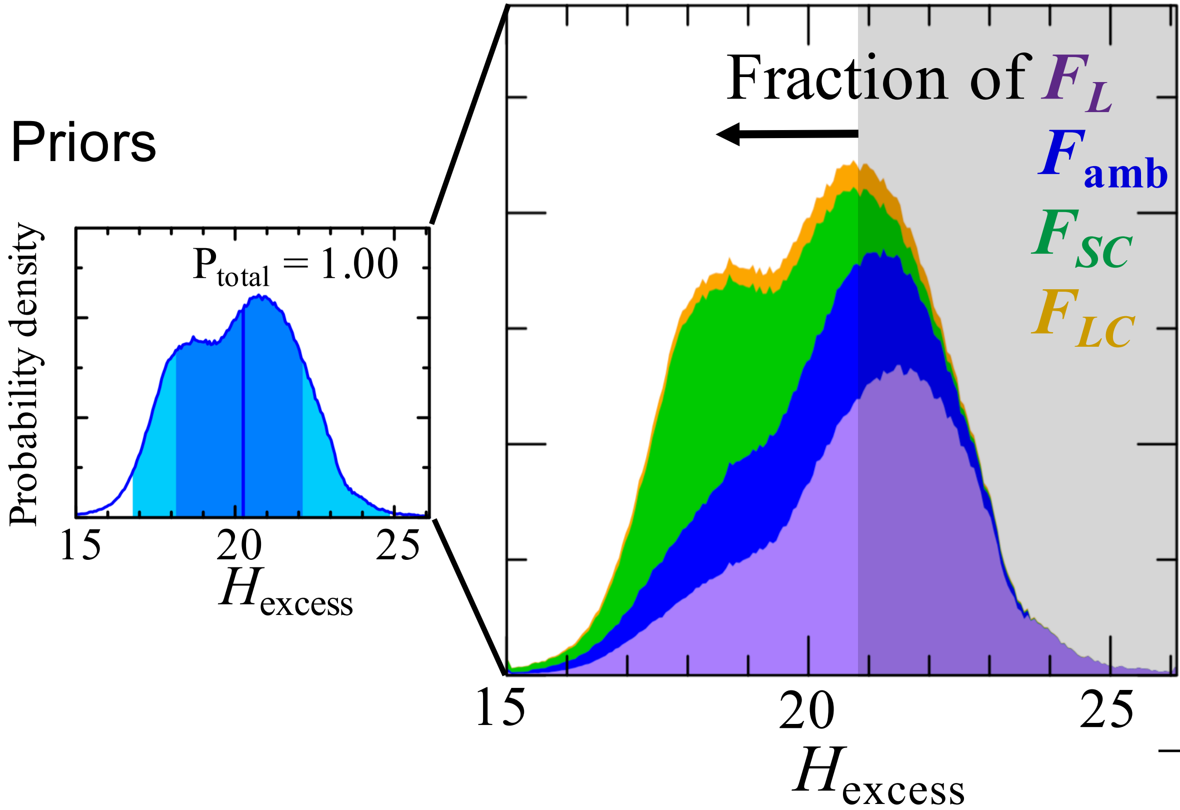


We can determine the lens mass well if the H_{excess} > ~23

But, we have detection limit :

3σ detection limit for H_{excess}

$$F_{target} \geq F_S + 3\sigma_{F_S}$$

$$\Rightarrow F_{excess} = F_{target} - F_S$$

$$\geq 3\sigma_{F_S}$$

$$\Rightarrow H_{excess} \leq 20.89$$

→ cannot expect the lens detection

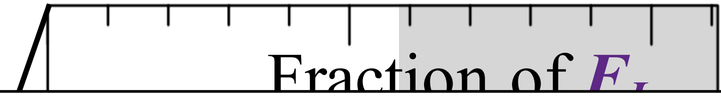
We can know, before any follow-up observations,

with which H_{excess} value, how much we can constrain the lens mass.

Contributions of Each Possibilities to the Excess

-What can we know before the follow-up observation?-

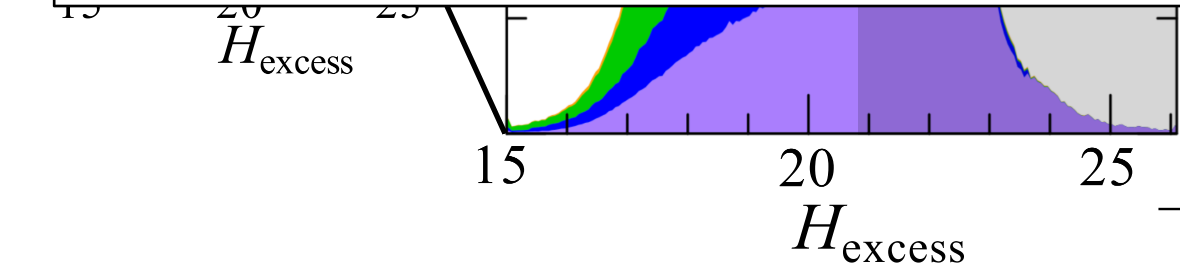
We can calculate the prior distributions before the follow up high-angular resolution imaging assuming a FWHM value.



We can determine the lens mass well if the $H_{excess} > \sim 23$

Pr

Of course, these predictions depend on the prior we assumed,
so we can assess the prior by a lot of observed excess fluxes statistically on the contrary.



$\leq 30 F_S$

$\Rightarrow H_{excess} \leq 20.89$

→ cannot expect the lens detection

We can know, before any follow-up observations, with which H_{excess} value, how much we can constrain the lens mass.

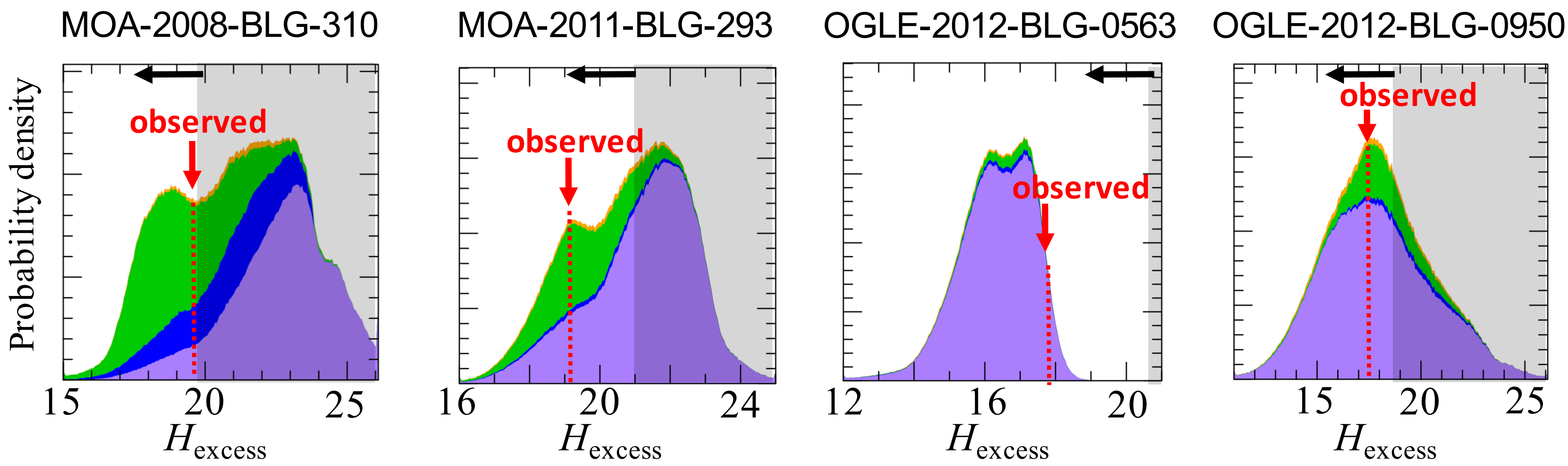
The Results for Previous Events

PRELIMINARY

Event	M_L (this work)	M_L if $H_{excess} = H_L$	θ_E (mas)	π_E	Paper
MOA-2016-BLG-227	$0.31^{+0.25}_{-0.17}$	0.63 ± 0.09	0.23	–	This work
MOA-2008-BLG-310	$0.15^{+0.31}_{-0.08}$	0.67 ± 0.14	0.155	–	Janczak et al. (2010)
MOA-2011-BLG-293	0.47 ± 0.27	0.86 ± 0.06	0.26	–	Batista et al. (2014)
OGLE-2012-BLG-0563	0.36 ± 0.13	$0.34^{+0.12}_{-0.20}$	1.34	–	Fukui et al. (2015)
OGLE-2012-BLG-0950	$0.54^{+0.12}_{-0.17}$	$0.63^{+0.04}_{-0.11}$	–	0.26	Koshimoto et al. (2016)

Priors assuming the same number density as that of the MOA-2016-BLG-227 field

Colors: F_L F_{amb} F_{SC} F_{LC}



The Results for Previous Events

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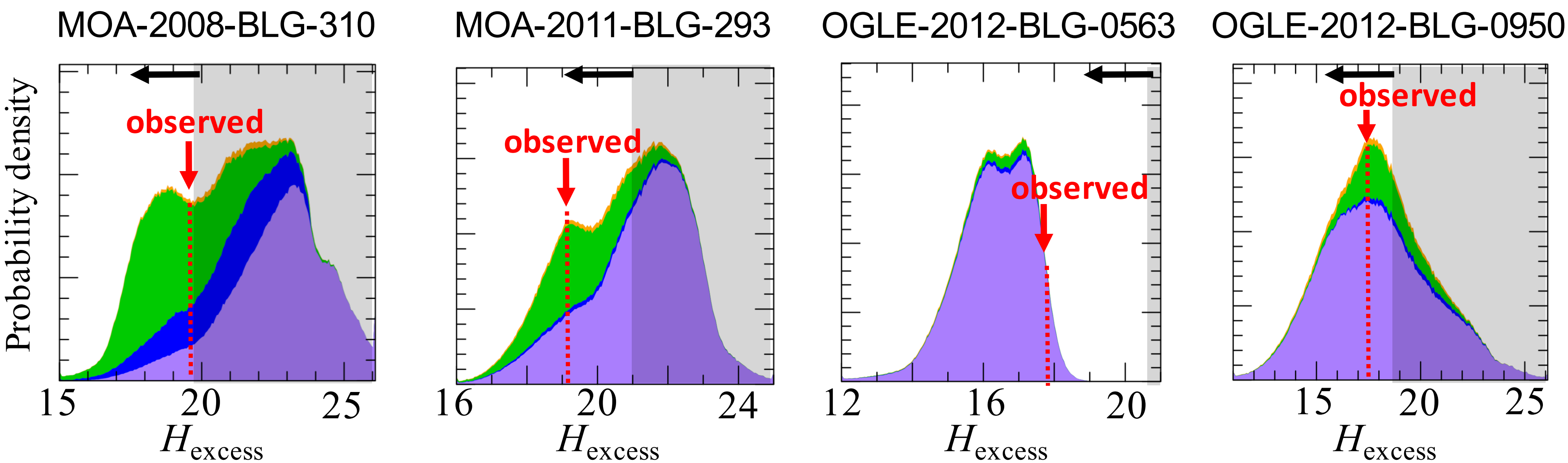
Colors: F_L $F_{\rm amb}$ F_{SC} F_{LC}



→ To distinguish each scenarios for the events with small θ_E, we need information of the proper motion of the objects which provide excess by additional follow-up observations .

Summary

We found

- New approach to evaluate the contamination probabilities through the Bayesian analysis .

