Quadarature Phase Interferometery: Squeezing More Out Of Rotation Shearing Interferometry

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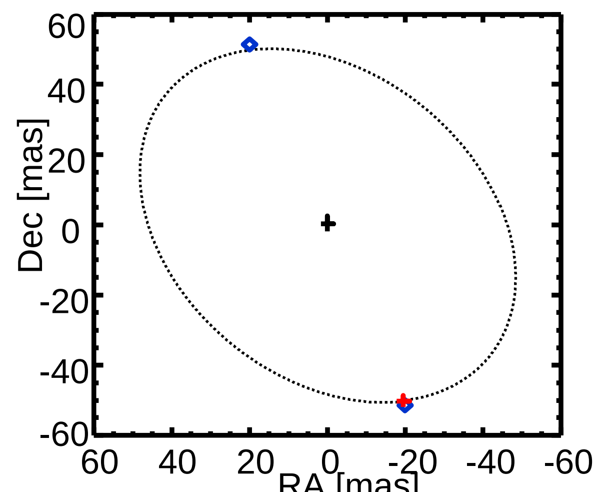
Michelson Fellows Symposium



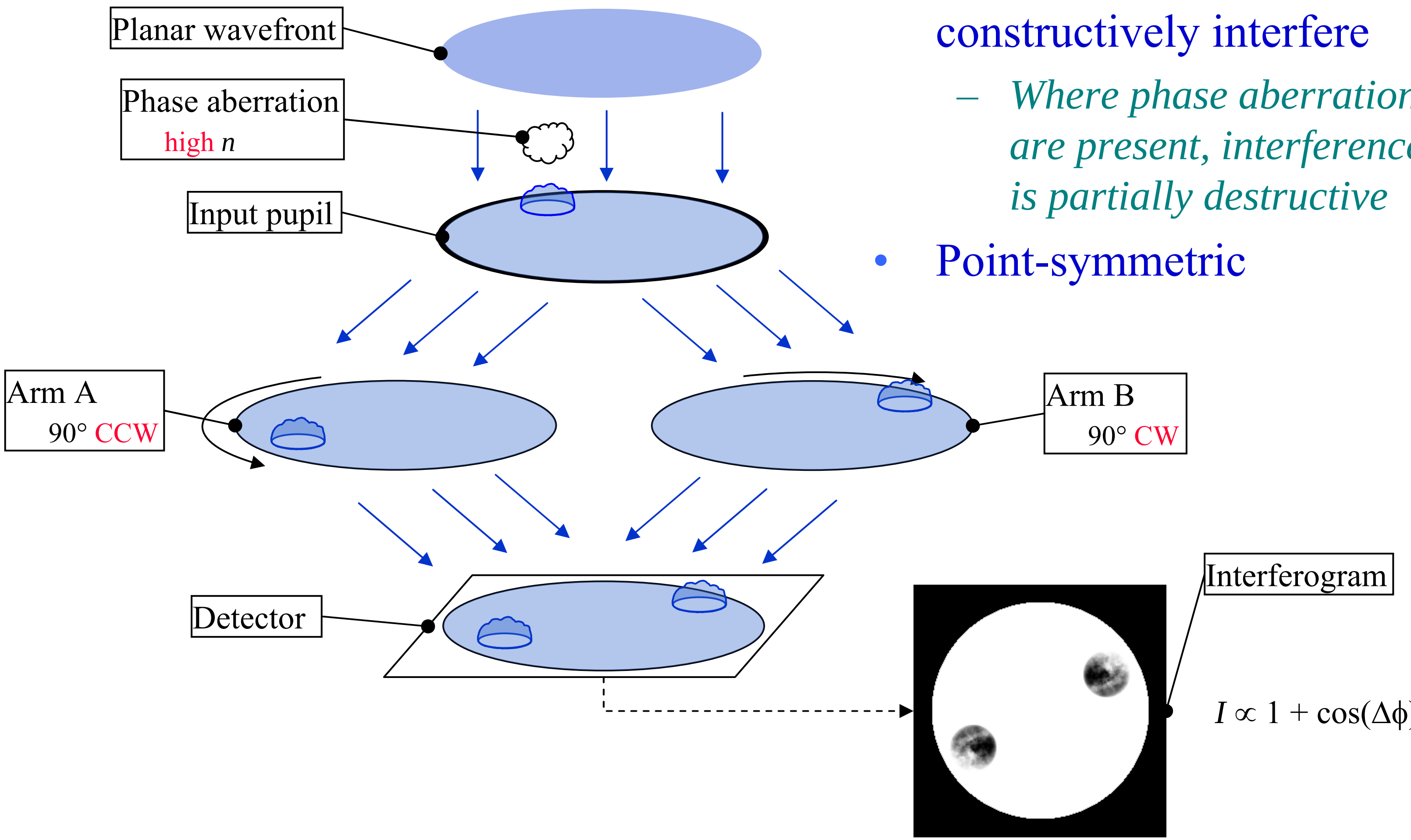
Project team — *Research personnel and collaborators*

- PIs:
  - *Dimotakis, Paul E. — Aeronautics and Applied Physics*
  - *Martin, Christopher — Physics*
- Postdoctoral Fellow
  - *Kern, Brian — Aeronautics*
    - QPI design, assembly and optics integration
    - Overall system integration
    - Lab/Palomar experiments lead
- Research staff
  - *Katzenstein, Garrett — Aeronautics*
    - Mechanical design
  - *Lang, Daniel B. — Aeronautics*
    - Camera head
    - Data-acquisition system
    - Imaging/electronic system integration
- Undergraduate Research Assistant:
  - *Thessin, Rachel — Applied Physics*
    - QPI analysis / alignment / calibration
- Digital-imaging collaborators
  - *Wadsworth, Mark, Collins, S. A., and Elliott, Tom — JPL*
    - CCD design
    - Imager system integration

Outline

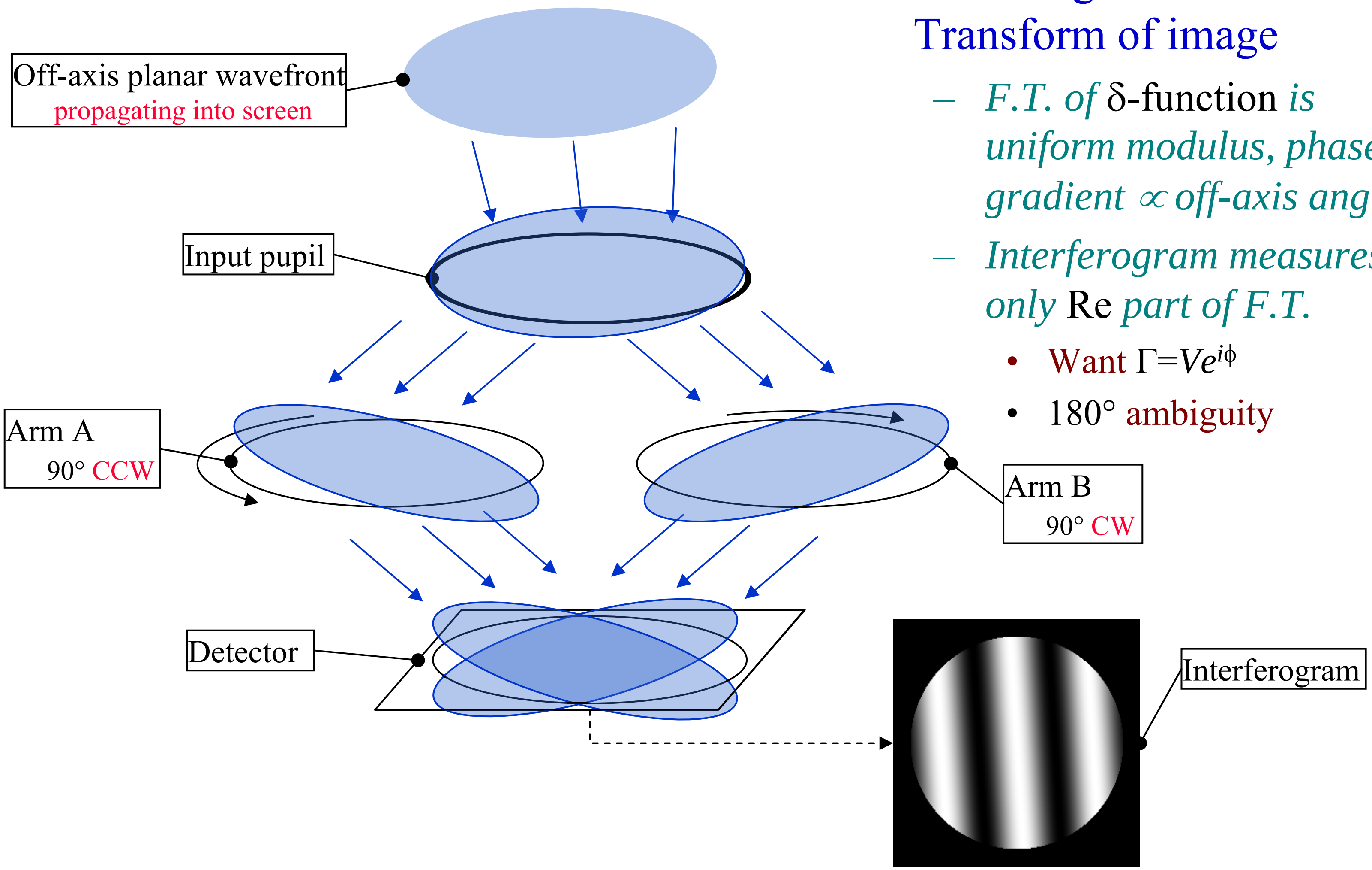
- How we do it
- What we did
- What we'd like to do

