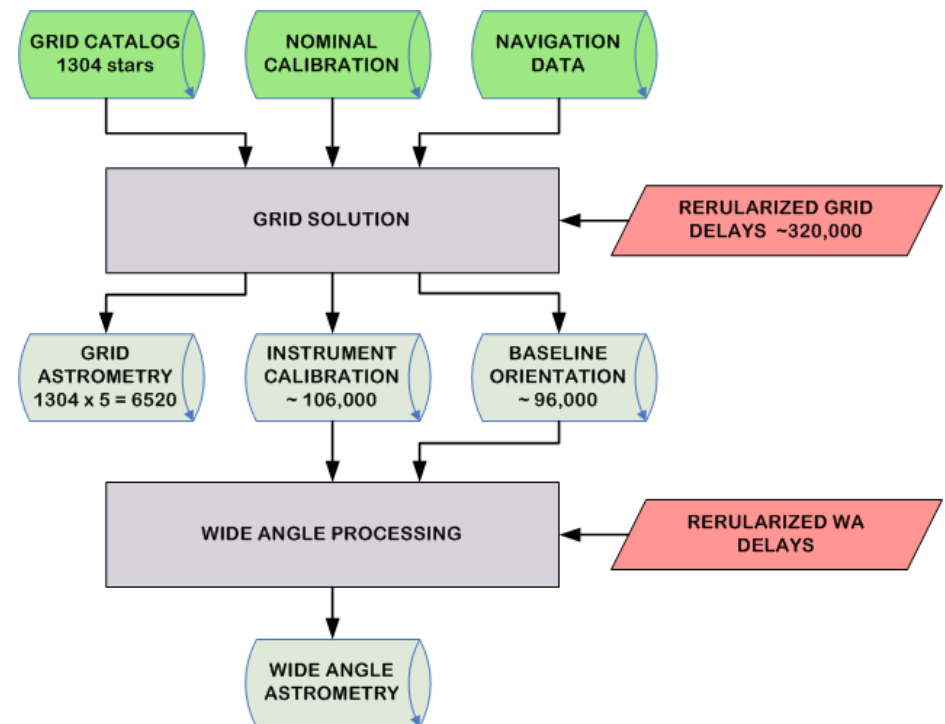


# Grid and Reference Frames Global Astrometry with SIM

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# Grid

- SIM Grid solution is a global, one step LS adjustment of ~200K unknowns in ~300K equations
- Purpose of the Grid:
  - Determine instrument calibration and baseline orientation parameters for subsequent use in Narrow Angle and Wide Angle data processing
  - Establish SIM Reference Frame (SIMRF) better than 1  $\mu$ s
  - Additional science (e.g., gravitational deflection) – **you are encouraged to invent your own!**
- Grid objects:
  - 1304 basic grid stars, RV vetted
  - 25—50 optically bright quasars
  - optionally, all NA targets and all reference stars



## Regularized Delay Equation

$$\delta d \cong (B \cdot \delta s) + (\delta B \cdot s) + C + \delta F + \varepsilon$$

- Unknown baseline orientation ( $\delta B$ , 2-vector), apparent position of star ( $\delta s$ , 2-vector), path delay offset  $C$ , calibration parameters ( $\delta F$ , up to  $\sim 40$ )
- Condition equation is severely underdetermined and can not be solved from a single measurement or a single tile, hence all sky, global solution needed

