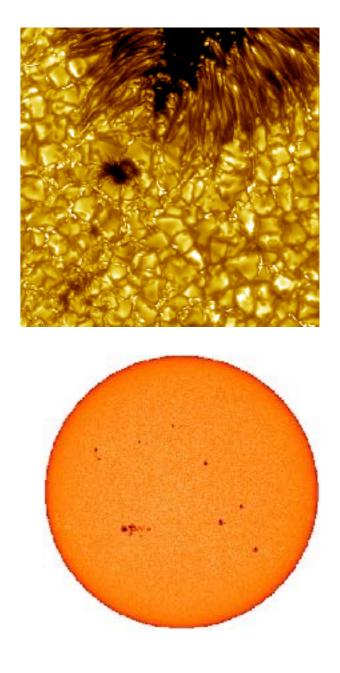
Perils of false signals

Stellar activity or Earth-mass planet?

Raphaëlle D. Haywood Sagan Fellow, Harvard College Observatory Accounting for stellar activity (or any kind of unknown correlated systematics) in RV analyses

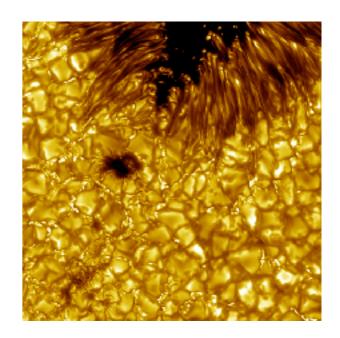


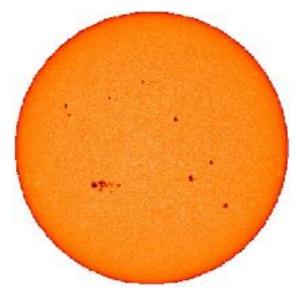
Granulation: Swedish Telescope, V. Henriques

Full Sun: SDO/HMI continuum

Accounting for stellar activity (or any kind of unknown correlated systematics) in RV analyses

• The main **goal** of RV follow-up is to determine accurate, precise planet masses, in the presence of stellar activity



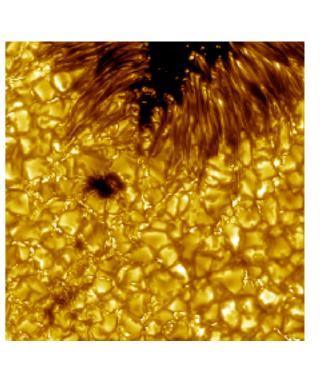


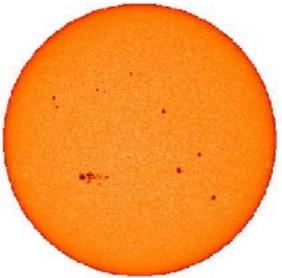
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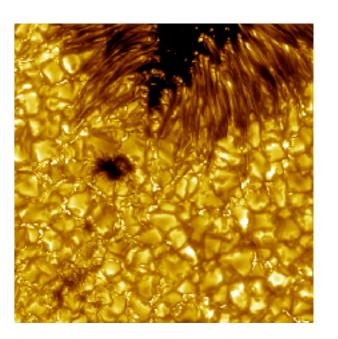
Full Sun: SDO/HMI continuum

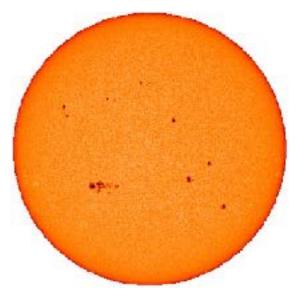
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- Often, we do not know enough about the physical processes responsible for activity-driven RV variations, so we cannot **model** them.





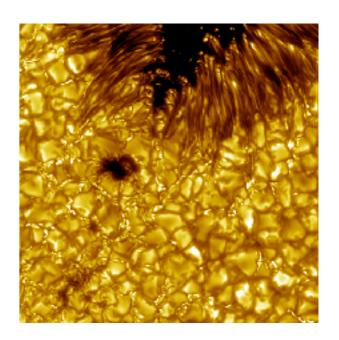
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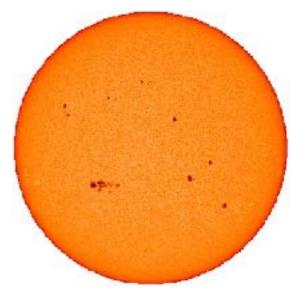




Full Sun: SDO/HMI continuum

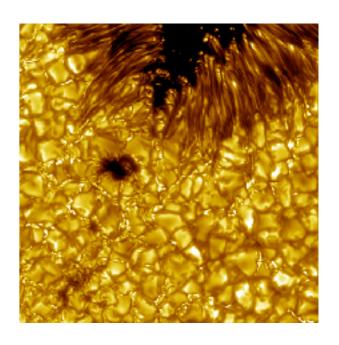
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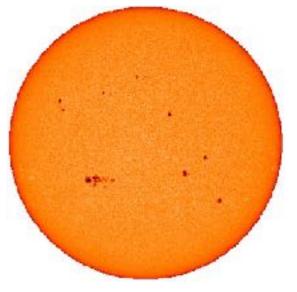




Full Sun: SDO/HMI continuum

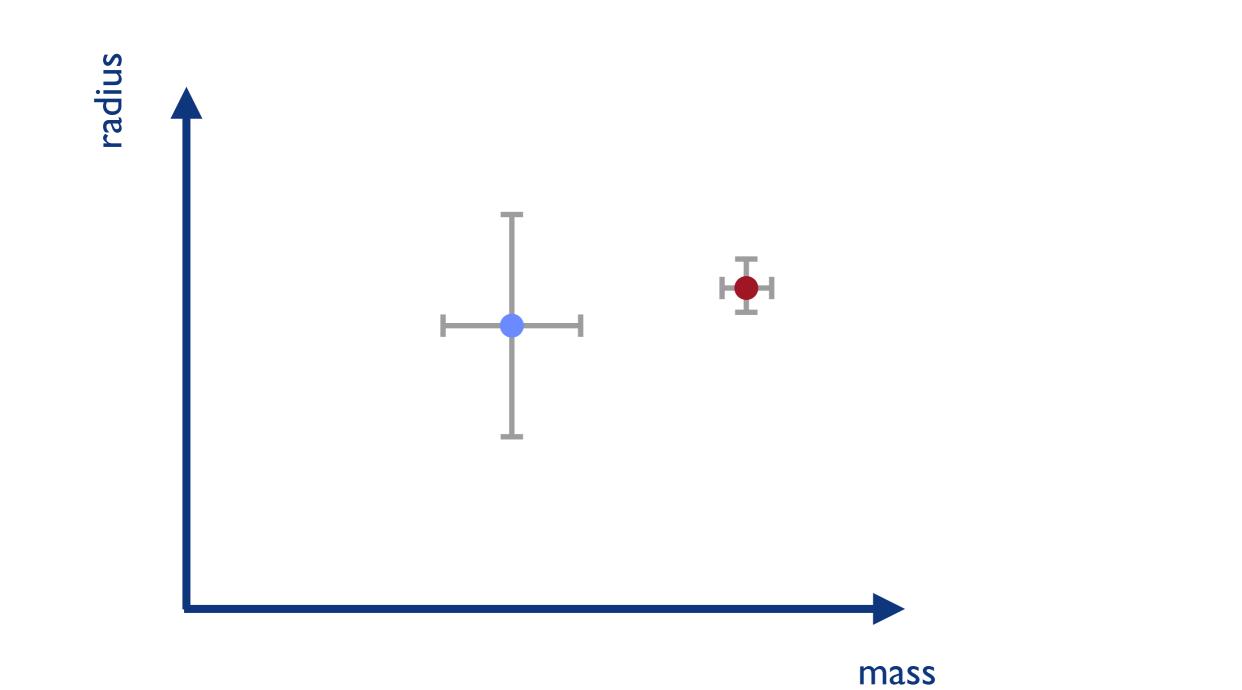
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- Solution: we account for the uncertainty induced by stellar activity by treating activity as **noise**.
- Stars are rotating and their surfaces are constantly evolving: we must treat their activity as correlated noise.