Rotation shearing interferometry



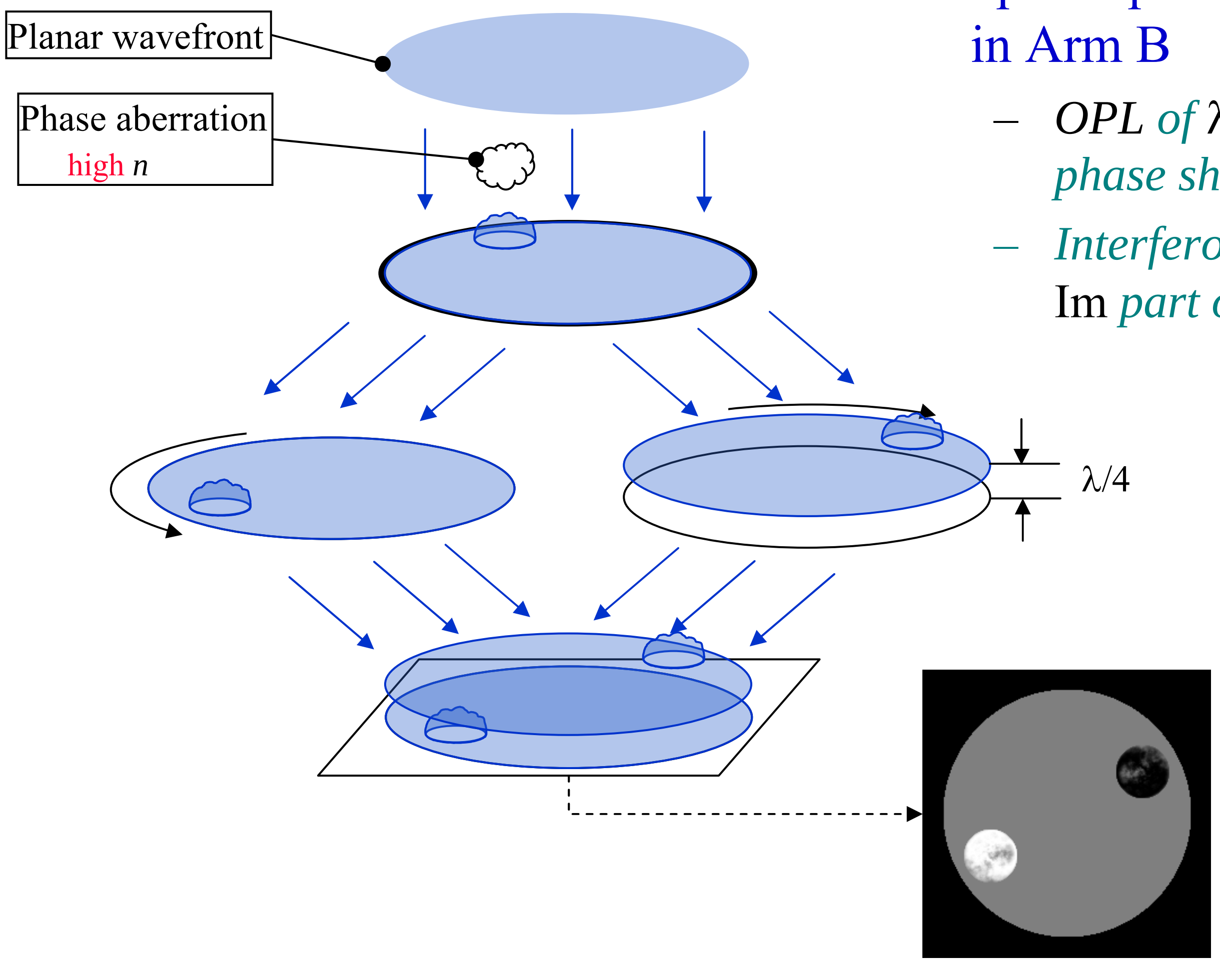
- Interferogram is bright where wavefronts constructively interfere
  - *Where phase aberrations are present, interference is partially destructive*
- Point-symmetric

Interferogram relationship to image



- Interferogram is Fourier Transform of image
  - *F.T. of* δ-function *is uniform modulus, phase gradient* ∝ *off-axis angle*
  - *Interferogram measures only* Re *part of F.T.*
    - Want $\Gamma = Ve^{i\phi}$
    - 180° ambiguity

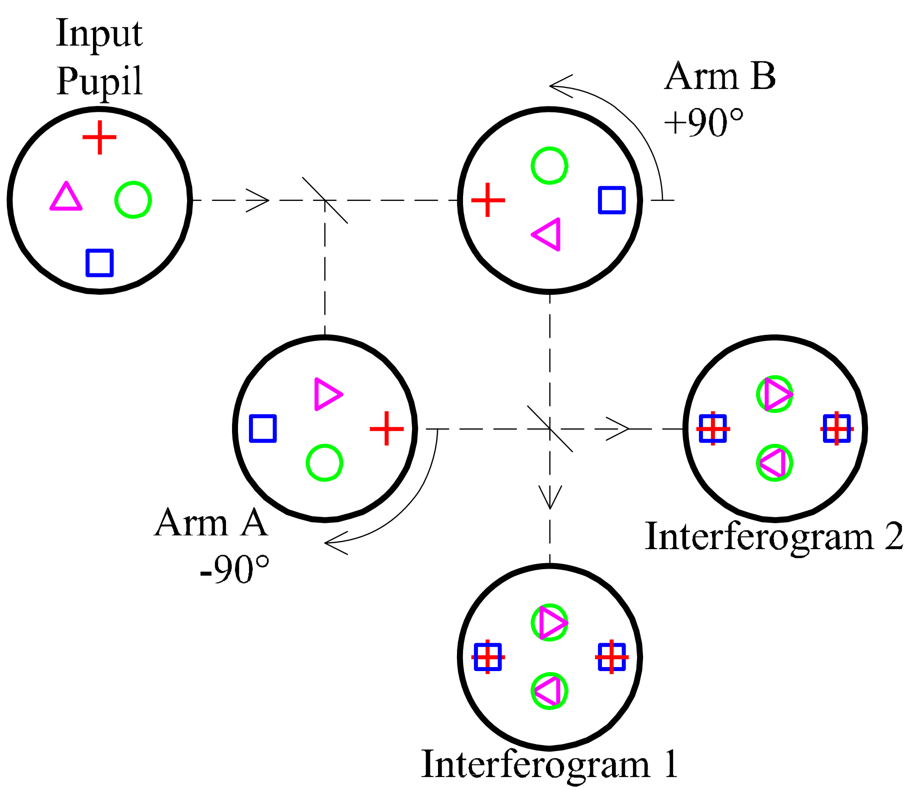
Control of instrumental phase



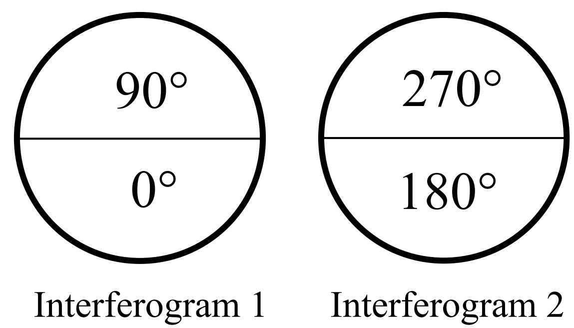
- Introduce additional optical path length (*OPL*) in Arm B
  - *OPL of* λ/4 *gives* 90° *phase shift*
  - *Interferogram measures* Im *part of F.T. of image*

Mach-Zehnder arrangement – two interferograms

- Interferometer makes 2 copies of pupil
  - *Each copy is rotated* 90º
  - *When recombined, relative rotation shear is:* 180º



- Arrange instrumental phase terms to give four quadrants
  - *Each interference pair simultaneously measured in all four quadrants*



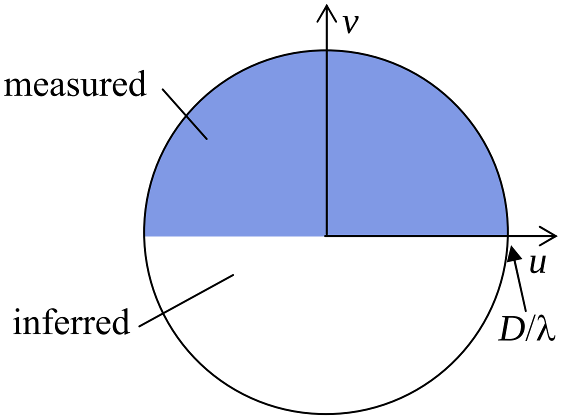
  - *Passively determines intensity, visibility modulus,* V*, and visibility phase,* ϕ_{meas}*, for each pixel,*

$$I / I_0 = 1 + V \cos(\phi_{meas})$$

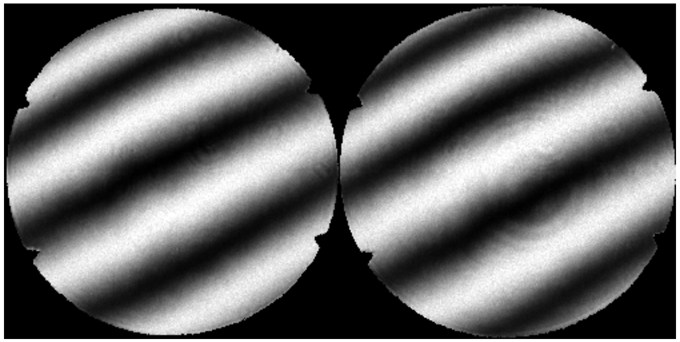
  - *Image is the Fourier Transform of the complex visibility,* $\Gamma = V e^{i\phi}$

Mach-Zehnder arrangement – two interferograms

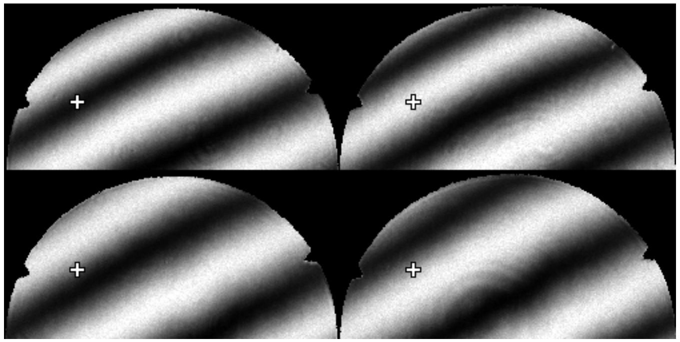
- Sample interferogram
  - *Source is pinhole,* $V=1$ *everywhere*
  - *Phase is linear function of position*
- Four measurements give I, V, ϕ
- Coverage of (u,v)-plane is perfect out to telescope cutoff frequency
  - *Complex* Γ *is Hermitian (object is real)*



raw



registered

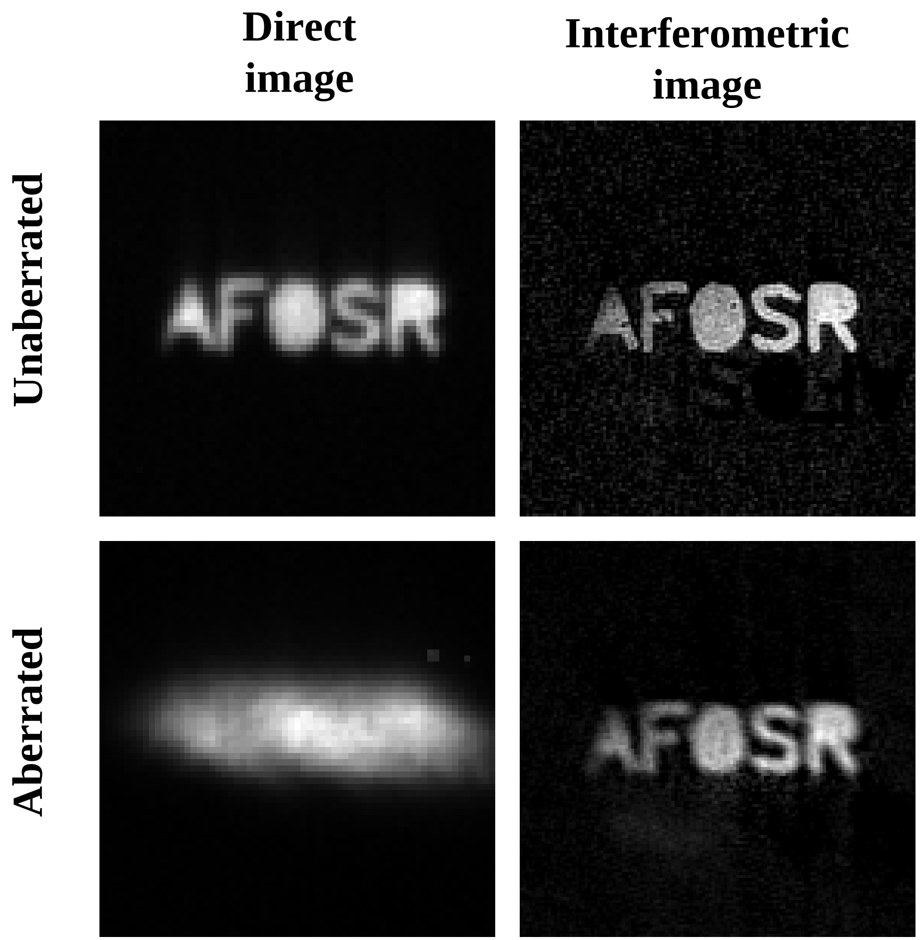


Comparison of QPI to AO

	QPI	AO
Equivalent number of actuators	50,000	1,500?
Does not require reference wavefront	✓	✗
Unaffected by amplitude fluctuations	✓	✗
Unlimited effective actuator stroke	✓	✗
Acts as wavefront sensor	✗	✓
Produces corrected wavefronts	✗	✓
Produces real-time image	?	✓
Accommodate turbulent timescales shorter than correction timescales	✓	✗

