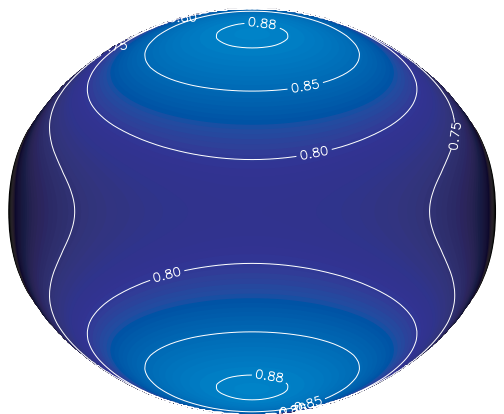
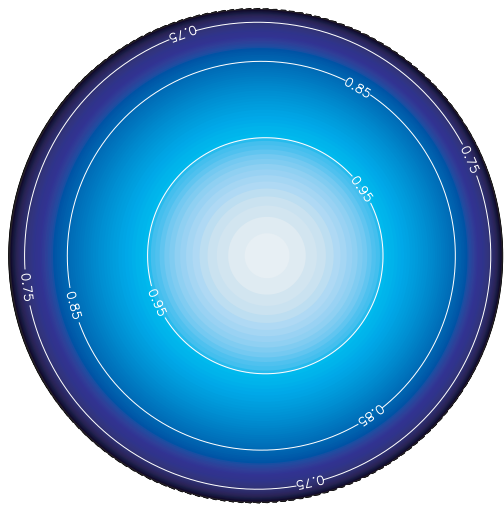
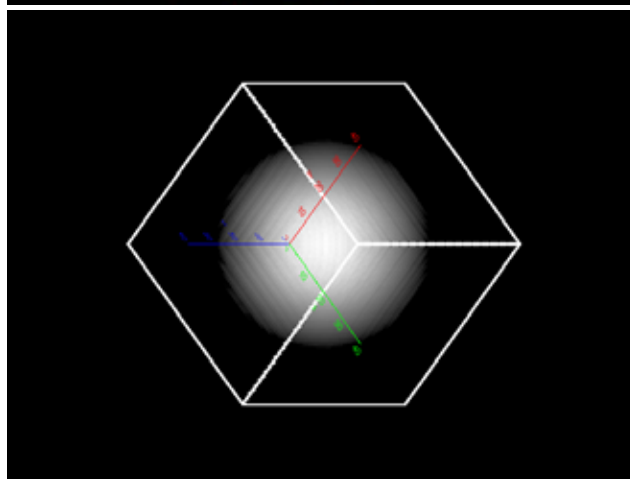
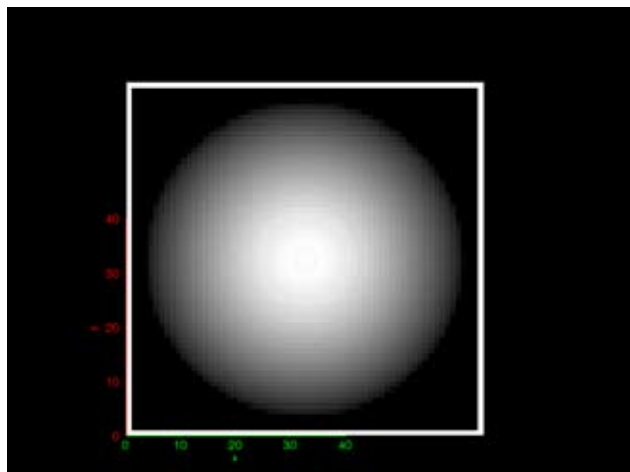


# What's the Matter with Vega?: The Next Generation of Model Atmospheres for Rapidly Rotating Stars

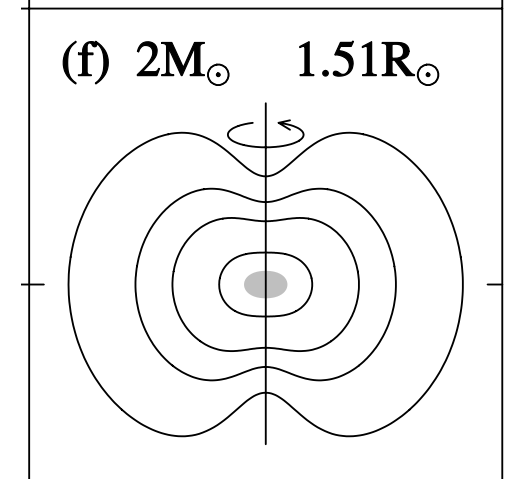
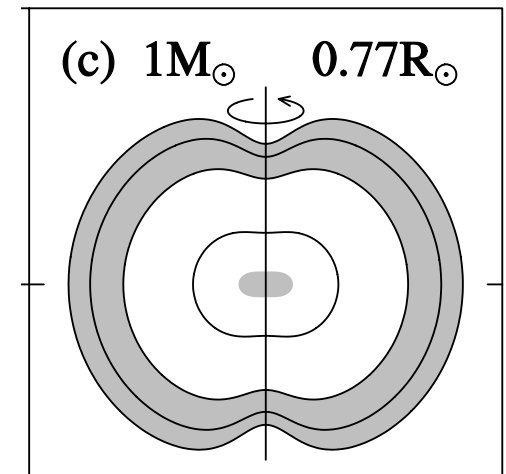
Jason Aufdenberg  
Embry-Riddle Aeronautical University



Aufdenberg et al. (2006) ApJ, 645, 664

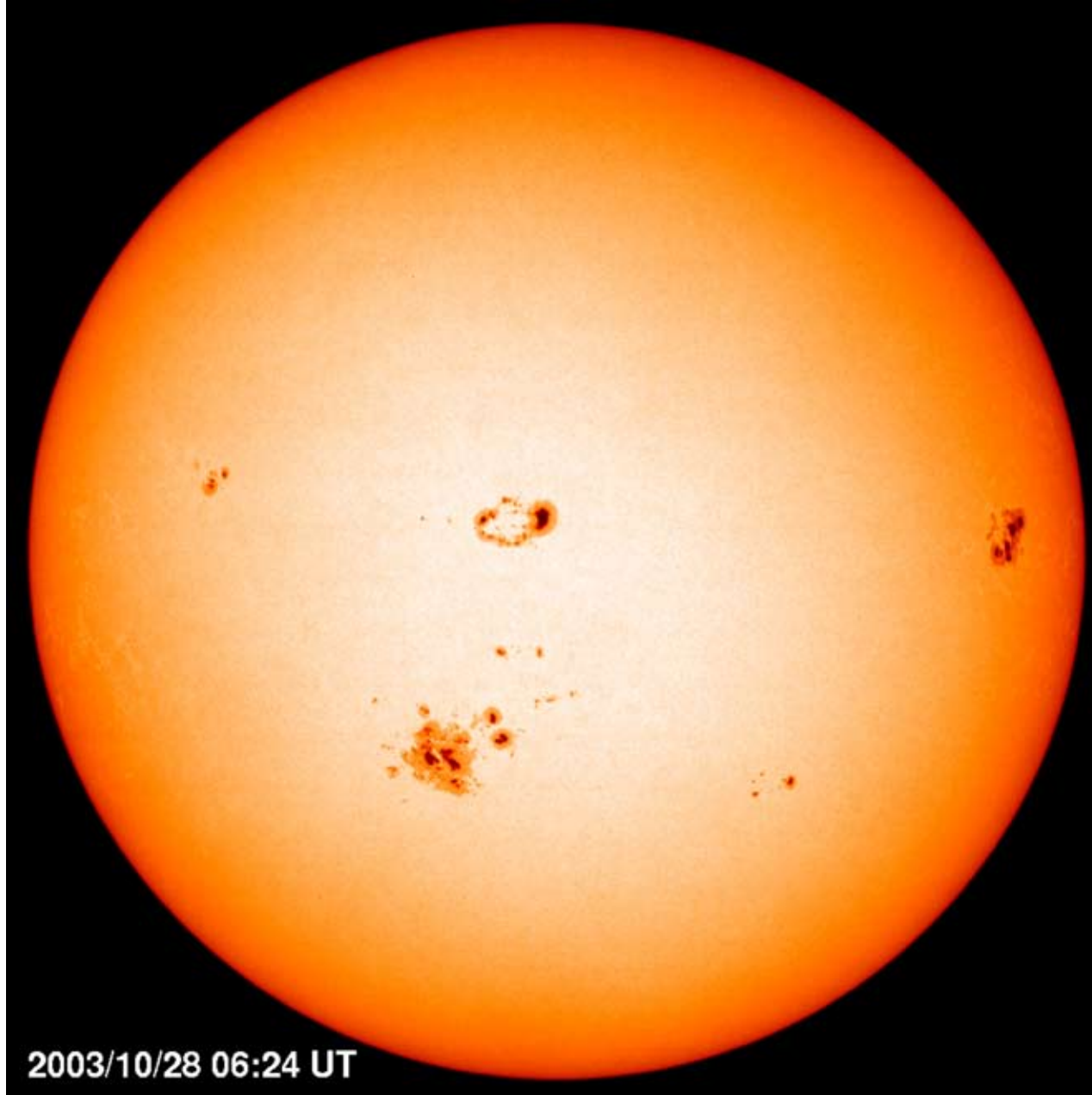


Hauschildt, P. H. and Baron, E. (2006) A&A, 451, 273



MacGregor et al. (2007) ApJ, 663, 560

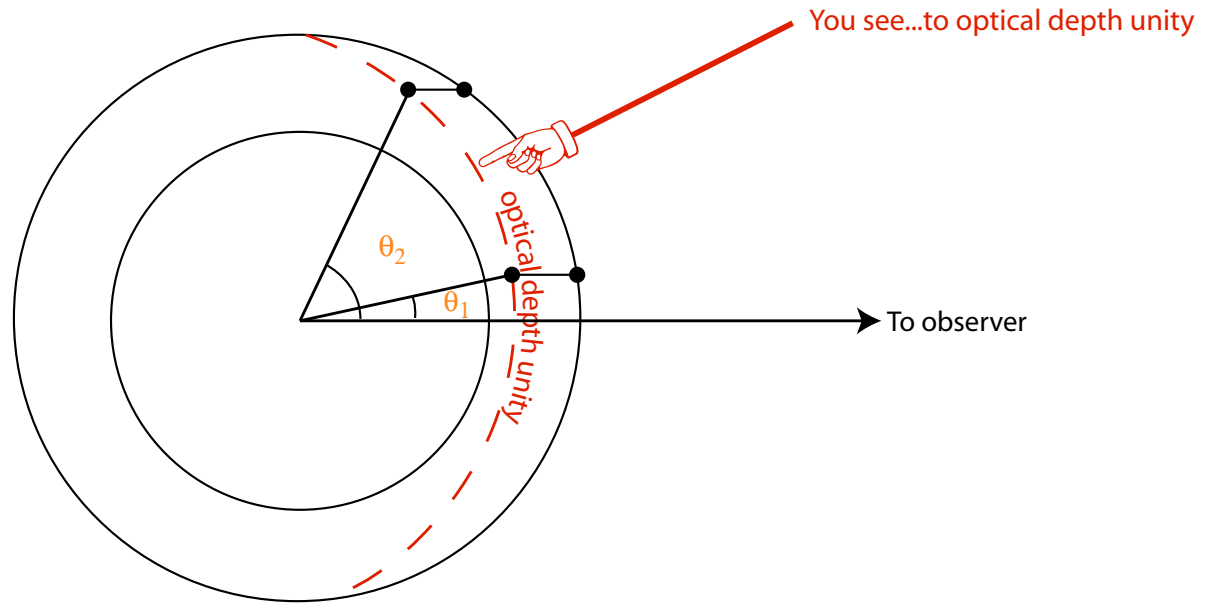
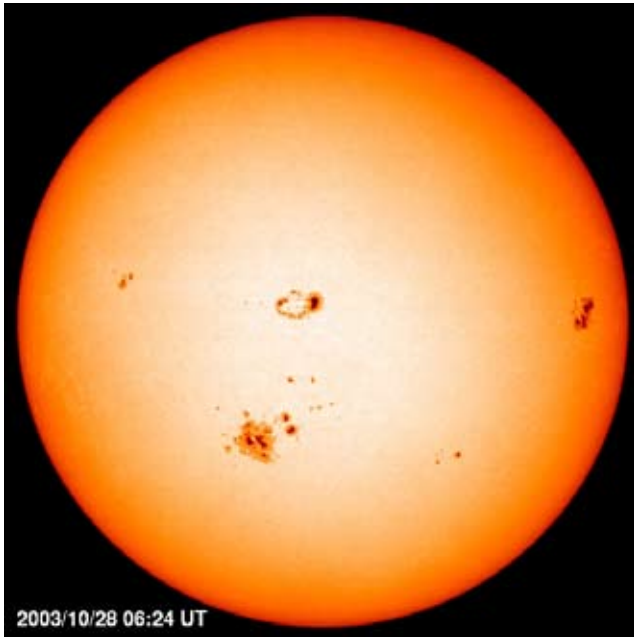
# Solar and Heliospheric Observatory (SOHO) Michelson Doppler Imager (MDI)



**Very narrow  
band  
Ni I 6768 Å**

<http://sohowww.nascom.nasa.gov/bestofsoho/images/large/mdi20031028.jpg>

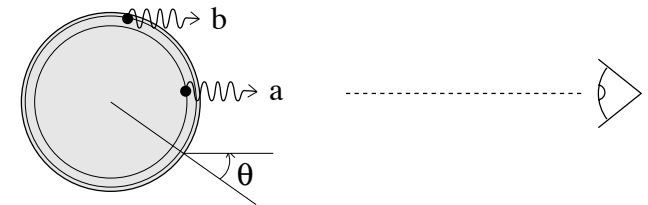
# Limb Darkening Basics



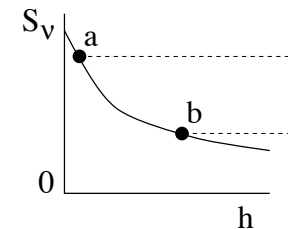
(a) Deeper, hotter layers are visible near the disk center

(b) Shallower, cooler layers are visible near the disk limb

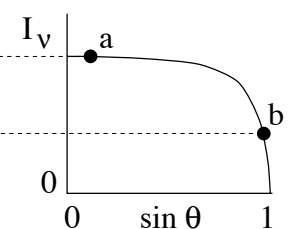
*isothermal atmospheres do not exhibit limb darkening*



Rutten (2003)

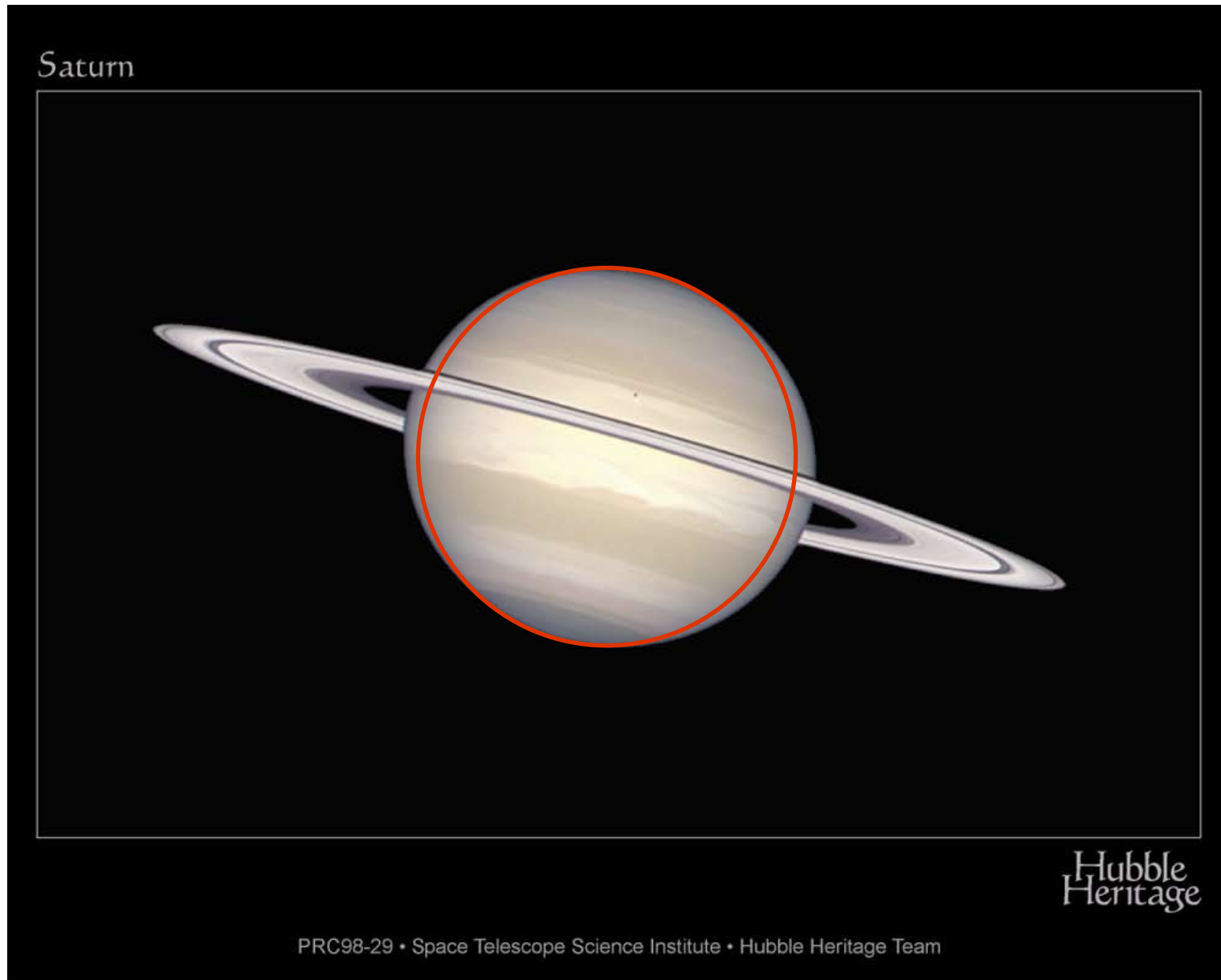


source function as a function of depth



center to limb intensity profile

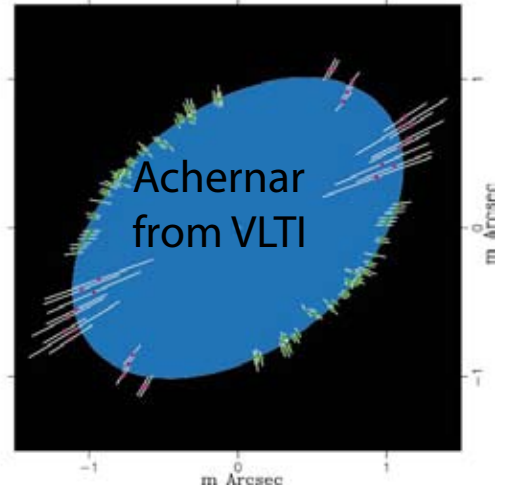
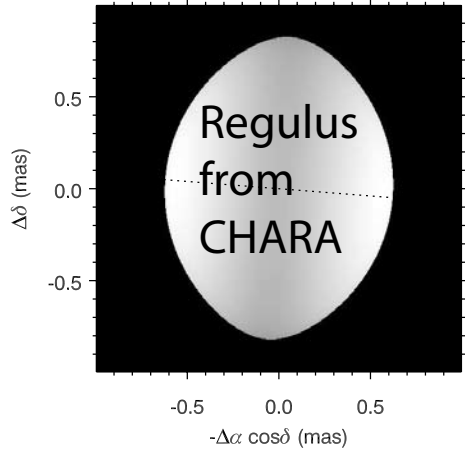
# Oblate Saturn Viewed from the Hubble Space Telescope



