

1% Luminosity-Independent Distances to Nearby Galaxies with the Rotational Parallax Technique

A SIM Science Studies Project

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Outline

- Calibrating the Distance Scale
- Astrophysics in the next Century: Precision & Accuracy
 - Example: H_0 , the CMB & Dark Energy
- Rotational Parallax
- Conclusions
- Backup slides
 - US Astrometry Funding for Gaia-based research?
 - RP extras
 - H_2O Masers & Eccentric Orbits
 - Stellar Ages, Gaia & SIM
 - More H_0 , Cosmology & Dark Energy

Most of this talk is based on an 2007 paper [Olling, 2007, MNRAS, 378, 1385] & OR:
http://www.astro.umd.edu/~olling/Papers/RP_H0_2007_Colloquium.pdf

Calibrating the Extragalactic Distance Scale

- **“Standard Candle Methods:”**

- Extinction & [Fe/H] may be greatest difficulties

- For known Galactic Cepheids: $\langle A_V \rangle \sim 1.7$ mag

- GAIA expects: $\epsilon(A_V) \sim 0.1$ mag. Much better in NIR

- **BUT: Standard Candles will be calibrated much better by Gaia/SIM than the current state-of-the-art**

- **GAIA:**

- 17,000 binaries (21 10⁶ stars) with masses (distances) $< \sim 1\%$ and $V < \sim 15$
- Radii for $\sim 360,000$ stars in Eclipsing Binary systems with 1% distances
- Uniform metallicity & extinction scale: photometric & spectroscopic

Extragalactic Distance Scale, cntd

- “Geometric” Methods are still problematic
 - Baade-Wesselink-type methods for Cepheids [p-factor]
 - Velocity Gradient, [Applied to LMC by GAIA]
 - (H₂O) Masers in extra-galactic star formation regions [Few systems per galaxy: depends on external velocity-field data]
 - Extra-galactic (nuclear) Mega masers [Just 3 lines of sight: sensitive to systematics]
 - “Licht Echo” method; X-ray scattering of background sources; Expanding Photospheres of SNe (non=LTE) [Special events]
 - (Detached) Eclipsing Binaries; Gravitational Waves Close WDs [No calibrators in HIPPARCOS (fixed by GAIA?)]
- [summarized in Olling 2007; and see: Gould 2000; Argon *et al* 2004, Brunthaler *et al* 2005, Braatz *et al* 2006; Panagia *et al* 1991, Gould 2000, Sparks 1994, Sugerman 2006; Draine & Bond 2004; Nugent *et al* 2006; Paczynski & Sasselov 1997, Fitzpatrick *et al* 2004, Stanek *et al* 1998; Cooray & Seto 2005; Freedman *et al.* 2008]

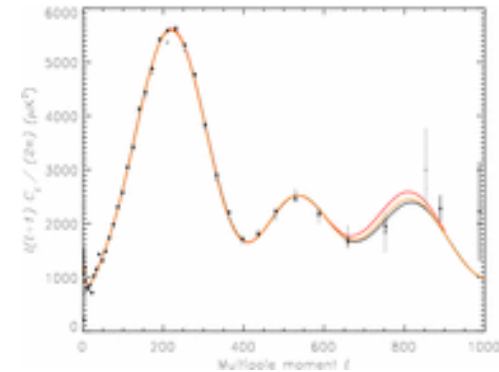
Astrophysics in next Century

- from **“Astronomy”** to **“Physics”**
 - from Precision \Rightarrow Accuracy
 - from Model-Dependent \Rightarrow “Truth”
- **Most important is reliability of errors**
 - Need be **precise** = small (pretty “easy”)
 - **And accurate** = reliable (very hard)

- **Need independent cross-checks:**
 - **different methods & objects**
to measure same parameter(s)
 - **Otherwise: results + errors can not be trusted**
- **Absolute distance (H_0) errors important for cosmology & dark energy work**
- **An absolute distance to a LG galaxy will eventually lead to an accurate H_0**

H_0 , the CMB & Dark Energy

- From the shape of the power spectrum, **WMAP “directly”** [e.g., Hu 2005] **measures the physical densities** (ρ_{matter} and ρ_{baryon})



– i.e., NOT the ρ_{crit} -normalized densities

– $\rho_{\text{crit}} = 3 H_0^2 / (8\pi G)$ is the critical density of Universe

- $\omega_b = \Omega_b h^2 \propto$ the physical baryon density
- $\omega_m = (\omega_b + w_{DM}) = \Omega_m h^2 \propto$ the physical matter density
- $h = H_0 / 100$

$$\Omega_m = \frac{\rho_b + \rho_{DM}}{\rho_{\text{crit}}} = \frac{\omega_m}{h^2} \quad \text{and} \quad \epsilon_{\Omega_m} = \sqrt{\left(\frac{\epsilon_{\omega_b}}{\omega_b}\right)^2 + \left(2 \frac{\epsilon_h}{h}\right)^2}$$

H_0 , CMB and Dark Energy (cntd)

- IF one wants to determine w , THEN one needs to know h :
 - A fit to the current WMAP & other data yields [Spergel et al 2006]:

$$-w \approx 1.59 - 2.56 \Omega_m = 1.59 - 2.56 \frac{\omega_m}{h^2}$$

- Now: EOS of Dark Energy is known to +/- **7%**, **and** ω_m and H_0 contribute about equally
- Decreasing error on ω_m leaves constant contribution from H_0 ==> hardly any decrease in ϵ_w
- Better (x 8) determination of ω_m with *PLANCK*,
- Need better (x10) determination of H_0 (e.g., with *SIM-Lite*)

Rotational Parallax Distances

- Distance (D) to *Local Group Spirals* can be determined via the [Rotational Parallax Method](#)
[Peterson & Shao, 1997; Olling & Peterson, APH/0005484; Olling, 2007, MNRAS, 378, 1385]
- Principle **very** straightforward:
 - Measure circular rotation via radial-velocities (V_c)
 - Measure circular rotation via proper motions ($\mu_c \propto V_c / D$)
- **Distance $\propto V_c / \mu_c$**
 - **EXPECT: Unbiased Distances**

Accuracy of several % out to ~1 Mpc

- Requires:
 - Large-scale ordered motions (e.g., rotation)
 - Ground-based radial velocities and
 - Space-based proper motions at the $< \sim 10 \mu\text{as/yr}$ level