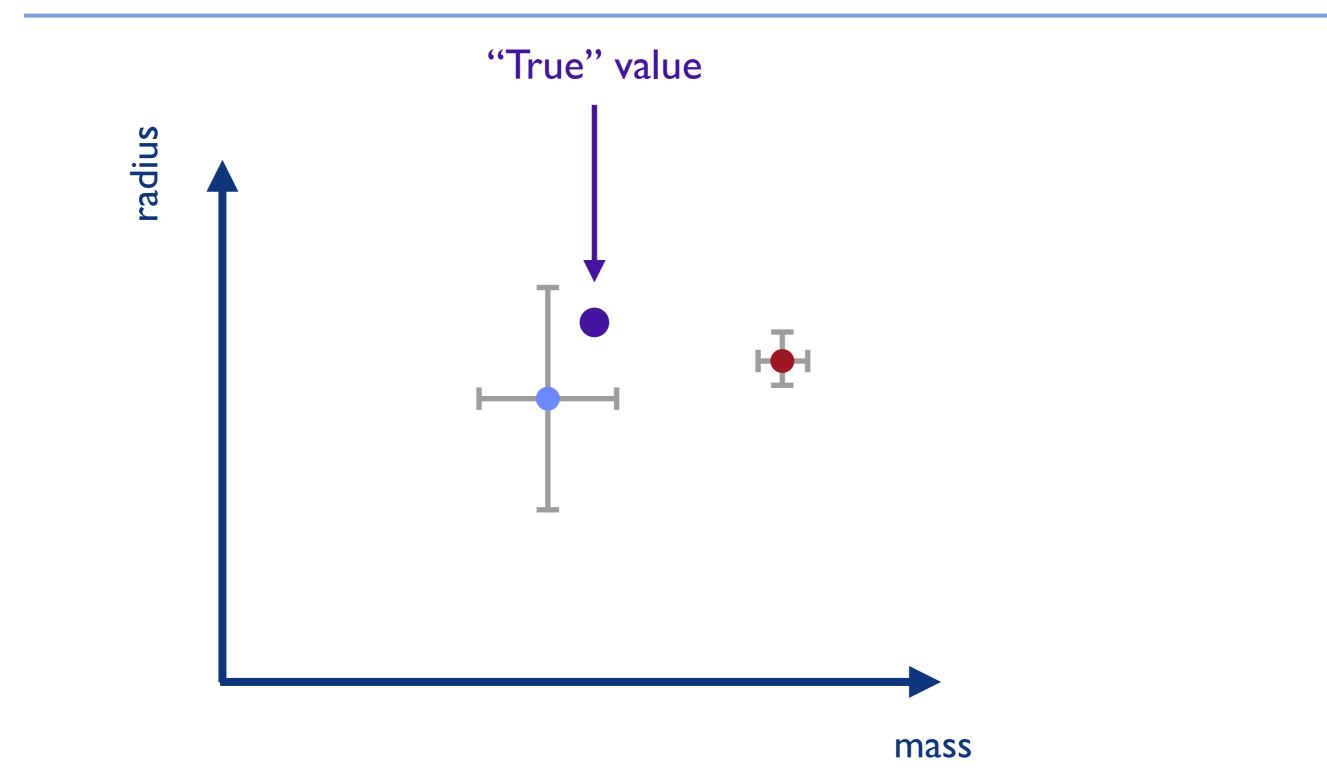


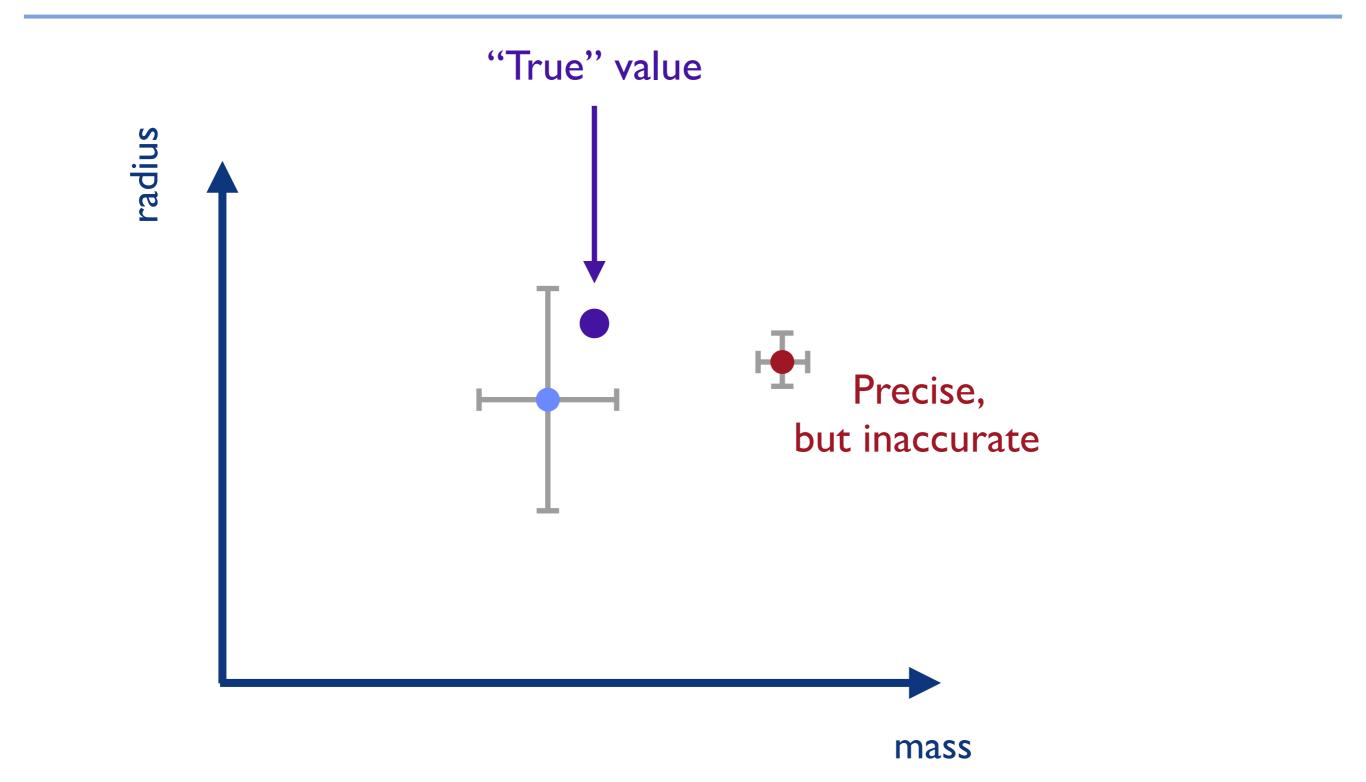


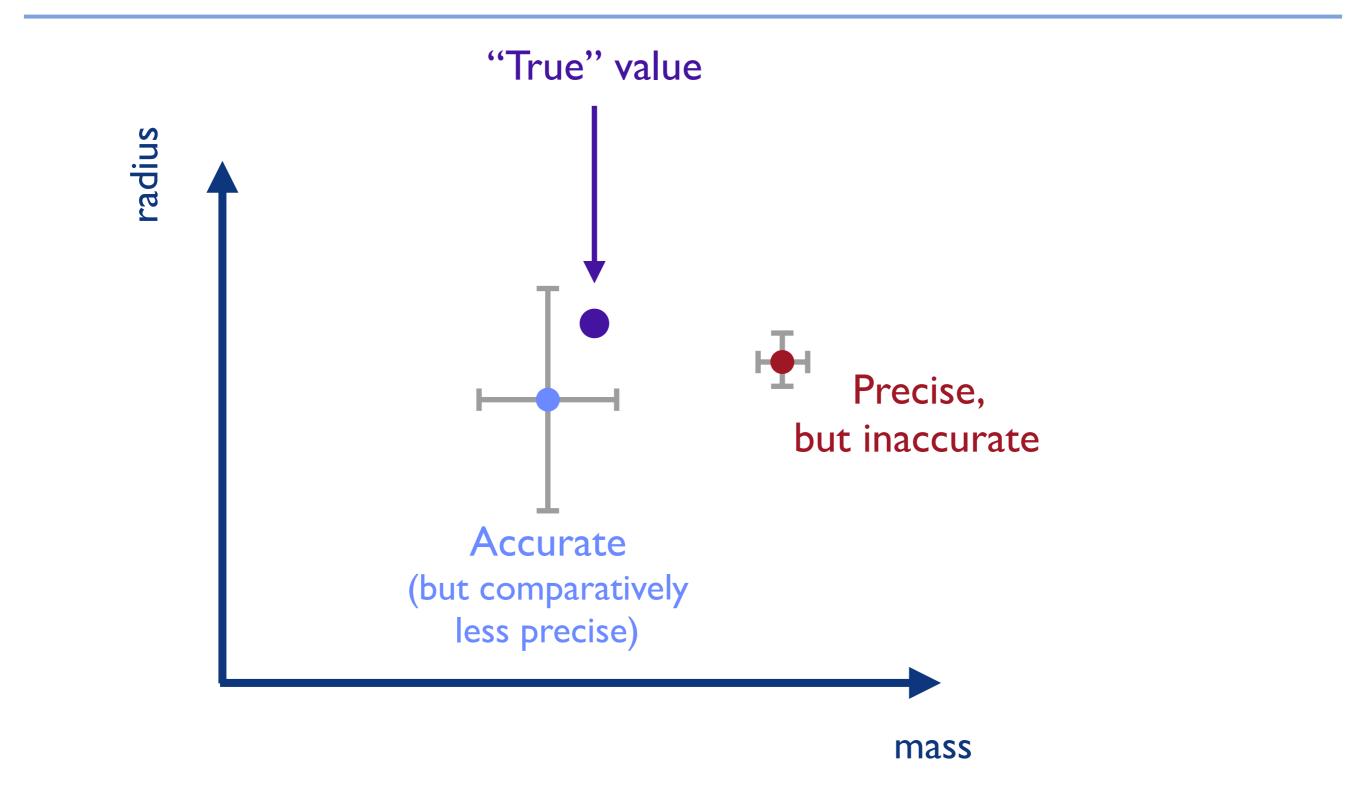
Outline

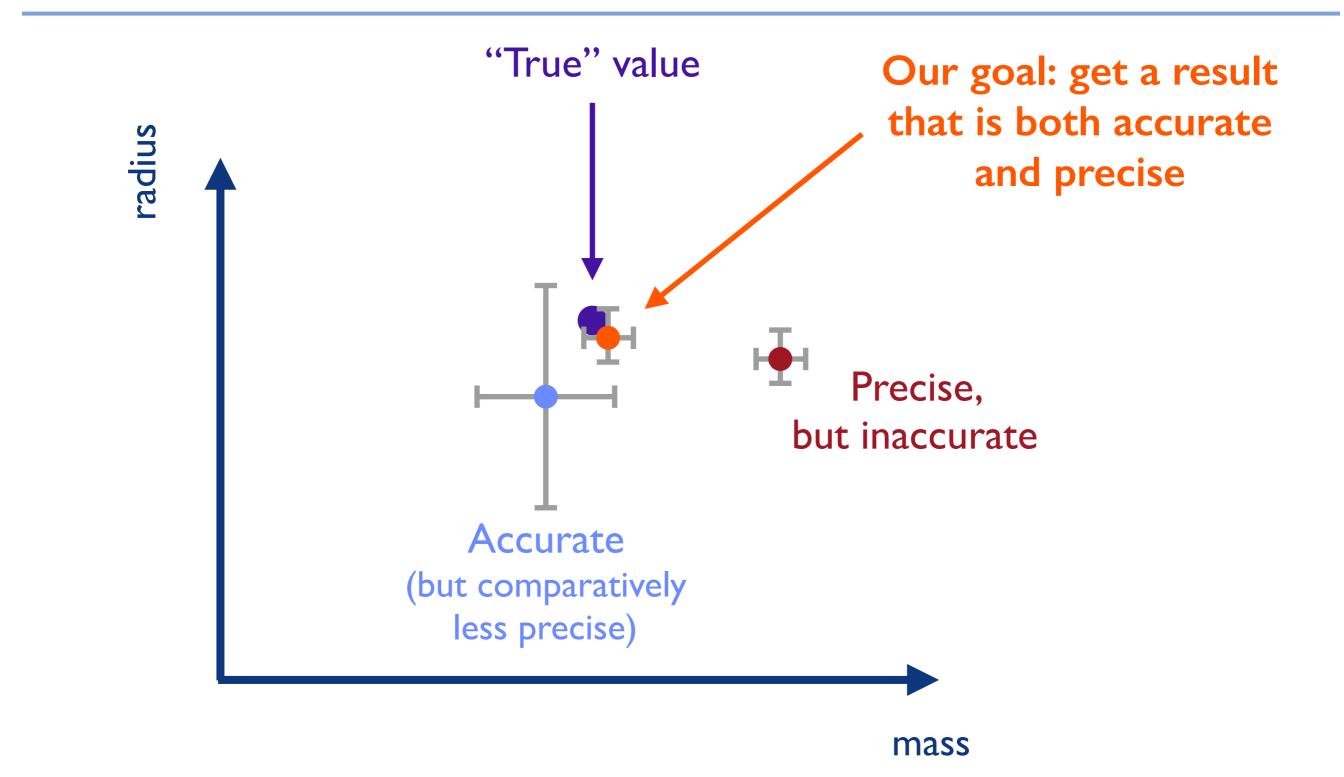
- Accuracy and precision
- Signal and noise
- Uncorrelated and correlated noise
- (A brief intro to) Gaussian process regression
- Astrophysically motivated GPs to account for stellar activity in RV observations
- Conclusions
- References for further reading/coding











I. Signal: carries the information of interest (eg. the planet's reflex motion)

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→ We account for **noise** in our **goodness** of fit (the chi square or the likelihood)

No noise:

$$\chi^2 = \frac{(\text{data-model})^2}{\text{errorbars}^2}$$

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Uncorrelated noise (Gaussian-distributed, "white" noise, "jitter"):

$$\chi^{2} = \frac{(\text{data-model})^{2}}{\sqrt{(\text{errorbars}^{2} + \text{jitterterm}^{2})}}$$

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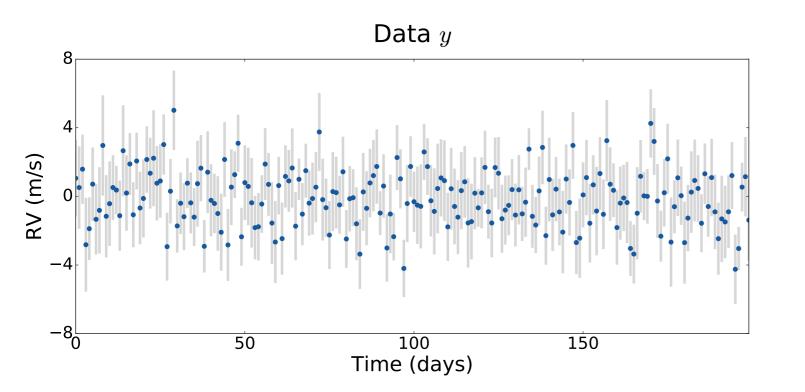
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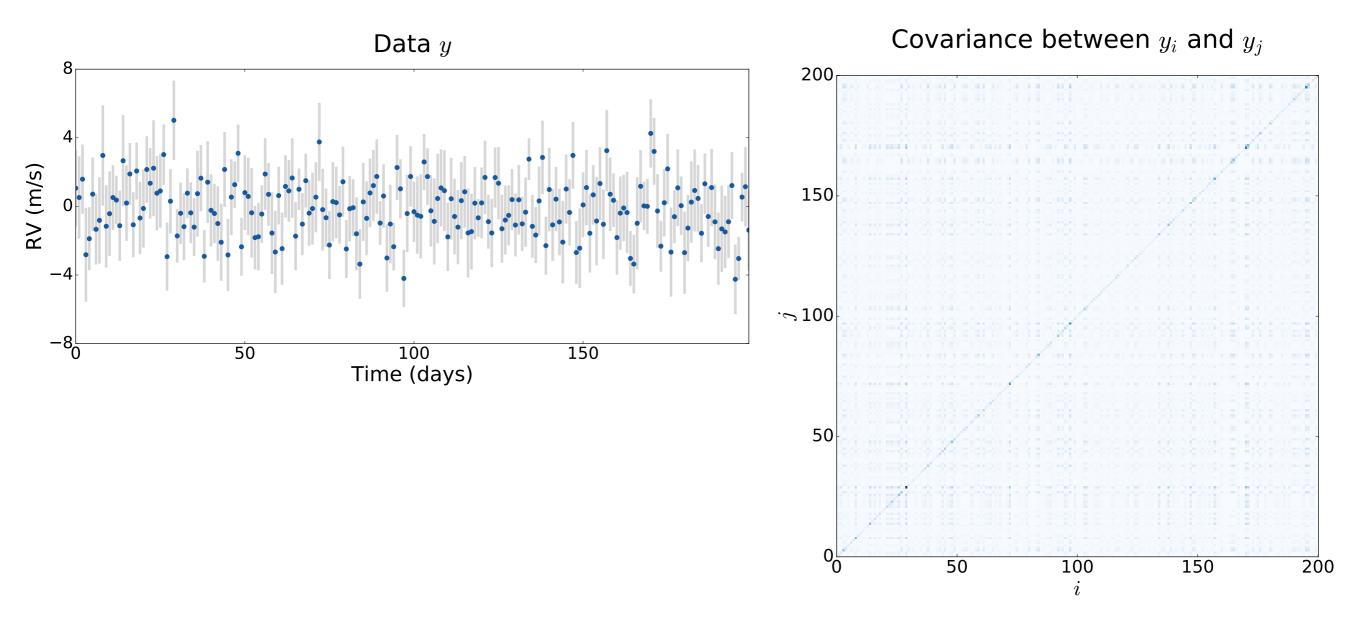
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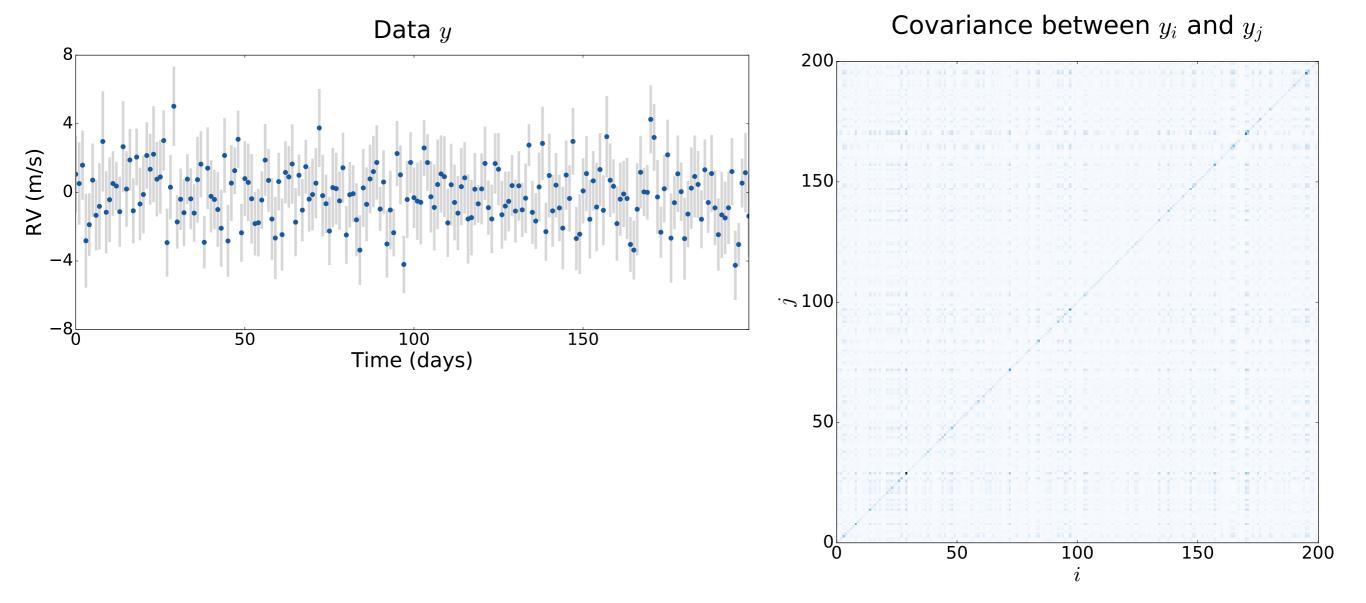
Correlated noise (non-random, "red" noise):