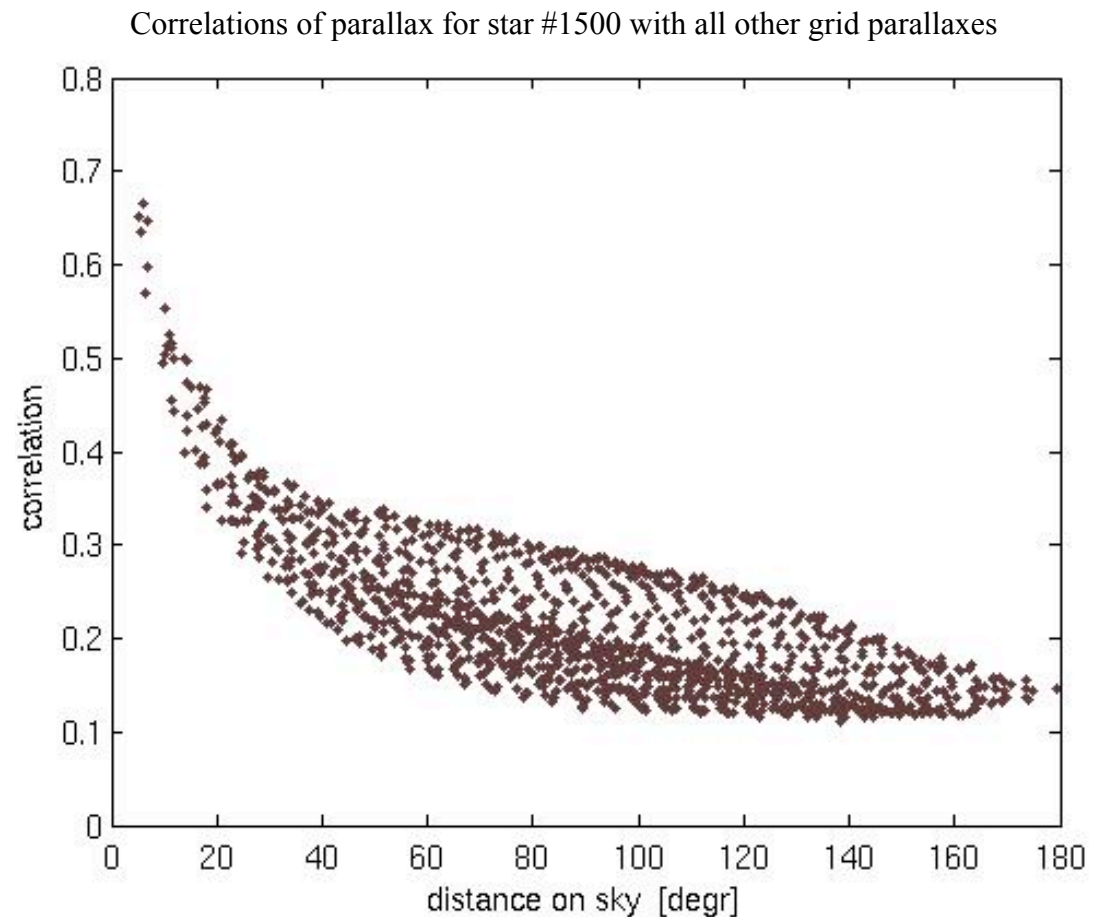
$$\sigma_{\text{mission average per star}} = M \cdot \sigma_0$$

where  $\sigma_0$  is single delay measurement precision ( $\sim 14 \mu\text{as}$ ),  $M$  is grid multiplier

- If the condition equations were perfectly conditioned,  $M \sim 1/\sqrt{N} \sim 0.07$ , but in reality  $M \approx 0.26$  – the loss of condition comes from a coupling of  $\delta B$  and  $\delta s$  unknowns in the finite FOR

