ExoPlex Group Projects

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For this exercise we will be using the ExoPlex mass-radius-composition solver to try and determine the compositions of the TRAPPIST-1 planets. The mass and radius of the TRAPPIST-1 planets are some of the most precise available (both measured to <10%) due to the nature of the transit timing variation (TTV) methods used to measure these properties.

Each group will be utilizing the same dataset and varying a different planetary parameter. To start, here are the basic data available for the TRAPPIST-1 planets from Grimm et al., 2018 (The Astrophysical Journal):

Planet	Mass [Earth Masses] (+σ/-σ)	Radius [Earth Radii] (+σ/-σ)	Density [Earth Density] (+σ/-σ)
TRAPPIST-1b	1.07 (+0.154, -0.143)	1.121 (+0.031, -0.032)	0.726 (+0.092, -0.091)
TRAPPIST-1c	1.156 (+0.142, -0.131)	1.095 (+0.030, -0.031)	0.883 (+0.083, -0.078)
TRAPPIST-1d	0.297 (+0.039, -0.035)	0.784 (+0.023, -0.023)	0.616 (+0.067, -0.062)
TRAPPIST-1e	0.772 (+0.079, -0.075)	0.910 (+0.026, -0.027)	1.024 (+0.076, -0.070)
TRAPPIST-1f	0.934 (+0.080, -0.078)	1.046 (+0.029, -0.030)	0.816 (+0.038, -0.036)
TRAPPIST-1g	1.148 (+0.098, -0.095)	1.148 (+0.032, -0.033)	0.759 (+0.034, -0.033)
TRAPPIST-1h	0.331 (+0.056, -0.049)	0.773 (+0.026, -0.027)	0.719 (+0.117, -0.102)

Some useful values:
1 Earth mass = $5.97 * 10^{24}$ kg
1 Earth radius = 6371 km
Earth Density = 5512 kg m^{-3} = 5.512 g cm^{-3}

ExoPlex Mantle Validity Ranges (all values by mole):
$0.5 \leq Si/Mg \leq 2.0$ (steps of 0.1)
$0.02 \leq Ca/Mg \leq 0.1$ (steps of 0.01)
$0.04 \leq Al/Mg \leq 0.12$ (steps of 0.01)
$0.0 \leq$ Mass Fraction FeO ≤ 0.20 (0.0 – 0.1 steps of 0.02, 0.15, 0.2)
1400 K ≤ Mantle Potential Temperature ≤ 2000 K

For these exercises each group will be changing a planetary compositional parameter and calculating what range of these parameters are consistent with the observed densities of the TRAPPIST-1 planets.

For each model, every parameter can be changed in MR_Trappist.py. There is a duplicate file included for ease in undoing mistakes named duplicate_MR_Trappist.py.

All text editing will be done using the text editor nano and a Jupyter notebook. NOTE: if you want to make your data tables in Excel, that's totally okay!

Team Mantle Chemistry 2 (Fe)

1. Begin by making sure you are in the ExoPlex/Example folder in the terminal (cd ExoPlex/Examples).
   Make sure you activate the ExoPlex Python environment (source activate ExoPlex).

2. Let's begin by building a planet. To do this you need only to decide a composition (providing values for CaMg, FeMg, Mantle temperature etc.) and how many depth slices you'd like to appear in the core, mantle and water layers (num_core_layers, num_mantle_layers and number_h2o_layers). Note that if you do not include a water layer, number_h2o_layers should be set to zero.

    a. Type **python MR_Trappist.py** and press Return. A plot should appear that shows you the density, pressure, gravity and temperature profile within your planet. This is for a two-layer planet (mantle + core) with the same mass as TRAPPIST-1b and an Earth composition (note: no water). Write down the calculated radius for this mass and composition. This will be your baseline mass (for a T1b size planet).
        i. If you'd like to plot this again (minus gravity), open the Jupyter notebook Plots.ipynb in a separate terminal (ssh into the AWS instance, cd ExoPlex/Examples, type 'jupyter notebook --no-browser', copy the URL into a local browser window, launch Plots.ipynb).
    b. Now try changing things! Add water. Change FeMg and SiMg. Each time you change something, rerun 'python MR_Trappist.py' and see if you notice any changes. Compare your plots with those from others within your group and see if you can find changes. (Make a note of your changes so you can reset to the Earth composition). Make sure to change your filename to describe what you changed so you can compare. Feel free to use Plots.ipynb to plot pressure, temperature and density for your new planets.
    c. Write down what you've changed, and some of your observations on how the calculated radius is changing and other aspects of the planetary interior (core radius, pressure at middle of core etc.). Once you've tried a few different compositions do you notice any trends? Note these down!
        i. If ExoPlex breaks, double check that you haven't gone outside the compositional bounds and rerun. If it's still breaking, let me know and we'll find the fix.
    d. Go ahead and reset your compositions back to where they started. In case it is needed, I've provided a duplicate file with the default values (duplicate_MR_Trappist.py). Just create a copy of the duplicate file to get a reset version of MR_Trappist.py.
    e. If you don't want to see this plot EVERY time, follow instructions after the line that says "#Now let us plot"', roughly line 171 in MR_Trappist.py