- Difficulty in excluding the contamination scenarios for events with small θ_E

→ Requirement of the information of the proper motion

- We can calculate the expectation for the lens detection before any follow-up observations

PRELIMINARY

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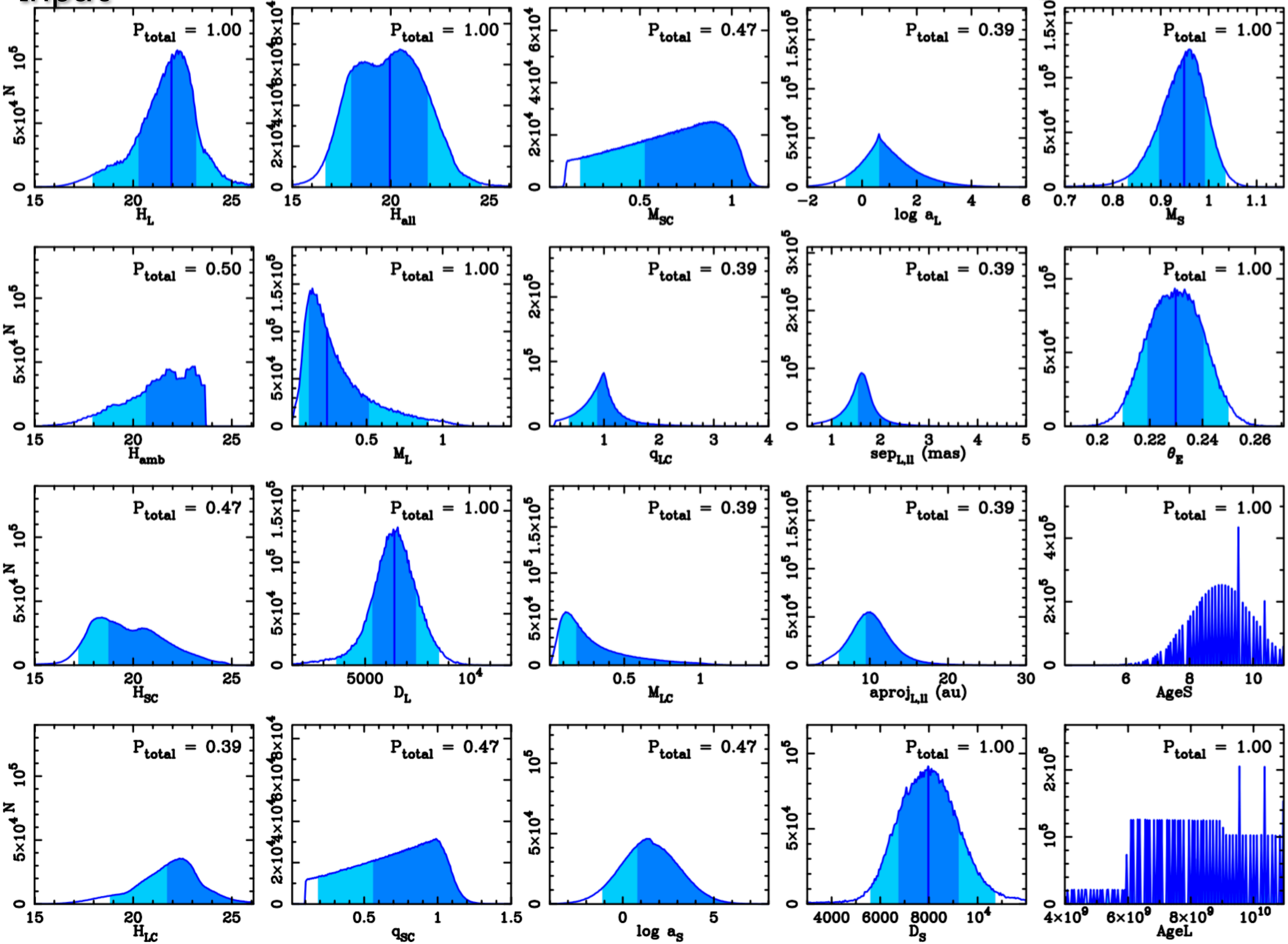
Back UP

Input



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log a_L
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M_S
P_total = 0.50
H_amb
P_total = 1.00
M_L
P_total = 0.39
q_LC
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H_SC
P_total = 1.00
D_L
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M_LC
P_total = 0.39
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P_total = 1.00
AgeS
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H_LC
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D_S
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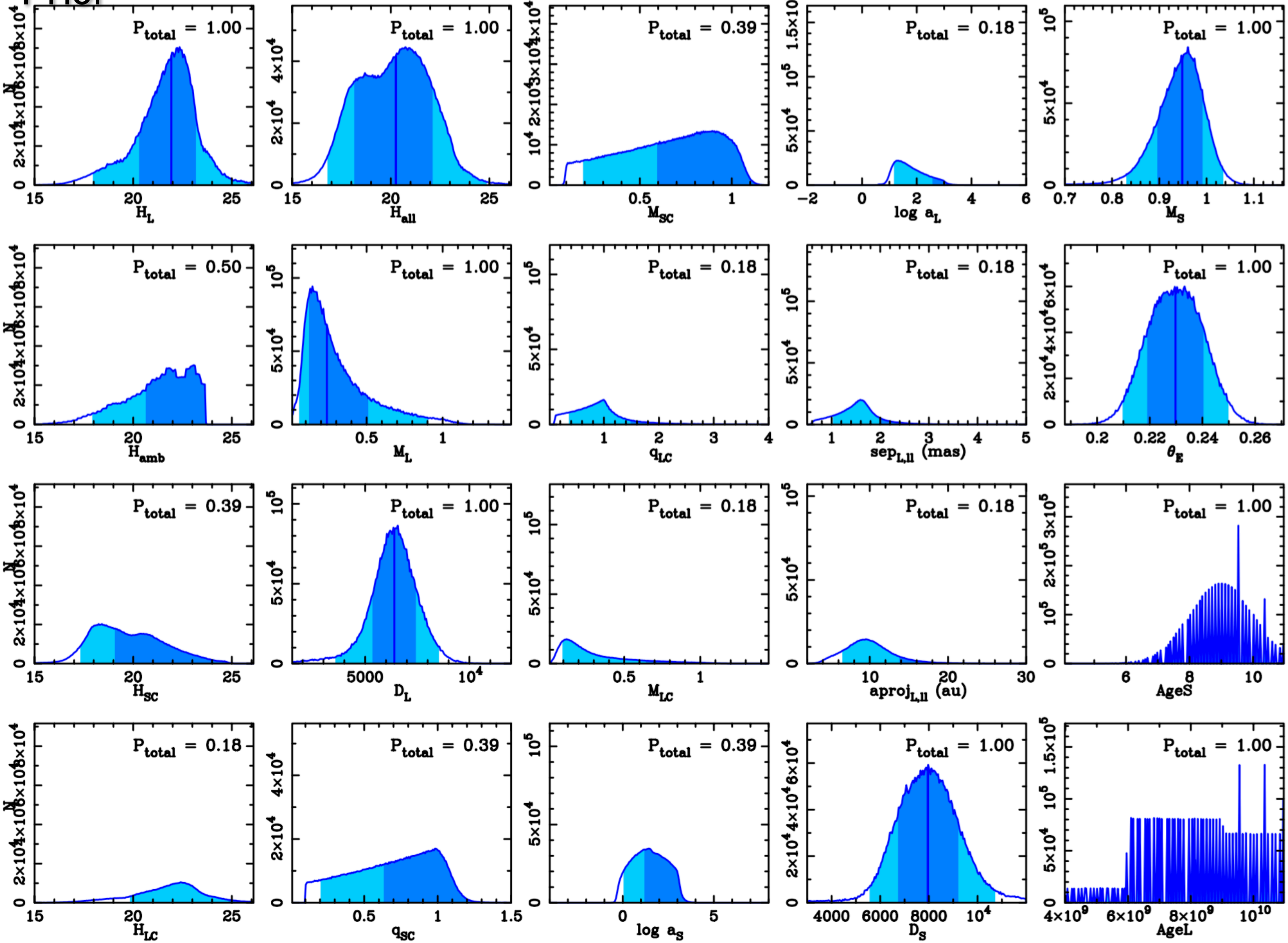
AgeL
N

Prior



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D_L
Ptotal = 0.18
M_LC
Ptotal = 0.18
aproj_L,ll (au)
Ptotal = 1.00
AgeS
Ptotal = 0.18
H_LC
Ptotal = 0.39
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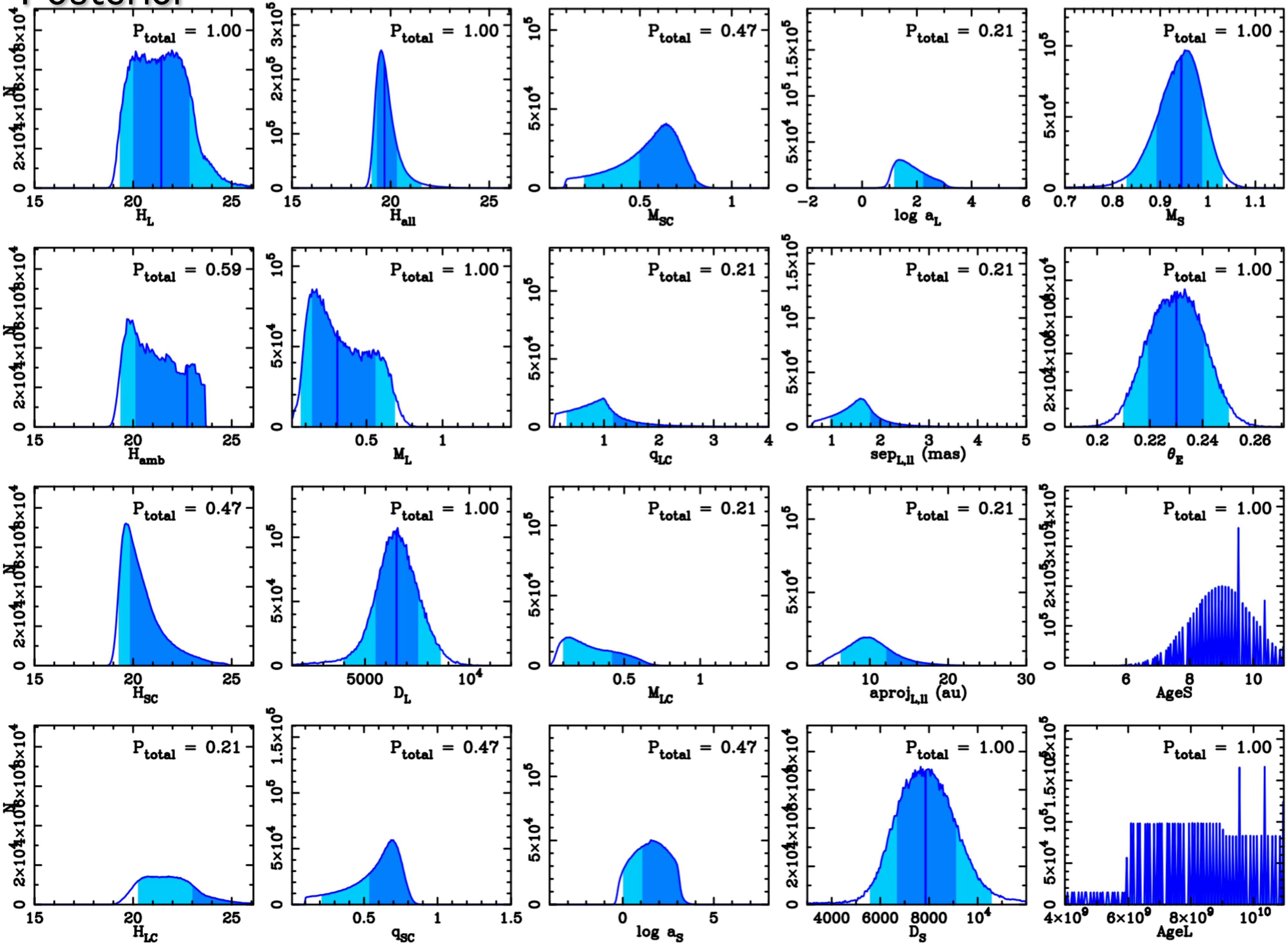
Ptotal = 0.39
log a_S
Ptotal = 1.00
D_S
Ptotal = 1.00
AgeL
N