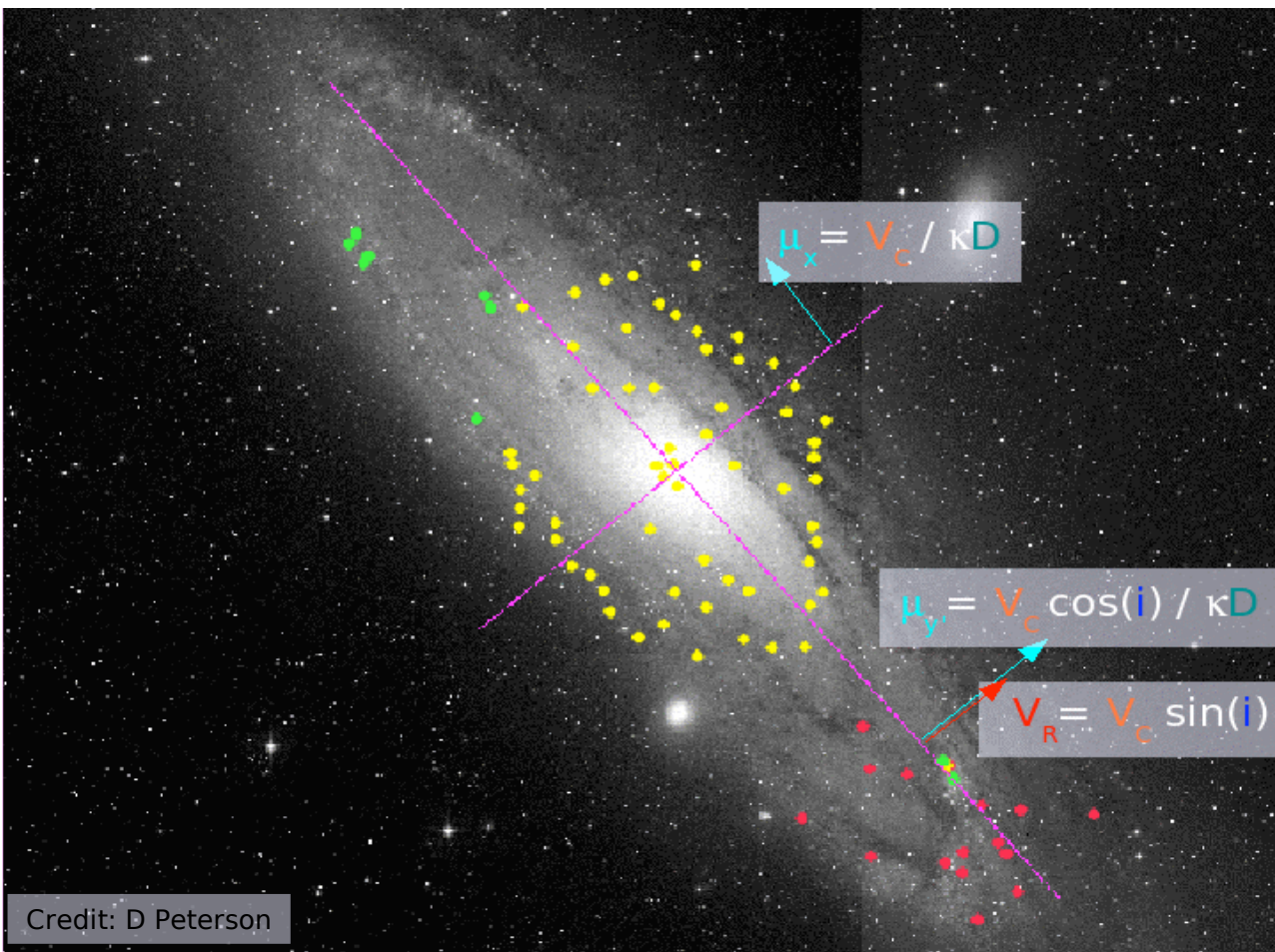
Rotational Parallax Illustrated

M 31: $i \sim 77^\circ$

$D \sim 0.84$ Mpc

$V_c \sim 270$ km/s

$\mu_c \sim 74$ μ as/yr



- **For Circular Orbits:**

- minor axis: $\mu_x = V_c / (\kappa D)$

- Major axis: $\mu_{y'} = V_c \cos(i) / (\kappa D)$

- Major axis: $V_R = V_c \sin(i)$

- **Three equations, (observables)**

- **Three unknowns,**

- **Three solutions**

- **Several Approaches**

The Rotational Parallax Method

- **GENERAL CASE, any position in galaxy**

$$\cos^2(i) = -(y' \mu_y) / (x \mu_x)$$

$$D_G = V_R / \kappa * [-(y'/\mu_y) / (x \mu_x + y' \mu_y)]^{-1/2}$$

- **Flat Rotation Curve, Circular Orbits, HI Inclination**

$$D_{\text{IHl}} = V_R(\text{major axis}) / [\kappa * \sin(i) * \mu_x(\text{minor axis})]$$

$$\epsilon(D_{\text{IHl}})^2 = D^2 [(\epsilon(V_R) / V_R)^2 + (\epsilon(\mu_x) / \mu_x)^2]$$

- **Flat Rotation Curve, Circular Orbits, Unknown Inclination**

$$\cos(i) = |\mu_{y'}(\text{major axis})| / |\mu_x(\text{minor axis})|$$

$$D_{\text{mM}} = V_R * [(\mu_{y'}(\text{major axis}))^2 - (\mu_x(\text{minor axis}))^2]^{-1/2}$$

The Rotational Parallax Method (cntd)

• How About?

- Space-motion of the galaxy
- Warp
- Non-circular motions
 - Spiral-arm streaming motions
 - Bar-induced motions
 - Tidal distortions
 - Et cetera
- Rotation of astrometric grid
 - Any physical process that produces proper motion will have a corresponding radial velocity
 - Grid translation: don't care $\implies V_{\text{SYS}}$
 - Grid Rotation: no V_{RAD} equivalent \implies take out

!! The RP method is VERY ROBUST !!

General Rotational Parallaxes

Unknowns:

Total Space Velocity:

- $\mathbf{V}_{\text{TOTAL}} = \mathbf{V}_{\text{SYS}} + (V_{\text{C}} + \mathbf{V}_{\text{P}}) + \mathbf{V}_{\sigma}$
- = systemic + circular + peculiar + random
 $\Rightarrow 3 + 1 + 3 + 3 = 10$

Coordinate system:

- Origin of coordinate system $\Rightarrow 2$
- Position angle of major axis (ϕ) $\Rightarrow 1$
- Distance and Inclination $\Rightarrow 2$
- **Star position in galaxy** $\Rightarrow 3$
- **TOTAL: 18 unknowns**

OBSERVABLES (per star):

2 positions + 2 proper motions + V_{RAD} = **5 knowns**

General Rotational Parallaxes (cntd)

- **However:**

- Many unknowns are “shared” between test particles:

- Center of galaxy + PA: 3 shared vars.
- Systemic velocity: 3 shared vars.
- Rotation Speed: 1 shared var.
- Distance & inclination 2 shared vars.
- Velocity dispersion: 3 shared vars.
- **TOTAL** 12 shared variables