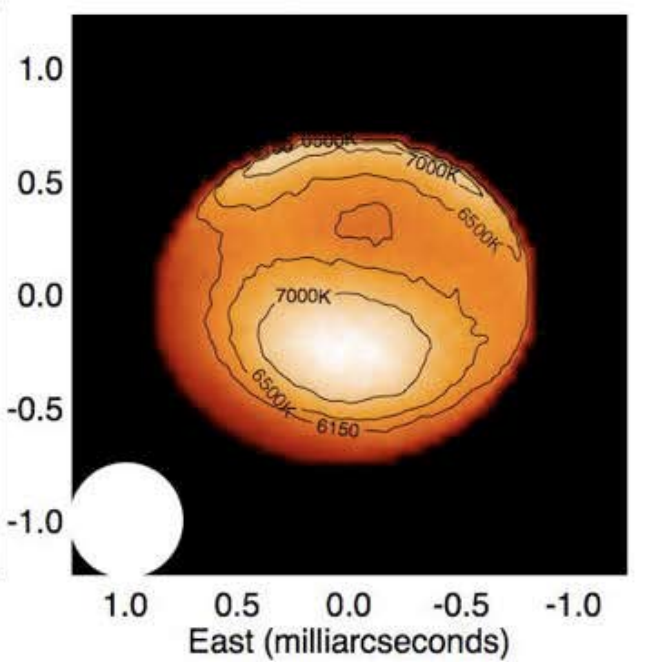
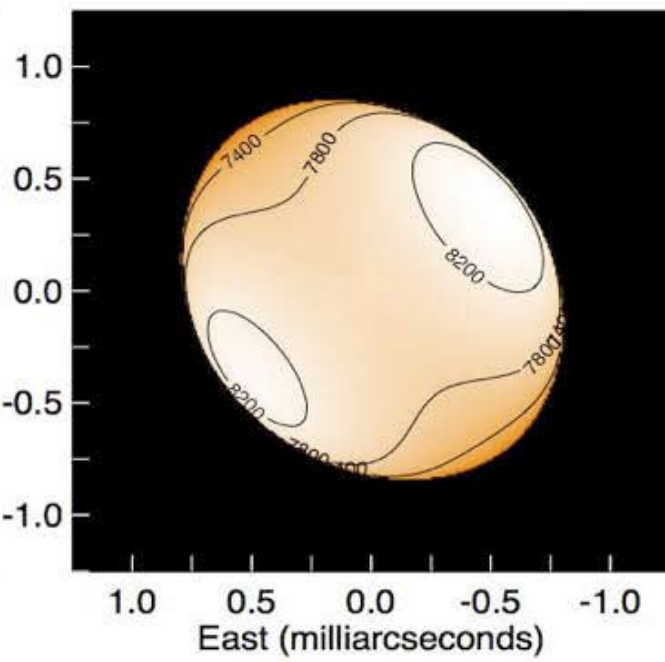
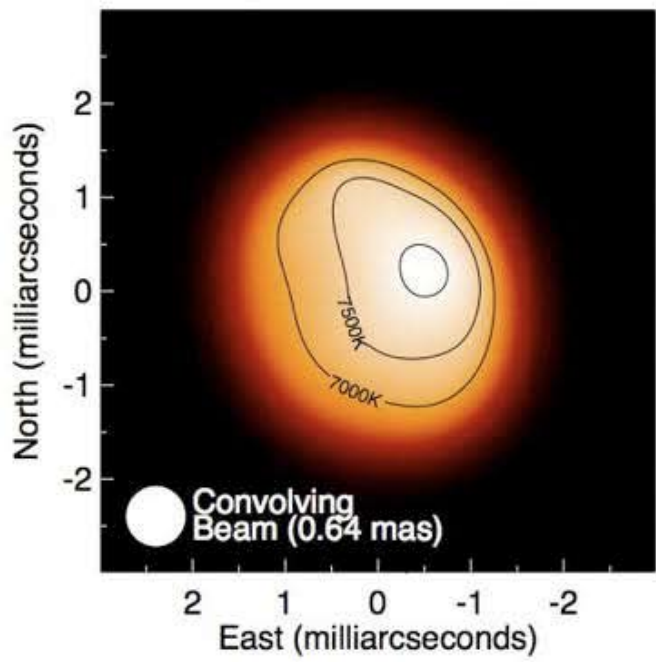


# Resolved Rapid Rotating Stars from Interferometry

\*Disk of **Regulus** (B7 V) resolved as ellipsoid by CHARA (McAlister et al. 2005). Axial ratio:  $1.32 \pm 0.02$



\*Disk of **Achernar** (B3 Vpe) resolved as ellipsoid by VLTI (A. Domiciano de Souza et al. 2003). Axial ratio:  $1.56 \pm 0.05$



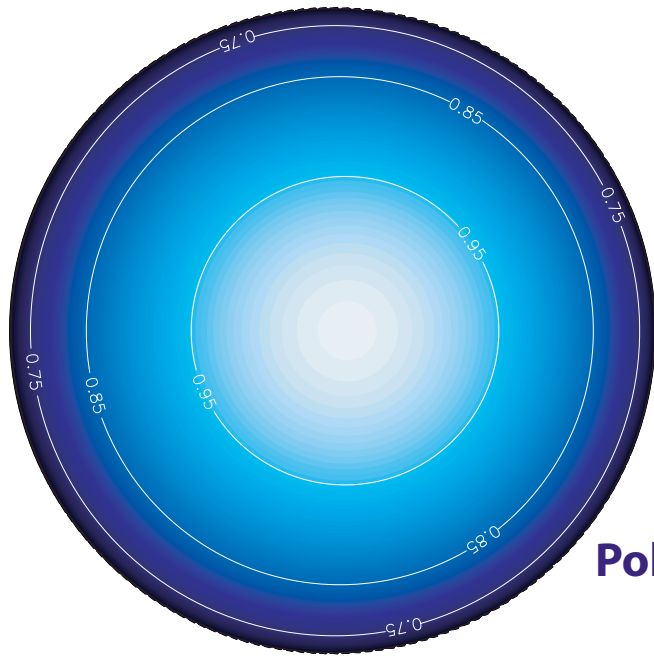
\*Disk of **Altair** (A7 V) resolved by CHARA (J. Monnier et al. 2007).

\*Disk of **Rasalhague** (A5 III) resolved by CHARA (M. Zhao et al. 2009).

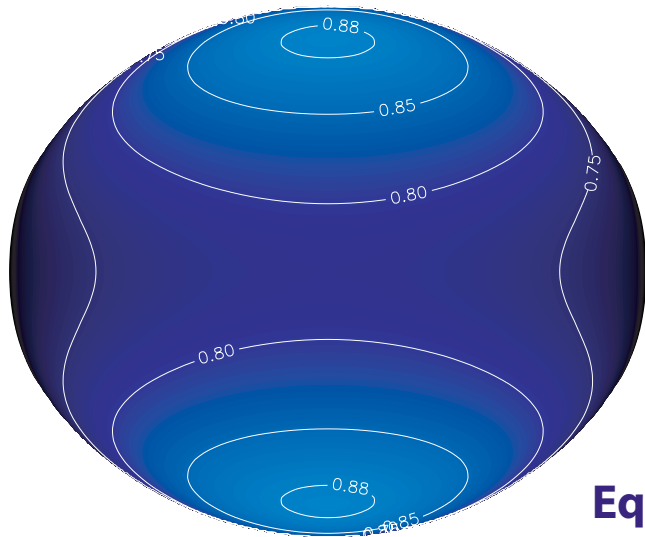
\*Disk of **Alderamin** (A7 V) resolved by CHARA (M. Zhao et al. 2009).

# Gravity Darkening vs. Limb Darkening

## Rapidly Rotating Model with Intensity Contours



Pole-on view



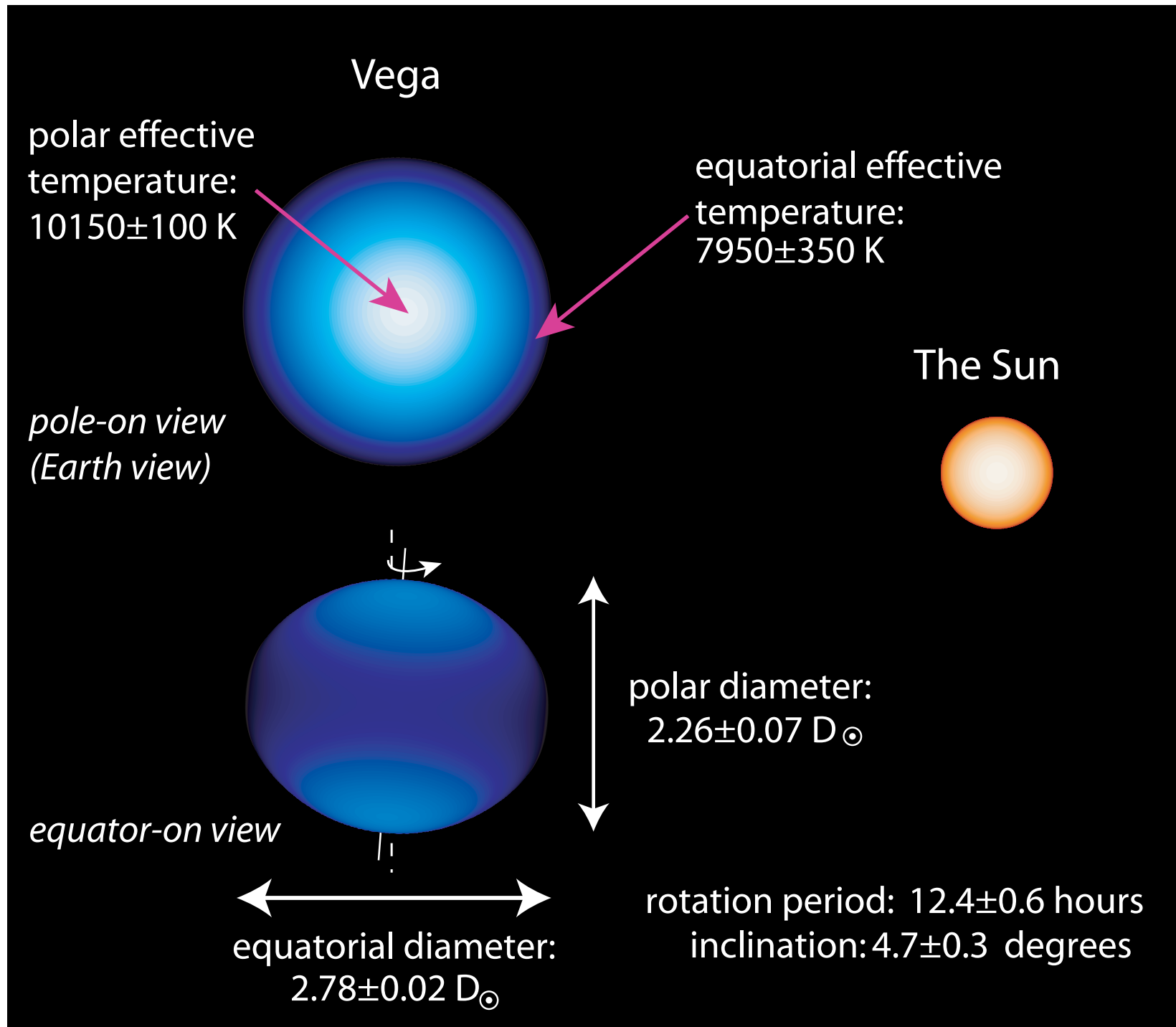
Equator-on view

**Gravity darkening:** *Intrinsic to the star, a pole-to-equator effective temperature gradient resulting from rapid rotation. Local  $T_{\text{eff}}$  on surface correlates with local gravity (e.g.,  $T_{\text{eff}} \propto g^{1/4}$ )*

**Limb darkening:** *An observer-dependent effect in which the intensity across a stellar surface varies due to a radial or depth dependent temperature gradient.*

Aufdenberg et al. (2006) ApJ, 645, 664

# A Pole-on Rapid Rotator: Vega (Aufdenberg et al. 2006)

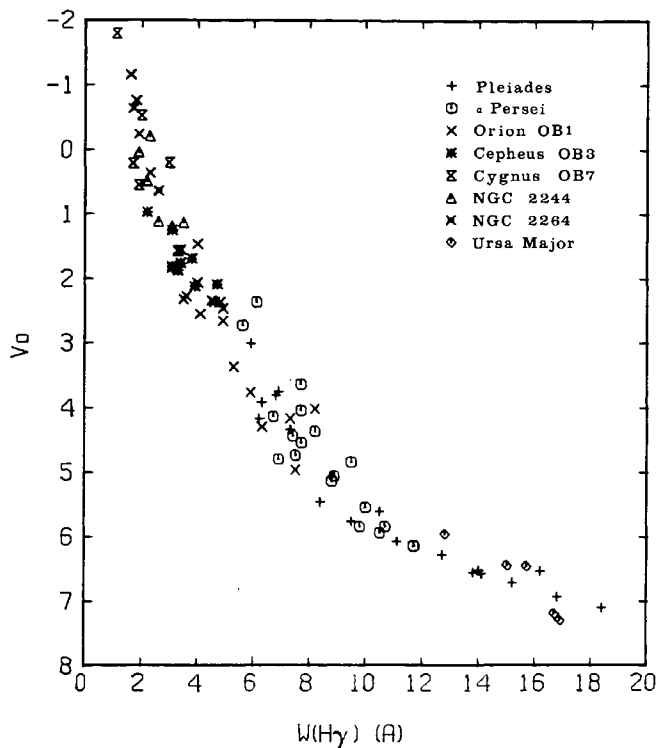
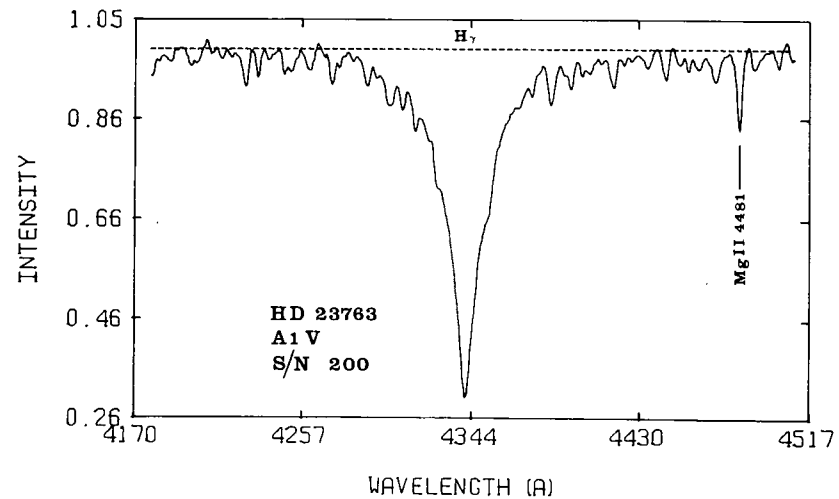


# Vega is 47% more luminous than expected for its spectral type (A0 V)

## AN EMPIRICAL $H\gamma$ LUMINOSITY CALIBRATION FOR CLASS V–III STARS

(1985)

CHRISTOPHER G. MILLWARD AND GORDON A. H. WALKER  
Geophysics and Astronomy Department, University of British Columbia



Vega brighter than mean  $W(H\gamma) - M_V$  relation

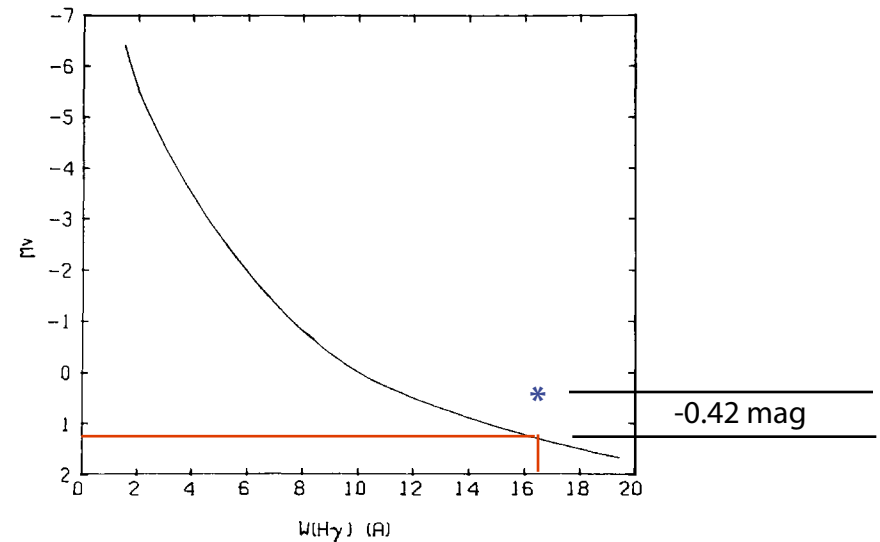
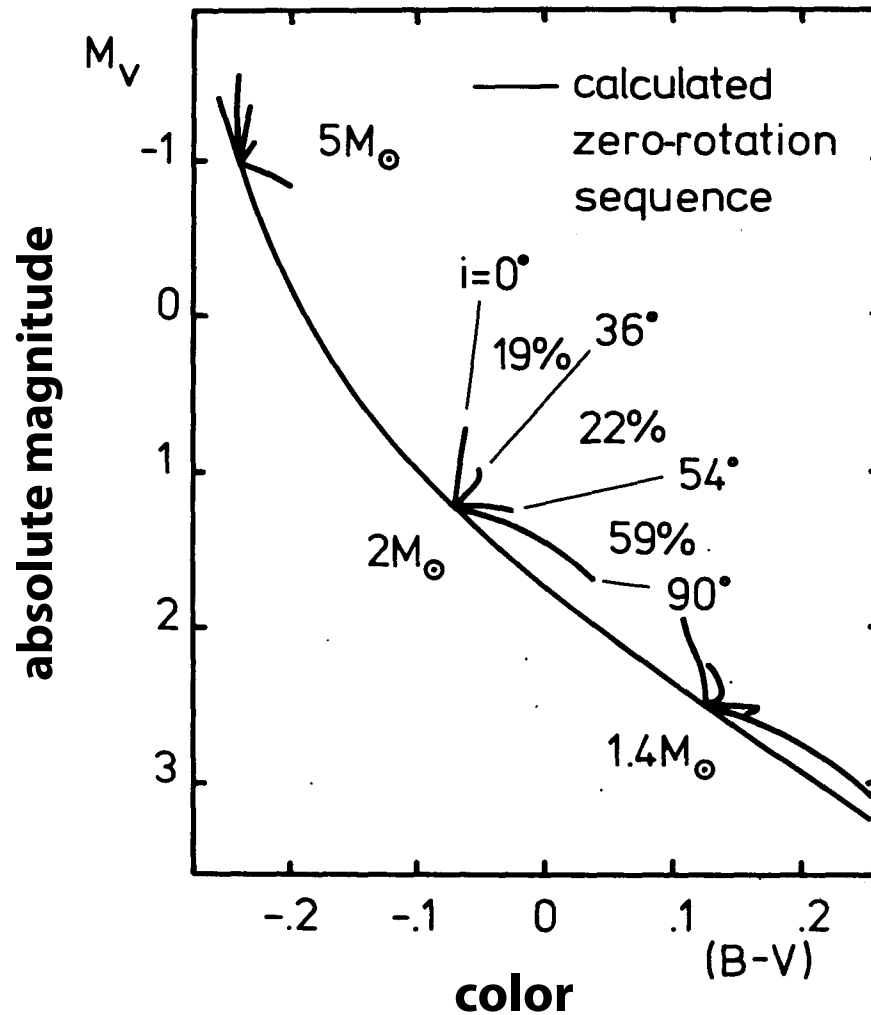