Posterior



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M_S
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H_amb
P_total = 1.00
M_L
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q_LC
P_total = 0.21
sep_L,ll (mas)
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P_total = 0.47
H_SC
P_total = 1.00
D_L
P_total = 0.21
M_LC
P_total = 0.21
aproj_L,ll (au)
P_total = 1.00
AgeS
P_total = 0.21
H_LC
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q_SC
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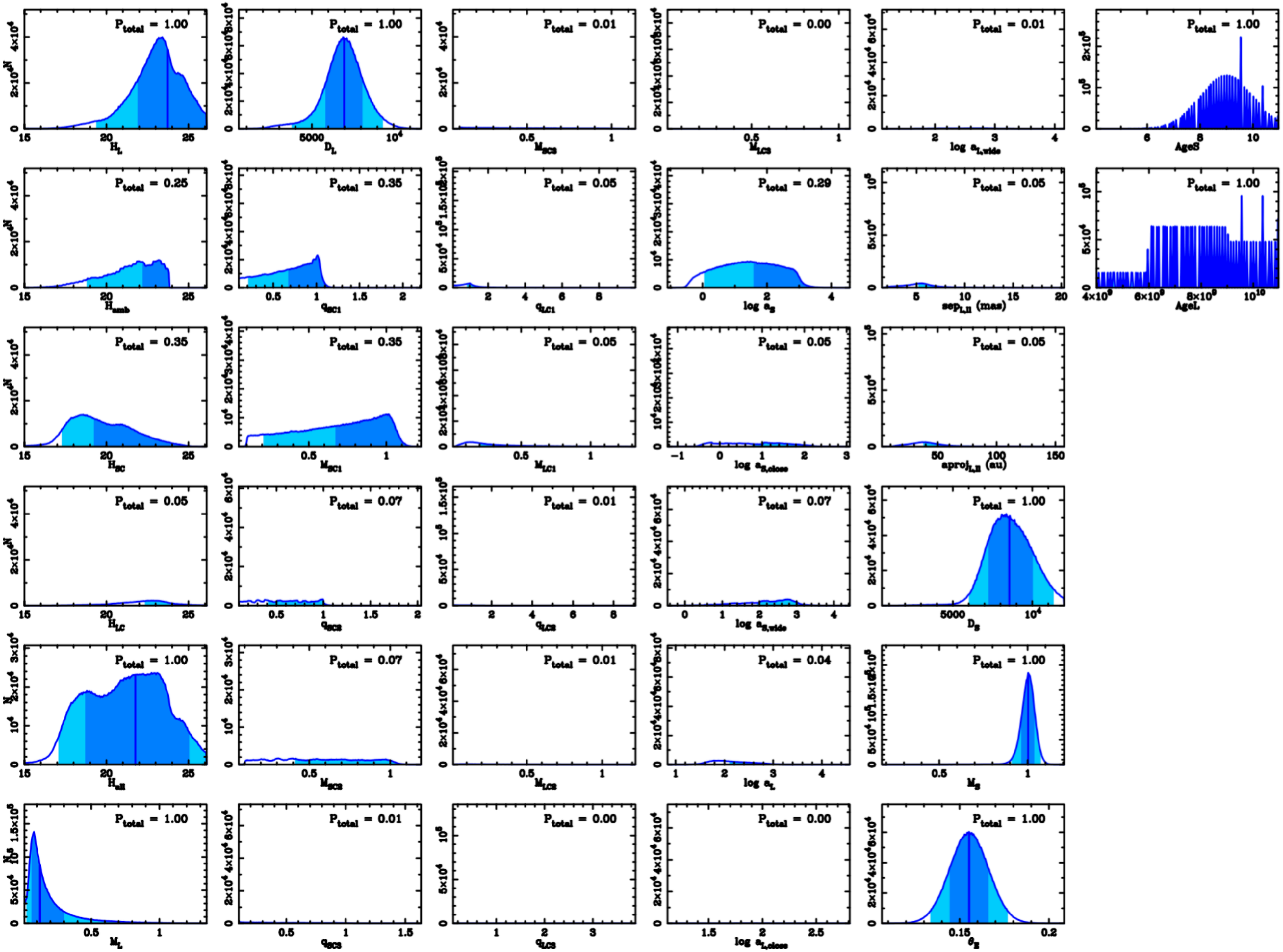
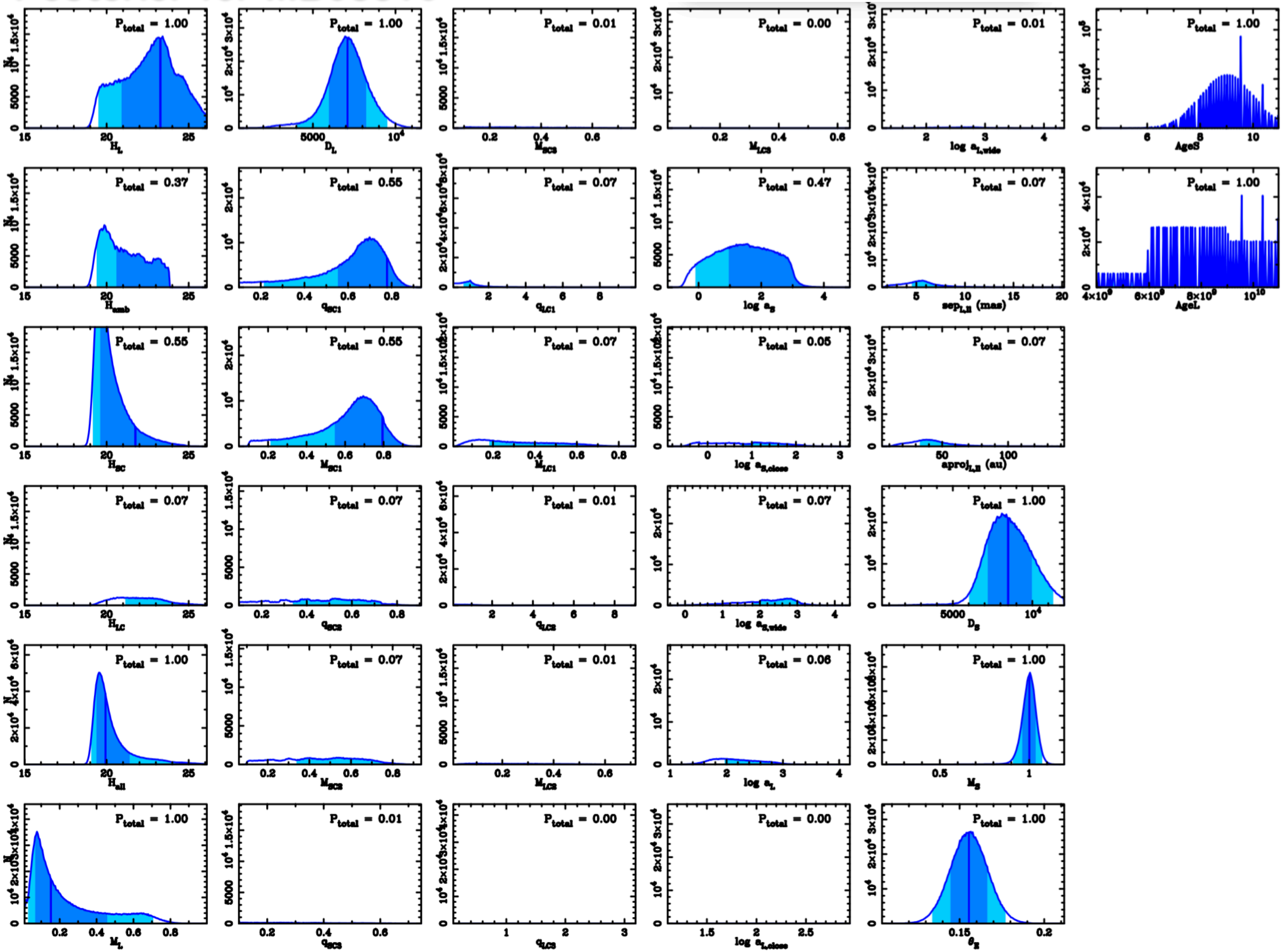
log a_S
P_total = 1.00
D_S
P_total = 1.00
AgeL
N