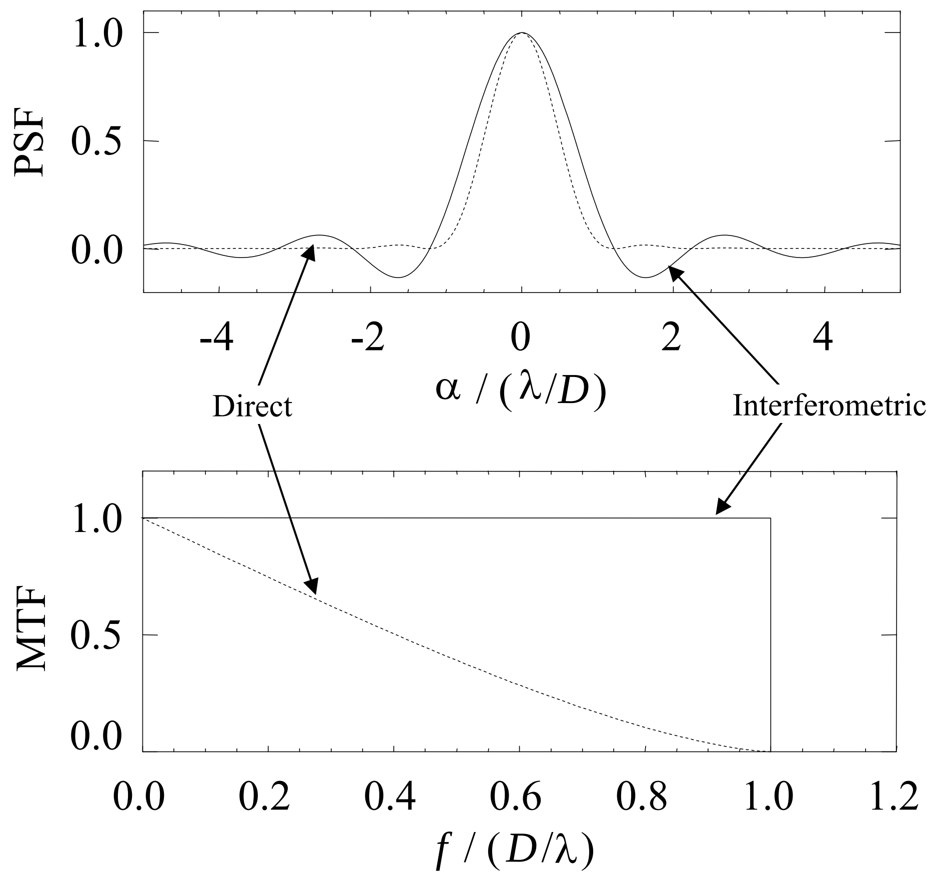
Laboratory experiment

- Direct images acquired simultaneously with interferometric images, through identical non-aberrating/turbulent conditions
  - *Exposures:* 100 μs
  - *Aberrated images (direct and interferometric) are* 12-*frame averages*
  - *Interferometric images are high-pass filtered*
    - Split between quadrants removes small **u**’s
- Interferometric images remain near-diffraction-limited
  - *Diffraction limit is approximately half the width of straight segments in individual letters*



MTF, PSF of unaberrated imaging systems

- Point-Spread Function (PSF) of imaging system (vs. angular measure) is response to point-source object
  - *Direct-imaging PSF is square of Airy function*
  - *QPI PSF is Airy function*
- Modulation Transfer Function (MTF) of imaging system (vs. angular frequency) is modulus of PSF Fourier Transform
  - *Direct-imaging MTF decreases to zero at high angular frequencies*
  - *QPI MTF is uniform vs. angular frequency*

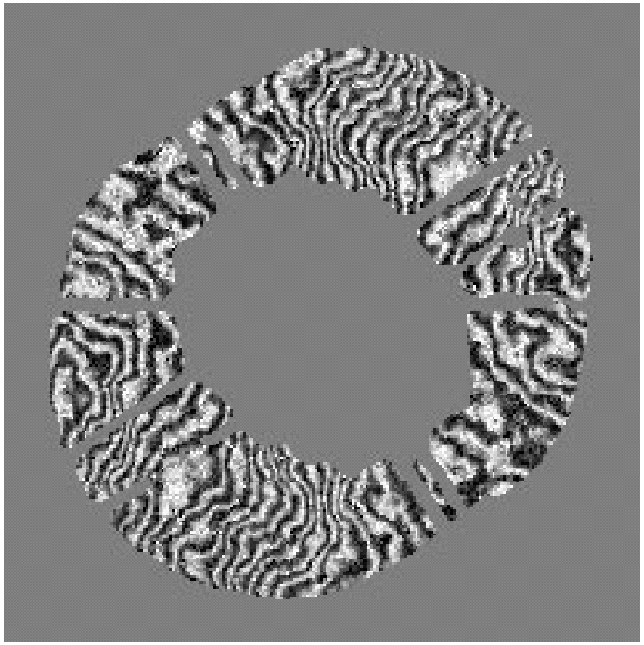


Laboratory experiment — *Discussion*

- Interferometric imaging can yield sharp images in the presence of phase aberrations that severely compromise direct imaging.
  - *Rotation-shearing interferometers are insensitive to phase aberrations that are even about the center of rotation (point-symmetric)*
    - Spherical aberration
    - Defocus
    - Astigmatism
  - *QPI measures amplitude fluctuations separately from phase fluctuations*
    - Amplitude fluctuations are not included in image reconstruction, mitigating image degradation
    - This is much more important when imaging horizontally through near-ground, atmospheric boundary layer turbulence, for example
- QPI technique is most powerful at high angular frequencies
  - *QPI Modulation Transfer Function (MTF) is uniform out to cutoff frequency, while Direct Imaging MTF decreases (nearly) linearly out to cutoff frequency*
    - This difference is much more pronounced in the presence of aberrations
    - QPI MTF is unaffected by turbulent aberrations

Palomar observations — *Vega*

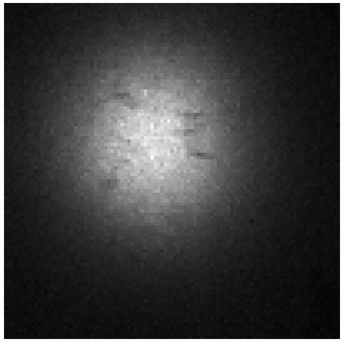
- Vega is bright, point-like star
  - *Vega's size is below diffraction limit, so* $V \sim 1$, *everywhere*



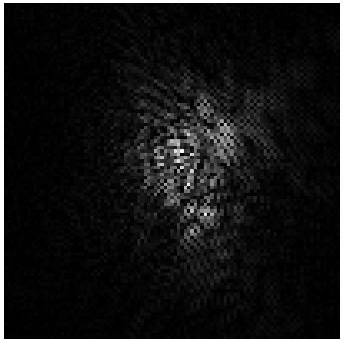
- Structure in interferogram is due to phase aberrations
  - *Scintillation has already been removed by differencing*

Palomar observations — *Vega image reconstruction*

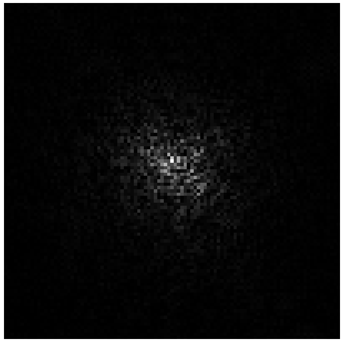
- Direct images show FWHM ~ 1 arcsec
  - *Compare to diffraction limit of* 0.029 arcsec
- Raw interferometric images from 10 s of data (400 frames) show FWHM of 0.5 arcsec
- "Calibrated" interferometric images show FWHM of 0.2 arcsec



Direct image



Interferometric image

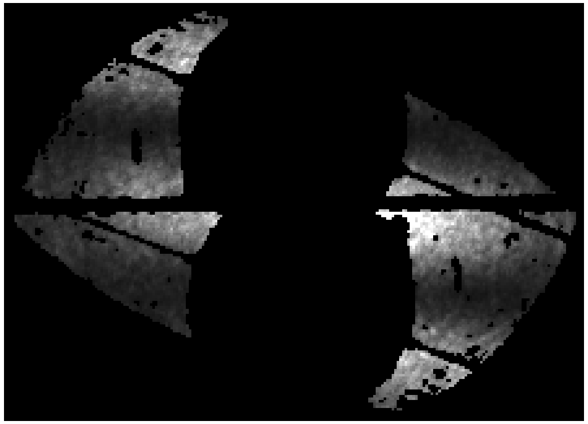


Calibrated interferometric image

- Averaged phase statistics worse than expected

Palomar observations — *Capella*

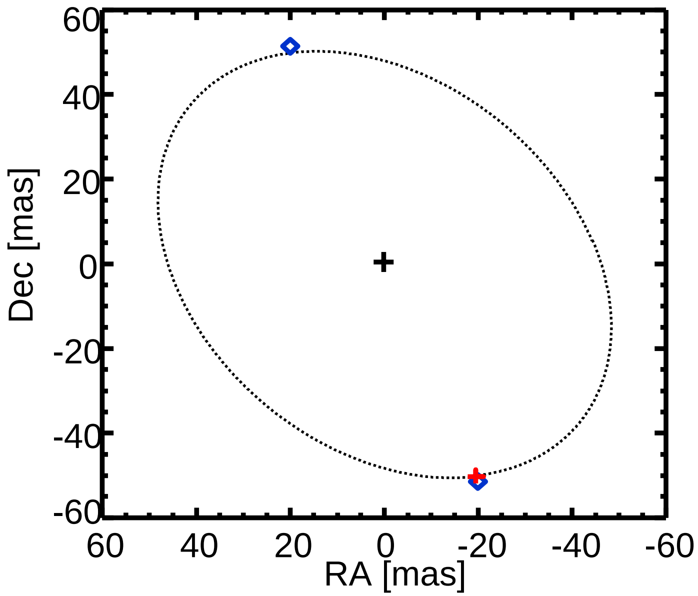
- Capella is bright, binary star system
  - *The two stars have equal brightness*
  - *Separation* 0.050 arcsec
    - Compare to 1 arcsec seeing, 0.029 arcsec diffraction limit
  - 100-day *orbit*
- Measured while QPI had alignment errors
  - *Due to flexure of telescope*
  - *No visibility phase measurements*
    - Modulus-only



