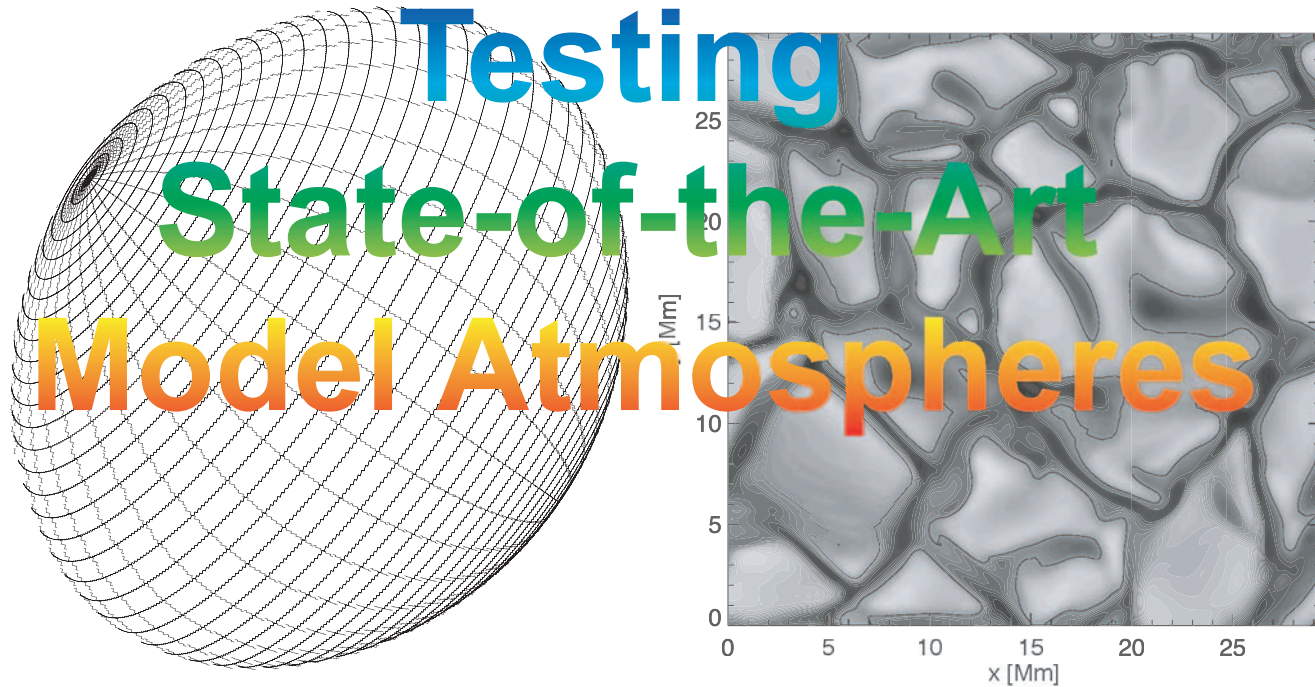
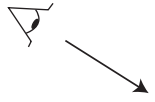


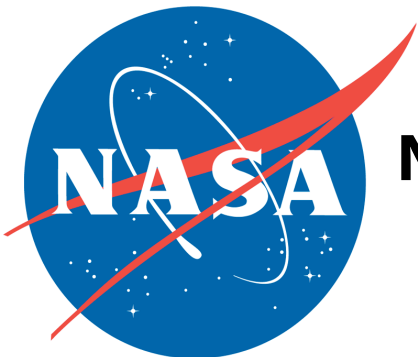
# Limb-Darkening Observations



Testing  
State-of-the-Art  
Model Atmospheres

**Jason Aufdenberg**

**Michelson Postdoctoral Fellow  
National Optical Astronomy Observatory  
Tucson, USA**



## Interferometry Collaborators

S. T. Ridgway (NOAO)

P. Kervella, A. Mérand, V. Coudé du Foresto,

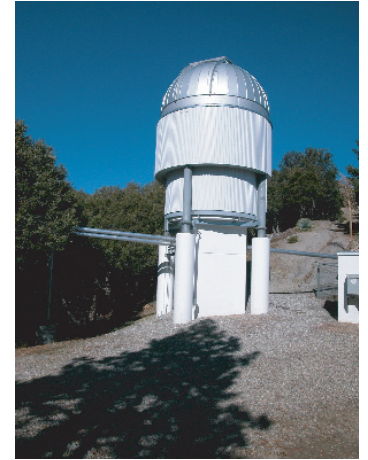
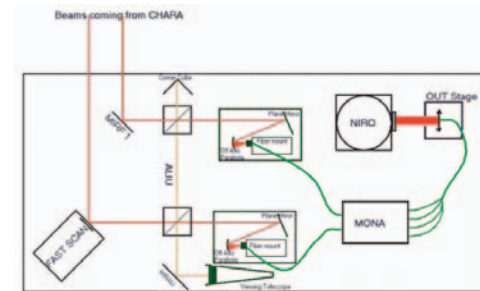
E. Di Folco, G. Romero

(LESIA, Observatoire de Paris-Meudon)

O. Absil (Université de Liège)

H. McAlister and the CHARA Team

(Georgia State University and Mount Wilson Observatory)



## Model Atmosphere Collaborators

H.-G. Ludwig (Observatoire de Paris-Meudon)

P. Hauschildt, A. Schweitzer (Hamburger Sternwarte)

F. Allard (CRAL-ENS, Lyon)

E. Baron (University of Oklahoma)

T. Barman (UCLA)

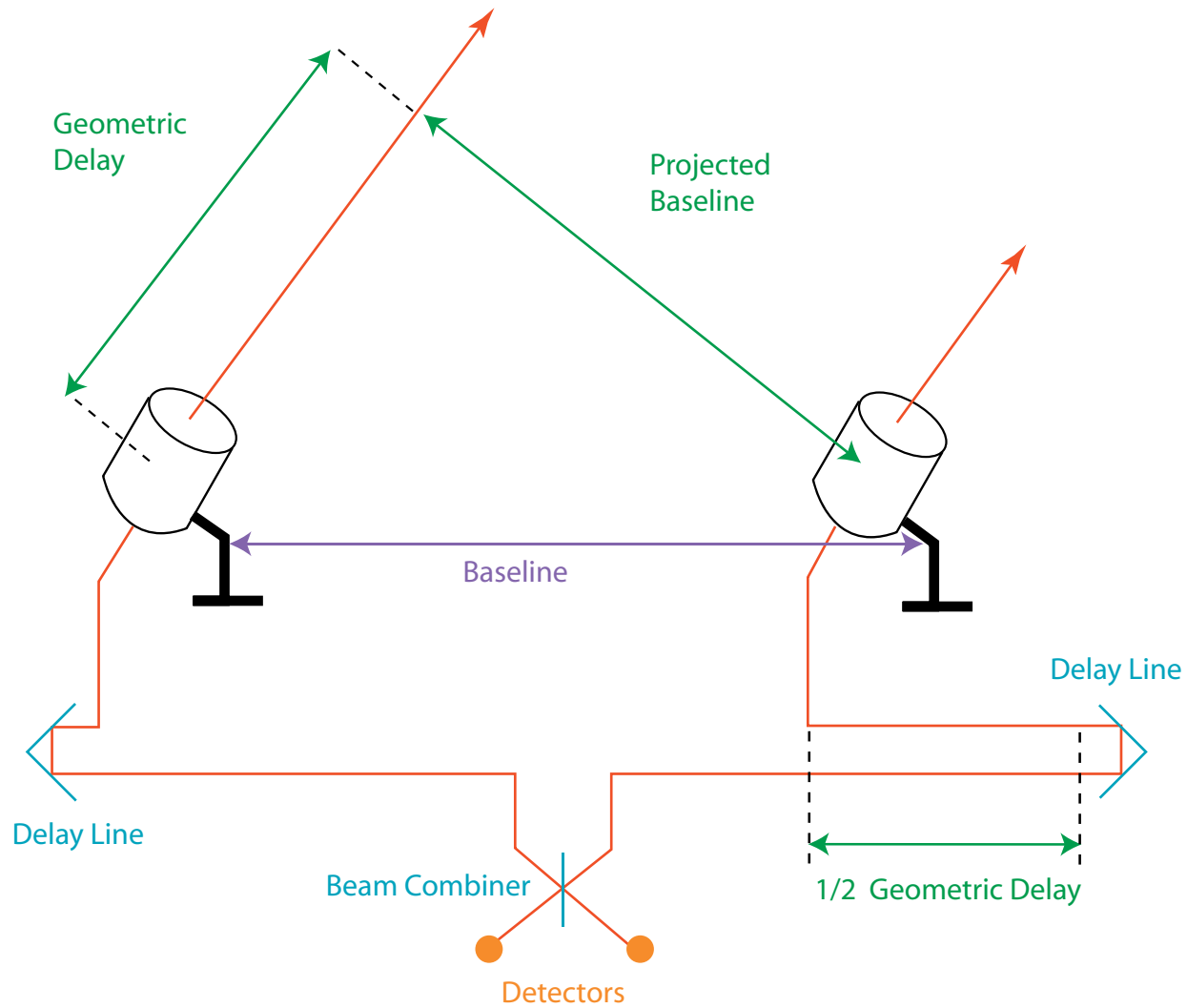


## What's to Come....

Vega's Gravity Darkening with CHARA/FLUOR

Procyon's Convection with VLT/VINCI and Mark III

# Schematic Two-Telescope Interferometer

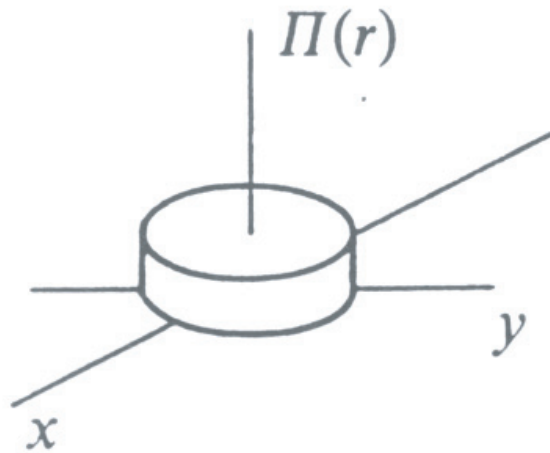
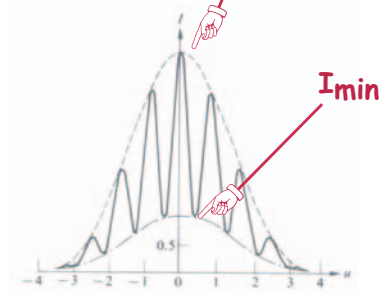
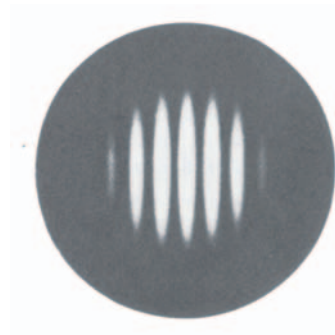