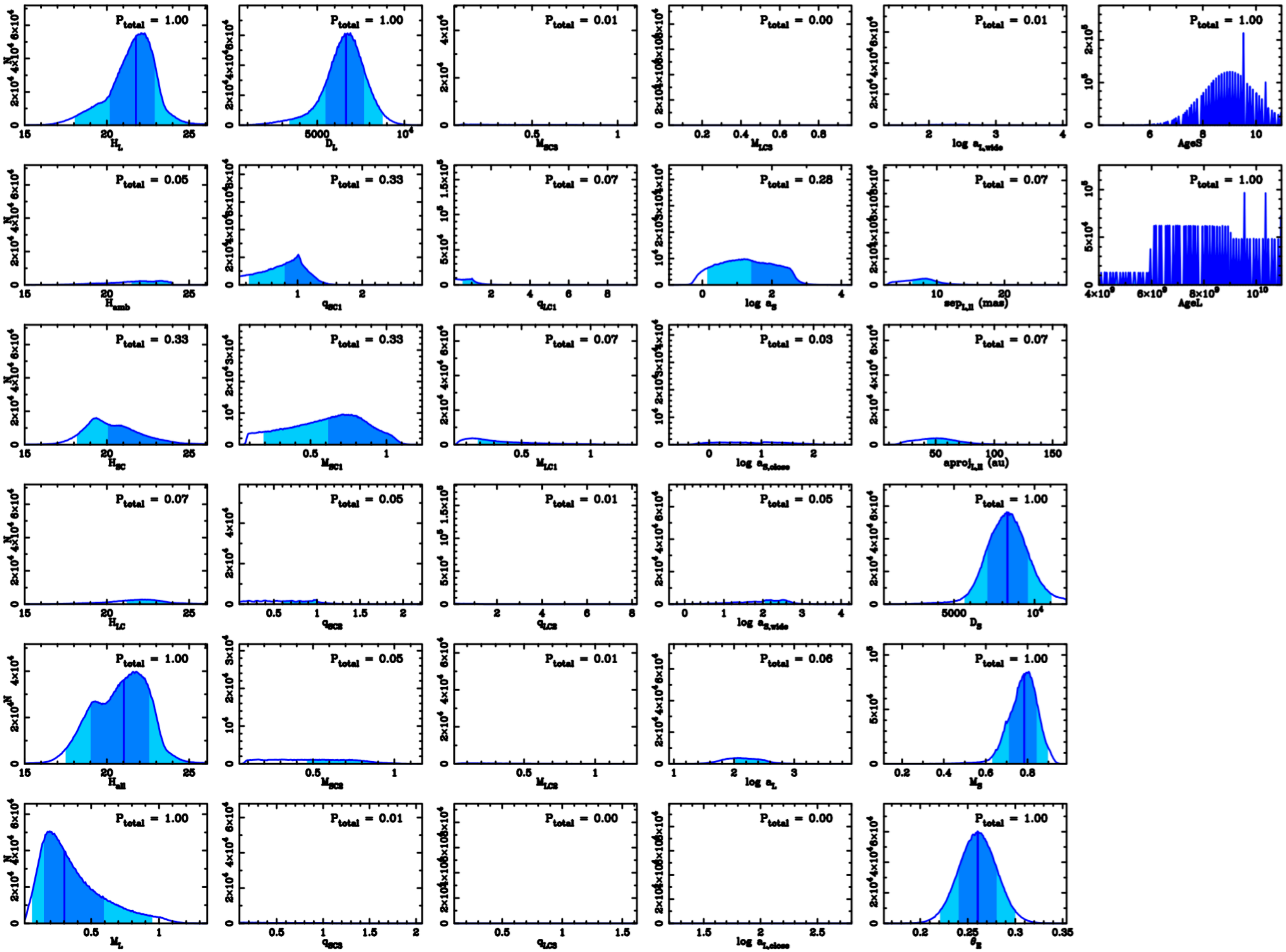
Prior for MB08310



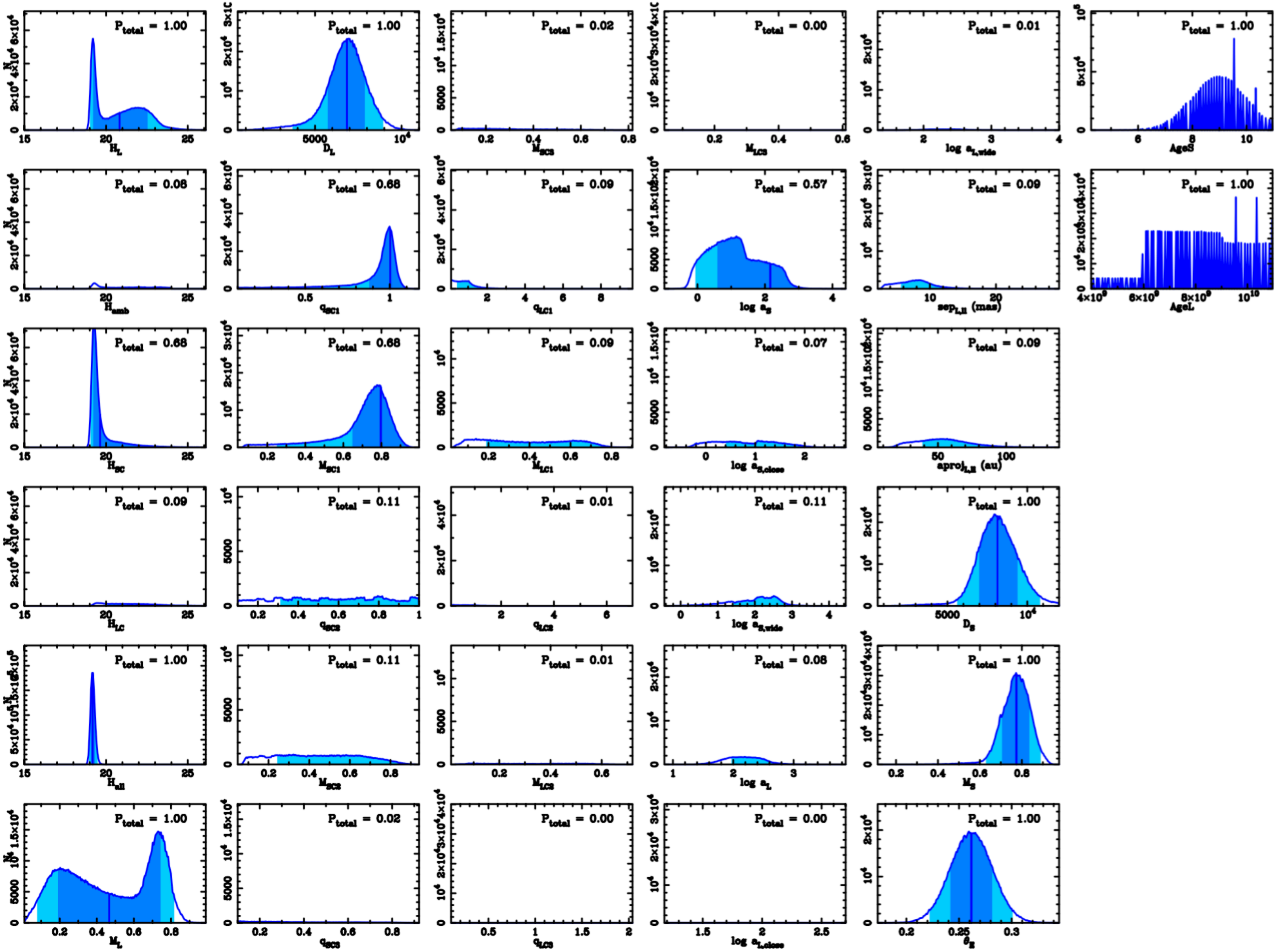
Posterior for MB08310



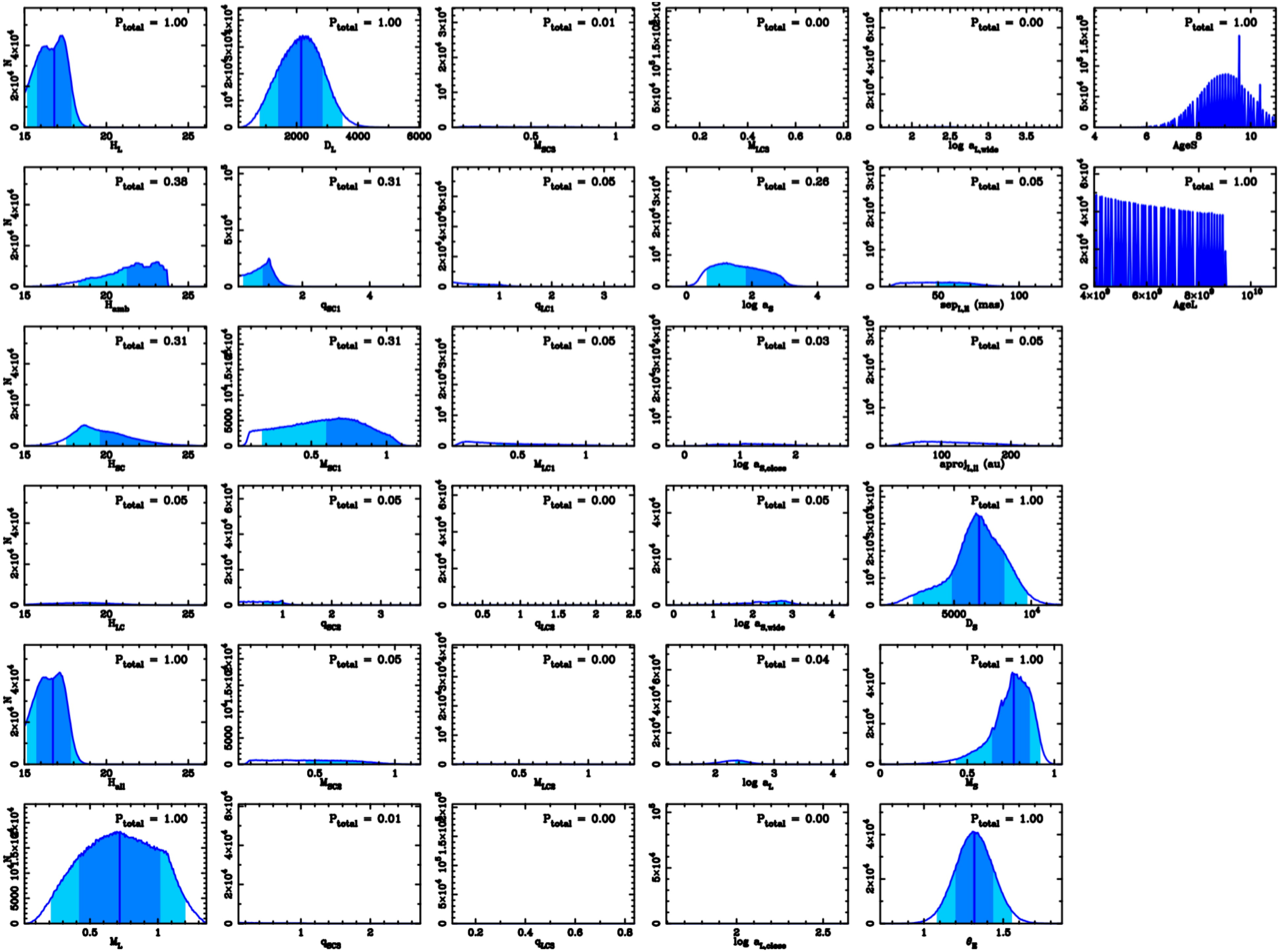
Prior for MB11293



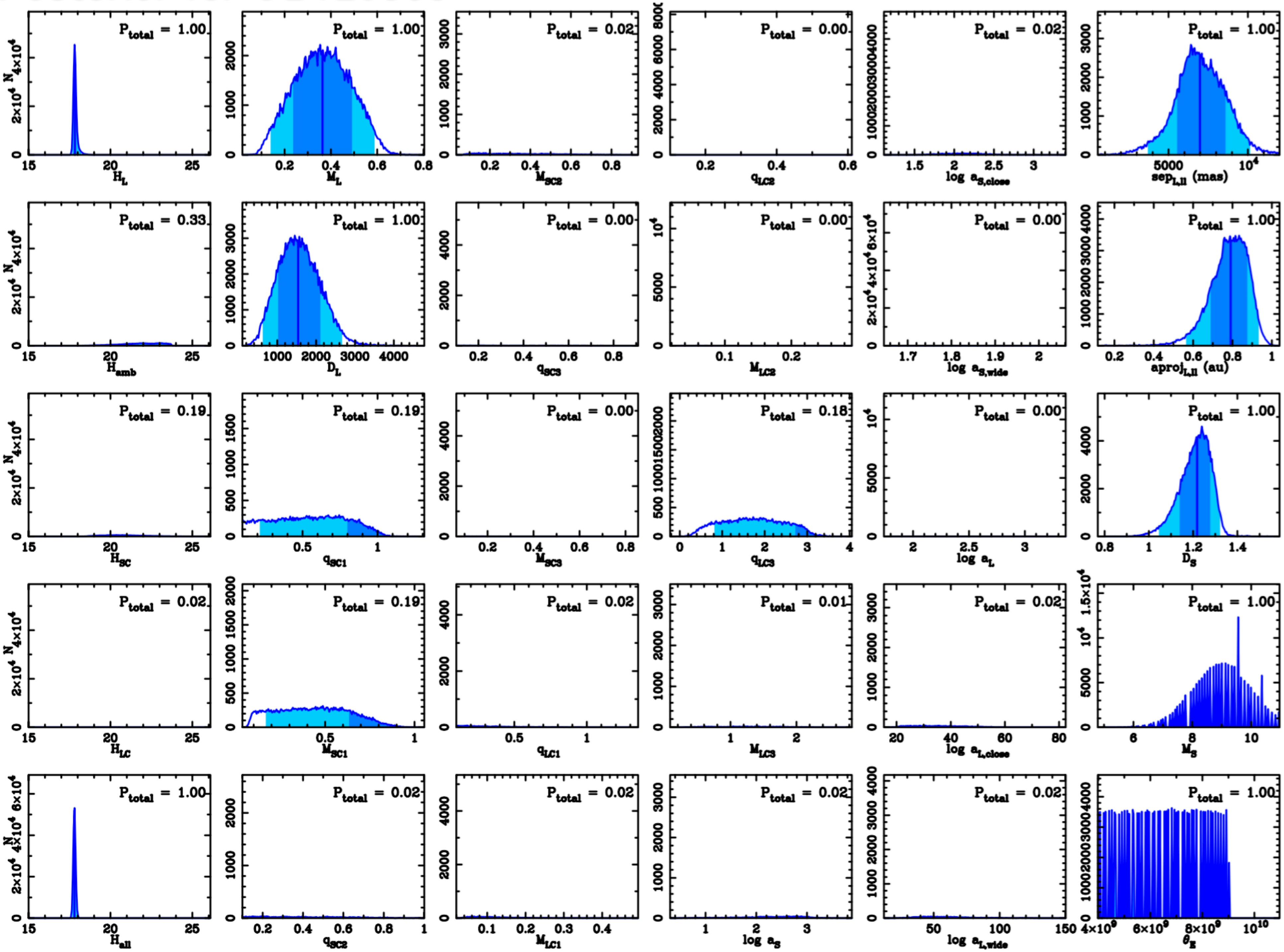
Posterior for MB11293



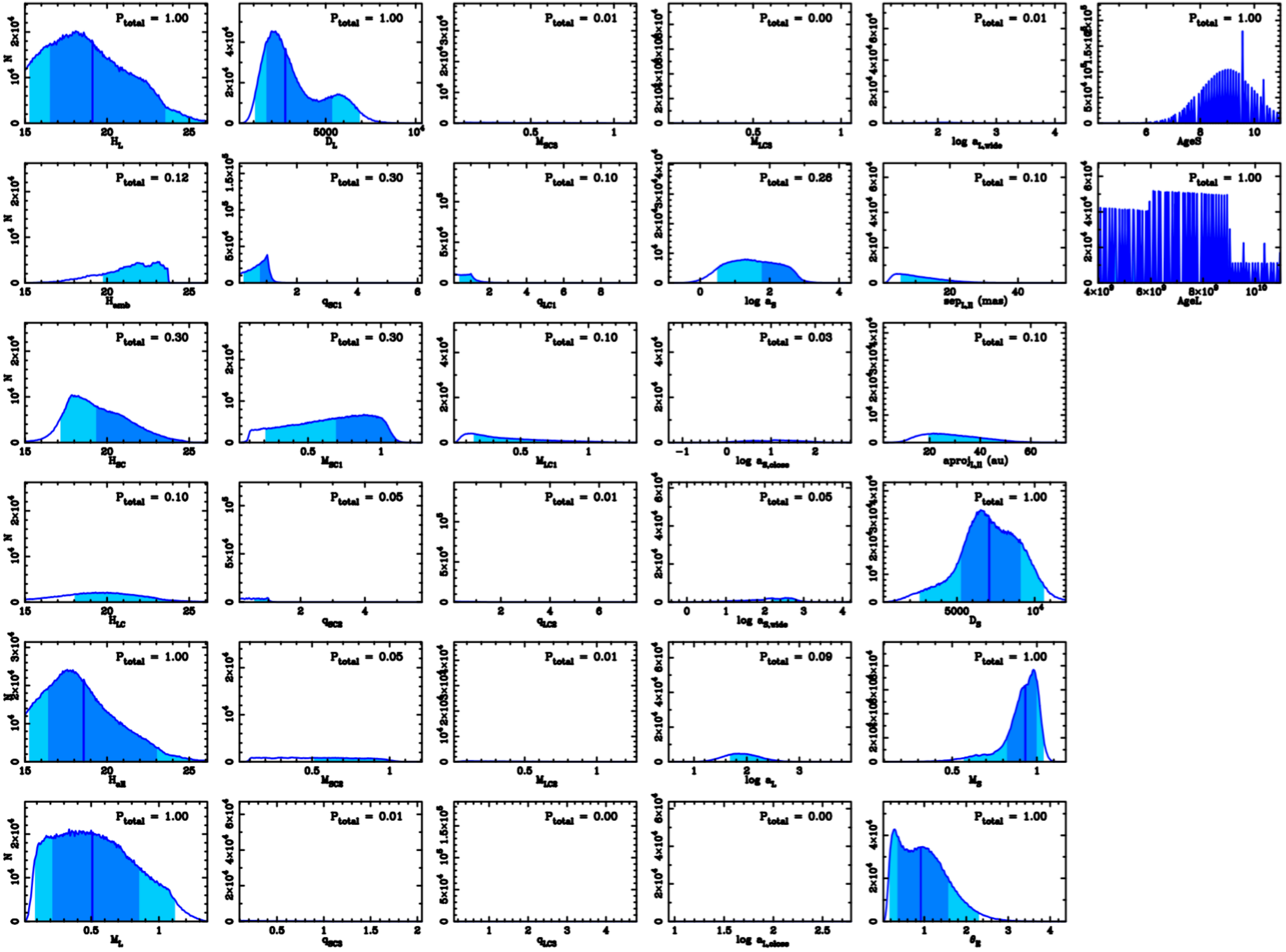
Prior for OB120563



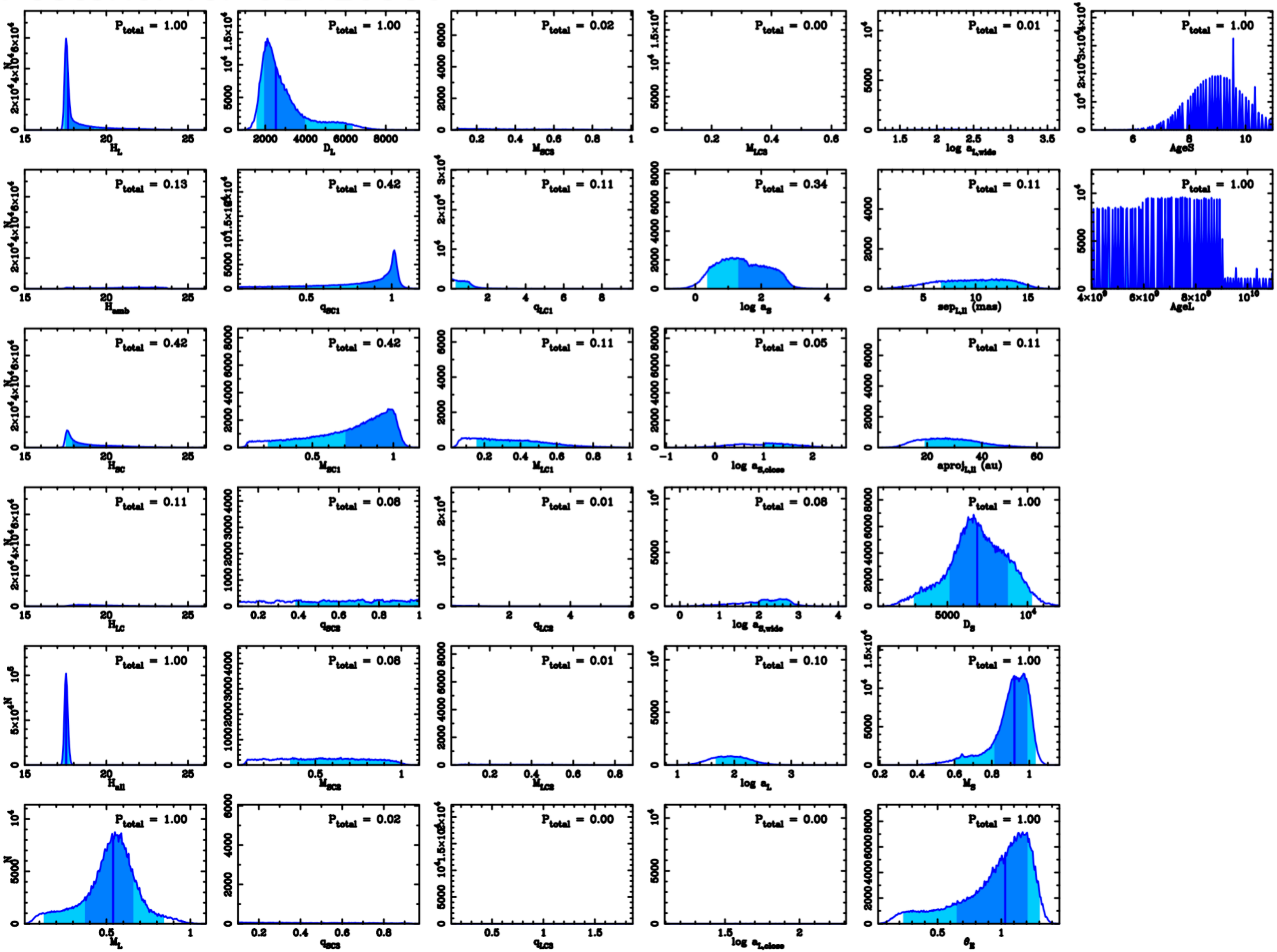
Posterior for OB120563



Prior for OB120950

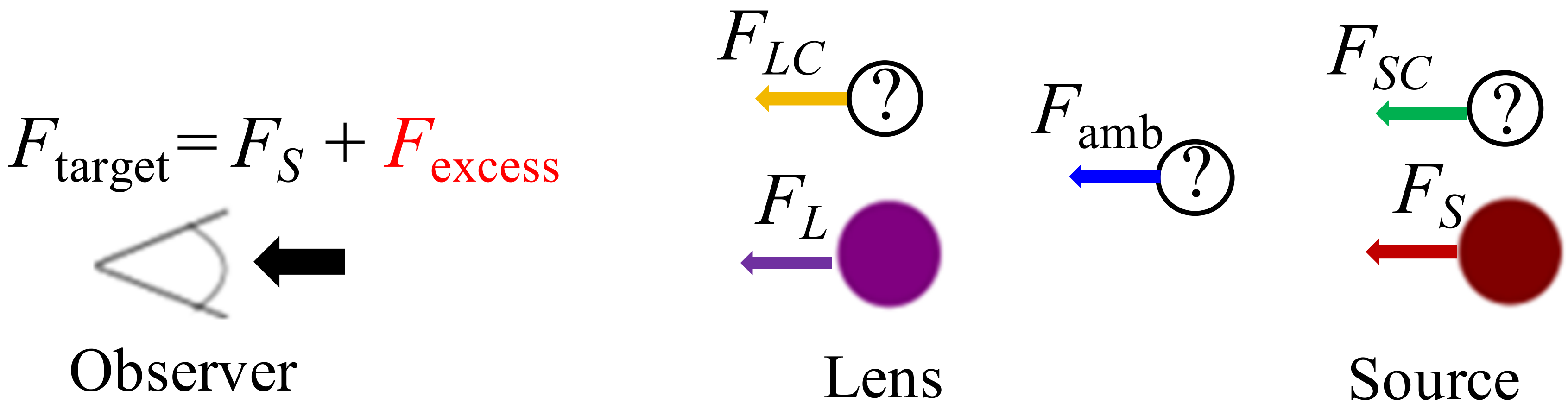


Posterior for OB120950



Event	M_L (this work)	M_L if $H_{\rm excess} = H_L$	$P_{H_{\rm excess}}$	$E_{F_L/F_{\rm excess}}$	Paper
MOA-2016-BLG-227	$0.31^{+0.25}_{-0.17}$	0.63 ± 0.09	0.62	0.33	This work
