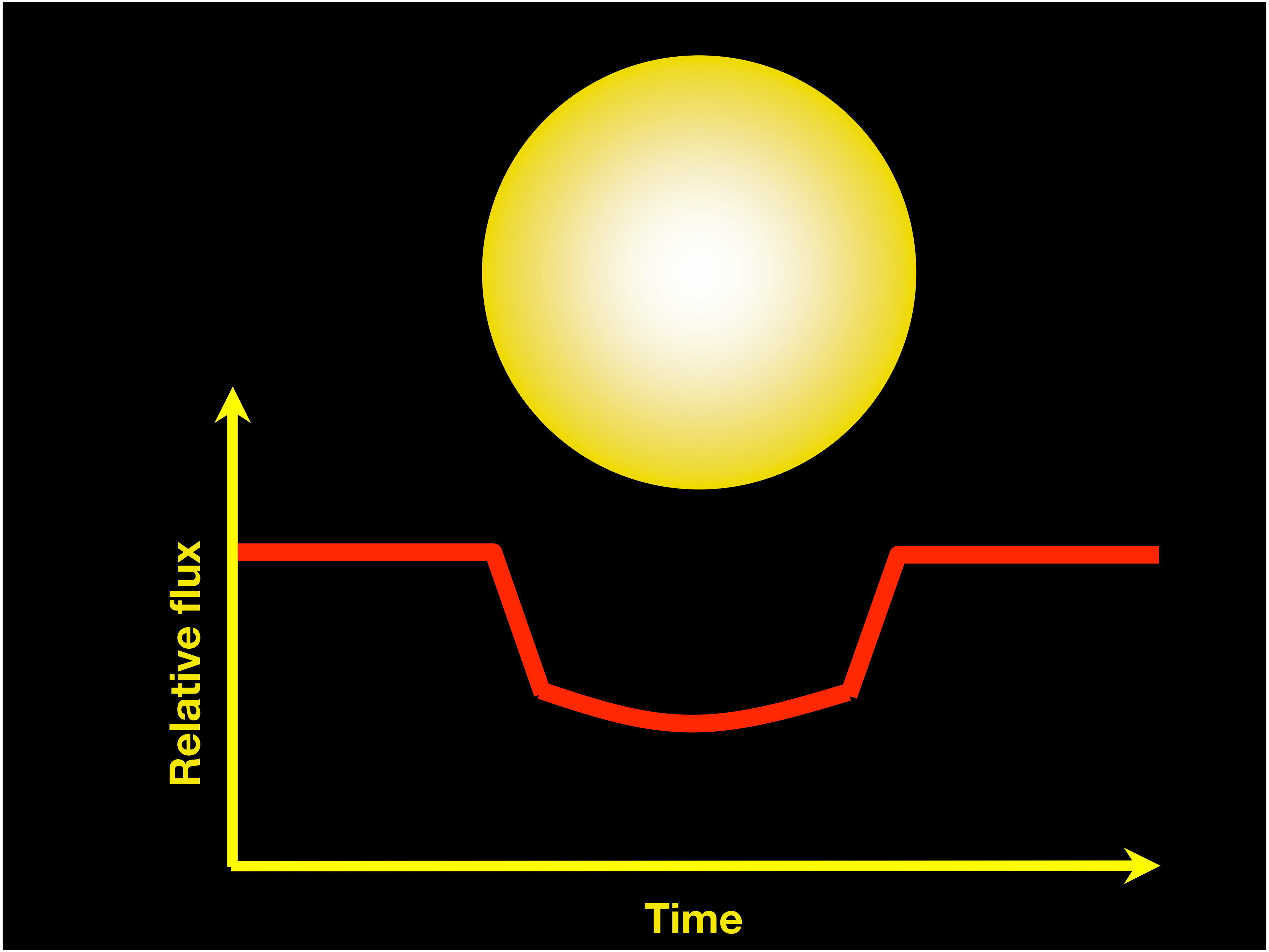
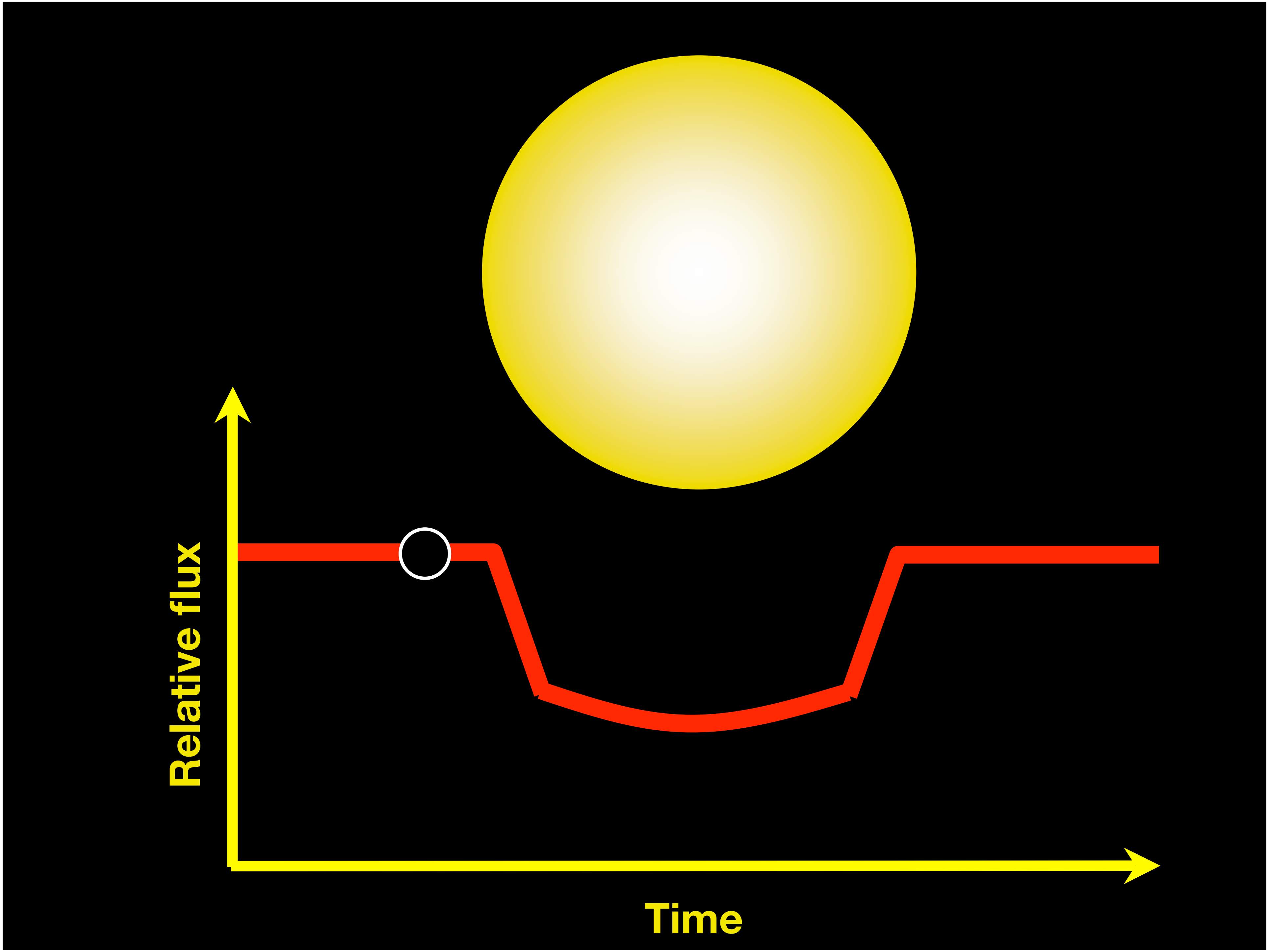
Measuring the Astrometric Signature of Transiting Planets with SIM

Scott Gaudi, Ohio State University


Relative flux
Time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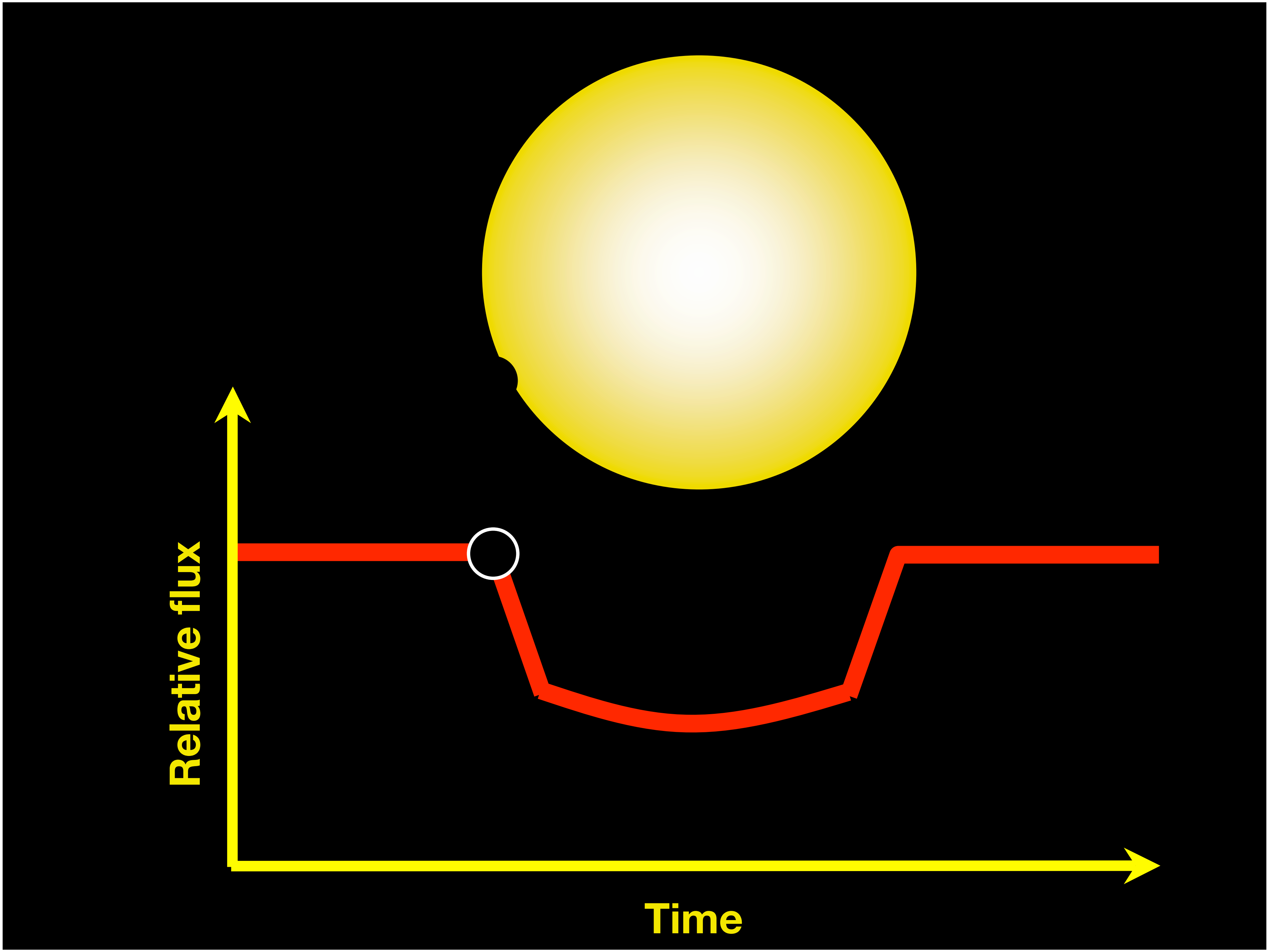
Time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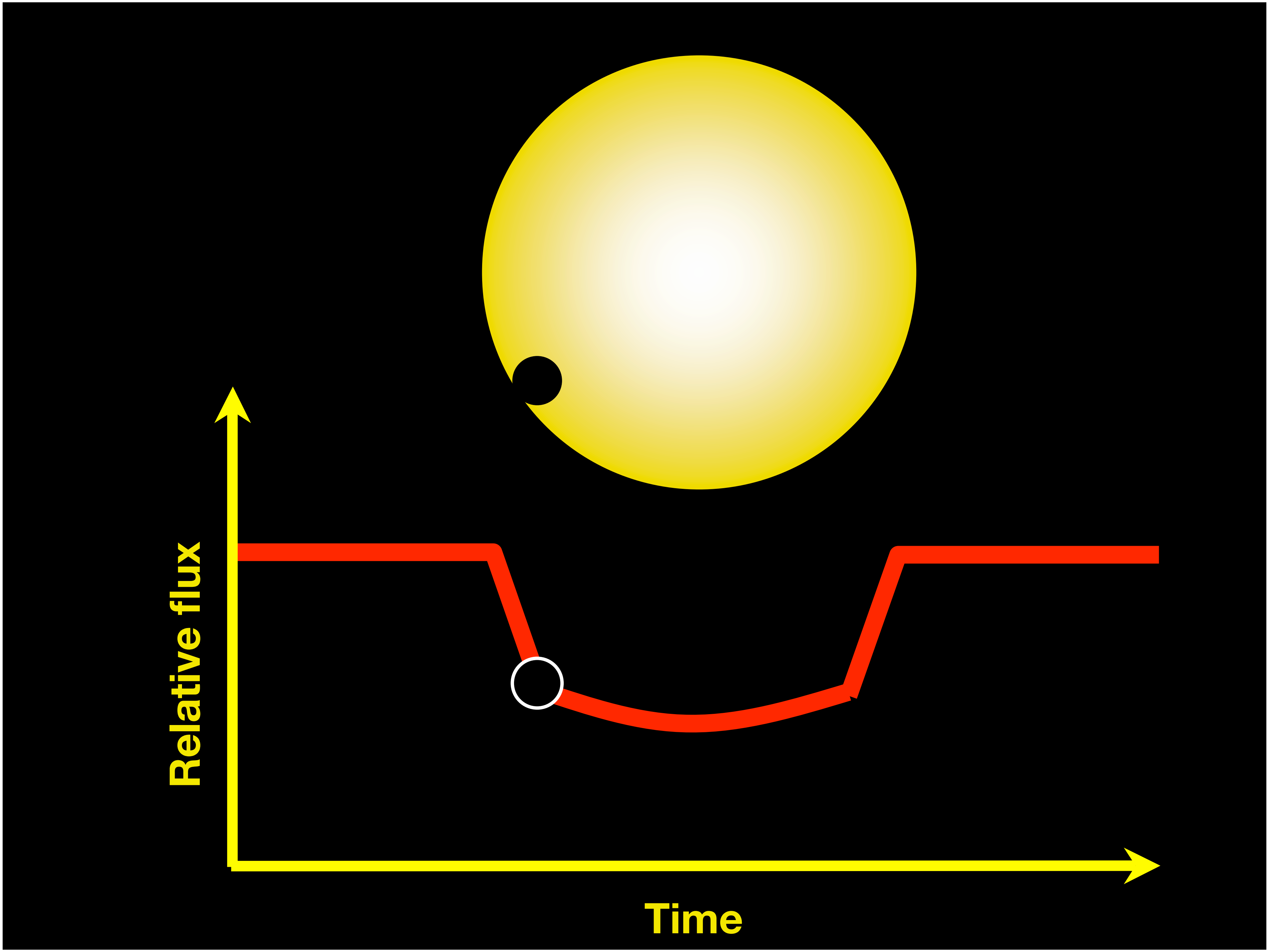
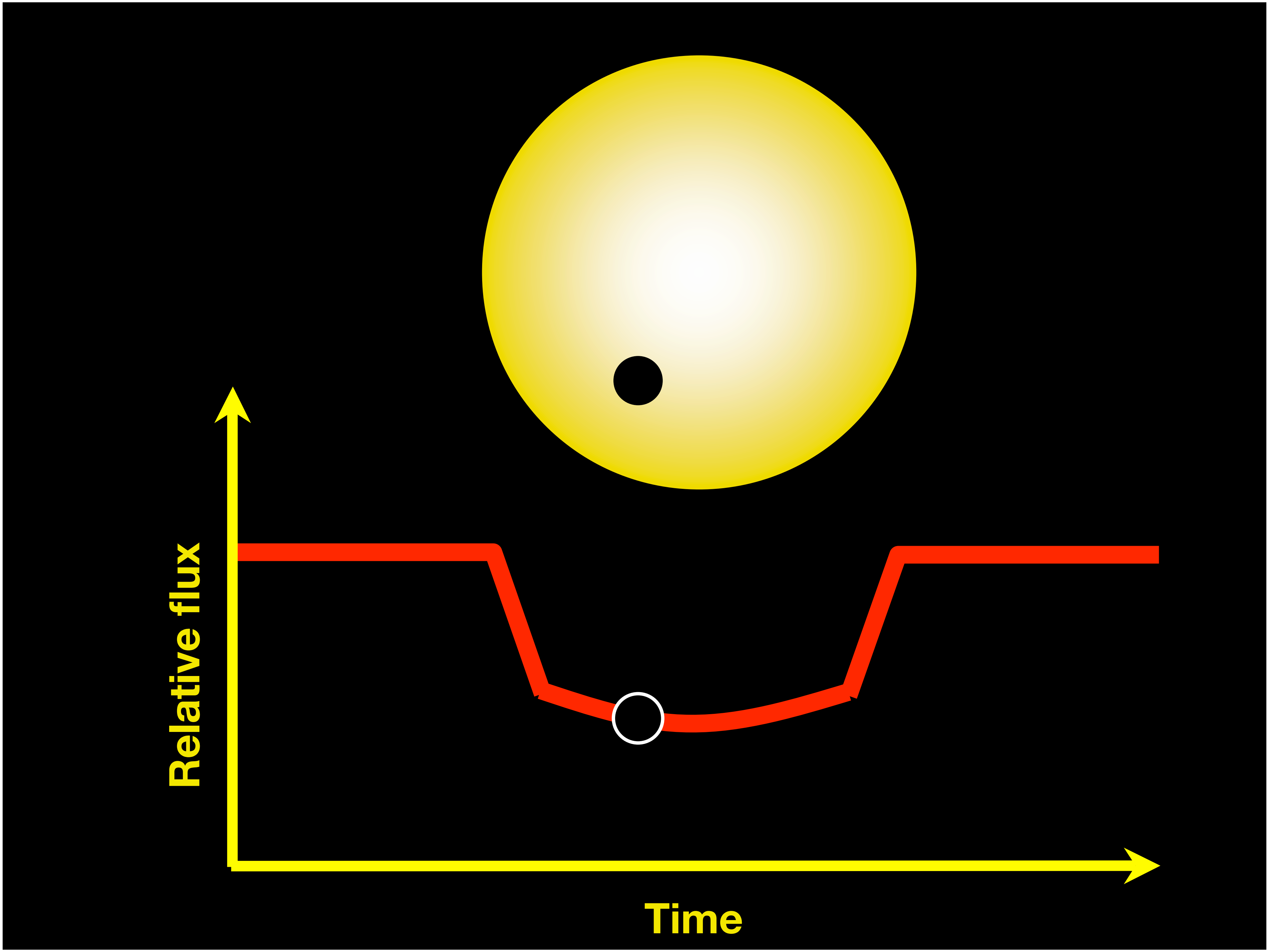
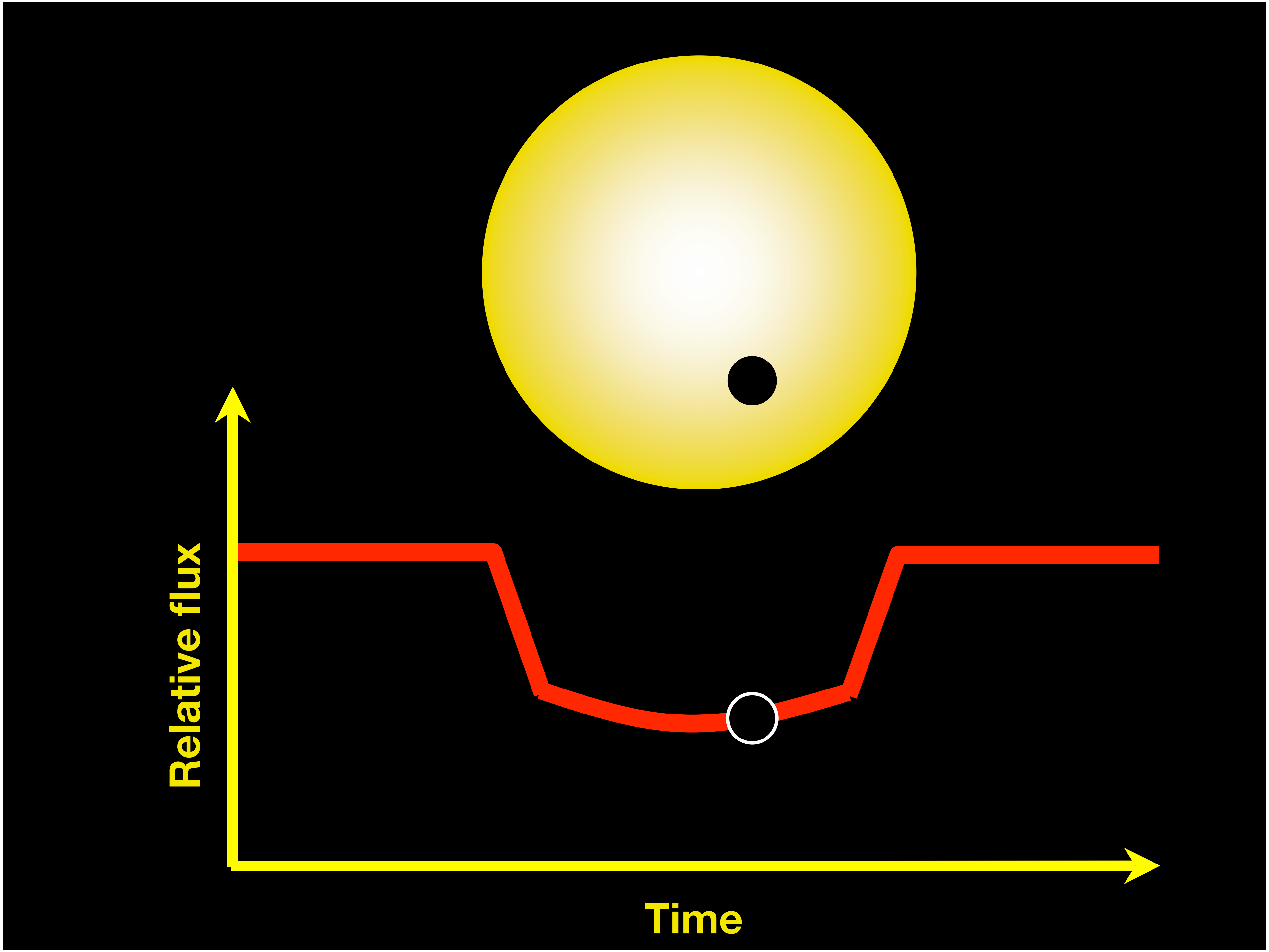
Time


