Understanding Speckle Noise and Dynamic Range in Coronagraphic Images

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Collaborators

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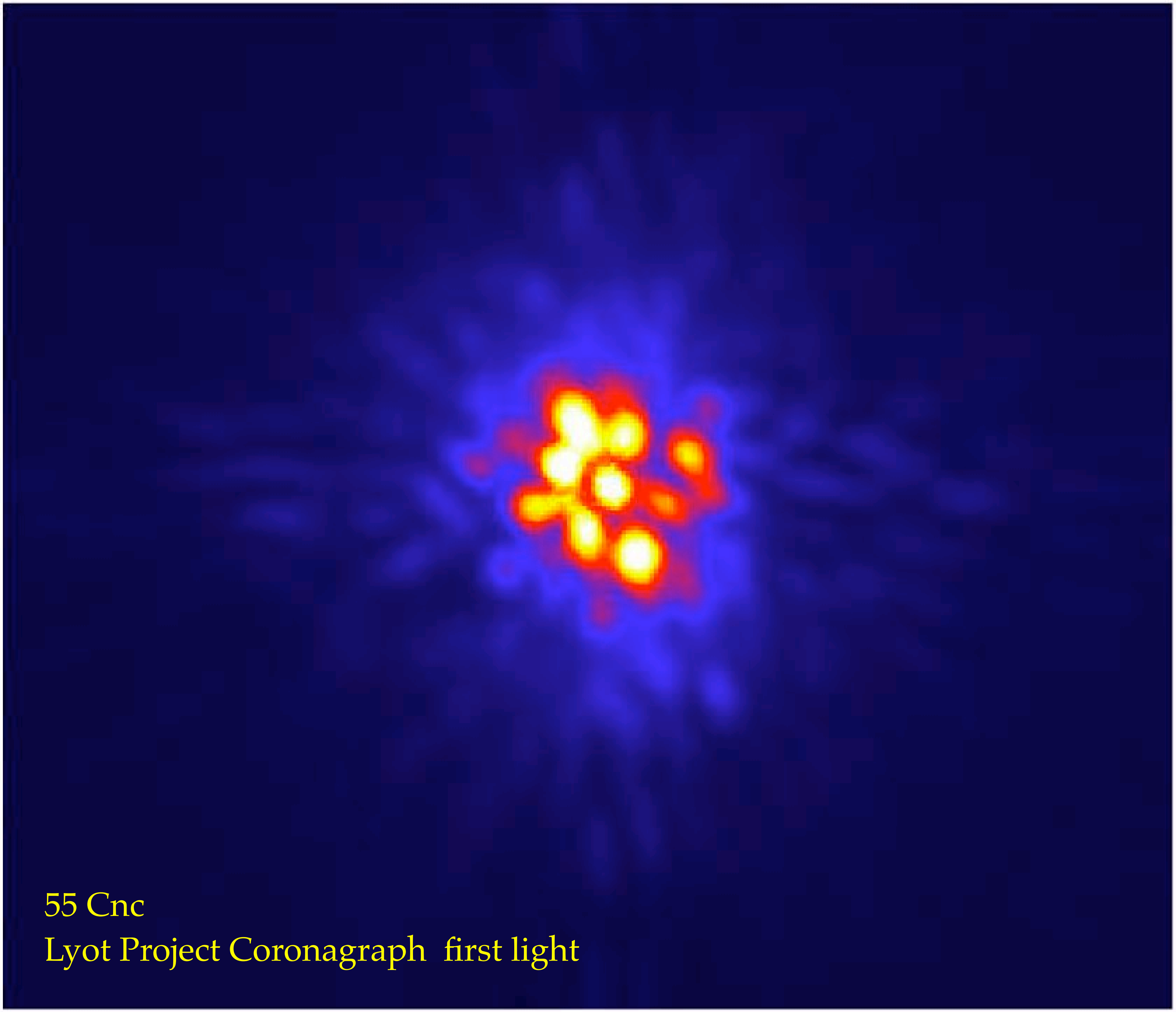
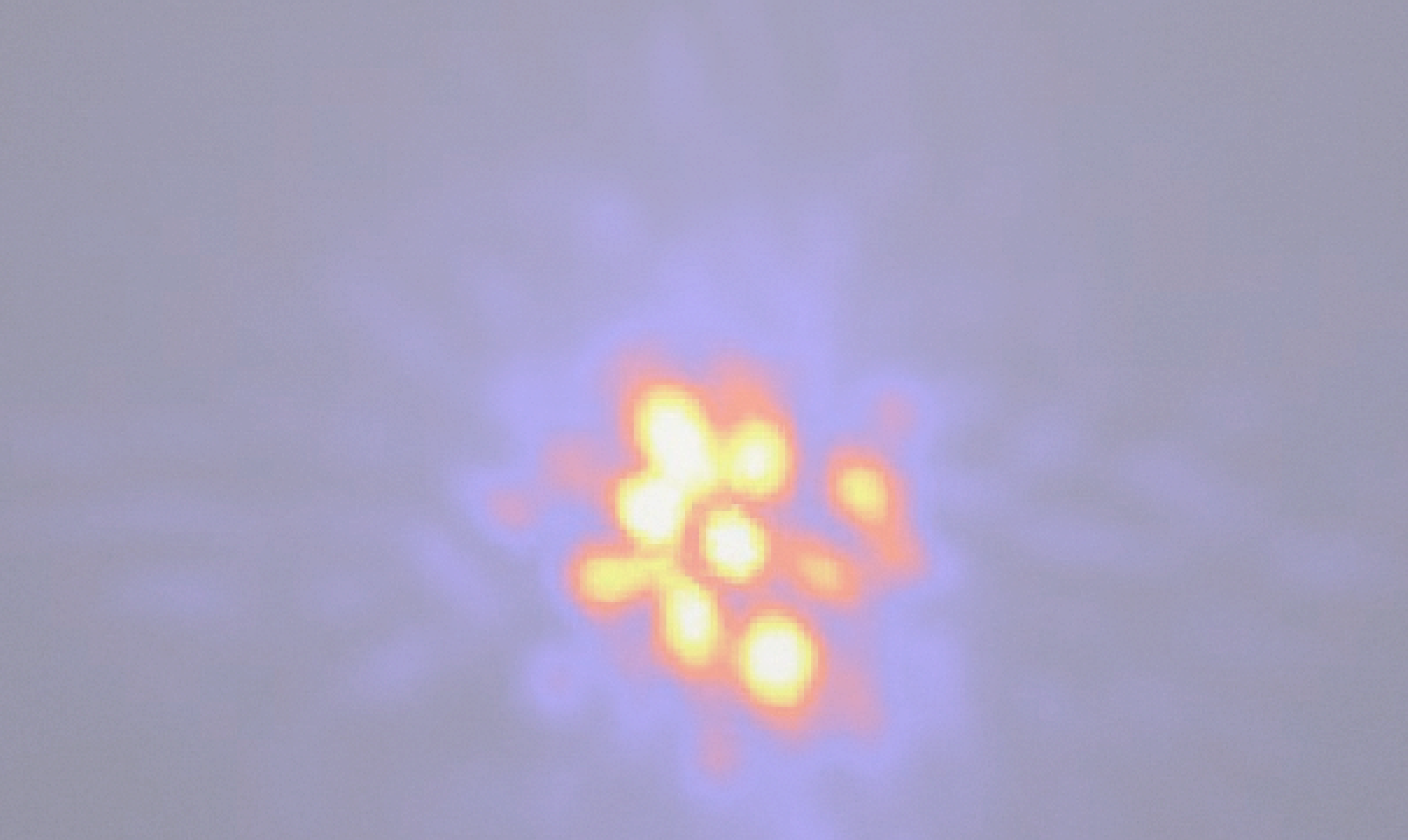
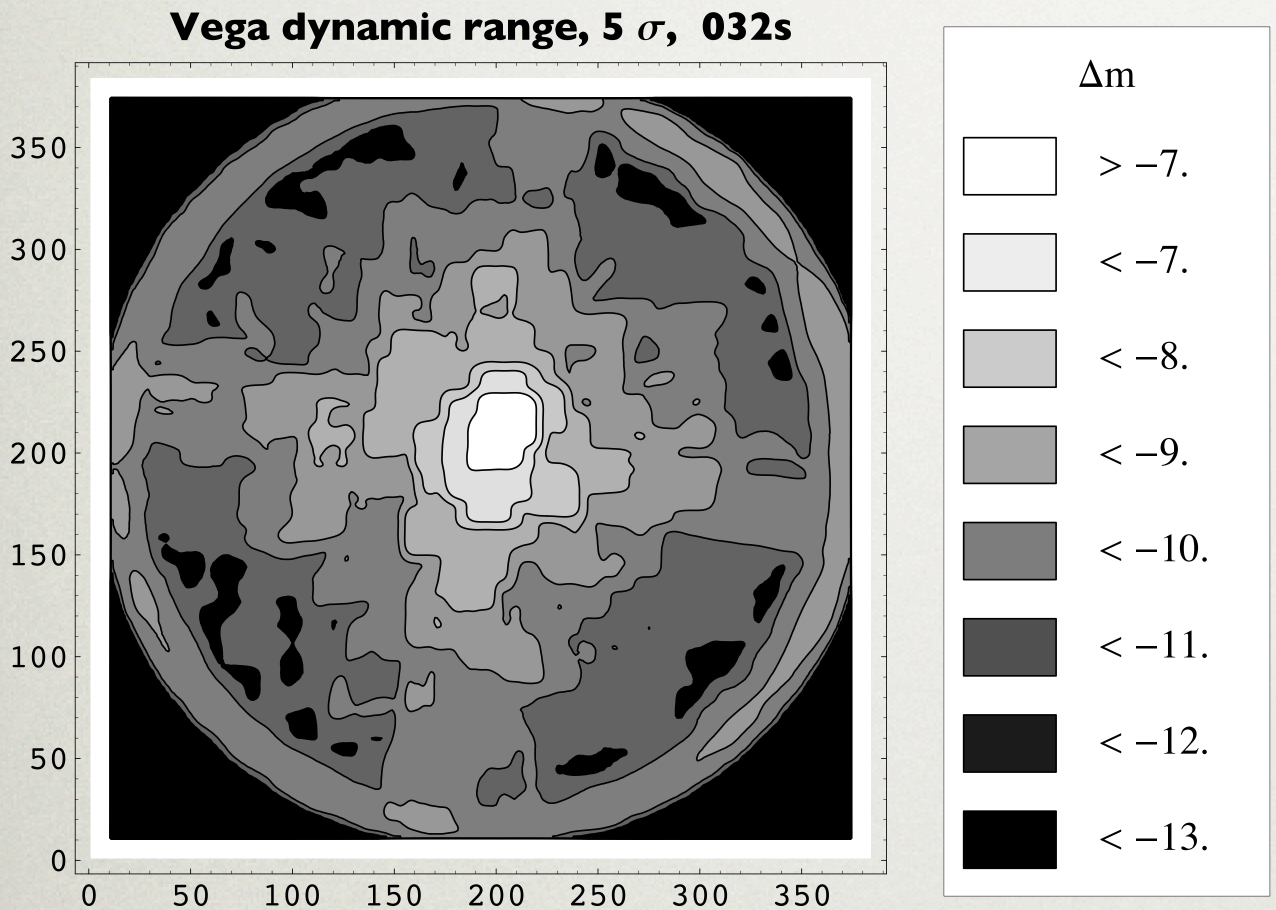
- AMNH:
  - B. Oppenheimer, A. Sivaramakrishnan, A. Digby, S. Hinkley
- Cornell
  - J. Lloyd
- STScI:
  - R. Makidon
- Berkeley
  - J. Graham, M. Perrin, M. Fitzgerald
- HIA
  - L. Jolissaint (PAOLA package)