adapted from Oskar van der Lühe

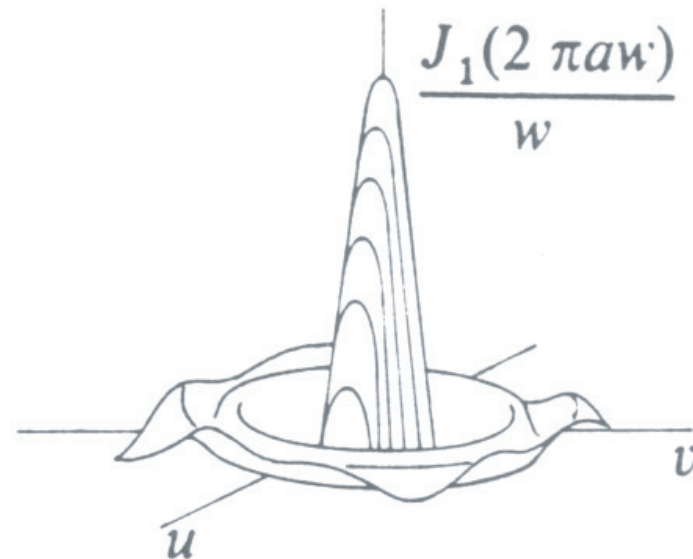


# The Observable: Visibility

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

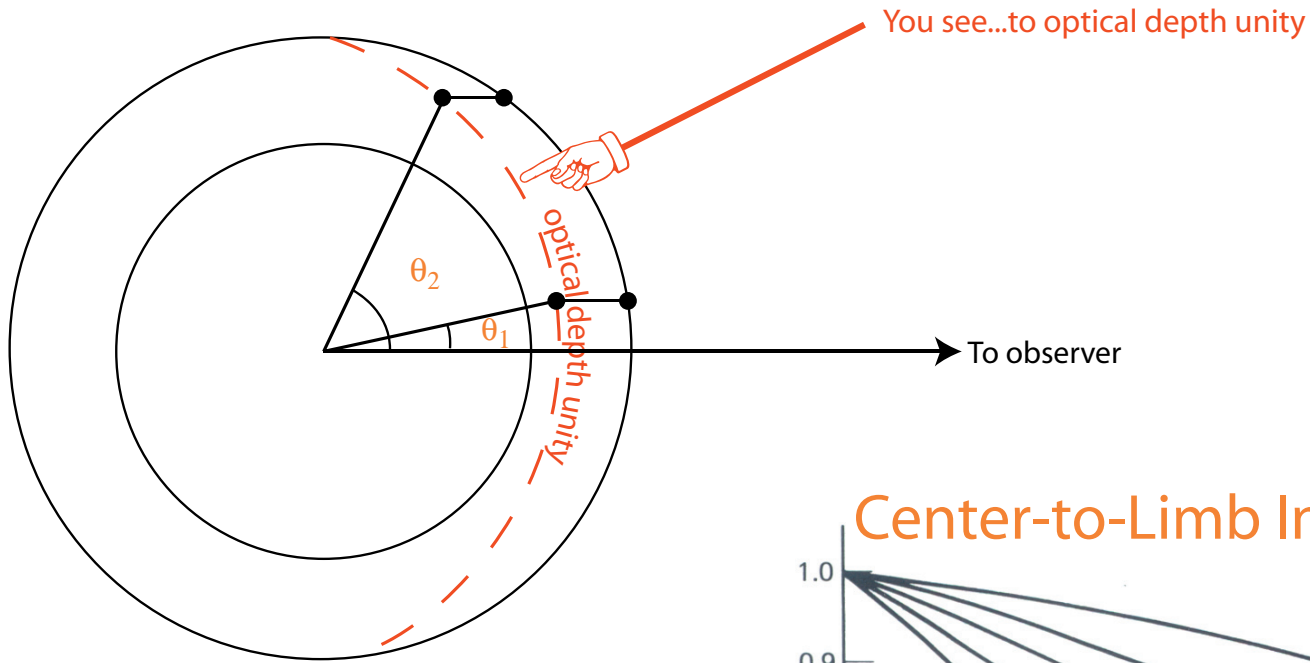


on the sky



what the interferometer measures

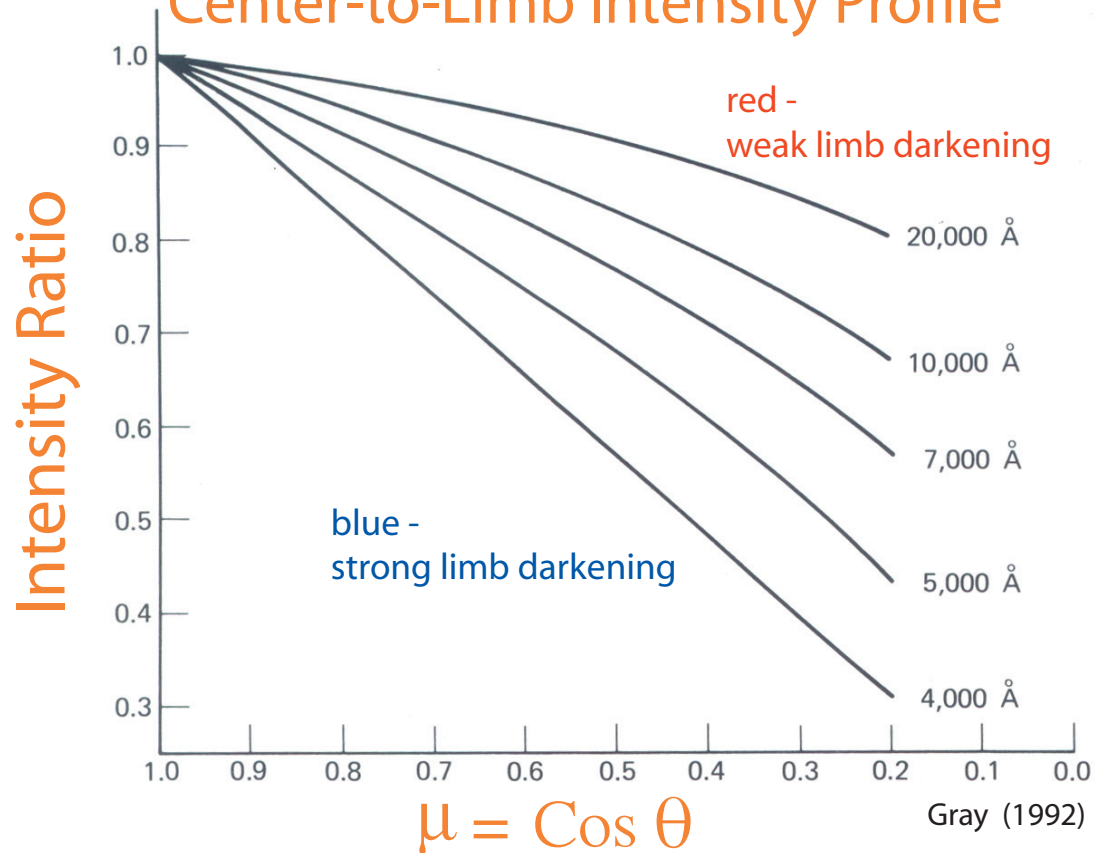
# Limb Darkening Basics



$$B_\lambda = 2 ckT \lambda^{-4}$$

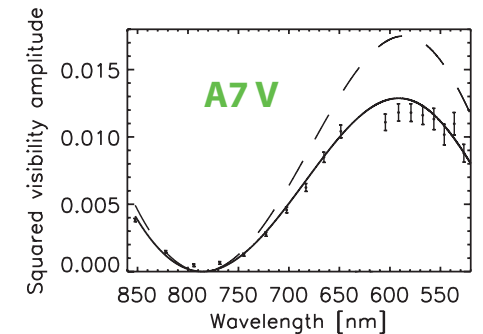
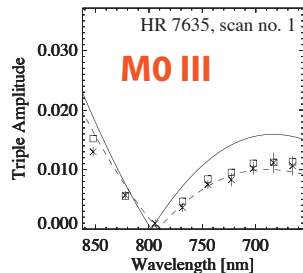
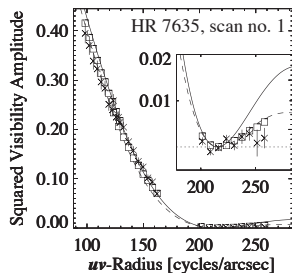
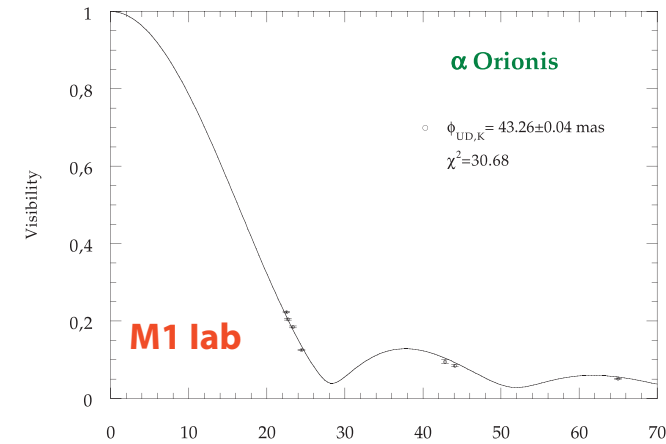
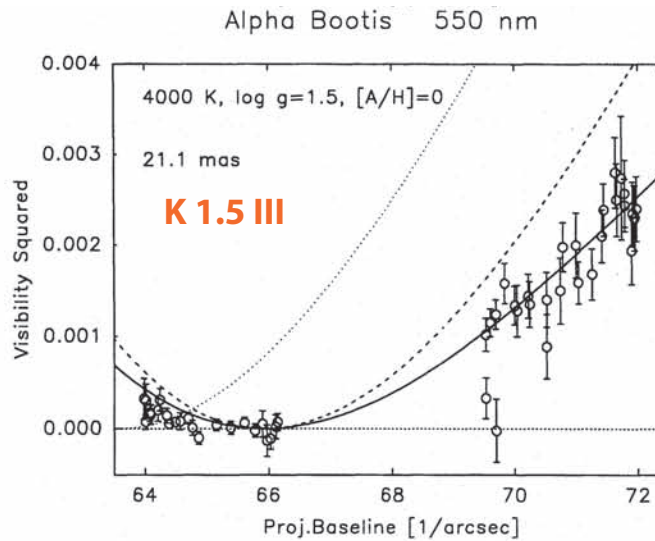
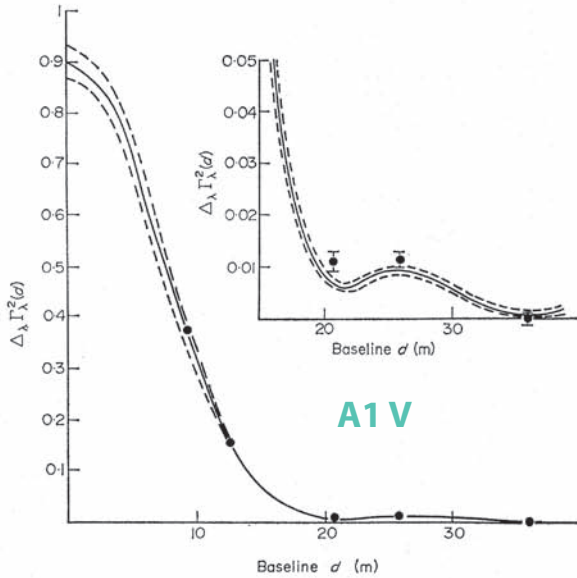
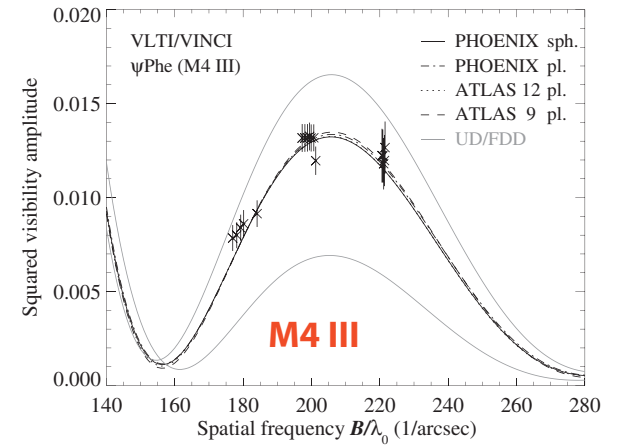
$$\frac{dB_\lambda}{dT} = \frac{2}{c} k \lambda^{-4}$$

## Center-to-Limb Intensity Profile



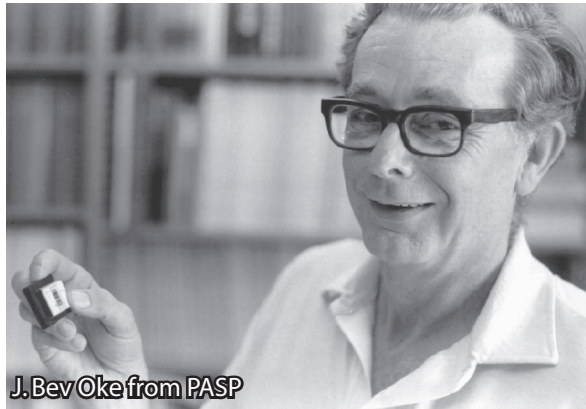
# “Direct” Limb darkening measurements from interferometry

- M5 II**  $\alpha$  Her (Perrin et al., 2004)
- M4.5 III** BY Boo (Wittkowski et al., 2001)
- M4 III** V416 Lac,  $\psi$  Phe (Wittkowski et al., 2001; Wittkowski et al., 2004)
- M1lab** Betelgeuse (a Ori; Burns et al., 1997; Perrin et al., 2004)
- M0 III**  $\gamma$  Sge (Wittkowski et al., 2001)
- K2 III**  $\alpha$  Ari (Hajian et al., 1998)
- K1.5 III** Arcturus ( $\alpha$  Boo; Quirrenbach et al., 1996)
- K0 III**  $\alpha$  Cas (Hajian et al., 1998)
- A7 V**  $\alpha$  Aql (Ohishi et al., 2004)
- A1 V** Sirius ( $\alpha$  CMa; Hanbury Brown et al., 1974)



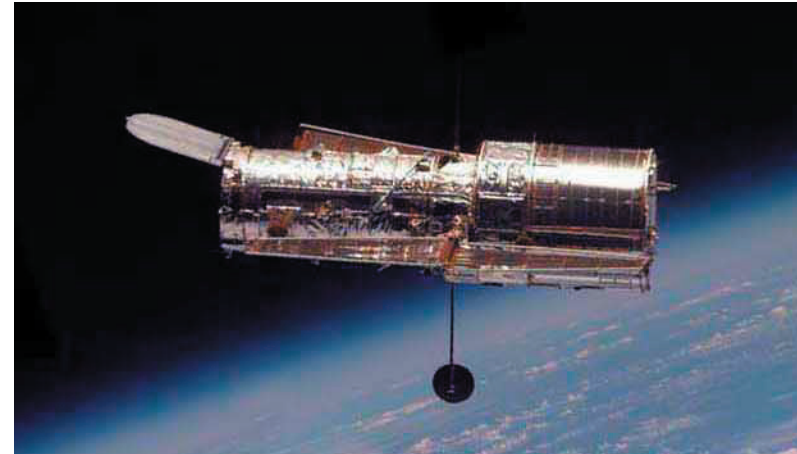
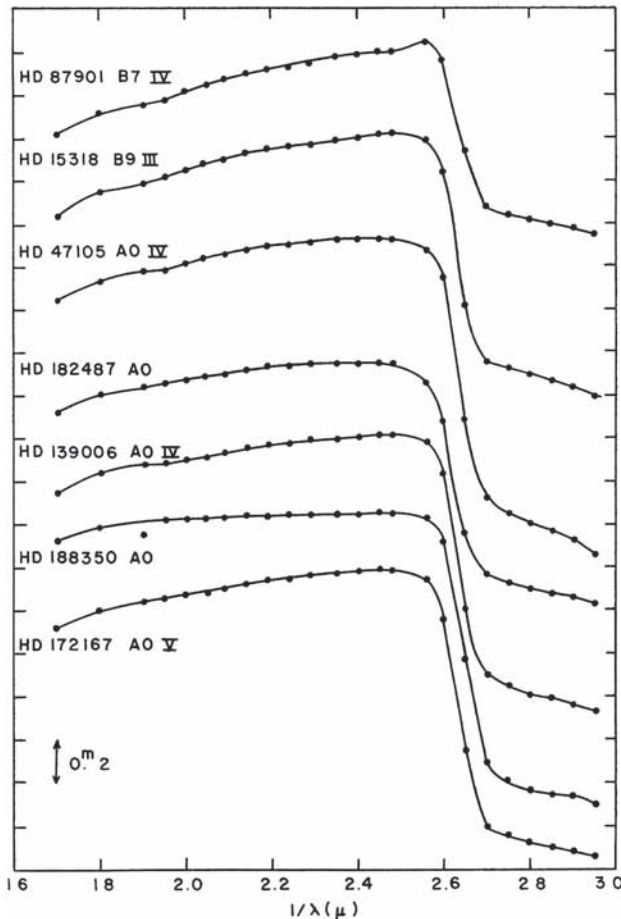
# Vega

# Vega as *the* photometric standard star (1862 - )

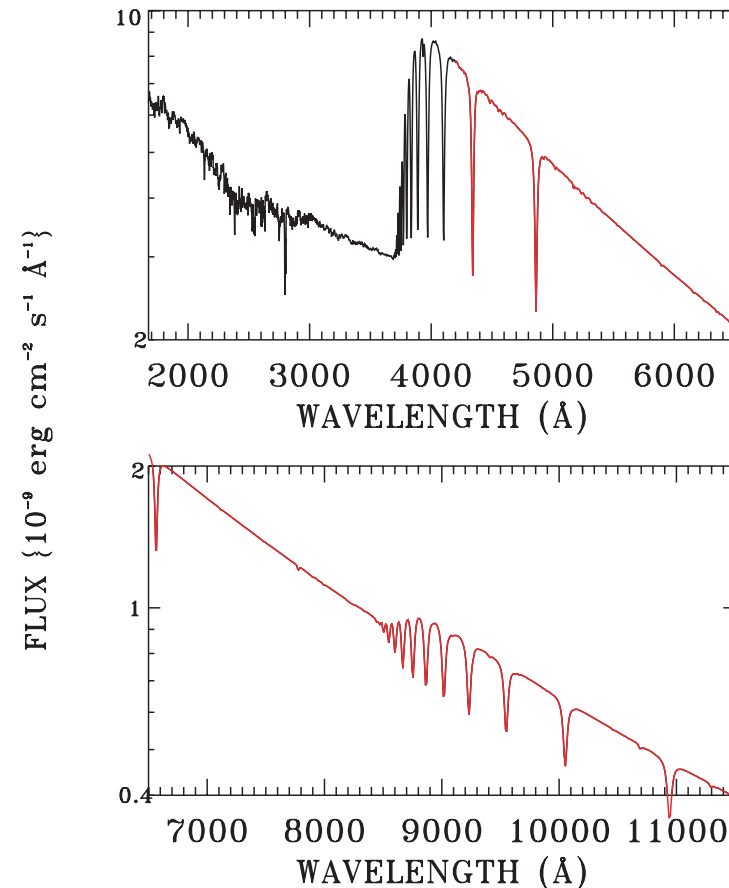


J. Bevan Oke from PASP

A. Code and J. B. Oke put Vega and other stars fluxes into absolute units (1960)



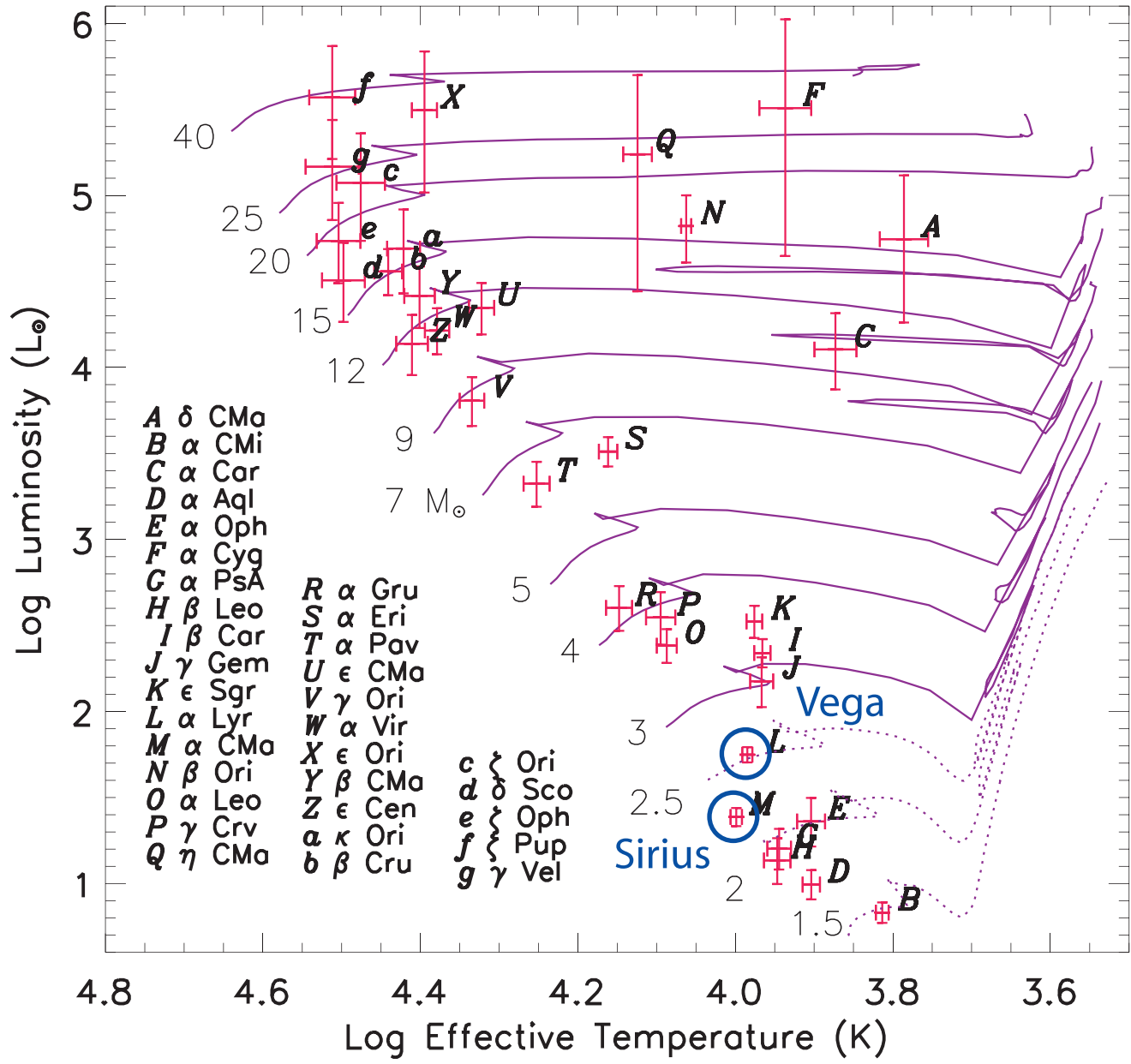
Vega with HST/STIS (Bohlin & Gilliland, 2004)



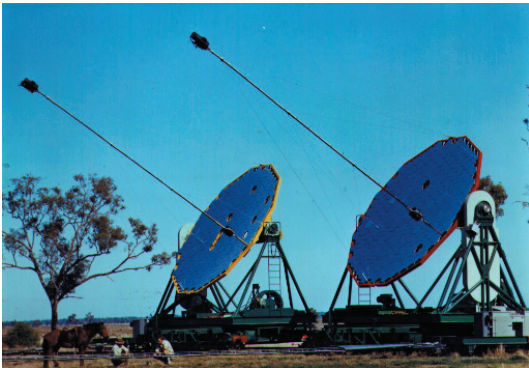
# Vega is 60% larger (in Radius) than Sirius. Mass-Radius relation predicts ~12%.