## Correlated astrometric parameters

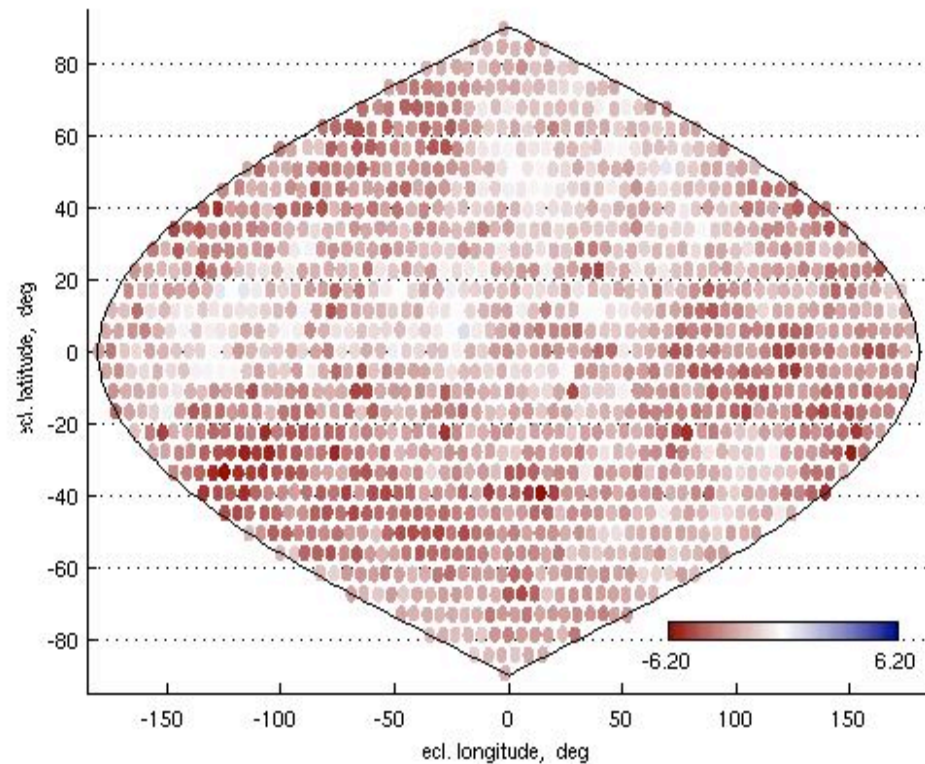
- The power spectrum of random errors defined by scalar or vector spherical harmonics is “red”, i.e., most error comes in large-scale perturbations
- Astrometric errors are *positively* correlated across the sky
- **Differential Wide Angle** mode can give a factor of 2 improvement in precision wrt global accuracy



# Parallax zero-point error

- In a typical realization of grid, almost all parallax errors have the same sign because of a dominating zero-point error
- A relatively small number of quasars (25—50) constrain low-order spherical harmonics and lead to dramatically better grid accuracy in parallax

A single realization of grid parallax error



## Quasars in the Grid

- USNO selected ~110 optically bright, low-variable quasars
- Simulations and covariance analysis of grid solutions with only 23 quasars reveal the benefits of quasar constraints:
  - Overall parallax accuracy improves ~ 17%
  - Parallax zero-point error improves ~60%
  - Greatly improved confidence intervals of parallax mission performance, e.g., the 0.99 confidence level on parallax error drops from 8.05  $\mu\text{as}$  without quasars to 3.97  $\mu\text{as}$  with only 23 grid quasars
  - The SIM Reference Frame (SIMRF) will be inertial to ~ 1  $\mu\text{as/yr}$  in residual spin and ~ 1.7  $\mu\text{as}$  in residual rotation
  - Some harmful systematic errors reduced by a factor of 5, e.g., certain systematic navigation errors and stellar aberration corrections

## Why Grid performance is important?

- Correlated zonal errors propagate 100% into Wide Angle astrometry
- Accurate and inertial SIMRF entails fundamental (and free for you) science, for example
  - Galactic rotation and Galactocentric acceleration of the Sun
  - Gravitational bending of light
  - Constraints on relic gravitational waves
  - Speculatively, rotation of the Universe



$$g = h \cos \varpi t (\mathbf{x}\mathbf{x} - \mathbf{y}\mathbf{y})$$

# Wide Angle astrometric performance



Q: Can WA accuracy be better than the Grid?