55 Cnc
Lyot Project Coronagraph first light

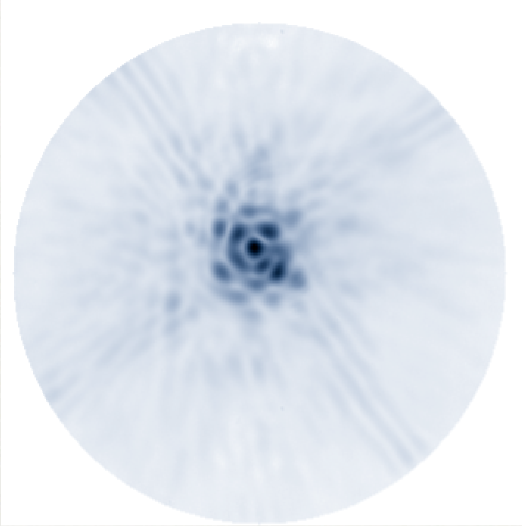


- Planet is a faint point~like source over a noisy background
- Several sources of noise: speckle photon detector etc.
- The stability and statistical properties of these noise source will define the actual dynamic range

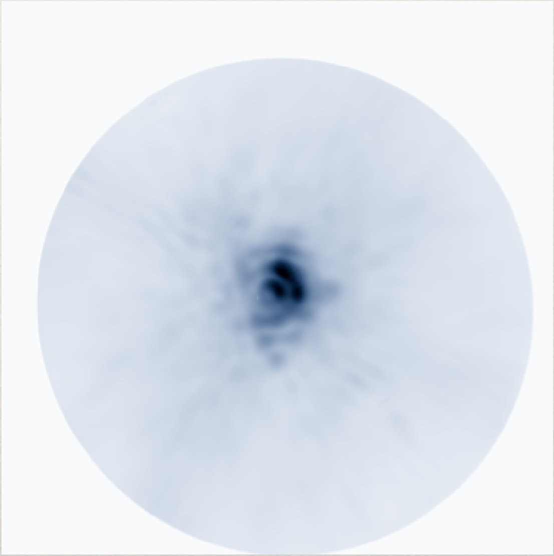
Lyot project dynamic range



Contrast levels (dynamic range) for a 20 sec exposure



Short Exposure (20 sec)



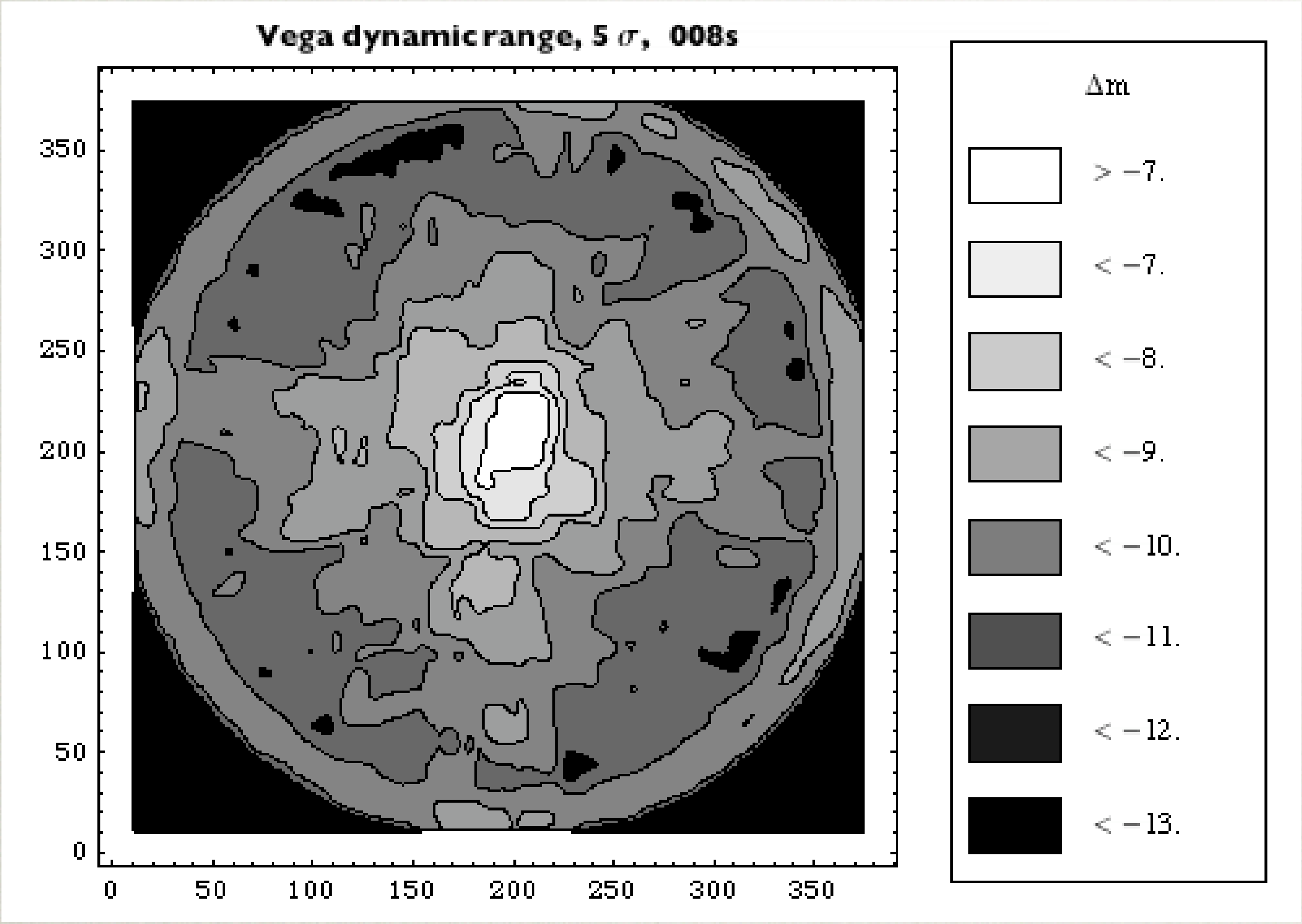
Long exposure

1400 sec

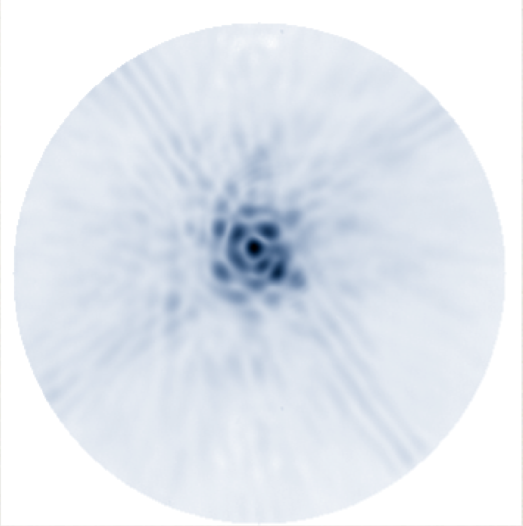
(tip~tilt off)

Hinkley etal in prep

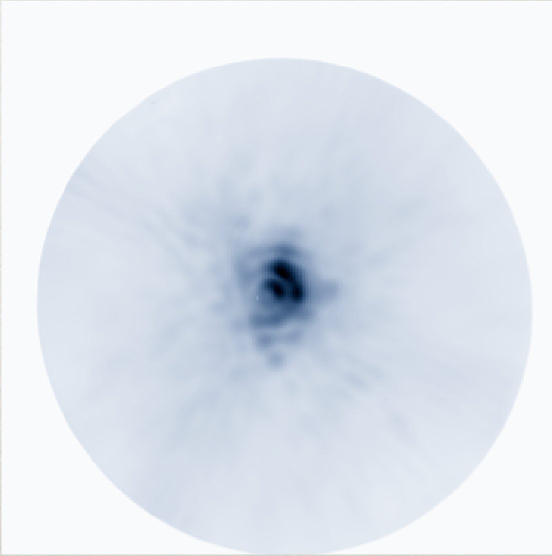
Lyot project dynamic range



Contrast levels (dynamic range) for a 20 sec exposure



Short Exposure (20 sec)



Long exposure

1400 sec

(tip-tilt off)

Main limitation in this case: Quasi-Static Speckles

Hinkley etal in prep

A few questions...

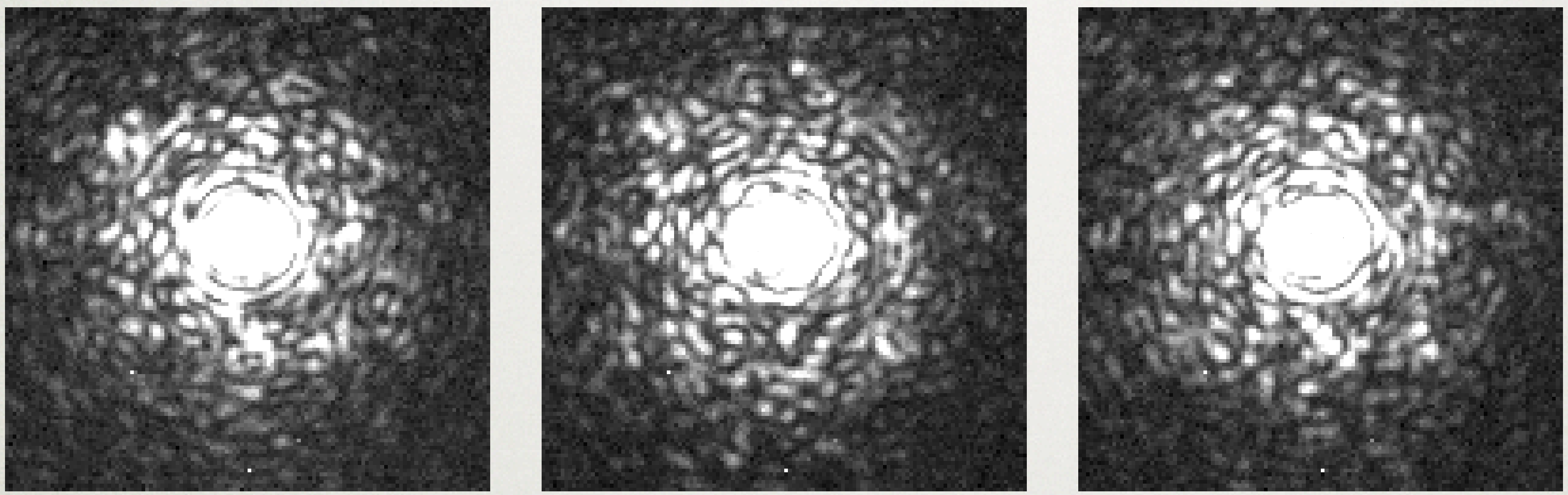
- What is the statistics of the speckle noise in direct high Strehl images?
- What is the effect of a coronagraph on the statistics of the noise?
- What is the interaction between coronagraph, static, quasi-static and residual atmospheric speckles?