Relative flux
Time


Relative flux
Time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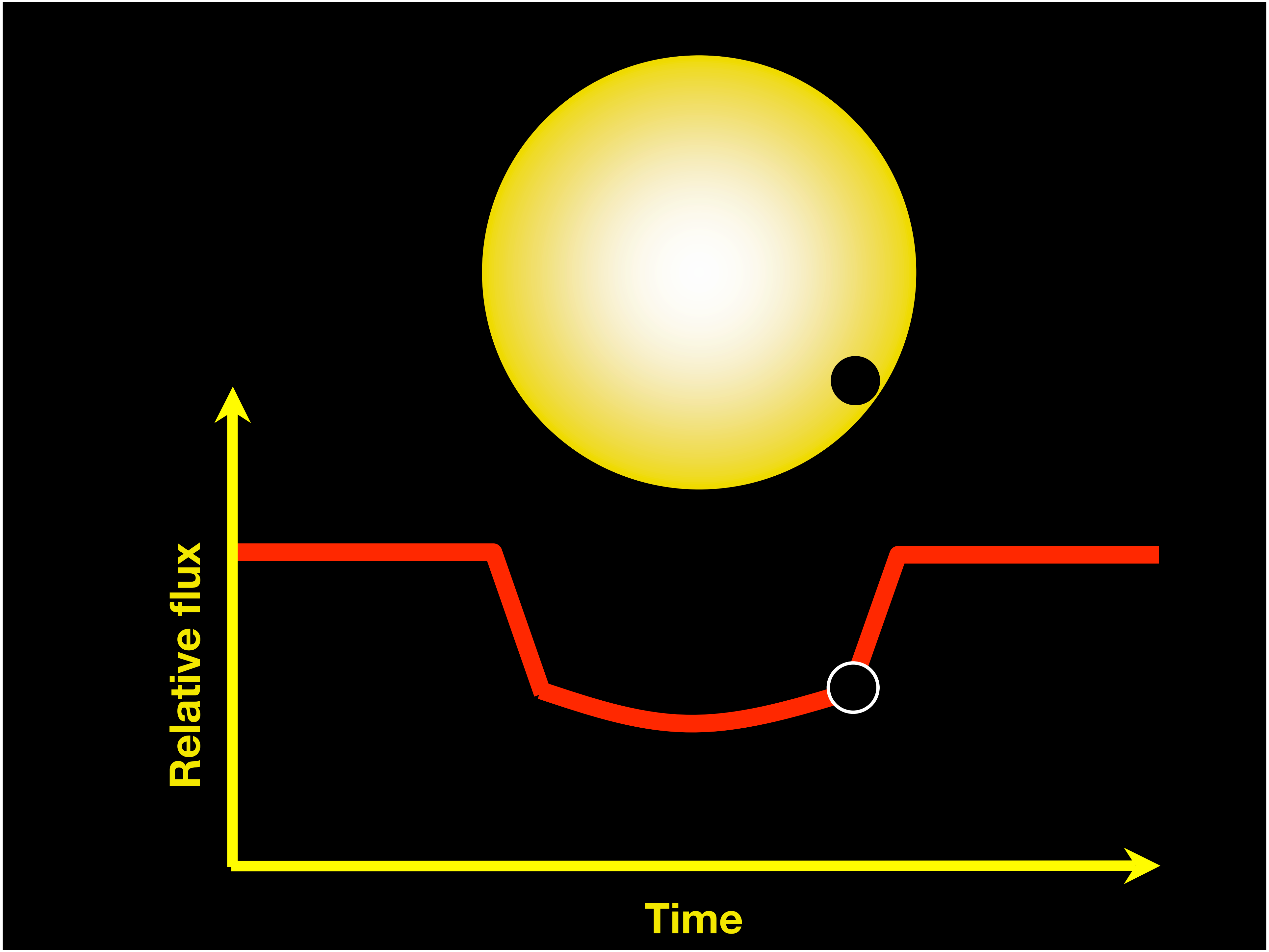
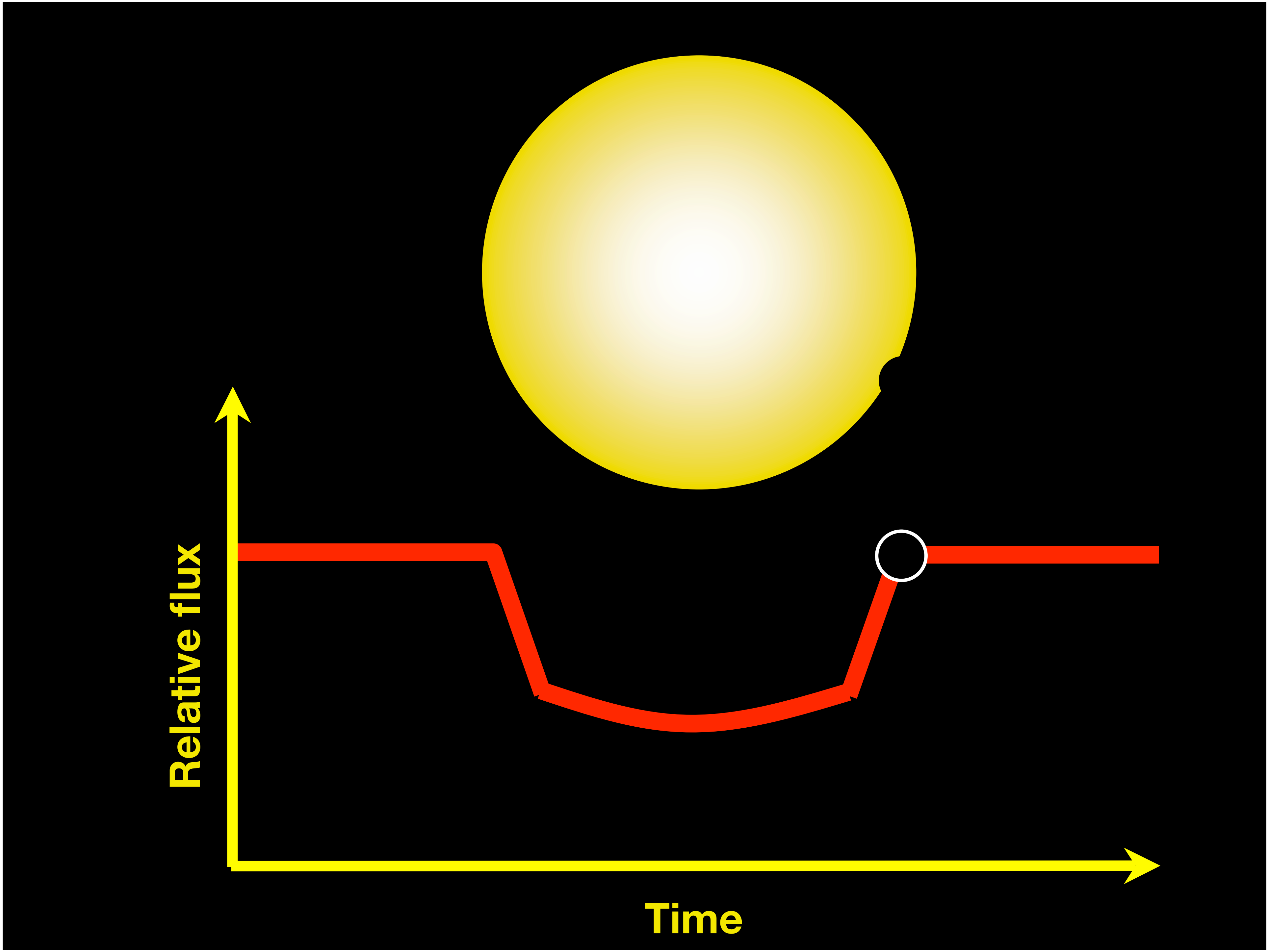
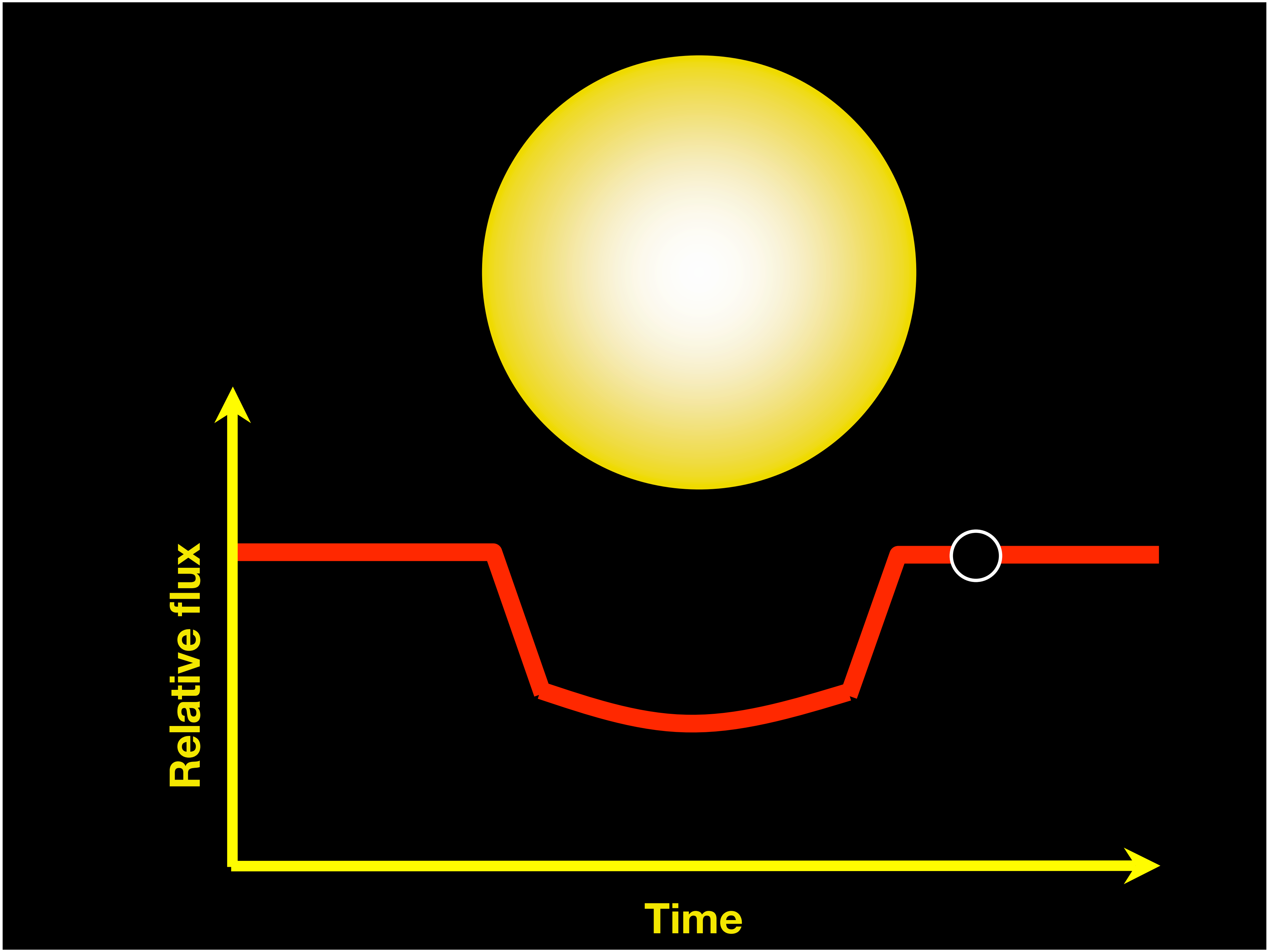