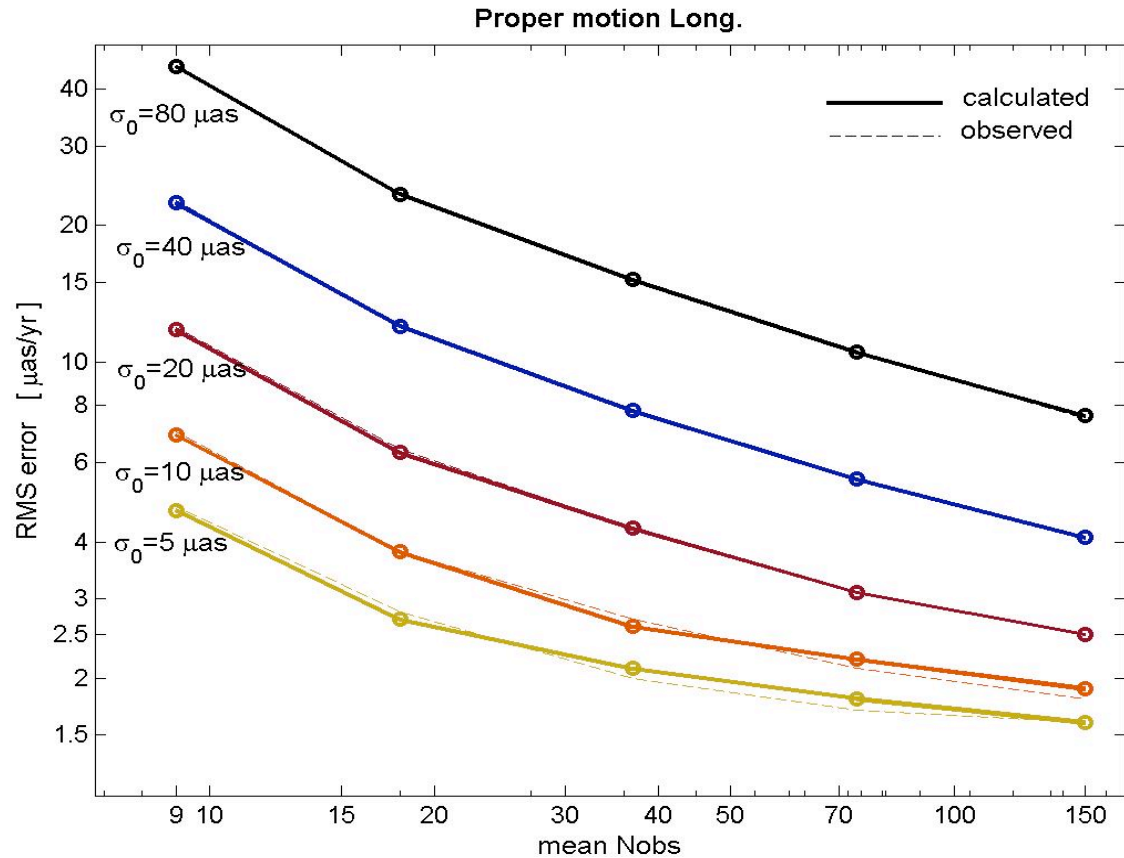
A: Yes, but only slightly, only for bright stars and at a cost

Q: Can single visit integration time be traded for number of observations?

A: Yes, as this plot shows

Q: Does the measurement precision depend on the position within a tile?