- Need for a statistical model
  - Start with direct images
  - Study the effect of a coronagraph
  - Study the effect of static or quasi-static aberrations

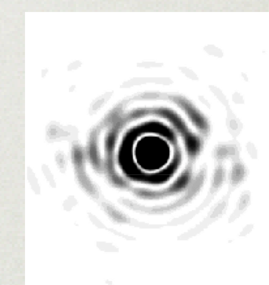
Speckle and photon noise in direct images

Imaging in the presence of wavefront errors

- Speckle pinning (Bloemof etal 2001, Sivaramakrishnan etal 2002)

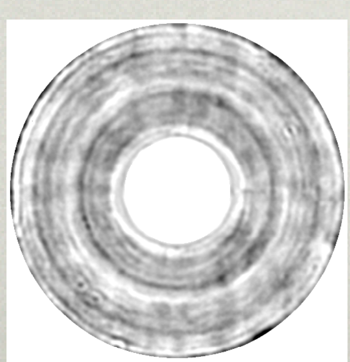


Palomar AO
juin 2004
K band

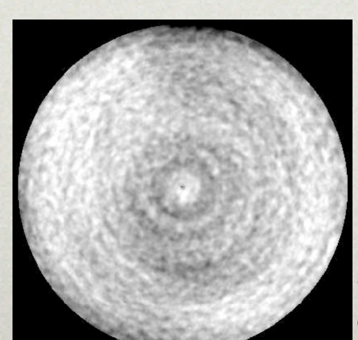


Simulated
AO PSF

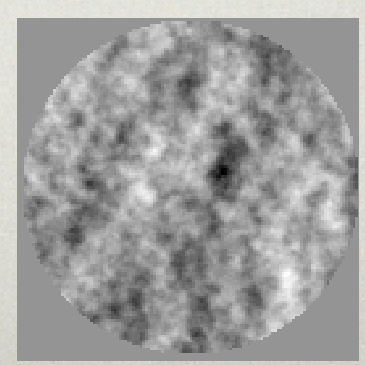
- origin: wavefront errors
  - atmospheric residuals and/or quasi static aberrations (space & ground)



HST
(Krist)



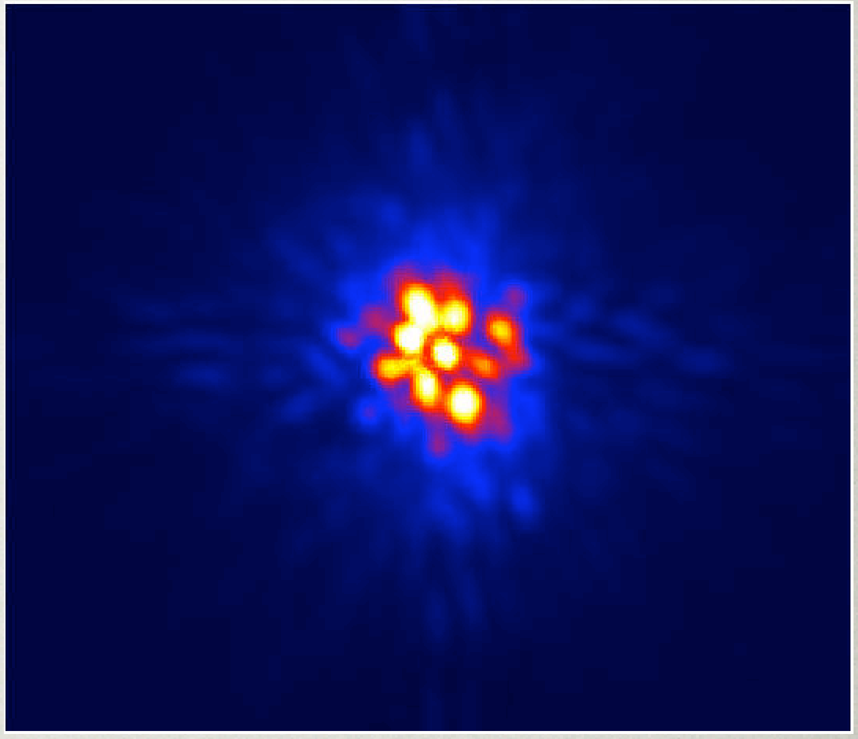
Tinsley
(Krist)



AO
simulation using PAOLA

Speckles

- Two complementary approaches:
  - Wavefront propagation using Tayor expansion (Sivaramakrishnan etal. 2002, Perrin etal. 2003).
  - Statistical optics approach (Aime & Soummer 2004, Soummer & Aime 2004)



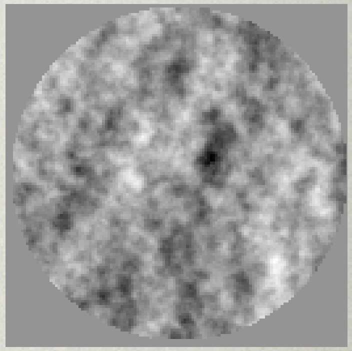
Statistical model: pupil plane

- Wave amplitude at the pupil plane:

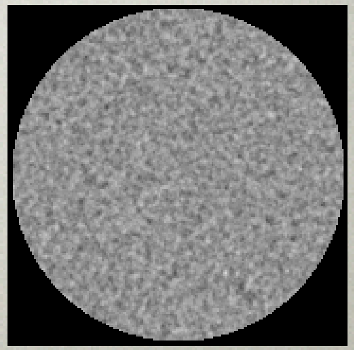
$$\Psi_1(x,y) = [A + a(x,y)]\ P(x,y)$$

Plane wave — uncorrected part of the wavefront

Phase simulation



Amplitude (scintillation)



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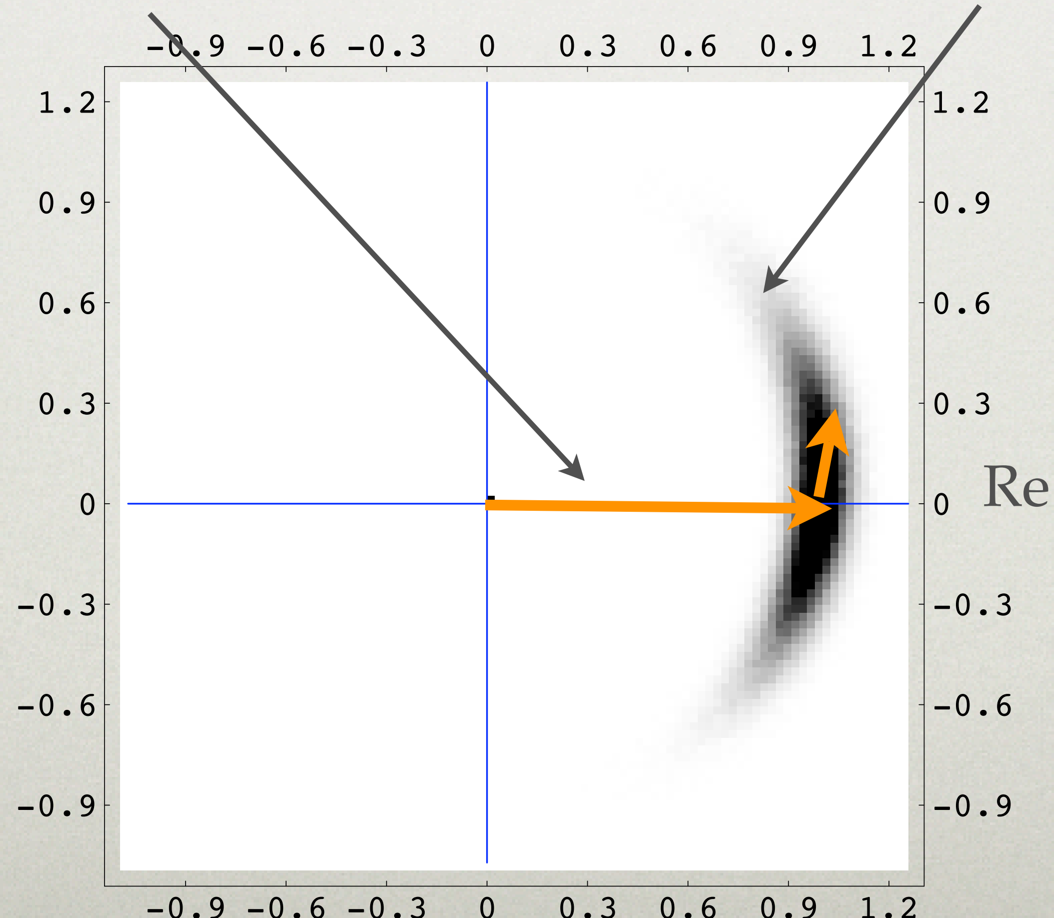
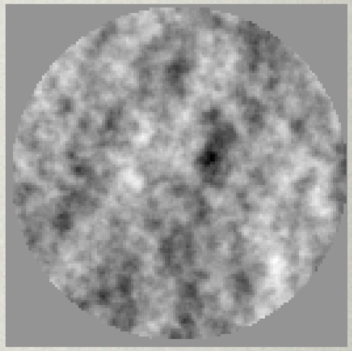


This decomposition is also valid in space

Complex PDF
AO simulation
phase + amplitude

Phase simulation



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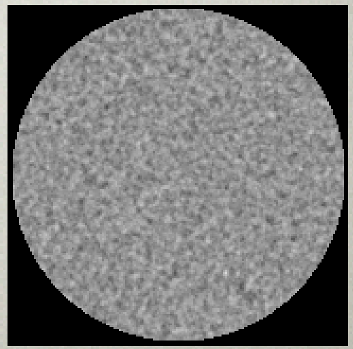


Statistical model: focal plane

$$\Psi_1(x,y) = [A + a(x,y)]\; P(x,y)$$

$$\Psi_2(x,y) = A \times \mathcal{F}[P(x,y)] + \mathcal{F}[a(x,y) \times P(x,y)]$$