- Left with: 3 \mathbf{V}_p 's, x,y,z: **6 star-dependent unknowns**

- No solution because we have **5 observables per star**

- **No solution \Rightarrow Eliminate 2 more variables**

- e.g., assume $\langle V_{p;z} \rangle = 0$ and $\langle z \rangle = 0$

- **4 star-dependent unknowns left**

General Rotational Parallaxes (cntd)

- **Then:** $(4 N_* + N_{SV})$ unknowns
 $5 N_*$ observables
- \Rightarrow **Solution if:** $5 N_* \geq (4 N_* + N_{SV})$
 $N_* \geq N_{SV}$
- **In our example, if $N_* \geq N_{SV} = 12$**
- **Get all parameters from 12 stars/galaxy**
- **Alternatively,** allow for corrugations
 - $z(\theta) = z_0 + \sum_1^{n_z} A_n \cos(2n\pi\theta) + B_n \cos(2n\pi\theta)$
 - $V_{p;z}(\theta) = V_{p;z;0} + \sum_1^{n_{vpz}} C_n \cos(2n\pi\theta) + D_n \cos(2n\pi\theta)$
 - \Rightarrow **Increase N_{SV} & N_* by: $2 * (n_z + n_{vpz} + 1)$**

General Rotational Parallaxes (cntd)

- **Alternatively:**
 - **Model in-plane peculiar motions,**
 - Either physically
 - Or as Fourier series for V_x and V_y (and iterate)
 - Follow OP2000's approach: split problem into linear correlations between observables
- *If peculiar motions have a “long-range” component that can be determined from a number of stars, a proper solution for the distance will be possible.*
- **Similar Procedures are/will-be employed for:**
 - Distance determination with maser-regions in galaxies
 - ~ 17 H₂O Masers in M31 & M33 at SKA sensitivity (Barely exceeds the minimum number of shared variables)
 - Velocity-field/Rotation Curve determination of Milky Way

Rotational Parallax: Observability

- **Need bright sources:**
 - Minimize confusion & Maximize observing speed
 - All stars share (almost) the same proper motion
 - **Enough bright stars available”**
 - M 31's “ring of fire” ($\sim 0.6^\circ - 1.5^\circ$)
 - ~ 300 GSC2 ($V \leq 17.5$)
 - ~ 300 Massey et al (2006) ($V \lesssim 16.5$)
 - M 33: 300 2MASS stars ($K_s \lesssim 15$)
 - LMC: 23,000 UCAC stars ($V \lesssim 16$)
- **Also: Need least disturbed galaxy**
- **Our Preference for SIM: M31, M33**
- **Low SIM “cost,”**
 - M31 in 16-32 days ($\frac{1}{4} - \frac{1}{2}$ Key Project)**
(upper limit to time estimate)

Conclusions

- **1% Galaxy Distances will be possible**
 - **Luminosity-Independent**
 - SIM can do M31 & M33 in reasonable amount of time
 - Gaia will do LMC, SMC?? & M31?
(Gaia/M31 needs 10,000 stars down to $V \sim 19$ & 1 km/s RVs)
- RP-targets in LG galaxies: 5D phase-space data
- 1% Distance to LG galaxies will calibrate 2^{ndary} calibrators (Cepheids, TRGB, EBs, ...) $\Rightarrow H_0$
 - **Transfers Solar Neighborhood (<1%) stellar calibration to LG galaxies**
- **Other methods will become available for cross-checks: very important**
- **H_0 is important for Cosmology**

Backup Slides

The Future of Astrometry-enabled Astrophysics (in the US?)

- **Gaia flies before SIM-Lite!**
 - ==> there will be many (10,000's) interesting objects too look at with SIM to get better data
- **Is there going to be dedicated US funding to work with Gaia data?**
 - This would be required to prepare GAIA-follow-up SIM-Lite programs
- **Would it be useful to upgrade SIM-Lite?**
 - Make a larger difference with GAIA
 - Deal with the GAIA follow-up (faster for faint sources)
 - Improve extra-galactic capabilities
- **Are DARWIN & TPF-I potential SIM follow-up?**

General Rotational Parallaxes (cntd)

• Warp?

$$-V_c(R) = V_c(R_0) + dV/dR * (R-R_0)$$

$$i(R) = i(R_0) + di/dR * (R-R_0)$$

- Each relation adds 2 unknowns (zpt & slope)

• Would require $>(12+2+2)=16$ stars

The Rotational Parallax Method (cntd)

• Order of magnitude Estimates:

- **M 33:** $i \sim 56^\circ$, $D \sim 0.84$ Mpc, $V_c \sim 97$ km/s $\Rightarrow \mu_c \sim 24 \mu\text{as/yr}$
- **M 31:** $i \sim 77^\circ$, $D \sim 0.84$ Mpc, $V_c \sim 270$ km/s $\Rightarrow \mu_c \sim 74 \mu\text{as/yr}$
- **LMC:** $i \sim 35^\circ$, $D \sim 0.055$ Mpc, $V_c \sim 50$ km/s $\Rightarrow \mu_c \sim 192 \mu\text{as/yr}$

• Importance of Random Motions (σ) ~ “measurement errors”

- **M 33:** $V_c/\sigma = 9.7 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/9.7 \sim 14.5$ % (per star)
- **M 31:** $V_c/\sigma = 27.0 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/27.0 \sim 5.2$ % (per star)
- **LMC:** $V_c/\sigma = 2.5 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/2.5 \sim 56.5$ % (per star)

Rotational Parallaxes: Accuracy

The equations to solve

$$\begin{aligned}\kappa D \mu_x &= V_{\text{sys},x} + V_{\sigma,x} + V_{c,x} + V_{p,x} \\ \kappa D \mu_{y'} &= V_{\text{sys},ry'} \sin i_s - (V_{p,z} + V_{\sigma,z}) \cos(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \sin(i) \\ V_r &= V_{\text{sys},ry'} \cos i_s + (V_{p,z} + V_{\sigma,z}) \sin(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \cos(i)\end{aligned}$$

are **mildly non-linear** with reasonably **well-known initial conditions: Good solutions expected**

• Problem investigated by Olling & Peterson

[2000, aph/0005484]