MOA-2008-BLG-310	$0.15^{+0.31}_{-0.08}$	0.67 ± 0.14	0.26	0.12	Janczak et al. (2010)
MOA-2011-BLG-293	0.47 ± 0.27	0.86 ± 0.06	0.51	0.56	Batista et al. (2014)
OGLE-2012-BLG-0563	0.36 ± 0.13	$0.34^{+0.12}_{-0.20}$	0.99	0.94	Fukui et al. (2015)
OGLE-2012-BLG-0950	$0.54^{+0.12}_{-0.17}$	$0.63^{+0.04}_{-0.11}$	0.65	0.82	Koshimoto et al. (2017)

How to Evaluate The Possible Contaminations



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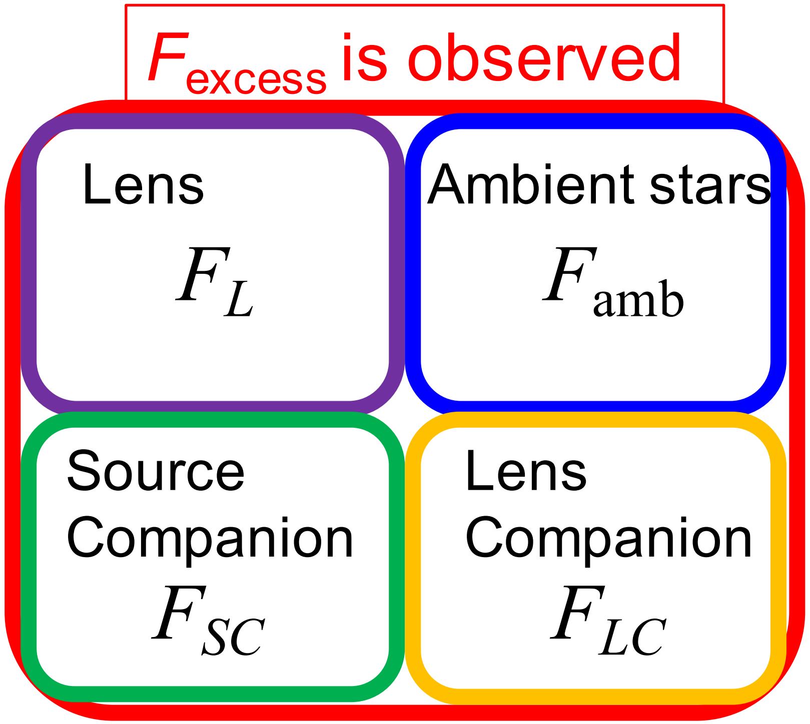
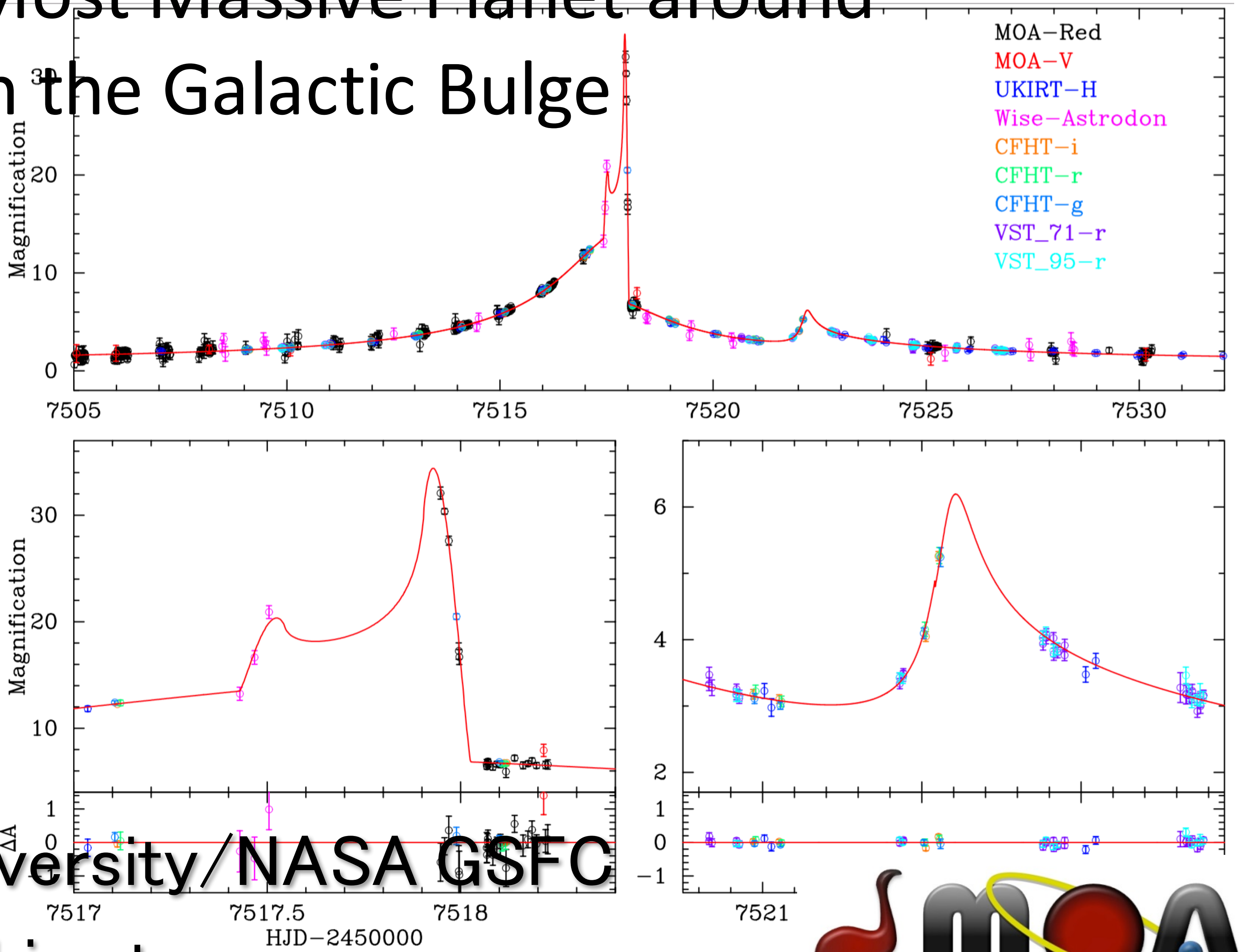
Considering four possibilities of the excess separately,



MOA-2016-BLG-227Lb: Possible Most Massive Planet around K-Dwarf in the Galactic Bulge