$\tilde{C}(\mathbf{r})$

Constant
deterministic term

$S(\mathbf{r})$

Speckle
random term

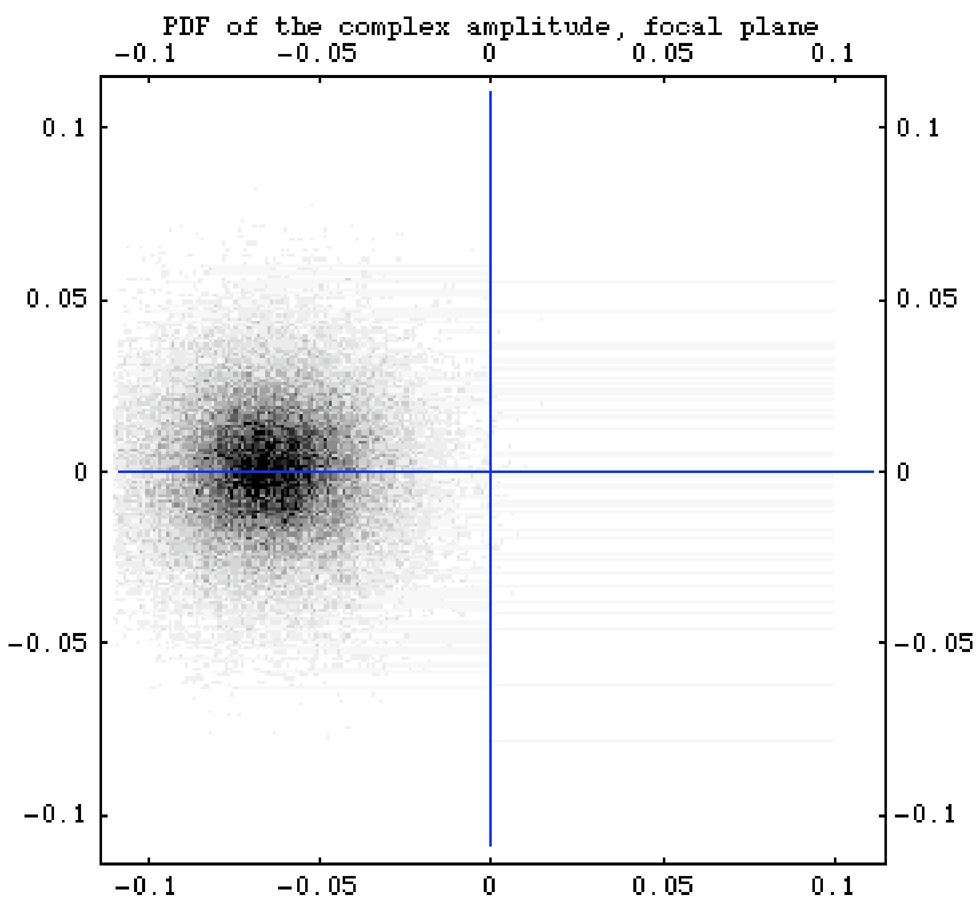
$$\mathcal{P}(\xi,\eta) = \frac{1}{\pi < |S(\mathbf{r})|^2 >} \exp\left(\frac{-(\xi - \mathrm{Re}[\tilde{C}(\mathbf{r})])^2 + (\eta - \mathrm{Im}[\tilde{C}(\mathbf{r})])^2}{< |S(\mathbf{r})|^2 >} \right)$$

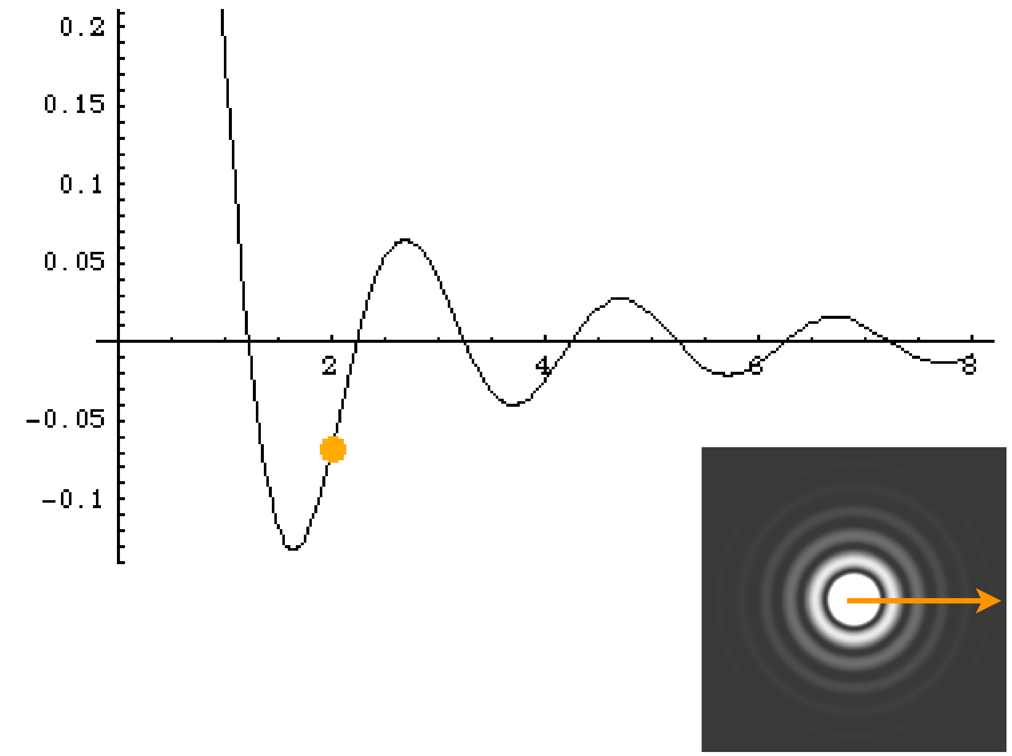
decentered Gaussian statistics
(complex amplitude in the focal plane)

same solution as for holographic speckle (Goodman 1975)

Statistical model: focal plane

$$\mathcal{P}(\xi,\eta) = \frac{1}{\pi < |S(\mathbf{r})|^2 >} \exp\left(\frac{-(\xi - \mathrm{Re}[\tilde{C}(\mathbf{r})])^2 + (\eta - \mathrm{Im}[\tilde{C}(\mathbf{r})])^2}{< |S(\mathbf{r})|^2 >} \right)$$





decentered gaussian statistics
(complex amplitude in the focal plane)

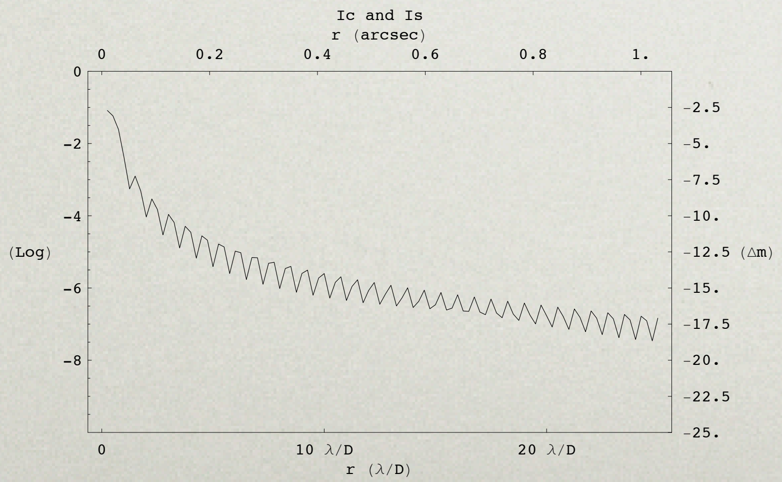
same solution as for holographic speckle (Goodman 1975)

Statistical model: focal plane

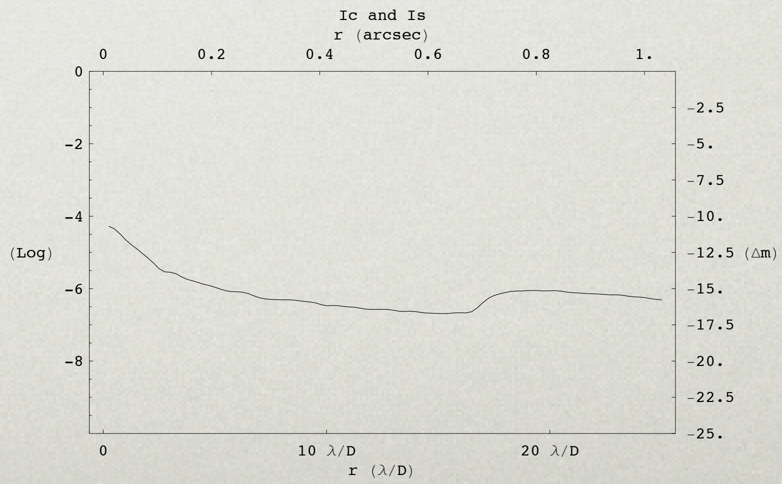
$$\mathcal{P}(\xi, \eta) \longrightarrow \mathcal{P}_I(I)$$

- Statistics of the intensity in the focal plane: modified Rice distribution

$$\mathcal{P}_I(I) = \frac{1}{I_s} \exp\left(-\frac{I + I_c}{I_s}\right) \mathrm{I}_0\left(\frac{2\sqrt{I}\sqrt{I_c}}{I_s}\right)$$



$I_c = |\tilde{C}(\mathbf{r})|^2$

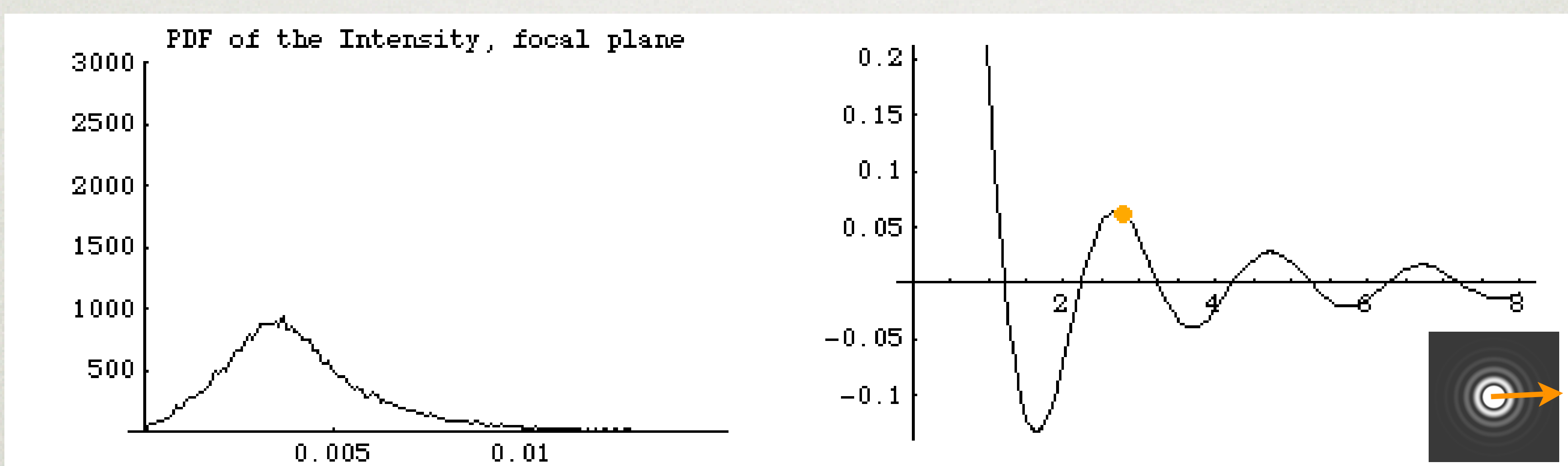


$I_s = < |S(\mathbf{r})|^2 >$

Statistical model: focal plane

- Modified Rice distribution

$$\mathcal{P}_I(I) = \frac{1}{I_s} \exp\left(-\frac{I + I_c}{I_s}\right) \mathrm{I}_0 \left(\frac{2\sqrt{I}\sqrt{I_c}}{I_s}\right)$$