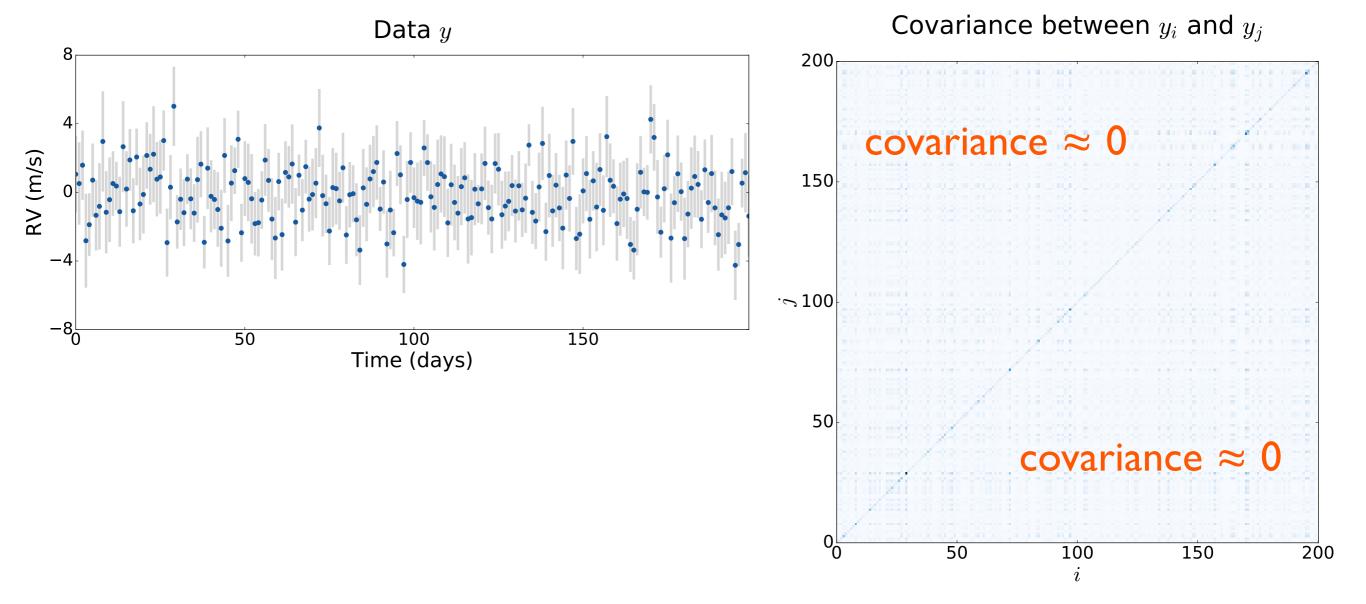
 $\mathcal{L} \neq \exp(-\chi^2/2)$

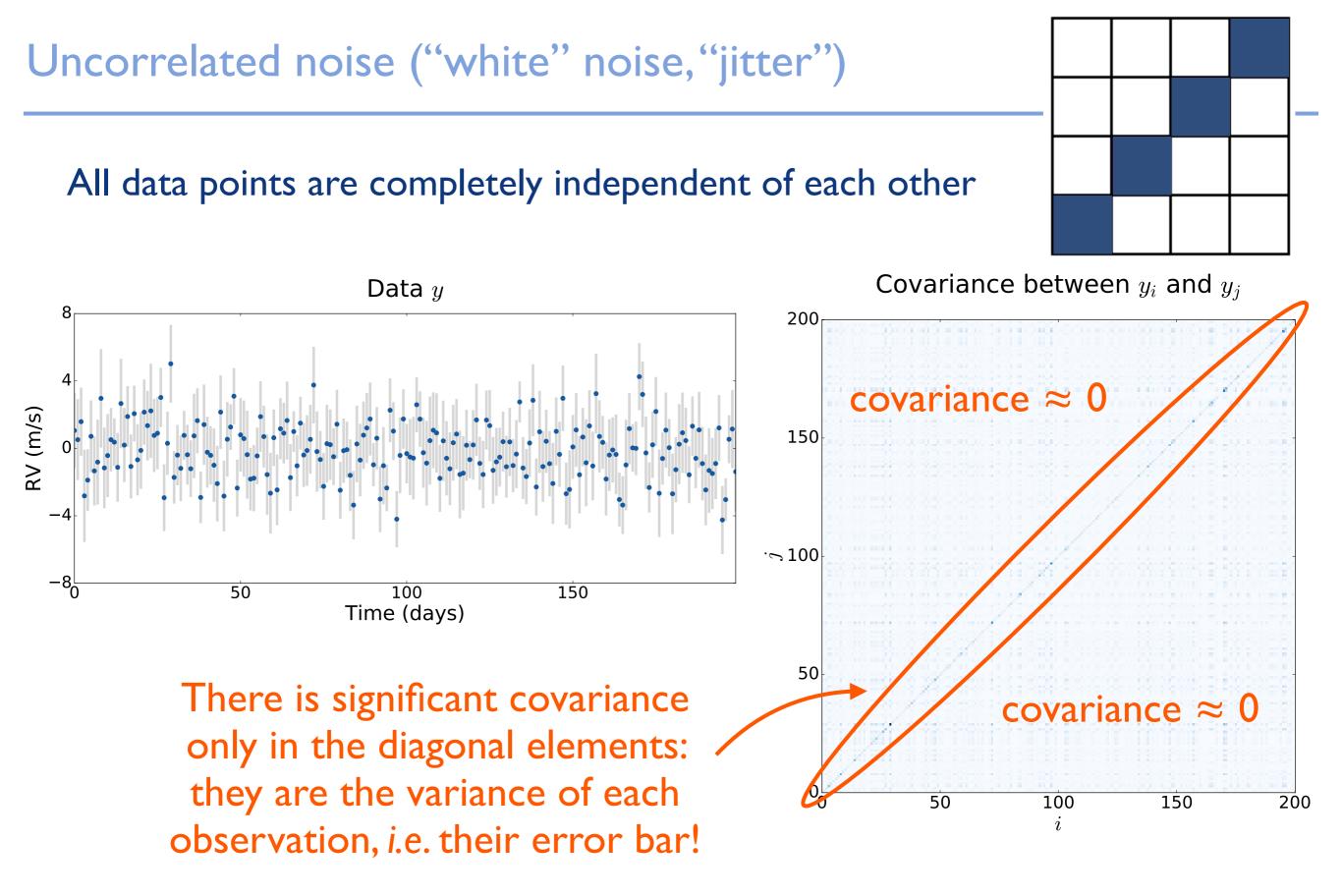
 $= \exp(-\chi^{2}/2)$



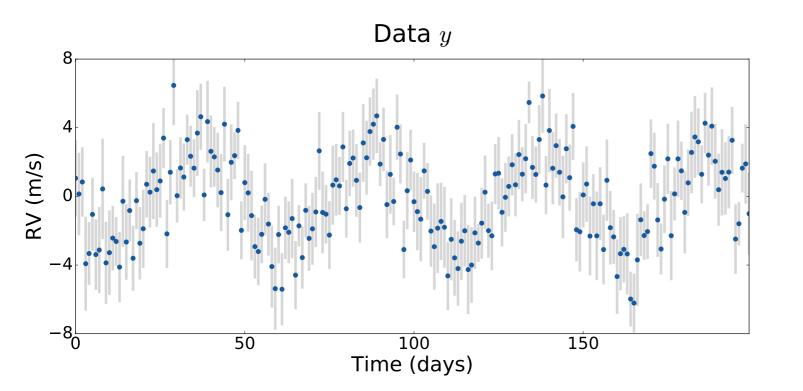




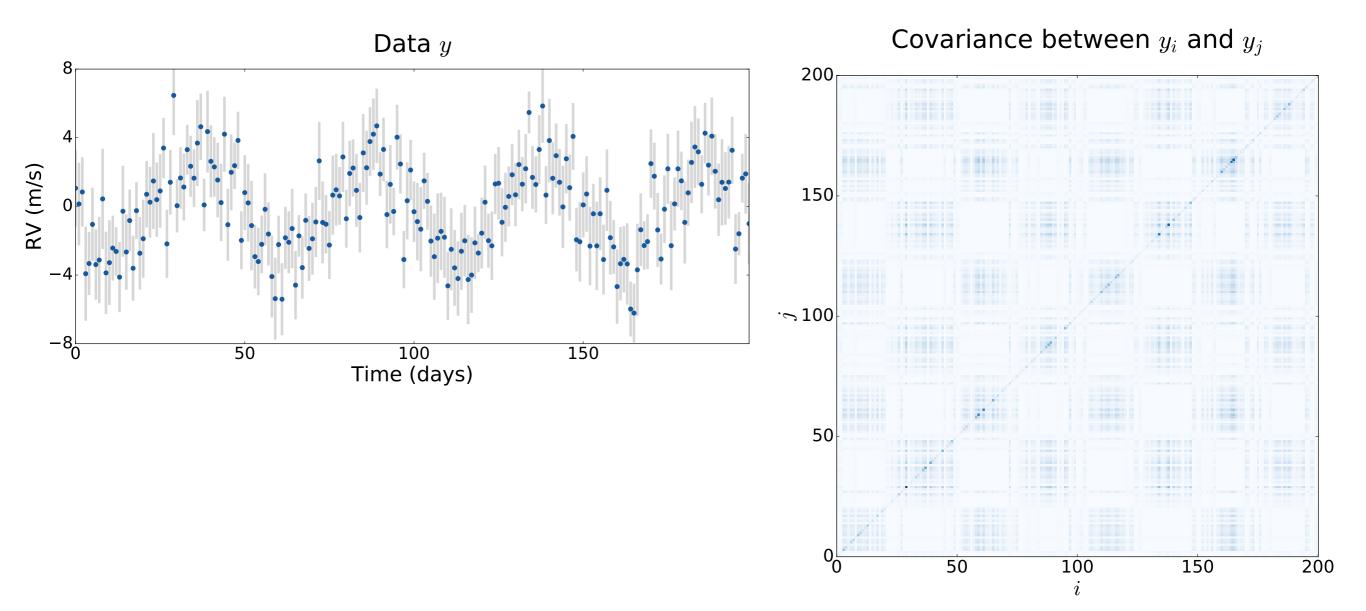




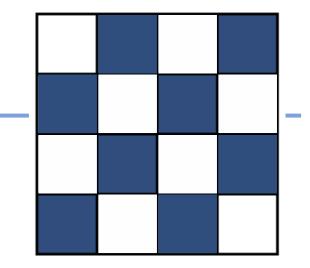
Data points are correlated with each other