FIG. 6.—Mean  $W(H\gamma) - M_v$  relation

# Vega is a Pole-on Rapid Rotator (R. Gray 1985, 1988)

Pole-on rapid rotators appear more luminous, have same color as slow rotators



From Tassoul (1978) "Theory of Rotating Stars"  
originally from Maeder & Peytremann (1970)



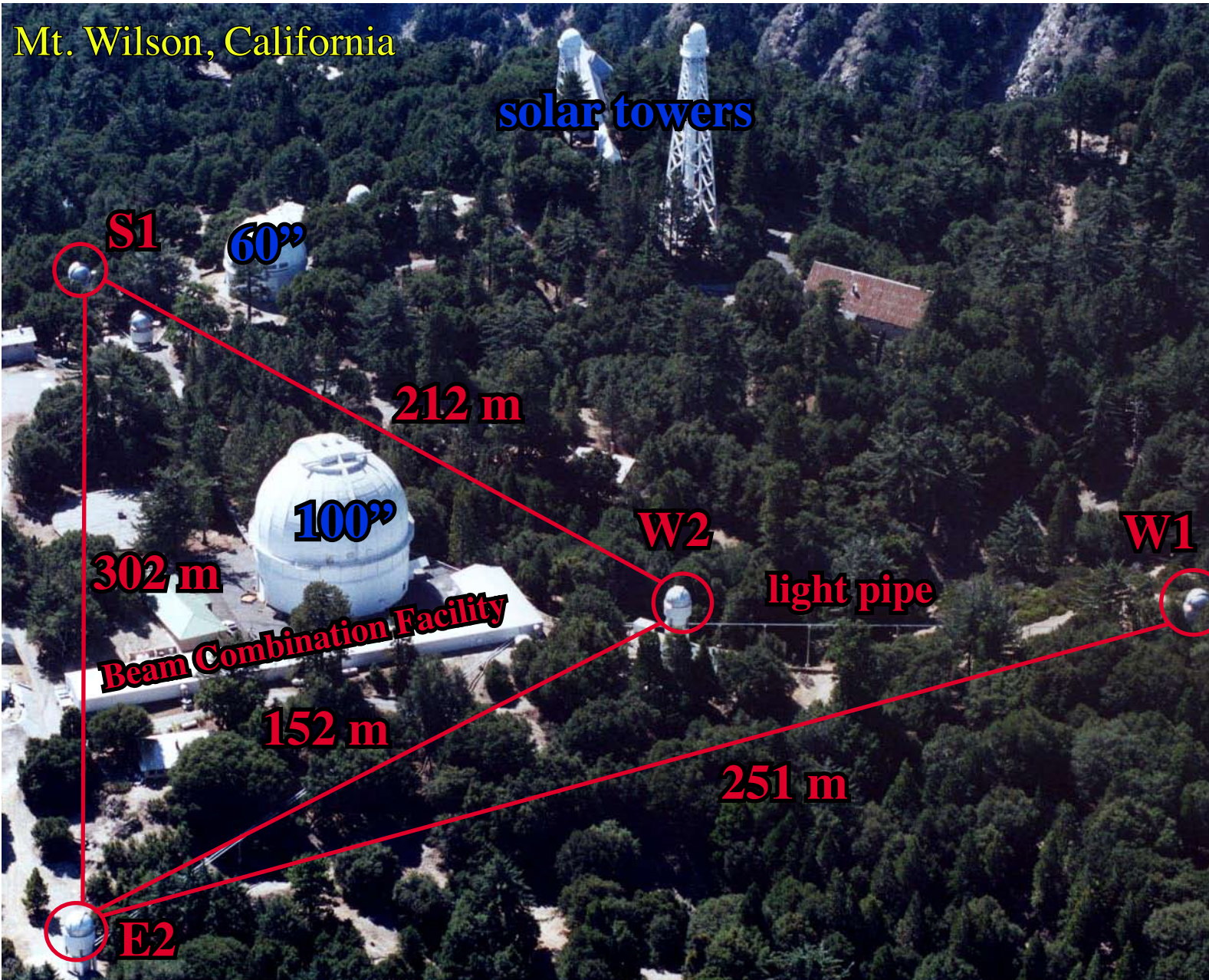
# CHARA (Center for High Angular Resolution Astronomy) Array

Vega  
Observations  
May, June 2005

K-band  
 $\lambda = 2.2 \mu\text{m}$

Fiber  
Linked  
Unit for  
Optical  
Recombination

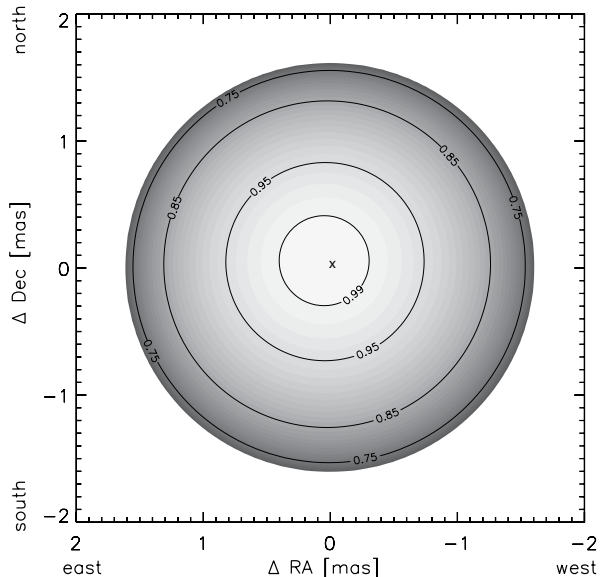
beam combiner



<http://www.chara.gsu.edu/CHARA/Slides/CHARAoverview.pdf>



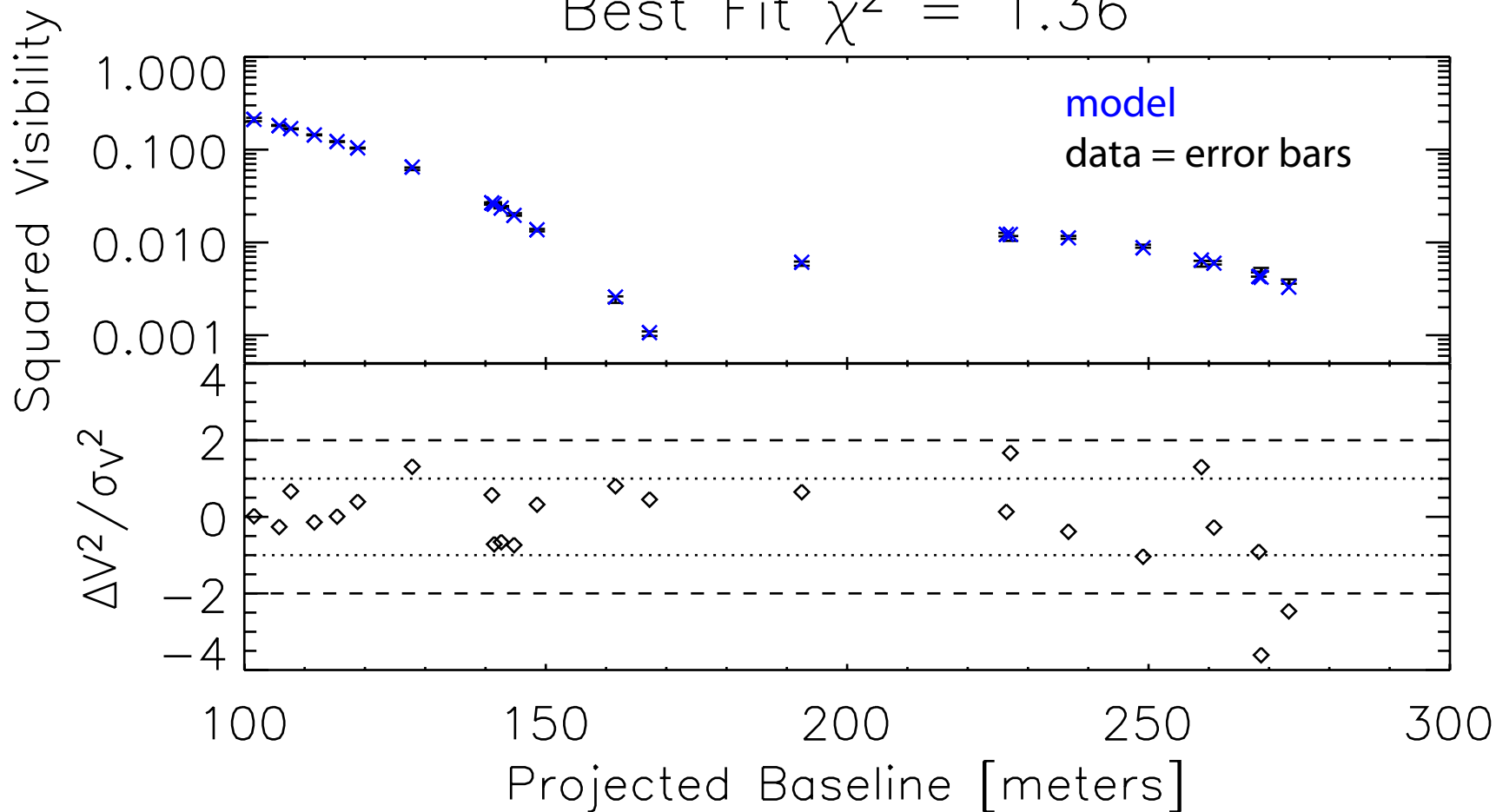
# A Rotating Model Fit to the Visibility Data



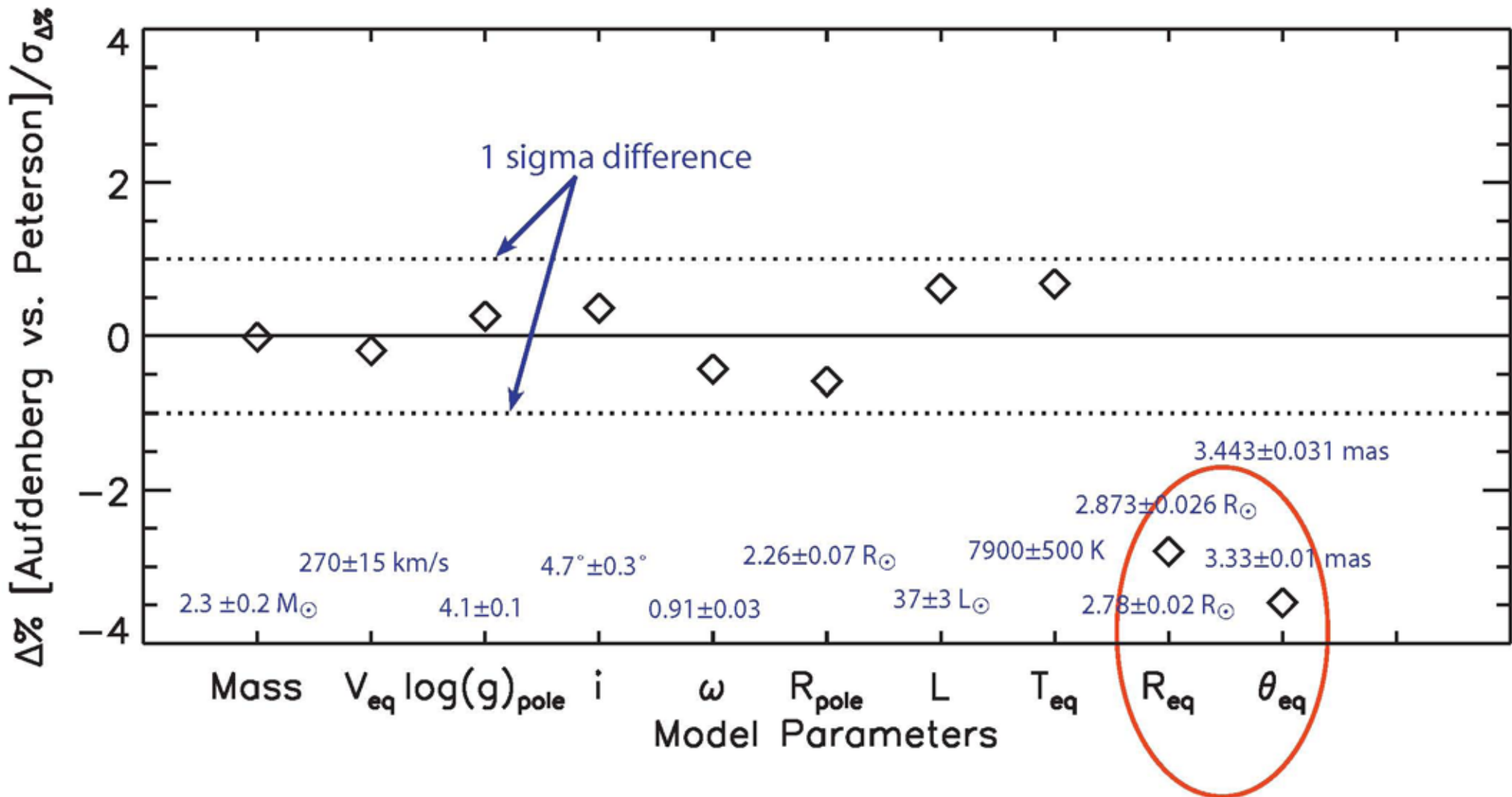
## Best Fit Parameters

$T_{\text{eff}}$ (pole)	= 10250 K	B9 spectral type
$T_{\text{eff}}$ (equator)	= 7900 K	A8 spectral type
Log(g) (pole)	= 4.10	
$\omega$ (angular break-up fraction)	= $0.91 \pm 0.03$	
$\theta_{\text{equator}}$	= $3.33 \pm 0.03$ milliarcsecs	

Best Fit  $\chi^2 = 1.36$



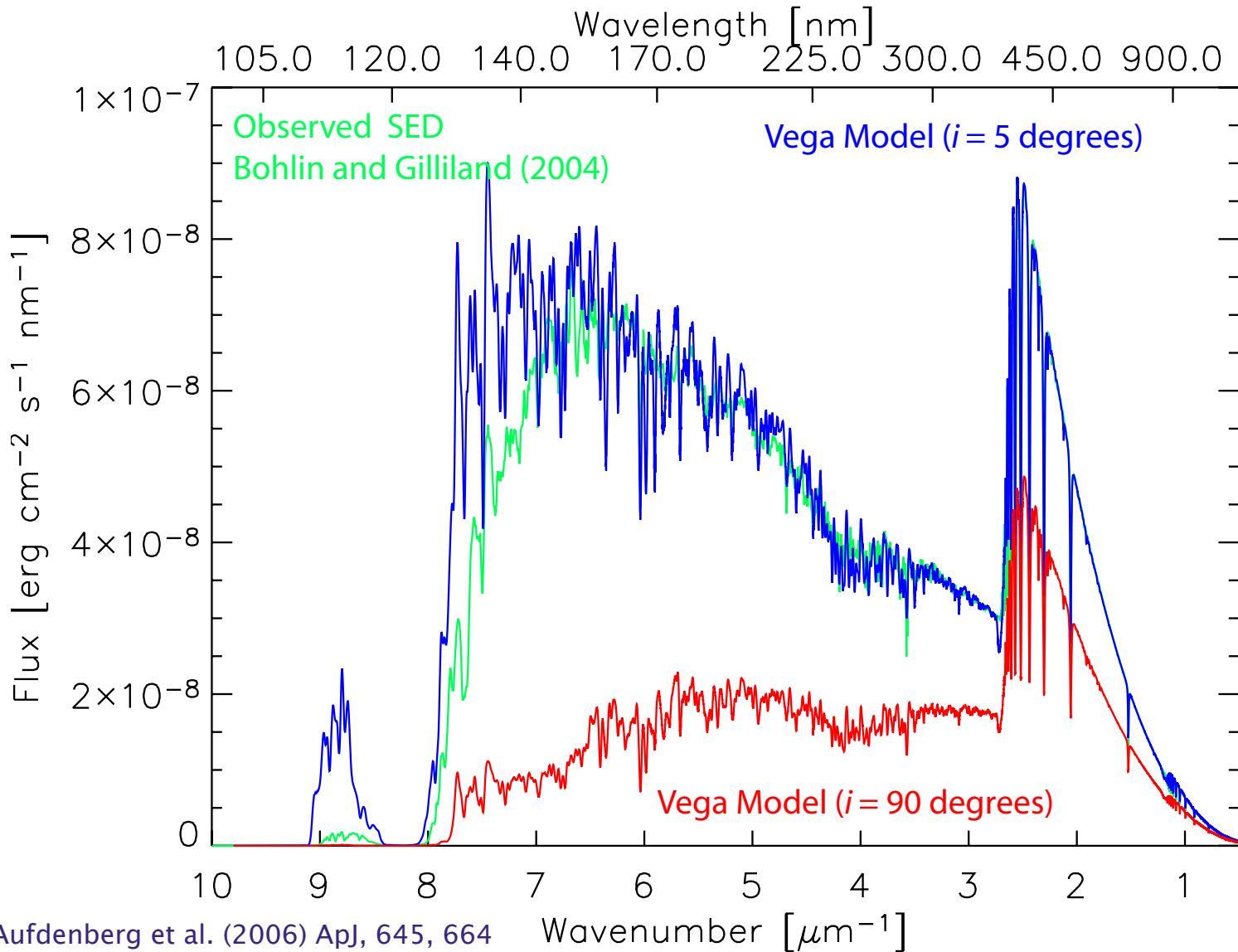
# A Comparison of Parameters for Vega from Aufdenberg et al. and Peterson et al.