Relative flux
Time



Relative flux
Time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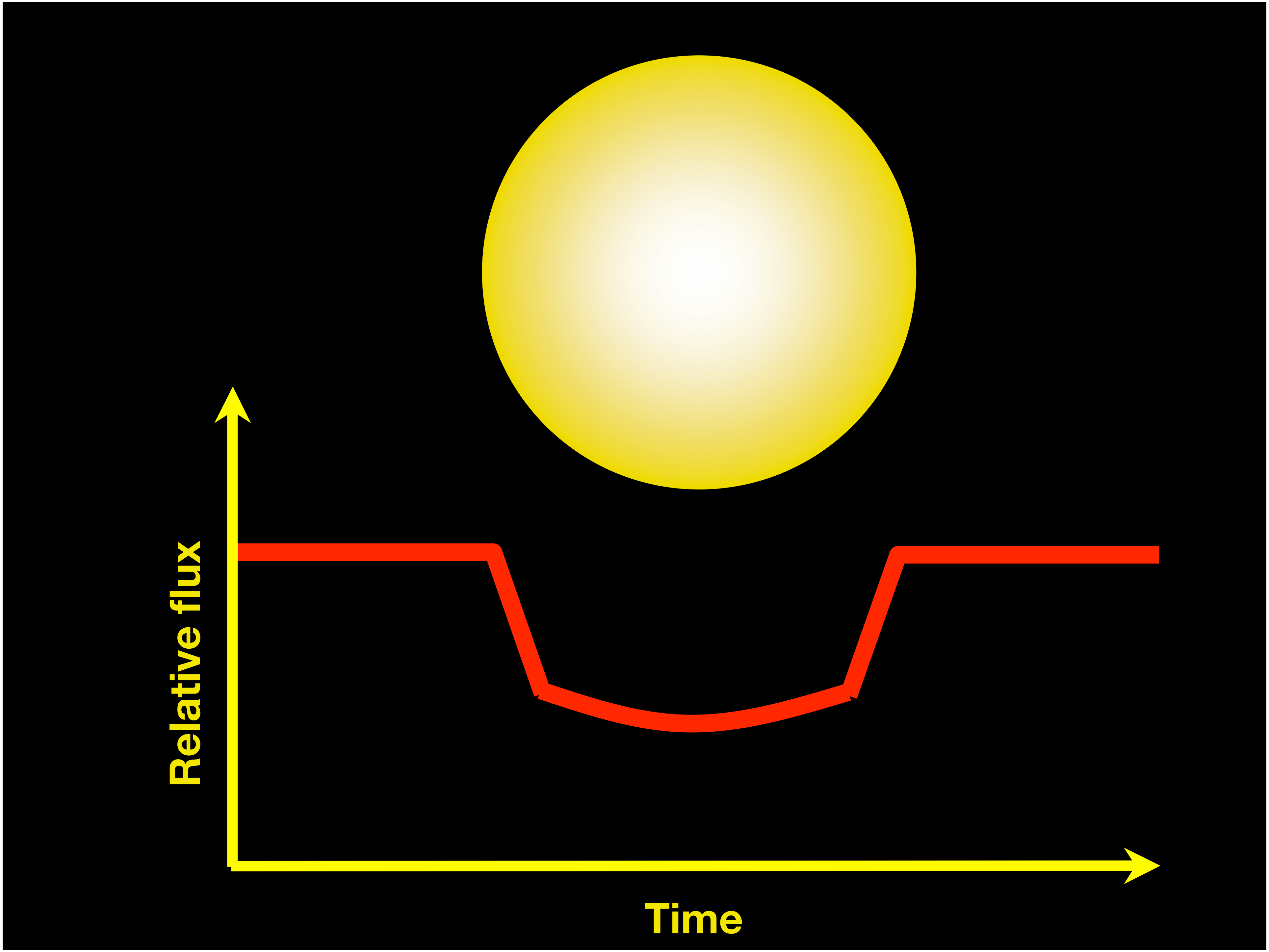
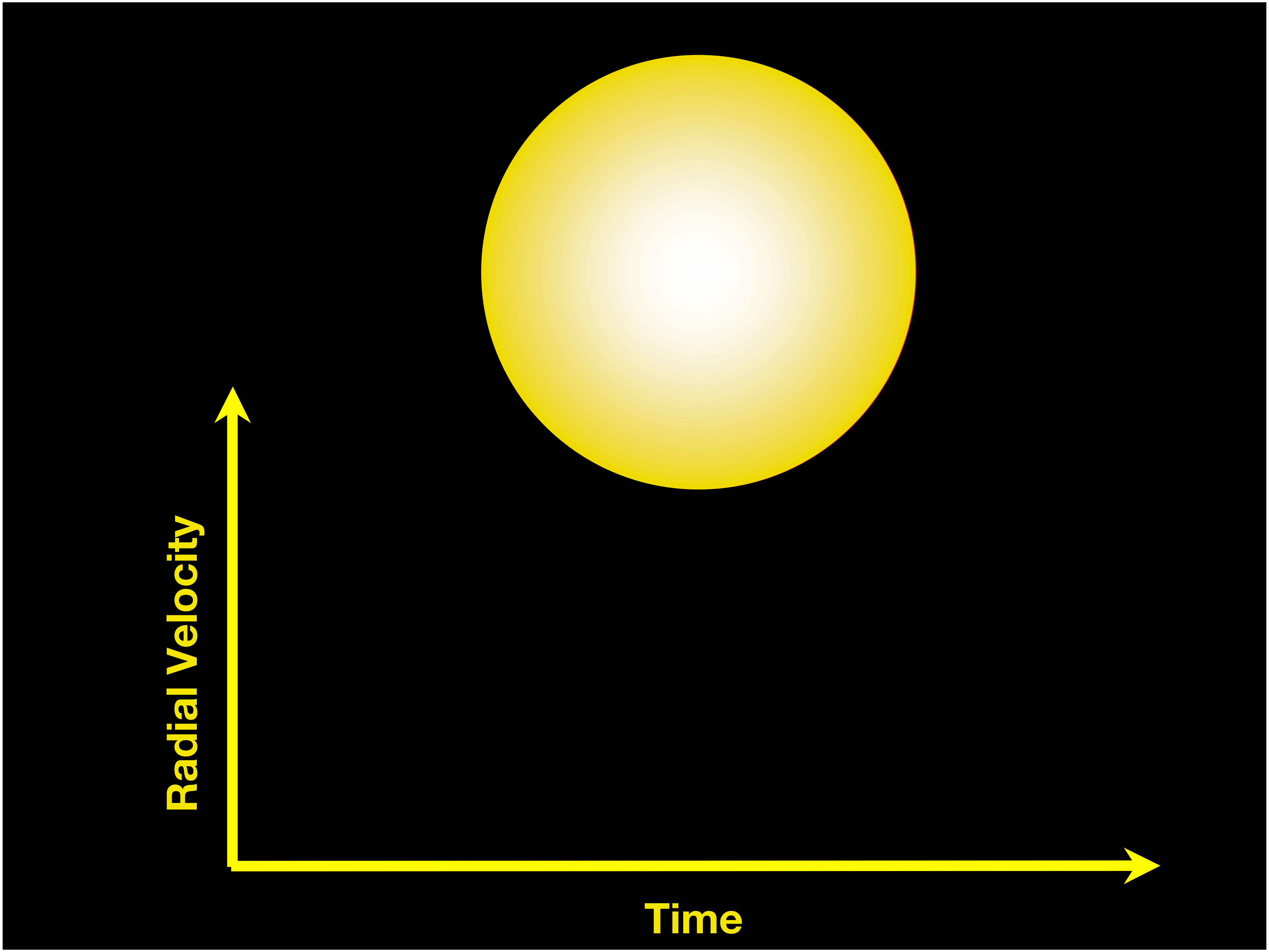
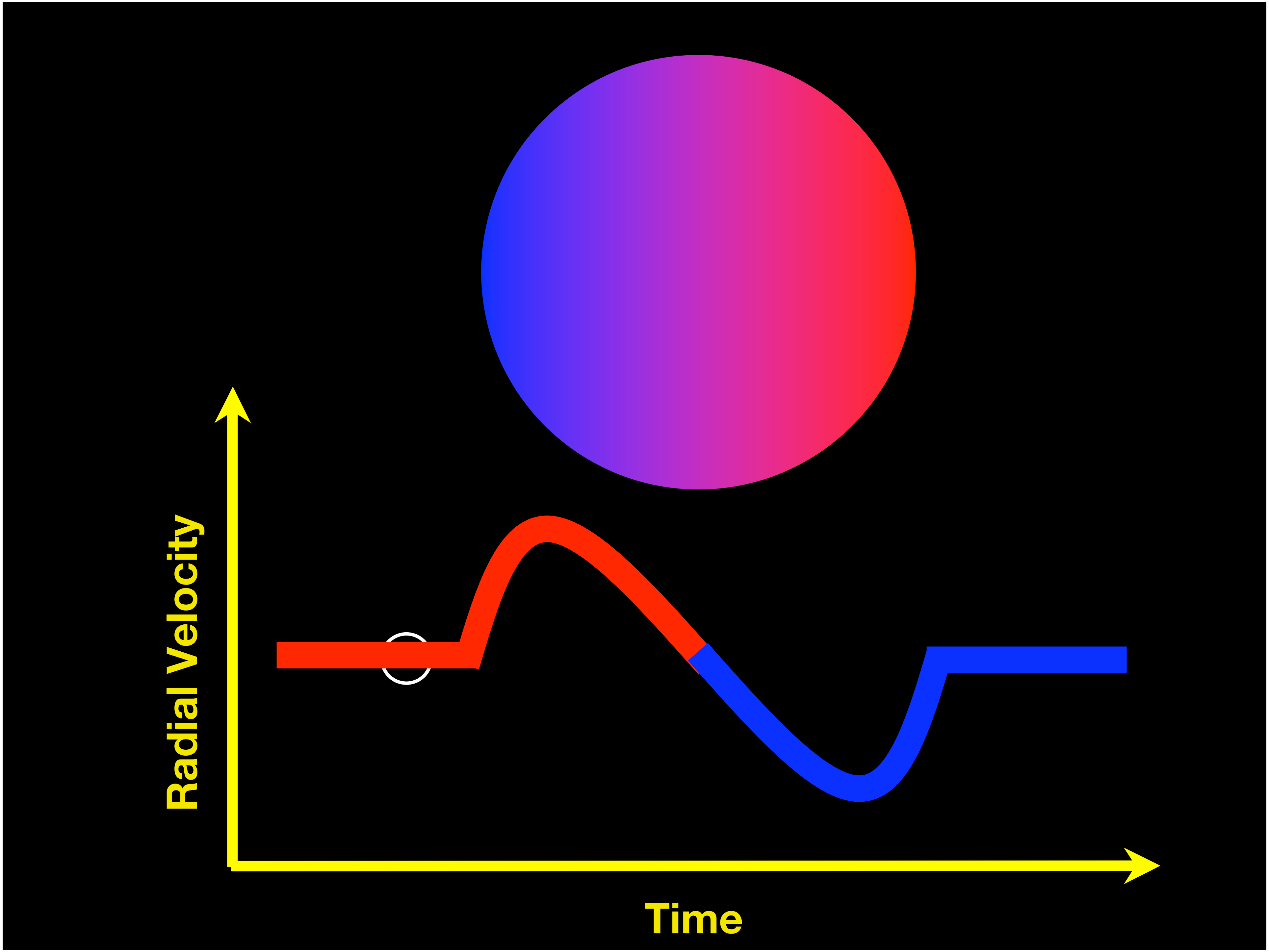
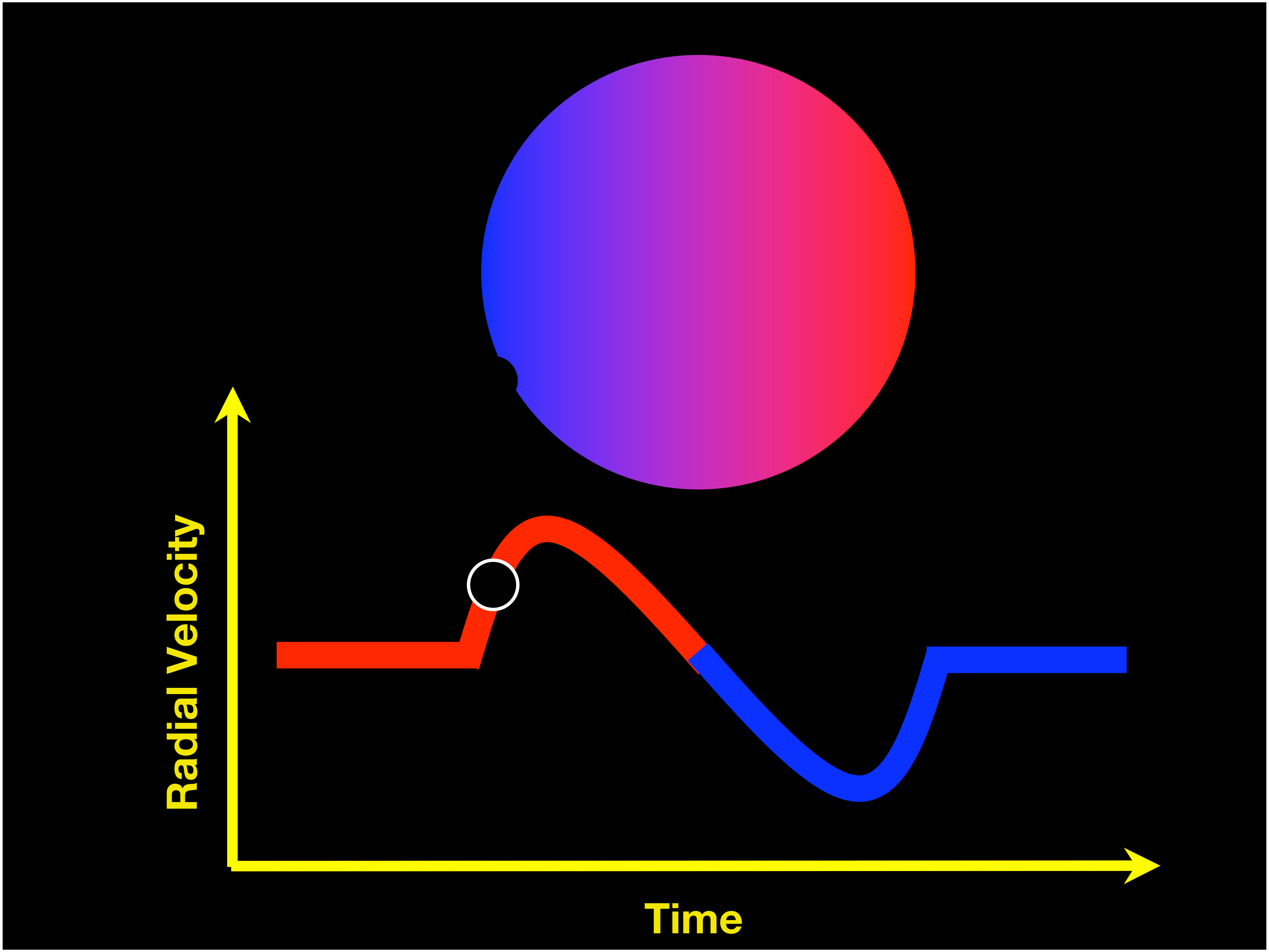
Time