$R(\text{Vega}) = 2.73 \pm 0.01 R_{\odot}$  (Ciardi et al. 2001)

$R(\text{Sirius}) = 1.711 \pm 0.013 R_{\odot}$  ;  $\text{Mass}(\text{Sirius}) = 2.12 \pm 0.06 M_{\odot}$  (Kervella et al. 2003)



R. Hanbury Brown et al. (1967) measure angular diameters of Vega and Sirius with the Intensity Interferometer.



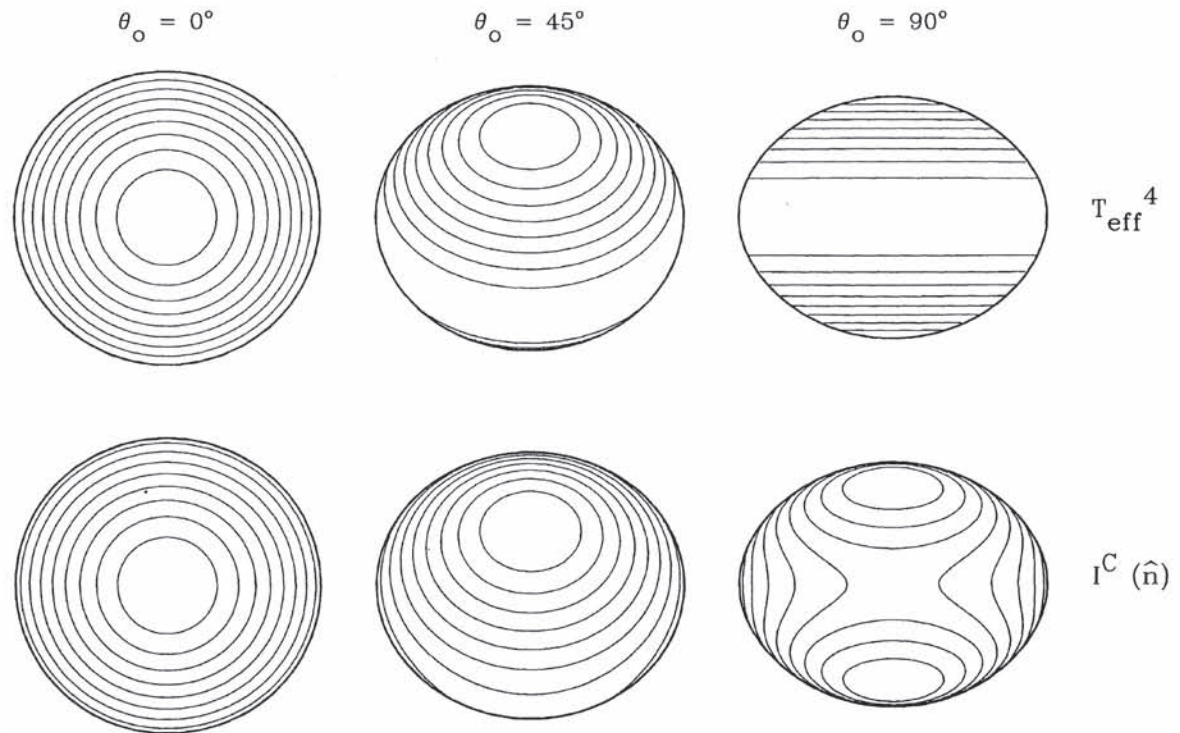
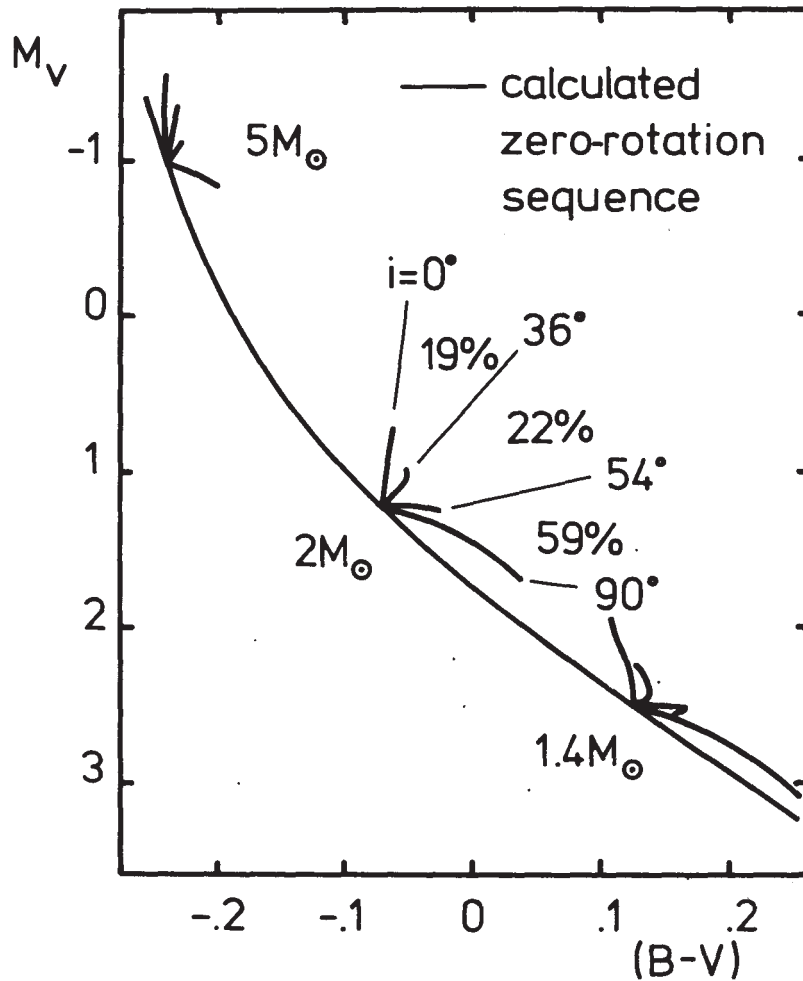
The homology relation  $R \approx M^{0.715 \pm 0.035}$  says that  $R(\text{Vega})/R(\text{Sirius})$  should be  $\sim 1.12$

**Also:** Vega is over luminous for its spectral type *and* has *weird* line profiles (Millward & Walter 1985; Gulliver et al. 1991).



# Vega is a Pole-on Rapid Rotator (R. Gray 1985, 1988)

\*Pole-on rapid rotators appear more luminous, have same color as slow rotators



From Cranmer & Owocki (1995)

*The Radiative Equilibrium of a Rotating System of Gaseous Masses.*  
By H. v. Zeipel, Assoc.R.A.S.

$$T_e^4 = \frac{\tilde{F}}{\pi\mu} = \frac{4}{ac} \tilde{F} = \frac{4}{aC} \frac{d\tilde{\Phi}}{dn_i} \quad T_{\text{eff}} \propto g^{1/4}$$

From von Zeipel (1924)

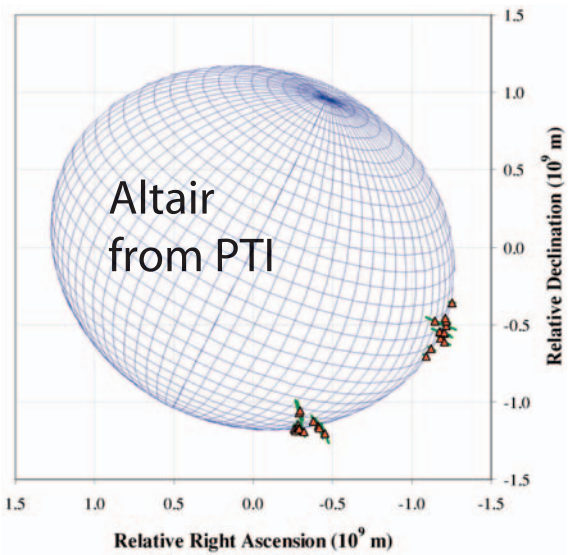
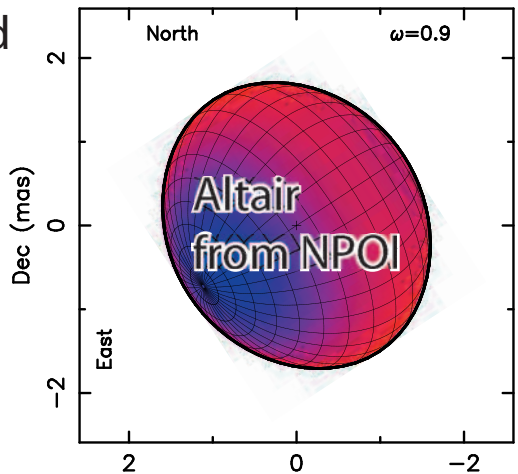
From Tassoul (1978) "Theory of Rotating Stars"  
originally from Maeder & Peytremann (1970)



# Resolved Rapid Rotators from Interferometry

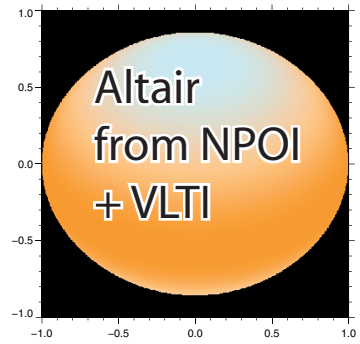
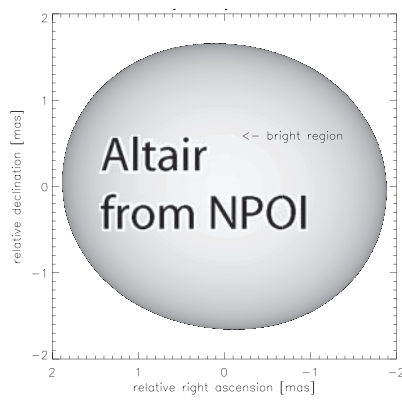
\*Disk of Altair (A7 V) resolved as ellipsoid by the Palomar Testbed Interferometer (van Belle et al. 2001) Axial ratio:  $1.140 \pm 0.029$

NPOI 3-telescope observations reveal 'hot spot' near pole (Ohishi et al. 2004)



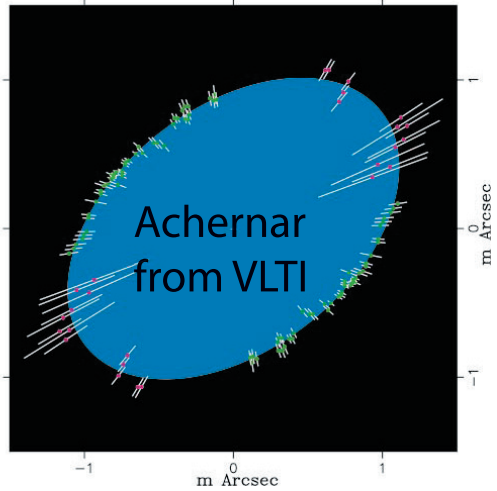
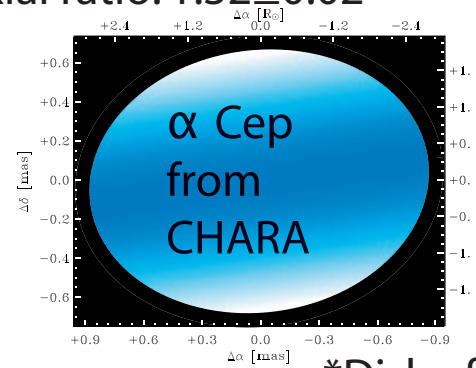
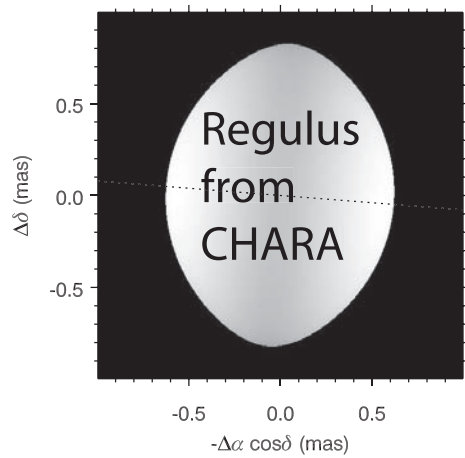
Forthcoming Altair papers:

A. Domiciano de Souza et al. (VLTI+NPOI); Peterson et al. 2006 (NPOI)



\*Disk of Alderamin (A7 IV-V) resolved as an ellipsoid by CHARA (van Belle et al. 2006). Axial ratio:  $1.298 \pm 0.051$

\*Disk of Regulus (B7 V) resolved as ellipsoid by CHARA (McAlister et al. 2005). Axial ratio:  $1.32 \pm 0.02$



\*Disk of Achernar (B3 Vpe) resolved as ellipsoid by VLTI (A. Domiciano de Souza et al. 2003). Axial ratio:  $1.56 \pm 0.05$



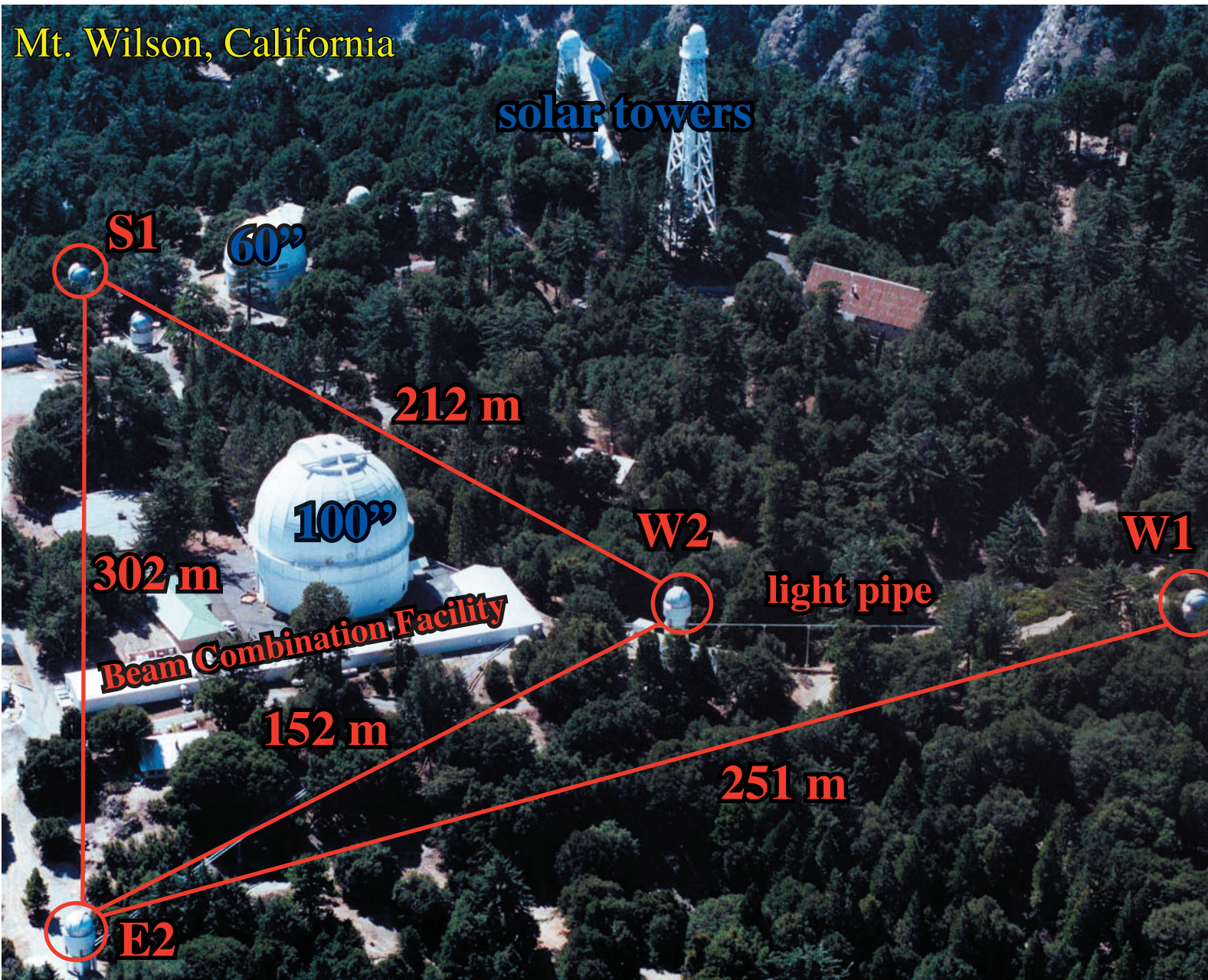
# CHARA (Center for High Angular Resolution Astronomy) Array