Capella visibility modulus *V*

Palomar observations — *Capella analysis*

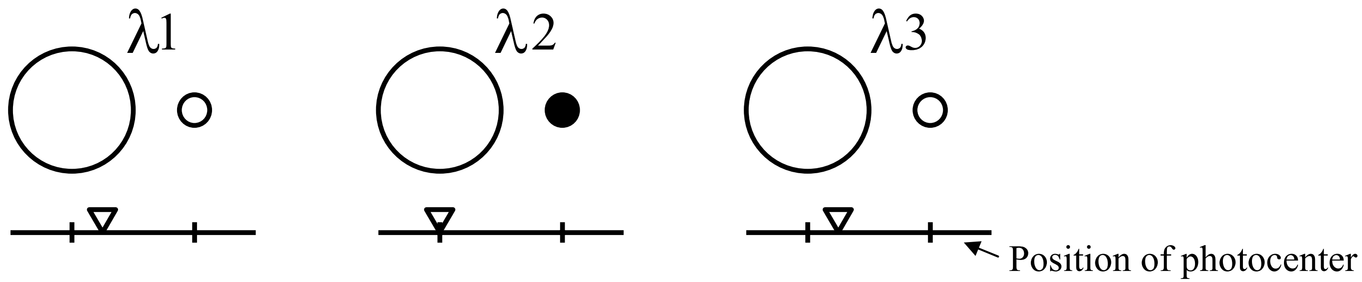
- Capella's orbit previously measured interferometrically
  - *Extrapolate to time of Palomar observations*
- Compare measured binary separation to inferred binary separation

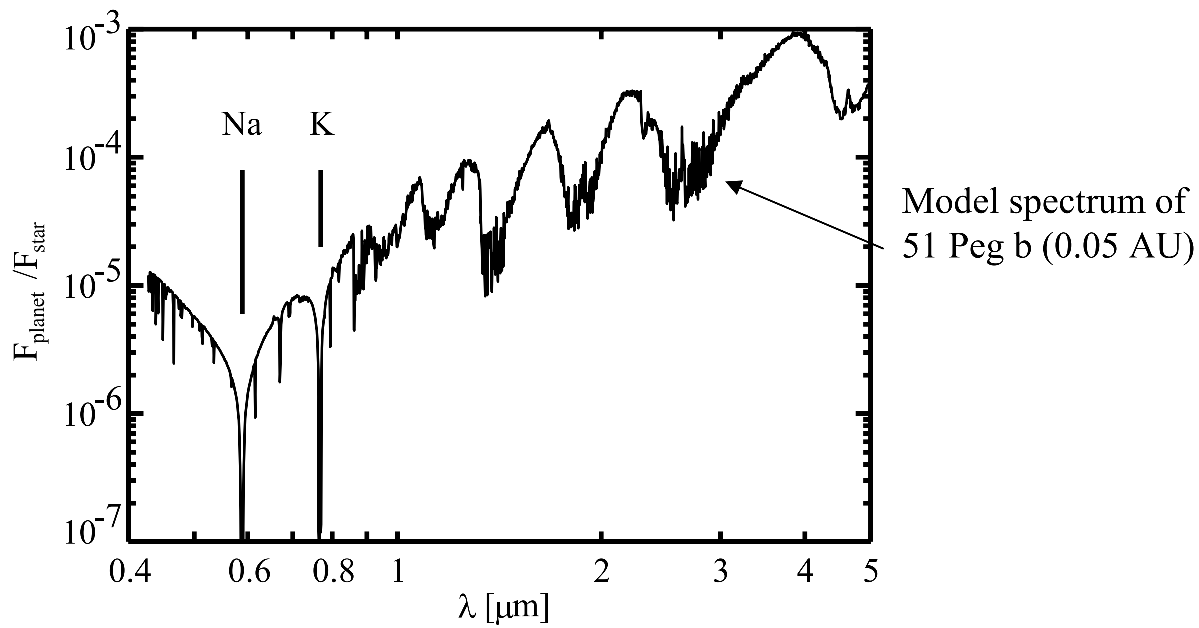


- Relative error 0.001 arcsec
  - *Compare to diffraction limit of* 0.029 arcsec
  - *"Super-resolution" due to fitting of* ~ 10,000 *independent measurements*

Differential phase

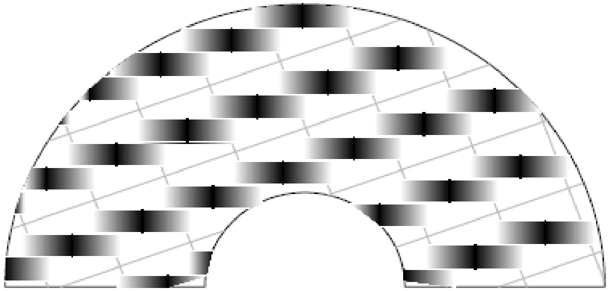
- Measure phase difference between nearby wavelengths

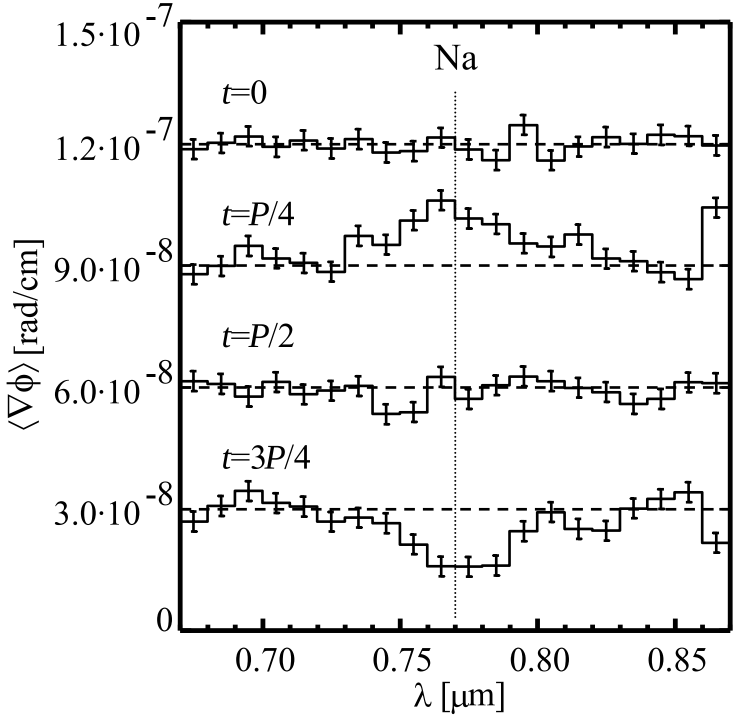




Differential phase sensitivity

- Introduce spectral information
  - *Lenslet array, reflection grating*
- Measure phase at each wavelength
  - 2 hours *at Keck*
  - *Observe* τ Boo, 51 Peg b, υ And

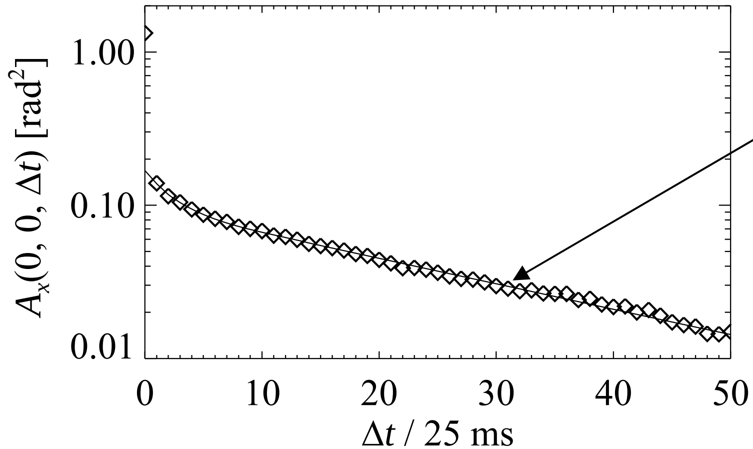




- Get orbital inclination, unambiguous mass
- Measure spectra!

Palomar Observations — *Characterization of turbulence*

- First statistic comes from variance of phase gradients
  - *Variance of* x*-gradient is* $\langle(\nabla_x\phi)^2\rangle \sim 2D_\phi(\Delta x)$, *same for* y*-gradient*
    - Using finite differences, Δx (pixel scale at pupil) is 2 cm
  - *Kolmogorov turbulence gives* $D_\phi(r) = 6.88\,(r/r_0)^{5/3}$ rad^2
  - *Compare with estimate from seeing,* $r_0 \sim \lambda/\omega$
    - Seeing of $\omega = 1$ arcsec, $\lambda = 0.7$ μm gives $r_0 \sim 14$ cm
- Variance of phase gradients ~ 1.3 rad^2, corresponds to $r_0 \sim 8$ cm
  - *Underestimate by* ×2
- Measurement noise (read & shot/Poisson noise) must be removed
  - *Examine temporal variation of spatio-temporal autocorrelation of measured phase (~atmospheric-turbulence) gradients*

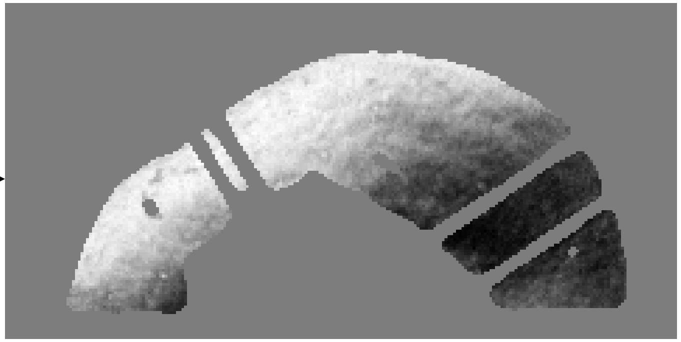


Solid line is two-component exponential fit for $\Delta t > 0$.

Palomar Observations — *Determination of phase offset*

- Instrumental phase terms in "quadrants" are not exactly 90° apart
  - *Realizing QPI potential when imaging through* 5m *telescope and full-spectrum turbulent medium requires* ~ 1° *phase accuracy, i.e.,* ~10 nm *surface specification*
- Determine instrumental phase offset from covariance of intensities
  - *Covariance gives* $2\,\langle \cos(\phi_{turb})\,\cos(\phi_{turb}+\Delta\phi)\,\rangle \;=\; \cos(\Delta\phi)$
    - Assumes ϕ_{turb} is has uniform statistical distribution
    - Pixel-by-pixel self-calibration

Pixel-by-pixel normalized covariance based on 400-frame sample.