The equatorial radii derived from CHARA (IR) and NPOI (Visible) data are inconsistent

# Vega's Spectrum from Different Points of View

$T_{\text{eff}}$  (pole) = 10250 K B9 spectral type  
 $T_{\text{eff}}$  (equator) = 7900 K A8 spectral type

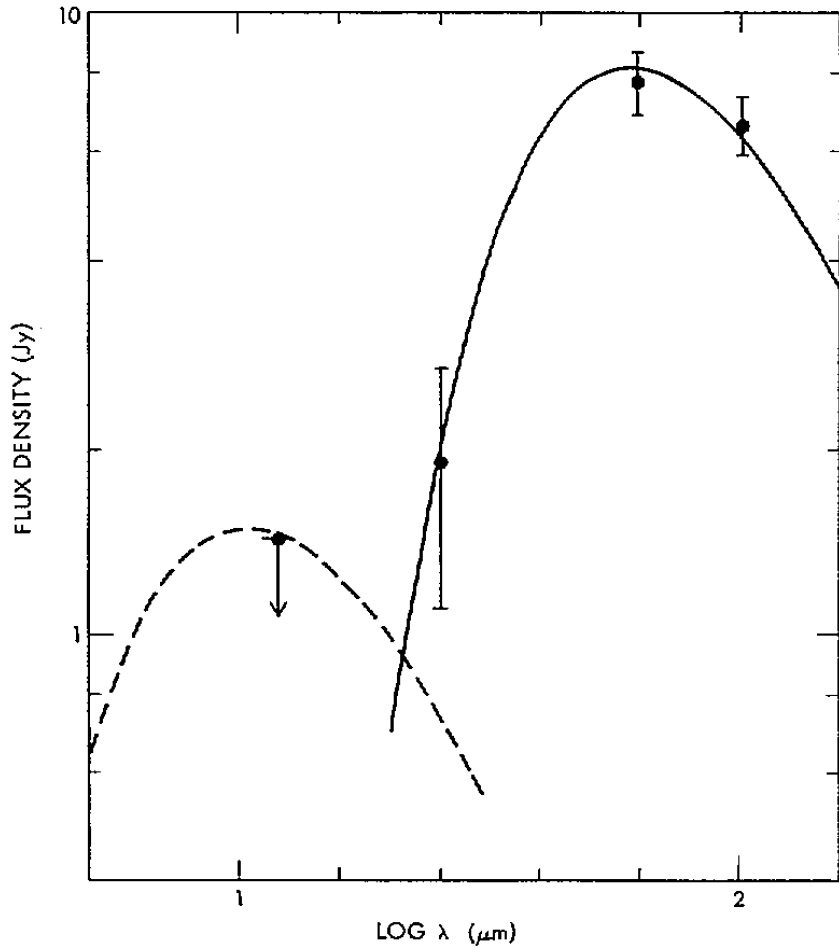


Aufdenberg et al. (2006) ApJ, 645, 664

Wavenumber [ $\mu\text{m}^{-1}$ ]

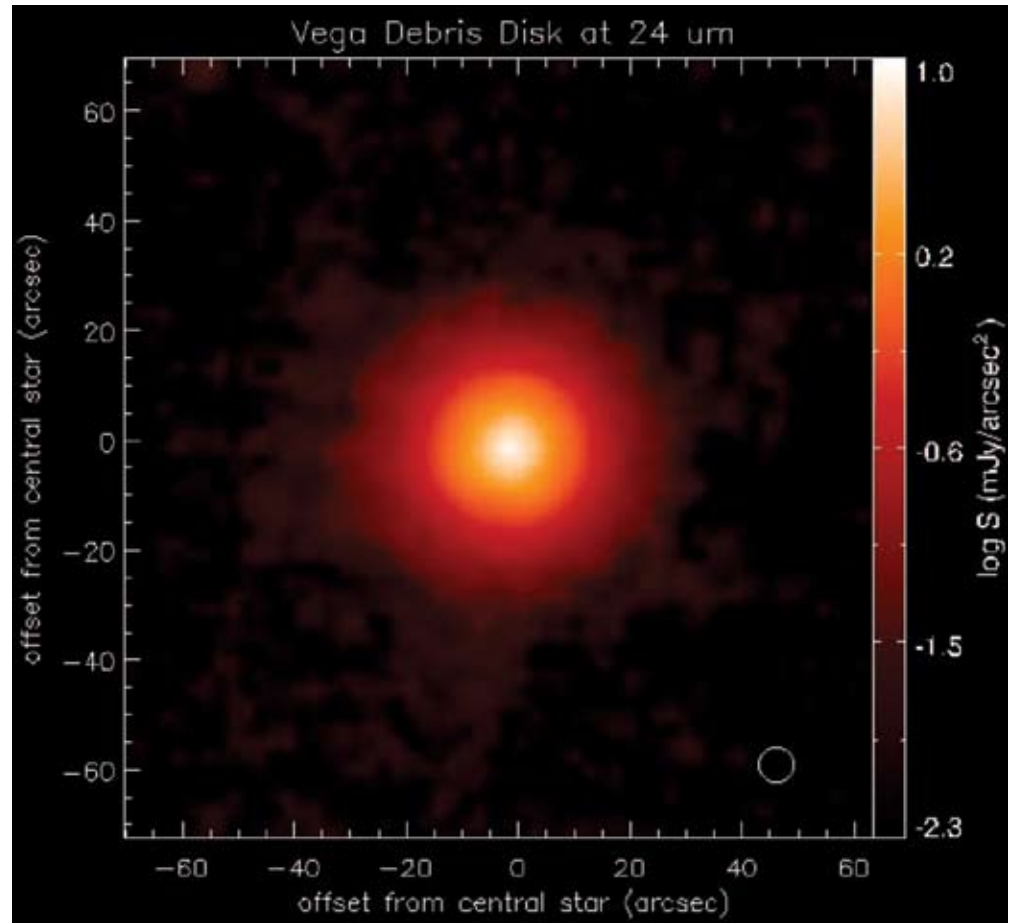
# Vega's Disk

IRAS



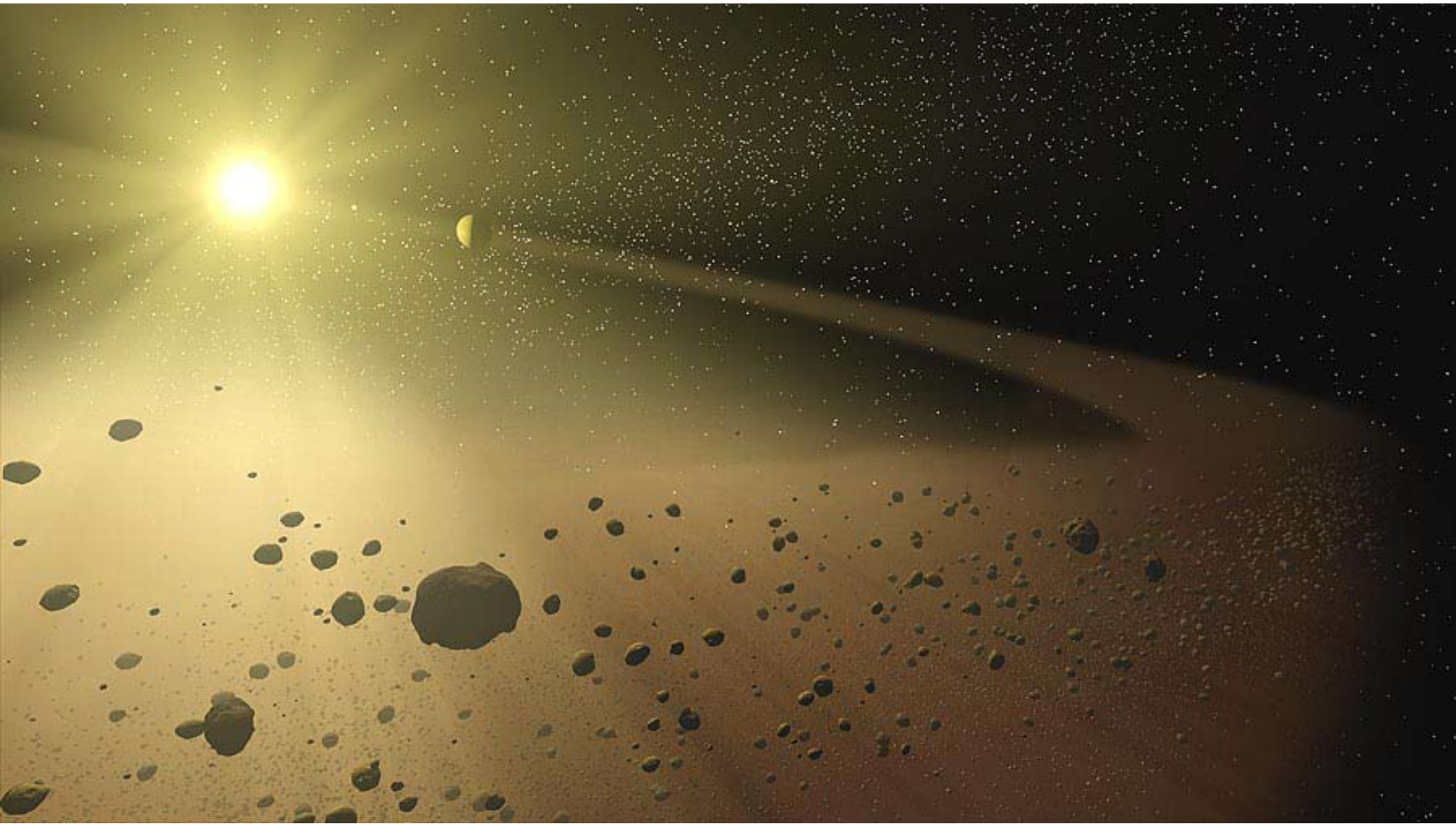
Aumann et al. (1984)

Spitzer Space Telescope



Su et al. (2005)

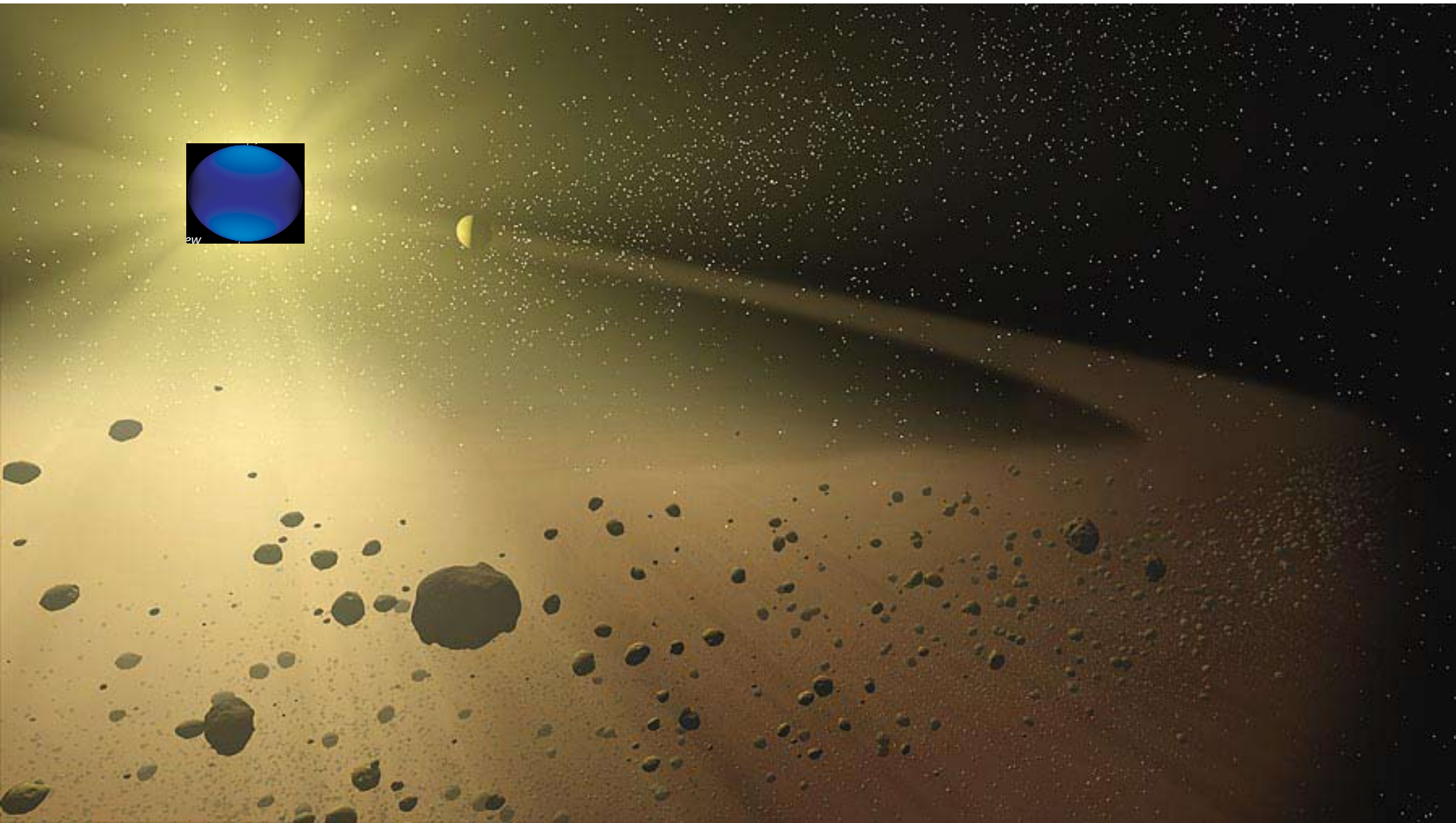
## A Debris Disk Around a Young Star (artist conception)



NASA/JPL-Caltech/T. Pyle



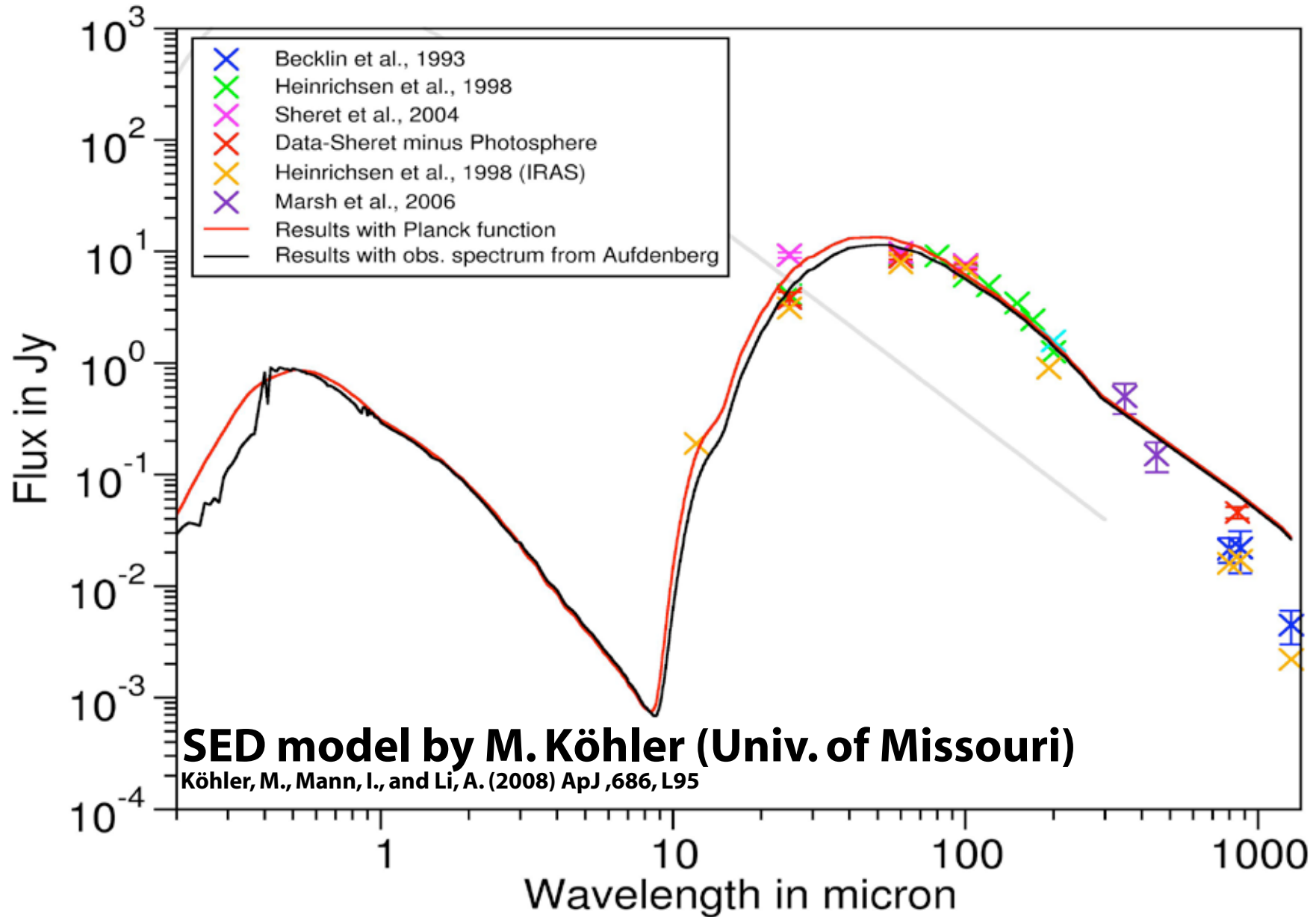
# A Debris Disk Around a Rapidly Rotating Young Star