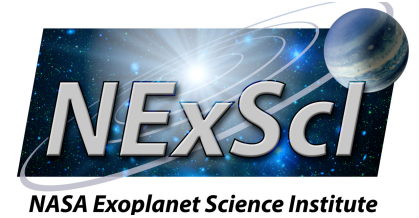
A: Yes, see backup slide



Mission-average WA astrometric accuracy of proper motions as function of number of observations and single measurement precision

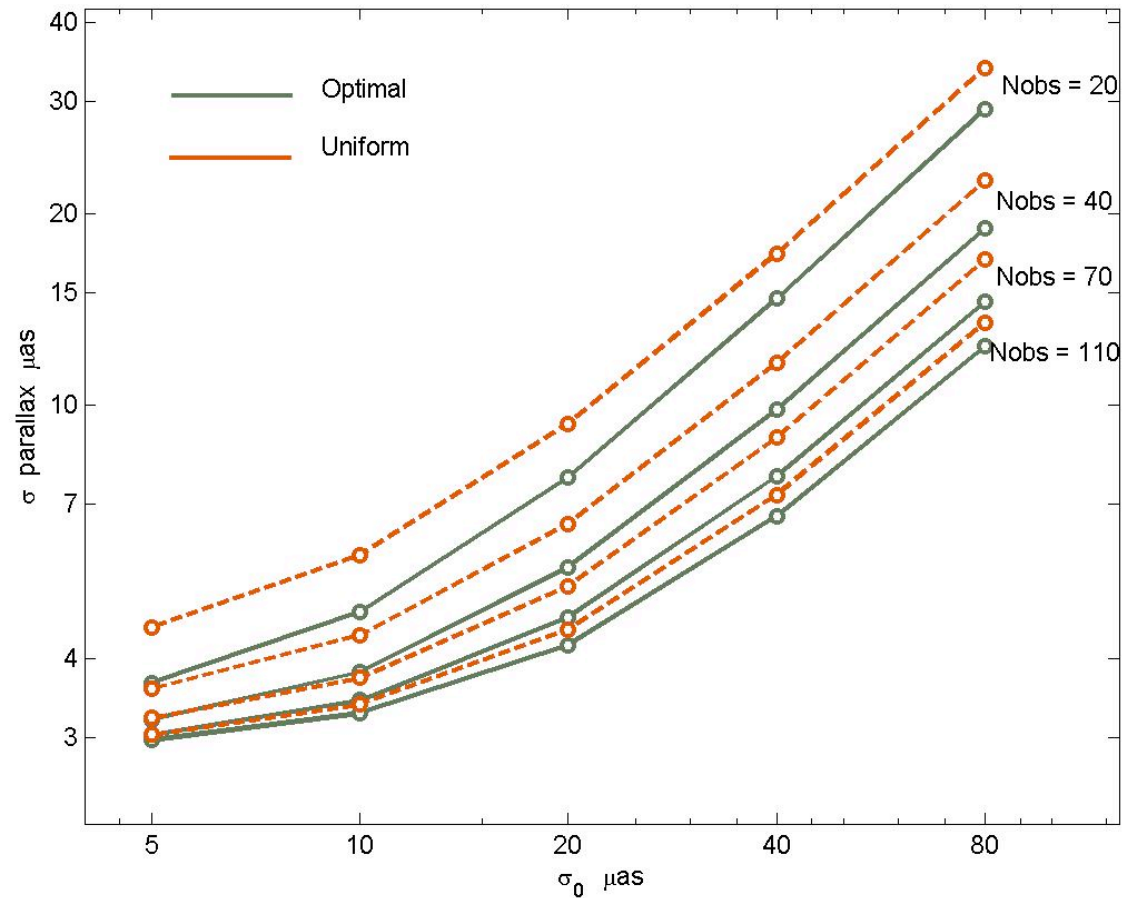


# Optimized schedules



Q: Can accuracy be gained by clever scheduling of a given number of observations?