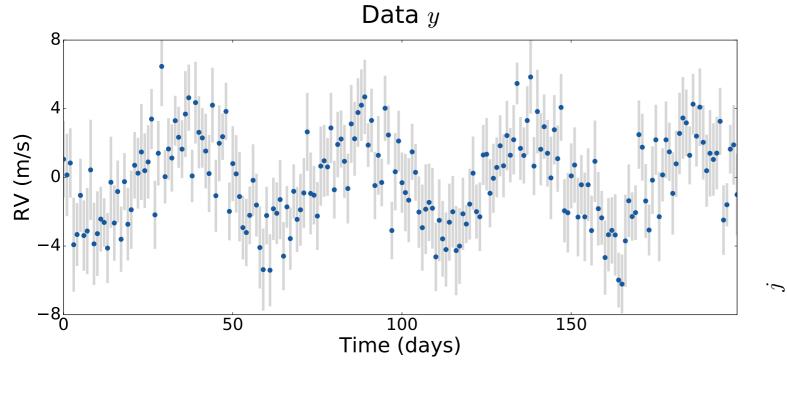


Data points are correlated with each other

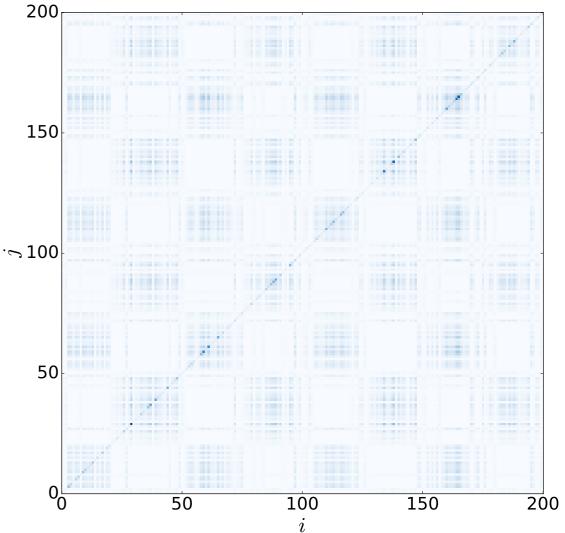


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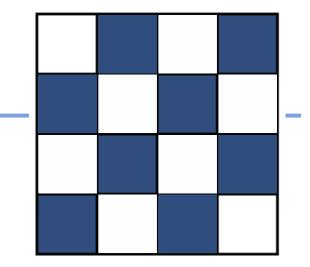


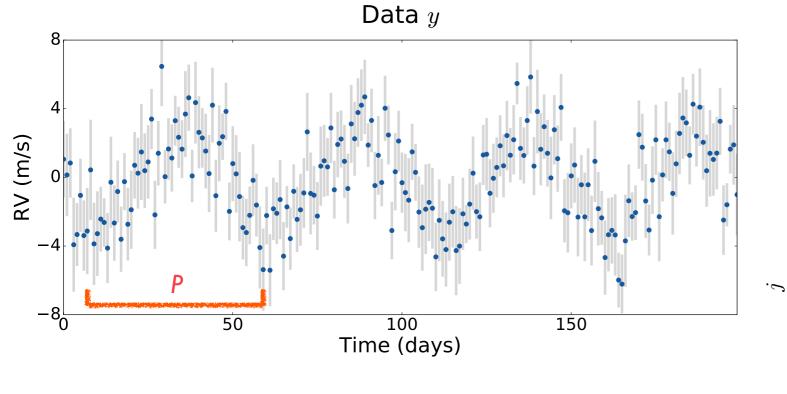


Covariance between y_i and y_j

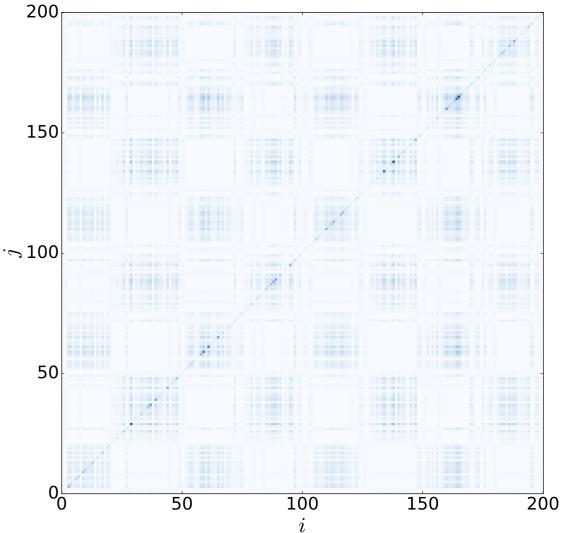


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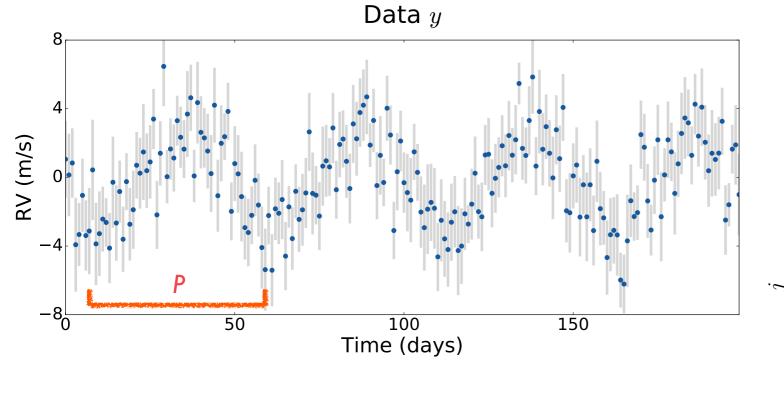




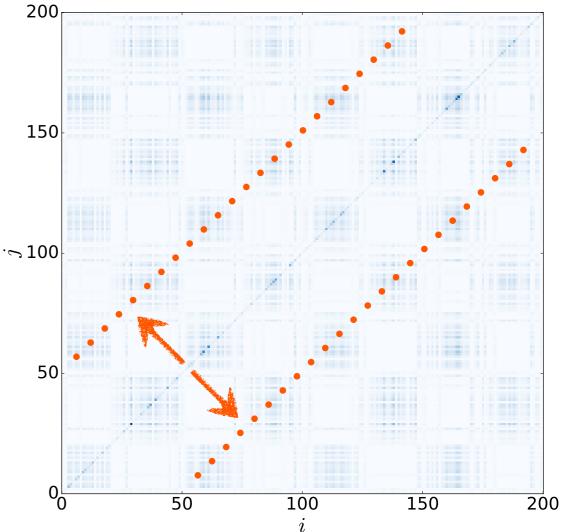
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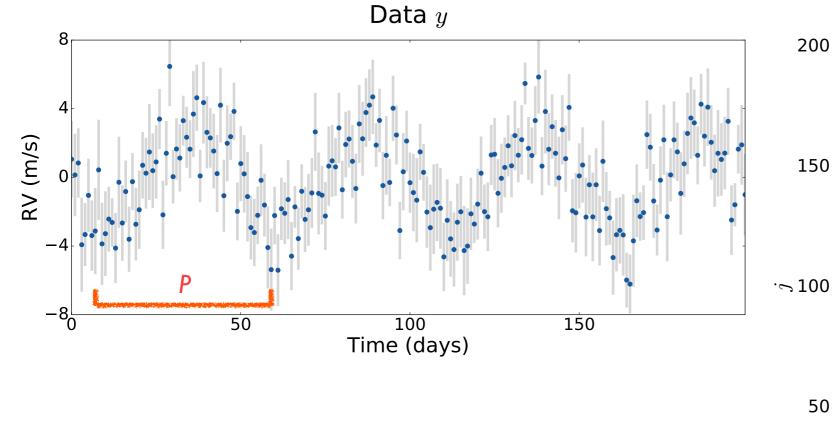
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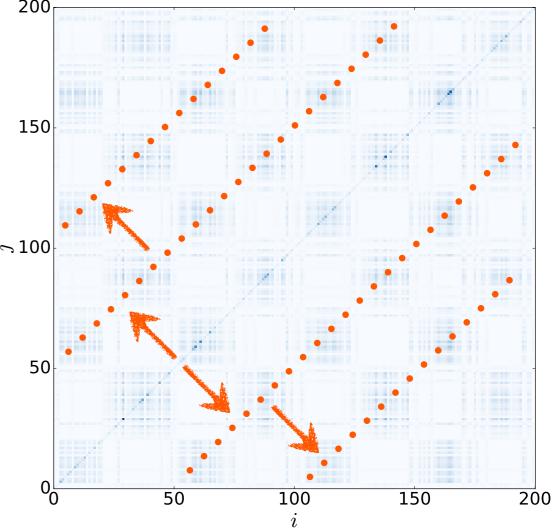
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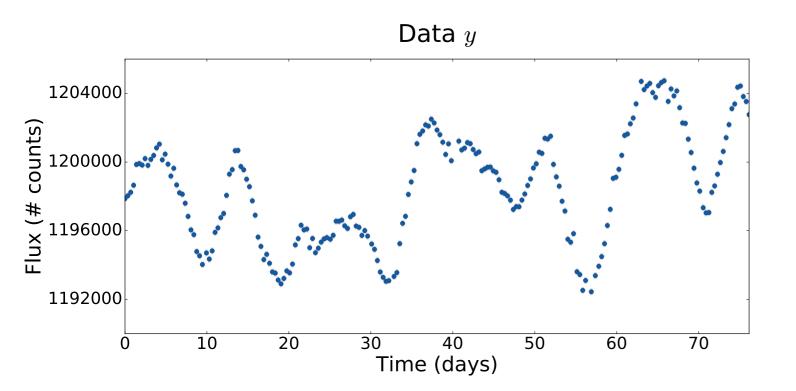


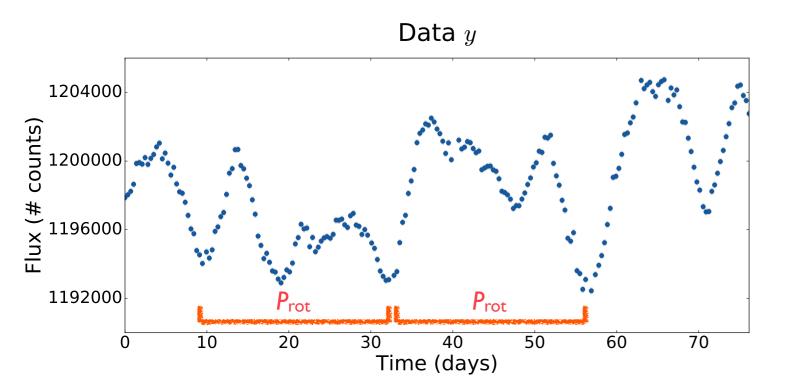
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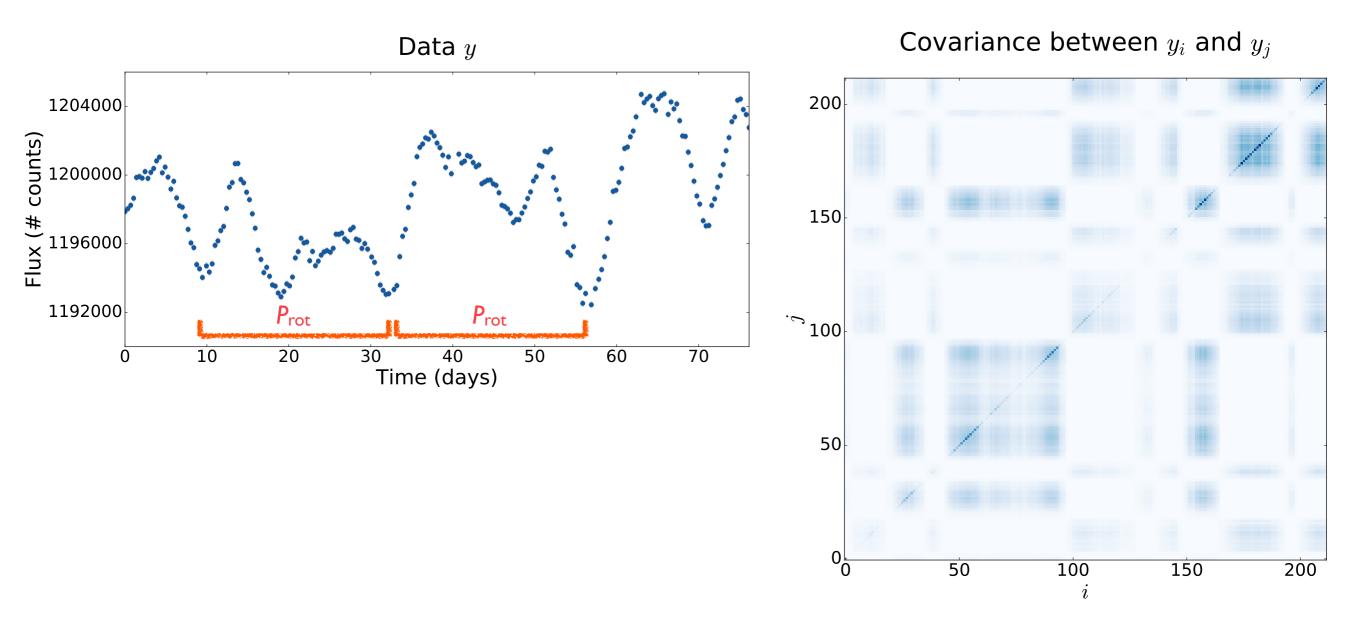