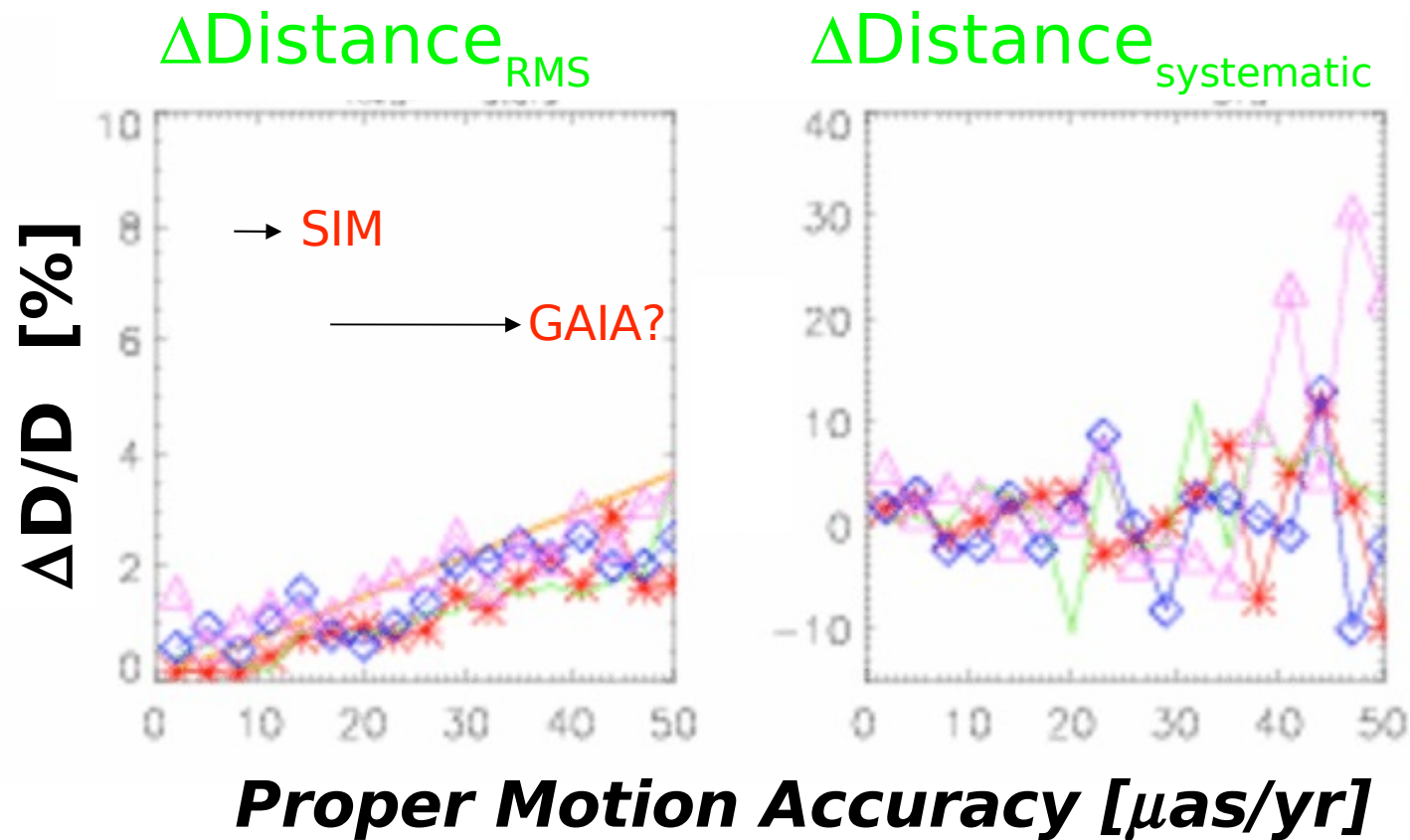
- Solve V_r relation for $(V_{p,z} + V_{\sigma,z})$ and substitute in $\mu_{y'}$
- Or solve V_r relation for $(V_{c,y} + V_{p,y} + V_{\sigma,y})$ and substitute in $\mu_{y'}$
- Or solve $\mu_{y'}$ relation for $V_{c,y} = V_{c,x} * x/y$ and substitute in μ_x

Rotational Parallaxes: Accuracy (cntd)

Rewrite equations employing observables x, y'

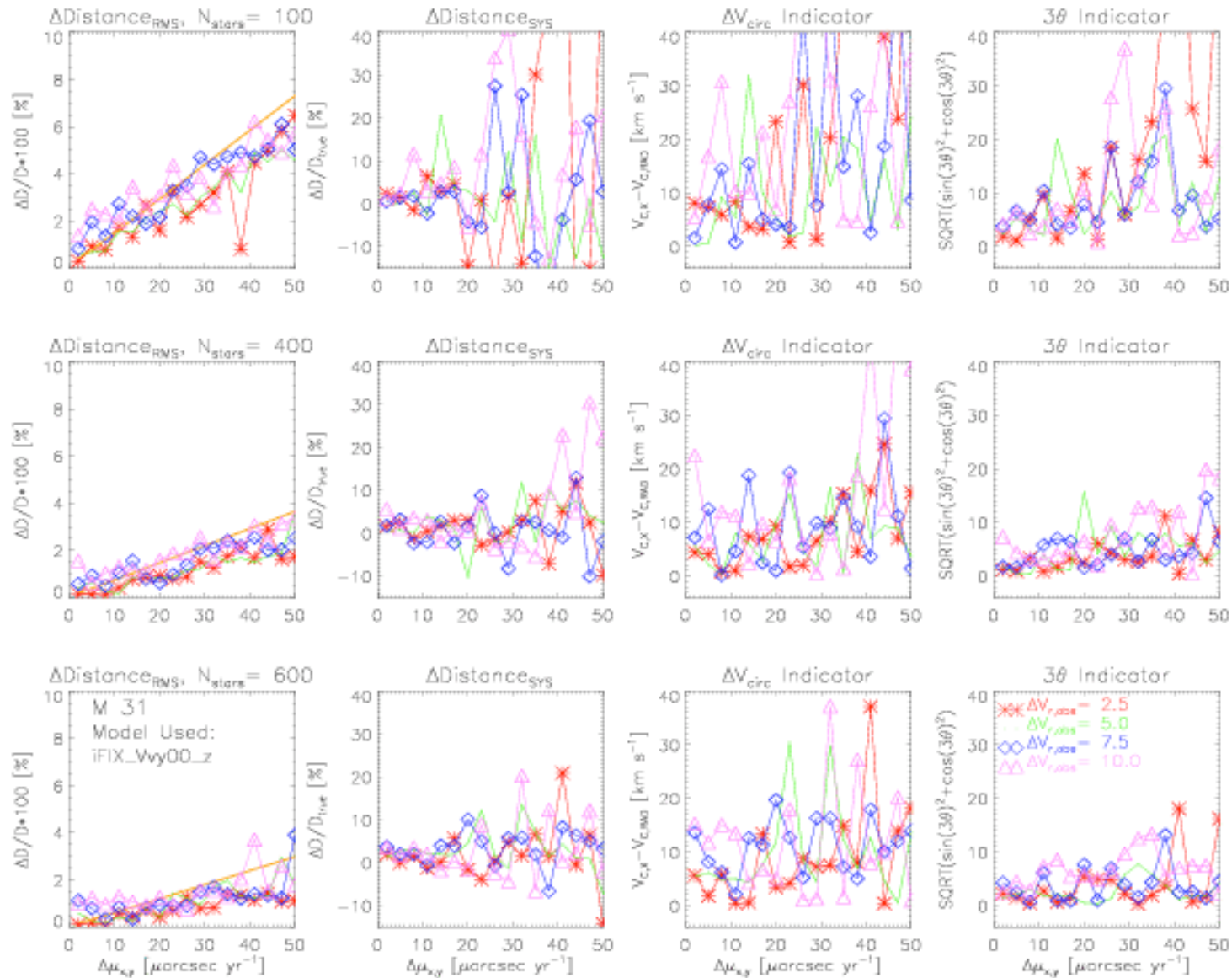
- $\mu_{y'}(\mathbf{V}_R) = \alpha_{y'r} * \mathbf{V}_R + \gamma_{y'r}$
- $\mu_x(\mathbf{V}_R; y'/x) = \alpha_{xr} * \mathbf{V}_R * y'/x + \beta_{xr} * y'/x + \gamma_x$
- $\mu_x(\mu_{y'}; y'/x) = \alpha_{xy'} * \mu_{y'} * y'/x + \beta_{xy'} * y'/x + \gamma_x$
- Solve for unknown α , β and γ coefficients
- The α and γ coefficients yield the desired parameters
 - $\cos^2(i) = -1 / \alpha_{xy'}$
 - $D = 1 / [\alpha_{y'r} \kappa \tan(i)]$
 - Non-circular motions and V_{SYS} appear only in $\gamma_{y'r}$ and β s
- Accuracy of fitted parameters follows from back-substitution and Fourier analysis of velocity field