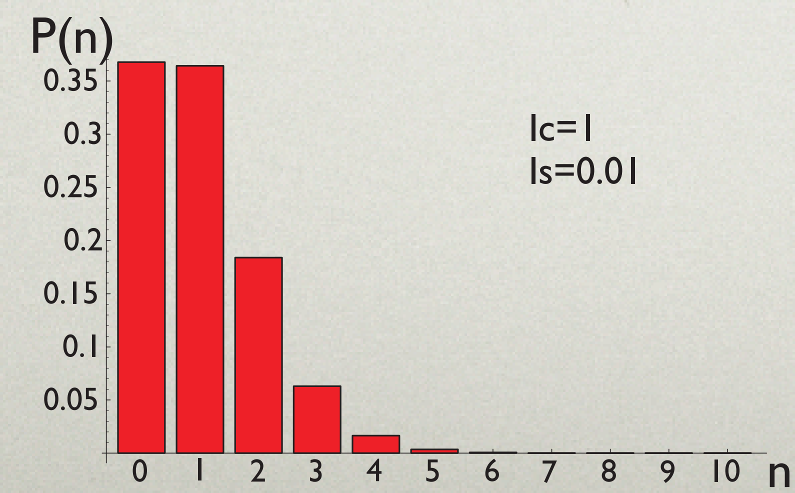
Photon counting statistics

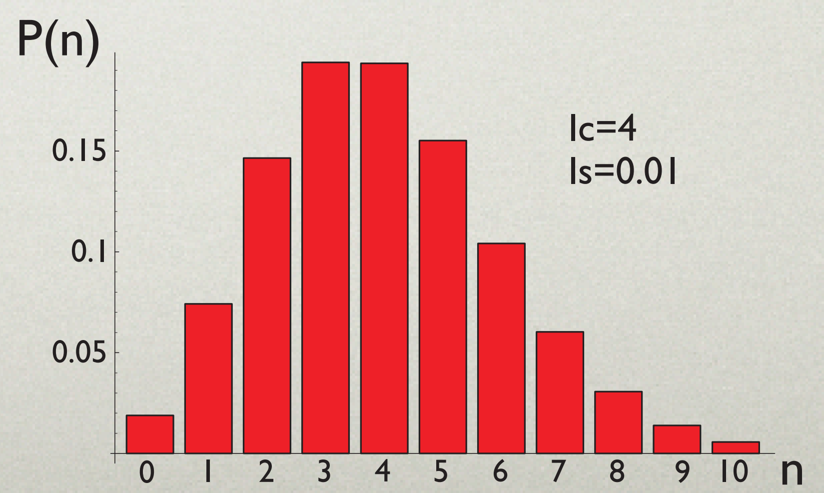
- Poisson Mandel Transformation

$$\mathcal{P}(n) = \frac{1}{n!}\int_0^\infty \mathcal{P}_I(I) I^n \exp(-I)\mathrm{d}I$$

- Photon counting statistics:

$$\mathcal{P}(n) = \frac{1}{I_s+1}(1+\frac{1}{I_s})^{-n} \exp\left(-\frac{I_c}{I_s}\right)\ {}_1F_1\left(n+1;1;\frac{I_c}{I_s^2+I_s}\right)$$





Mean and Variance

- Mean intensity: diffraction pattern + halo

$$< |\Psi_2(x,y)|^2 >= |C(x,y)|^2 + < |S(x,y)|^2 >= I_c + I_s$$

- Variance speckle:

$$\sigma_I^2 = I_s^2 + 2 I_s I_c$$

Mean and Variance

- Mean intensity: diffraction pattern + halo

$$< |\Psi_2(x,y)|^2 >= |C(x,y)|^2 + < |S(x,y)|^2 >= I_c + I_s$$

- Variance speckle:

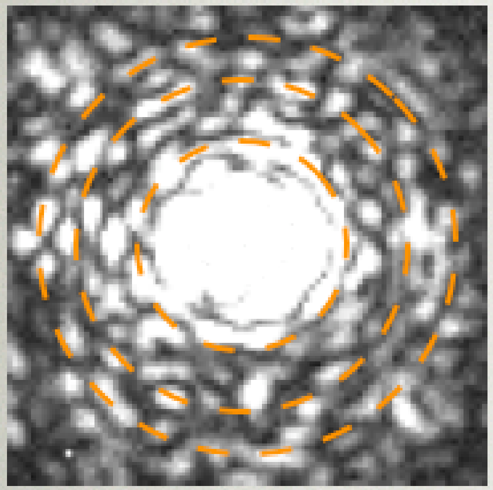
$$\sigma_I^2 = I_s^2 + 2 I_s I_c$$

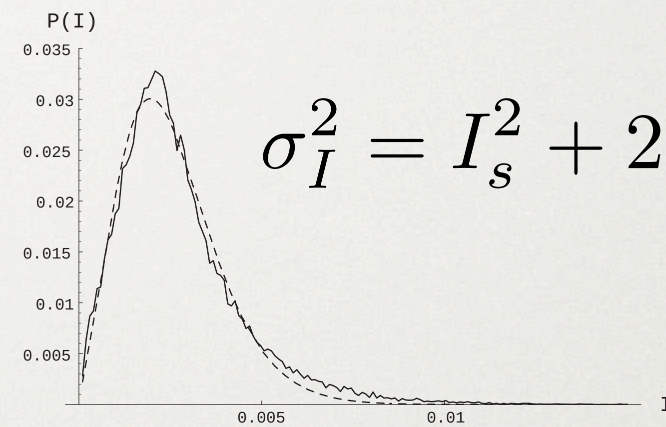
- Variance speckle + photon noise:

$$\sigma^2 = I_s^2 + 2 I_s I_c + I_c + I_s$$

Same variance expression used in the TPF error budget by Shaklan 2004

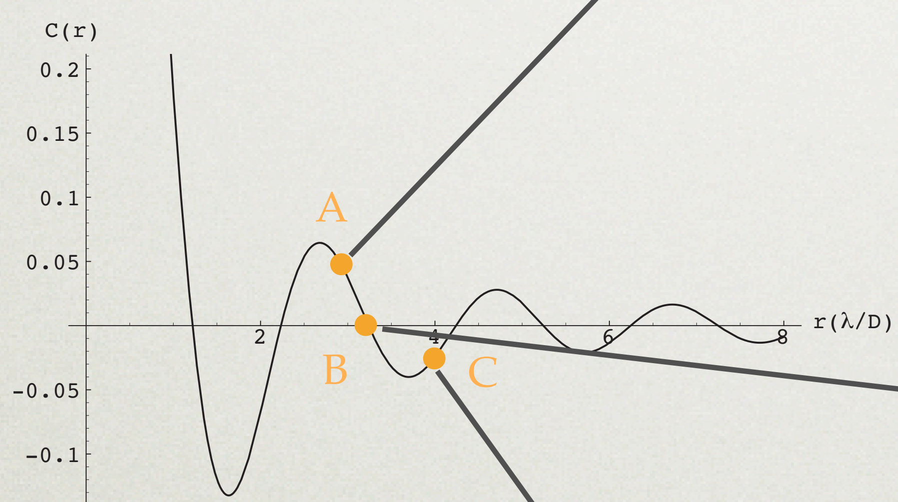
Explanation of speckle pinning



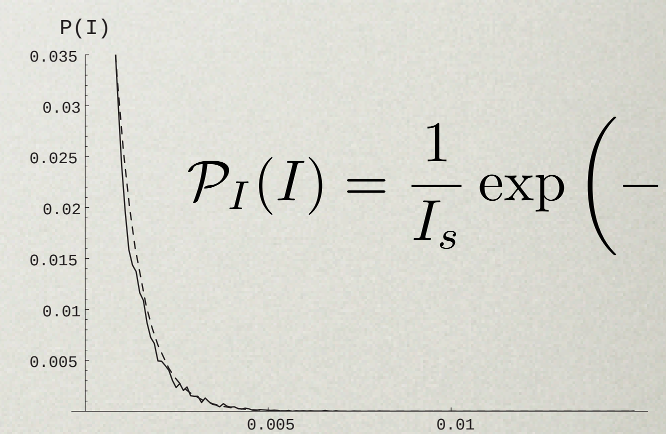


$$\sigma_I^2 = I_s^2 + 2I_sI_c$$

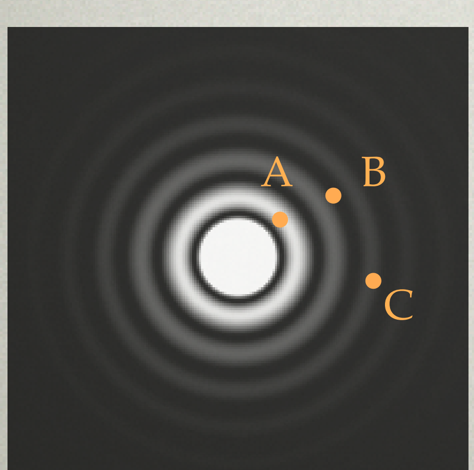
High continuous value amplification of the fluctuation