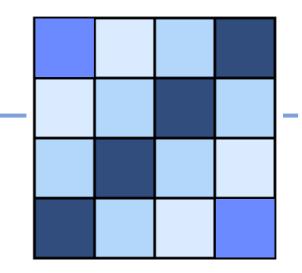
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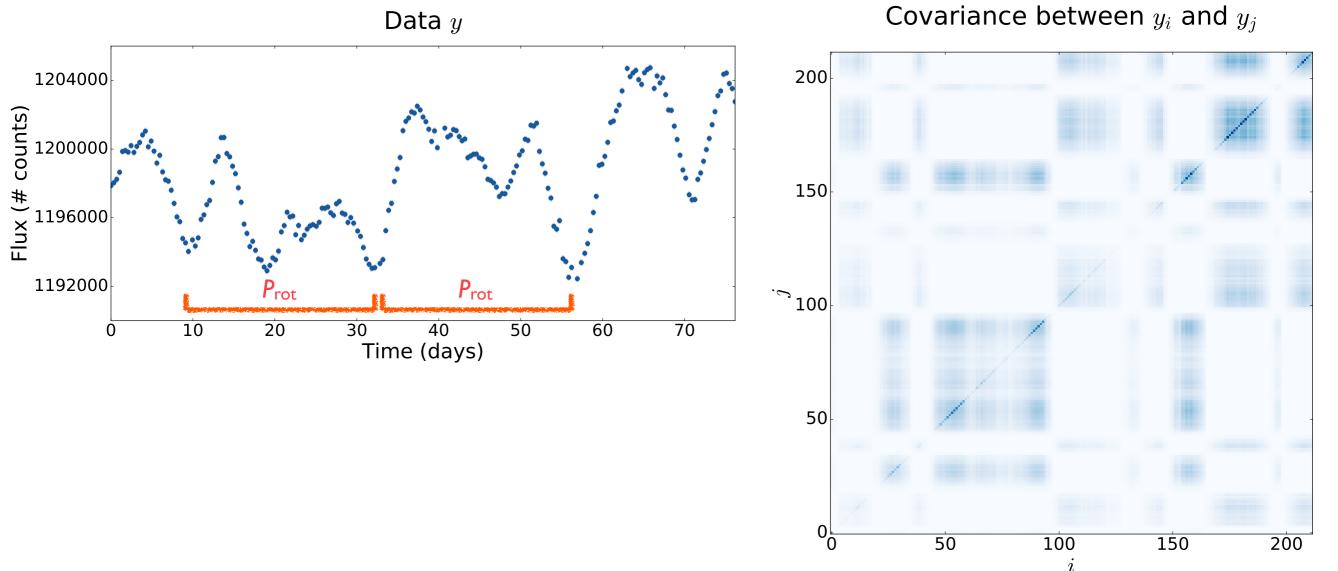


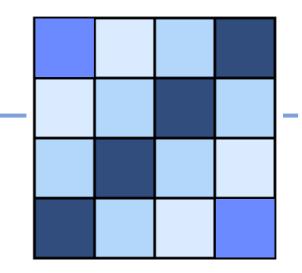


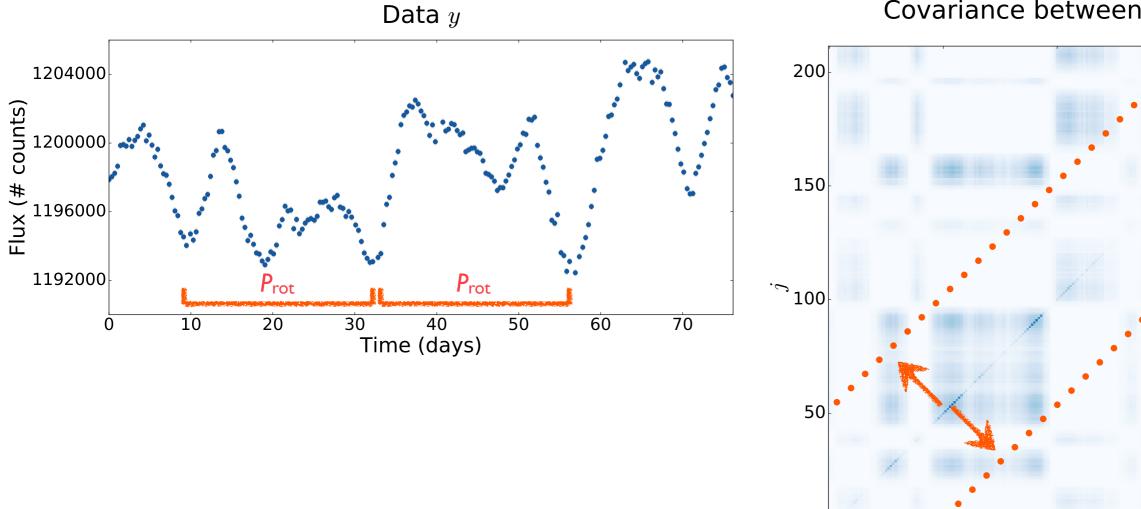












Covariance between y_i and y_j

See Rassmussen & Williams (2006), Haywood (2015, Chap. 2) and others

50

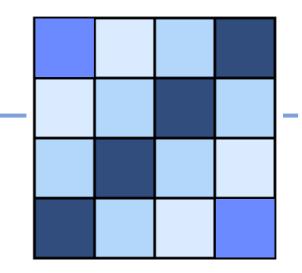
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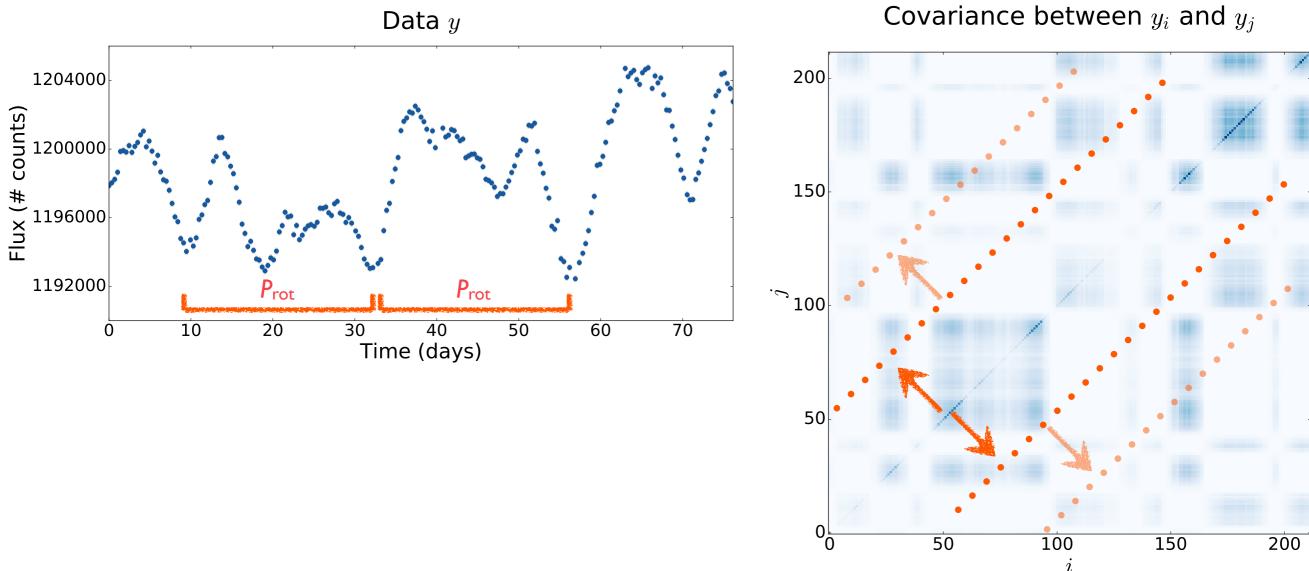
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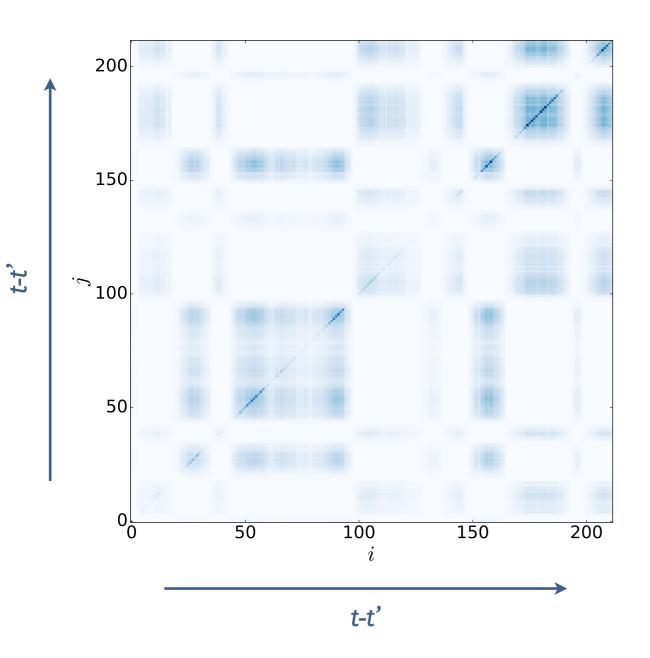
150

200

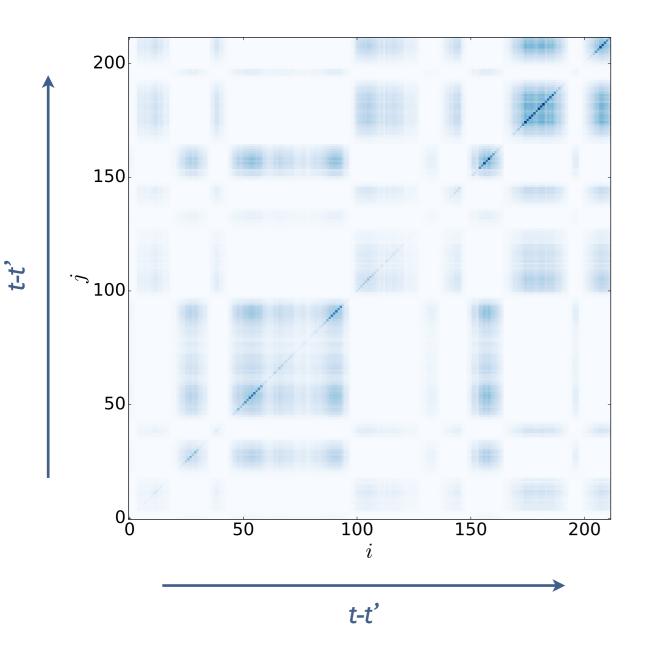
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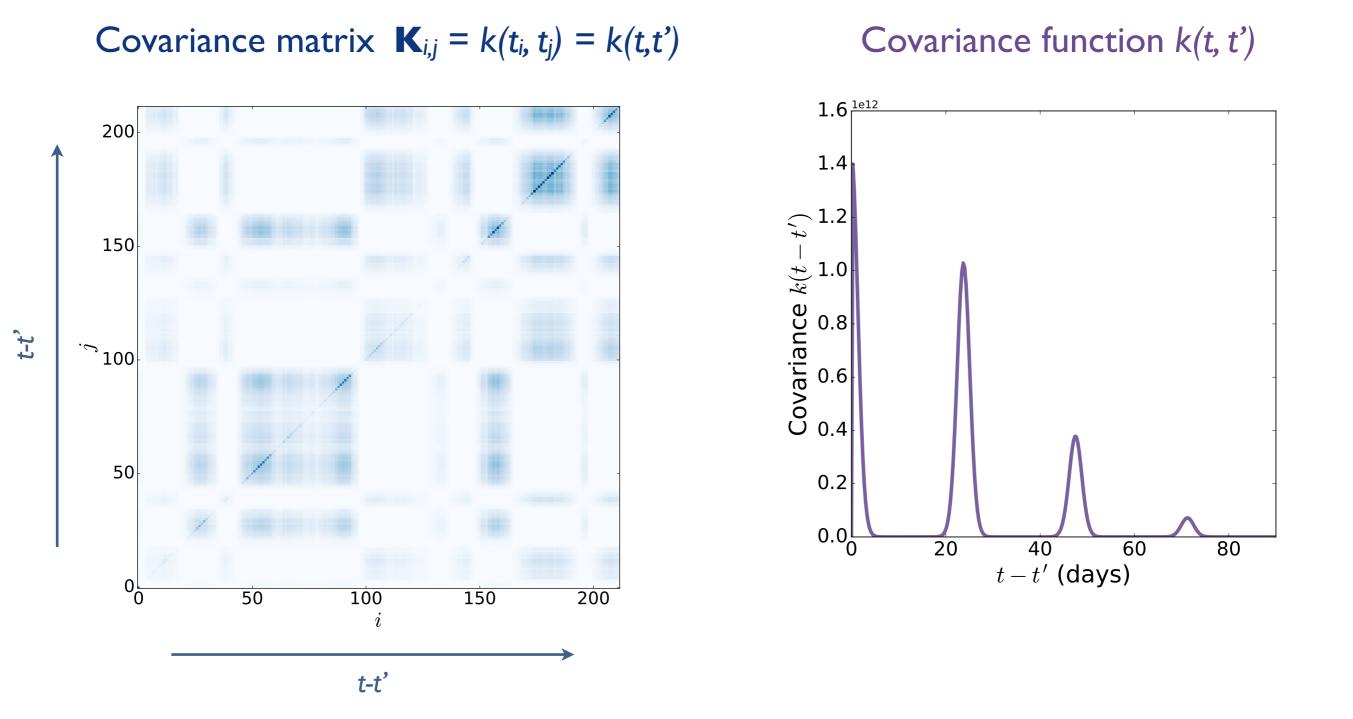