Rotational Parallax: Expected Results.



Achievable distance errors as a function of proper motion errors:
LEFT: random errors, RIGHT: systematic component

Symbols: accuracy of radial velocity data (2.5 - 10 km/s)
400 Stars used
from: Olling & Peterson (2000)



Distance accuracy (LEFT panel) and Systematic Effects (3 RIGHT panels) as a function of proper motion accuracy

Probing the Hubble Flow:

- **Need to go to >100 Mpc**

$$\epsilon(H_0) \sim V_{\text{pec}}/V_{\text{Hubble}} \sim 200 \text{ km/s} / (100 \text{ Mpc} * 75 \text{ km/s/Mpc}) \sim 2.6\%$$

- **The only known geometric method that probes that far:**

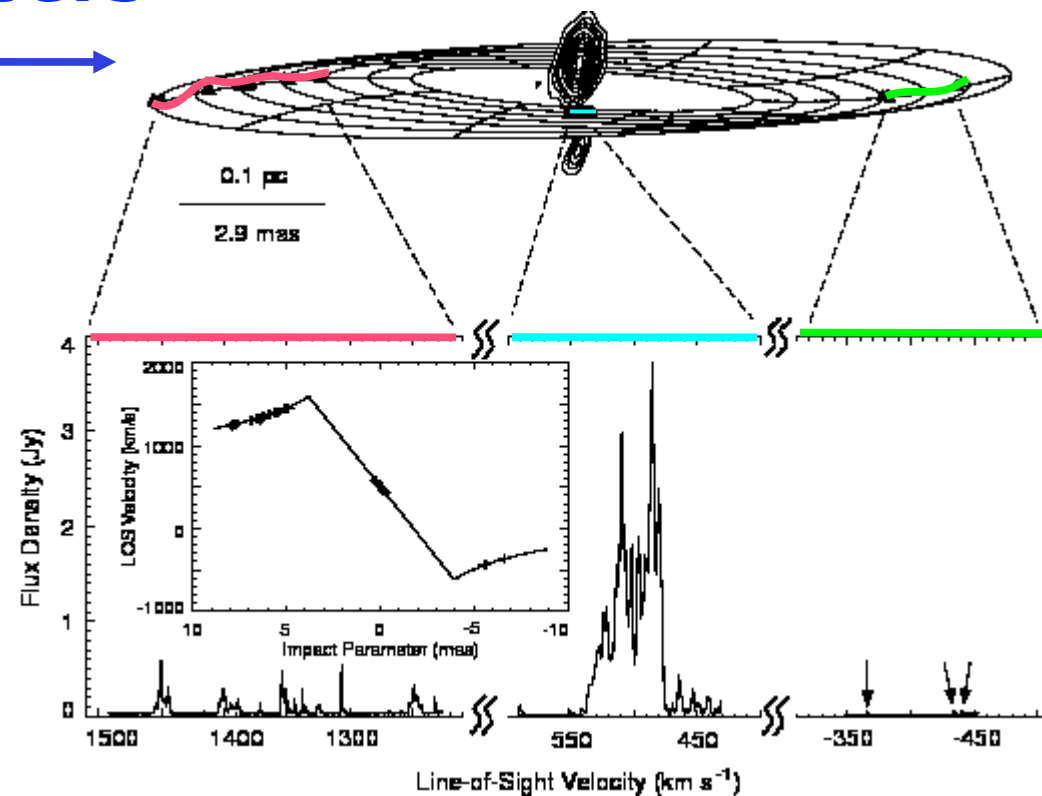
- **Extra-galactic H₂O Masers**

Thin, edge-on disks \longrightarrow

- NGC 4258: $D \sim 7.3$ Mpc
 $\Delta D/D \sim 5\%$

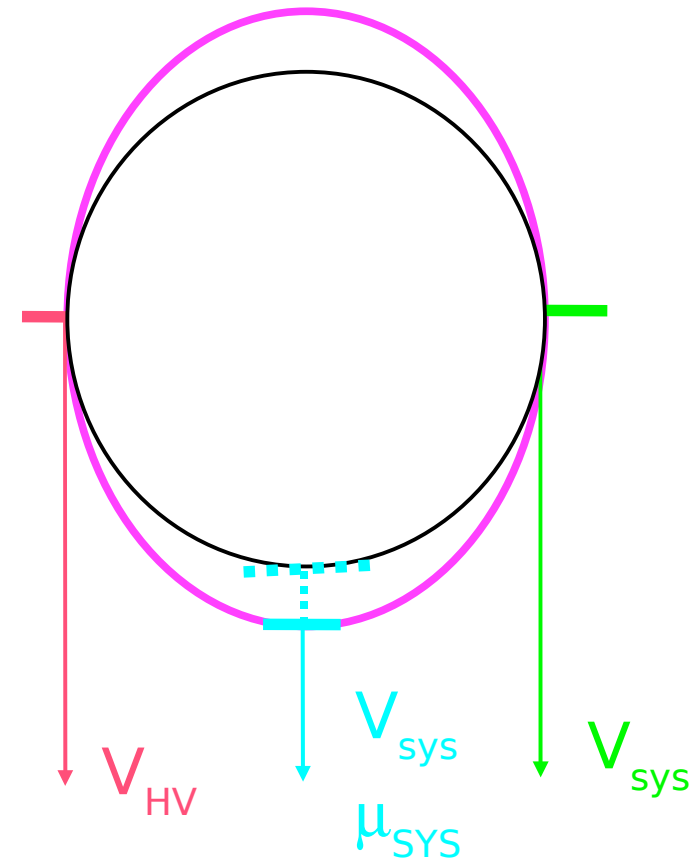
- NGC 1068 $D \sim 14$ Mpc

- $D \sim 200$ Mpc
[e.g., Argon et al, 2007, ApJ, 659, 1040]