Vega  
Observations  
May, June 2005

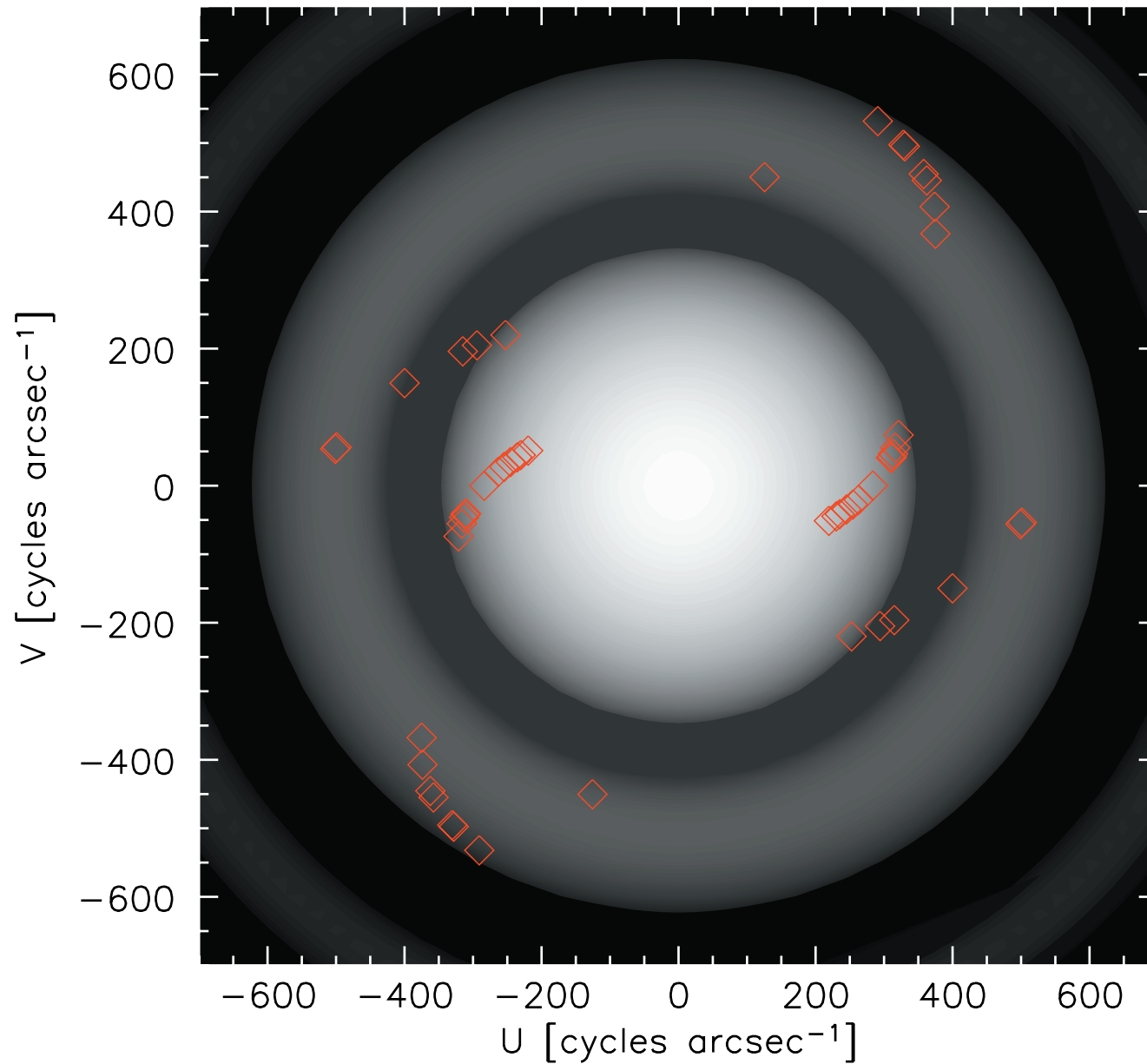
K-band  
 $\lambda = 2.2 \mu\text{m}$

Fiber  
Linked  
Unit for  
Optical  
Recombination

beam combiner



# (u,v) Coverage of Vega with the CHARA Array



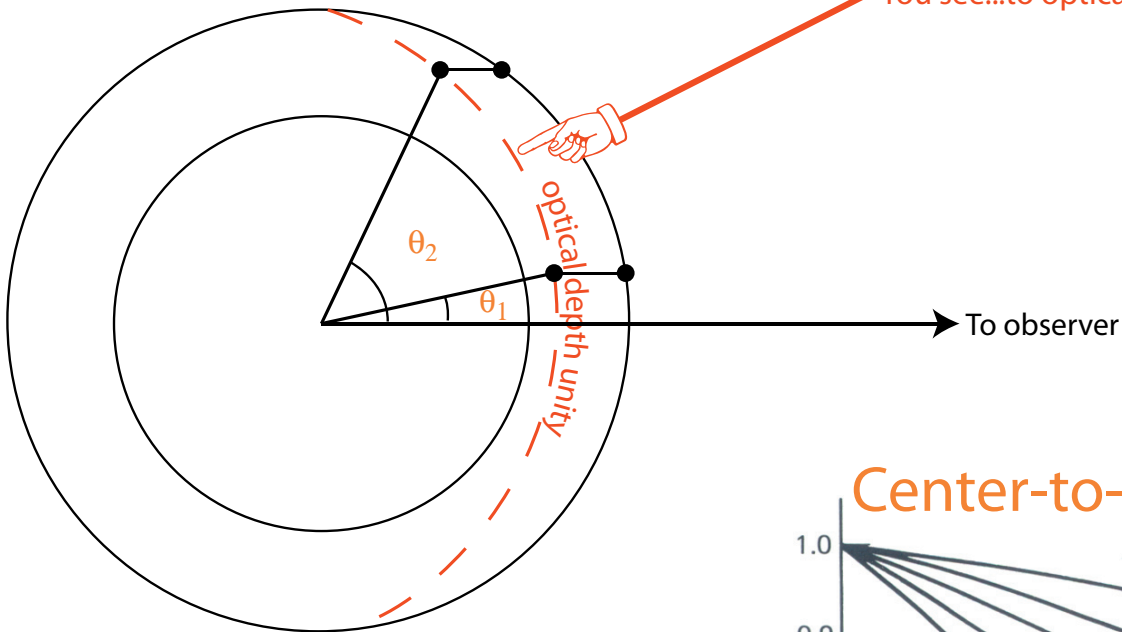


# Fringes with CHARA/FLUOR!



# Limb Darkening Basics

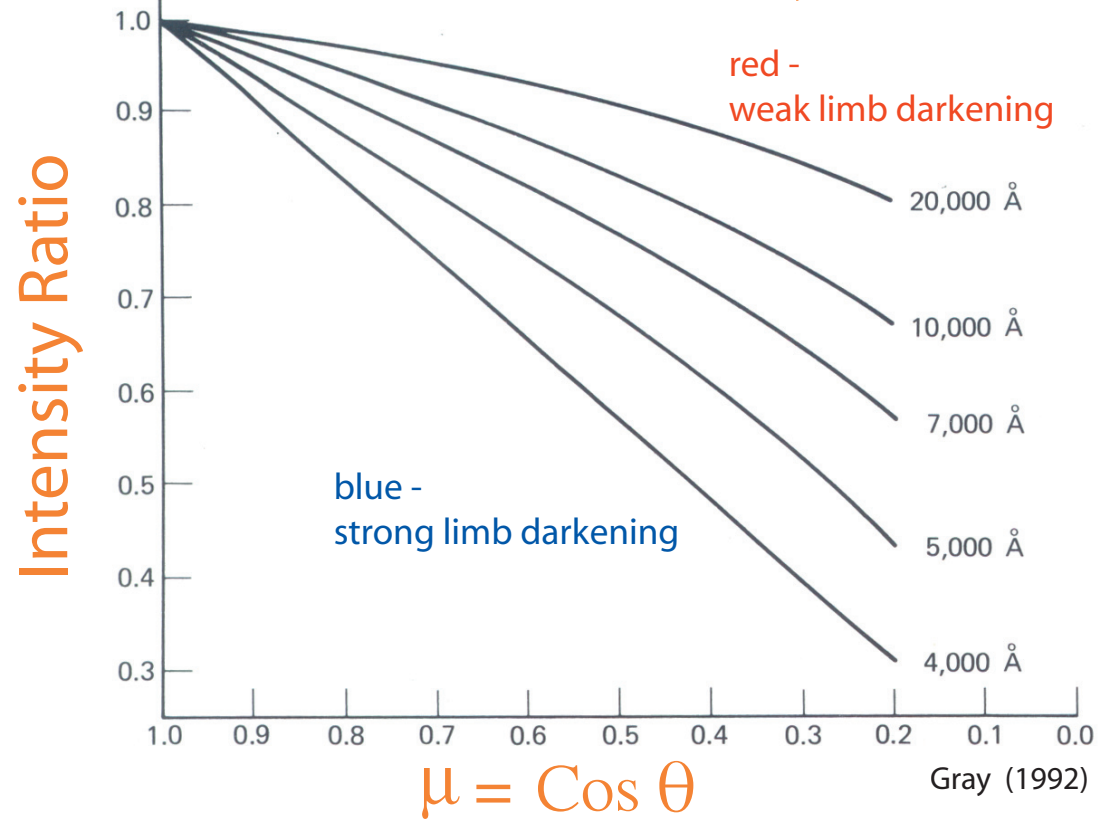
You see...to optical depth unity



$$B_\lambda = 2 \text{ ckT } \lambda^{-4}$$

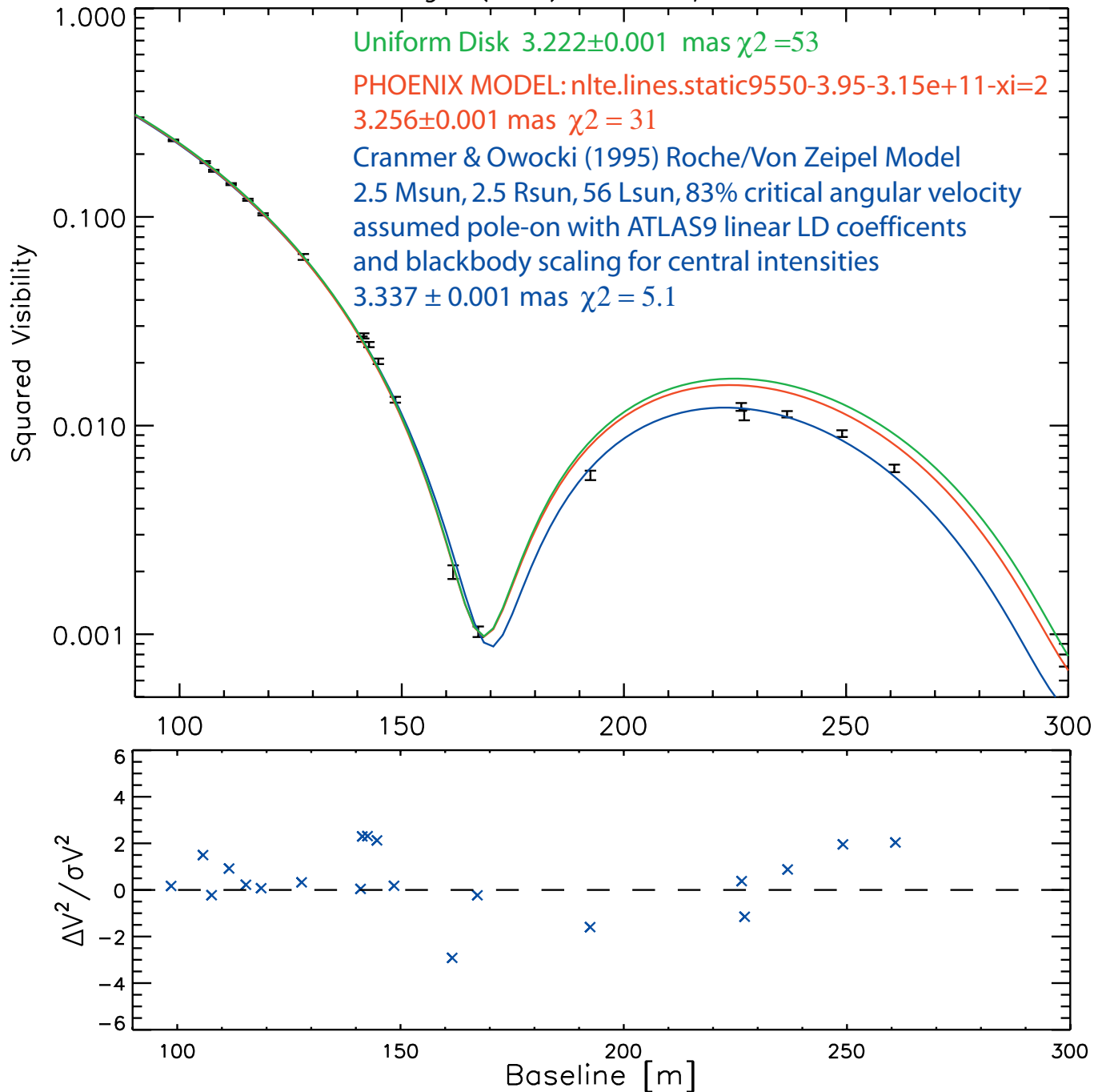
$$\frac{dB_\lambda}{dT} = \frac{2}{c} k \lambda^{-4}$$

## Center-to-Limb Intensity Profile



# Observations and 1st Modeling

Vega (AOV): CHARA/FLUOR



# Parameterizing the Problem: 4 Variables and lots of equations!

$$\omega \theta_{\text{equ}} T_{\text{eff}}^{\text{pole}} g_{\text{pole}}$$

$\omega$  Fraction of the angular break-up speed

$\theta_{\text{equ}}$  Equatorial Angular Diameter (physical AD of Vega viewed exactly pole-on)

$T_{\text{eff}}^{\text{pole}}$  Effective Temperature at the pole

$g_{\text{pole}}$  Surface gravity at the pole

$$R_{\text{equ}} = 107.48 \frac{\theta_{\text{equ}}}{\pi_{\text{hip}}}$$

$$M = \frac{g_{\text{pole}} R_{\text{pole}}^2}{G}$$

$$R_{\text{pole}} = \frac{\omega R_{\text{equ}}}{3 \cos \left[ \frac{\pi + \cos^{-1}(\omega)}{3} \right]}$$

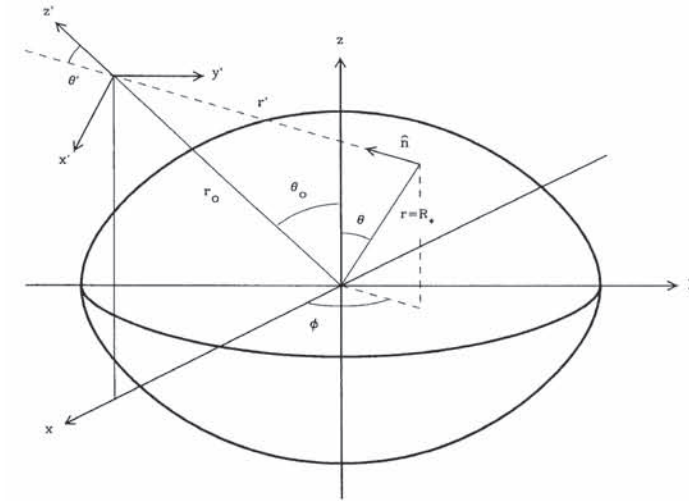
$$L = \frac{\sigma \Sigma (T_{\text{eff}}^{\text{pole}})^4}{g_{\text{pole}}}$$

$$R(\vartheta) = 3 \frac{R_{\text{pole}}}{\omega \sin \vartheta} \cos \left[ \frac{\pi + \cos^{-1}(\omega \sin \vartheta)}{3} \right]$$

$$\frac{T_{\text{eff}}(\vartheta)}{T_{\text{eff}}^{\text{pole}}} = \left( \frac{g(\vartheta)}{g_{\text{pole}}} \right)^\beta$$

$$g(\vartheta) = \left[ g_r(\vartheta)^2 + g_\vartheta(\vartheta)^2 \right]^{1/2}$$

$$T_{\text{eff}}(\vartheta) = \left[ \frac{L}{\sigma \Sigma} g(\vartheta) \right]^\beta$$



$$\Omega = \omega \Omega_{\text{crit}} = \omega \left[ \frac{8 GM}{27 R_{\text{pole}}^3} \right]^{1/2}$$

$$V_{\text{equ}} = R_{\text{equ}} \Omega$$

$$i = \sin^{-1} \left[ \frac{v \sin i}{V_{\text{equ}}} \right]$$

$$g_r(\vartheta) = \frac{-GM}{R(\vartheta)^2} + R(\vartheta) (\Omega \sin \vartheta)^2$$

$$g_\vartheta(\vartheta) = R(\vartheta) \Omega^2 \sin \vartheta \cos \vartheta$$

$$\Sigma \approx 4\pi GM \left[ 1.0 - 0.19696 \omega^2 - 0.094292 \omega^4 + 0.33812 \omega^6 - 1.30660 \omega^8 + 1.8286 \omega^{10} - 0.92714 \omega^{12} \right]$$



# Accurate Synthetic Visibilities with High Dynamic Range

2-D Fourier Transform

$$V_{\lambda}^2(u, v) = \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{\lambda} I_{\lambda}(x, y) e^{i2\pi(u x + v y)} dx dy \right]^2$$

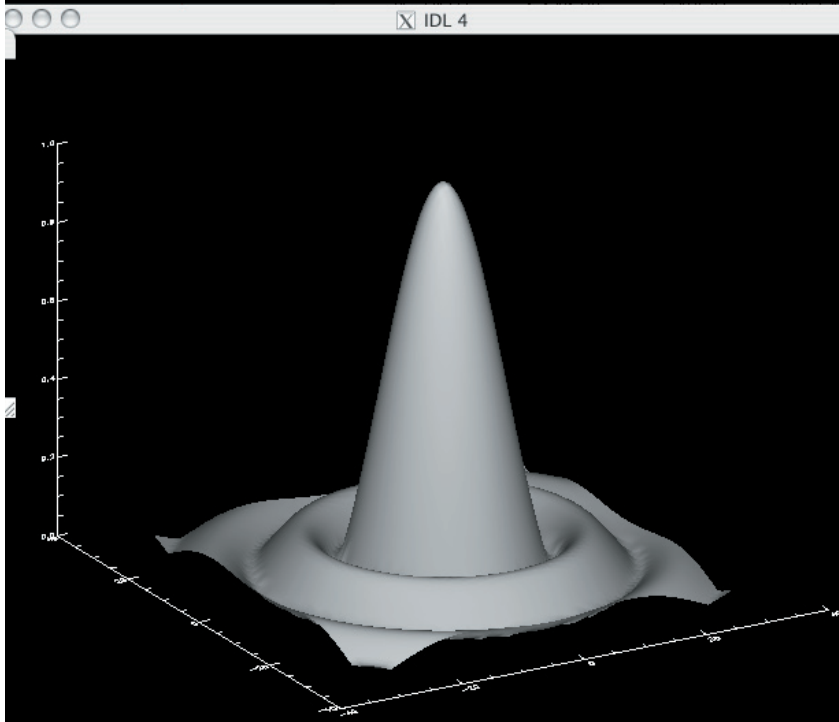
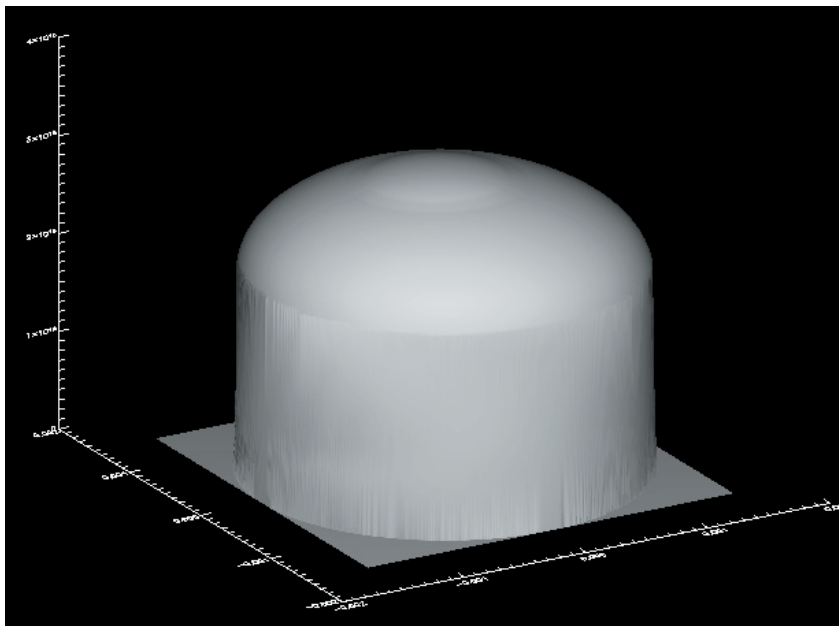
Numerical "Cubature"  
N=512

$$V^2(u_k, v_k) \approx \left[ \sum_{i=1}^N A_i \sum_{j=1}^N A_j S_k I_k(x_i, y_j) \cos(2\pi(u_k x_i + v_k y_j)) \right]^2 + \left[ \sum_{i=1}^N A_i \sum_{j=1}^N A_j S_k I_k(x_i, y_j) \sin(2\pi(u_k x_i + v_k y_j)) \right]^2$$