Covariance matrix

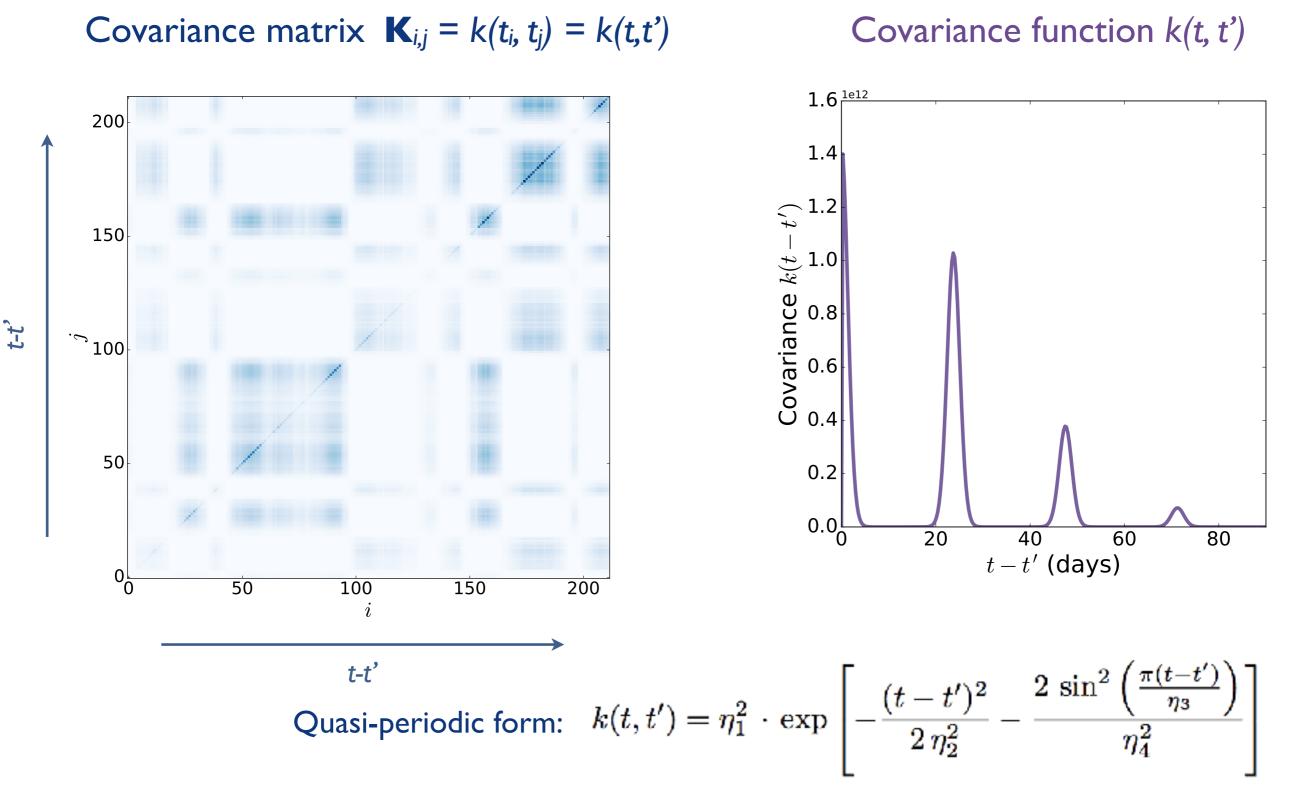


Covariance matrix $\mathbf{K}_{i,j} = k(t_i, t_j) = k(t,t')$

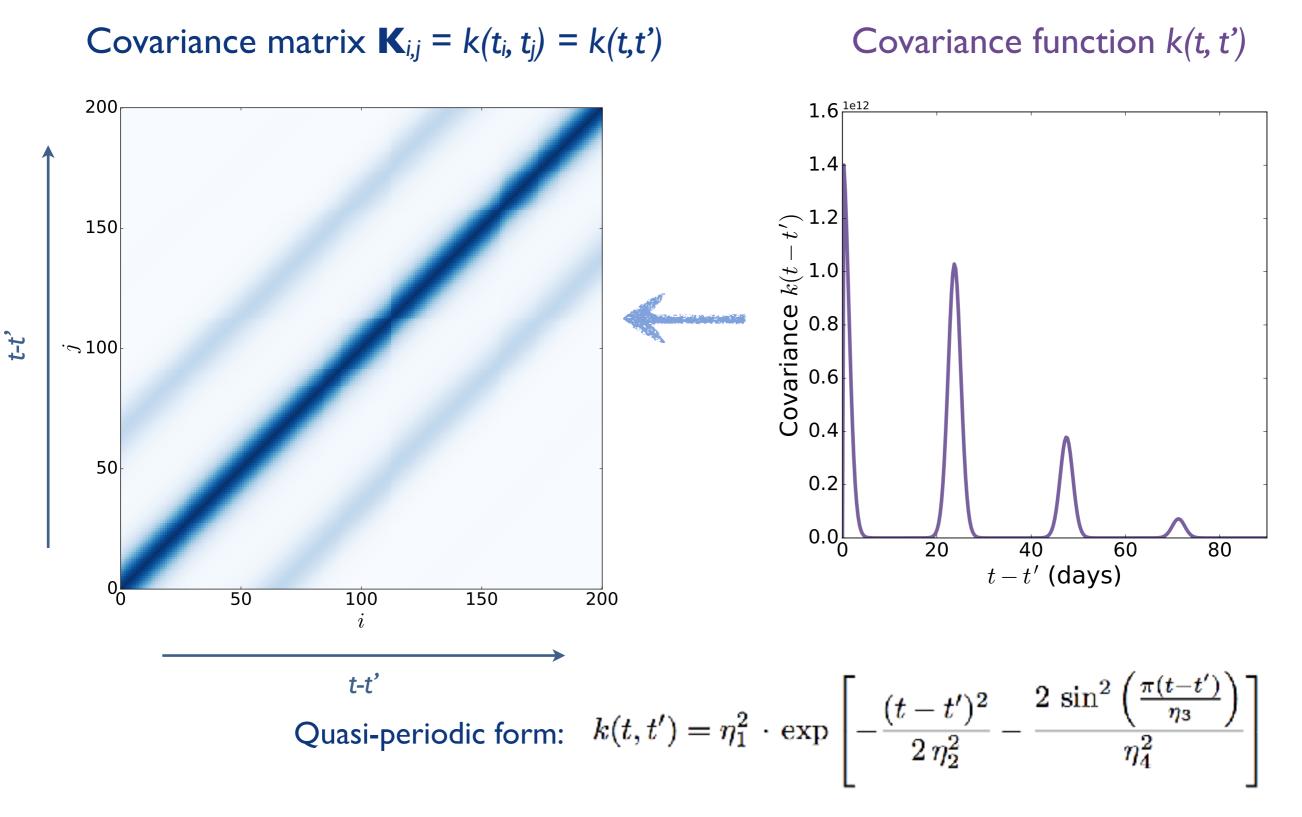
Non-parametric frameworks: we fit in "correlation (covariance) space"



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See Rassmussen & Williams (2006), Haywood (2015, Chap. 2) and others



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The covariance function encodes the correlation/covariance/frequency structure of the data series