Mega Maser Distance Uncertainties:

- N 4258 Distance:
 7.2 ± 0.3 (random)
 ± 0.4 (systematic)
- mostly due to orbital eccentricity [Argon 2007],
 - Up to $e \sim 0.3$ due to, e.g., binary black holes [Eracleous, etal 1995]
 - But ruled out by monitoring [Gezari, Halpren, Eracleous, 2007]
 - Not clear that elliptical orbits exist, if not >60% has emissivity variations [Storchi-Bergmann etal, 2003]



- **Distance error in case of unmodeled eccentricity:**
 - $D_{CIRC} = D_{TRUE} [(1 \mp e)^3 / (1 \pm e)]^{1/2} \sim D_{TRUE} [1 \mp 2e]$

Stellar Ages & Astrometry

- Astrophysics of stars is primarily based on:
THE SUN
 - Fundamental parameters well determined:
Mass, Radius, Luminosity, He-abundance (Y), [Fe/H]
 - ~100 Binaries with M and R better than 1%
 - but see: Kurucz' "**Some things we do not know about stars**" (2002nqsa.conf....3K)
- **M, R, [Fe/H] and Y "set" the rate of evolution**
 - Precise Age Determination of Individual Stars
- **⇒ Detailed Formation History of Galaxy**
 - Star Formation + Oldest Stars (< age of Universe?)
- **⇒ Galaxy Formation & Cosmology**

Stellar Ages & Astrometry (cntd)

• Rate of Evolution

- Luminosity: $(\Delta L/L)^{\text{theory}} \sim (10 + 2 L/L_{\odot}) \pm 5$ [% / Gyr]
Age: $\Delta\tau^{\text{theory}} = (\Delta L/L)^{\text{obs}} / (0.1 + 0.02 L/L_{\odot})$ [Gyr]
 $= (2\Delta\pi/\pi)^{\text{obs}} / (0.1 + 0.02 L/L_{\odot})$ [Gyr]

• **Mid G-type stars : $(\Delta L/L) \sim 10$ %/Gyr**

• **Hipparcos: $(\Delta\pi/\pi)^{\text{obs}} \sim (1 \text{ mas} / 100 \text{ pc}) \Rightarrow \Delta\tau^{\text{theory}} \sim 1,800 \text{ Myr}$**

• **SIM : $(\Delta\pi/\pi)^{\text{obs}} \sim (5 \mu\text{as} / 100 \text{ pc}) \Rightarrow \Delta\tau^{\text{theory}} \sim 9 \text{ Myr}$**
SIM @ 1 kpc $\Rightarrow \Delta\tau^{\text{theory}} \sim 90 \text{ Myr}$
SIM @ 5 kpc $\Rightarrow \Delta\tau^{\text{theory}} \sim 450 \text{ Myr}$
SIM @ 10 kpc $\Rightarrow \Delta\tau^{\text{theory}} \sim 900 \text{ Myr}$

• **Ages are model-dependent**

Will be calibrated with highly accurate GAIA/SIM, Seismology & Ground-Based data

Stellar Ages: GAIA and SIM

- **NEED RADIUS** \Rightarrow **Eclipsing Binaries** ($\sim 1\%$ of Population)

- Photometry $\Rightarrow R_*, m_V, A_V$
Spectroscopy $\Rightarrow V_{\text{ORB}}, M_* \text{ \& } [\text{Fe}/\text{H}]$
Astrometry (π) + m_V & $A_V \Rightarrow$ **Luminosity**

TWO stars on same Isochrone \Rightarrow Age & Helium

[Ribas (2006ASPC..349...55R), Lebreton (2005tdug.conf..493L),
Lastennet (2002A&A...396..551L); Metcalfe et al (2006ASPC..349...55R)]

- **GAIA** is survey mission will determine overall SF History

≤ 600 pc has $\Delta\pi/\pi \leq 1\%$ for G7 star ($V \sim 14.5$)

$\sim 6,254,000$ thin-disk stars $\Rightarrow \sim 190,000$ EBs $\Rightarrow \Delta\tau \sim 4$ Myr

$\sim 385,000$ thick-disk stars $\Rightarrow \sim 12,000$ EBs $\Rightarrow \Delta\tau \sim 5$ Myr

$\sim 12,000$ spheroid stars $\Rightarrow \sim 300$ Ebs $\Rightarrow \Delta\tau \sim 9$ Myr

- **SIM** should do the rare **Special Cases** at larger distances
Binary cousins of **Old Uranium Stars** with $[\text{Fe}/\text{H}] \sim < -3$

$\tau \sim 13.2 \pm 2.7$ Gyr HE 1523-0901, $d \sim 1$ kpc, $V \sim 11$ [Frebel et al 2007]

$\tau \sim 14.9 \pm 3.0$ Gyr CS 22892-0529, $d \sim 1.5$ kpc, $V \sim 12$ [Snedden et al 1996; Hill et al 2002]

$\tau \sim 13.2 \pm 2.7$ Gyr HE 1424-0241, $d \sim 8$ kpc, $V \sim 14$ [Cohen et al 2007]

[See Beers & Christlieb 2004, ARA&A for a review]

Astrometry & Cosmology

- CMB, high-z galaxy data, Ly- α forest & BBN yield:

Hubble Constant = H_0 = 71 ± 2 ± 7 [km/s/Mpc]

Age = t_0 = 13.7 ± 0.2 [Gyr]

Matter Density = Ω_m = 0.27 ± 0.02 [ρ_{CRIT}]

Total/Baryon Matter = Ω_m / Ω_b = 6.1 ± 1.1

Primordial Helium = Y_p = 0.2482 ± 0.0004

- **Astrometry** of *M31 (M33)* \Rightarrow strong limits on H_0
- **Astrometry** of *Galactic Objects* can set relevant limits on t_0 , Y_p and Star Formation History

[Spergel *et al*, 2003, 2006; Freedman *et al* 2001; Mathews *et al*, 2005; Madau *et al*, 1996, this talk]

H_0 , CMB and Dark Energy (cntd)

- WMAP yields: location (l_A) of the acoustic peak:
- Cosmology yields:
 - 1) size of the acoustic oscillation
 - 2) the angular-size distance (D_A) relation

$$D_A = a_* \int_{a_*}^1 \frac{1}{a^2 H(a)} da \quad \text{with} \quad \frac{H(a)}{100} = \sqrt{\frac{\omega_m}{a^3} + \frac{h^2 - \omega_m}{a^{3(1+w)}}} \quad \text{and} \quad \Omega_{tot} = 1$$

- This is an integral equation with two unknowns:
 - H_0 and
 - “ w ” the Equation of State (EOS) of Dark Energy
- IF the Cosmological Constant is the Dark Energy, THEN $w = -1$ and: **WMAP determines H_0**