A: Absolutely, see the plot for a SVD-based optimization on parallax accuracy



# Backup Slides





NASA Exoplanet Science Institute

# Grid multipliers

| freq. | pos.  | par.  | p.m.  | BLL    | Z4     | Z5      | Z6     | Z7     | Z8     | Z9     | Z10    | Z11    | Z12    | Z13    | Z14    | Z15    | Z16-Z28       |
|-------|-------|-------|-------|--------|--------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------------|
| 1/480 | 0.217 | 0.249 | 0.150 | 0.0908 | 0.0305 | -       | 0.0351 | 0.0271 | 0.0957 | 0.0952 | 0.0266 | 0.0332 | 0.0485 | 0.0211 | 0.0492 | 0.0316 | 0.0262-0.0859 |
| 1/480 | 0.210 | 0.243 | 0.146 | 0.0323 | 0.0209 | -       | 0.0253 | 0.0202 | 0.0408 | 0.0345 | 0.0196 | 0.0222 | 0.0295 | 0.0199 | 0.0328 | 0.0218 | -             |
| 1/480 | 0.209 | 0.242 | 0.145 | 0.0320 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/480 | 0.209 | 0.242 | 0.145 | 0.0320 | -      | 17.05   | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/480 | 0.209 | 0.242 | 0.145 | 0.0320 | -      | -       | 0.0183 | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/100 | 0.286 | 0.279 | 0.199 | 0.2023 | 0.0755 | -       | 0.0898 | 0.0781 | 0.1826 | 0.1863 | 0.0509 | 0.0800 | 0.1032 | 0.0473 | 0.1163 | 0.0766 | 0.0630-0.1742 |
| 1/100 | 0.263 | 0.259 | 0.183 | 0.0837 | 0.0532 | -       | 0.0600 | 0.0567 | 0.0922 | 0.0777 | 0.0353 | 0.0532 | 0.0704 | 0.0461 | 0.0759 | 0.0528 | -             |
| 1/100 | 0.255 | 0.250 | 0.177 | 0.0819 | 0.0396 | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/48  | 0.291 | 0.270 | 0.202 | 0.1212 | 0.1296 | -       | 0.1263 | 0.0947 | 0.1791 | 0.1602 | 0.0984 | 0.1206 | 0.1468 | 0.0957 | 0.1500 | 0.1053 | -             |
| 1/48  | 0.275 | 0.251 | 0.191 | 0.1113 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/48  | 0.275 | 0.253 | 0.191 | 0.1114 | -      | 59.06   | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/48  | 0.276 | 0.256 | 0.191 | 0.1114 | -      | -       | 0.0583 | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.321 | 0.292 | 0.223 | 0.1601 | 0.1978 | -       | 0.1813 | 0.1375 | 0.2517 | 0.2455 | 0.1364 | 0.1671 | 0.2226 | 0.1512 | 0.2152 | 0.1633 | -             |
| 1/30  | 0.293 | 0.258 | 0.203 | 0.1364 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.294 | 0.259 | 0.204 | 0.1366 | 0.0702 | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.294 | 0.259 | 0.204 | 0.1366 | -      | 72.43   | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.295 | 0.265 | 0.205 | 0.1366 | -      | -       | 0.0719 | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.294 | 0.259 | 0.204 | 0.1366 | -      | -       | -      | 0.0902 | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/30  | 0.196 | 0.237 | 0.136 | -      | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | 0.0808 | -             |
| 1/30  | 0.325 | 0.295 | 0.226 | 0.1868 | 0.2443 | 1.61e+5 | 0.2357 | 0.1615 | 0.3012 | 0.2924 | 0.1603 | 0.2049 | 0.2904 | 178.8  | 0.3238 | 0.2015 | -             |
| 1/24  | 0.303 | 0.266 | 0.211 | 0.1497 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/24  | 0.307 | 0.276 | 0.214 | 0.1501 | -      | -       | 0.0792 | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/12  | 0.363 | 0.328 | 0.252 | 0.2018 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |
| 1/6   | 0.482 | 0.479 | 0.335 | 0.2898 | -      | -       | -      | -      | -      | -      | -      | -      | -      | -      | -      | -      | -             |

## Propagation of FDE

- Field-dependent instrument parameters will be determined in the grid and applied to WA delays
- Field-dependent errors sharply increase at the edge of the FOR (tile)