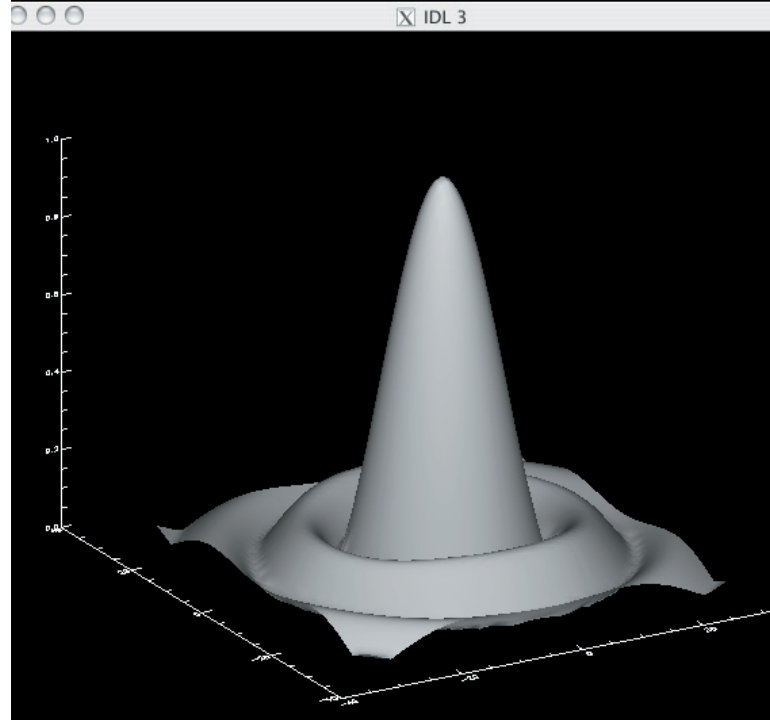
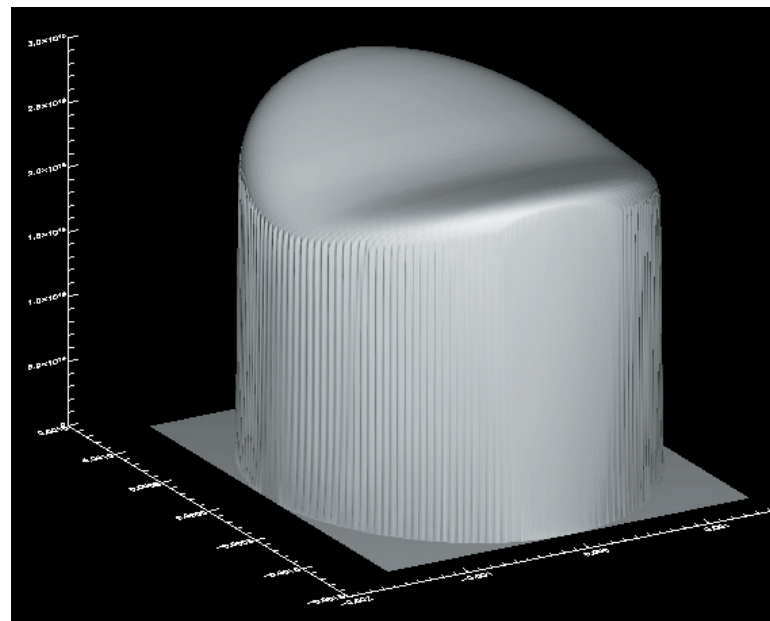
CPU time for 26 (u,v) points is ~6x faster than a 4096x4096 FFT

# The 2-D Modeling Problem

Intensity Map: Vega (pole-on)



Intensity Map: Vega (equator-on)



Fast Fourier Transform —»

# Rotating Model Fit to the CHARA/FLUOR Visibility Data

## Model Input Parameters

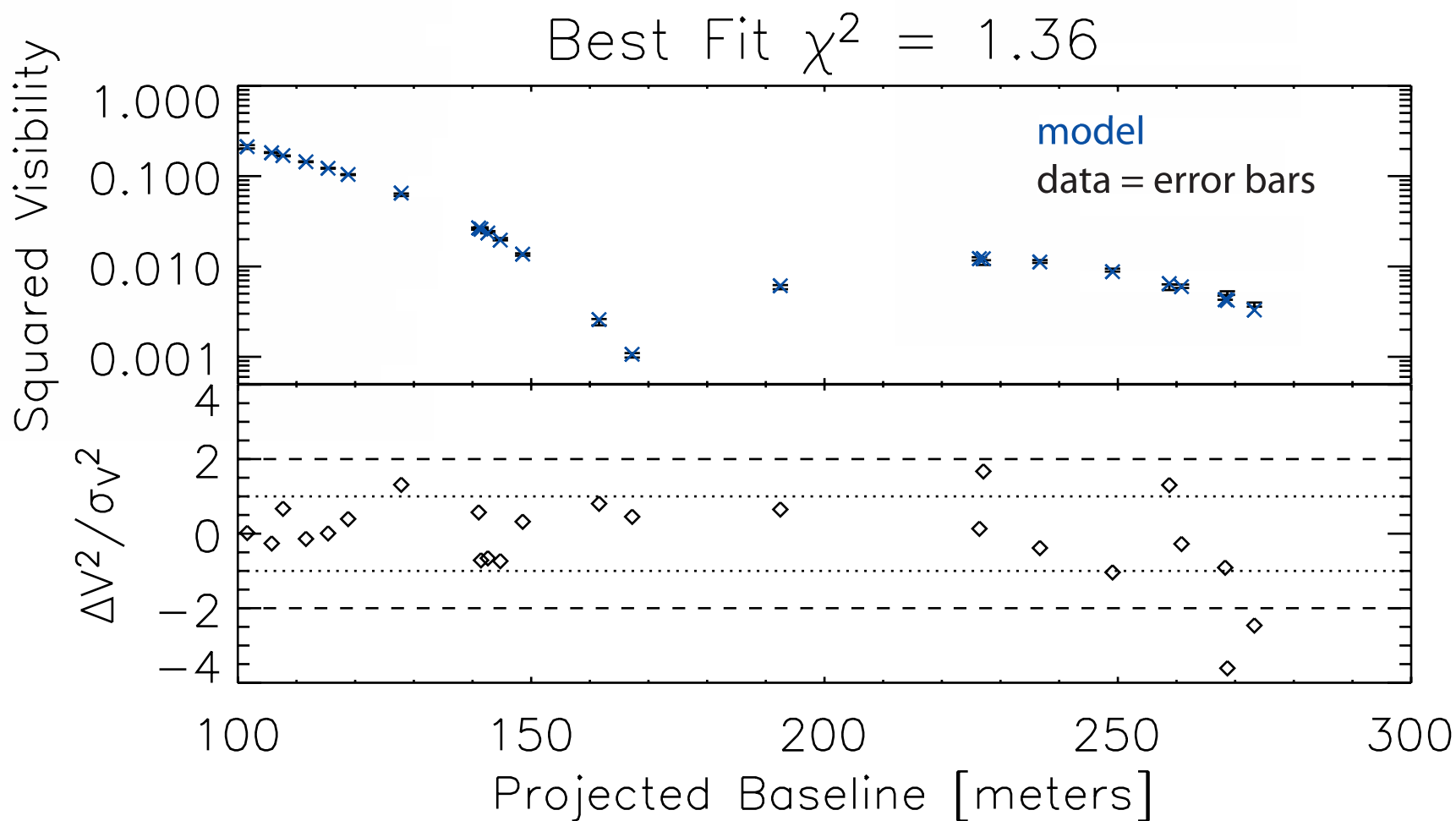
$T_{\text{eff}}$  (pole) = 10250 K B9

$T_{\text{eff}}$  (equator) = 7700 K A9

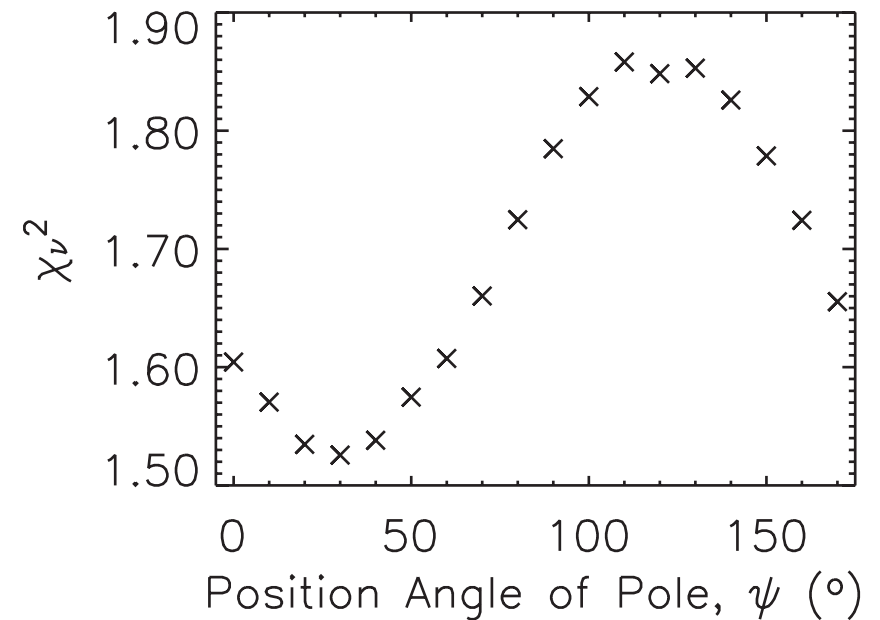
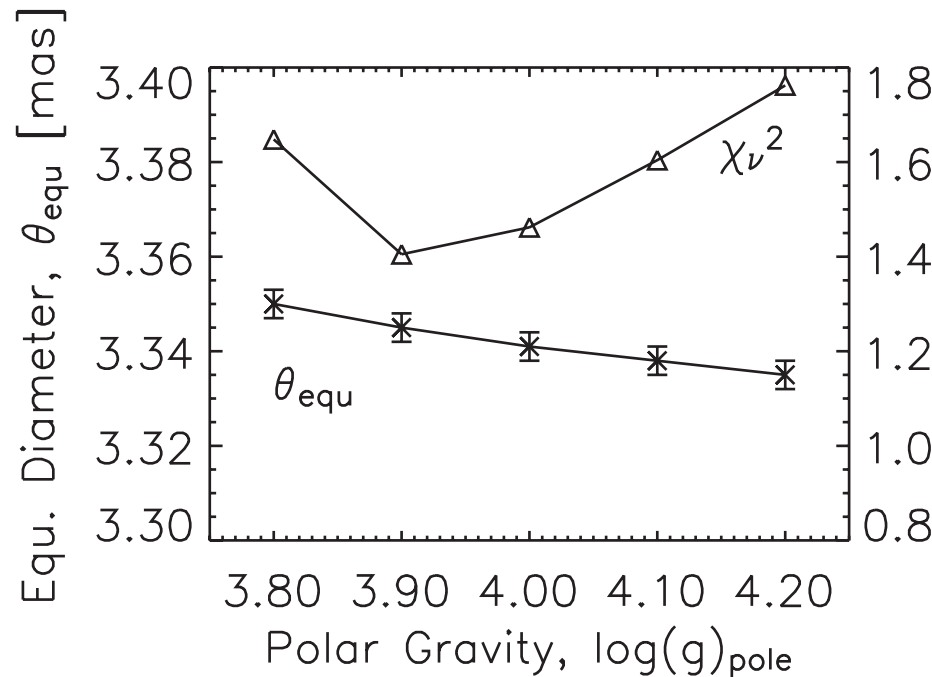
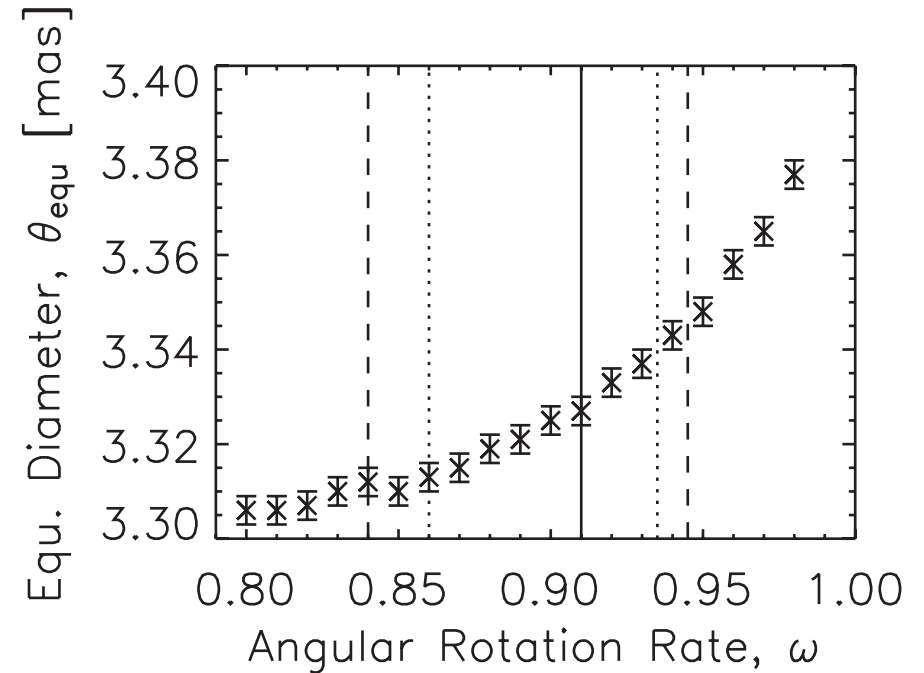
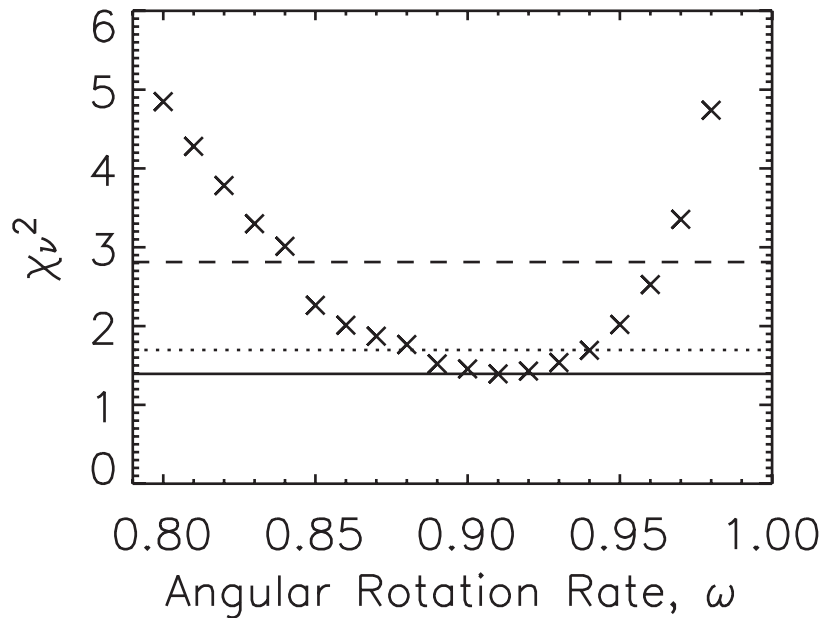
Log(g) (pole) = 4.10