  - *Sign uncertainty in* $\Delta\phi$ *(cosine is an even function)*
    - Assume all $\Delta\phi$ positive
  - *Can eliminate sign ambiguity by fitting smooth function to entire map*
- Covariance does not calibrate all of instrumental phase term
  - *Phase offset is difference between upper and lower instrumental terms*
  - *Upper instrumental phase term requires separate calibration for full phase retrieval*

Visibility phase statistics — *Averaging of turbulent phase terms*

- Statistics of turbulent phase terms specified by structure function
  - *Assumes turbulence is isotropic and homogeneous*
  - *Kolmogorov assumption gives* $D_\phi(r) = 6.88\,(r/r_0)^{5/3}\ \mathrm{rad}^2$
  - *Average turbulent phase term is zero*
- rms turbulent phase terms are $> 2\pi$ for $r \sim 30$ cm
  - *Compare to* $r = 500$ cm *at edge of pupil*
- Measured phase terms are wrapped (modulo 2π)
- Use directional statistics
  - *Mean direction* $$\theta^{av} = \mathrm{atan}\left[\frac{\sum_i \sin(\theta_i)}{\sum_i \cos(\theta_i)}\right]$$
  - *Variance of sample mean,* $\langle(\theta^{av})^2\rangle = (e^{\sigma^2} - e^{-\sigma^2})\,/\,2n$
    - If averaging turbulent phase terms, σ^2 comes from D_ϕ
    - Can't simply average measured phase
      - *Baseline of* $r = 1$ m, *with* $r_0 = 10$ cm, *requires* $> e^{300}$ *exposures for* 1 rad *rms error*

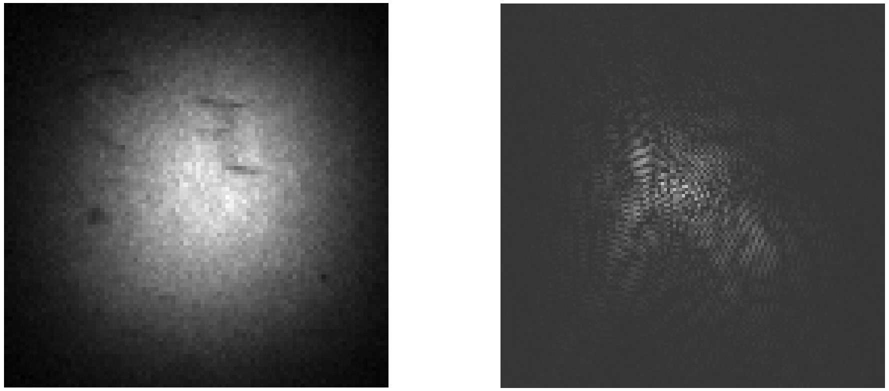
Visibility phase — *Averaging of phase gradients*

- Variance of sample mean $\langle(\theta^{av})^2\rangle = (e^{\sigma^2} - e^{-\sigma^2}) / 2n$
- Instead of averaging phase terms, average phase gradients
  - *Define* $\nabla_x\phi = \phi(x+\Delta x,y) - \phi(x,y)$, $\Delta x = 2$ cm
  - *Variance of turbulent phase gradient* $\sigma^2 = 2D_\phi(\Delta x) \sim 1$ rad^2 (*for* $r_0 = 10$ cm)
  - *Turbulent phase-gradient terms average out quickly*
  - *For small* σ^2, *variance of sample mean looks like* σ^2/n
- Create $\nabla^2\phi$ from average phase gradient components $(\nabla_x\phi)^{av}$ and $(\nabla_y\phi)^{av}$
  - *Solve Poisson's Equation,* $\nabla^2\phi = f(x,y)$, *to find average phase,* ϕ^{av}
  - *Poisson's Equation solution is automatically unwrapped*
- Solution from $\nabla^2\phi$ reduces turbulent phase term rms by $1/n^{1/2}$
  - *Separation of timescales*
  - *Time-varying phase terms reduced to their mean by this process*
  - *Makes use of correlation of nearby points to reduce variance*

Palomar Observations — *Image reconstruction*

- Form $\Gamma(u,v) = V(u,v) \exp\{i\, \phi(u,v)\}$, take Fourier Transform
  - *Measure* V *and* ϕ *only in upper quadrant,* $v > 0$
    - $F(\alpha,\beta)$ is real, so $\Gamma(u,v)$ is Hermitian
    - $V(-u,-v) = V(u,v)$, $\phi(-u,-v) = -\phi(u,v)$

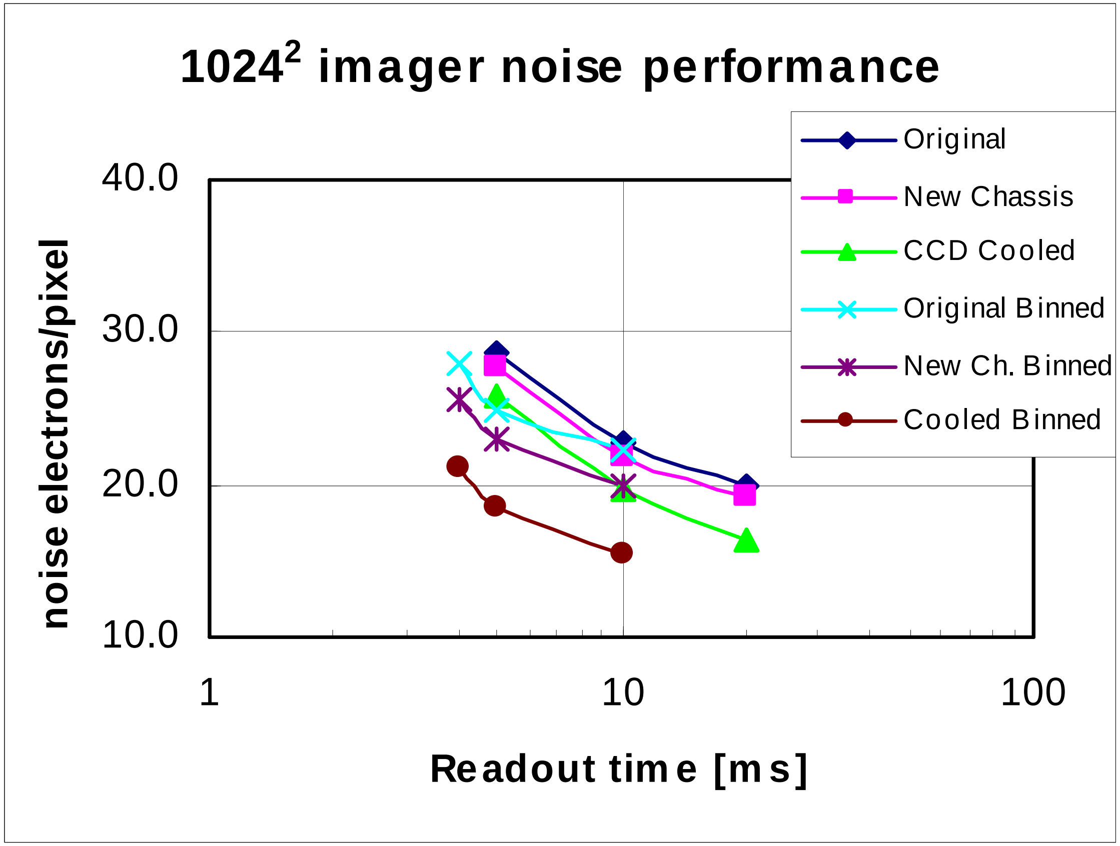
Direct image | Interferometric image



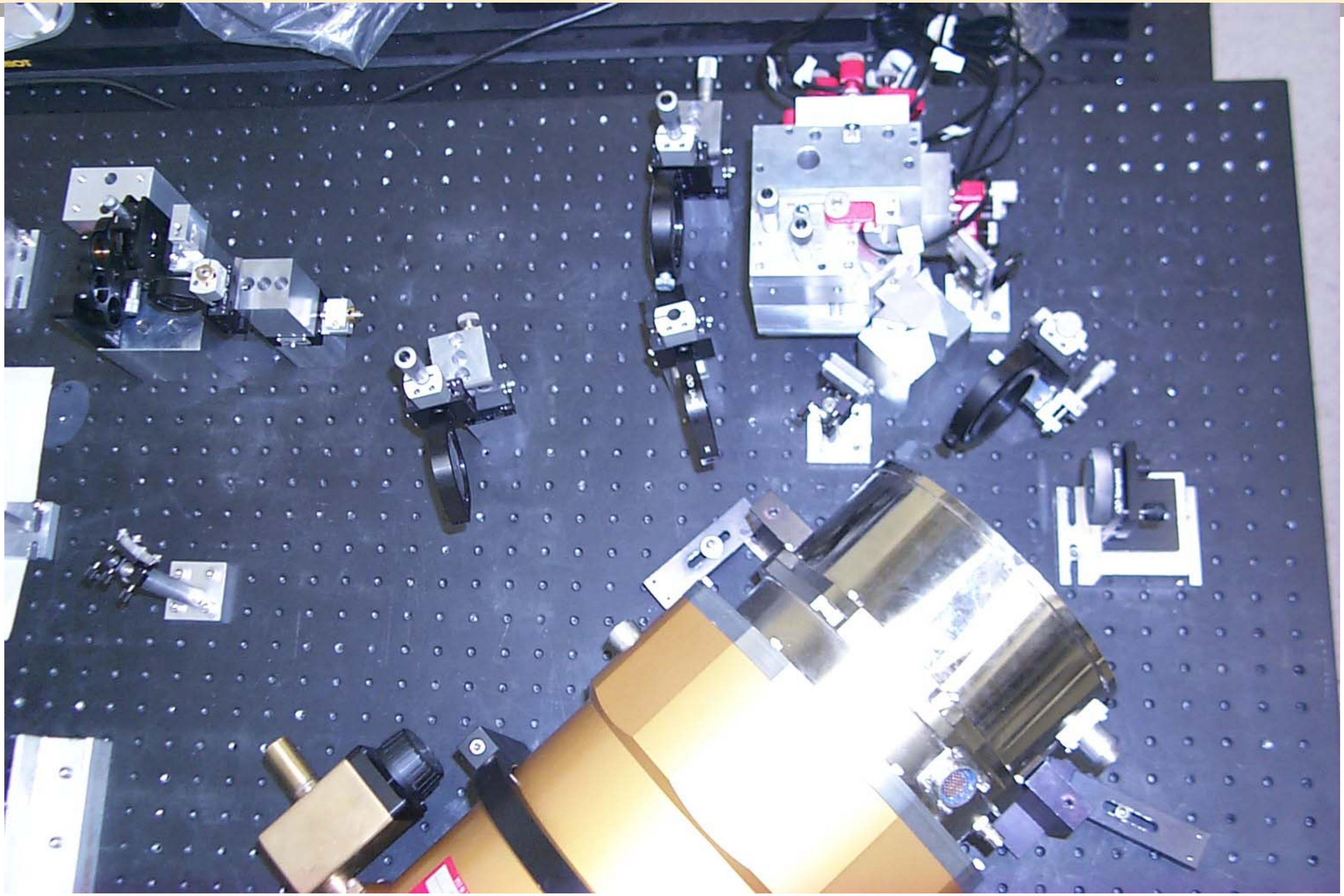
- Both images are averages of 400 exposures (10 s)
- FWHM of direct image 1.0 arcsec, interferometric image 0.3 arcsec
- Average phase ϕ^{av} still contains instrumental phase term