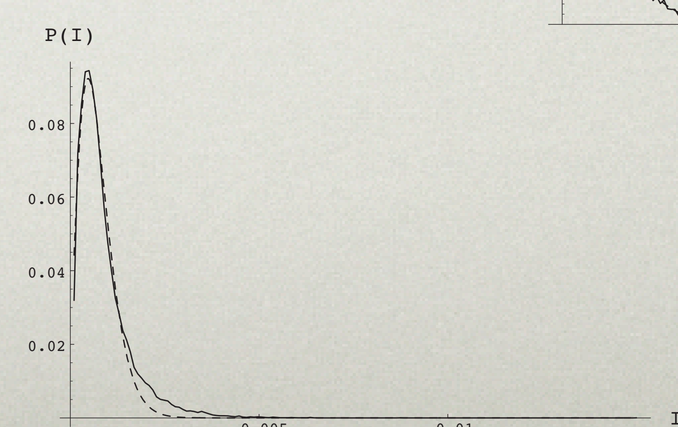
Zero of the PSF exponential statistics



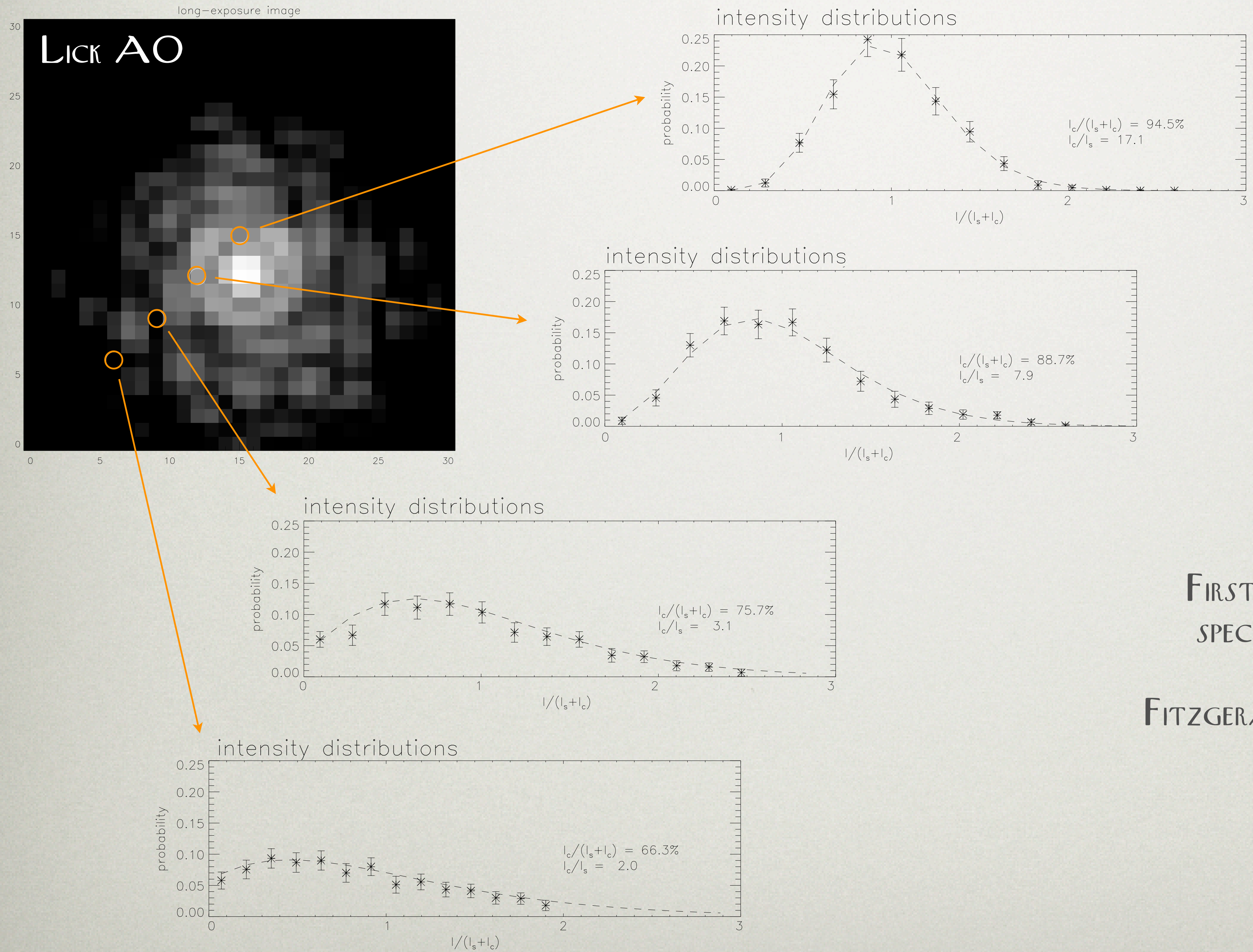
$$\mathcal{P}_I(I) = \frac{1}{I_s} \exp\left(-\frac{I}{I_s}\right)$$



Low value of the continuous level



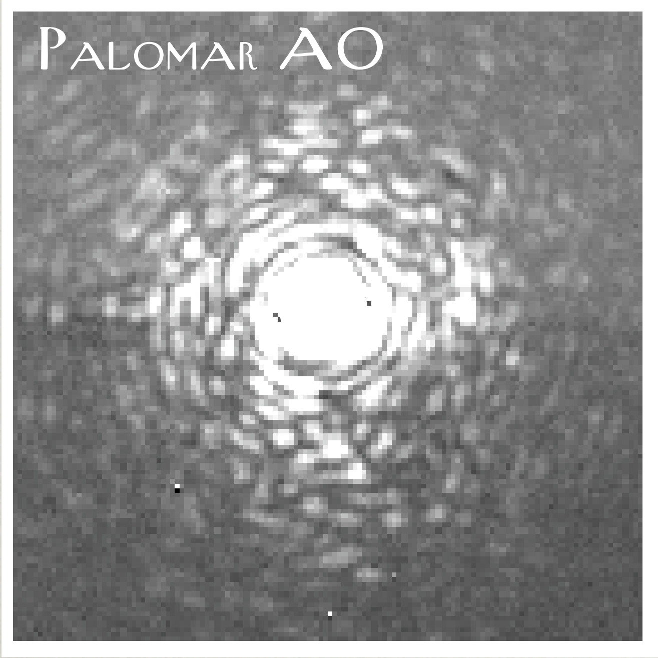
Lick AO data

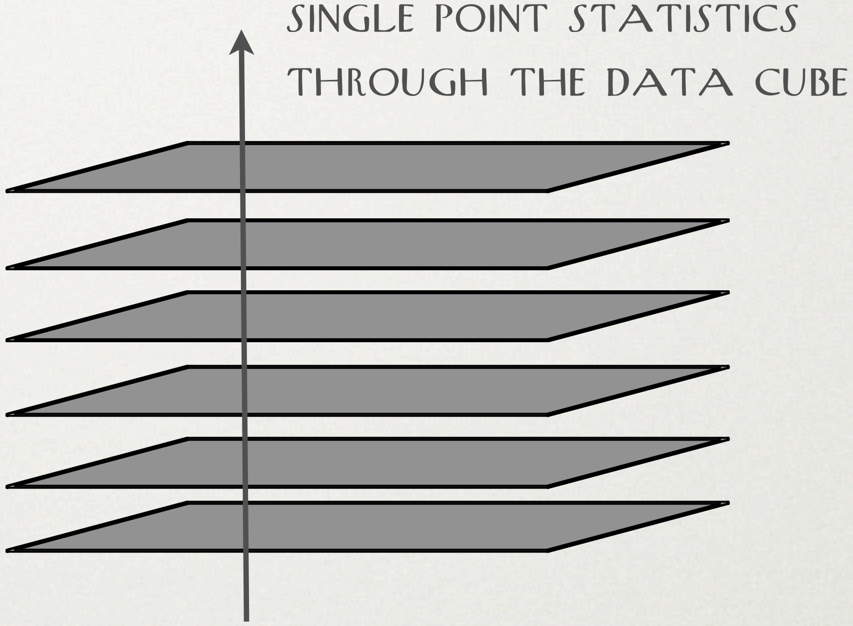


First verification on sky of speckles Rician statistics:

Fitzgerald & Graham (2005)

Palomar AO data





Exposure time 120ms in K band, need for a model of integrated speckles

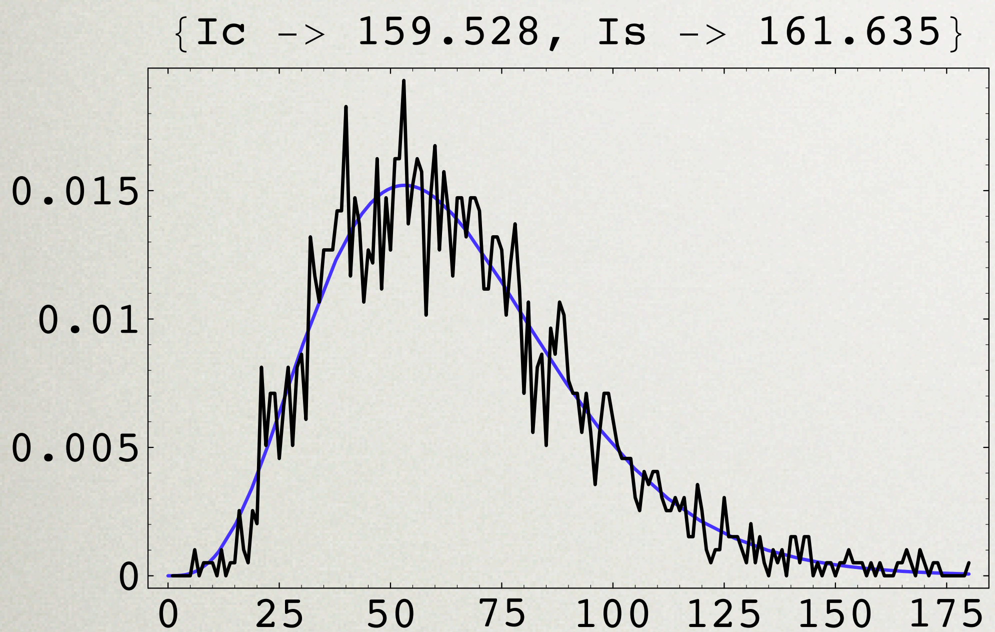
$$\mathcal{P}_I(I) = \frac{M}{i_s}\left(\frac{i}{i_c}\right)^{\frac{M-1}{2}} e^{-\frac{M(i+i_c)}{i_s}} I_{M-1}\left(\frac{2\sqrt{i}\sqrt{i_c}M}{i_s}\right)$$

Non central chi square distribution of order M

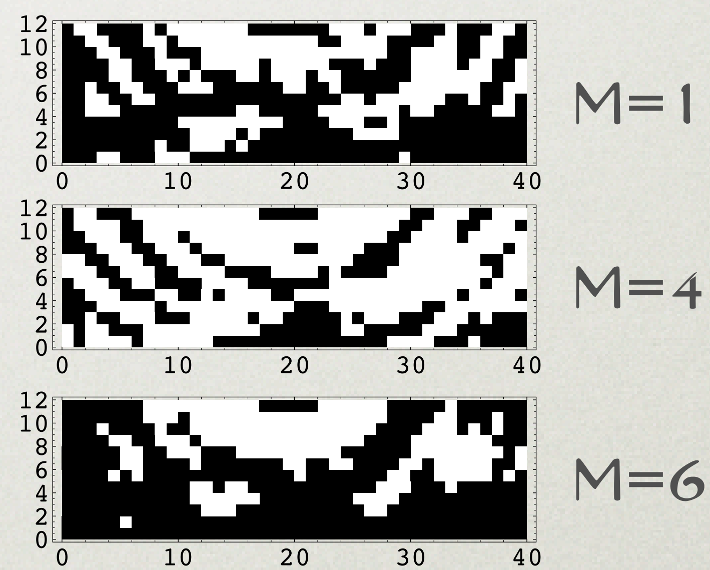
Soummer etal in prep

Palomar AO data

Maximum Likelihood:
Non central chi square vs. data



Kolmogorov Smirnoff test



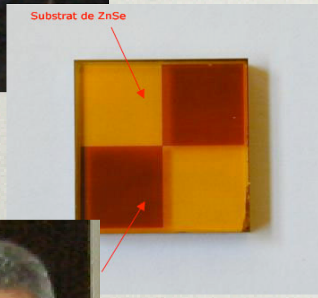
i.e. lifetime of 30ms
consistent with K band