3. Now that we have a sense of what things can happen, let's focus on your group's individual parameter. Begin with TRAPPIST-1 b and assign different team members the other planets.

a. It'd be naïve to think that planetary mantles all have the Earth's composition. The Earth and Mars differ in mantle Fe content by a factor of two.
    i. Your group's task is to change the mantle chemistry, but only by adjusting the mantle iron content (all other elements are for another group). By including Fe in the mantle, it must be removed from the core (in order to conserve moles). In doing so, the density of the entire planet is different, having an effect on our mass-radius-composition interpretations.
    ii. You have access to one parameter to vary in your mantle: mol_frac_Fe_mantle. This parameter essentially outlines what fraction of all the iron in a planet ends up in the mantle (as FeO) instead of the core. The maximum is roughly mol_frac_Fe_mantle = 0.2 currently.
b. Run ExoPlex for 10 different mol_frac_Fe_mantle. Make sure to not go beyond the compositional range of ExoPlex noted above. If you're proficient in Python, you can write a loop to do this for you. If not, you can copy things by hand into the Plots.ipynb "individual runs" section of the Jupyter notebook.
    i. Keep track of the resulting radii and your input mol_frac_Fe_mantle for each element; these will be used in 4a and b.
c. What are you noticing about the core mass fraction reported in the terminal as you increase mol_frac_Fe_mantle?
d. How much do you estimate mol_frac_Fe_mantle must change in order to change the radius by ~2% for each planet? Is a 5% increase possible? 10%?
e. Which mol_frac_Fe_mantle looks like the best fit for each TRAPPIST-1 planet? Write these values down for each T1 planet below.

Planet	Best-fit Mantle Fe content (mole fraction)
TRAPPIST-1 b	
TRAPPIST-1 c	
TRAPPIST-1 d	
TRAPPIST-1 e	
TRAPPIST-1 f	
TRAPPIST-1 g	
TRAPPIST-1 h	

4. Let's be more precise with our determination of "best-fit" mol_frac_Fe_mantle content. To do this we will adopt a chi-squared test. We define this test by:

$$\chi^2 = \frac{(Radius\_Actual - Radius\_Model)^2}{Radius\_Uncertainty^2}$$

a. When $\chi^2 \leq 1$, this is considered a good fit. For each of the radii you calculated in 3b, calculate χ^2. Either automate this using Python, or proceed by hand. Make sure to do this for each T1 planet.

b. Input these χ^2 values into the chi-squared section of Plots.ipynb. Note to keep track of your x-axis (mol_frac_Fe_mantle) and y-axis (χ^2).
c. You may find MANY compositions produce $\chi^2 \leq 1$. What are the minimum and maximum mol_frac_Fe_mantle that produce $\chi^2 \leq 1$? Is mol_frac_Fe_mantle = 0.2 large enough?
    i. It is also possible to find no amount of variation in your parameter causes χ^2 to fall below one. This just means that our model is insensitive to changes in your parameter. Does this mean we can constrain this aspect of a planet's composition using only mass and radius? Do you think if you were able to increase mol_frac_Fe_mantle for these planets the fit would be better?
    ii. Fill out the table below, and make sure to include it in your talk!

Planet	Min. Mantle Fe Mole Fraction ($\chi^2 \leq 1$)	Max. Mantle Fe Mole Fraction ($\chi^2 \leq 1$)
TRAPPIST-1 b		
TRAPPIST-1 c		
TRAPPIST-1 d		
TRAPPIST-1 e		
TRAPPIST-1 f		
TRAPPIST-1 g		
TRAPPIST-1 h		

5. Would you say the TRAPPIST-1 planets are consistent with being "Earth-like?"

Complete 6 and 7 if you have time.

6. Unfortunately planetary mass has uncertainty that we aren't accounting for in the above models. In order to account for this we must randomly sample the mass within the observational uncertainty.
    a. Currently Mass_planet_sigma (line 43) is set to zero. Change this value to your individual T1 planet's respective mass uncertainty (available in the comments of MR_Trappist.py).
    b. Now instead of running a single iteration, let's run 50 total samplings. Change number_of_runs (line 85) to 50.
    c. Currently MR_Trappist.py is set up to save a file that contains the data produced from all 50 runs. If you'd like to give this file a special name, change Output_filename (line 83) to whatever you like. Note the code automatically adds ".txt" to the end.
    d. Run MR_Trappist.py for your planet and maximum and minimum best fit compositions you found above (100 runs total, 50 for max, 50 for min).
    e. In Plots.ipynb, upload this datafile and plot the respective histogram.

i. How much does the radius vary for this "best-fit" composition just from the uncertainty in mass? Is this variation larger or smaller than the uncertainty on the radius itself?

7. If you have even more time, calculate and plot χ^2 for each of these 100 runs (same as 4a-c). Did your range of "best fit" compositions expand? If so, how much?

Planet	Min. Mantle Fe Mole Fraction ($\chi^2 \leq 1$)	Max. Mantle Fe Mole Fraction ($\chi^2 \leq 1$)
TRAPPIST-1 b		
TRAPPIST-1 c		
TRAPPIST-1 d		
TRAPPIST-1 e		
TRAPPIST-1 f		
TRAPPIST-1 g		
TRAPPIST-1 h		