KFS CCD measured noise performance

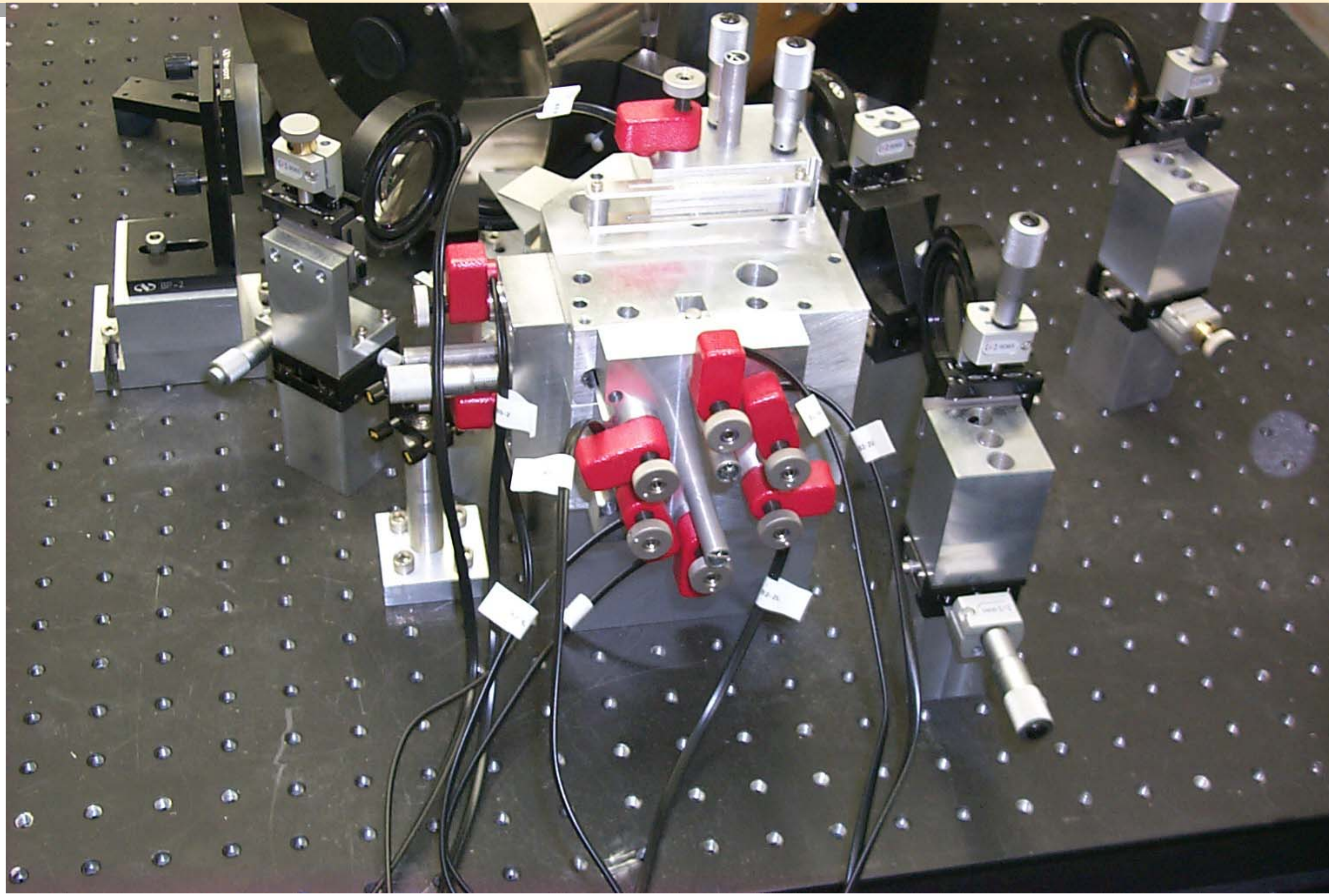
- Noise performance optimized for each read time
- Options:
  - *Read time*
  - *Binning*: 2×2
  - *Cooling:* -10°C
- Shortest read time is 3 ms (333 fps) unbinned
- Lowest noise performance (that we are aware of) at such frame rates
  - *These figures represent system noise*
    - CCD + amplifiers + A/D + DAS



Interferometer layout

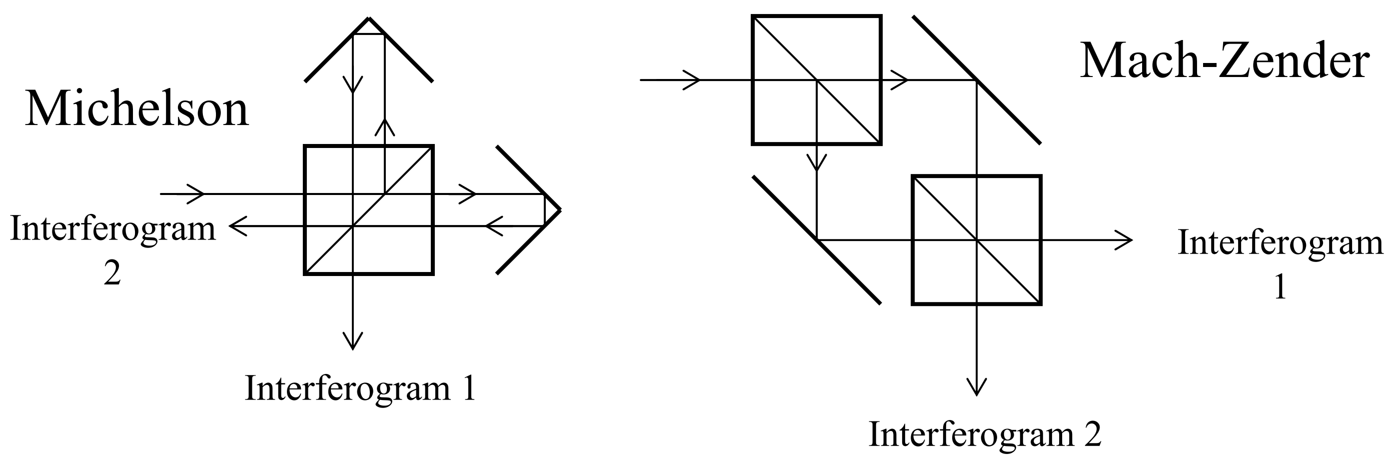


Interferometer closeup



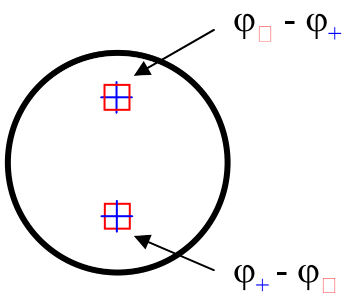
Optical system — *Present implementation**

- Quadrature-Phase Interferometer
  - *Rotation-shearing pupil-plane interferometer*
  - *Rotation shear by* 180º
  - *Measures full complex-visibility, over entire pupil, in a single exposure*
- Two important developments
  - *Mach-Zender interferometer*
    - Two interferograms available
    - Higher efficiency (energy conservation)
  - *Phase shifts in each exposure*: 0º, 90º, 180º, 270º



* Interferometer instrumentation development previously cosponsored by NSF Grant AST9618880.

Quadrature phase in interferogram — *I*

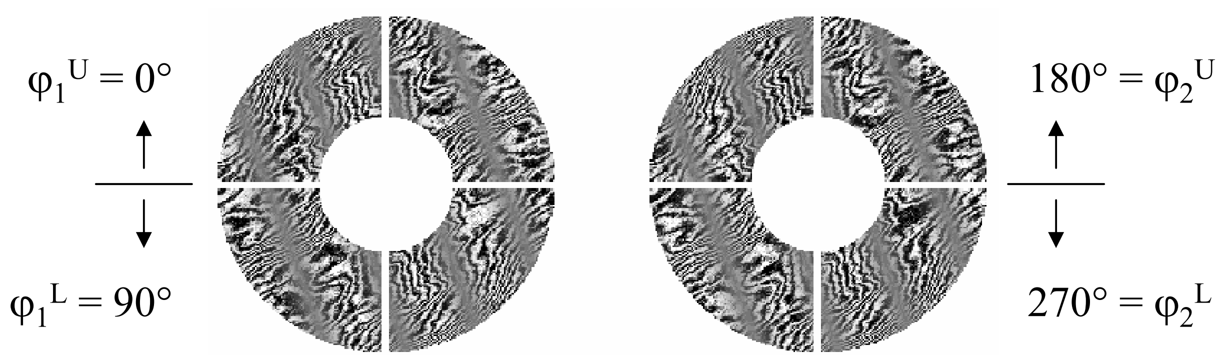


Interferogram

- Interferogram intensity depends on

  $$\cos(\varphi_{\square} - \varphi_{+}) = \cos(\varphi_{+} - \varphi_{\square})$$

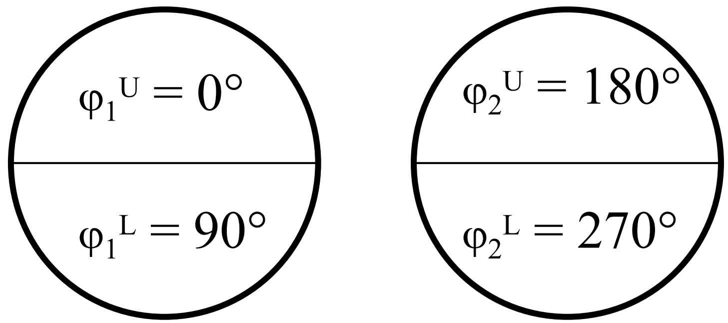
  Each interferogram has redundant information.
- Splitting Mirror A3 into two halves introduces a path-length difference, set to 90º-phase shift in half of each interferogram.
- The two interferograms, each with two halves, allow concurrent measurement of $V(\mathbf{u})$ and $\varphi(\mathbf{u})$



Simulated 10 ms exposure frame of double-star Capella with Palomar 5 m telescope, coherence length, r_0 = 10 cm (amplitude fluctuations removed).

Quadrature phase in interferogram — *II*

- Split interferograms into two halves
  - *Set instrumental path-length differences to multiples of 90°*



Interferogram 1 Interferogram 2

$$I_1^U(\mathbf{x}) / \langle I \rangle = 1 + \mathrm{Re}\{\gamma(\mathbf{u}) e^{i0}\} = 1 + \mathrm{Re}\{\gamma(\mathbf{u})\}$$

$$I_1^L(\mathbf{x}) / \langle I \rangle = 1 + \mathrm{Re}\{\gamma(\mathbf{u}) e^{i\pi/2}\} = 1 - \mathrm{Im}\{\gamma(\mathbf{u})\}$$