Relative flux
Time


Radial Velocity
Time


Radial Velocity
Time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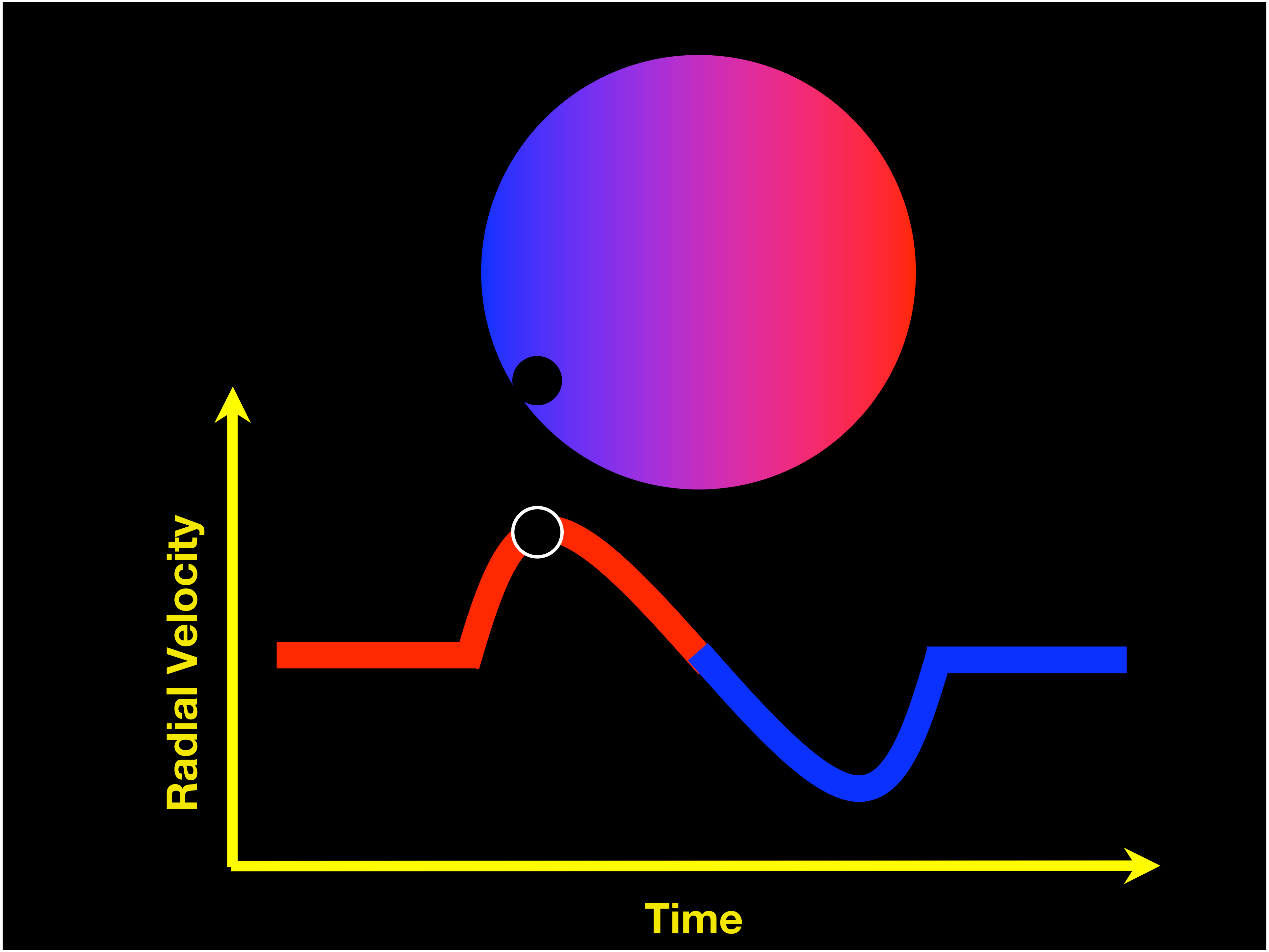
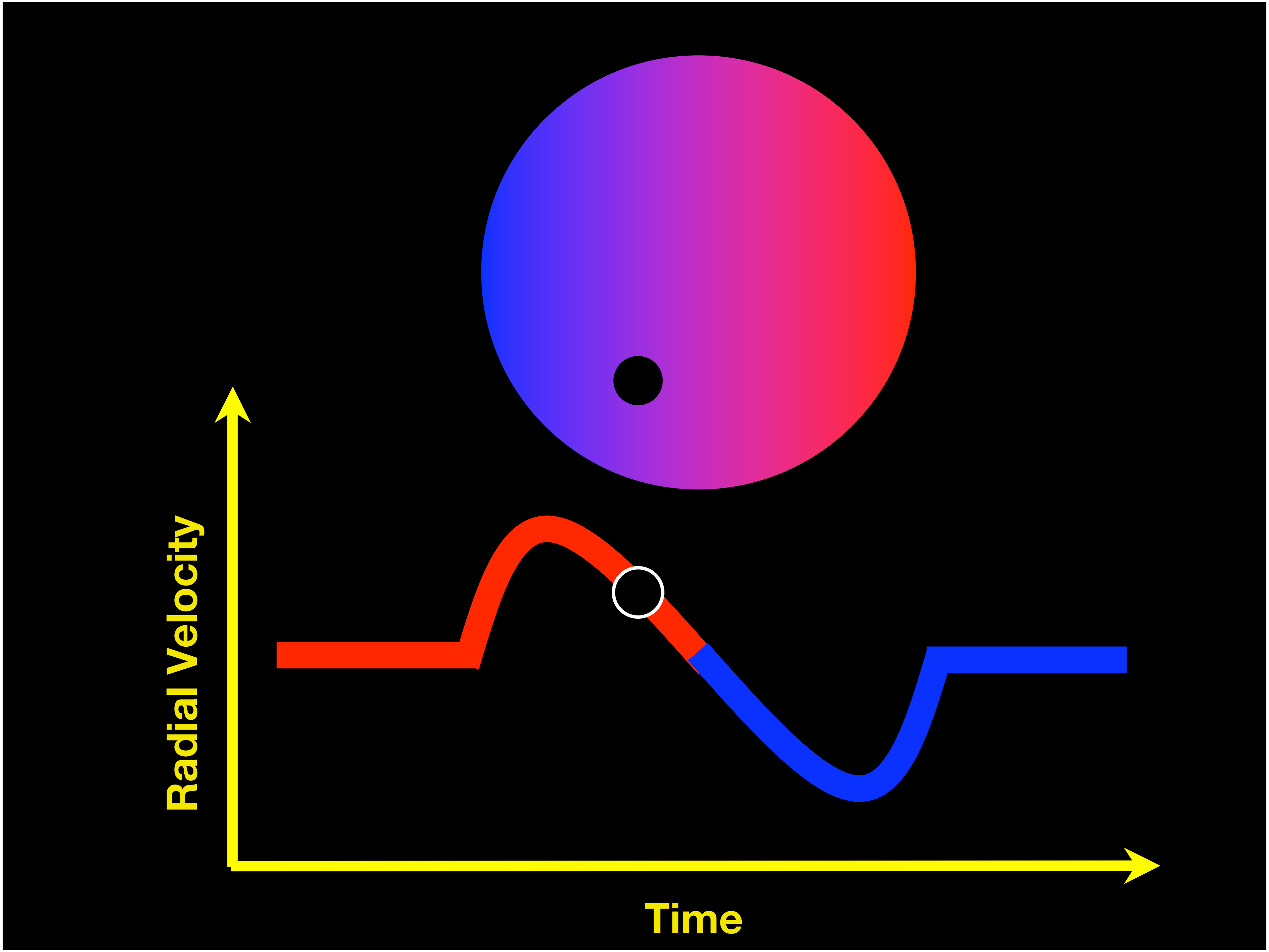
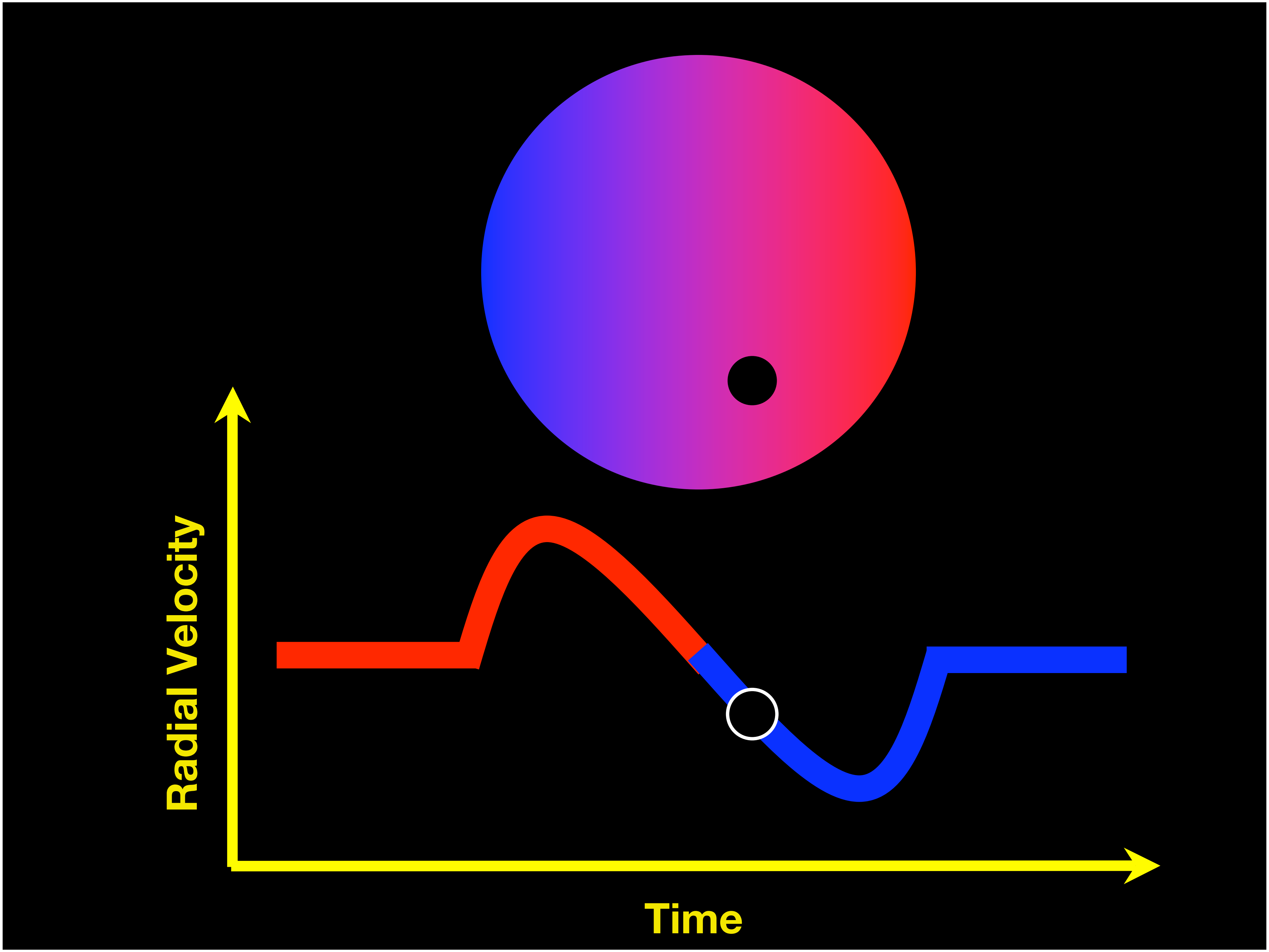
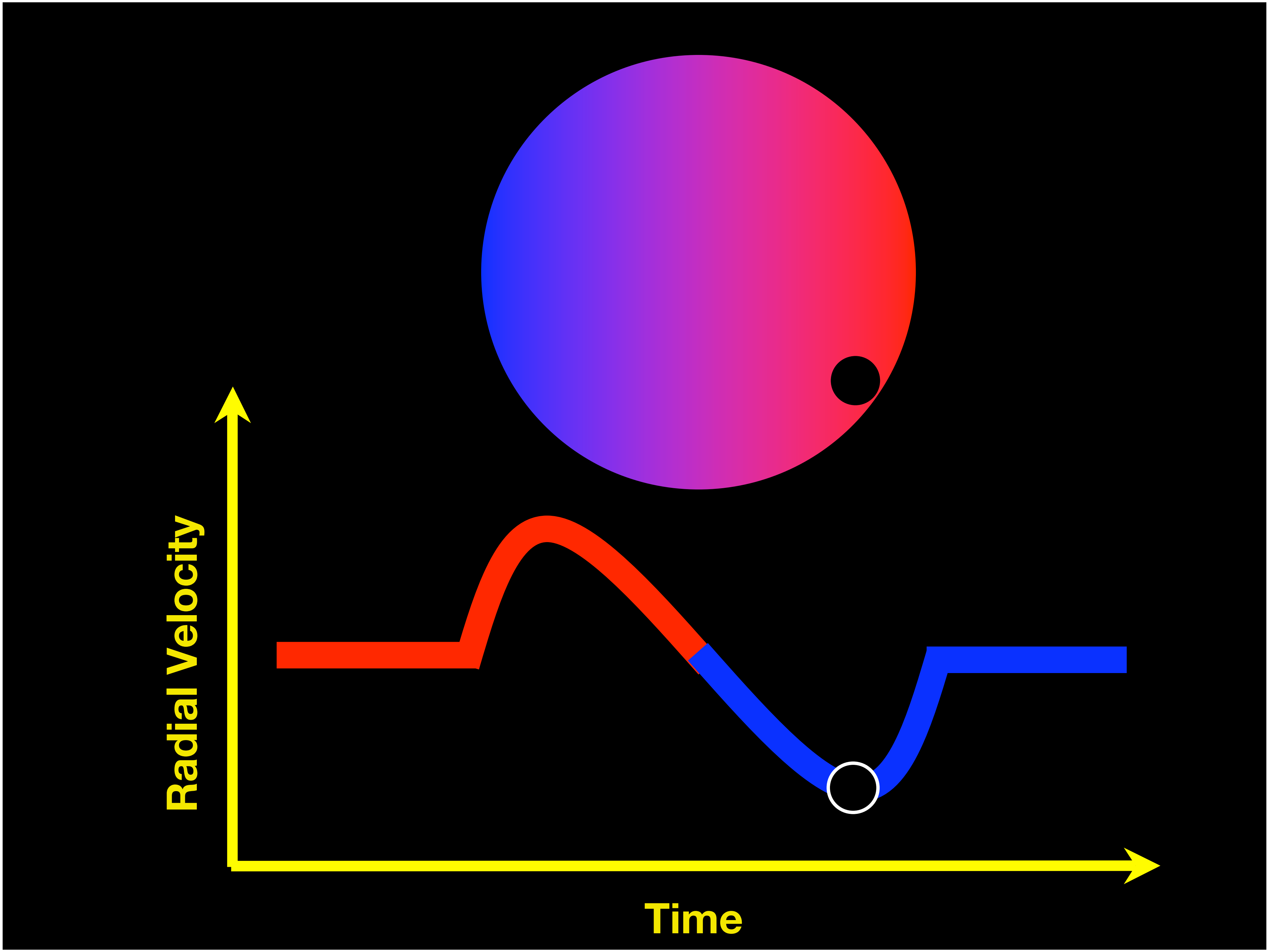
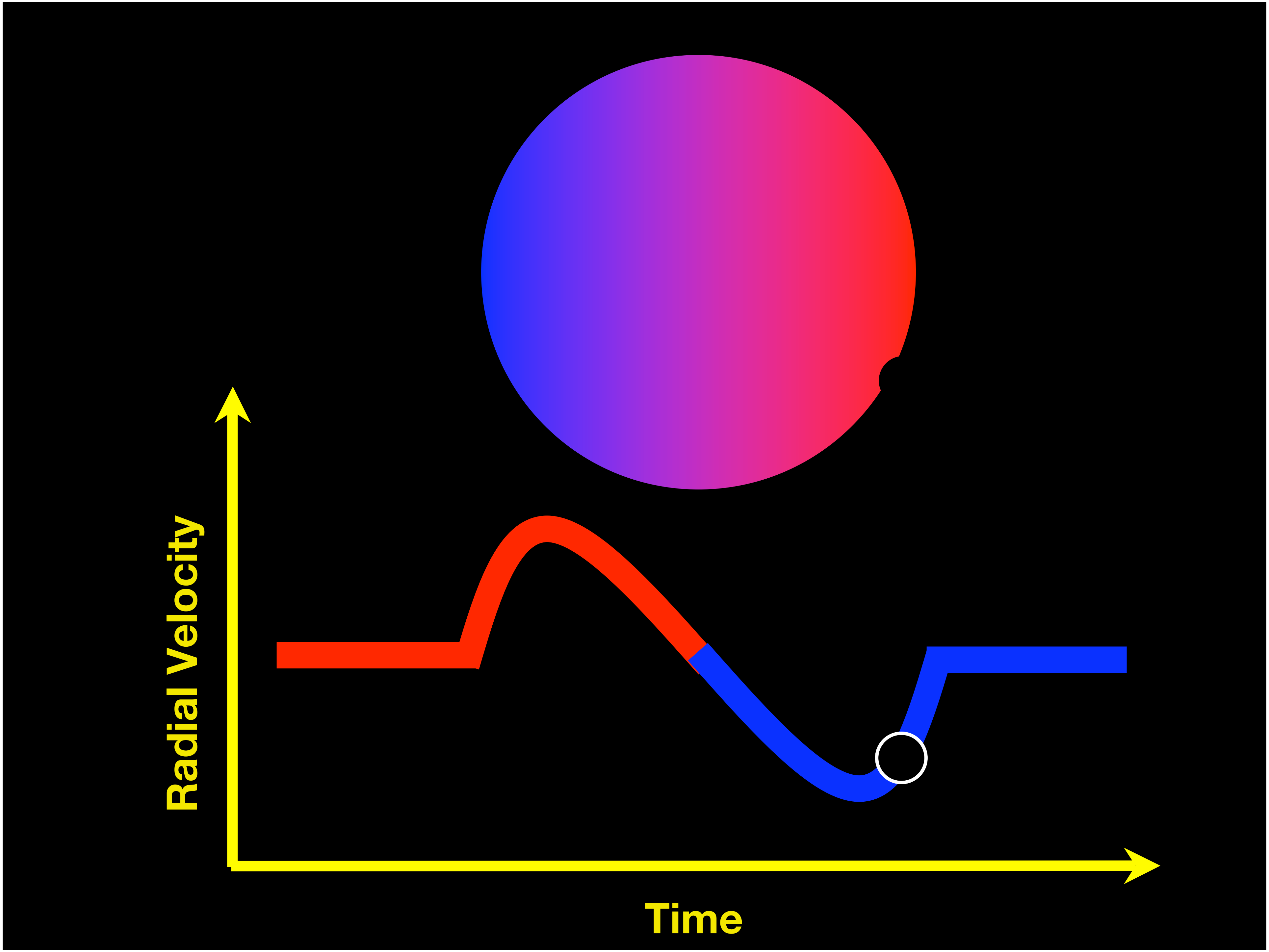
Time