$\omega$  (angular break-up fraction) = 0.93

$\theta_{\text{equator}}$  = 3.34 mas

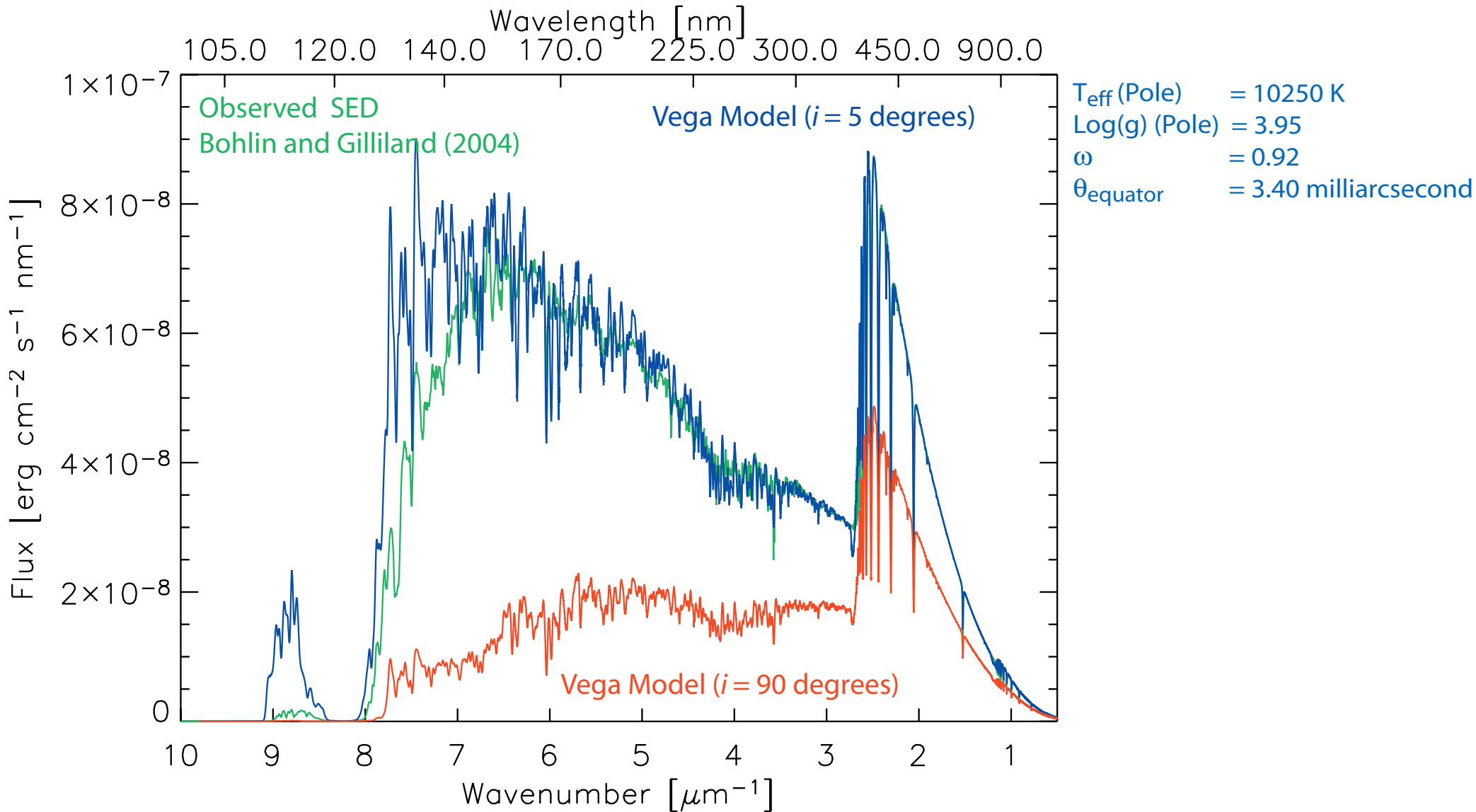


# Searching the 5-D Hypercube! Preliminary Fit Results for Vega



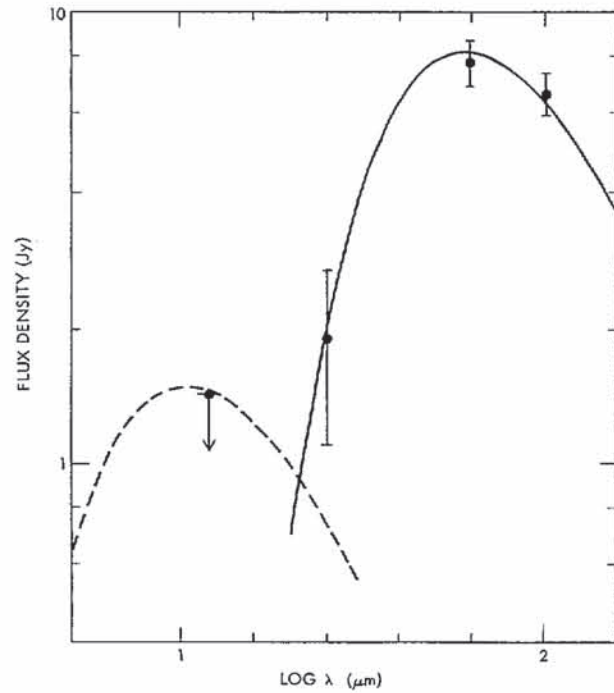
# Spectral Energy Distribution Comparison

$$F_\lambda = \int_0^\pi \int_0^{2\pi} -\frac{g(\vartheta)}{g_r(\vartheta)} I_\lambda(R, \vartheta, \varphi) R(\vartheta)^2 \sin \vartheta \mu(\vartheta, \varphi, i) d\varphi d\vartheta$$



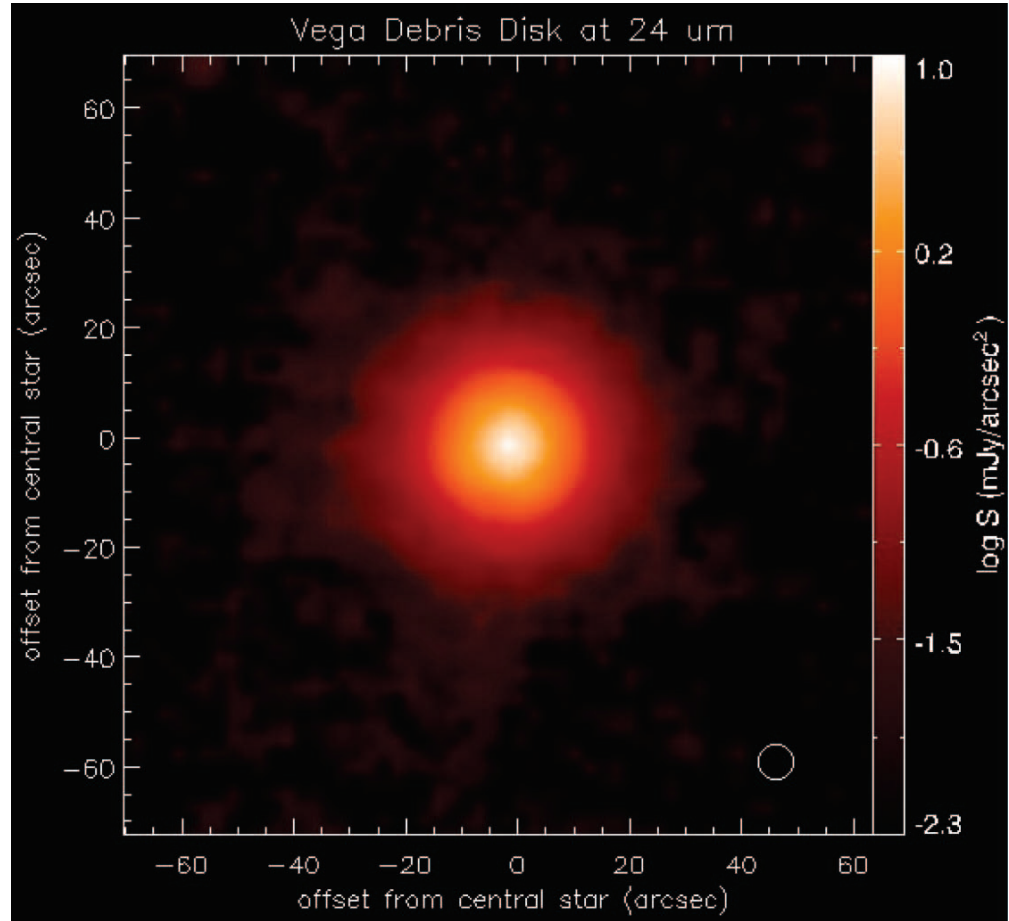
# Vega's disk sees a $\sim 7700$ K SED

IRAS



Aumann et al. (1984)

Spitzer



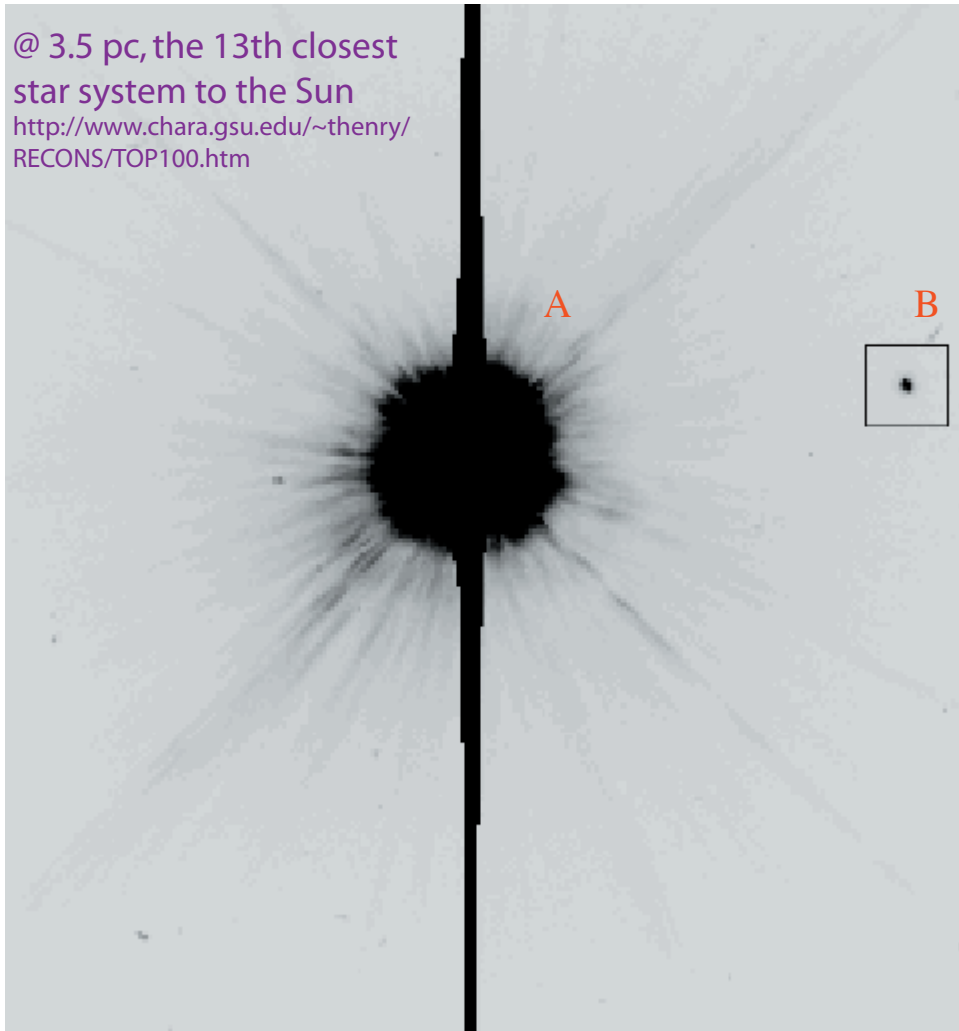
Su et al. (2005)

# Procyon



# Procyon: The Visual Binary (P = 40.82 yr)

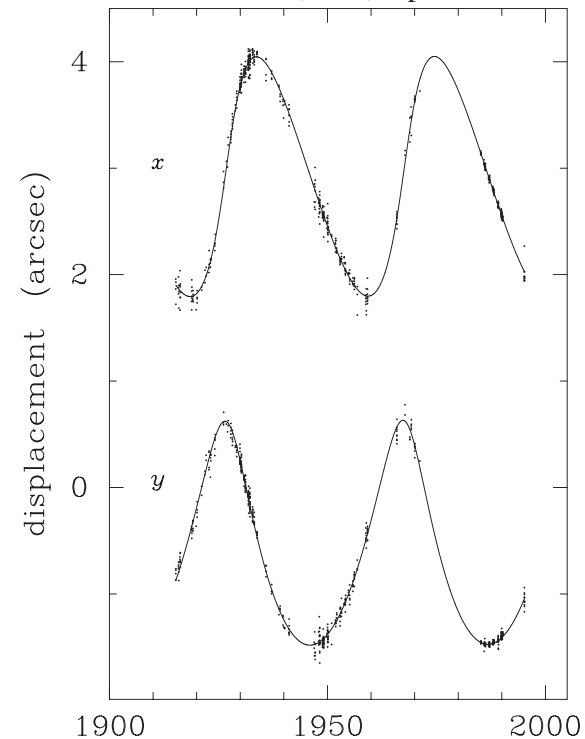
@ 3.5 pc, the 13th closest  
star system to the Sun  
[http://www.chara.gsu.edu/~thenry/  
RECONS/TOP100.htm](http://www.chara.gsu.edu/~thenry/RECONS/TOP100.htm)



Girard et al (2000) ApJ 119, 2428

HST/WFPC2 PC image (160 s)  
F218W filter

Girard et al (2000) ApJ 119, 2428



$$\text{Mass}_A = 1.497 \pm 0.037 M_{\odot}$$

$$\text{Mass}_B = 0.602 \pm 0.015 M_{\odot}$$

## Procyon A (F5 IV): Fundamental Parameters

Angular diameter =  $5.45 \pm 0.05$  mas (Kervella et al. 2003)

Parallax =  $285.93 \pm 0.88$  mas (Hipparcos: Perryman et al.)

Radius =  $2.05 \pm 0.02 R_{\odot}$

Log(g) =  $3.95 \pm 0.02$  cgs

Bolometric flux =  $17.8 \pm 0.9 \times 10^{-9} \text{ W m}^{-2}$

Effective Temperature =  $6516 \pm 87$  K

# Early Model Limb Darkening Models

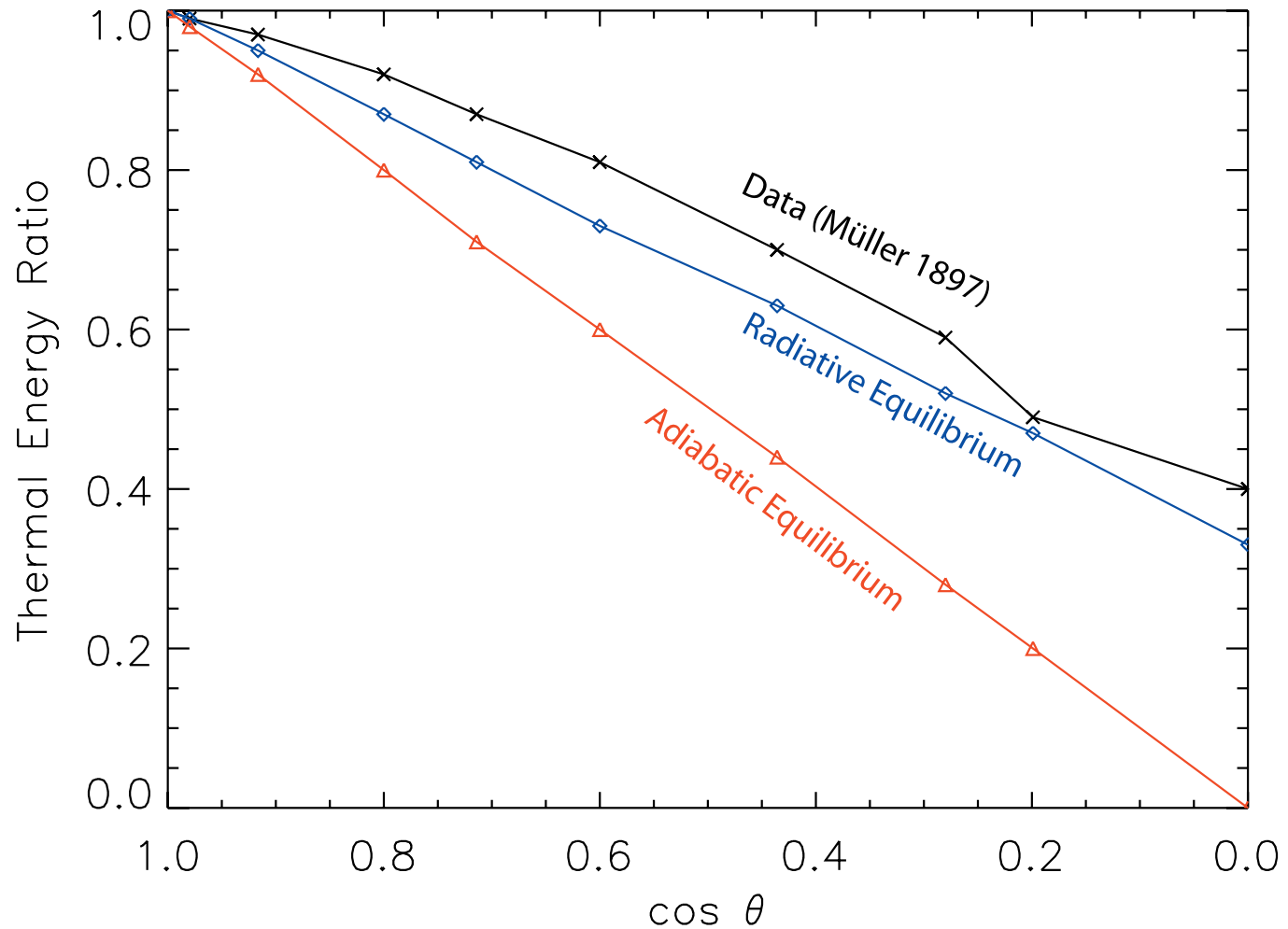


## 1906 - K. Schwarzschild

Derived a center-to-limb profile for the Sun with a radiative equilibrium temperature structure. He showed this to be consistent with observations, ruling out an adiabatic equilibrium temperature structure.

**Assumptions:** mass absorption coefficient is both wavelength and depth independent. Angular-dependent intensity is replaced by its mean.