Osaka University/NASA GSFC

Naoki Koshimoto

MOA-2016-BLG-227Lb: Possible Most Massive Planet around K-Dwarf in the Galactic Bulge

-A New Approach to Evaluate Contamination Probabilities-

The probability of *A* given that *B* is true:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Naoki Koshimoto

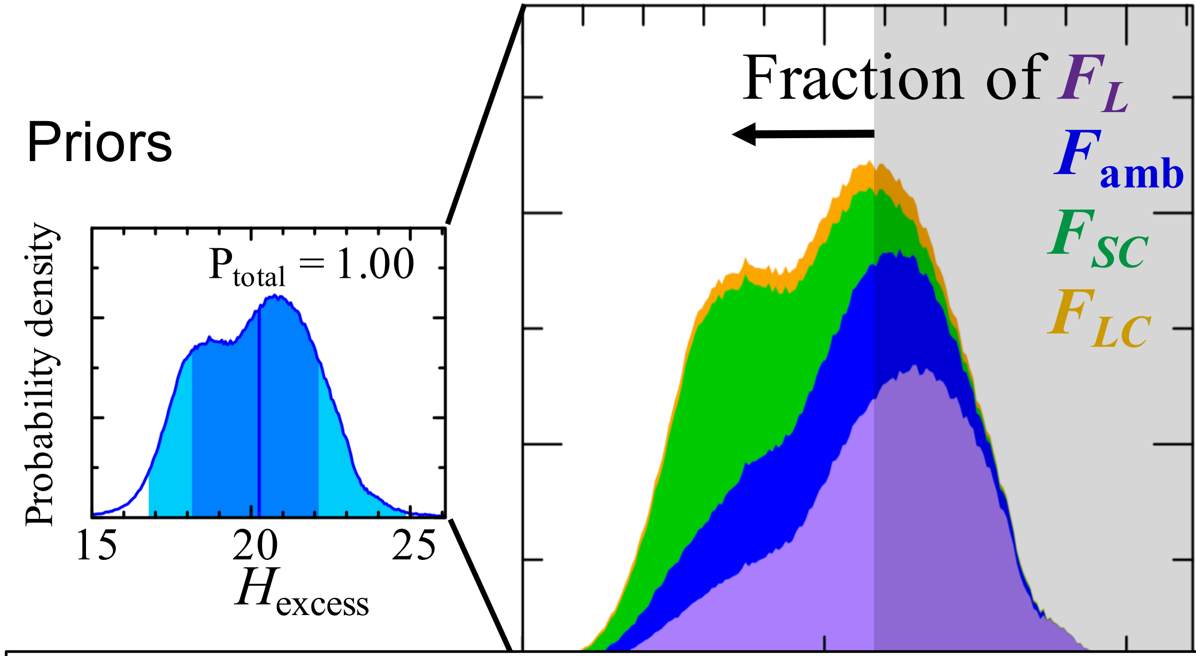
Osaka University/NASA GSFC

Contributions of Each Possibilities to the Excess

-What can we know before the follow-up observation?-

We can calculate the prior distributions before the follow up high-angular resolution imaging assuming a FWHM value.

Priors



We can exclude contamination scenarios if the H_{excess} > ~23

But, we have detection limit :

3σ detection limit for H_{excess}

$$F_{target} \geq F_S + 3\sigma_{F_S}$$

$$\Rightarrow F_{excess} = F_{target} - F_S$$

$$\geq 3\sigma_{F_S}$$

$$\Rightarrow H_{excess} \leq 20.89$$

We can know, before any follow-up observations,

-The prior probability of detecting H_{excess} :

$$P\,(\,H_{excess} \leq 20.89\,) = 0.62$$

-The expected fraction of F_L in the excess when we detect H_{excess} :

$$E\,(F_L\,/\,F_{excess}\,|\,H_{excess} \leq 20.89\,) = 0.33$$

Contributions of Each Possibilities to the Excess
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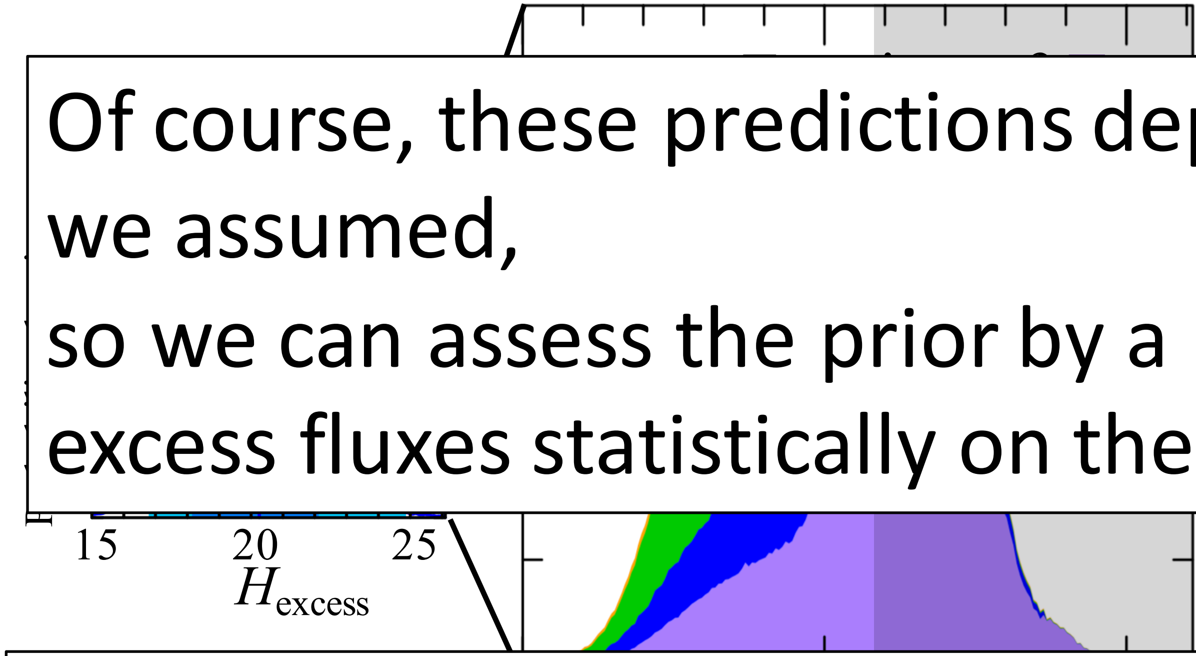

We can exclude contamination

Of course, these predictions depend on the prior we assumed,
so we can assess the prior by a lot of observed excess fluxes statistically on the contrary.

$\geq 3\sigma_{F_S}$

$\Rightarrow H_{\mathrm{excess}} \leq 20.89$

We can know, before any follow-up observations,

-The prior probability of detecting H_{excess} :

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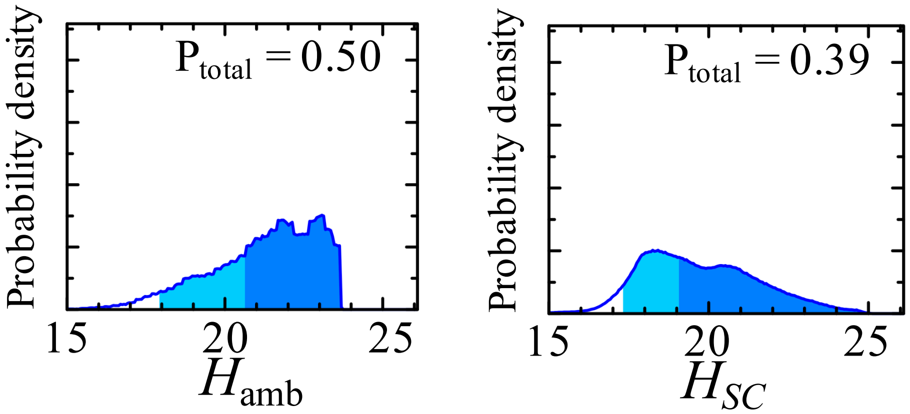
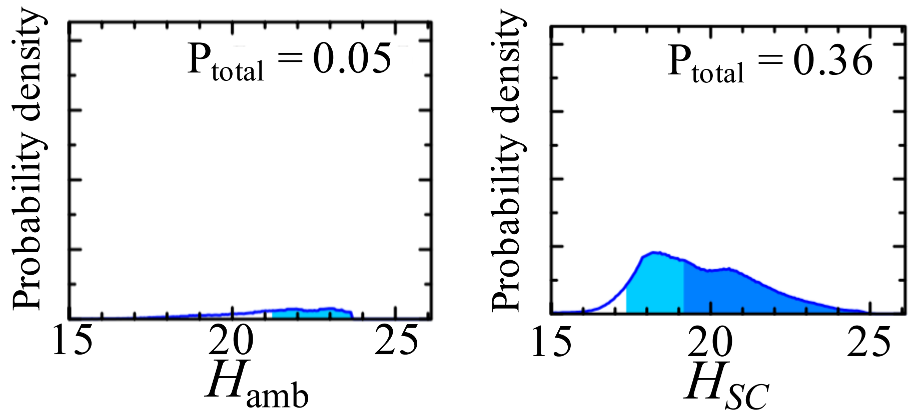
Comparison with Better FWHM Value

Priors (w/ FWHM = 184 mas)



Priors (w/ FWHM = 60 mas)



<u>The probability of H_{SC} is almost same even with much better seeing !!</u> while that of H_{amb} gets much smaller

Comparison with Better FWHM Value

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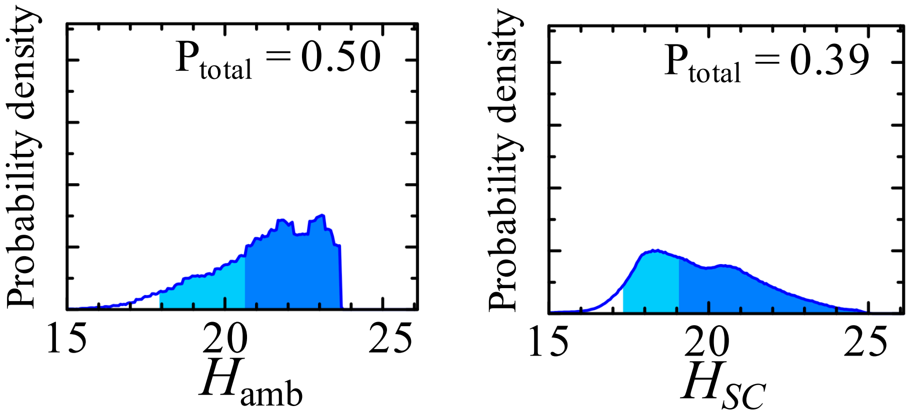

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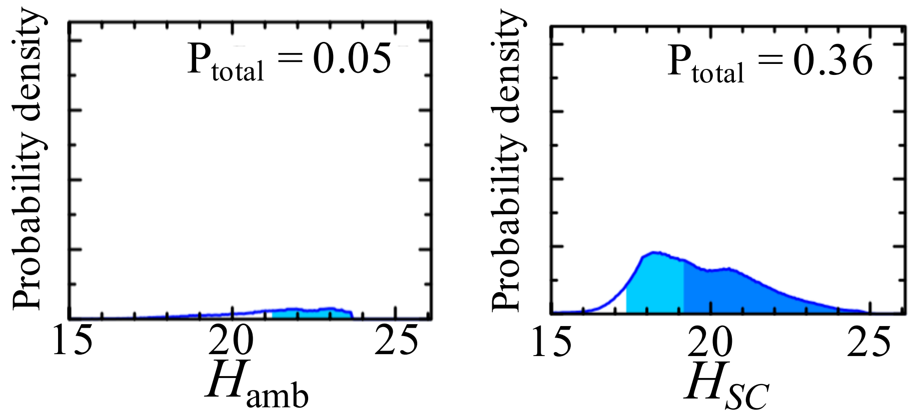
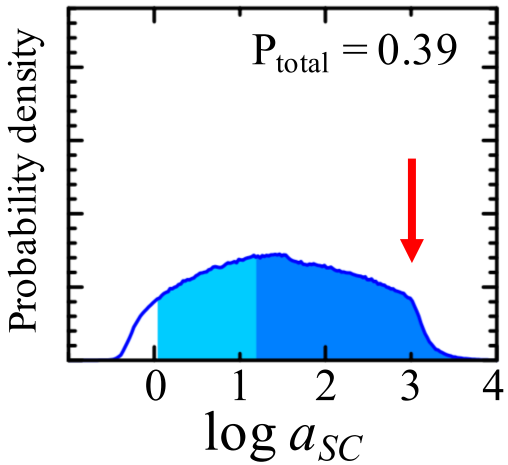


The probability of H_{SC} is almost same even with much better seeing !! while that of H_{amb} gets much smaller

Because the distribution of the semi-major axis is "log normal" !!