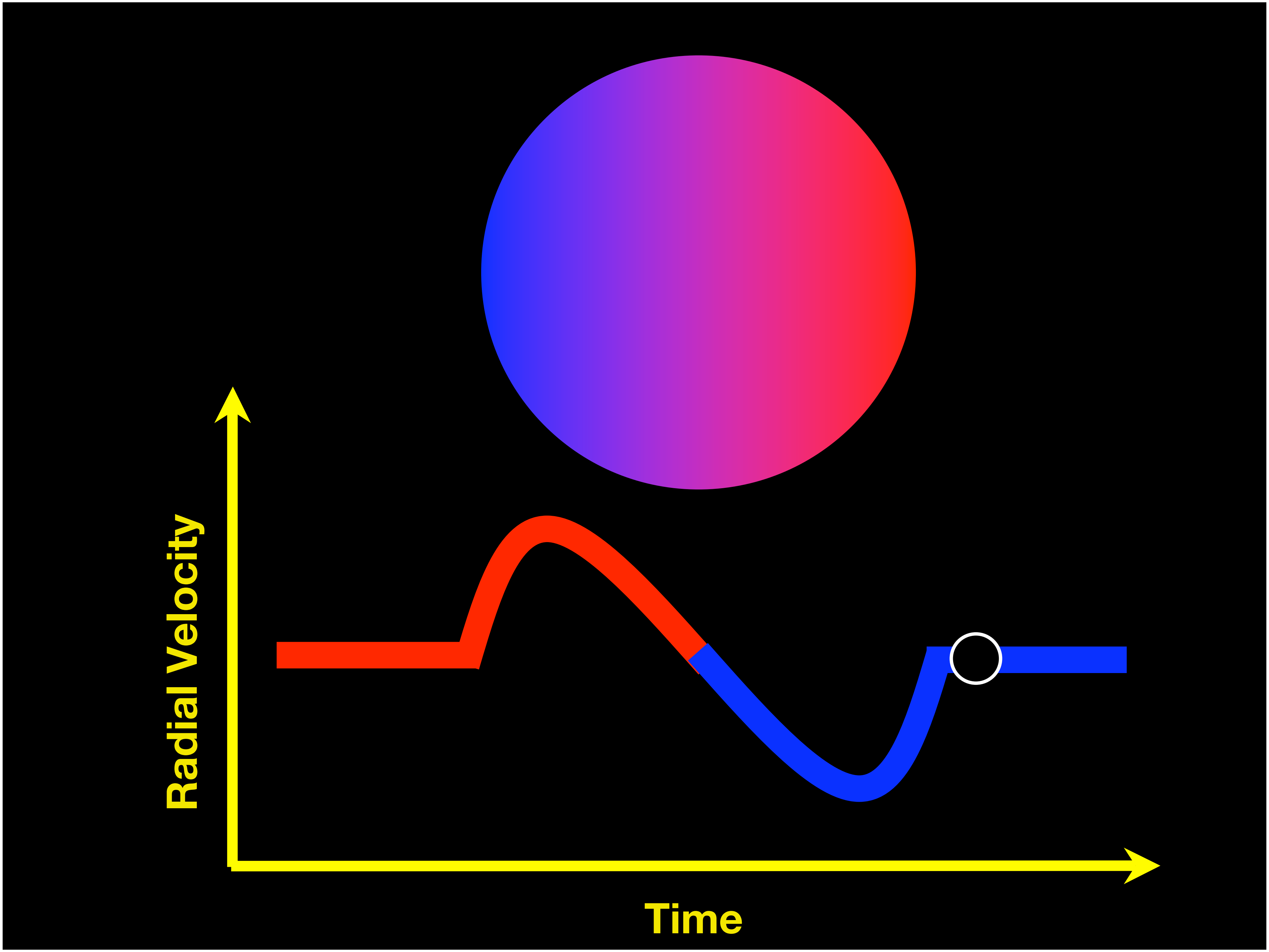

Radial Velocity
Time

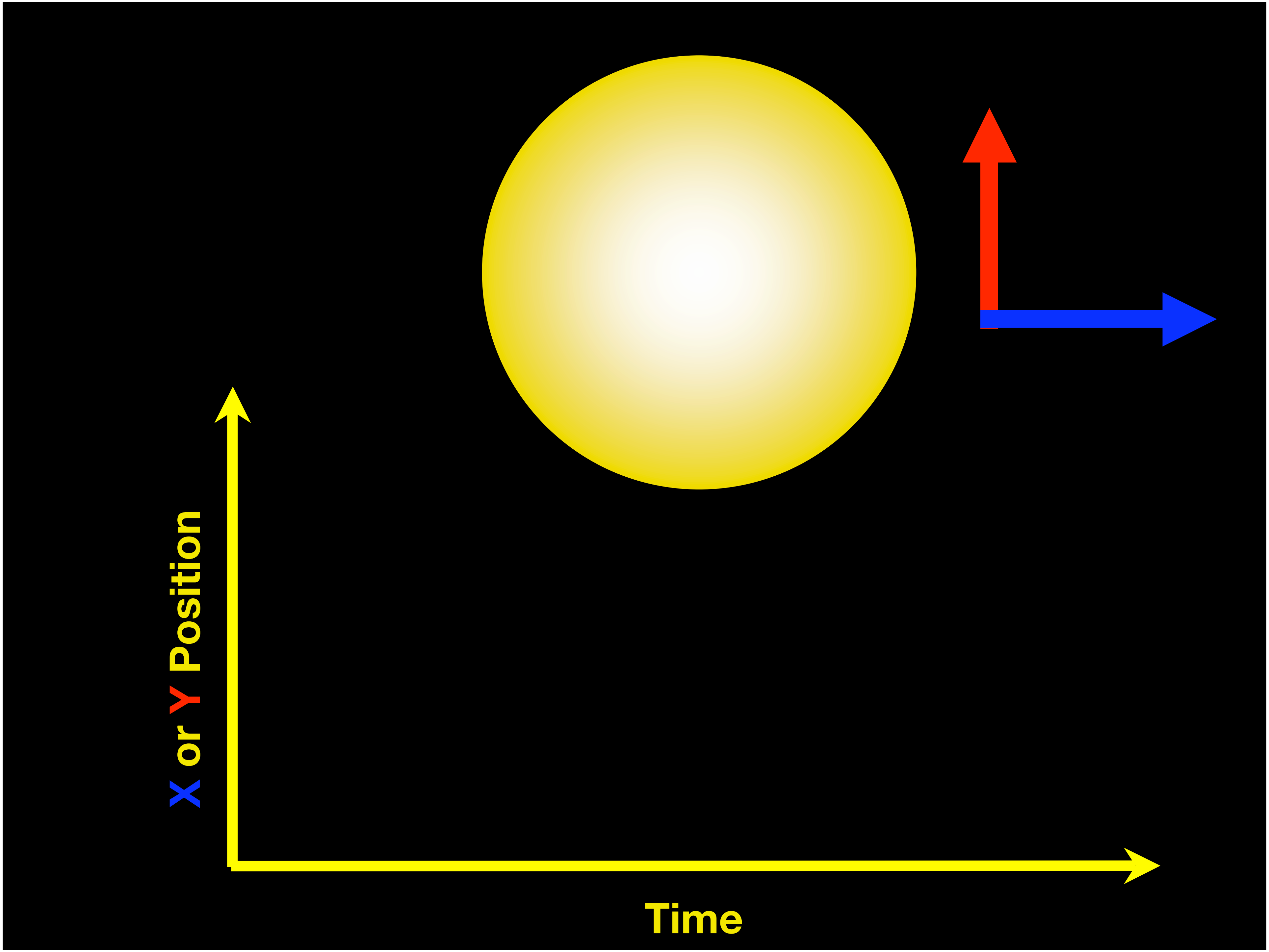

Radial Velocity
Time

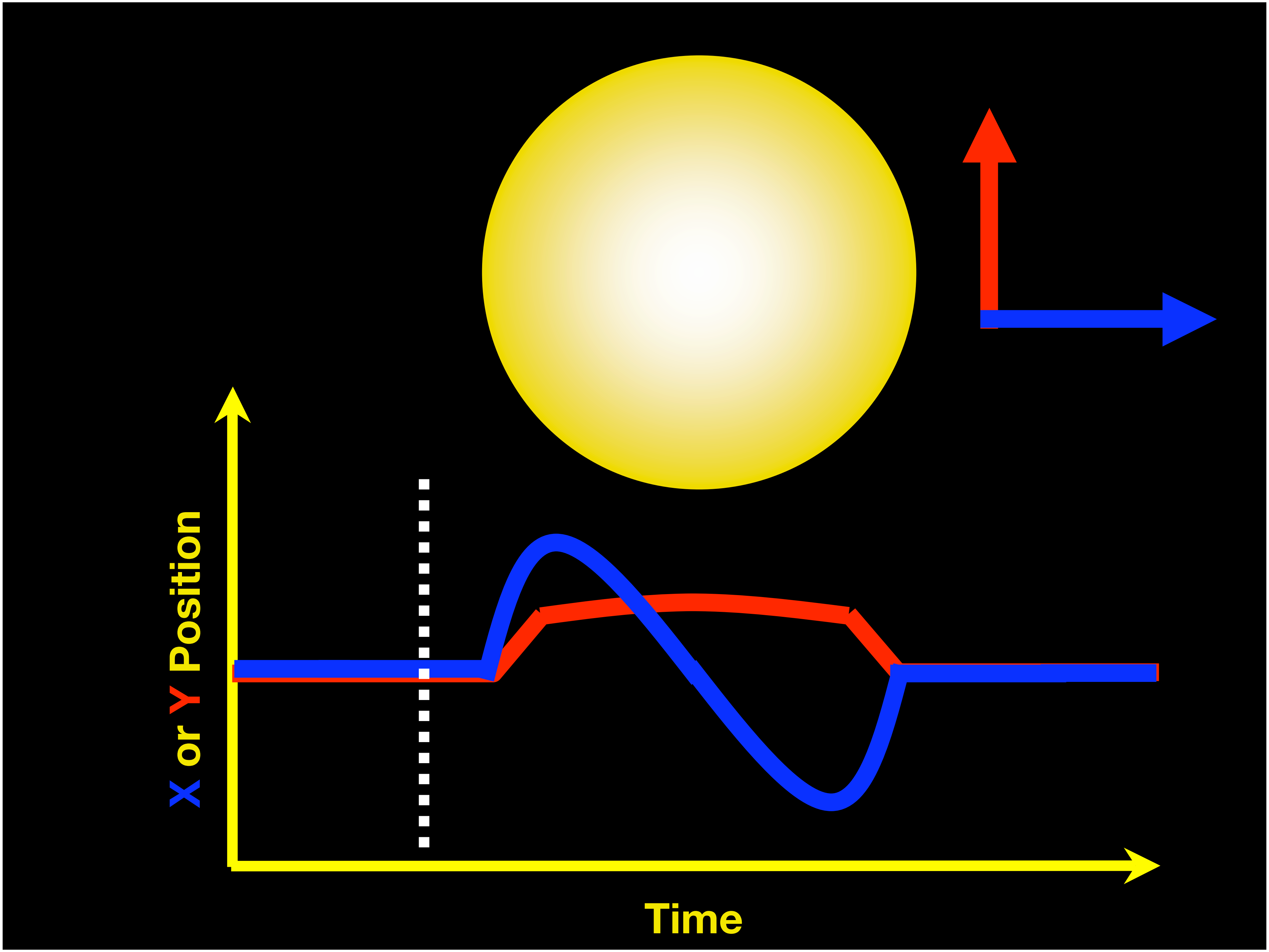

Radial Velocity
Time

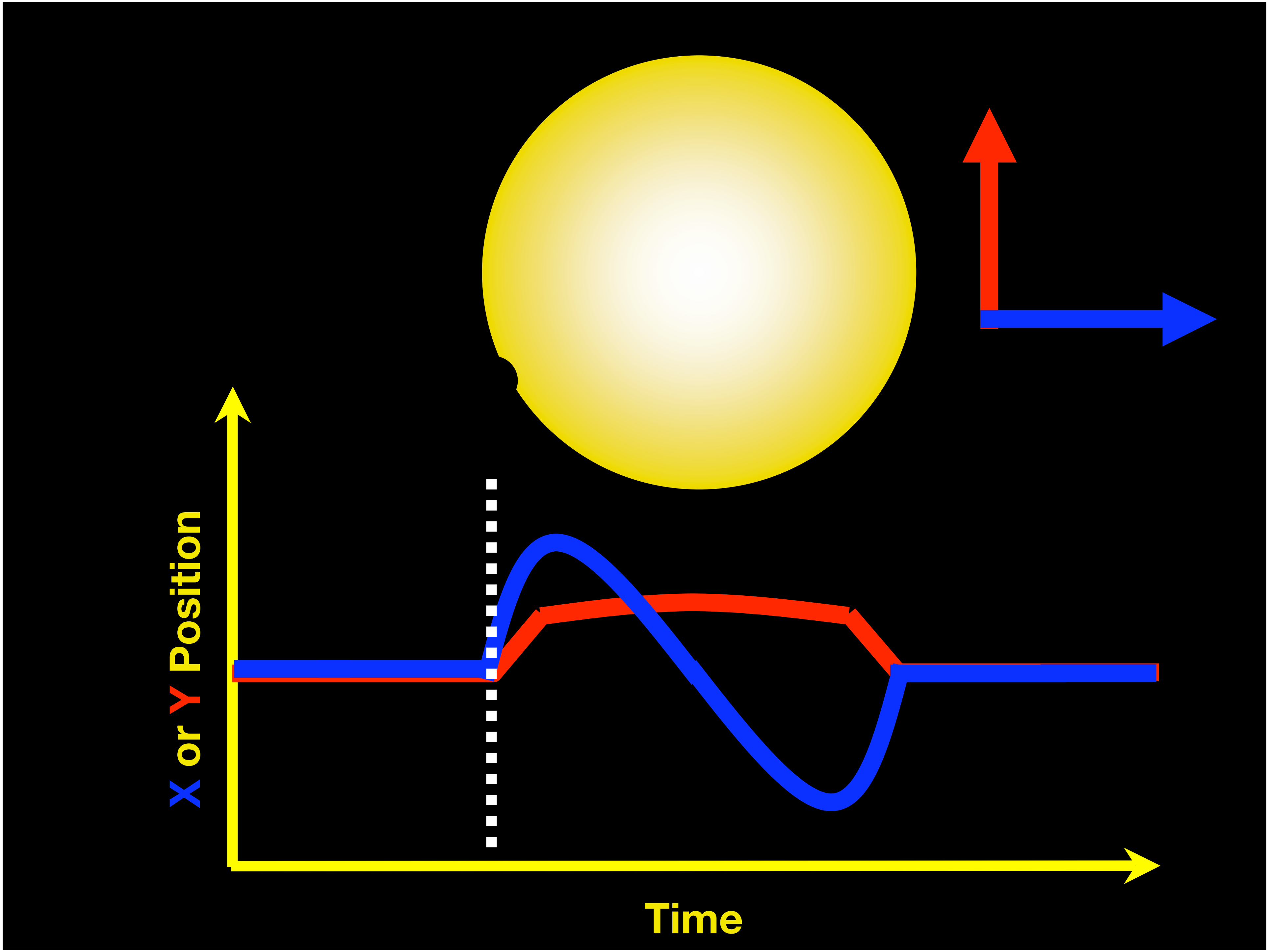

Radial Velocity
Time


Radial Velocity
Time


Radial Velocity
Time


X or Y Position
Time


X or Y Position
Time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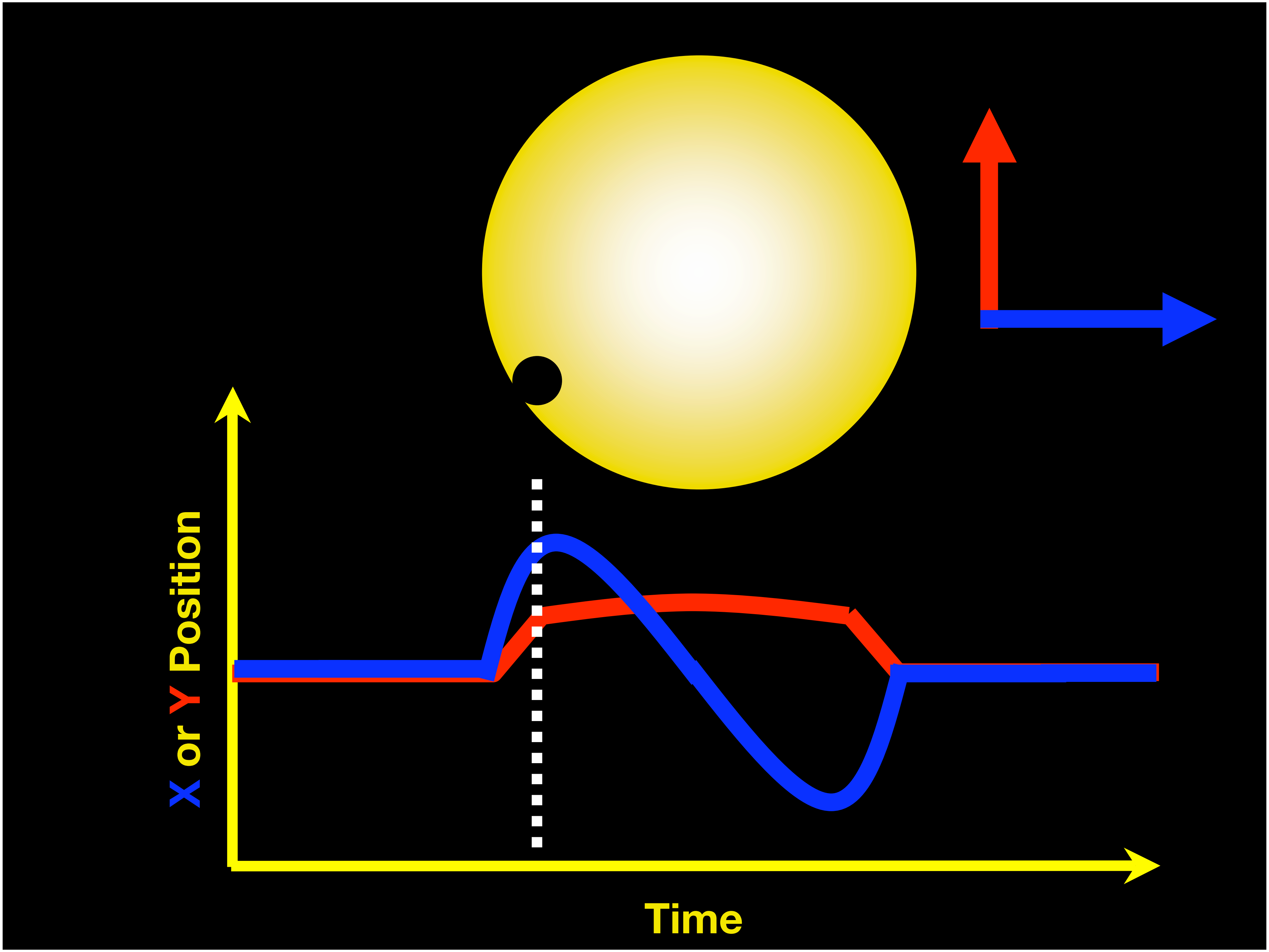