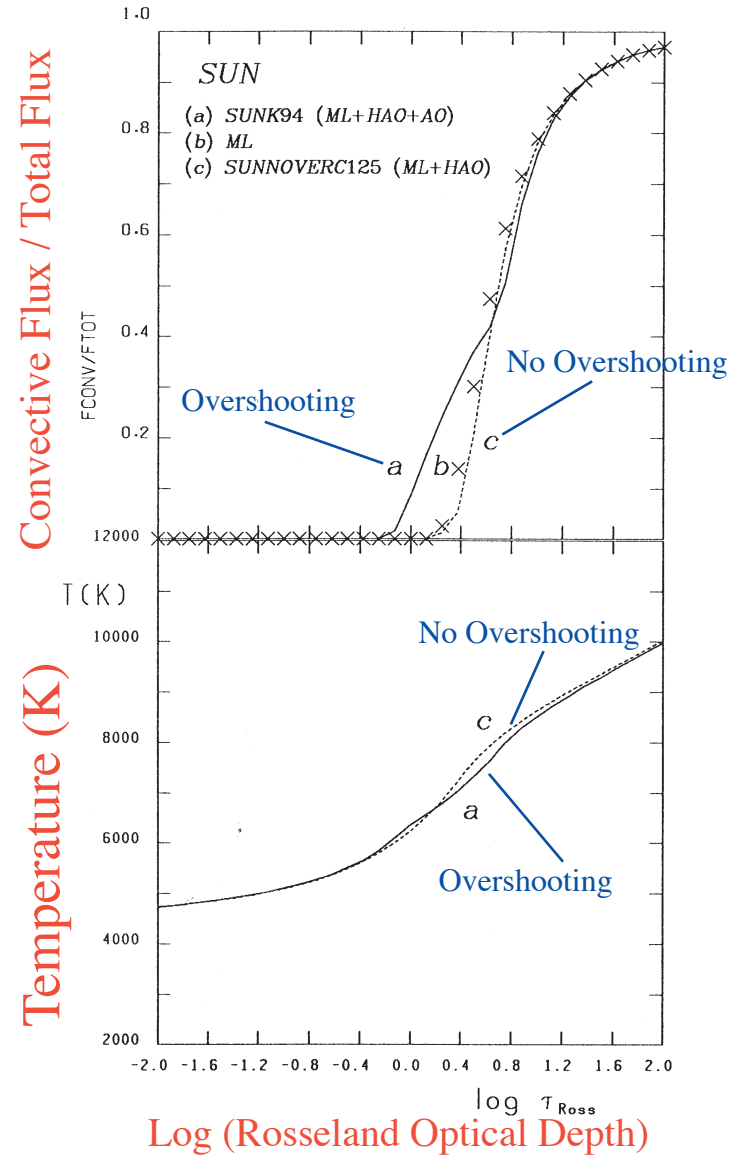
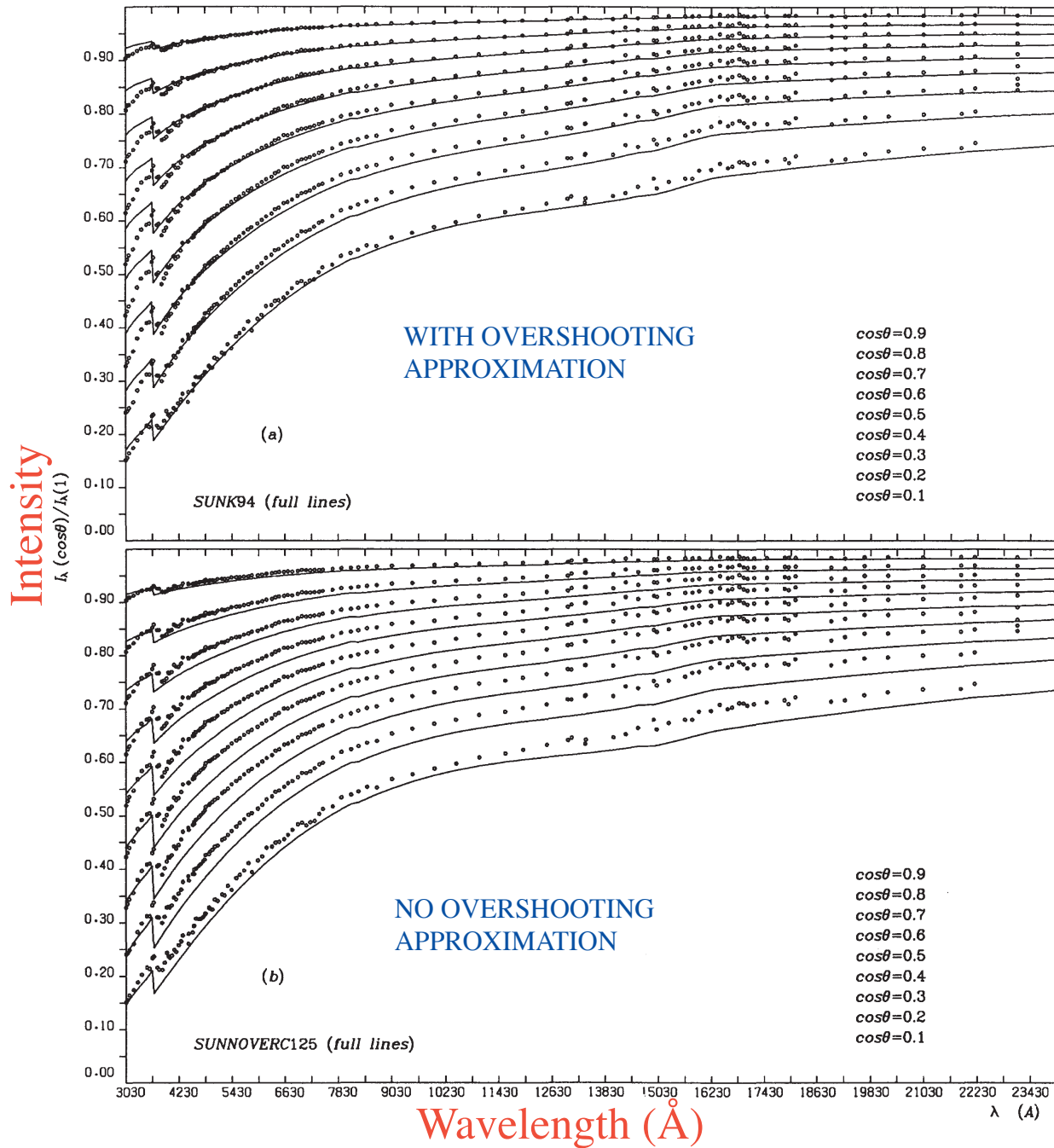
Schwarzschild (1906) Models vs. Contemporary Data



Adapted from K. Schwarzschild (1906) "Über das Gleichgewicht der Sonnenatmosphäre" *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen. Math.-phys. Klasse*, 295, 41  
Translation in D. H. Menzel, Ed., *Selected Papers on the Transfer of Radiation* (1966) NY: Dover

# Solar Limb Darkening and the Overshooting Approximation

Castelli, Gratton & Kurucz (1997) A&A 318, 841



# Asplund et al. (2004) A&A 417, 751

## Line formation in solar granulation

### IV. [O I], O I and OH lines and the photospheric O abundance

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**Abstract.** The solar photospheric oxygen abundance has been determined from [O I], O I, OH vibration-rotation and OH pure rotation lines by means of a realistic time-dependent, 3D, hydrodynamical model of the solar atmosphere. In the case of the O I lines, 3D non-LTE calculations have been performed, revealing significant departures from LTE as a result of photon losses in the lines. **We derive a solar oxygen abundance of  $\log \epsilon_{\text{O}} = 8.66 \pm 0.05$ .** All oxygen diagnostics yield highly consistent abundances, in sharp contrast with the results of classical 1D model atmospheres. This low value is in good agreement with measurements of the local interstellar medium and nearby B stars. This low abundance is also supported by the excellent correspondence between lines of very different line formation sensitivities, and between the observed and predicted line shapes and center-to-limb variations. Together with the corresponding down-ward revisions of the solar carbon, nitrogen and neon abundances, the resulting significant decrease in solar metal mass fraction to  $Z = 0.0126$  can, however, potentially spoil the impressive agreement between predicted and observed sound speed in the solar interior determined from helioseismology.

**Key words.** convection – line: formation – Sun: abundances – Sun: granulation – Sun: photosphere

**We derive a solar oxygen abundance of  $\log \epsilon_{\text{O}} = 8.66 \pm 0.05$ .**

## 1. Introduction

Oxygen is the most abundant element in the Universe with a non-Big Bang nucleosynthesis origin. As a consequence, oxygen plays a central role in many different fields of astrophysics ranging from supernova physics and galaxy evolution to dating stars and production of the light elements through cosmic ray spallation. Yet it appears that in many crucial objects for which accurate knowledge of the oxygen abundances is necessary the oxygen content is hotly debated. Recent disputes revolve around the overabundance of oxygen in metal-poor halo stars (see Asplund & García Pérez 2001; Nissen et al. 2002, and references therein), the Galactic radial abundance gradient (Rolleston et al. 2000; Cunha & Daflon 2003), and, astonishingly, the solar oxygen abundance. Partly these disagreements stem from differences in the adopted input data (e.g. *gf*-values, effective temperatures  $T_{\text{eff}}$ , surface gravities  $\log g$ ) but more importantly they reflect the choice of spectral lines to derive the

abundances using classical 1D stellar model atmospheres. In particular in the solar case, the freedom of parameters to obtain consistency is very restricted yet the discrepancy is present in full.

Until recently the commonly adopted solar oxygen abundance was  $\log \epsilon_{\text{O}} = 8.93 \pm 0.04$ <sup>1</sup> (Anders & Grevesse 1989). This historically high abundance was suggested by analyses of the forbidden [O I] 630.0 nm line (Lambert 1978) as well as OH vibration-rotation and pure rotation lines in the infrared (Grevesse et al. 1984; Sauval et al. 1984) using the 1D hydrostatic Holweger-Müller (1974) semi-empirical model of the solar atmosphere and LTE line formation. On the other hand, a much lower abundance is indicated by the permitted high-excitation O I lines, most noteworthy the IR triplet at 777 nm, when employing the same model atmosphere with non-LTE line formation. This discrepancy of about 0.2 dex between different abundance indicators have often been blamed on over-estimated departures from local thermodynamic

<sup>1</sup> On the customary abundance scale defined as  $\epsilon(X) = 10^{12} \times N(X)/N(\text{H})$ .

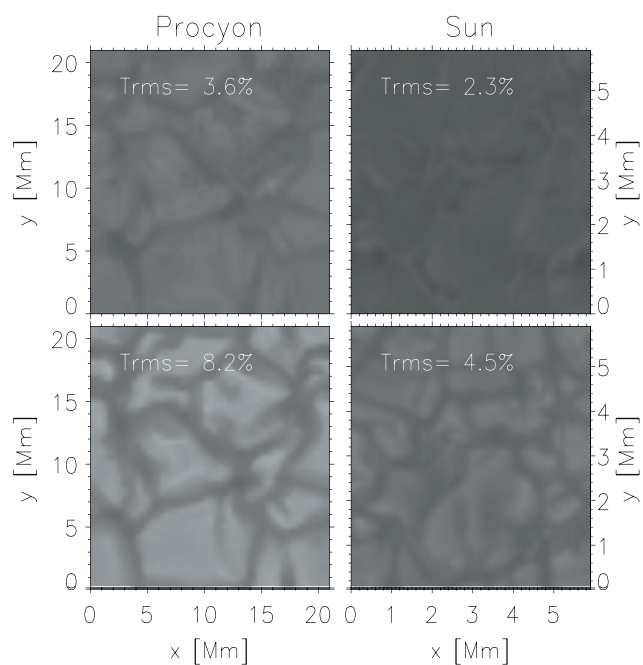
Send offprint requests to: M. Asplund,  
e-mail: martin@mso.anu.edu.au

**Until recently the commonly adopted solar oxygen abundance was  $\log \epsilon_{\text{O}} = 8.93 \pm 0.04$ <sup>1</sup> (Anders & Grevesse 1989).**

# 3-D Hydrodynamical Simulations of Procyon

Allende Prieto et al (2002) ApJ 567, 544

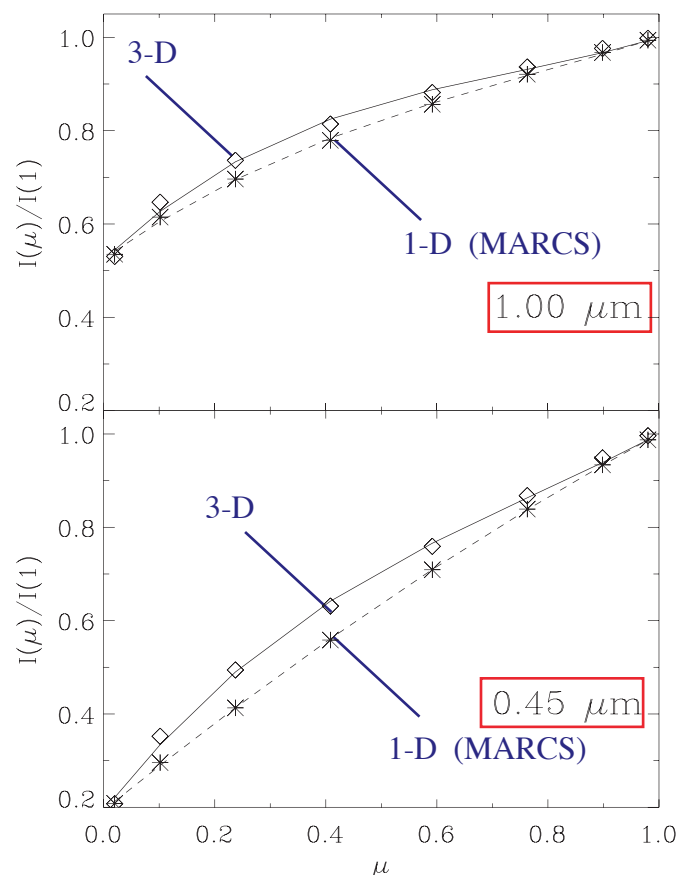
**500 K  $T_{\text{eff}}$**   
**Surface R.M.S.**



$\tau = 0.3$

$\tau = 1.0$

3-D versus 1-D  
Center-to-Limb Profiles



Prediction:  
1.6 %  
angular size  
difference  
between  
1 micron  
and  
450 nm

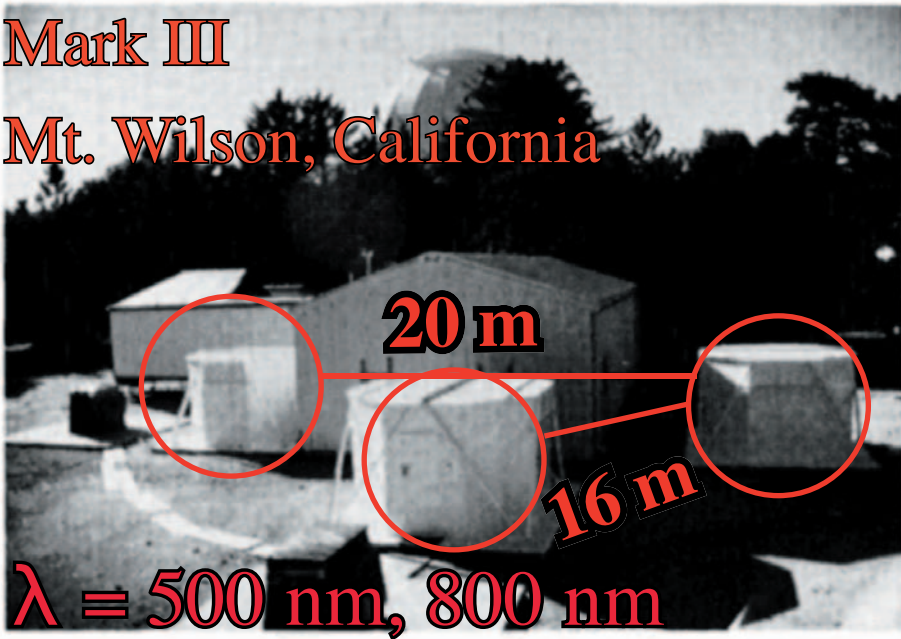
FIG. 19.—Limb darkening for the three-dimensional (*rhombi*) and one-dimensional (*asterisks*) models. Third-order polynomials have been fitted by regular least-squares fits to the data.



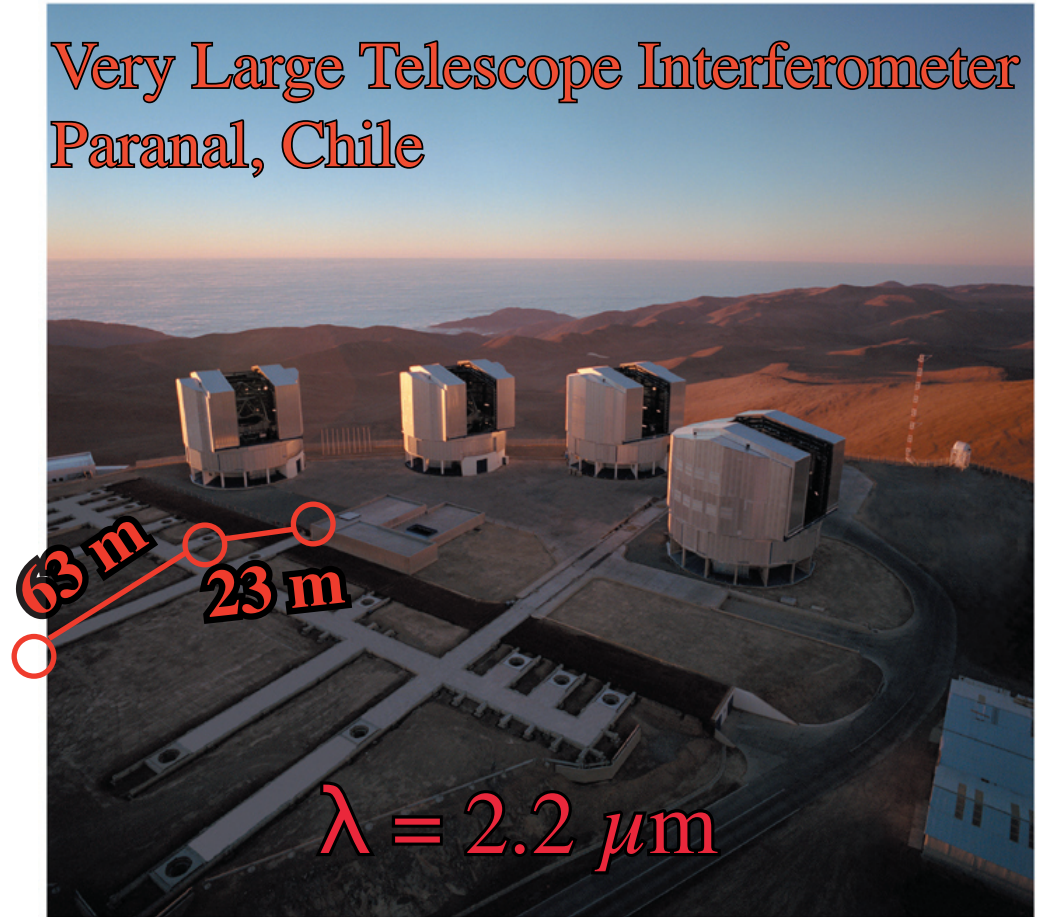
# Procyon Data from the Mark III and VLT/VINCI