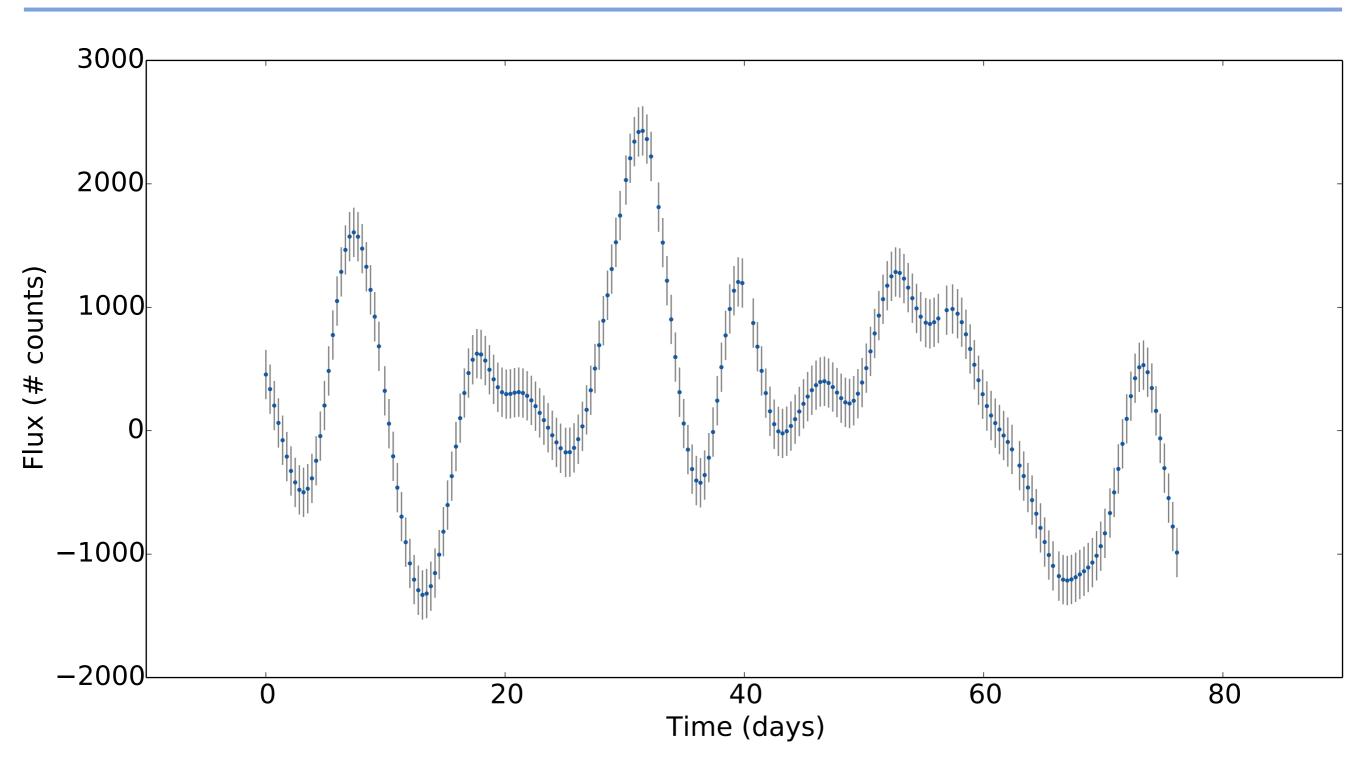


Image credit: C. Nava

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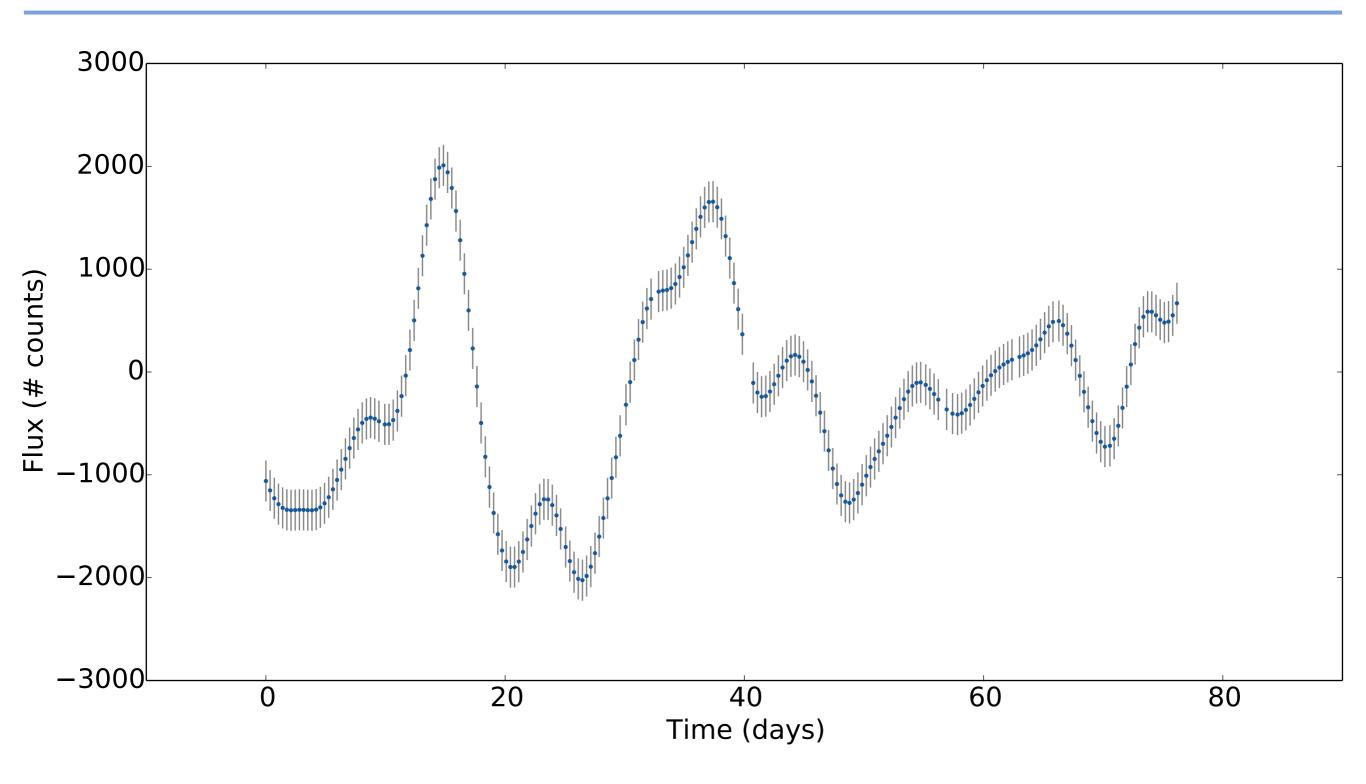


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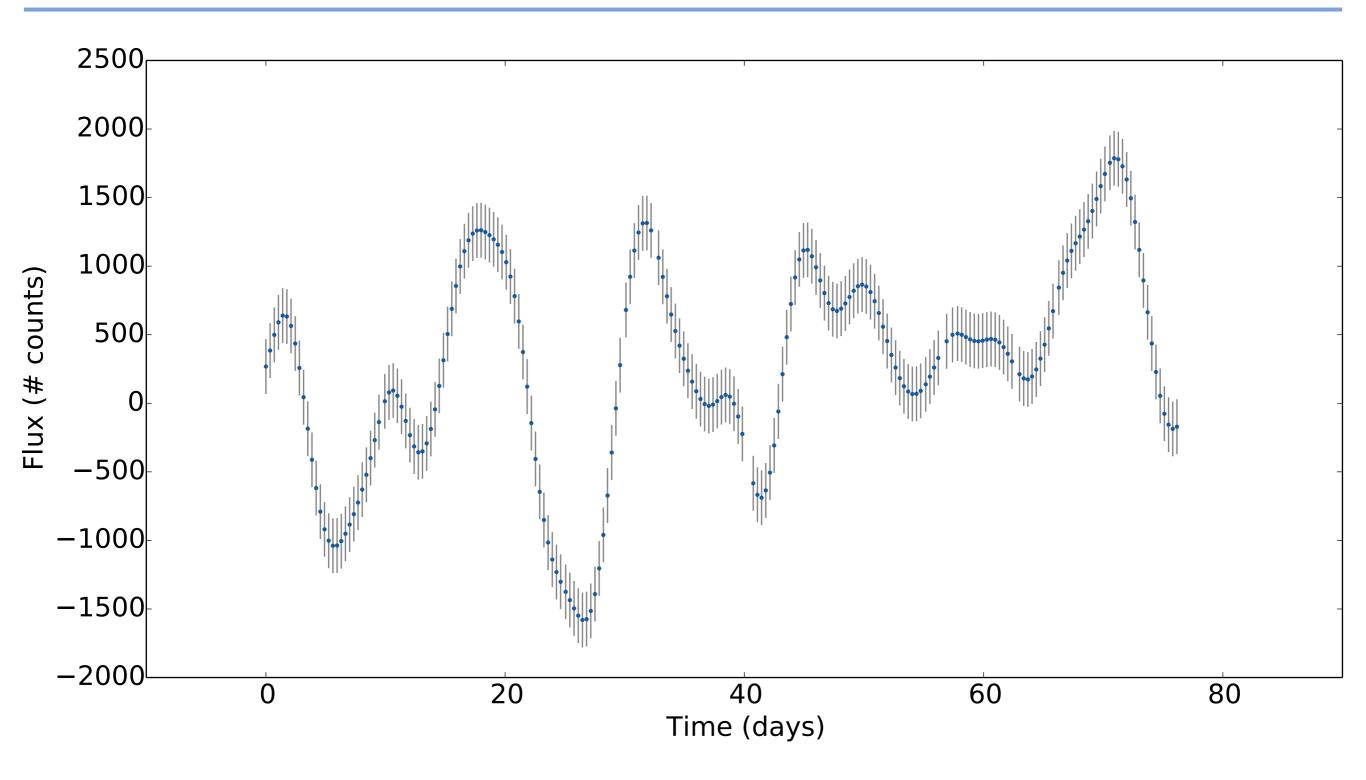
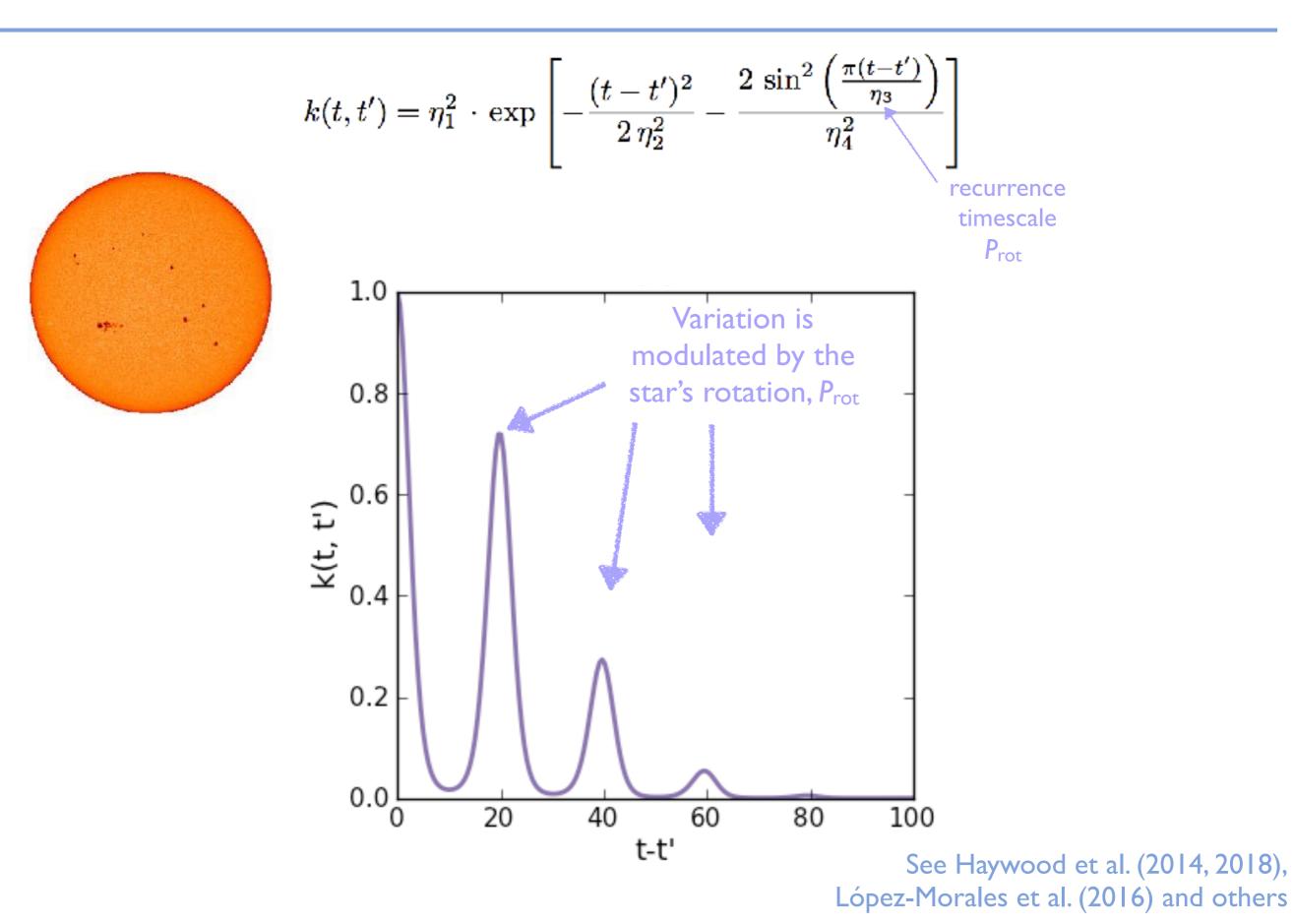
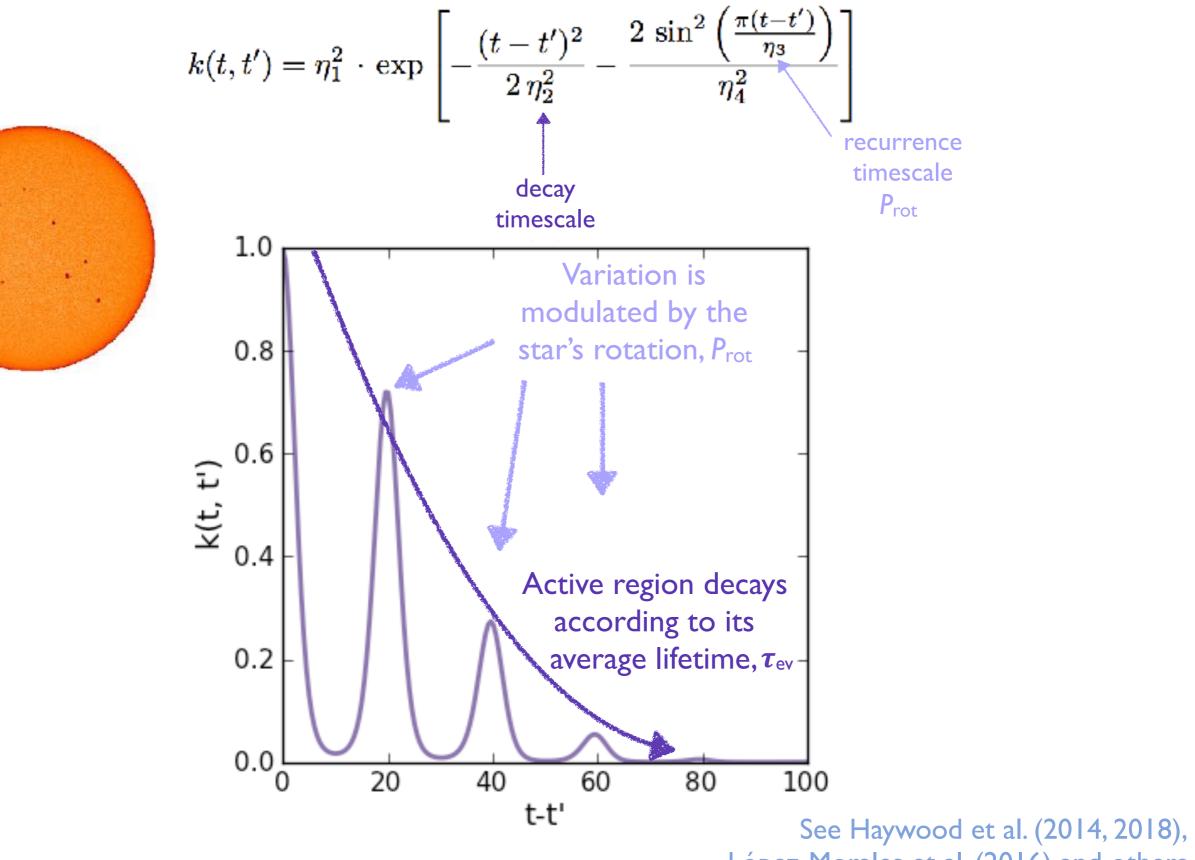