X or Y Position
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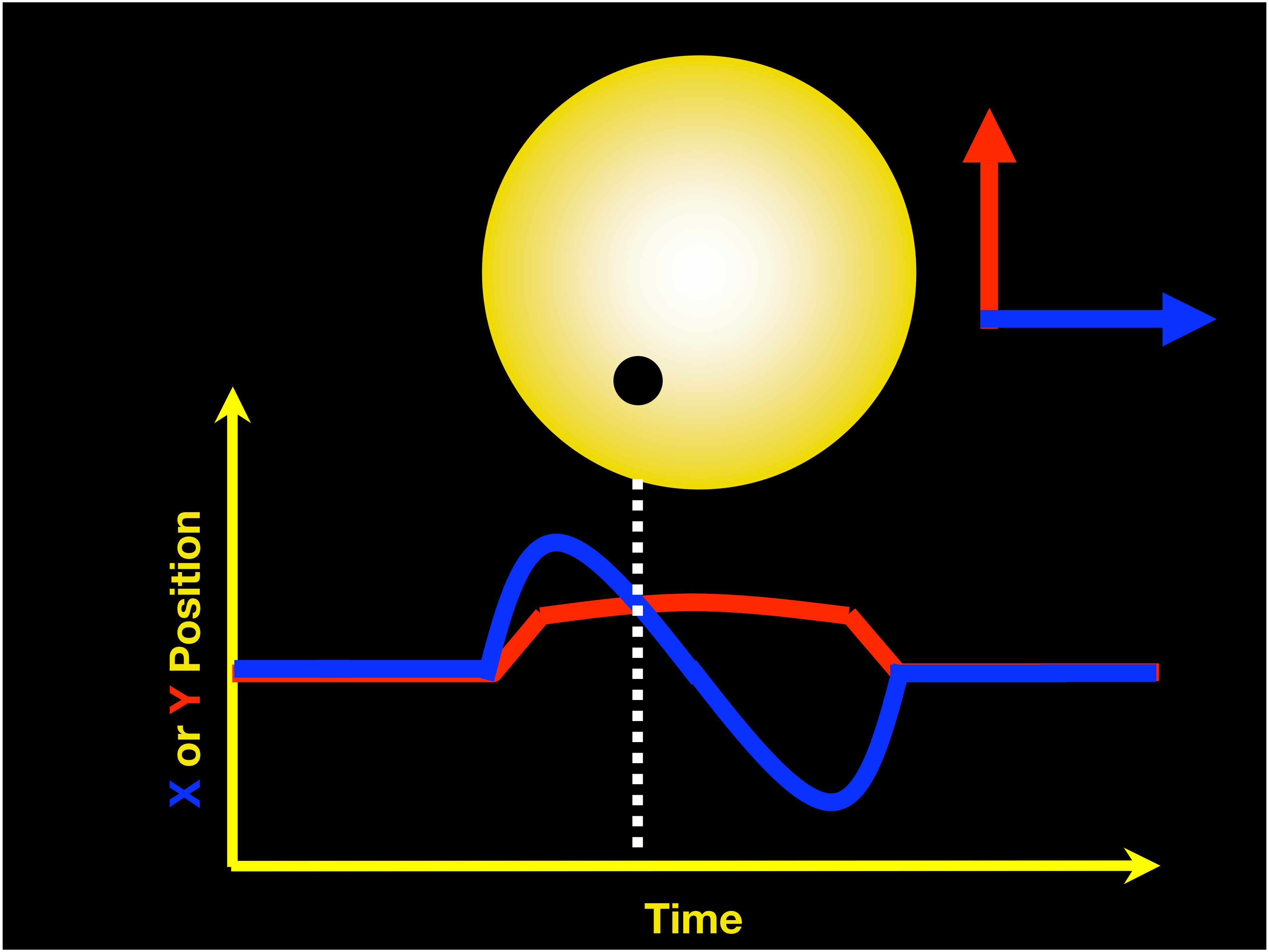
X or Y Position
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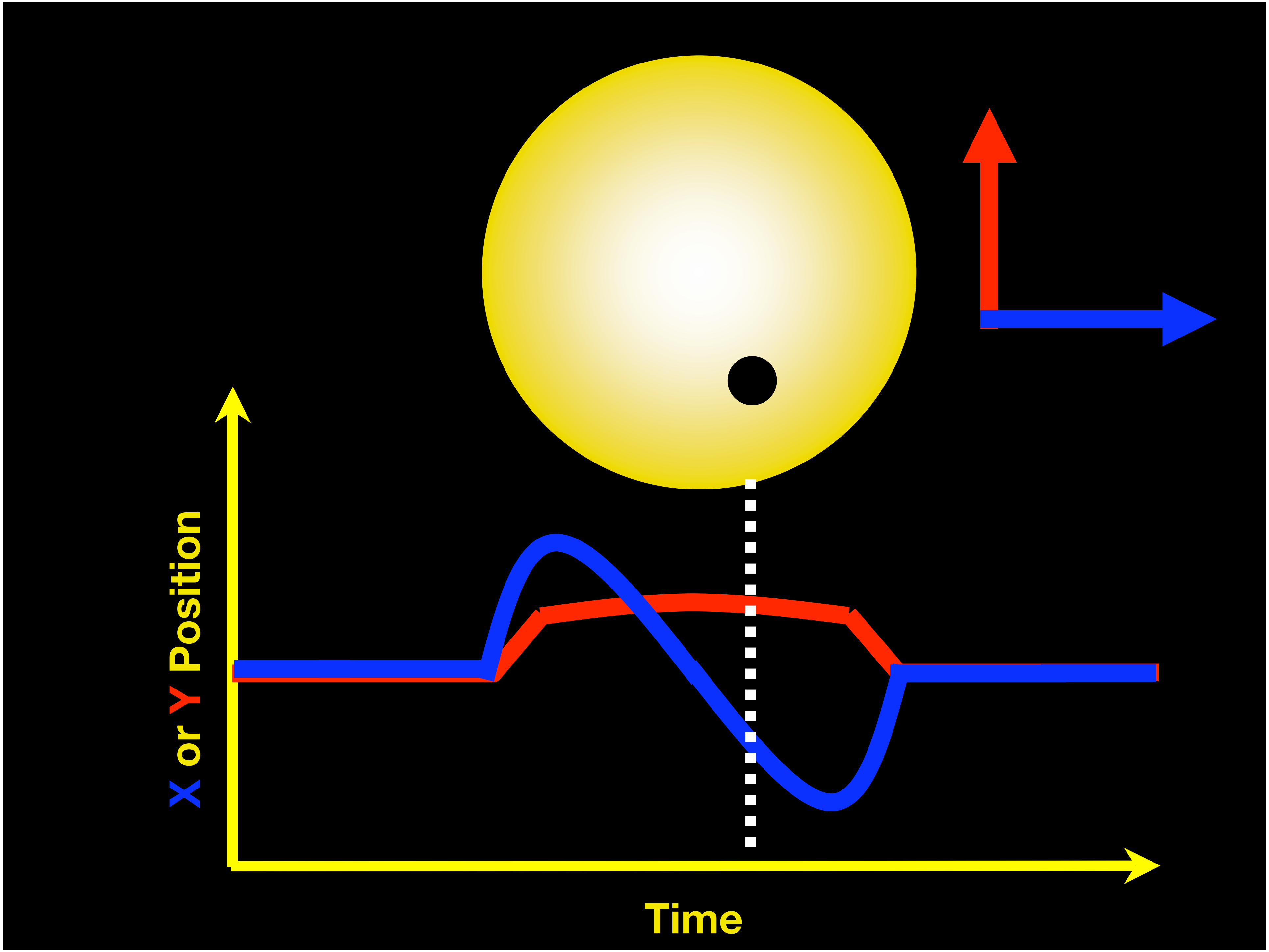
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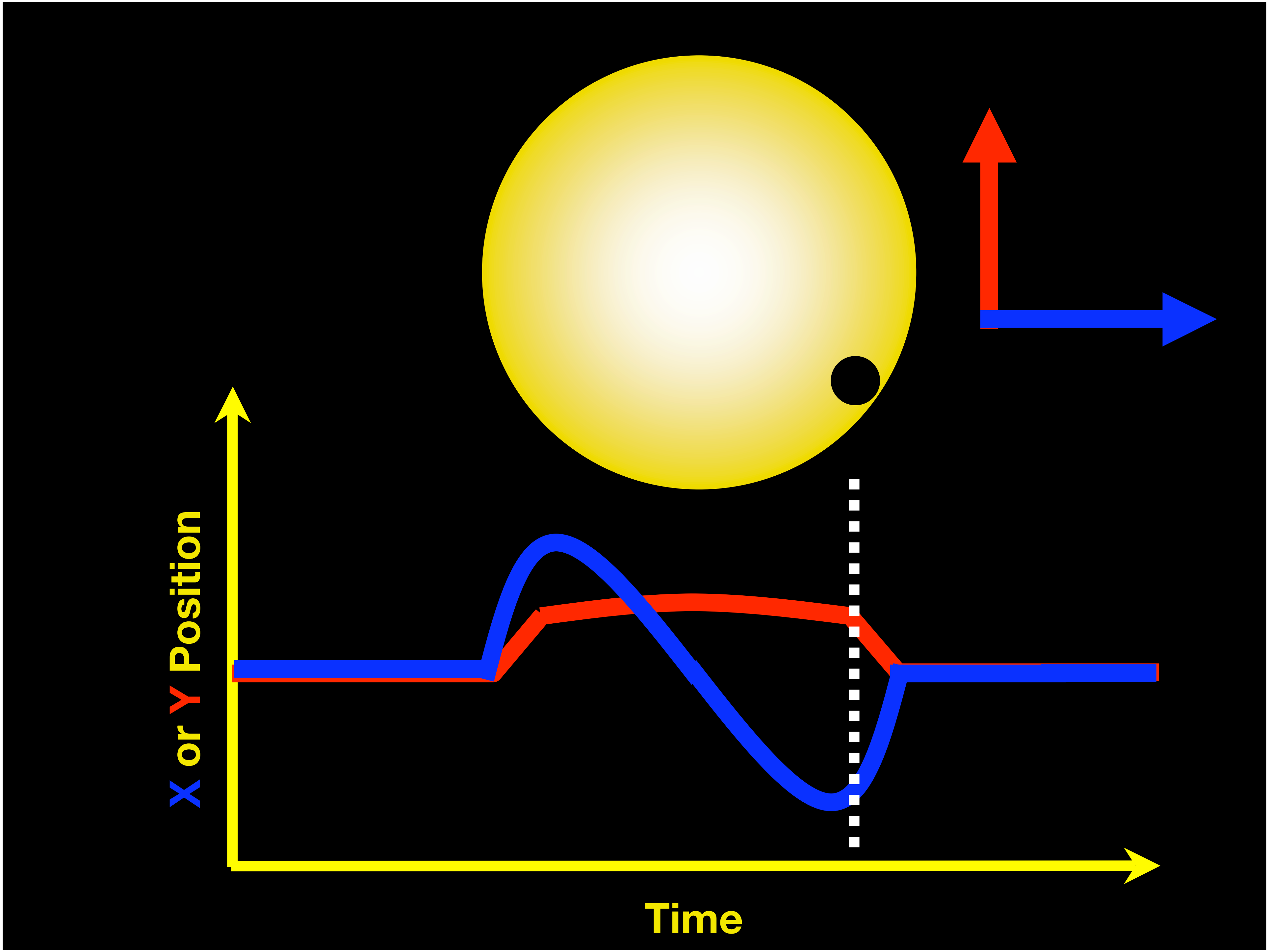
X or Y Position
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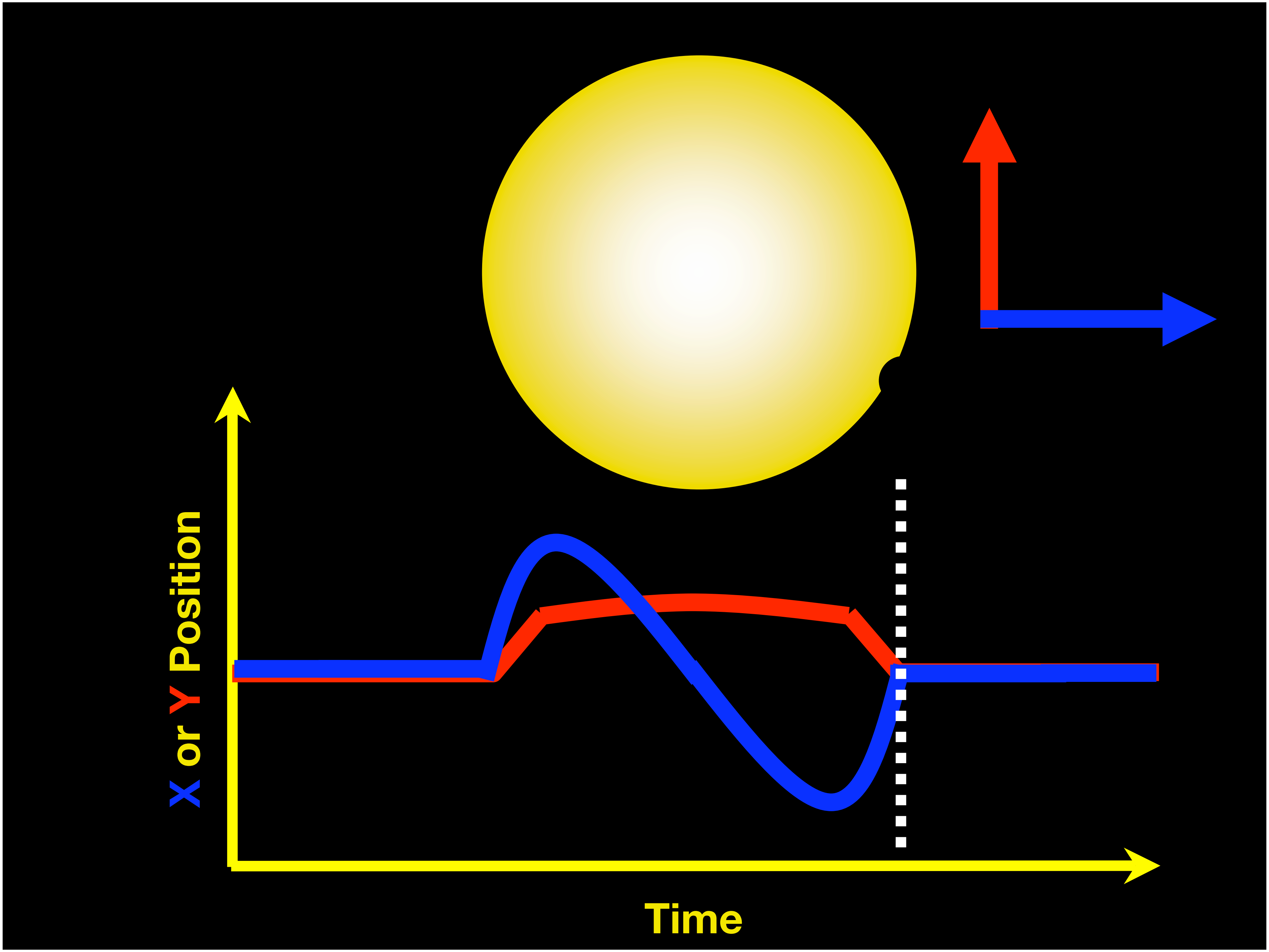