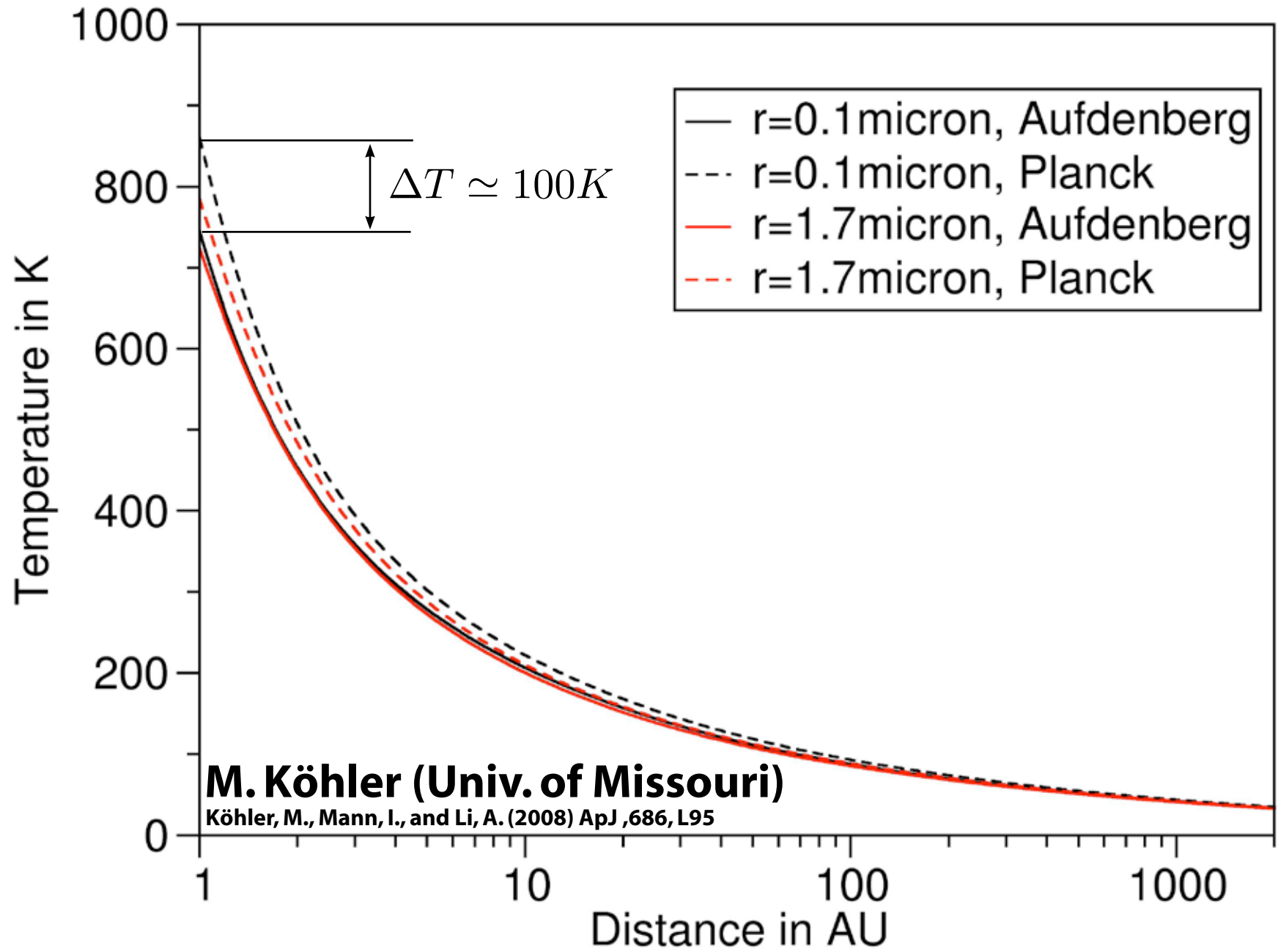
NASA/JPL-Caltech/T. Pyle



# Amorphous Silicate Illuminated by Vega (Equatorial) Model Spectrum



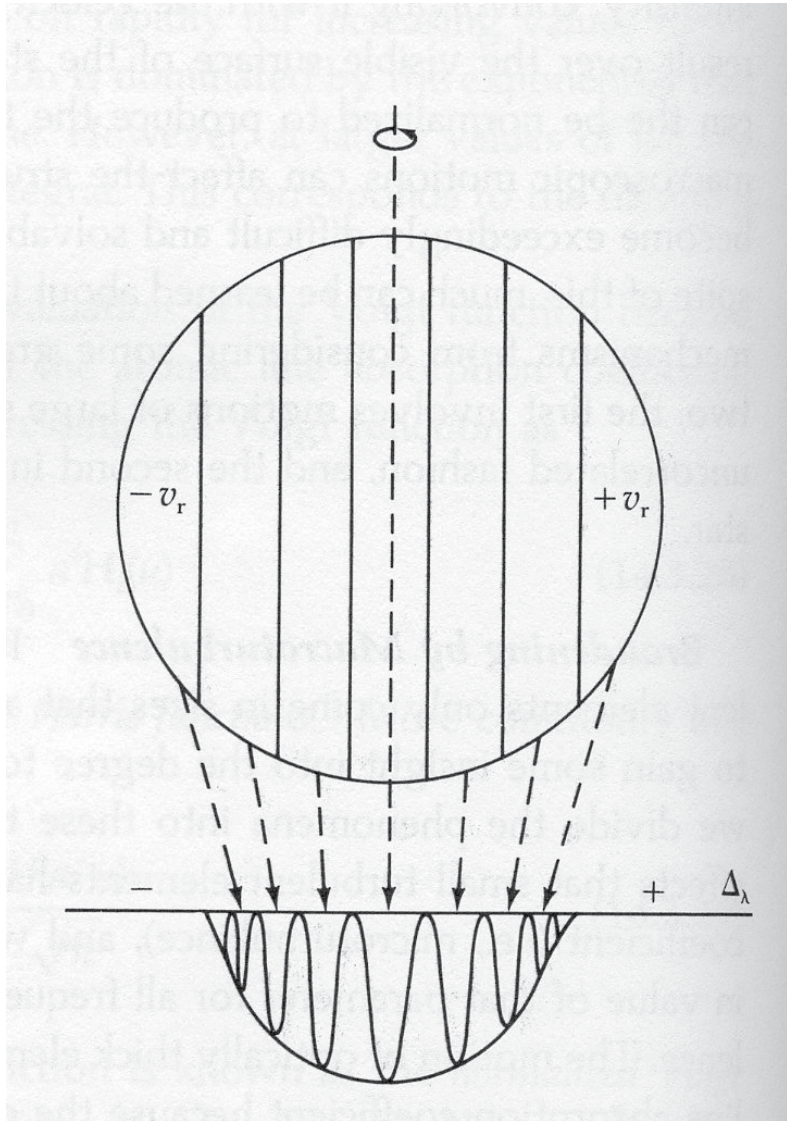
# Amorphous Silicate Illuminated by Vega (Equatorial) Model Spectrum



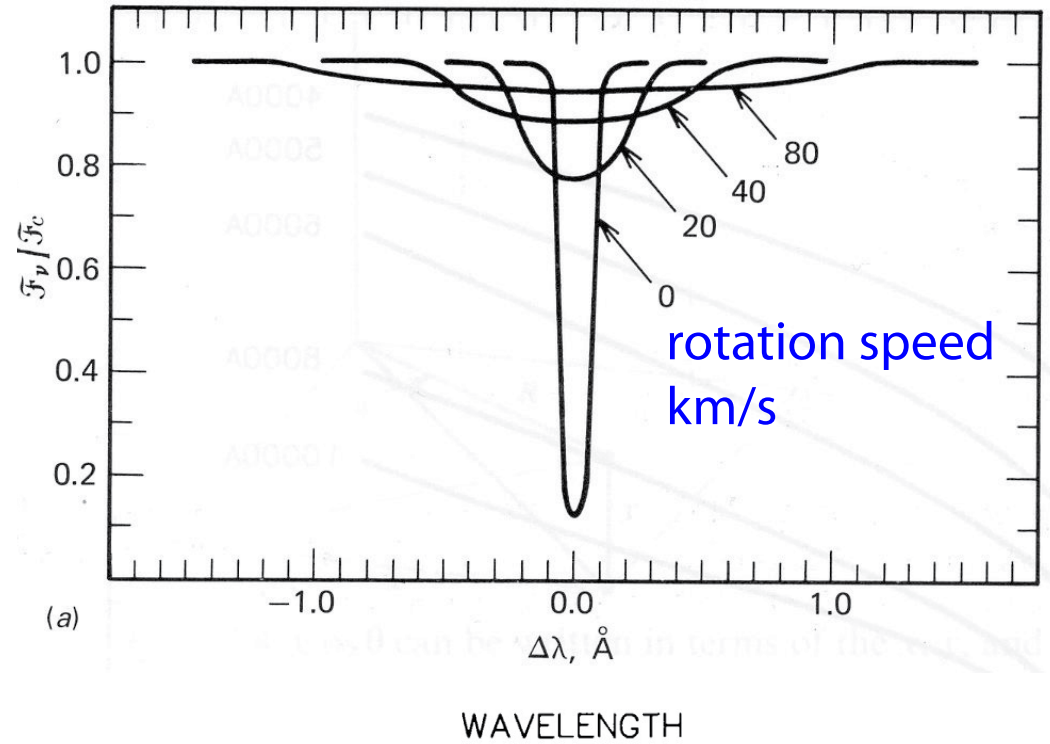
**M. Köhler (Univ. of Missouri)**  
Köhler, M., Mann, I., and Li, A. (2008) *ApJ*, 686, L95

# Clues from Rotational Broadening of Spectral Lines

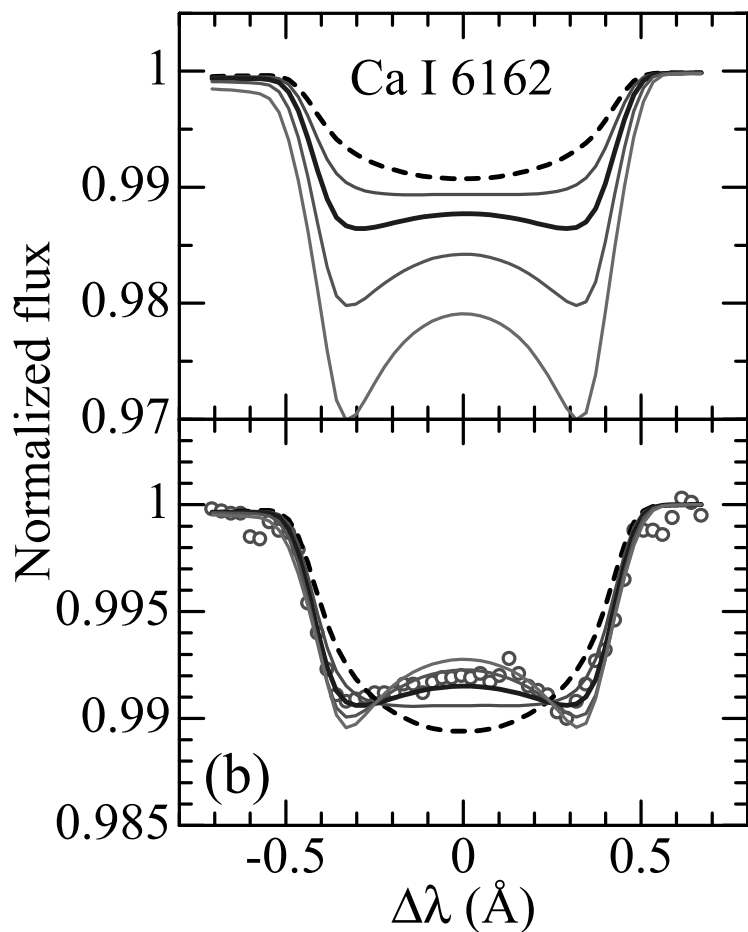
From Collins (1989)



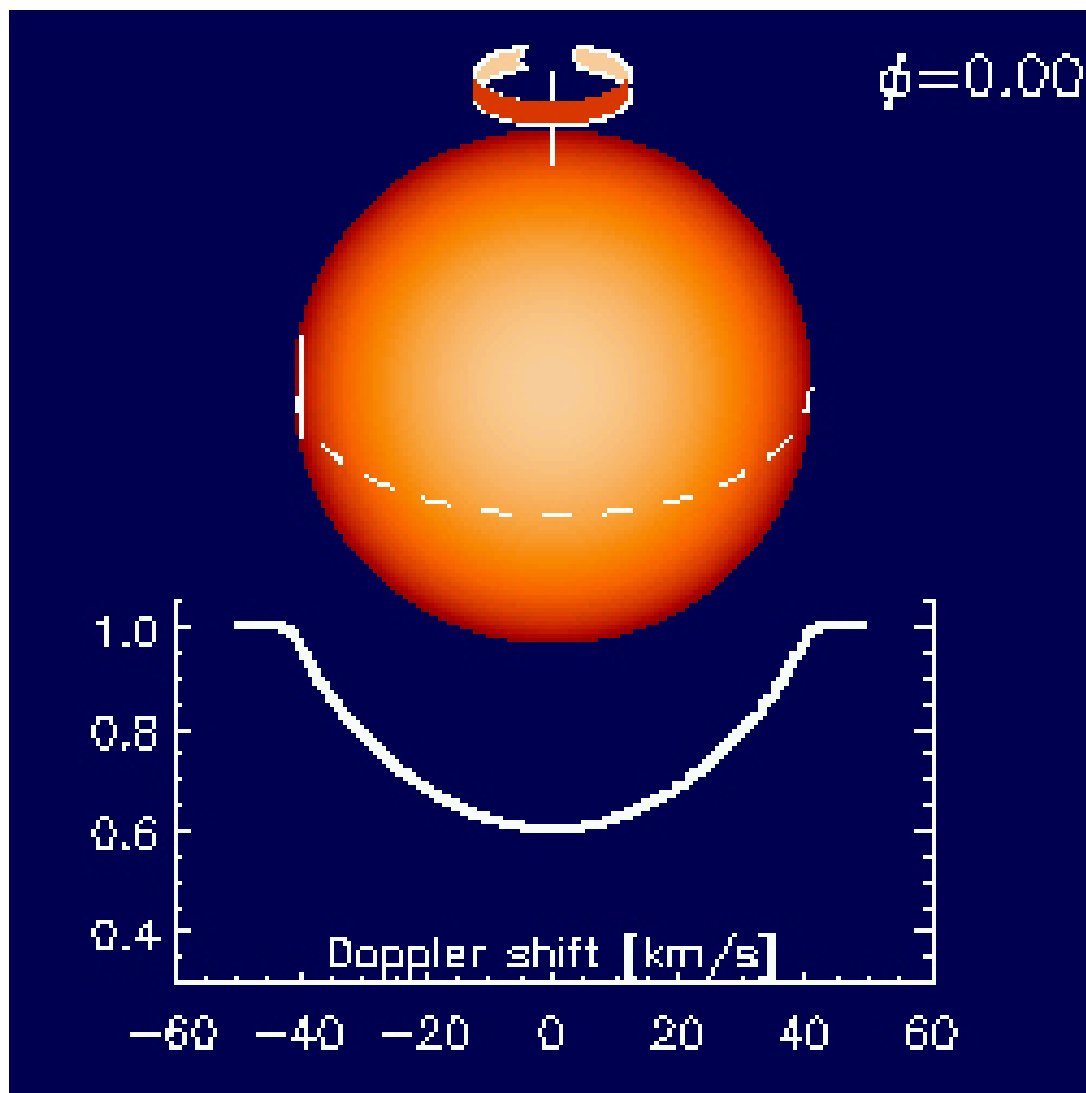
## Rotational Line Broadening



# Takeda et al. (2008) find 175 km/s Equatorial Velocity



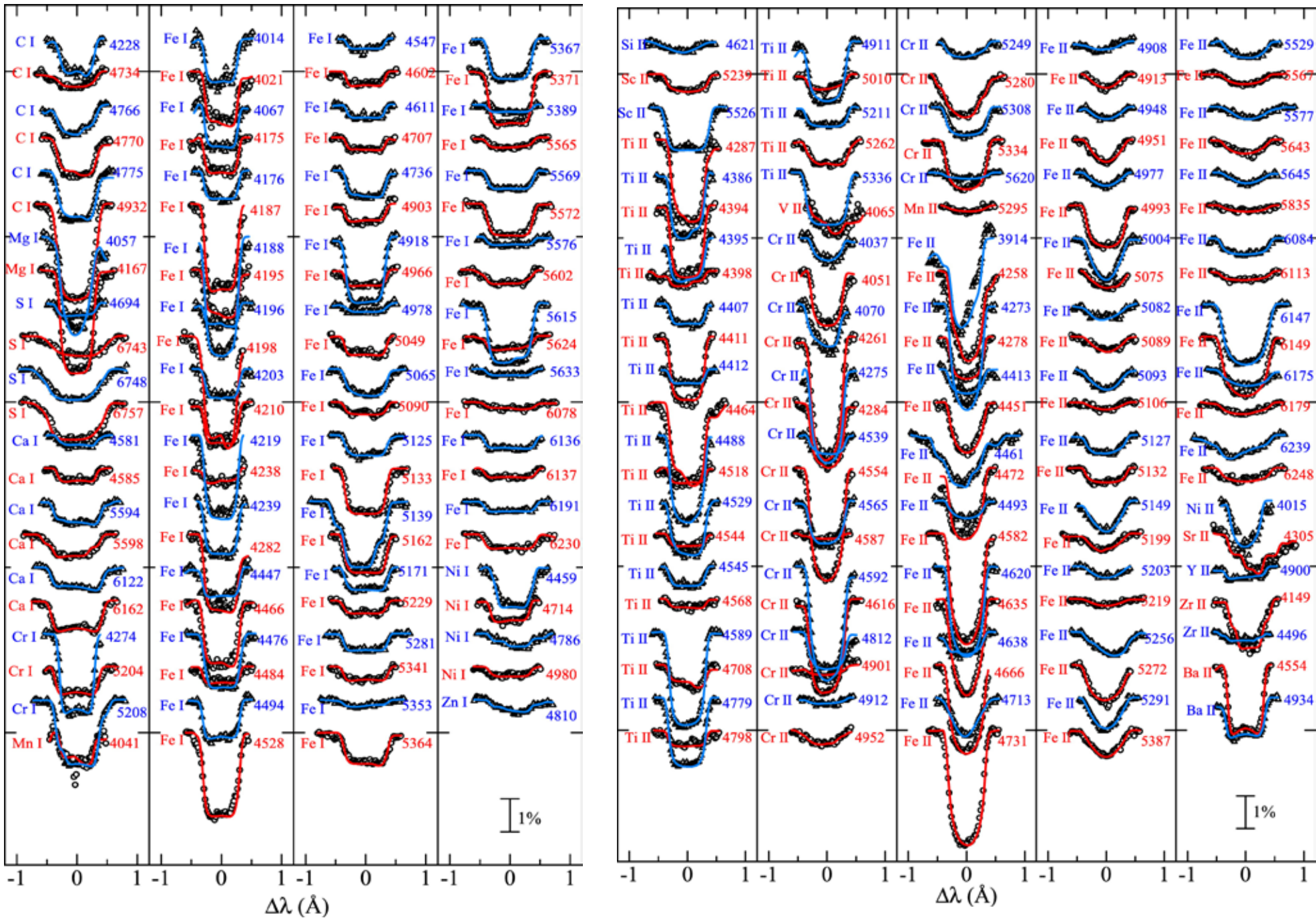
Takeda, Y., Kawanomoto, S., and Ohishi, N. (2008).  
Rotational Feature of Vega Revealed from Spectral  
Line Profiles. *ApJ*, 678, 446–462.