H_0 , CMB and Dark Energy (cntd)

- However, this may *not very accurate*.
The Assumptions were:
 - Flat Universe
 - Dark Energy has constant EOS
 - Dark Matter does not cluster, no tensor modes, no quintessence, no running spectral index, no strings, no domain walls, no non-Gaussian fluctuations, no deviations from GR, et cetera
- Allowing for a variable EOS of Dark Energy, Hu (2005) concludes that:
 - ``... the Hubble constant is the single most useful complement to CMB parameters for dark energy studies ... [if $H_0(z)$ is] ... accurate to the present level''

Alternatively, one can (try to) determine the **ages**:

$$-\tau = \int_0^1 da / [a H(a)] \Leftrightarrow \text{ages of oldest stars}$$

$$-\tau(z) = \int_0^{a(z)} da / [a H(a)] \Leftrightarrow \text{ages of high-}z \text{ galaxies}$$

[e.g., Bothum et al 2006, Jimenez et al 2003, Simon 2005]

Summarized in Figure 4 of Spergel et al, 2004

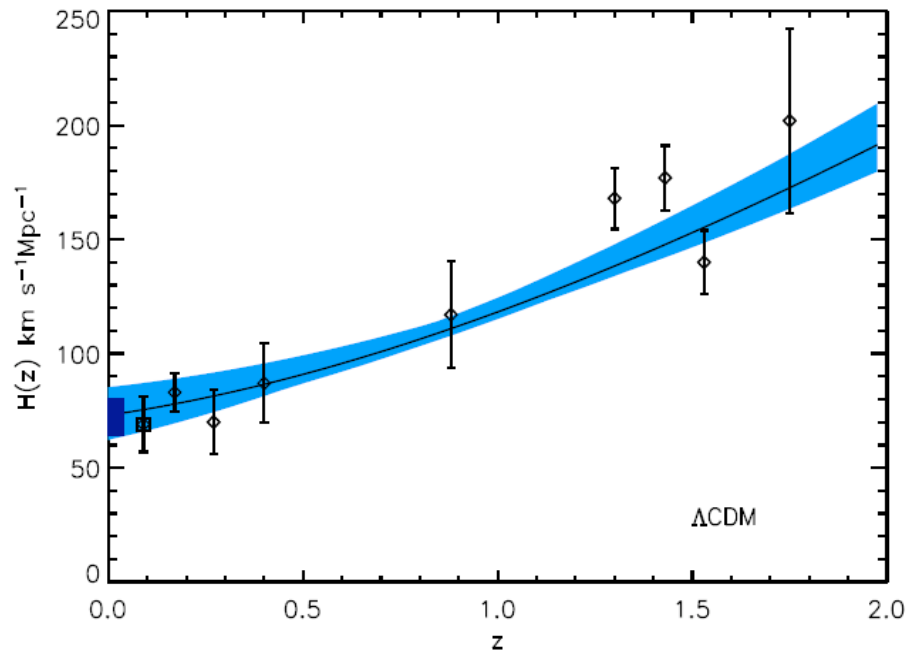
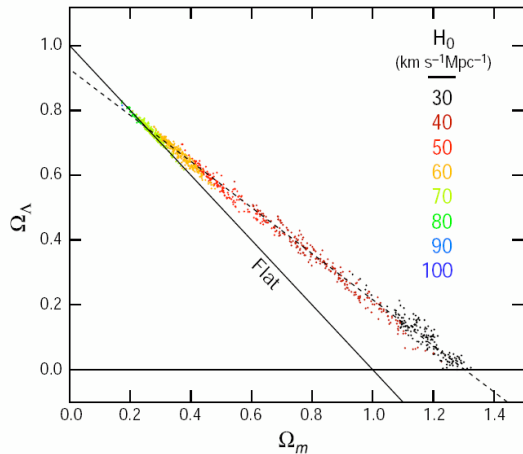


Fig. 4.— The Λ CDM model fit to the WMAP data predicts the Hubble parameter redshift relation. The blue band shows the 68% confidence interval for the Hubble parameter, H . The dark blue rectangle shows the HST key project estimate for H_0 and its uncertainties (Freedman et al. 2001). The other points are from measurements of the differential ages of galaxies, based on fits of synthetic stellar population models to galaxy spectroscopy. The squares show values from Jimenez et al. (2003) analyses of SDSS galaxies. The diamonds show values from Simon et al. (2005) analysis of a high redshift sample of red galaxies.

H_0 , CMB and Dark Energy (cntd)

- Many groups pursue other methods to determine some (combination of) parameter(s) that constrain the “integral”
 - **Luminosity-Distance relation from Supernovae Ia**
 - $D_L(z) = D_A(z) / a(z)^2$
 - **Baryon Oscillations (sensitive to local galaxy density)**
 - $Volume(z) = [D_A(z) / a(z)]^2 / H(z) * \Omega_{sky} \Delta z$
 - **Galaxy Cluster Abundance**
 - Depends on Volume(z) and non-linear structure growth
 - **Weak Lensing**
 - Depends on: $D_A(z)$, $H(z)$ and structure growth
- *[e.g., Albrecht et al 2006 = DETF]*

H₀ , CMB and Dark Energy (cntd)



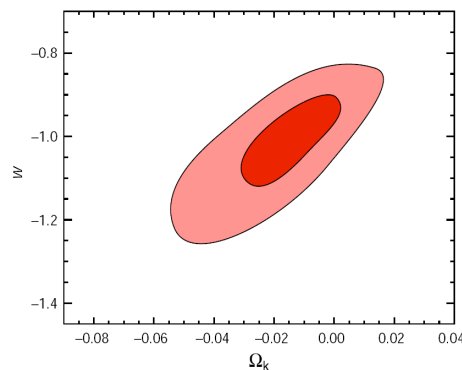
- We use the Spergel et al (2006/7) WMAP & “other data” to approximate the relations between the various parameters ($P_i = a_{ij} + b_{ij} P_j$) :