X or Y Position
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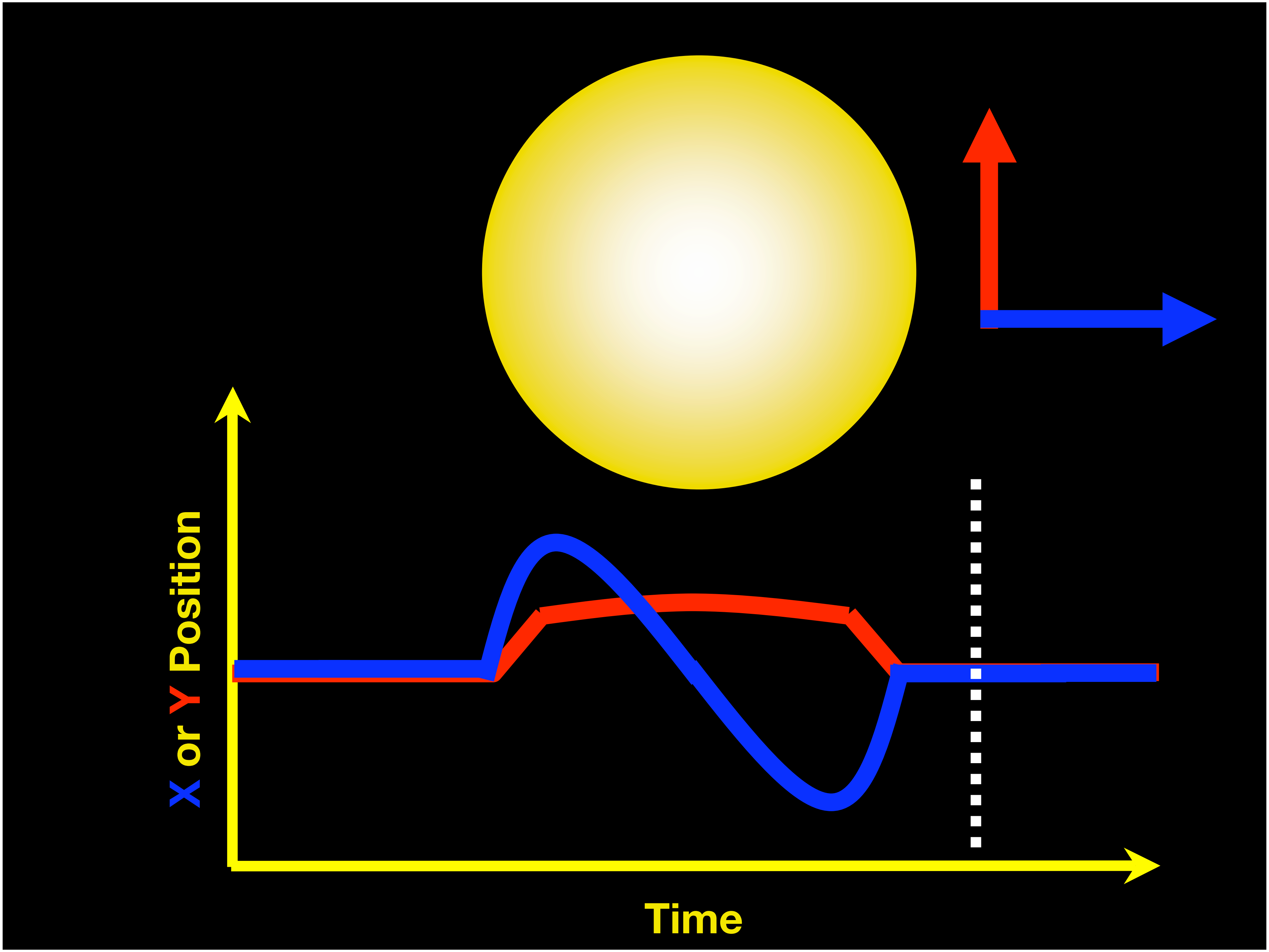
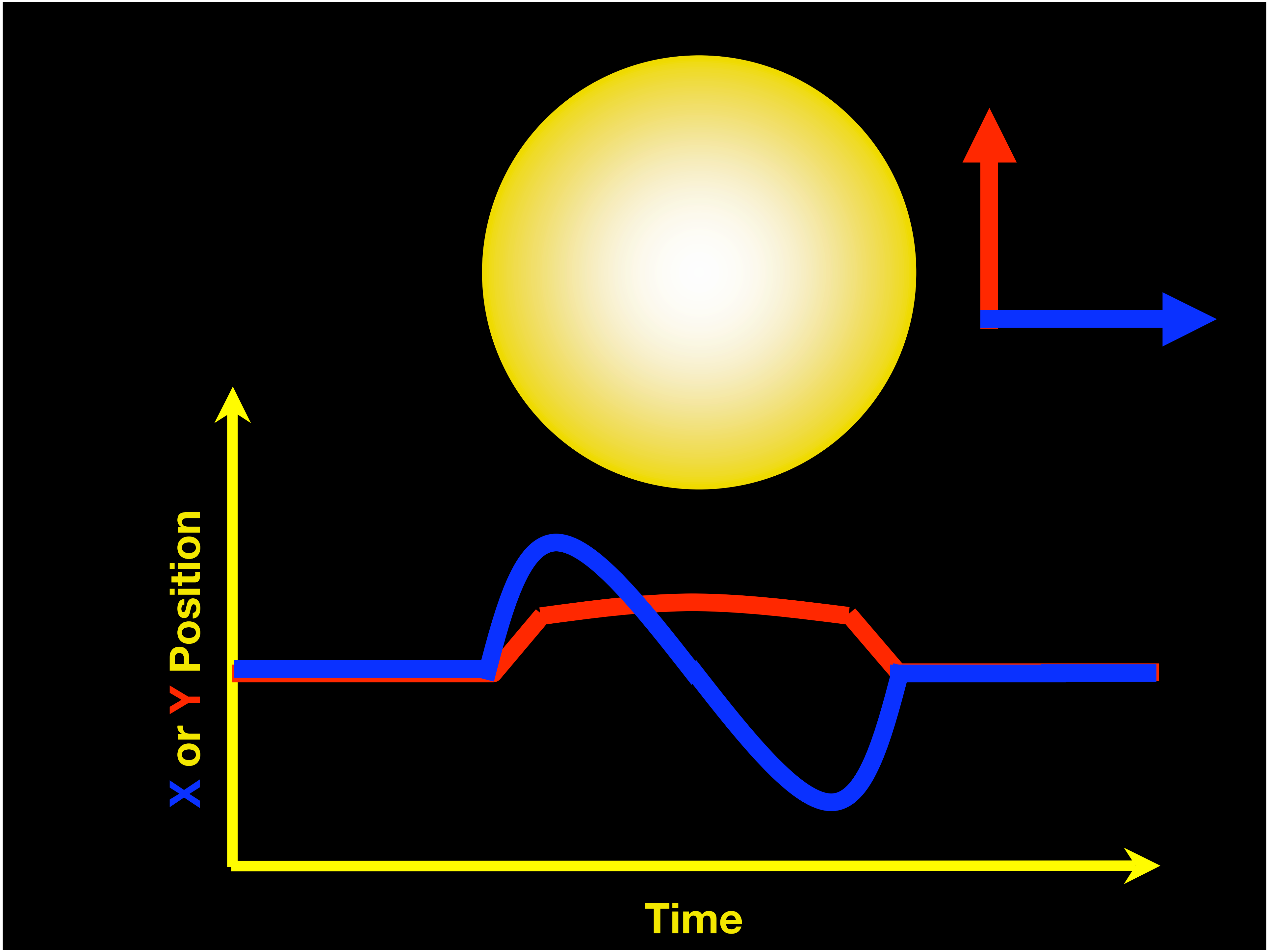

X or Y Position
Time



X or Y Position
Time

Order of Magnitude

- Centroid:

$$\Delta\alpha_x = \theta_* \frac{\iint x' I(x',y')dx'dy'}{\iint I(x',y')dx'dy'}, \quad \Delta\alpha_y = \theta_* \frac{\iint y' I(x',y')dx'dy'}{\iint I(x',y')dx'dy'}$$