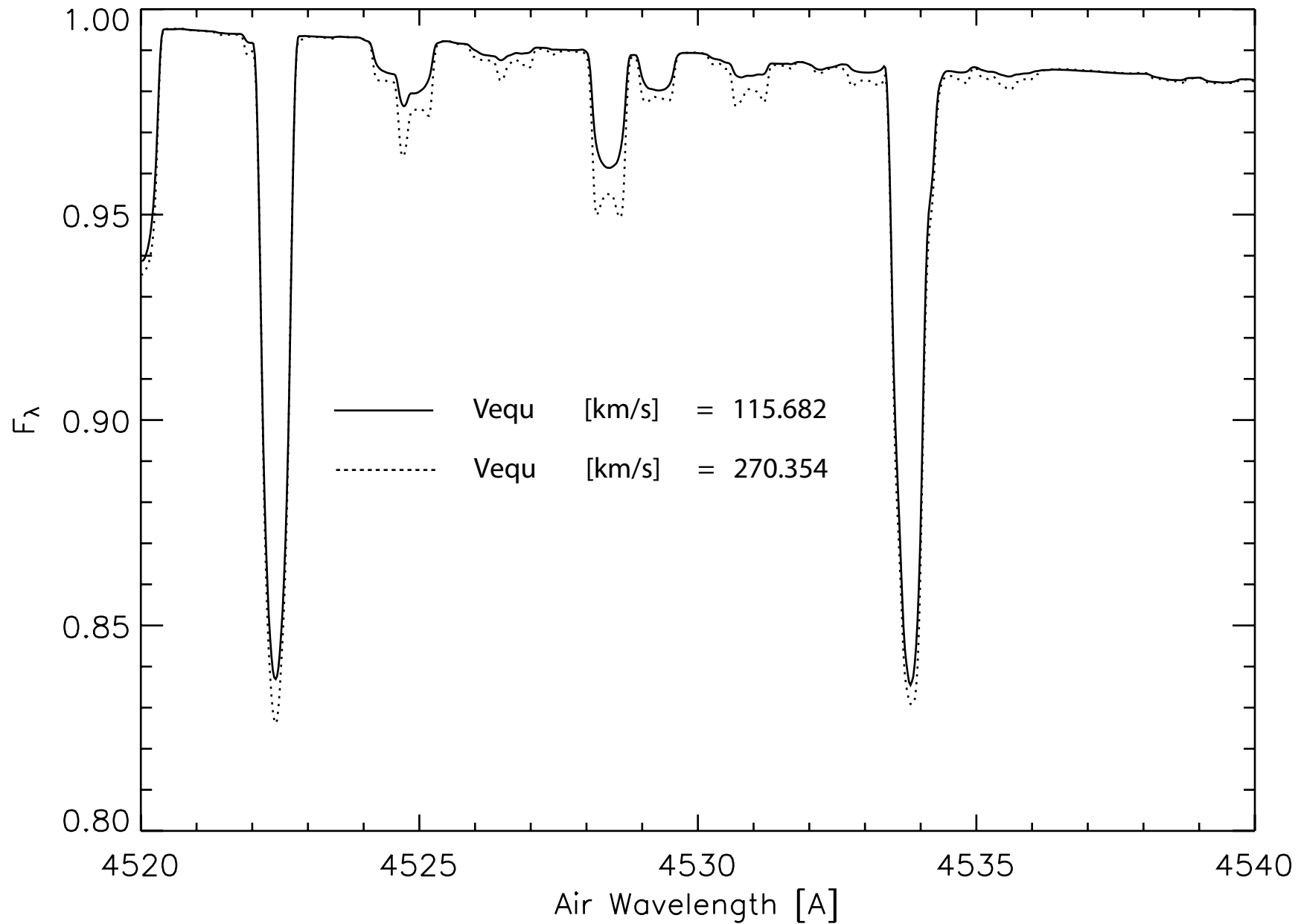
<http://www.astro.uu.se/~oleg/structures.html>



# Takeda et al. (2008) Model Fits Many Neutral and Singly Ionized Lines



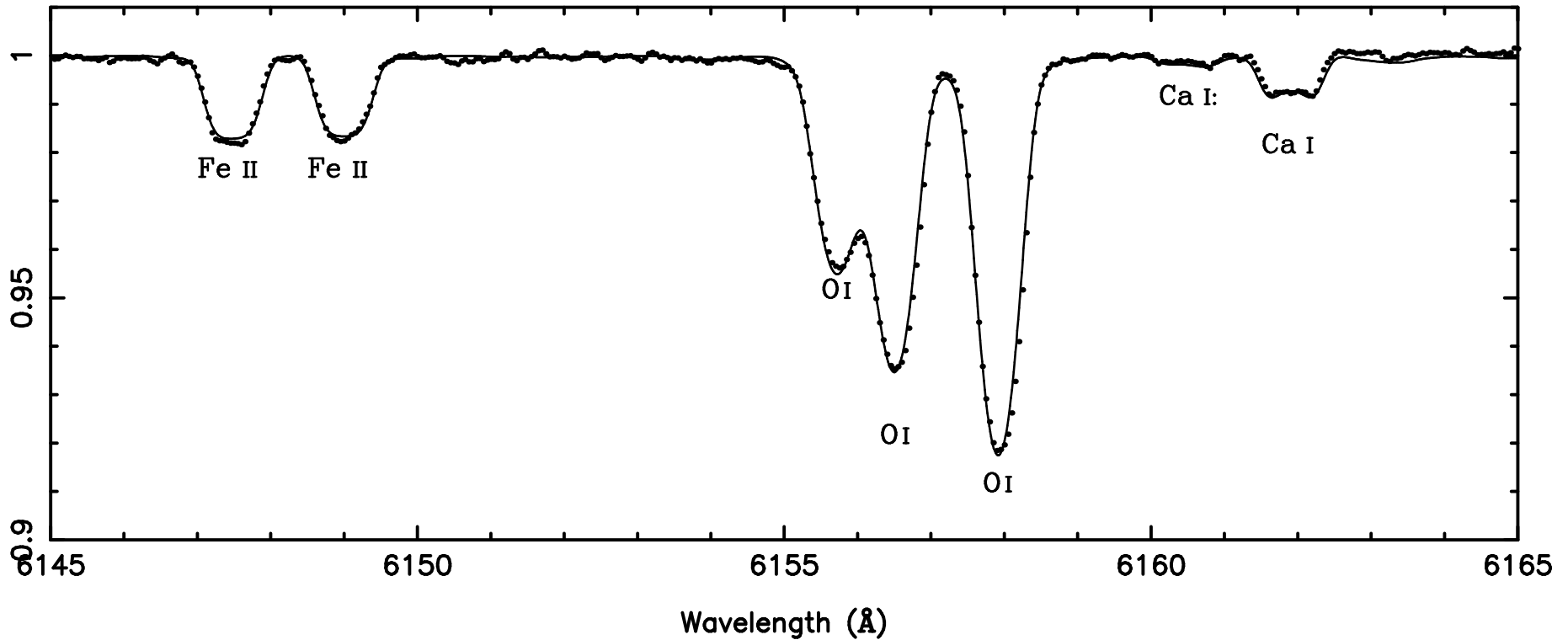
# Our calculations indicate 270 km/s equatorial speed too high





## ... But Yoon et al. (2008) find 275 km/s Equatorial Velocity

Yoon, J., Peterson, D. M., Zagarelli, R. J., Armstrong, J. T., and Pauls, T. (2008). The Effect of Rotation on the Spectrum of Vega. *ApJ*, 681, 570–578.



**The difference is additional macro-turbulence broadening of 10 km/s.**

# Meridional Circulation May Provide “Macro-turbulence”

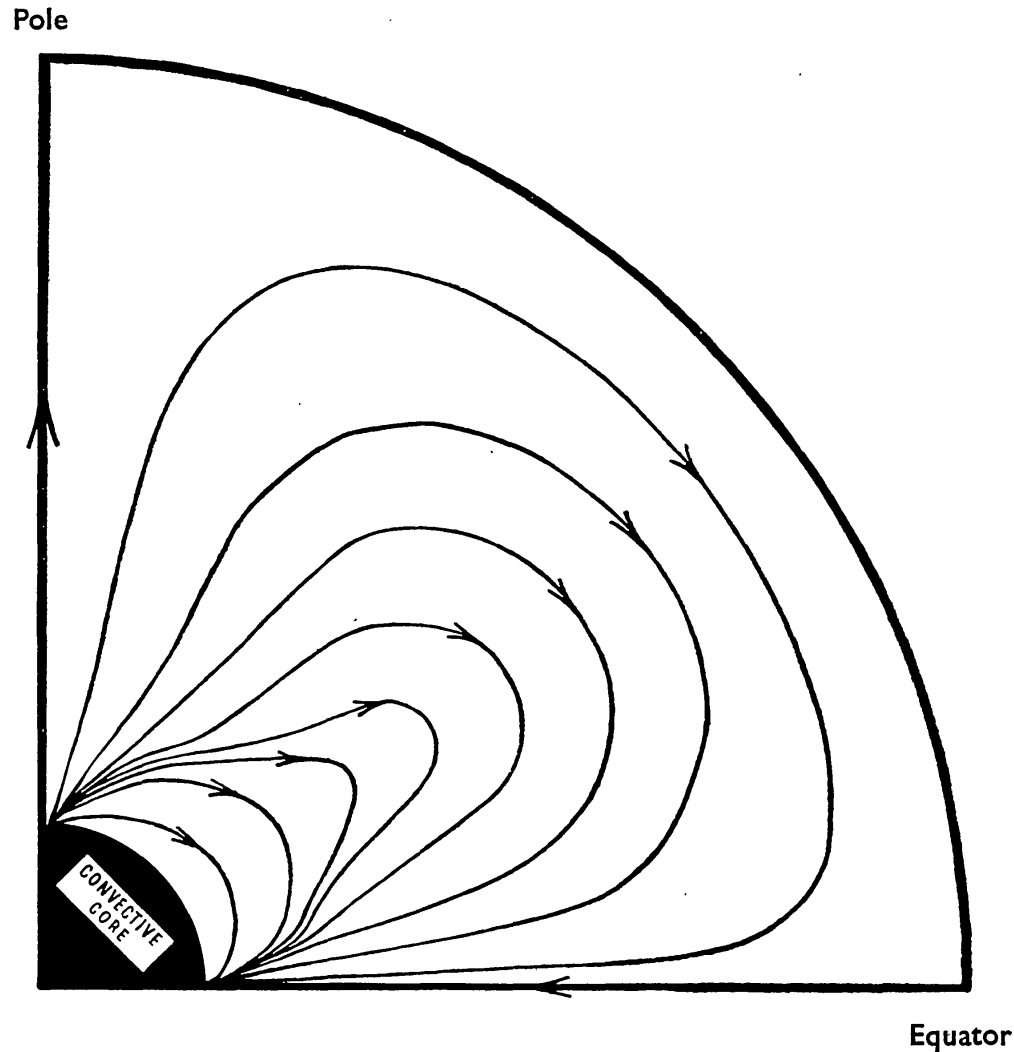


FIG. 1.—*Circulation in a uniformly rotating star.*  
Sweet (1950). *The Importance of Rotation in Evolution*, MNRAS, 110, 548.

**Gridding 1-D models on to 3-D stars neglects velocity shear between latitudes.**

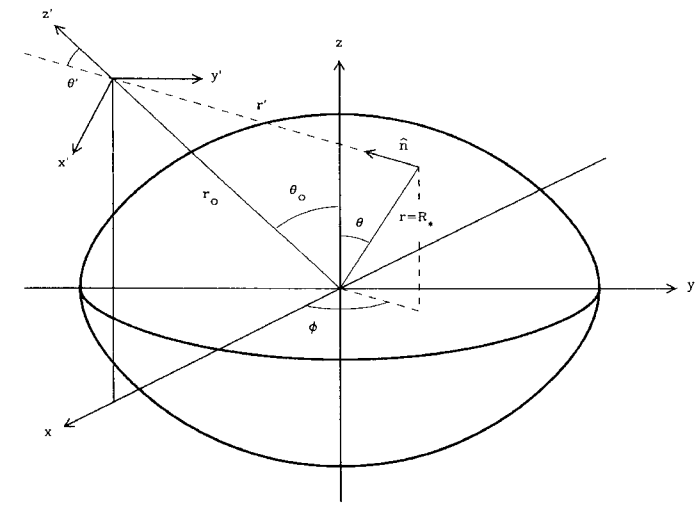
# Standard Roche-Von Zeipel Model Atmosphere

$\omega$  Fraction of the angular break-up speed

$\theta_{\text{equ}}$  Equatorial Angular Diameter

$T_{\text{eff}}^{\text{pole}}$  Effective Temperature at the pole

$g_{\text{pole}}$  Surface gravity at the pole



$$R_{\text{equ}} = 107.48 \frac{\theta_{\text{equ}}}{\pi_{\text{hip}}}$$

$$R_{\text{pole}} = \frac{\omega R_{\text{equ}}}{3 \cos \left[ \frac{\pi + \cos^{-1}(\omega)}{3} \right]}$$

$$R(\vartheta) = 3 \frac{R_{\text{pole}}}{\omega \sin \vartheta} \cos \left[ \frac{\pi + \cos^{-1}(\omega \sin \vartheta)}{3} \right]$$

$$g(\vartheta) = \left[ g_r(\vartheta)^2 + g_\vartheta(\vartheta)^2 \right]^{1/2}$$

$$\frac{T_{\text{eff}}(\vartheta)}{T_{\text{eff}}^{\text{pole}}} = \left( \frac{g(\vartheta)}{g_{\text{pole}}} \right)^\beta$$

$$\Omega = \omega \Omega_{\text{crit}} = \omega \left[ \frac{8 GM}{27 R_{\text{pole}}^3} \right]^{1/2}$$

$$V_{\text{equ}} = R_{\text{equ}} \Omega$$

$$i = \sin^{-1} \left[ \frac{v \sin i}{V_{\text{equ}}} \right]$$

$$g_r(\vartheta) = \frac{-GM}{R(\vartheta)^2} + R(\vartheta) (\Omega \sin \vartheta)^2$$

$$g_\vartheta(\vartheta) = R(\vartheta) \Omega^2 \sin \vartheta \cos \vartheta$$

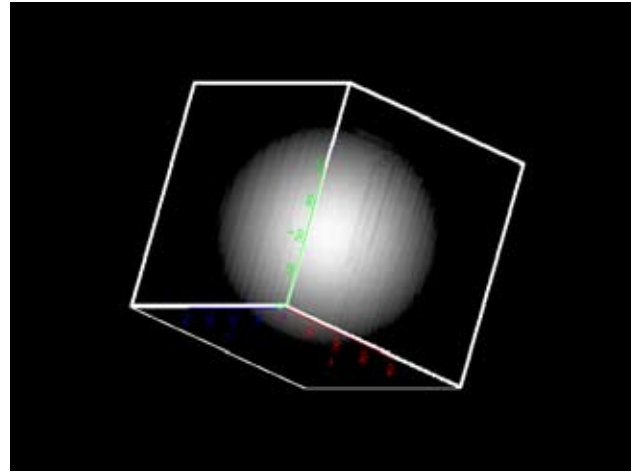
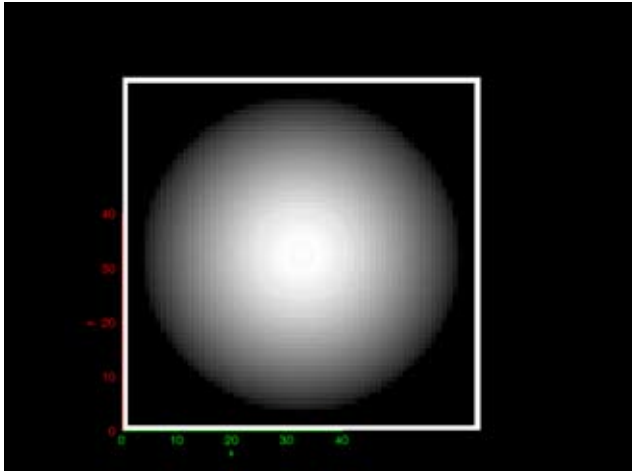
$$M = \frac{g_{\text{pole}} R_{\text{pole}}^2}{G} \quad L = \frac{\sigma \Sigma (T_{\text{eff}}^{\text{pole}})^4}{g_{\text{pole}}}$$

$$\Sigma \approx 4\pi GM \left[ 1.0 - 0.19696 \omega^2 - 0.094292 \omega^4 + 0.33812 \omega^6 - 1.30660 \omega^8 + 1.8286 \omega^{10} - 0.92714 \omega^{12} \right]$$

## Standard Rotating Star Model

1. Point-mass approximation for the potential
2. Uniform (non-differential) rotation
3. Stellar shape is an equipotential surface, the sum of gravitational and centrifugal potentials
4. Von Zeipel gravity darkening law
5. Interpolated 1-D atmosphere models

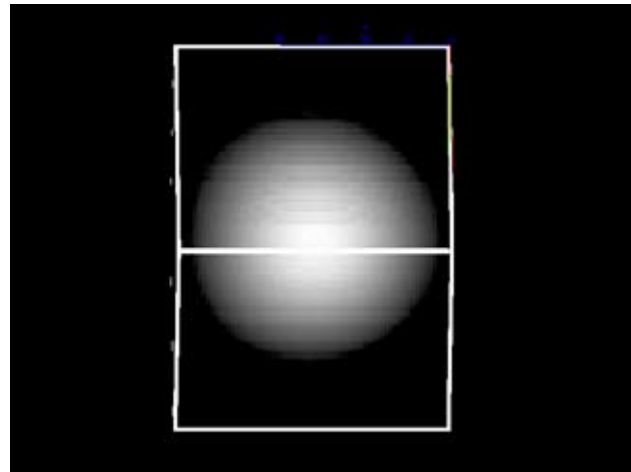
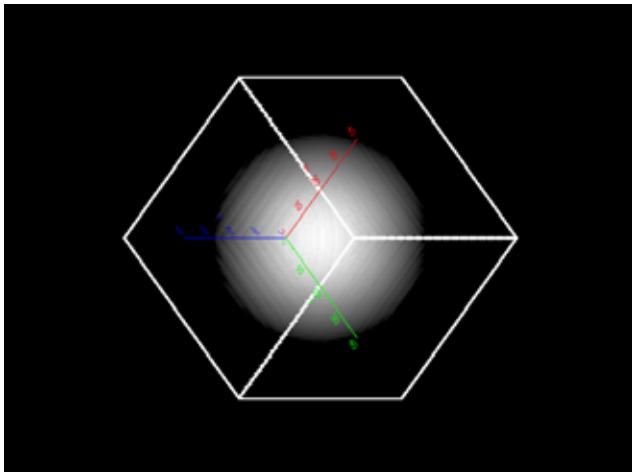
# Three-Dimensional Radiative Transport Required to Simulate Spectrum



Visualization of the results from the PHOENIX 3-D code (Hauschildt & Baron)

The voxel (volume element) grid has  $65^3$  elements.