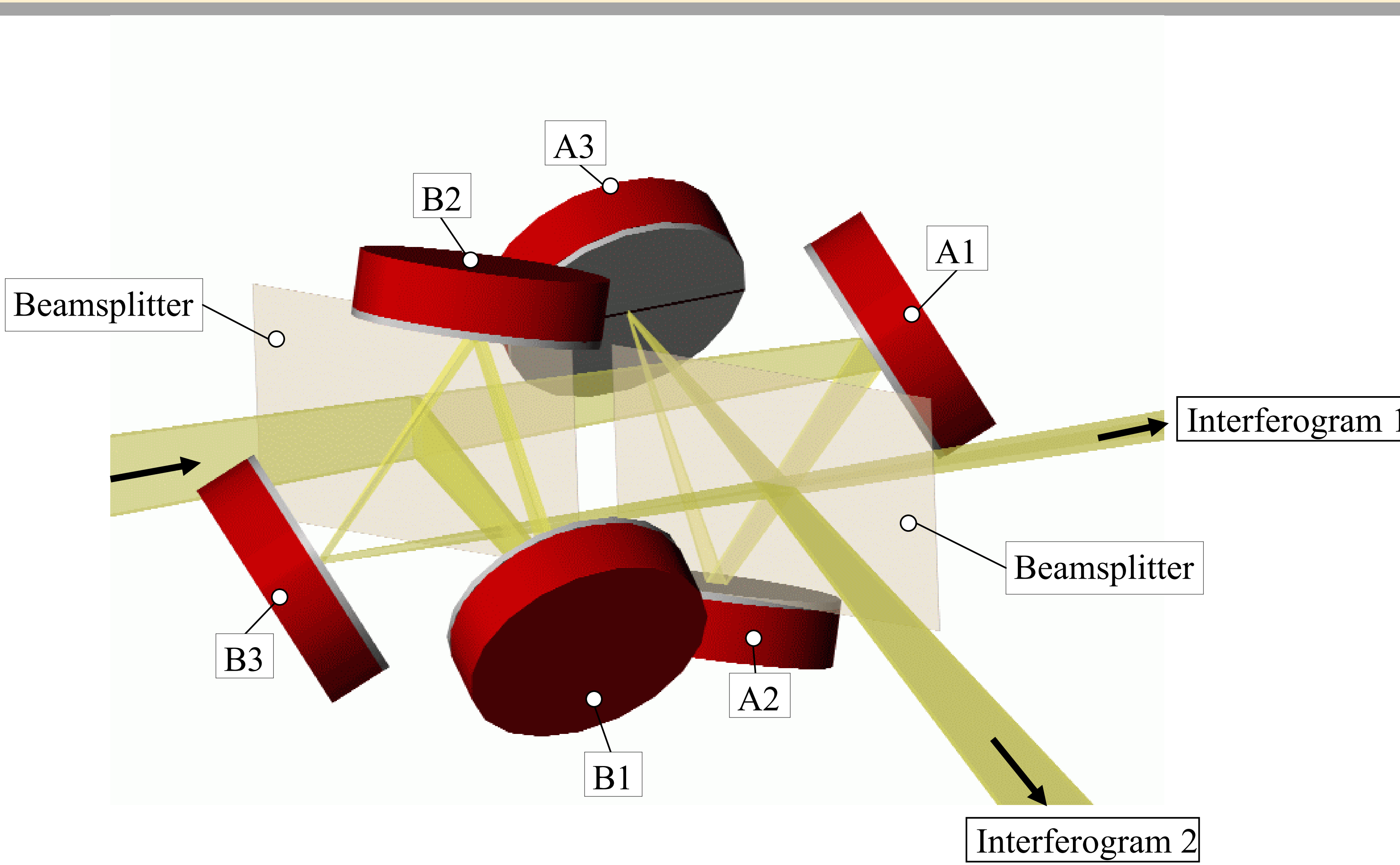
$$I_2^U(\mathbf{x}) / \langle I \rangle = 1 + \mathrm{Re}\{\gamma(\mathbf{u}) e^{i\pi}\} = 1 - \mathrm{Re}\{\gamma(\mathbf{u})\}$$

$$I_2^L(\mathbf{x}) / \langle I \rangle = 1 + \mathrm{Re}\{\gamma(\mathbf{u}) e^{i3\pi/2}\} = 1 + \mathrm{Im}\{\gamma(\mathbf{u})\}$$

$$I_1^U(\mathbf{x}) - I_2^U(\mathbf{x}) \Rightarrow \gamma_R(\mathbf{u})$$

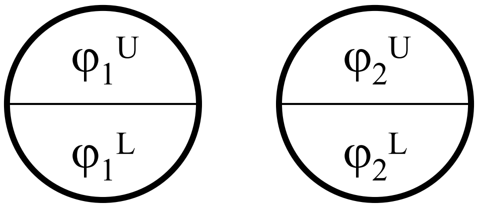
$$I_2^L(\mathbf{x}) - I_1^L(\mathbf{x}) \Rightarrow \gamma_I(\mathbf{u})$$

Interferometer geometry — *Instrument implementation*

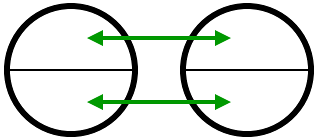


Alignment requirements — *Instrumental quadrant phases I*

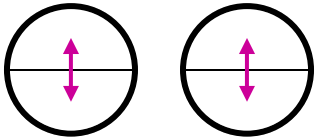
- Instrumental visibility phases (path-length differences) should be
  $\varphi_1^U = 0^o$, $\varphi_1^L = 90^o$, $\varphi_2^U = 180^o$, $\varphi_2^L = 270^o$.



  - *Guaranteed to have* $\varphi_2^U - \varphi_1^U = \varphi_2^L - \varphi_1^L = 180^o$ *(conservation of energy)*



  - *Quadrant offset,* $\Delta\varphi = \varphi_1^U - \varphi_1^L = \varphi_2^U - \varphi_2^L$, *should be* 90^o.



Alignment requirements — *Instrumental quadrant phases II*

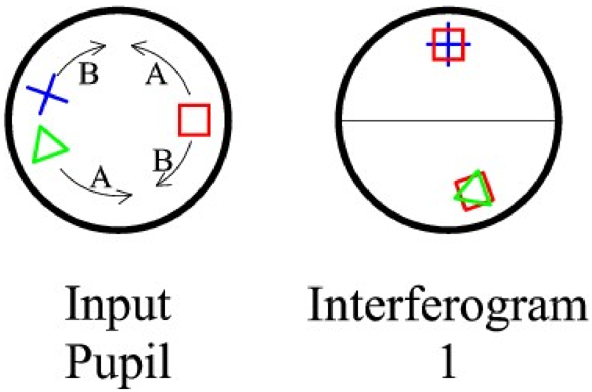
- Instrumental visibility phases (path-length differences) should be:
  $\varphi_1^U = 0°$, $\varphi_1^L = 90°$, $\varphi_2^U = 180°$, $\varphi_2^L = 270°$.
  - *Guaranteed to have* $\varphi_2^U - \varphi_1^U = \varphi_2^L - \varphi_1^L = 180°$ *(conservation of energy)*
  - *Phases* φ_1^U *and* φ_1^L *are independently controlled with* < 30 nm *precision*
  - *Can determine quadrant offset,* $\Delta\varphi = \varphi_1^U - \varphi_1^L = \varphi_2^U - \varphi_2^L$*, from the same exposures used for observations*
    - Deviations from $\Delta\varphi = 90°$ do not cause errors, as long as $\Delta\varphi$ is measured well
    - Need random phase aberrations for in-line calibration
    - Intensities in interferograms measure $\cos(\varphi_1^U + \varphi_{aber})$, $\cos(\varphi_1^L + \varphi_{aber})$, …
    - Correlation of upper- and lower-quadrant intensities gives $\Delta\varphi$, since,

$$\left\langle \cos(\varphi_1^U + \varphi_{aber,t})\cos(\varphi_1^L + \varphi_{aber,t}) \right\rangle_t = \cos(\varphi_1^U - \varphi_1^L)$$

    - No need for independent calibration of quadrant offset
  - *The free variable* φ^U_1 *can be calibrated offline*
    - Errors in φ_1^U contribute no errors in the visibility modulus, $V(\mathbf{u})$
    - Errors in φ_1^U contribute only overall phase shifts to the complex visibility, $\gamma(\mathbf{u})$.

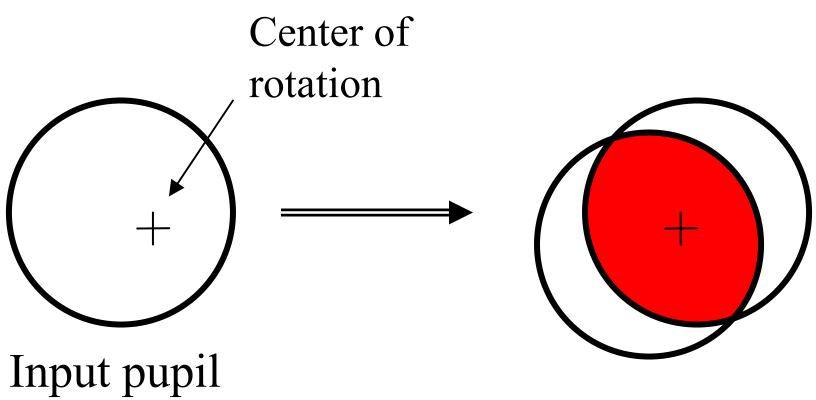
Alignment requirements — *Rotation shear*

- Rotation shear should be 180º
  - *Shear is a measure of relative geometric rotation about the interferometer's center of rotation (not phase)*
  - *Arm A rotates* +90º
  - *Arm B rotates* -90º
  - *Rotation shear is the net rotation,* 180º
- Effect of errors in rotation shear depend on spatial structure of measured phase (including object visibility phase and turbulent phase)
  - *Example: Arm A rotates* +90º*,*
    Arm B rotates -70º
  - *Upper and lower interferograms do not interfere same pairs of points in the input pupil*
  - *If phase changes on scales short enough to differ between noncommon points, interferometer will not measure quadrature phase*



Alignment requirements — *Pupil image positions*

- Ideally, center of rotation should coincide with center of pupil image
- Location of pupil with respect to center of rotation affects the measurements only through the angular frequency coverage in a single exposure

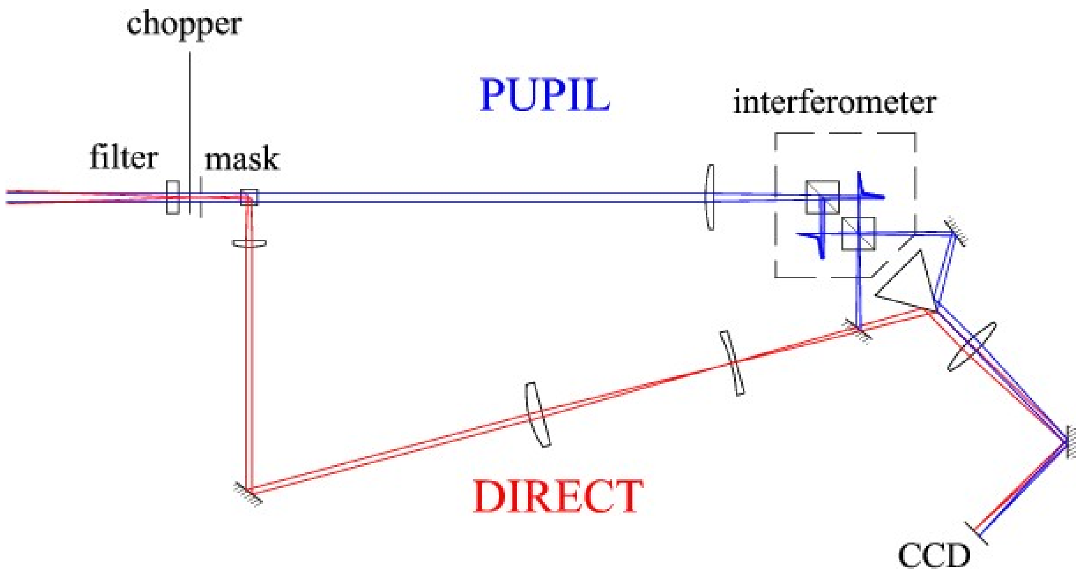


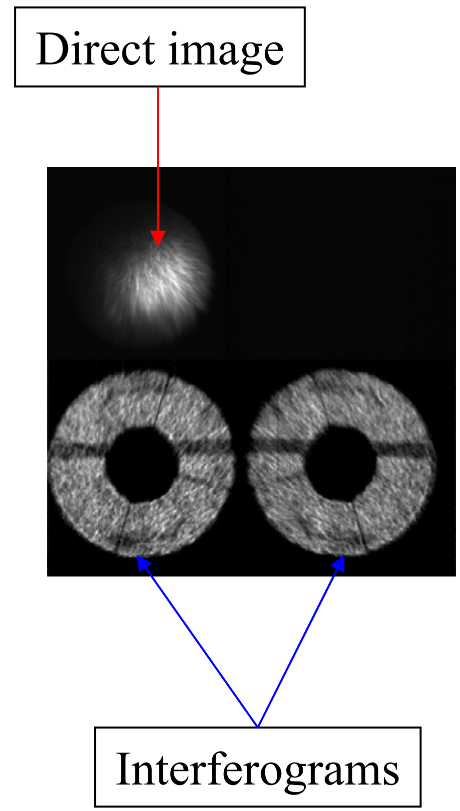
  - *Highest frequencies have no interferometric coverage*