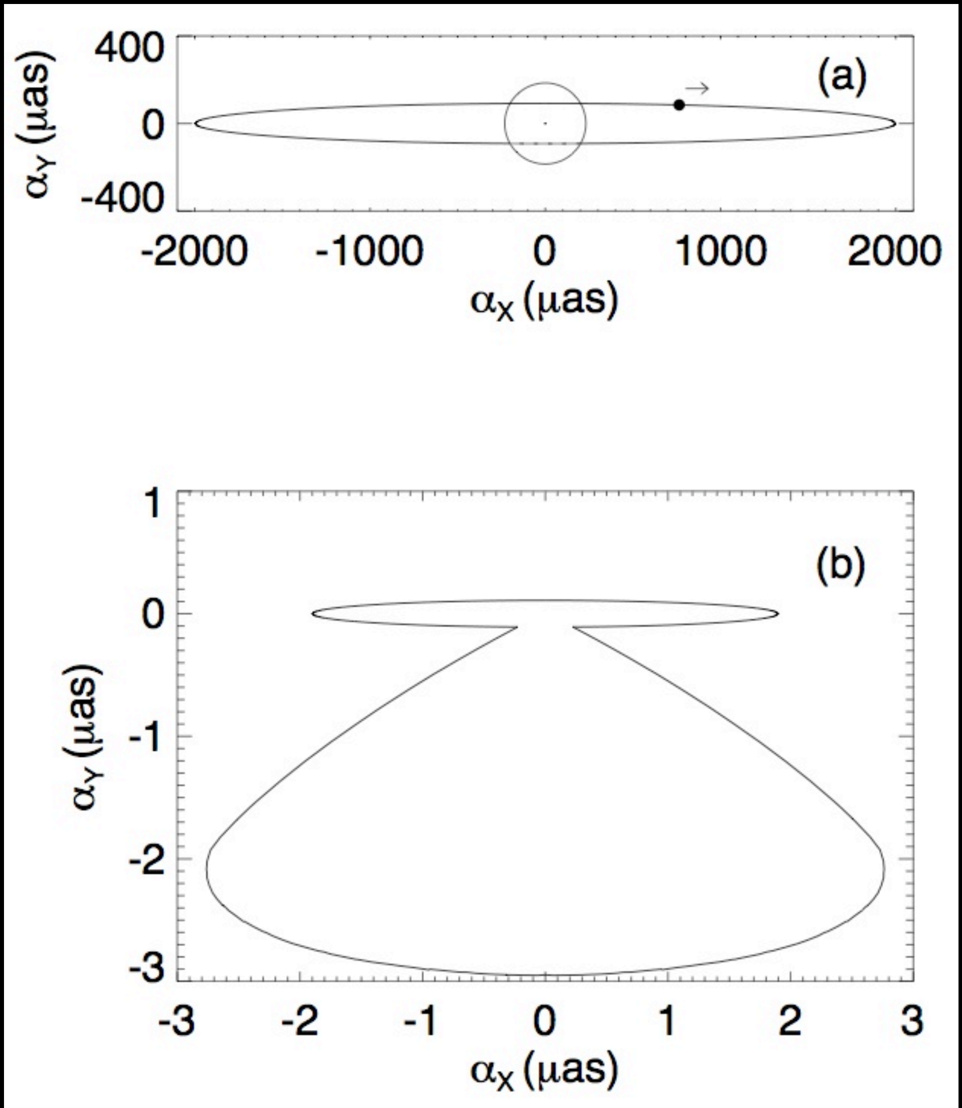
- Uniform surface brightness, full transit:

$$\Delta\alpha_x = \theta_* \frac{\delta}{1-\delta} x, \quad \Delta\alpha_y = \theta_* \frac{\delta}{1-\delta} y, \text{ where } \delta = \left(\frac{R_p}{R_*}\right)^2 \text{ is the transit depth}$$

- Amplitude of the centroid shift:

$$\alpha_T \equiv \theta_* \frac{\delta}{1-\delta} \approx 2.5\mu\text{as} \left(\frac{R_p}{R_J}\right)^2 \left(\frac{R_*}{R_\odot}\right)^{-1} \left(\frac{d}{20\text{pc}}\right)^{-1}$$

Astrometric Wobble and Transit Signal



- HD 189733
- M_*=0.81$M_\odot$, R_*=0.79$R_\odot$, d=19.3pc
- M_p=1.15M_J, R_p=1.16R_J, a=0.031AU
- Astrometric Wobble Amplitude

$$\alpha_W \equiv \frac{a}{d}\frac{M_p}{M_* + M_p} \approx 1.9\mu\text{as}\left(\frac{a}{0.04\,\text{AU}}\right)\left(\frac{M_p}{M_J}\right)\left(\frac{M_*}{M_\odot}\right)^{-1}\left(\frac{d}{20\text{pc}}\right)^{-1}$$

- Astrometric Transit Amplitude

$$\alpha_T \equiv \theta_* \frac{\delta}{1-\delta} \approx 2.5\mu\text{as}\left(\frac{R_p}{R_J}\right)^2\left(\frac{R_*}{R_\odot}\right)^{-1}\left(\frac{d}{20\text{pc}}\right)^{-1}$$

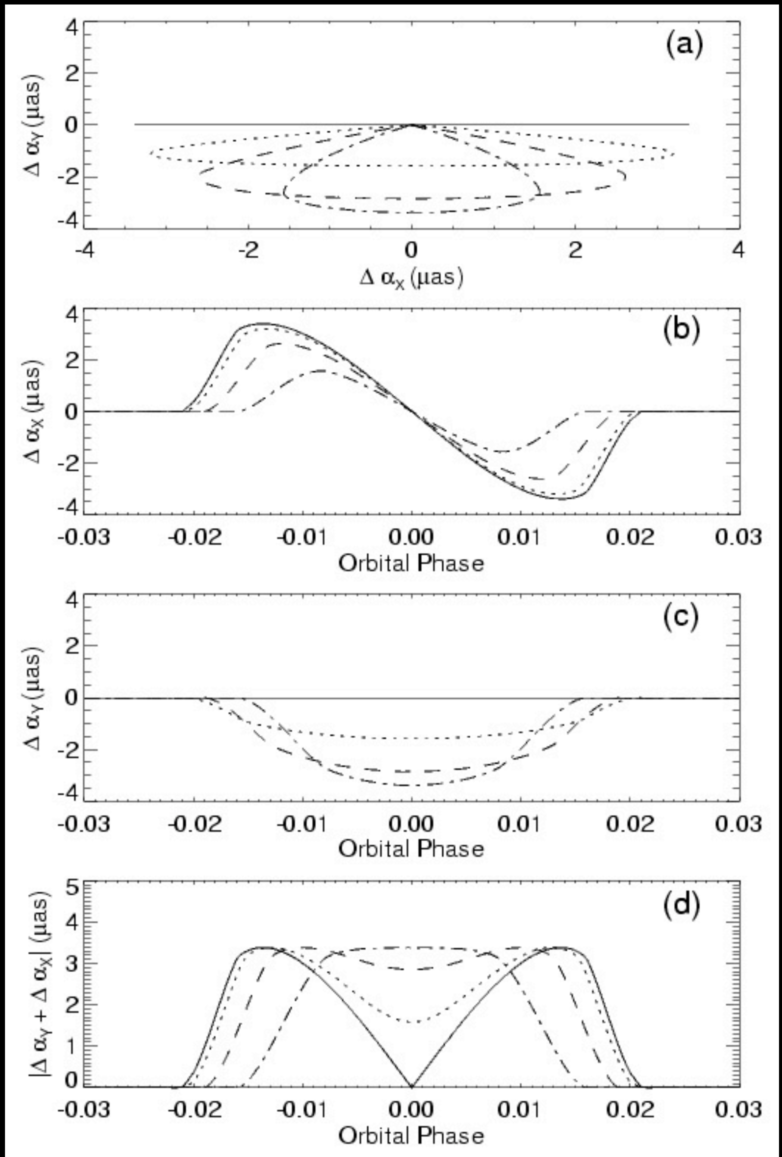
- Ratio of Amplitudes

$$\frac{\alpha_T}{\alpha_W} \approx 1\left(\frac{R_p}{R_J}\right)^2\left(\frac{R_*}{R_\odot}\right)^{-1}\left(\frac{a}{0.04\,\text{AU}}\right)^{-1}\left(\frac{M_p}{M_J}\right)^{-1}\left(\frac{M_*}{M_\odot}\right)$$

- Ratio of Durations

$$\frac{\text{Period of Orbit}}{\text{Transit Duration}} = \frac{\pi a}{R_*} \sim 30$$

Parameter Dependence



- Transit Duration

$$\text{Duration} = 2\tau_0 (1 - b^2)^{1/2}, \text{ where } \tau_0 \equiv \frac{PR_*}{2\pi a}$$

- Parallel Amplitude

$$\sim \alpha_T (1 - b^2)^{1/2}$$

- Perpendicular Amplitude

$$\sim \alpha_T b$$

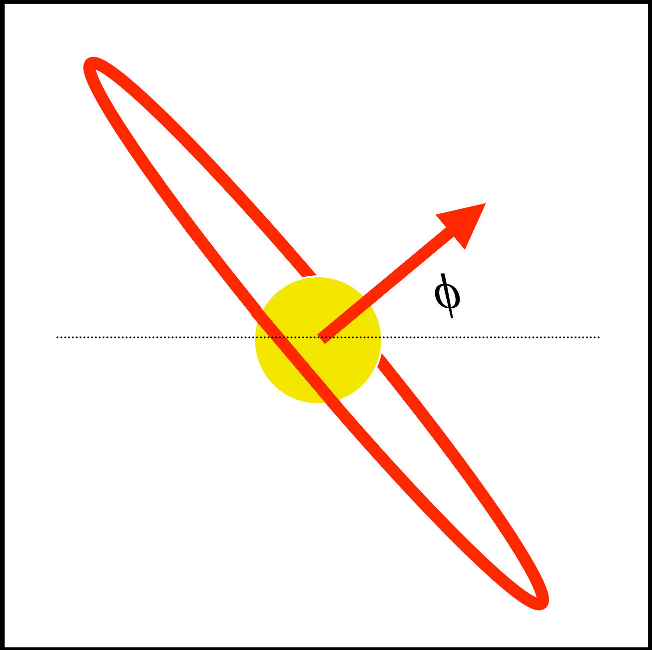
Observable Parameters

- Photometric

$$b,\ r \equiv \frac{R_p}{R_*},\ t_0,\ \tau_0$$

- Astrometric

$$b,\ r \equiv \frac{R_p}{R_*},\ t_0,\ \tau_0,\ \theta_*,\ \phi$$

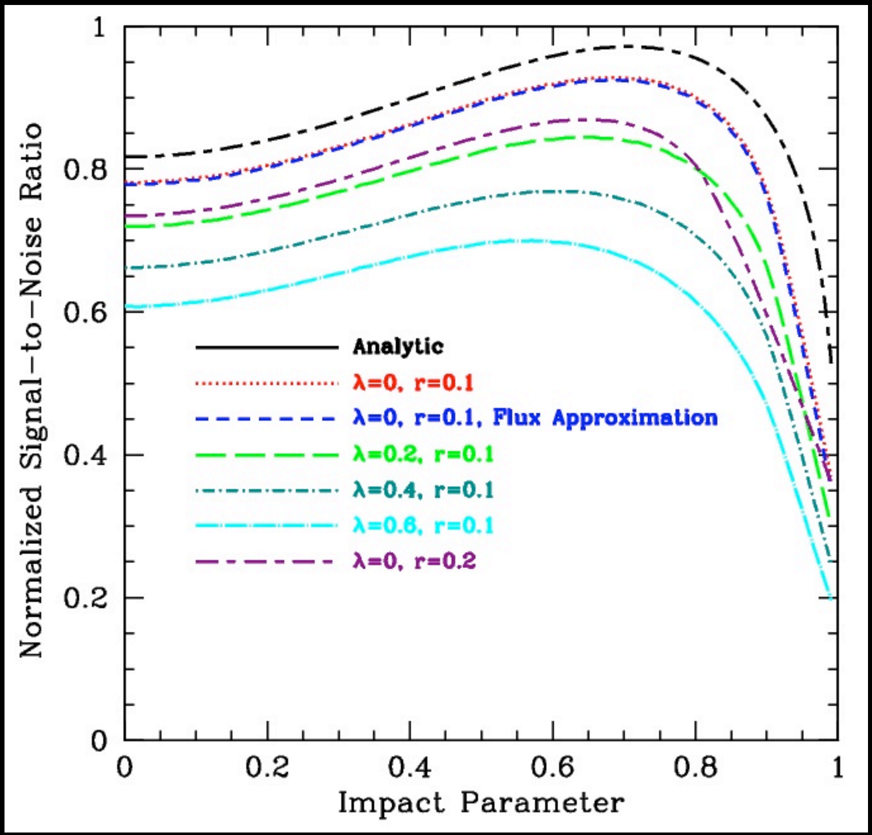


ϕ = angle of projected planet orbit normal w.r.t. sky coordinates

What is it good for?

- Measure M_* and R_*.
- Measure M_p and R_p.
- Confirm or constrain polarized reflected light measurements.
- Determine the inclination of the planet orbit w.r.t. more distant companions.
- Remove degeneracies is multiple transiting planets systems.

Signal-to-Noise Ratio and Uncertainties



- Signal-to-Noise Ratio
  - Duration decreases with *b*
  - Signal amplitude increases with *b*

$$\frac{\mathrm{S}}{\mathrm{N}} = Q\left[\frac{2}{3}(1-b^2)^{1/2}(1+2b^2)\right]^{1/2}, \quad Q \equiv (\Gamma\tau_0)^{1/2}\frac{\alpha_T}{\sigma}$$

- Parameter Uncertainties
  - Assume photometric parameters are known
  - Fisher matrix

$$\sigma_{\theta_*}/\theta_* \sim \sigma_{\varphi} \sim (S/N)^{-1}$$

Known Systems

- Estimated S/N per transit pair
  - 10% Out-of-transit data
  - Two transits with orthogonal baselines
  - Four V=10 ref. Stars
  - Three chop cycles
  - 30s integrations
- Also S/N for 30 total hrs
  - Three systems with S/N>3
    - GJ436b, S/N~7.3
    - HD189733b, S/N~16.9
    - HD209458b, S/N~3.9
  - Three with 1.5<S/N<3
    - HD17156b, S/N~2.3
    - WASP-10b, S/N~1.7
    - HAT-P-1b, S/N~1.5

Table 1. Signal-to-Noise Ratios for Transiting Planets Within 400 pc

Planet Name	V	b	r	θ_* [μas]	α_T [μas]	Time (per pair, hrs)	S/N (per pair)	S/N (30 hrs)
GJ436b	10.68	0.95	0.0968	211.5	2.00	1.06	1.38	7.29
HD189733b	7.67	0.65	0.1571	181.9	4.60	3.20	5.52	16.89
HD209458b	7.65	0.40	0.0929	144.4	1.26	7.84	1.99	3.90
HD17156b	8.17	0.68	0.0846	87.4	0.63	11.94	1.47	2.33
HD149026b	8.15	0.51	0.0518	88.2	0.24	6.71	0.37	0.78
WASP-10b	12.7	0.57	0.1680	40.5	1.18	4.21	0.63	1.69
HAT-P-2b	8.71	0.00	0.0711	48.8	0.25	9.10	0.36	0.65
HAT-P-1b	10.4	0.69	0.1109	37.3	0.46	5.11	0.61	1.48
HAT-P-3b	11.86	0.49	0.1110	27.4	0.34	4.19	0.25	0.66
WASP-7b	9.51	0.08	0.0769	41.1	0.24	7.72	0.31	0.62
WASP-2b	11.98	0.41	0.1279	26.9	0.45	4.16	0.29	0.79
XO-2b	11.18	0.20	0.1038	30.1	0.32	5.21	0.29	0.66
TrES-1	11.79	0.29	0.1355	24.3	0.45	4.73	0.32	0.80
WASP-14b	9.75	0.56	0.0999	37.7	0.38	5.10	0.49	1.19
XO-1b	11.3	0.14	0.1311	21.6	0.38	5.80	0.32	0.73
HAT-P-6b	10.5	0.60	0.0936	33.9	0.30	6.70	0.42	0.88
TrES-2	11.41	0.84	0.1254	21.1	0.34	2.86	0.30	0.96
WASP-3b	10.64	0.51	0.1028	27.3	0.29	5.14	0.33	0.79
TrES-3	12.4	0.82	0.1637	15.5	0.43	2.12	0.22	0.84
XO-3b	9.8	0.72	0.0909	24.6	0.20	5.28	0.30	0.72
XO-5b	12.13	0.63	0.1065	19.1	0.22	4.88	0.17	0.42
XO-4b	10.7	0.17	0.0889	24.6	0.20	8.95	0.24	0.44
WASP-5b	12.26	0.49	0.1092	16.1	0.19	4.25	0.12	0.32
CoRoT-Exo-2b	12.57	0.25	0.1670	14.0	0.40	4.25	0.19	0.51
WASP-4b	12.6	0.11	0.1296	17.8	0.30	5.20	0.15	0.36
HAT-P-4b	11.2	0.01	0.0821	23.9	0.16	8.51	0.17	0.31
HAT-P-7b	10.5	0.33	0.0761	26.7	0.16	7.94	0.20	0.39
HAT-P-5b	12	0.43	0.1110	16.0	0.20	5.65	0.15	0.35

To Do

- Practical application.
  - Refined S/N estimates.
  - Suitable reference stars?
  - Optimal strategies.
  - Expected uncertainties.
- Additional systems?
- Astrometric signature of eclipsing binaries.