The intensity image is shown for  $(\theta, \varphi) = (0^\circ, 0^\circ), (45^\circ, 45^\circ), (140^\circ, 250^\circ)$  and  $(89^\circ, 139^\circ)$ .



The intensities are mapped linearly to 255 shades of gray.

In spherical coordinates  $(n_r, n_\theta, n_\varphi) = (65, 33, 65)$  voxels, an angular resolution on the stellar surface of about 5.5 degrees.

Hauschildt, P. H. and Baron, E. (2006). A 3D radiative transfer framework. I. Non-local operator splitting and continuum scattering problems. *A&A*, 451, 273–284.

# Self-Consistent Field (SCF) Model for a Rotating Star

## Mass Distribution is Fully Self-Consistent with Effective Potential

$$g = \nabla \Psi$$

Effective gravitational acceleration

$$\Psi = \Phi + \Phi'$$

Effective potential

$$\nabla^2 \Phi = -4\pi G \rho$$

Gravitational potential (Poisson's Equation)

$$\Phi' = \int_0^{\varpi} \Omega^2(\varpi) \varpi d\varpi$$

Centrifugal potential

$$\eta = \Omega_0 / \Omega_{\text{cr}}$$

Ratio of the angular velocity at the pole to the critical angular velocity

$$\Omega_e / \Omega_0 = 1 / (1 + \alpha^2)$$

Ratio of the angular velocity at the equator to the angular velocity at the pole (differential rotation)

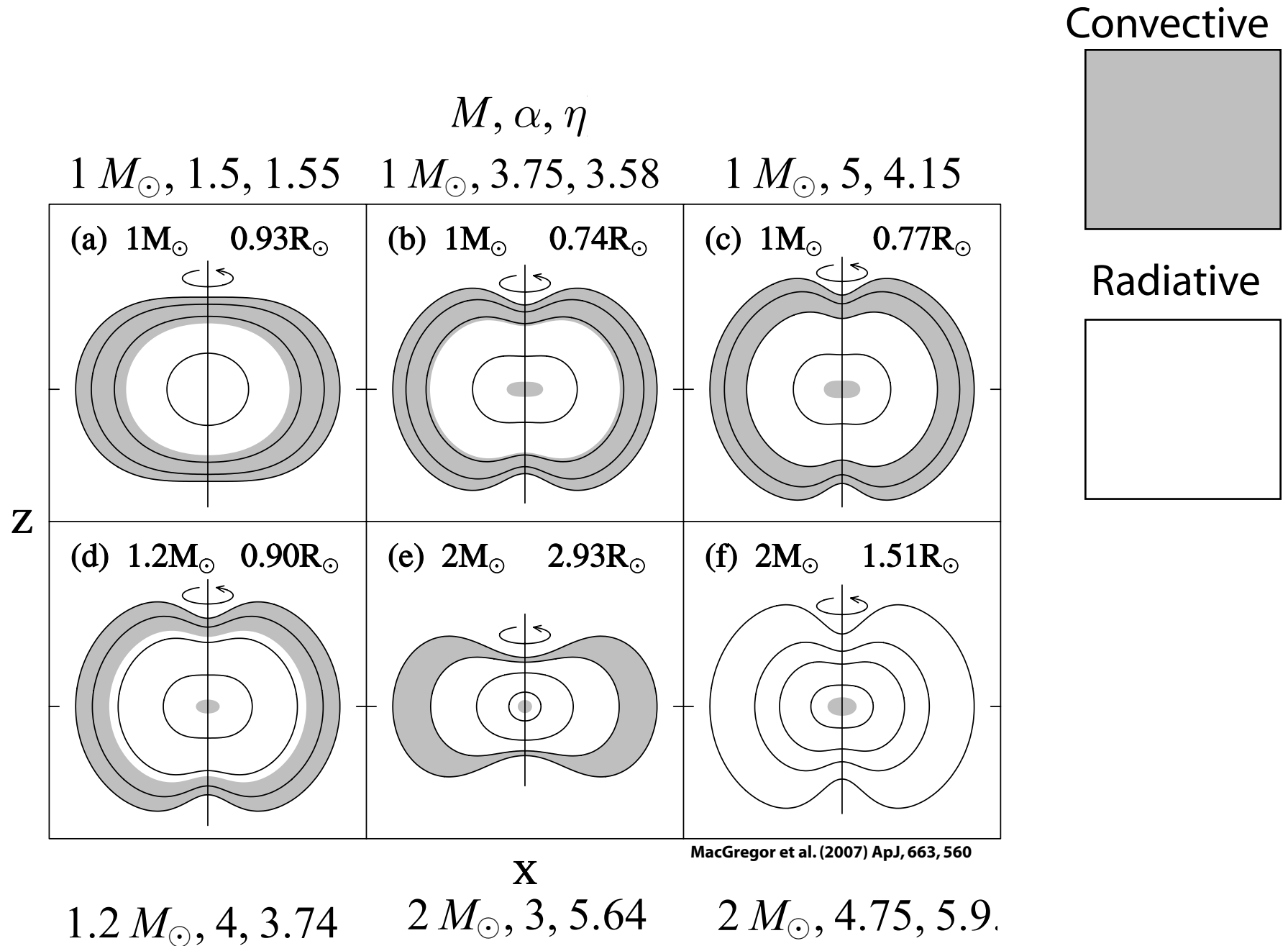
$$\Omega(\varpi) = \frac{\Omega_0}{1 + (\alpha\varpi/R_e)^2}$$

"Anti-solar" rotation: pole spins faster than equator

$$\Omega(\varpi) = \Omega_0 \left[ 1 + (\alpha\varpi/R_e)^2 \right]$$

"Solar" rotation: equator spins faster than pole

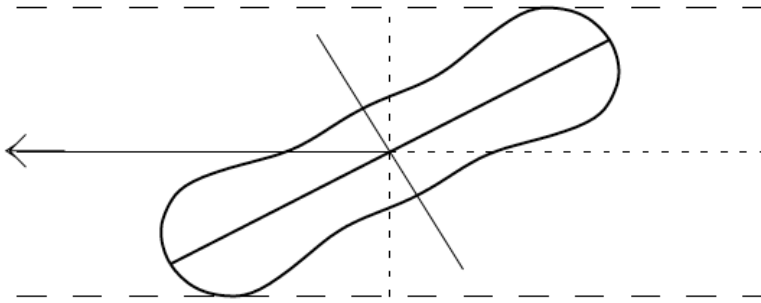
# One to Two Solar Mass SCF Differentially Rotating Models



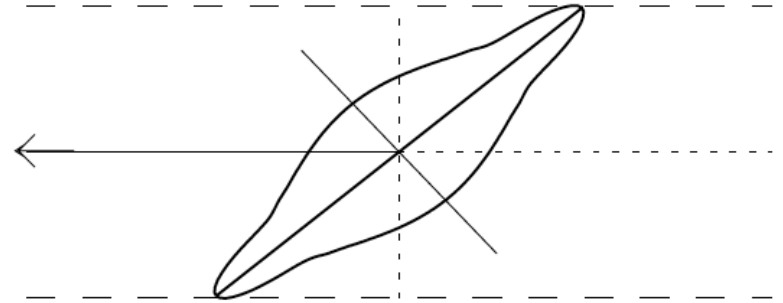


# More Massive SCF Models for Alpha Eri (Jackson et al.)

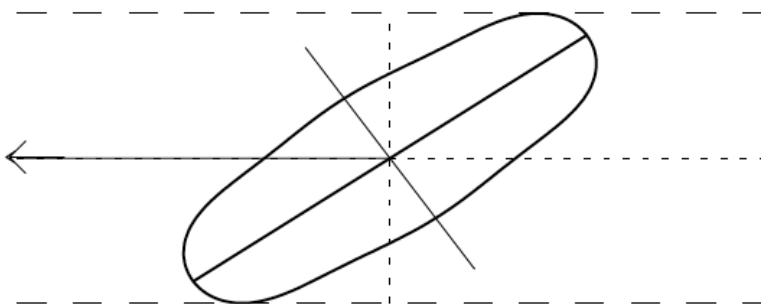
(A)  $M=6 M_{\odot}$  ,  $R_{eq}=12.19 R_{\odot}$   
 $i=61^{\circ}$



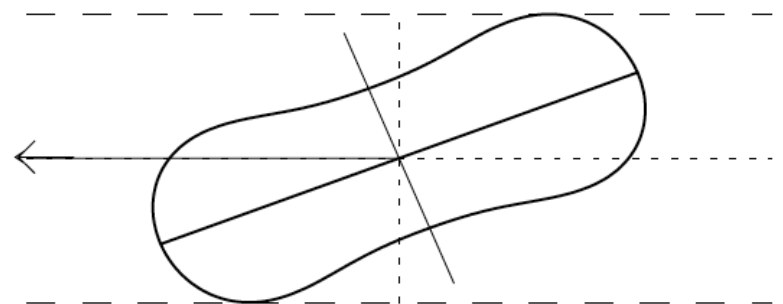
(B)  $M=9 M_{\odot}$  ,  $R_{eq}=11.73 R_{\odot}$   
 $i=49^{\circ}$



(C)  $M=12 M_{\odot}$  ,  $R_{eq}=11.57 R_{\odot}$   
 $i=55^{\circ}$

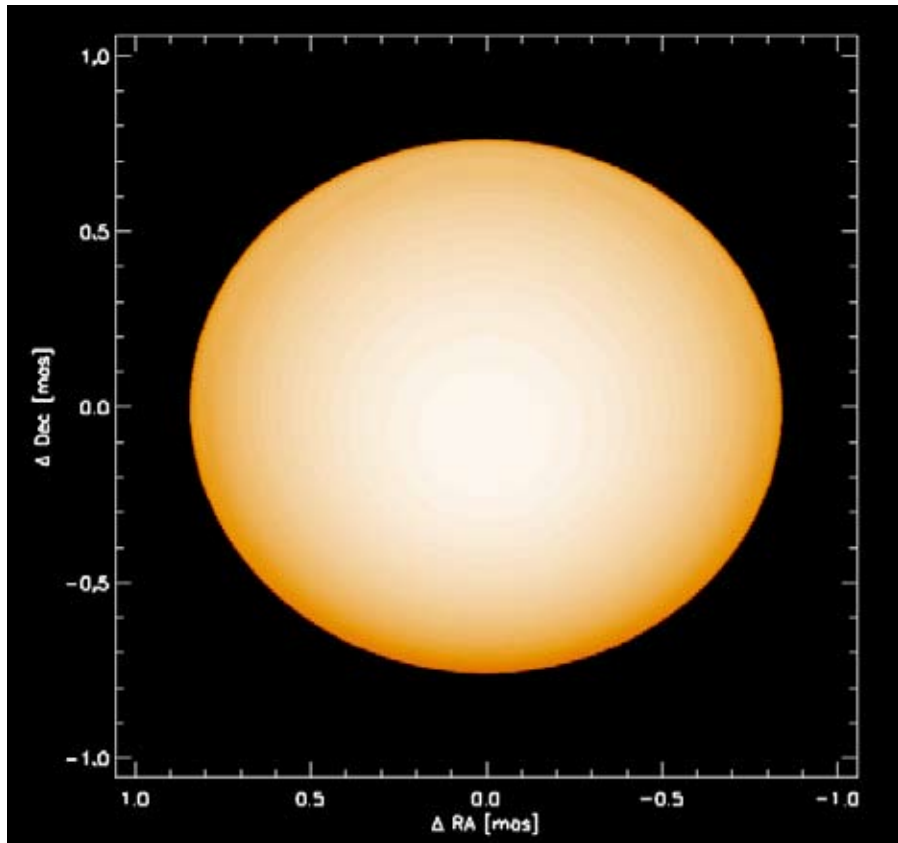


(D)  $M=15 M_{\odot}$  ,  $R_{eq}=12.41 R_{\odot}$   
 $i=68^{\circ}$



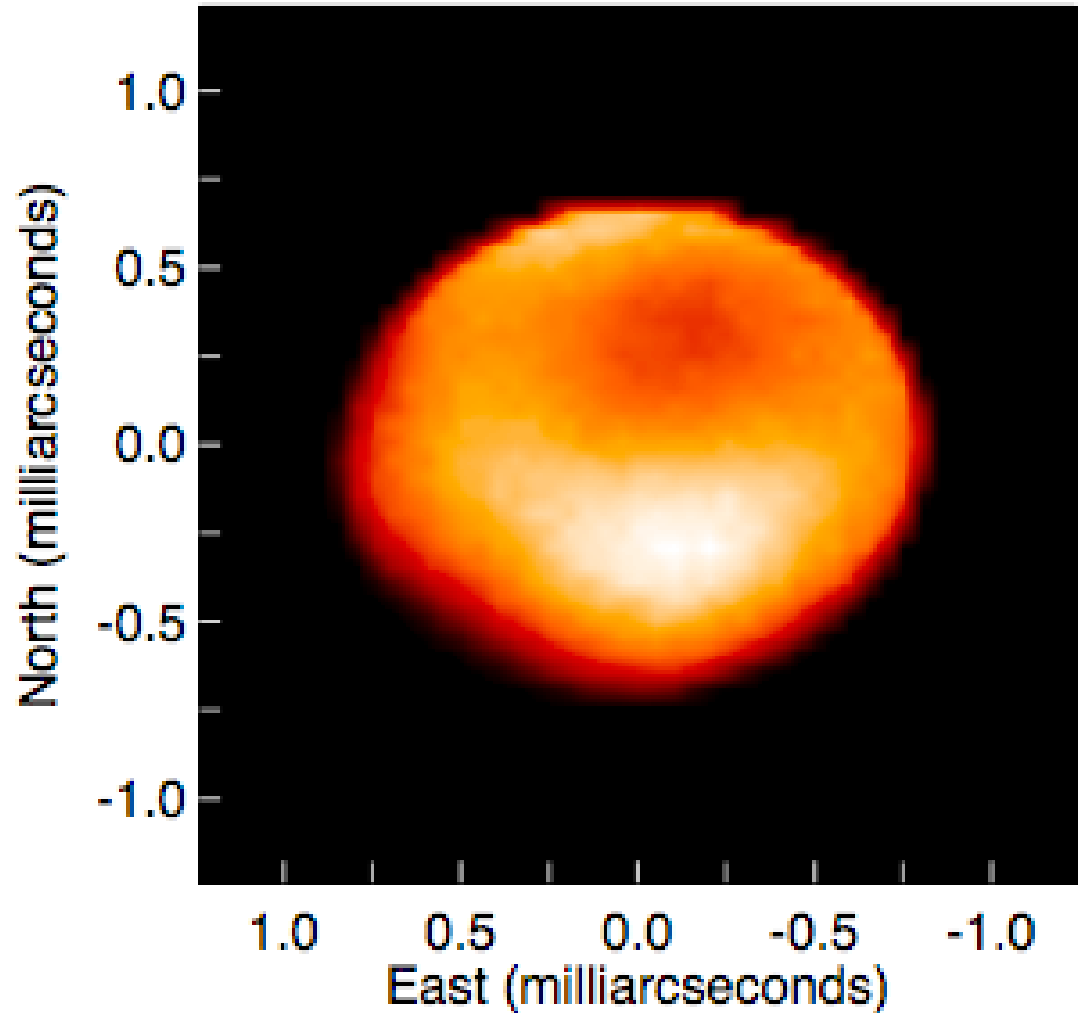
Jackson, S., MacGregor, K. B., and Skumanich, A. (2004). Models for the Rapidly Rotating Be Star Achernar. *ApJ* , 606, 1196–1199.