Alignment requirements — *Spectral coherence*

- Spectral coherence requirements are set by magnitude of maximum turbulent-phase terms
  - *Fractional bandwidth:* $\delta\lambda/\lambda \le 2\pi/\varphi_{max}$
  - *At Palomar (normal observing conditions), this is about* $\delta\lambda/\lambda \le 1/10$
  - *QPI set at* $\lambda \cong 700$ nm, $\delta\lambda \cong 20$ nm, $\Rightarrow \delta\lambda/\lambda \cong 1/35$
- Phase differences that exceed the limit imposed by the fractional bandwidth lead to decreases in visibility modulus
  - *Observations superpose fringes with different spacing (fringe spacing* $\propto 1/\lambda$ *),*
  - *Fringes at different wavelengths are out of phase with each other at large path length differences (large turbulent-phase terms)*

Optics system schematic

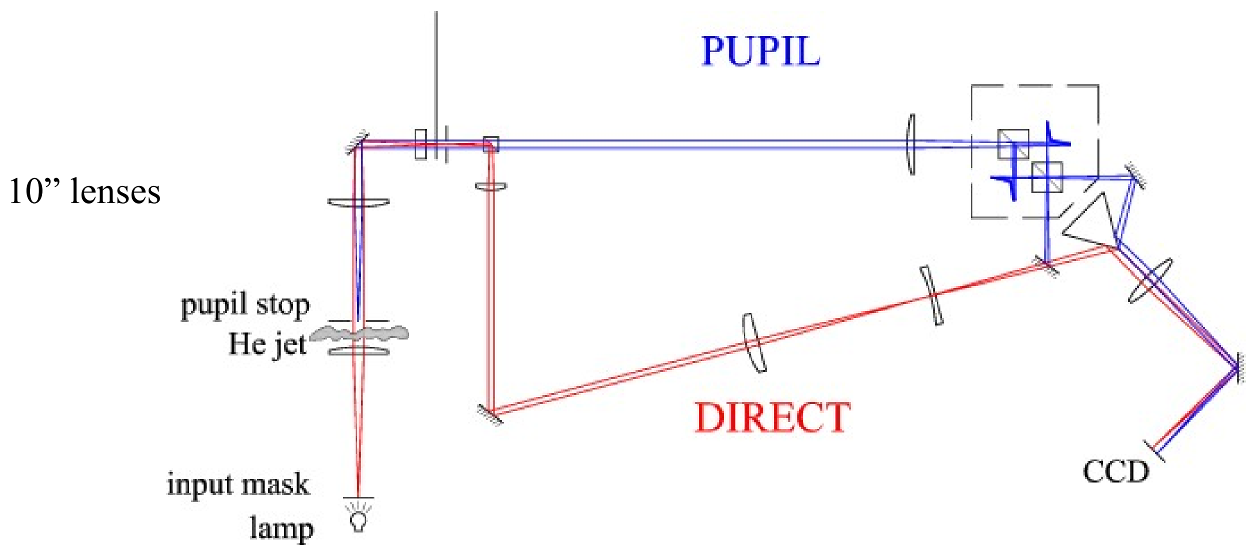




- Optical beam is
  - *filtered:* *band-passed to define* λ *and* δλ
  - *chopped:* *synchronized with KFS system*
- then split into a
  - *Pupil image*
    - Interferometer produces two interferograms
  - *Direct image*
    - Arranged with interferograms on single CCD array

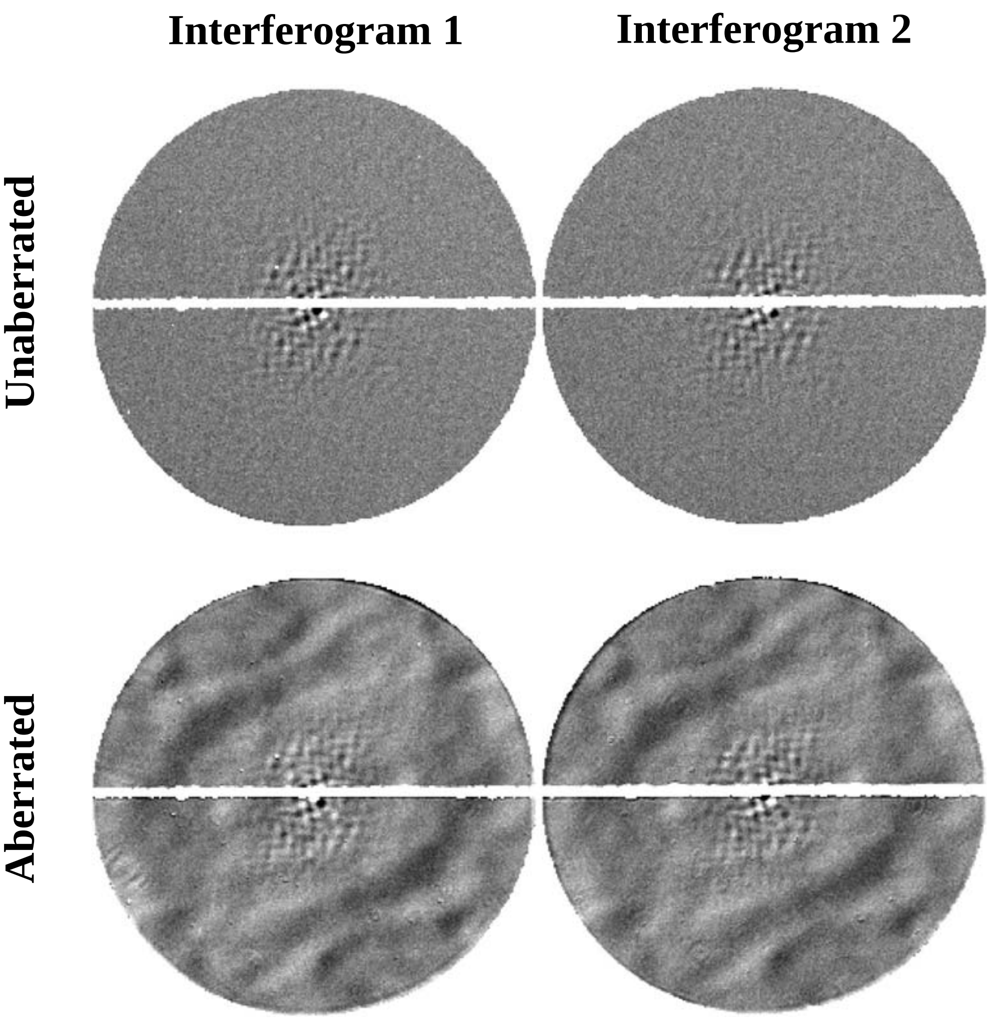
Laboratory experiment — *Optical layout*

- Input mask is illuminated by Hg lamp
  - *Input mask demagnified to have* 30 μm-*wide features*
    - Original machined with 0.005″ end-mill
  - *Demagnified overall dimensions*: 1000×200 μm^2
  - *Optics emulate input beam configuration from telescope*
  - *F/30 beam collimated by* 200 mm *lens; angular features*: 150 μrad
- He-air jet used to introduce turbulent aberrations



Laboratory experiment — *Interferograms*

- Each pair of interferograms is divided into four quadrants
  - *Top and bottom halves differ in phase by 90°*
  - *Interferograms 1 and 2 differ in phase by 180°*
- Interferograms with turbulence show amplitude fluctuations
  - *Amplitude fluctuations affect the two interferograms in the same sense*
  - *Phase fluctuations affect the two interferograms in the opposite sense*

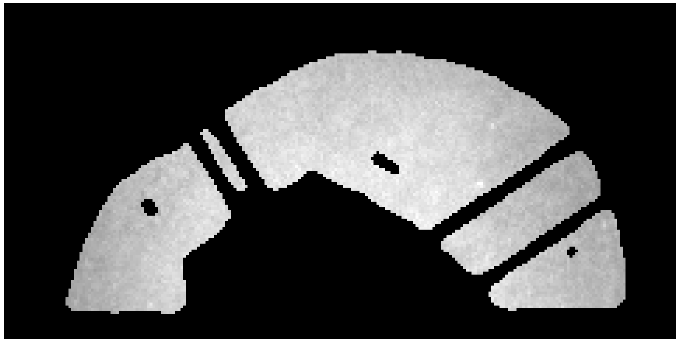


Laboratory experiment — *Discussion*

- In the presence of phase aberrations, which severely compromise direct imaging, interferometric imaging can yield sharp images
  - *Rotation-shearing interferometers are insensitive to phase aberrations that are even about center of rotation*
    - Spherical aberration
    - Defocus
    - Astigmatism
  - *QPI measures amplitude fluctuations separately from phase fluctuations*
    - Amplitude fluctuations are not included in image reconstruction, mitigating image degradation
    - This is much more important when imaging horizontally through near-ground turbulence, for example
- QPI technique is most powerful at highest angular frequencies
  - *QPI Modulation Transfer Function (MTF) is uniform out to cutoff frequency, while Direct Imaging MTF decreases (nearly) linearly out to cutoff frequency*
    - This difference is much more pronounced in the presence of aberrations
    - QPI MTF is unaffected by turbulent aberrations

Palomar Observations — *Visibility modulus*

- Visibility modulus is determined entirely by measured intensities and phase offset



- Visibility expected to be uniform across interferogram
  - *Average measured visibility* 0.61
  - *Visibility degraded by:*
    - Spectral bandwidth
    - Exposure time
    - Wavefront resolution
    - Amplitude fluctuations (second-order effect)

20-22 July 2002 Palomar observations — *Summary*

- Full quadrature-phase measurements made of Vega
  - *Vega is unresolved, so* $V = 1$ *everywhere (image is a point)*
- Measurements taken on Capella, with rotation shear error
  - *Capella is a binary star system, with separation* 50 milliarcsec ≈ ×2 *diffraction limit of Palomar* 5 m *at* 700 nm
  - *Effect of rotation shear error to be determined*
    - Error (≈ 1.5°) comparable to expected tolerance for error at medium to high frequencies
- Measurements taken on Mira, with rotation shear error
  - *Mira is a giant star, with diameter* 50 milliarcsec ≈ ×2 *diffraction limit of Palomar* 5 m *telescope*
  - *Effect of rotation shear error is the same as that for Capella*
- Successful integration of KFS camera, QPI, and Palomar telescope
  - *Pupil position not well controlled, not always centered*
  - *Rotation shear error not corrected until third (final) night*
    - New technique developed to correct rotation shear error