→ 3 times smaller FWHM reduces only ~ 0.48 dex in log scale.

Priors (w/ FWHM = 184 mas)



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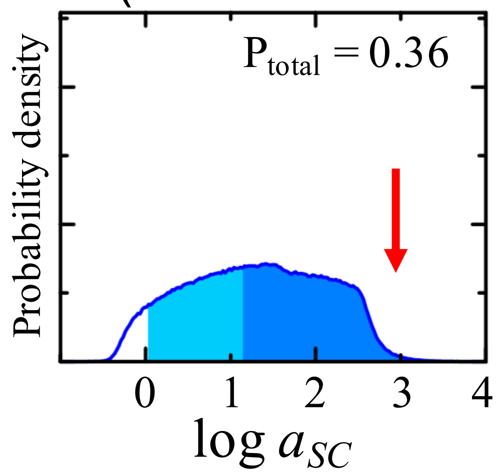
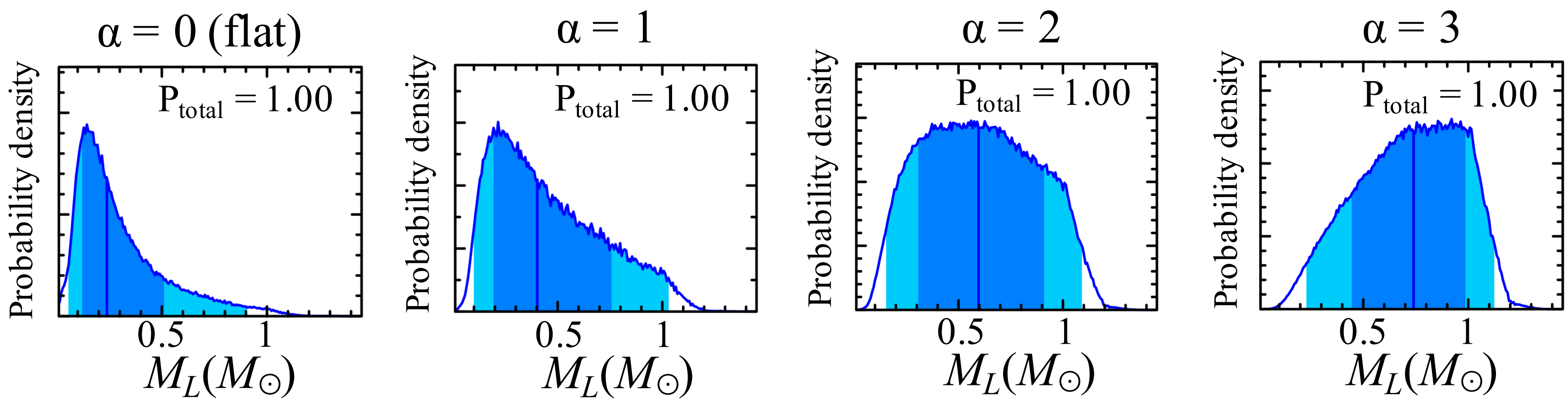


Comparison the Results with Other Priors of P_{host}

When we derive the prior distribution for M_L, we assume that the host mass dependency of the probability of hosting planet is flat.
→ Compare with the non-flat probabilities.

$$\text{Assuming } P_{host} \propto M^{\alpha}$$

Priors



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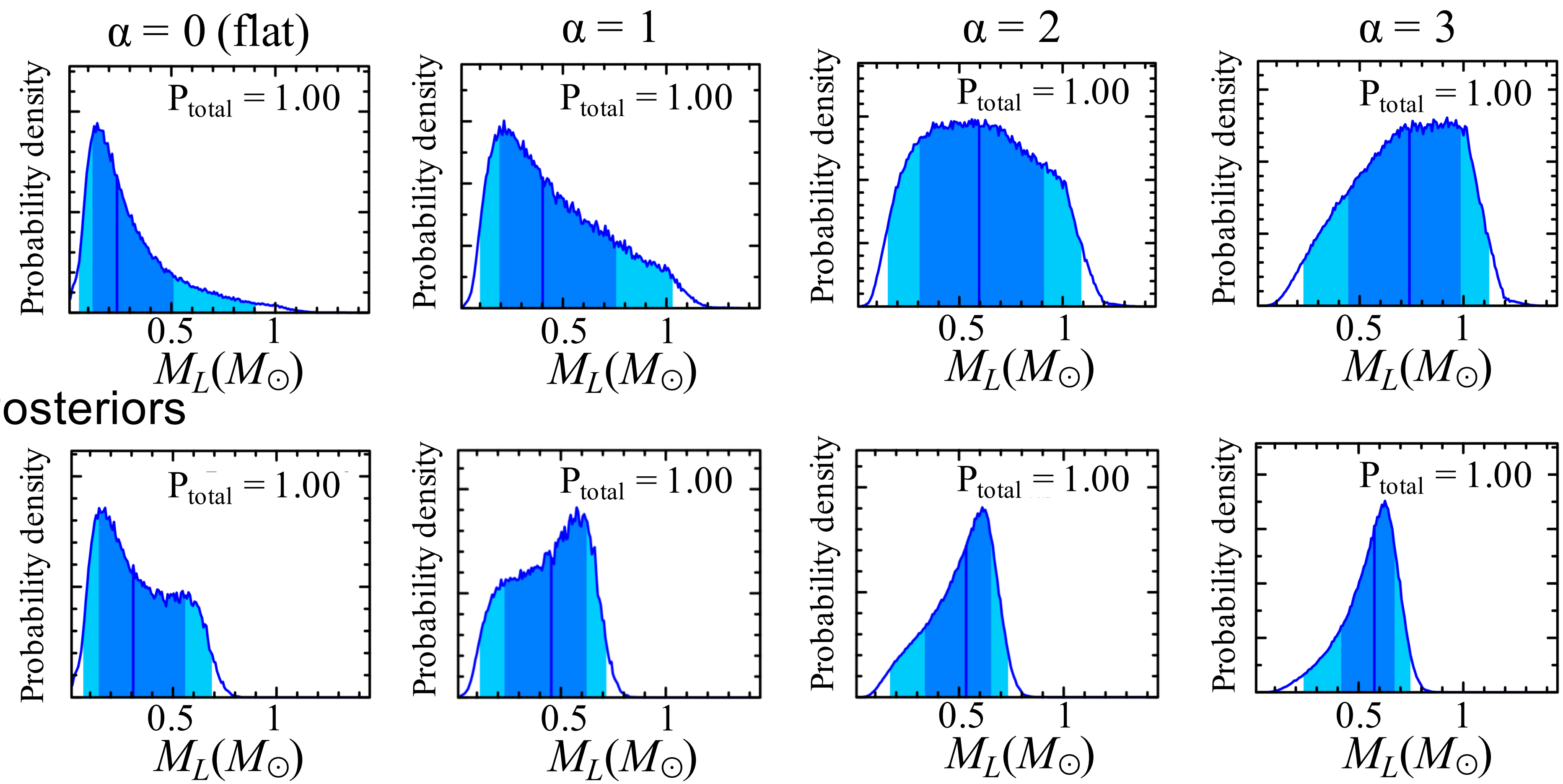


How to Evaluate The Possible Contaminations -Previous approach-

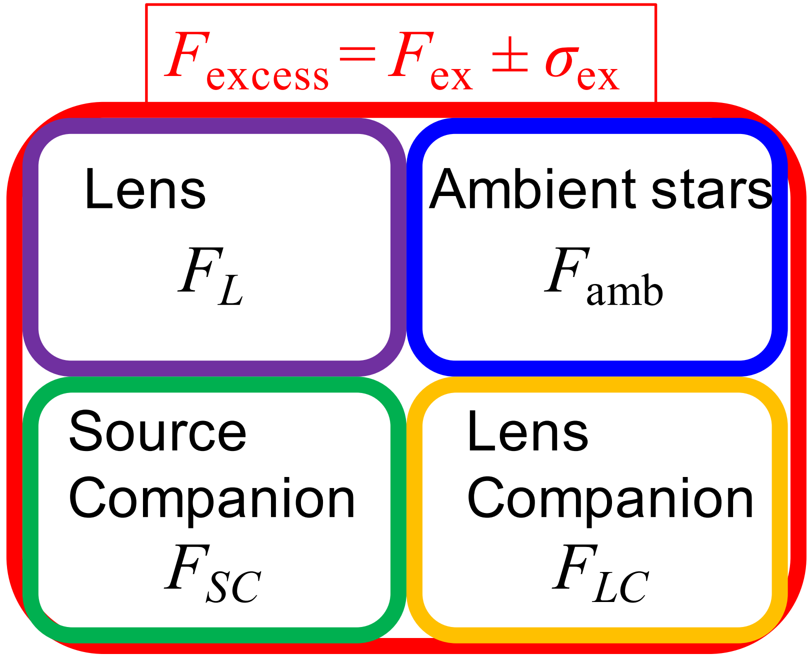
In some previous papers,
(e.g., Janczak+10, Batista+14, Fukui+15, Koshimoto+16)

Considering the four possibilities separately,

$$P_{\rm contami} = \sum_i P(F_i \sim F_{\rm ex}) \quad i = {\rm amb}, SC, LC$$

$$P(F_L \sim F_{\rm ex}) = 1 - P_{\rm contami}$$

where $F_i \sim F_{\rm ex} \iff F_{\rm ex} - \sigma_{\rm ex} < F_i < F_{\rm ex} + \sigma_{\rm ex}$



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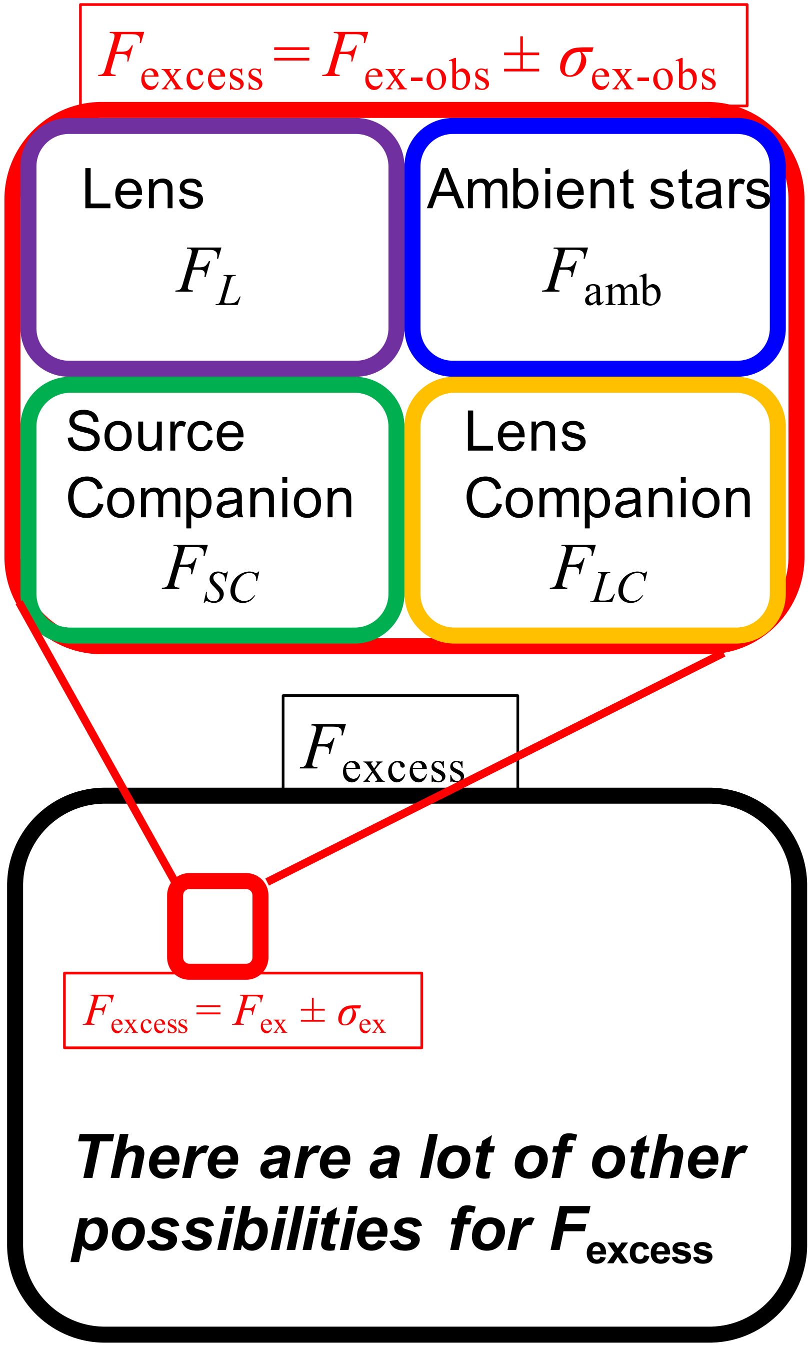