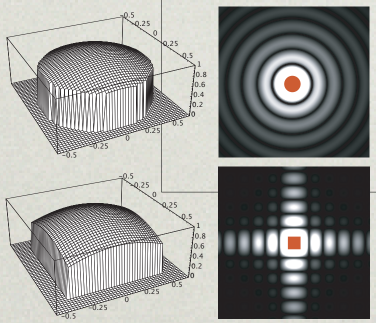
dark rings pixels fail the KS test
need to include detector noise in the model
.... work in progress

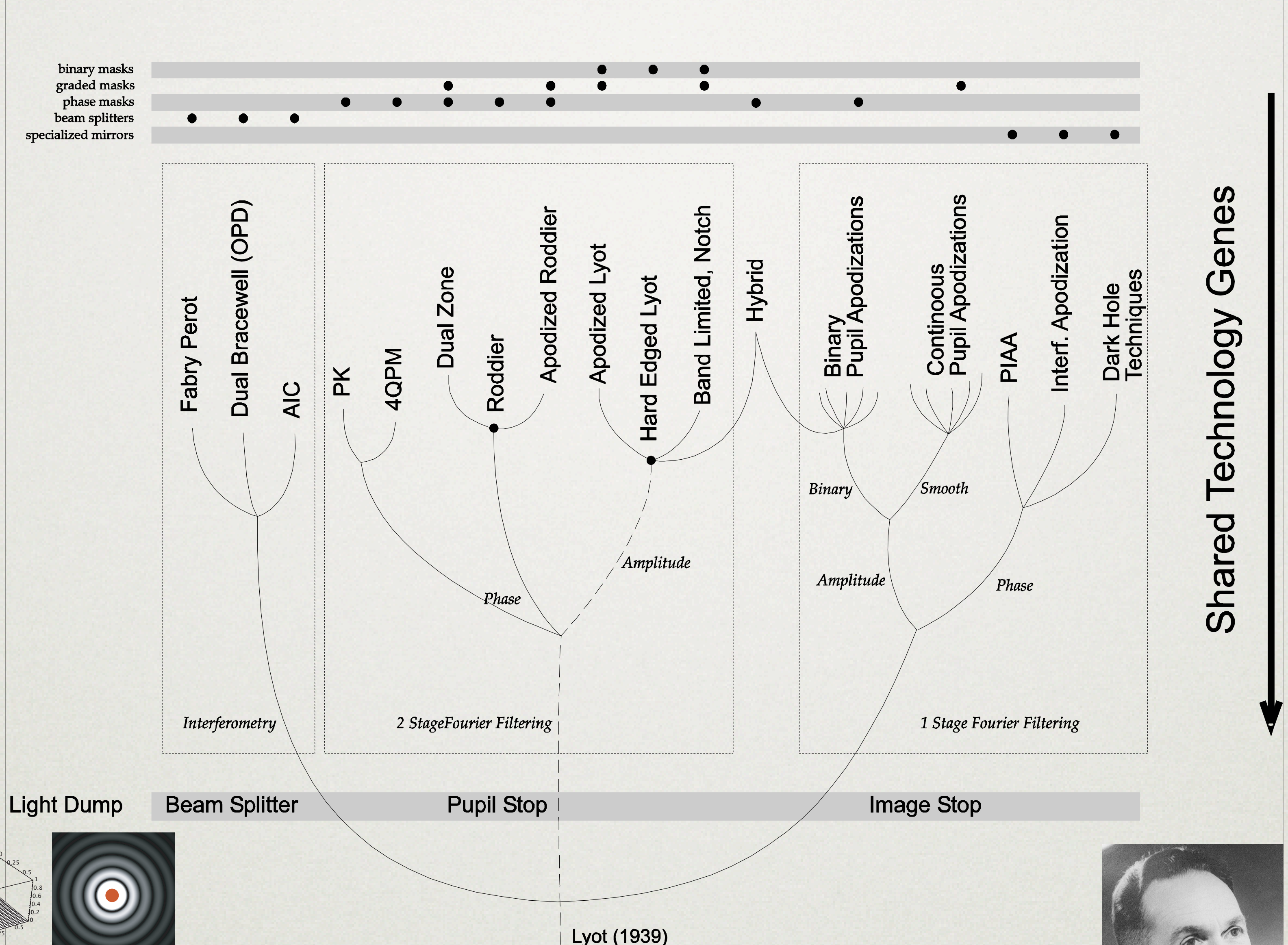
Soummer etal in prep

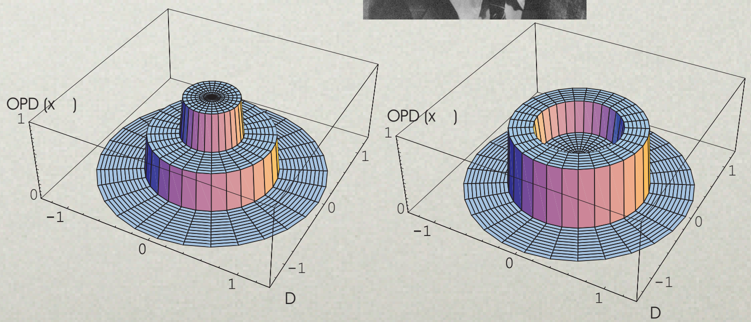
Effect of a coronagraph on the speckle and photon noise

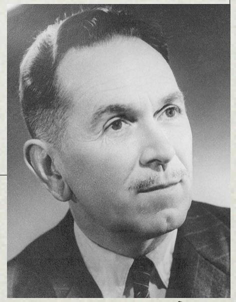
The Coronagraphic Tree of Life

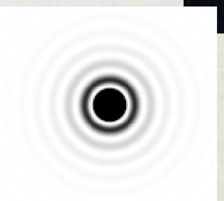


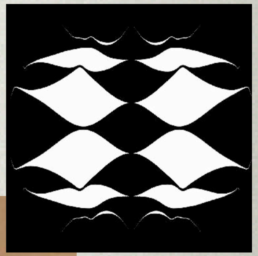












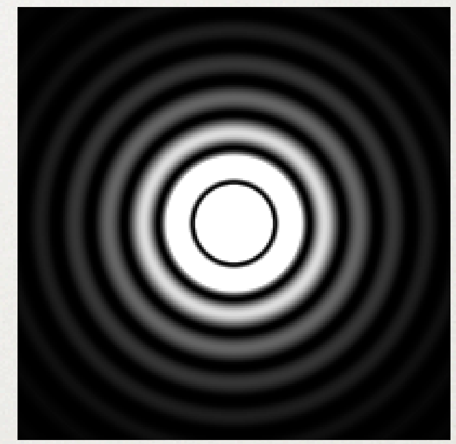
Effect of a coronagraph

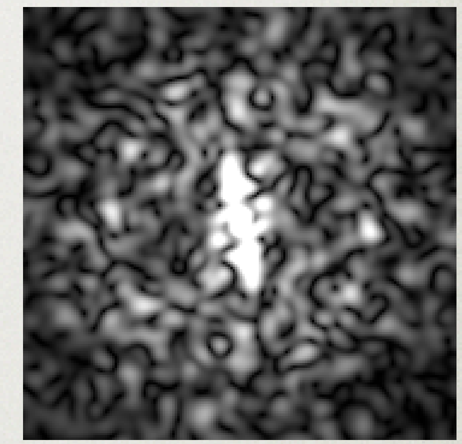
- The direct focal plane amplitude is:

$$\Psi_2(\mathbf{r}) = C_d(\mathbf{r}) + S(\mathbf{r})$$

Static direct response

Speckle term





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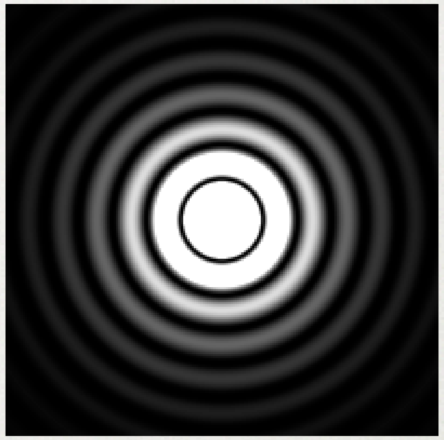
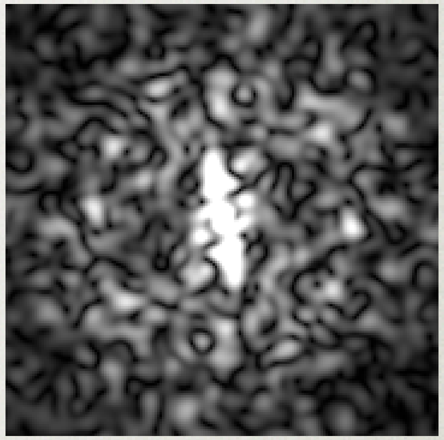


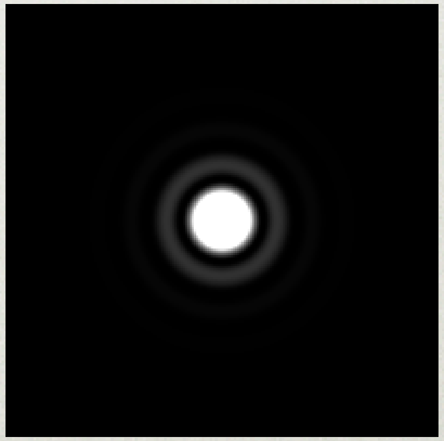


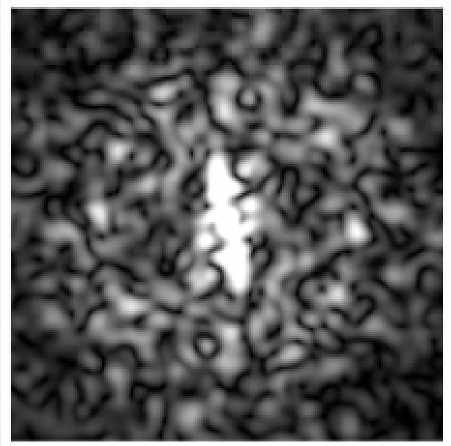
- The coronagraphic focal plane amplitude is:

$$\Psi_4(\mathbf{r}) = C_c(\mathbf{r}) + S(\mathbf{r})$$

Static coronagraph response

Speckle term

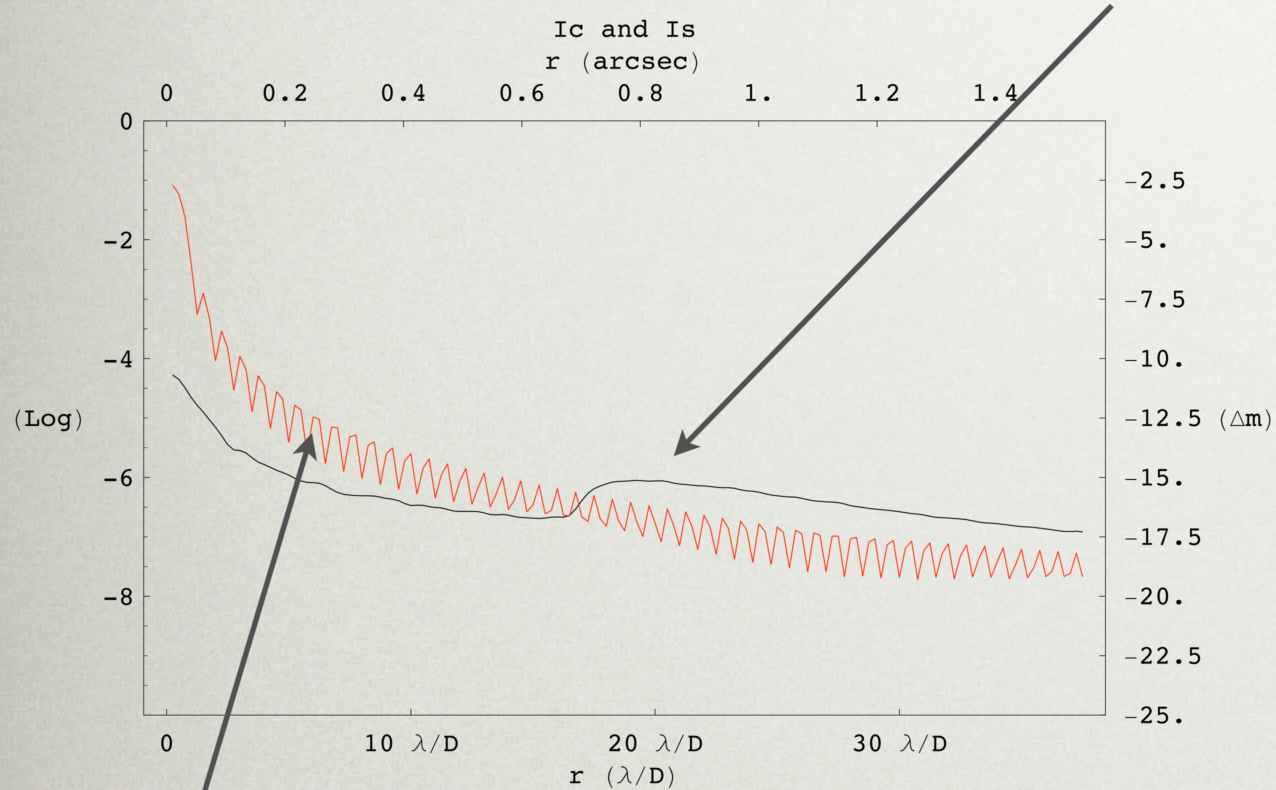




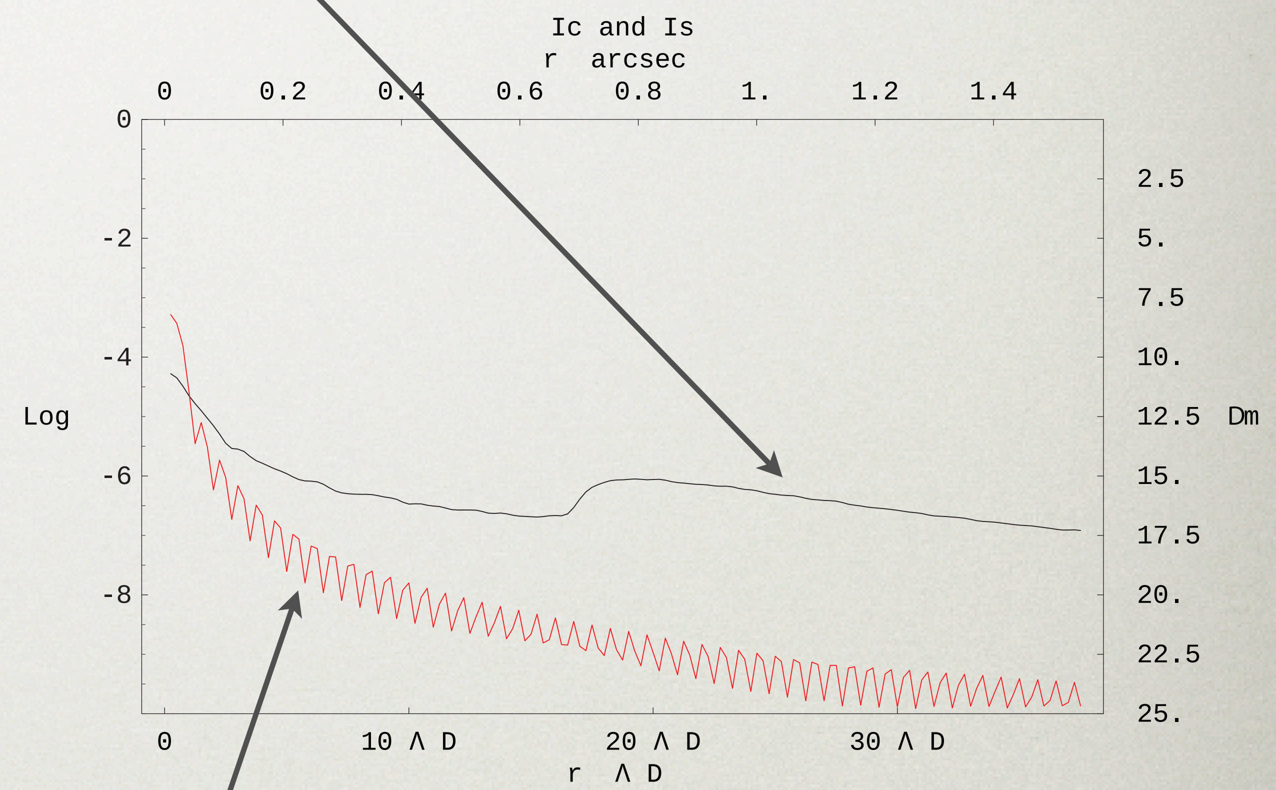
A coronagraph has a no effect on the speckle term outside the mask area

Suppression of speckle amplification

Speckle term: (not affected by a coronagraph)



Ic without coronagraph



Ic with coronagraph

Suppression of the speckle amplification due to the Ic term

Suppression of speckle amplification

- A coronagraph can suppress the speckle noise coherent amplification (speckle pinning)
- Direct coronagraphic gain where Ic>Is

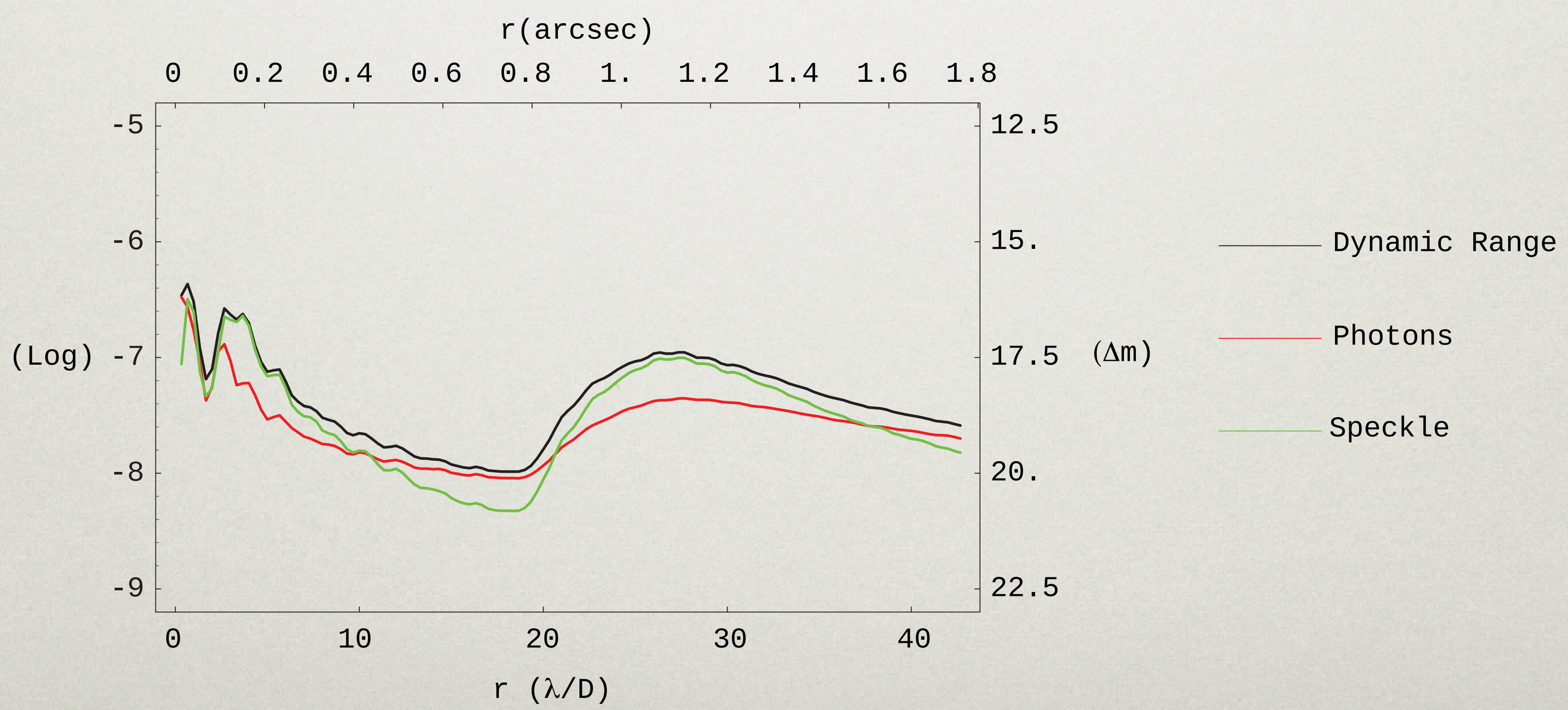
$$\sigma^2 = (2I_sI_c + I_c) + (I_s^2 + I_s) = \sigma_c^2 + \sigma_s^2$$

Can be removed by a coronagraph

Unaffected by a coronagraph

Semi~analytical method

- Ic and Is from simulations
- Variance from analytical expressions



Static aberrations

- The static coronagraph response includes the static aberrations

$$\Psi_4(\mathbf{r}) = C_c(\mathbf{r}) + S(\mathbf{r})$$

- Residual pinning amplification by static aberrations through the coronagraph!

- space observations like TPF

Static + Quasi-static

- Quasi-static speckles can also be included
- Decomposition into: perfect, static, quasi static and atmospheric terms:

$$\Psi_1 = A + A_s(x) + a_1(x) + a_2(x)$$

$$\Psi_4 = \tilde{C} + S_1(x) + S_2(x)$$

$$\sigma^2 = N_1 \tau_1^2 \left(I_{s1}^2 + k I_{s2}^2 + 2\tilde{I}_c \left(I_{s1} + k I_{s2} \right) + 2 I_{s1} I_{s2} \right)$$

Conclusions (I)

- The speckle statistics of direct and coronagraphic images is given by a modified Rice distribution
  - Ground based: (static + ao)
  - Space: (static + quasi static)
- Generalization possible for the ground (static + ao + quasi static)
- Model consistent with real data

Conclusions (II)

- Semi-analytical method to study/predict dynamic range
- Speckle calibration/cancellation (slow) necessary to reach the atmospheric variance level
- Performance of the speckle reduction can be derived from the analysis of the limiting noise contribution