$$\Omega_{\Lambda} = a_{\Lambda m} + b_{\Lambda m} \Omega_m$$

$$\Omega_K = a_{K\Lambda} + b_{K\Lambda} \Omega_{\Lambda}$$

$$w = a_{wK} + b_{wK} \Omega_K$$

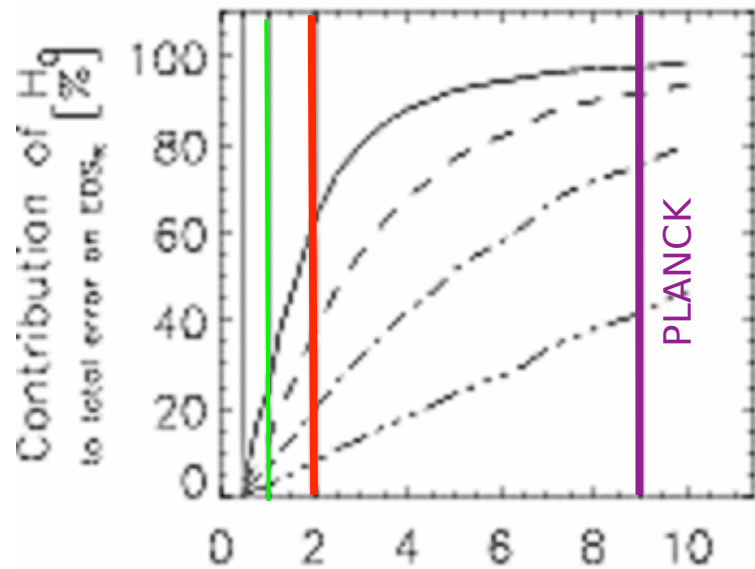
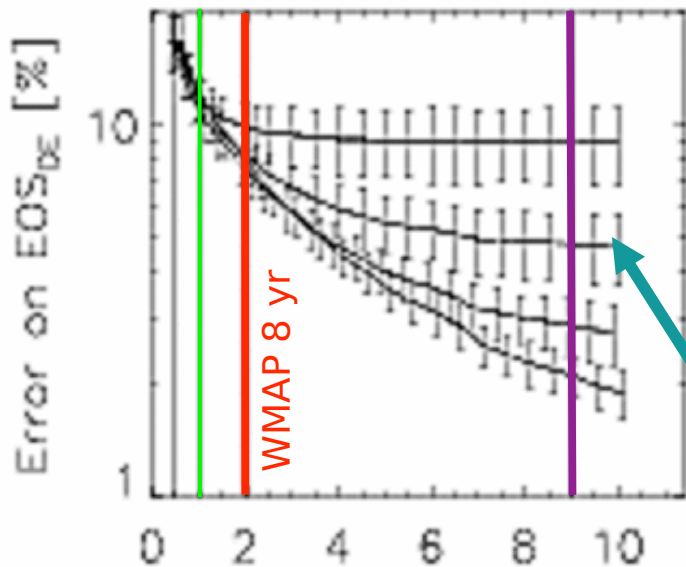
Data Set	Ω_K	Ω_{Λ}
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$



- For a constant EOS, but a Universe of general curvature**

Dark Energy Equation of State:

CMB and H_0 Error Dependencies



$\epsilon(\omega_m; \text{NOW}) / \epsilon(\omega_m) \rightarrow$

To arrive at:

- $$w = a_{wK} + b_{wK} (a_{K\Lambda} + a_{\Lambda m} b_{K\Lambda}) + b_{\Lambda m} b_{K\Lambda} b_{wK} * \omega_m / h^2$$

$$= (-0.83 \pm 0.11) - (0.56 \pm 0.06) \omega_m / h^2$$

- Error on EOS as a function of $\epsilon(\omega_m)$:

$$\epsilon_w^2 = \dots + b_{\Lambda m} b_{K\Lambda} b_{wK} \left[\left(\frac{\epsilon_{\omega_m}}{h^2} \right)^2 + \left(\frac{2 \omega_m \epsilon_h}{h^3} \right)^2 \right]$$

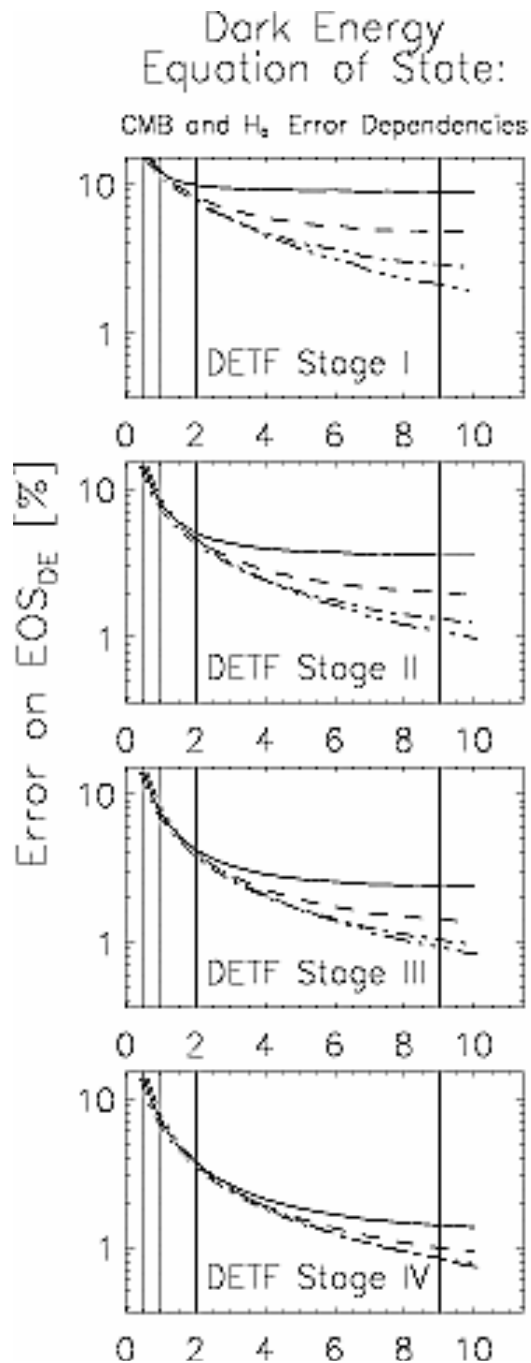
- In Figure: curves from top to bottom for $\epsilon(H_0) = \epsilon(H_0; \text{now}) * [1, 1/2, 1/4, 1/10]$

[Olling, 2007, MNRAS, 378, 1385]

H_0 , CMB and Dark Energy (cntd)

- The Dark Energy Task Force [\[Albrecht et al 2006\]](#) recommends several approaches to determine the “evolution” of the EOS:
 - Stage I: Current knowledge
 - STAGE II: Projects finishing soon (including *PLANCK*)
 - STAGE III: Photo- (spectro-) redshifts on 4^m (8^m) telescopes
 - STAGE IV: *Large Synoptic Telescope,*
Joint Dark Energy Mission,
Square Kilometer Array
 - At Stage IV, accurate H_0 knowledge matters $< \sim 50\%$
- **Unpublished Minority Opinion** (Freedman & Hu):
Spend effort on determination of H_0

H_0 & Dark Energy in various stages of the DETF:



• At intermediate stages, small H_0 errors matter more:

- Stage I: $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 8.9\%$
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 2.3\%$

- Stage II: $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 3.6\%$
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 1.2\%$

- Stage III: $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 2.4\%$
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 1.0\%$

- Stage IV: $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 1.5\%$
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 0.9\%$

$\epsilon(\omega_m; \text{NOW}) / \epsilon(\omega_m) \rightarrow$