# Trial SCF Model for Alderamin (Alpha Cep)



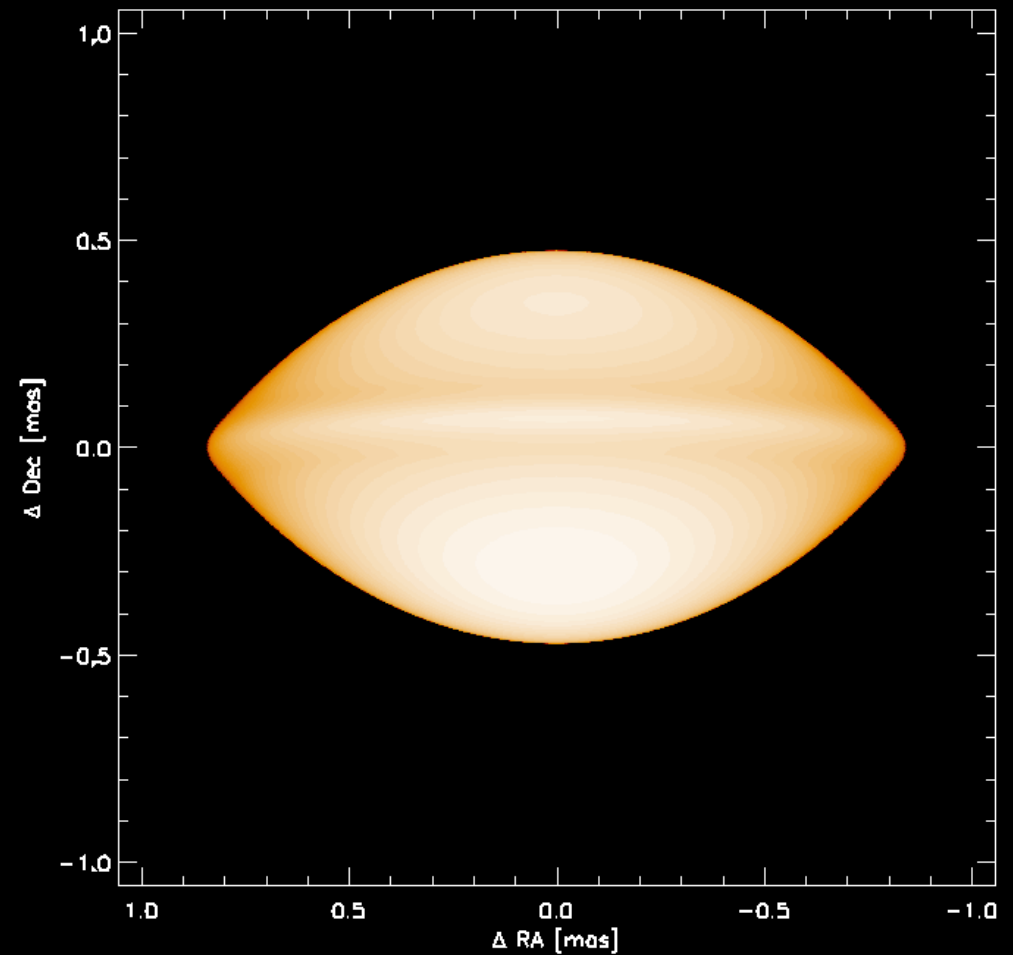
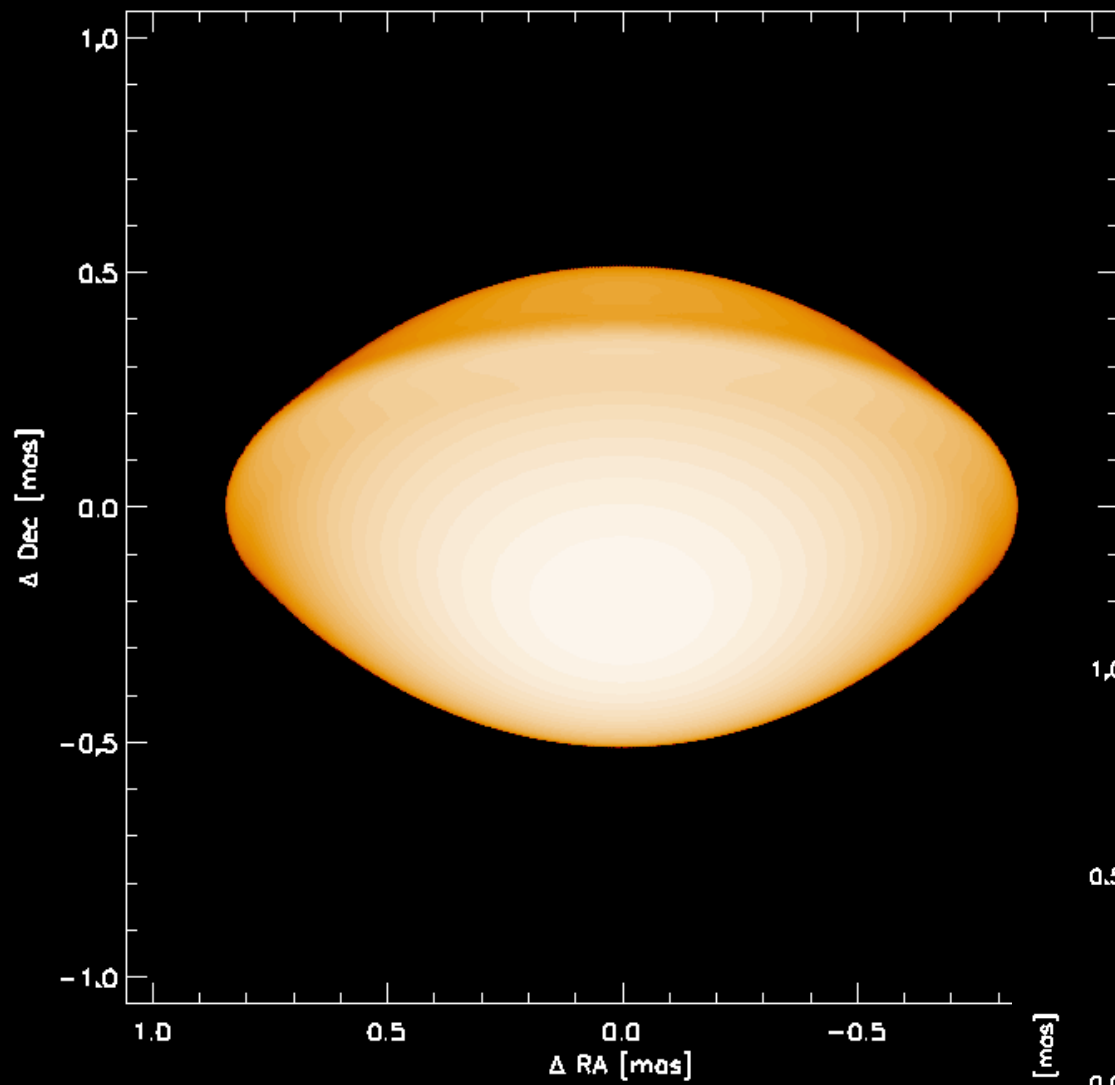
$M = 1.9M_{\odot}, \eta = 1.60, \alpha = 0.08$   
 $i = 26.5^{\circ}$  at 1.6 microns

## Alderamin Image Reconstruction



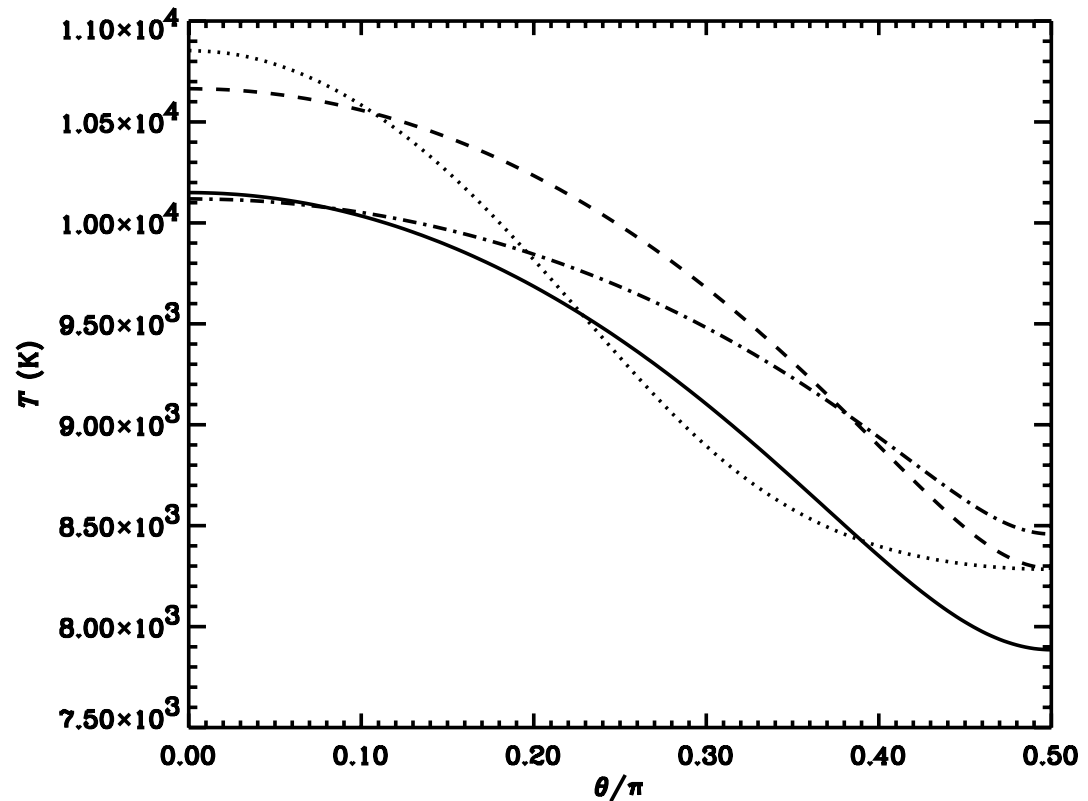
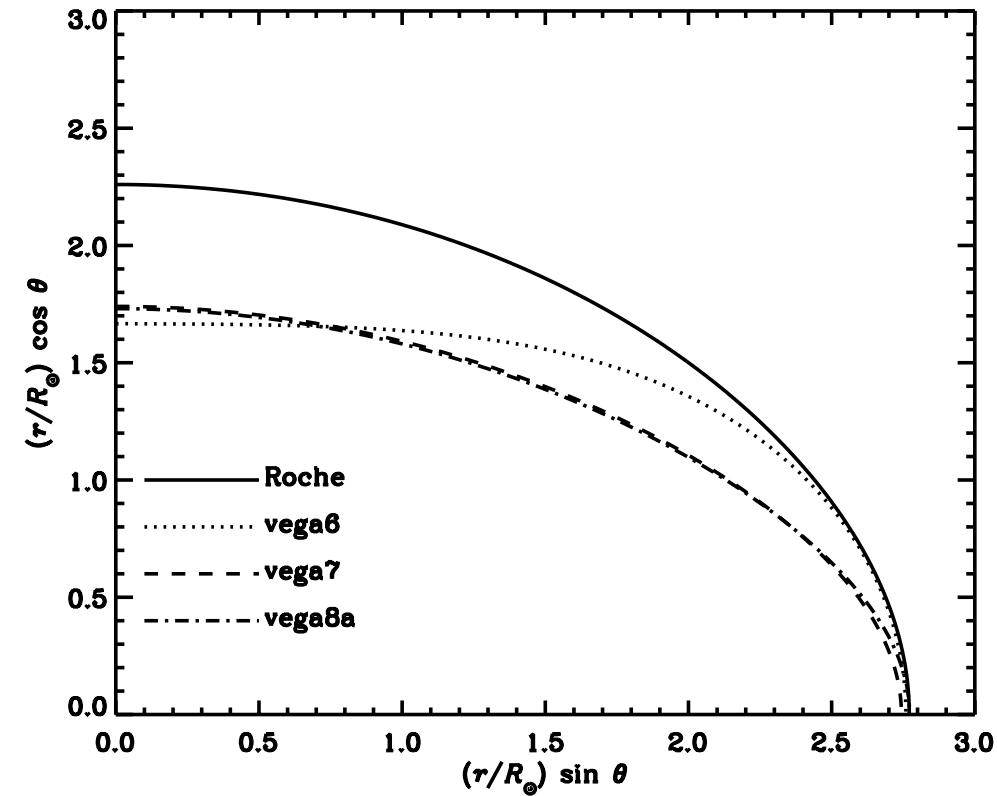
**What's the Matter with Vega?**

# Trial SCF Model at higher inclinations



# Searching for a Vega Model

## SCF Model Shapes and Pole-to-Equator Temperature Profiles



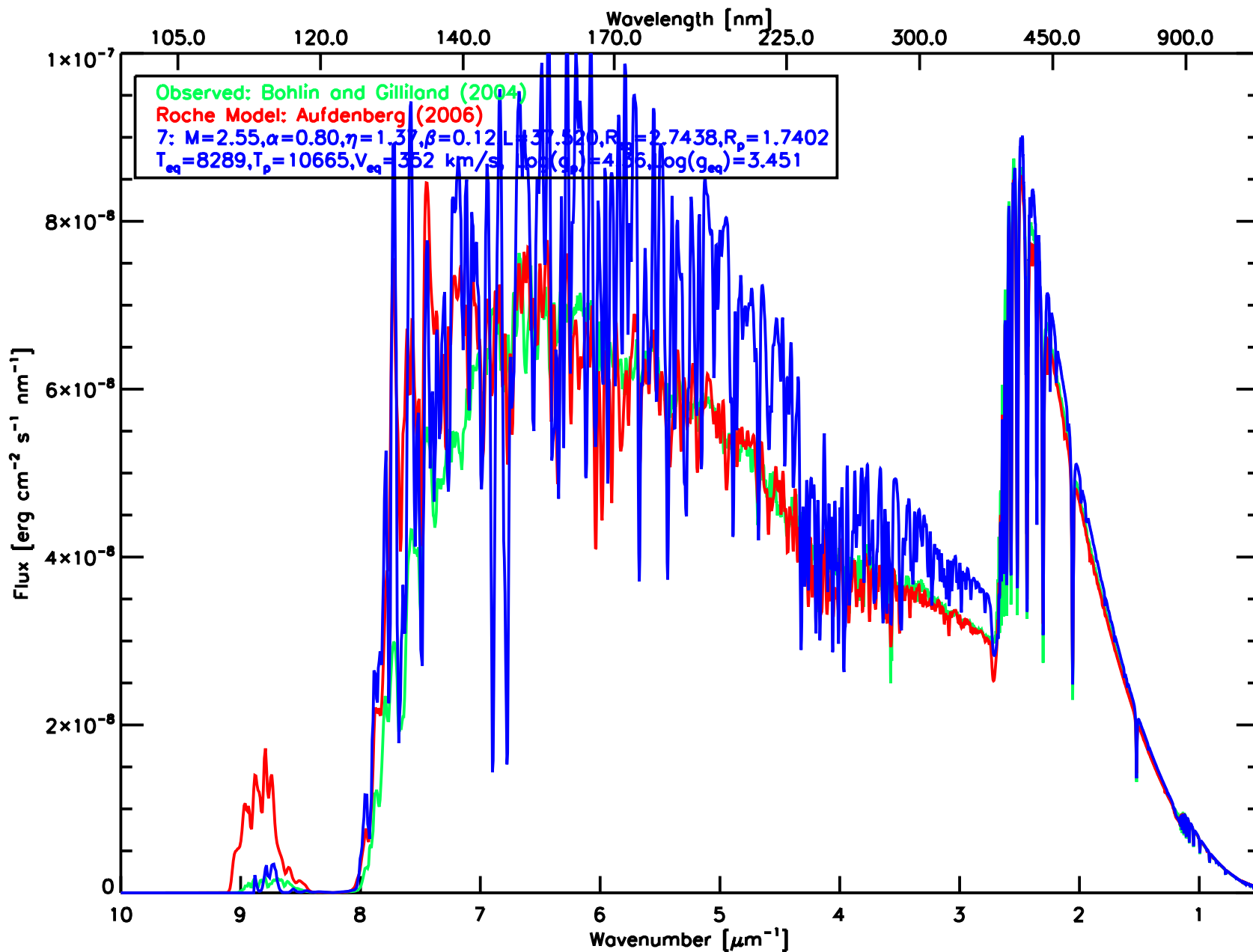
$$6 : M = 2.55 M_\odot, \eta = 2.05, \alpha = 1.50, \beta = 0.18$$

$$7 : M = 2.55 M_\odot, \eta = 1.37, \alpha = 0.80, \beta = 0.12$$

$$8a : M = 2.52 M_\odot, \eta = 1.40, \alpha = 0.80, \beta = 0.08$$

$$7 : M = 2.55M_{\odot}, \eta = 1.37, \alpha = 0.80, \beta = 0.12$$

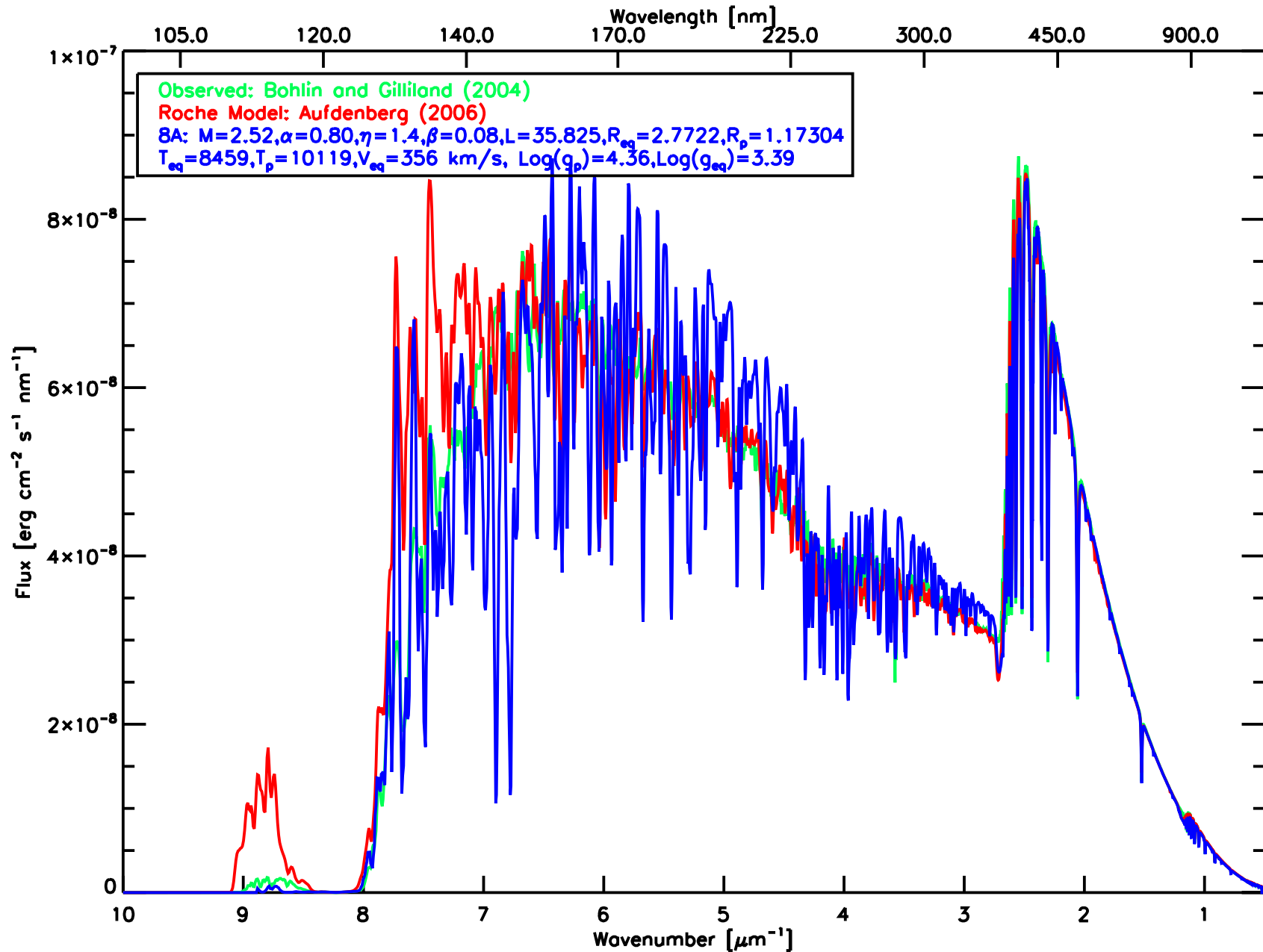
**Too much UV flux, Temperature profile too hot**





$$8a : M = 2.52M_{\odot}, \eta = 1.40, \alpha = 0.80, \beta = 0.08$$

**Better UV Match; line blanketing is too strong**



# Future Work

**1. Identify and quantify which radiative features are sensitive to which model features.**

**2. Find a best fit match of the SCF models to the spectrophotometric, spectroscopic, and interferometric data sets.**

**3. Include main-sequence evolutionary effects in the SCF rotating models.**

**4. Improve performance of parallel atmosphere interpolation algorithms.**