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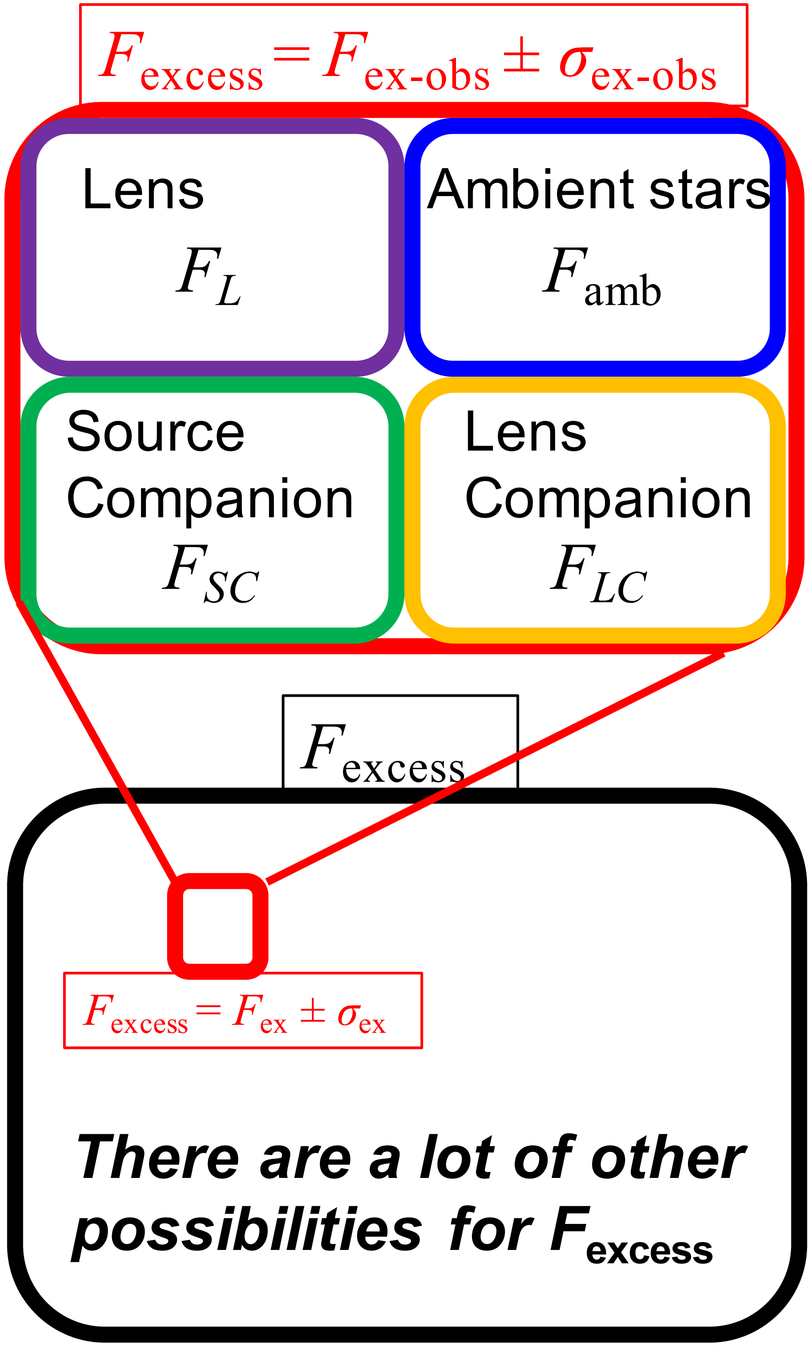

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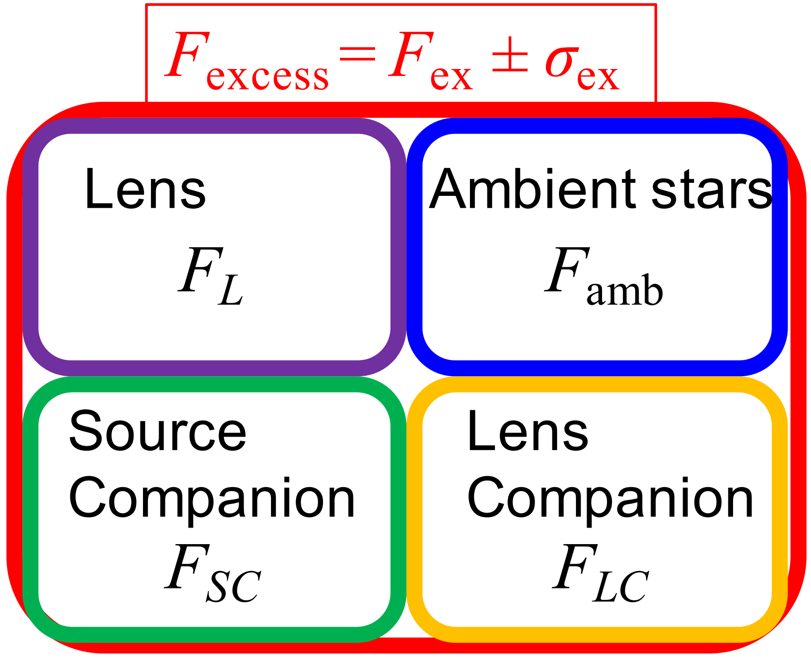

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- $P_{\rm contami} + P(F_L \sim F_{\rm ex})$ is not 1, but $P(F_{\rm excess} \sim F_{\rm ex})$
- What we want to know is not $P(F_L \sim F_{\rm ex})$, but $P(F_L \sim F_{\rm ex} \mid F_{\rm excess} \sim F_{\rm ex})$

$$P(F_L \sim F_{\rm ex} \mid F_{\rm excess} \sim F_{\rm ex}) = \frac{P(F_L \sim F_{\rm ex})}{P(F_{\rm excess} \sim F_{\rm ex})} = \frac{P(F_L \sim F_{\rm ex})}{\sum_i P(F_i \sim F_{\rm ex})} \quad i = \underline{L}, {\rm amb}, SC, LC$$

How to Evaluate The Possible Contaminations -Previous approach-

In some previous papers,
(e.g., Janczak+10, Batista+14, Fukui+15, Koshimoto+16)

Considering the four possibilities **separately**,

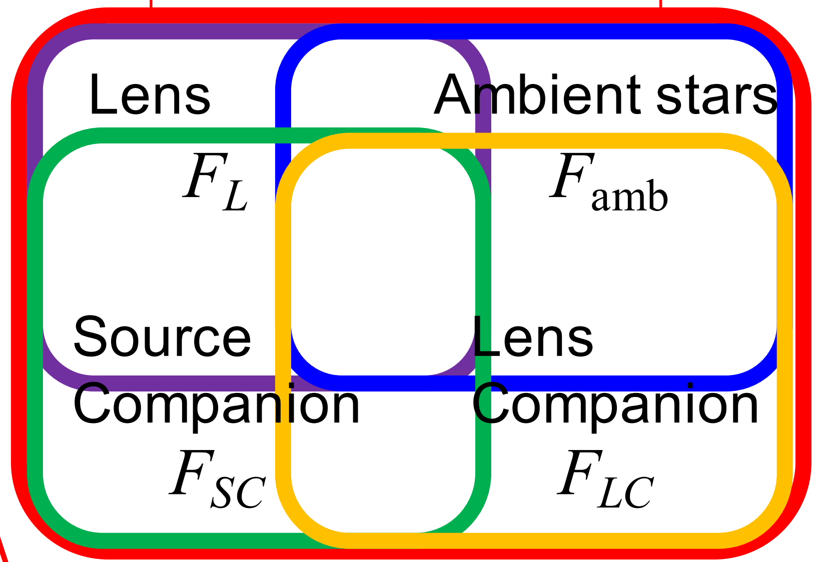
$$P_{\mathrm{contami}} = \sum_i P\,(F_i \sim F_{\mathrm{ex}})$$

$$i = \mathrm{amb},\ SC,\ LC$$

$$P\,(F_L \sim F_{\mathrm{ex}}) = 1 - P_{\mathrm{contami}}$$

where $F_i \sim F_{\mathrm{ex}}$ <=> $F_{\mathrm{ex}} - \sigma_{\mathrm{ex}} < F_i < F_{\mathrm{ex}} + \sigma_{\mathrm{ex}}$

$$F_{\mathrm{excess}} = F_{\mathrm{ex}} \pm \sigma_{\mathrm{ex}}$$



Excess can be came from any combinations!!

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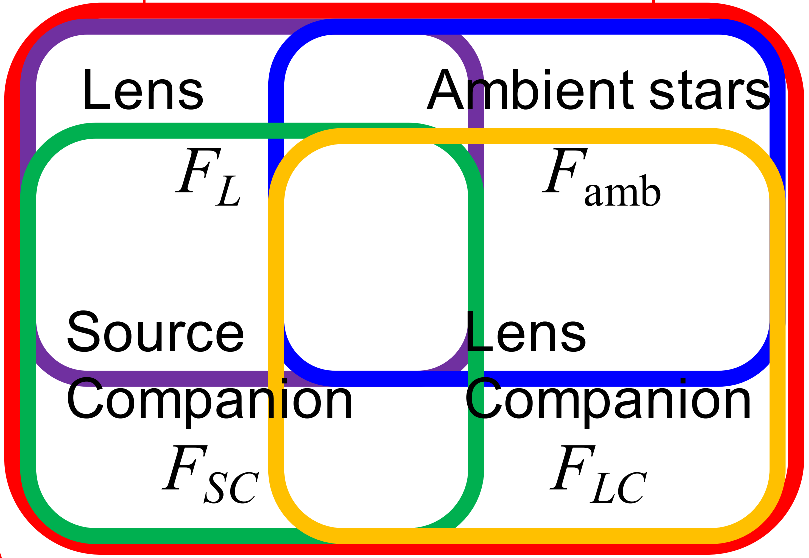
- We should consider all possibilities **simultaneously**

How to Evaluate The Possible Contaminations -Previous approach-

$$F_{\rm excess} = F_{\rm ex} \pm \sigma_{\rm ex}$$



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=> But

- $P_{\rm contami}$ + P ($F_L \sim F_{\rm ex}$) is not 1, but P ($F_{\rm excess} \sim F_{\rm ex}$)
- We should consider all possibilities **simultaneously**

- ~~What we want to know is not P ($F_L \sim F_{\rm ex}$), but P ($F_L \sim F_{\rm ex}$ | $F_{\rm excess} \sim F_{\rm ex}$)~~
- What we really want to know is probability distribution of the lens flux (mass): P (F_L | $F_{\rm excess} \sim F_{\rm ex}$)

Assumptions for The Prior Probability Distributions

We used similar prior probability distributions to those of the previous papers

Undetectable regions (s_i : separation between i and the lens)

Ambient stars : $s_{\rm amb} < 148$ mas ($= 0.8 \times 184$ mas)

Source companion: 0.058 mas $< s_{SC} < 148$ mas

Lens companion: ~ 1.5 mas $< s_{LC} < 148$ mas

Table 1. Assumptions and detectable limits used for the prior distributions

Prior probability for	Assumption	Closer limit	Wider limit	Used observed value	Paper for the assumption
H_L (Lens)	Galactic model	–	–	$t_{\rm E}$, $\theta_{\rm E}$	Han & Gould (2003)
$H_{\rm amb}$ (Ambient stars)	Luminosity function	–	0.8 FWHM	FWHM, Number density	Zoccali et al. (2003)
H_{SC} (Source companions)	Binary distribution	$\theta_E/4$	0.8 FWHM	FWHM, θ_E, H_S	Duchêne & Kraus(2013)
H_{LC} (Lens companions)	Binary distribution	w_c[1] $< u_0$	0.8 FWHM	FWHM, θ_E, H_L, u_0	Duchêne & Kraus(2013)

[1] The size of the caustic created by the hypothetical companion to the lens, $w_c = 4q_c/(s_c - s_c^{-1})^2$.

Table 2. Details of the binary distribution (Duchéne & Kraus 2013)

Parameter	Formula	Primary mass dependency
Multiplicity Fraction MF	-	$MF = 0.20 + 0.26 \times M$
Semi-major axis a	log normal, $N(\mu_{\log a}, \sigma^2_{\log a})$	$\mu_{\log a} = 0.57 + 1.01 \times M$
		$\sigma_{\log a} = 1.6 + 1.2 \times \log M$
Mass ratio q for $\log a < \mu_{\log a}$	Power law, $\propto q^{\gamma_c}$	$\gamma_c = 1.2 - 2.8 \times \log M$
Mass ratio q for $\log a > \mu_{\log a}$	Power law, $\propto q^{\gamma_w}$	$\gamma_w = 0$ (for $M > 0.34$)
		$\gamma_w = -3.1 - 6.7 \times \log M$ (for $M < 0.34$)