Mark III

Mt. Wilson, California



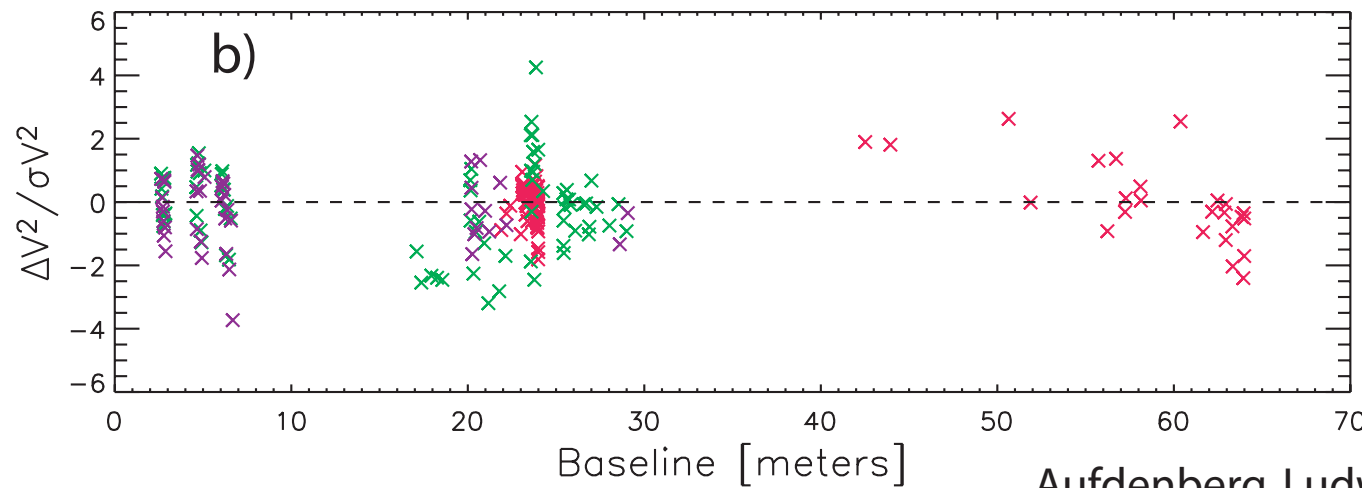
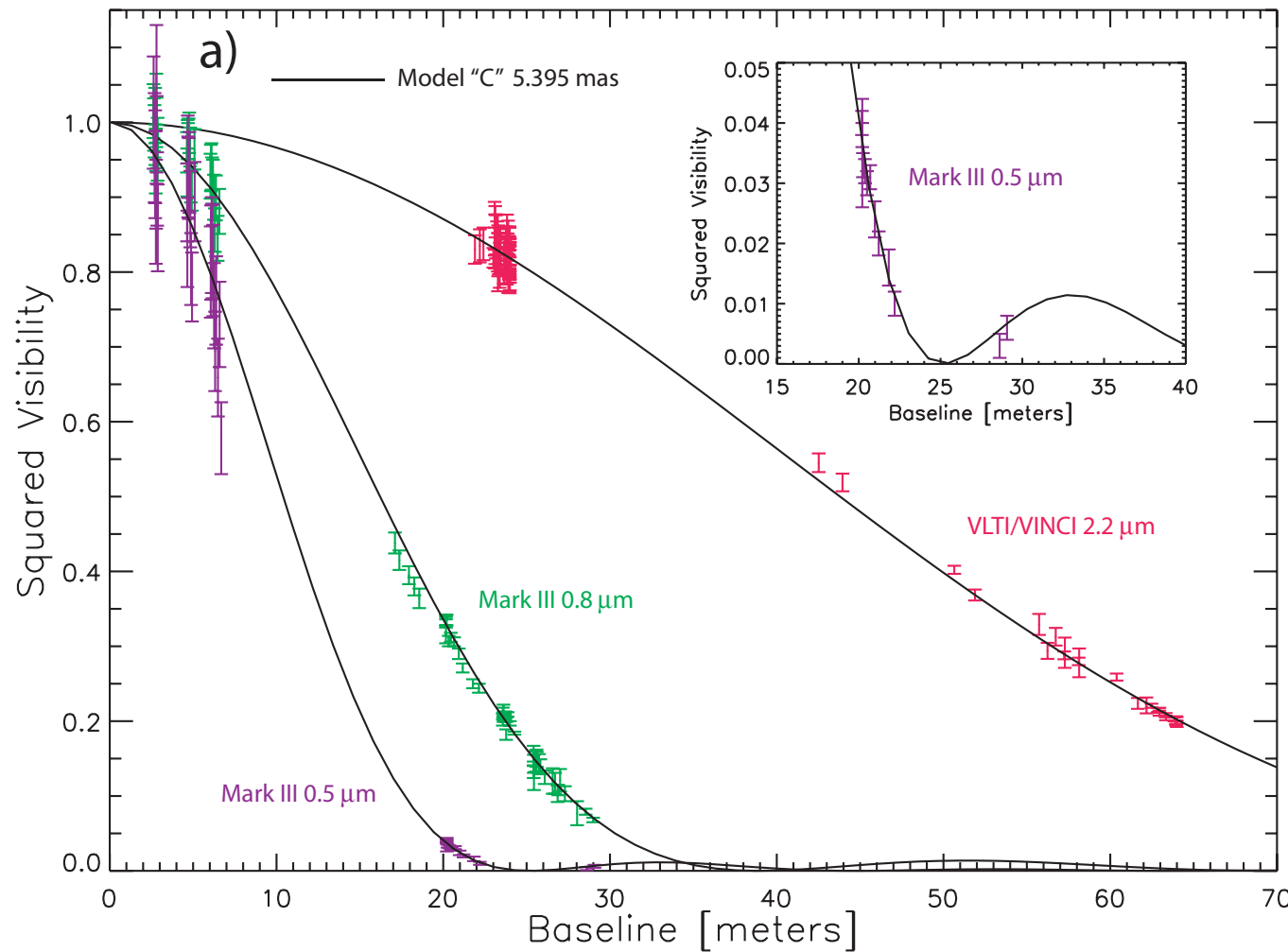
Very Large Telescope Interferometer  
Paranal, Chile



# 3-D CO<sup>5</sup>BOLD Model Atmosphere synthetic visibilities

vs.

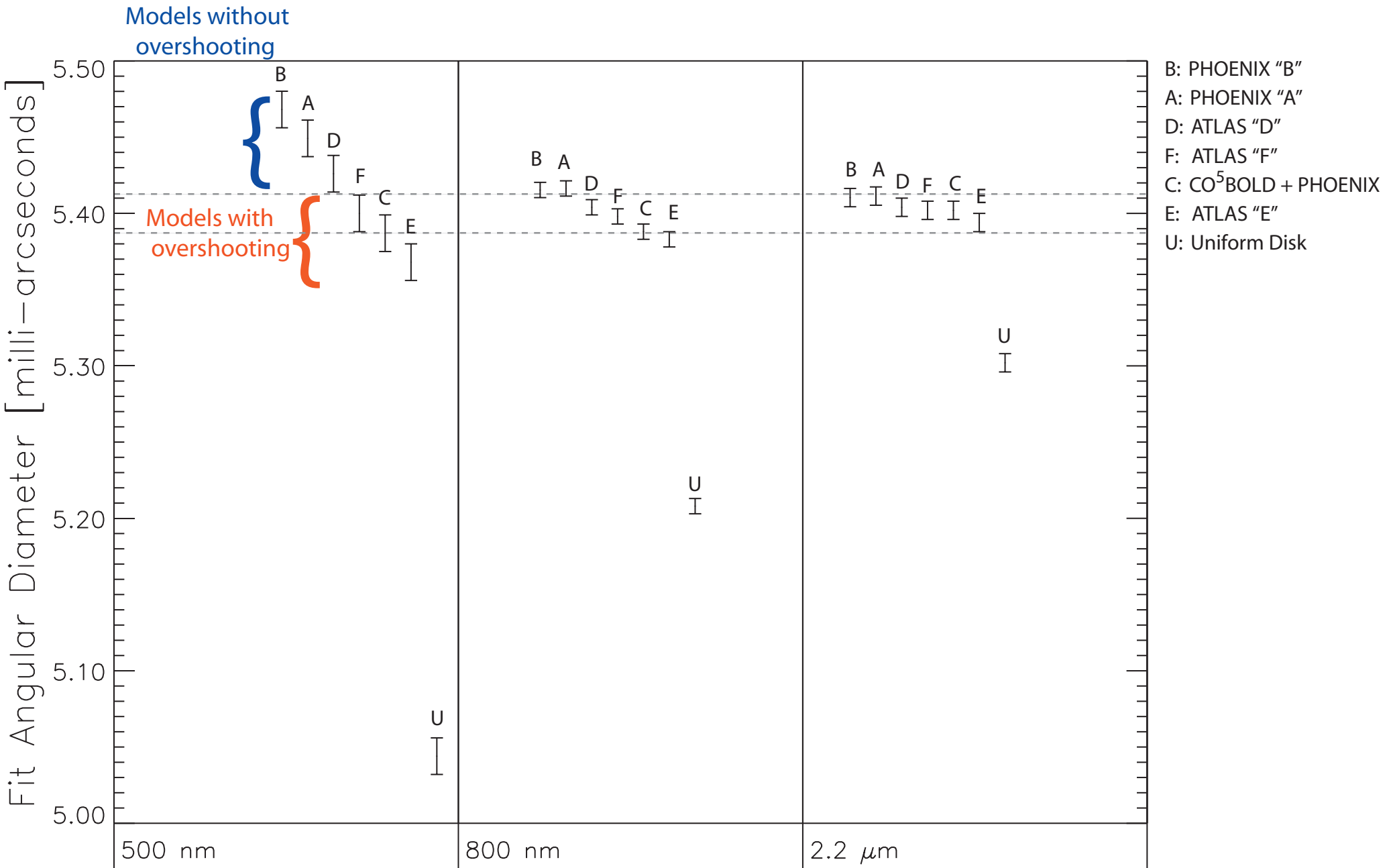
## Mark III 500 nm, 800nm VLTI/VINCI K-band visibility data



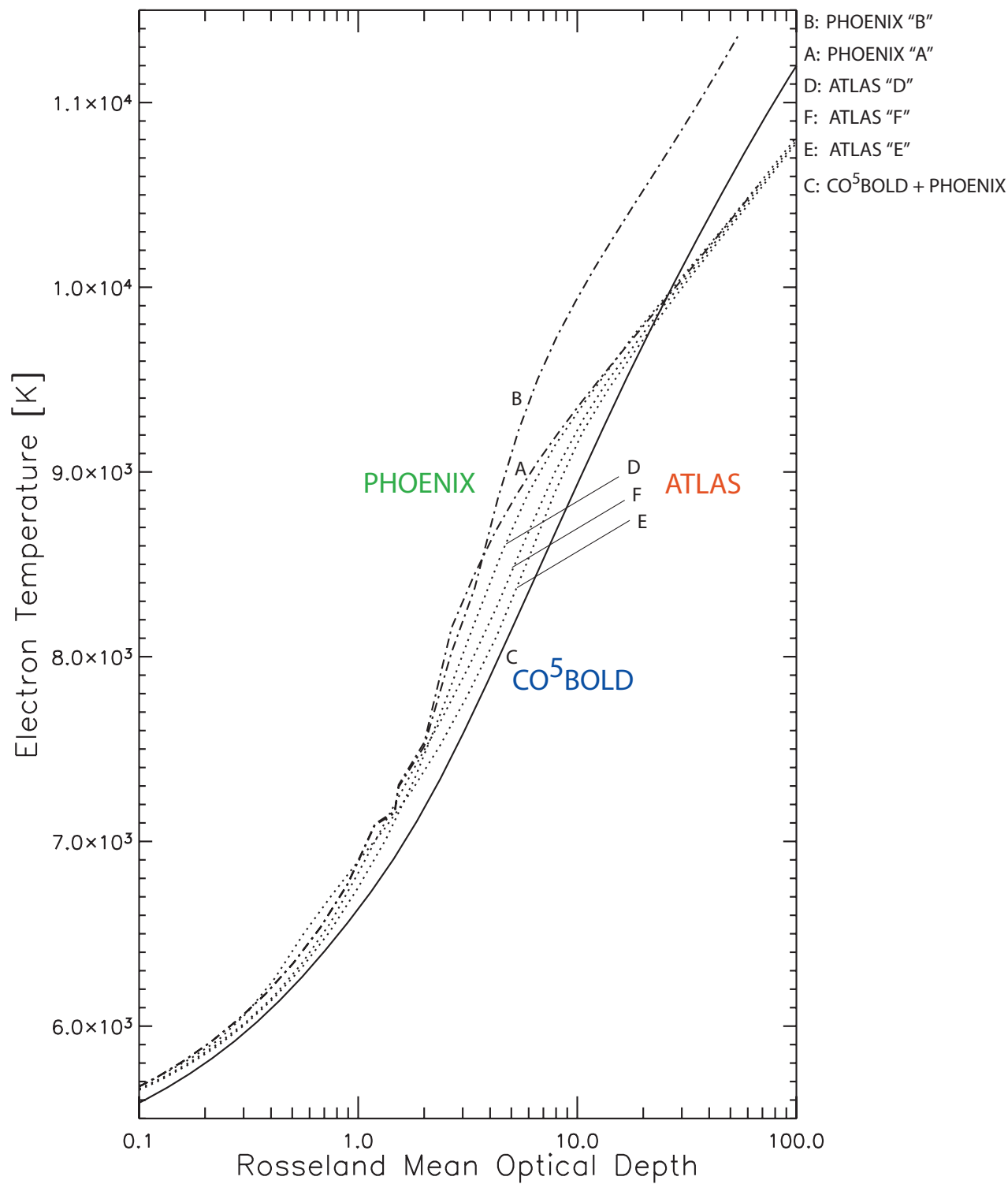
Aufdenberg, Ludwig, & Kervella (2005) ApJ, 633, 424



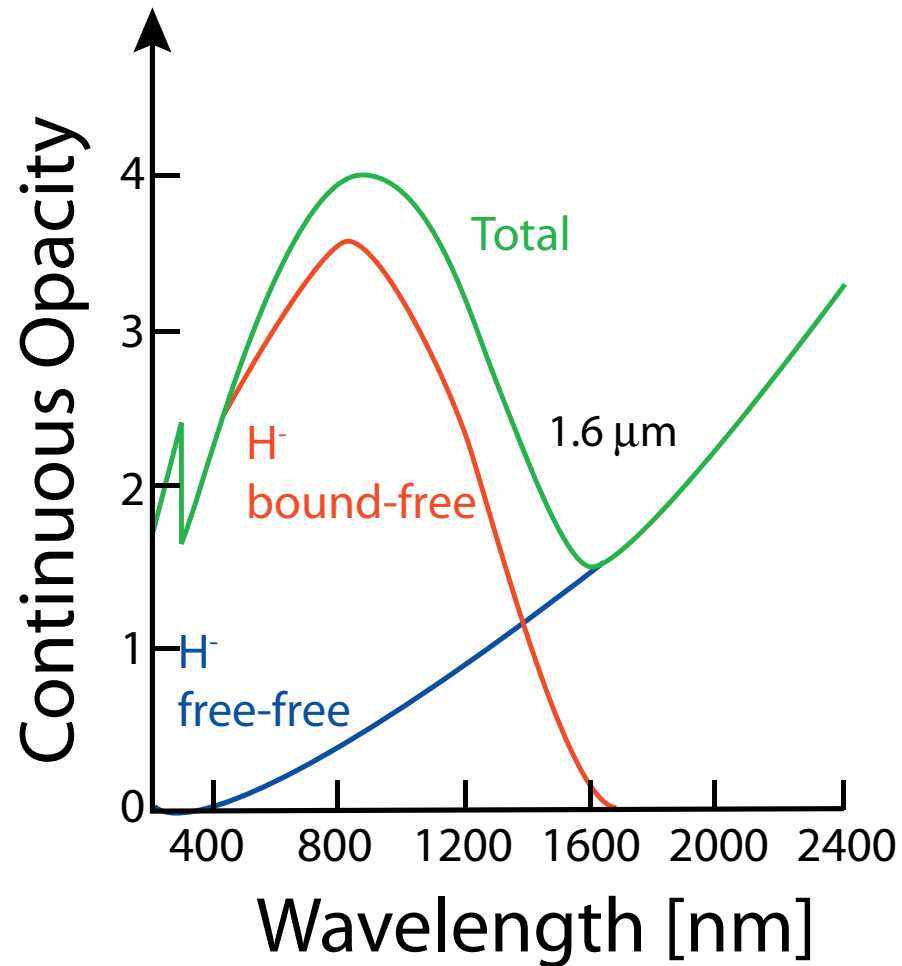
# Angular Diameters from 1-D & 3-D Model Fits



# Comparing 1-D and 3-D Model Temperature Structures



# Wavelength-Dependent Opacity of the Solar-like Atmosphere



# Summary

1. Interferometry is providing unique tests of multi-dimensional stellar atmosphere models
2. Beyond  $T_{\text{eff}}$  and  $\log(g)$ : gravity darkening, convection, winds, sphericity...
3. Vega: Rotating at  $\sim 91\%$  of angular breakup rate; large 2500 K pole-to-equator gradient; convection at equator; debris disk sees cooler SED than we do.

$$\omega = \frac{\Omega}{\Omega_{\text{crit}}} \neq \frac{V_{\text{equ}}}{V_{\text{crit}}} = 2 \cos \left[ \frac{\pi + \cos^{-1}(\omega)}{3} \right]$$

4. Procyon: Limb darkening is convection-dependent; confirmation of 3-D model predictions; multi-wavelength measurements important for atmosphere analyses, go to the blue!
5. Next step: Testing atmosphere models with closure phases.