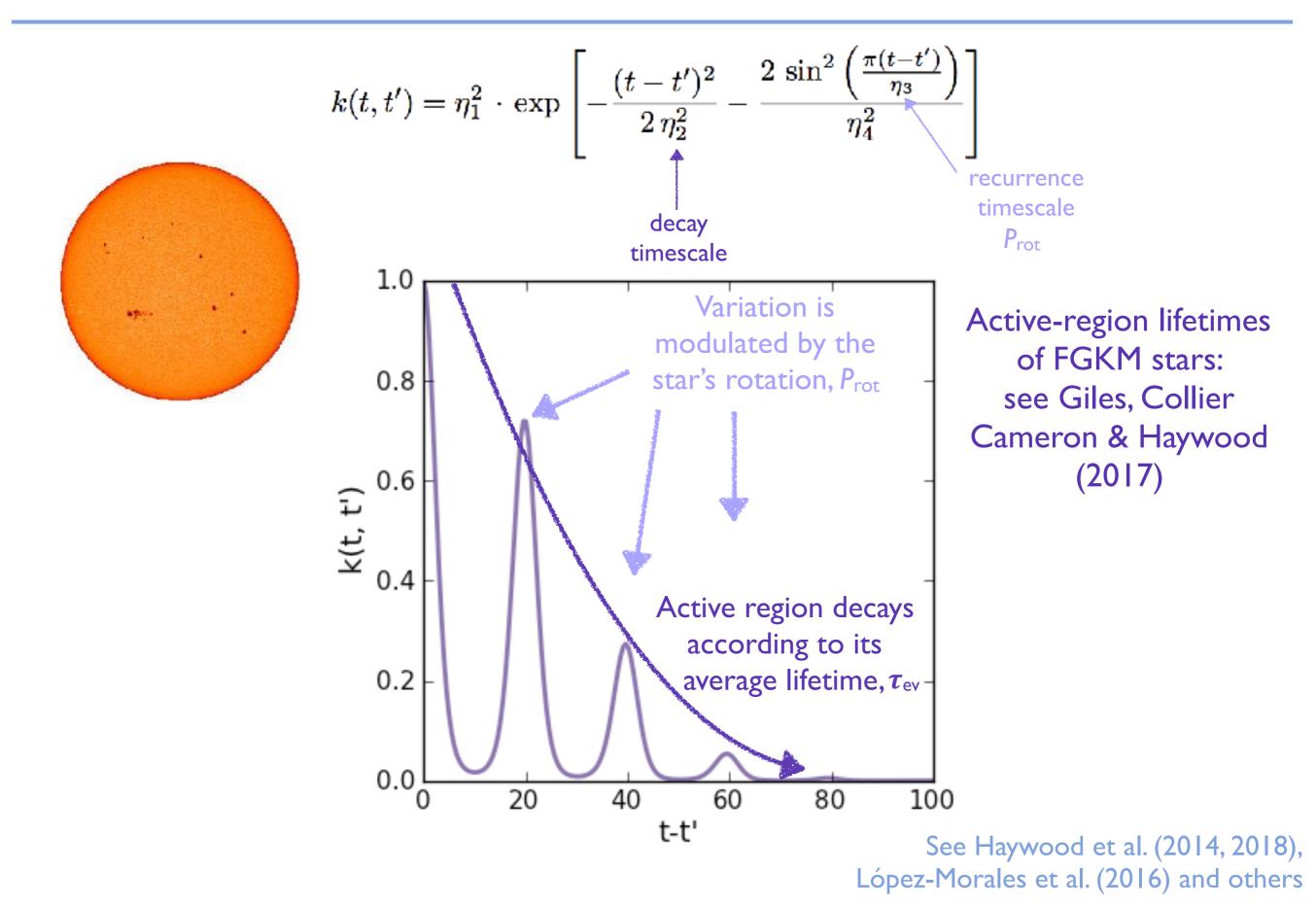
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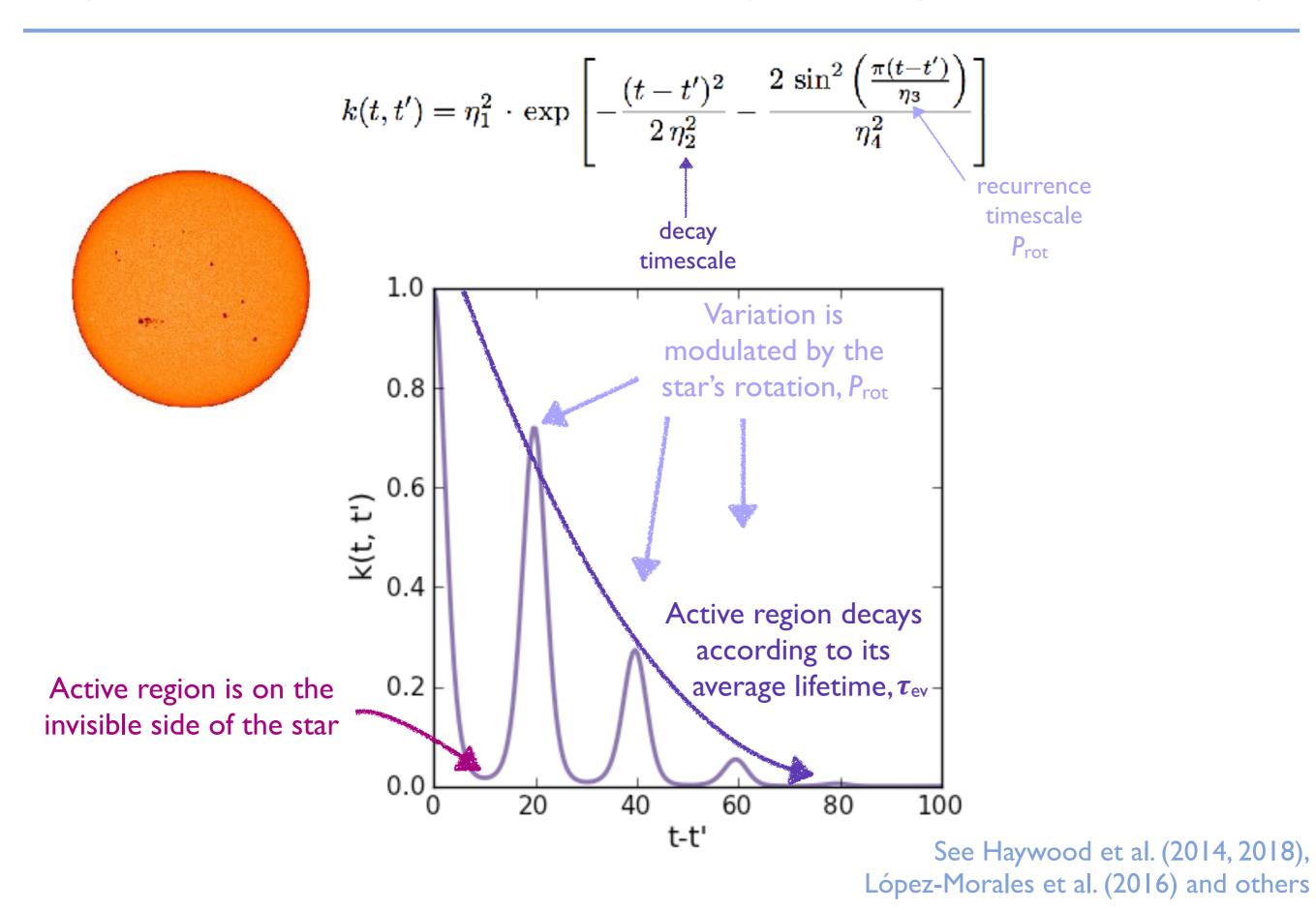
$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\eta_2^2} - \frac{2\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

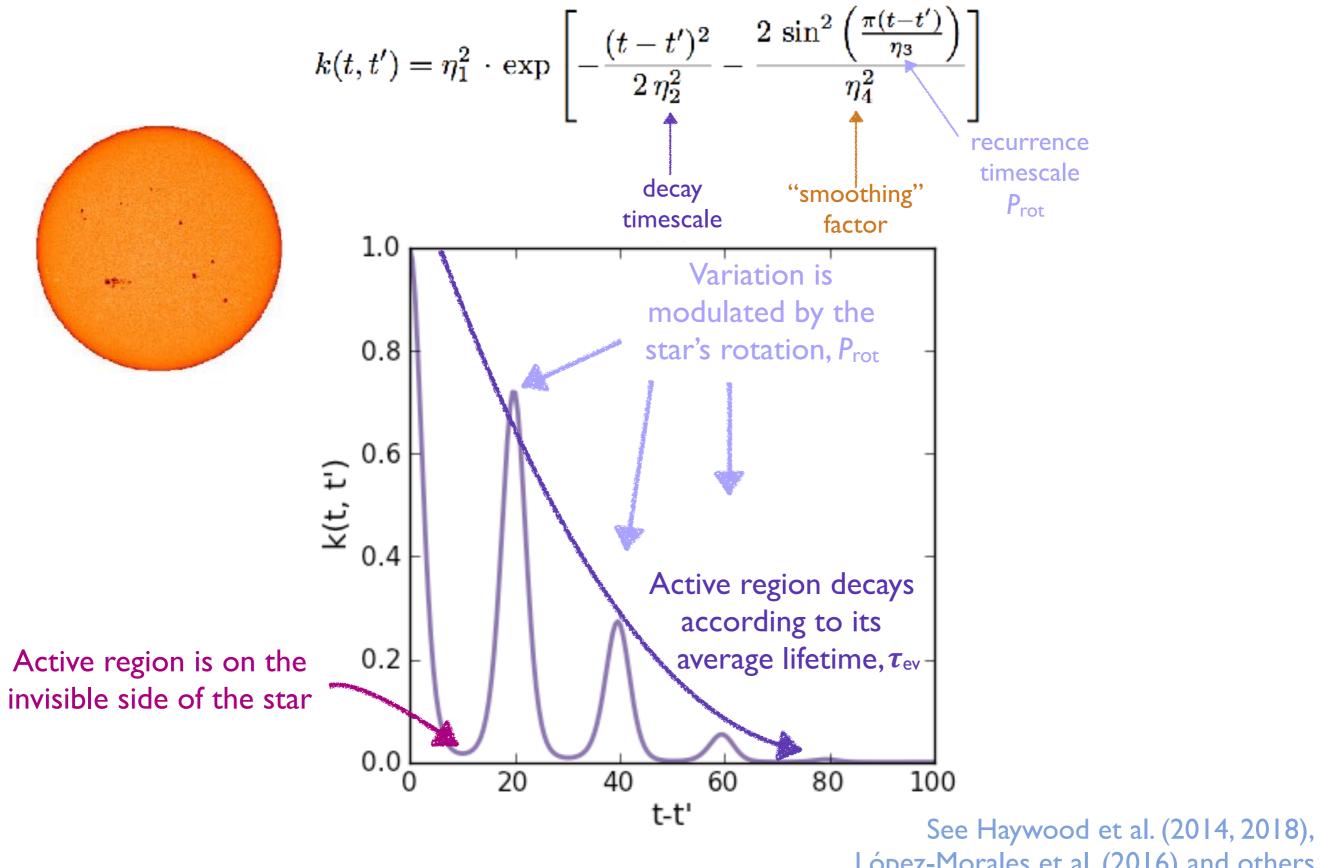




López-Morales et al. (2016) and others

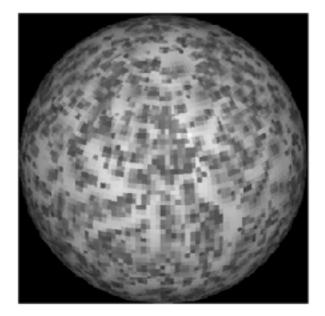




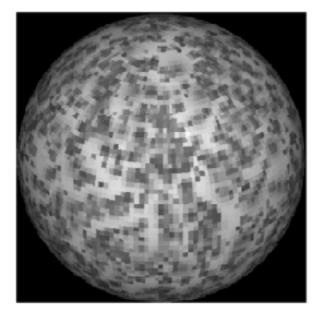


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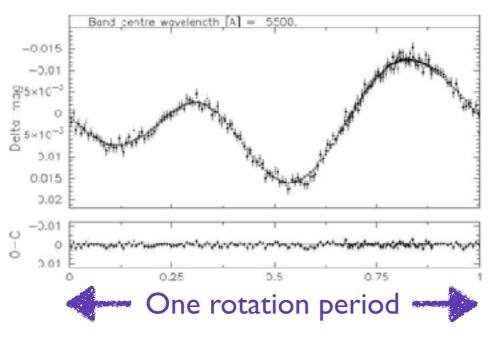
Synthetic stellar surface



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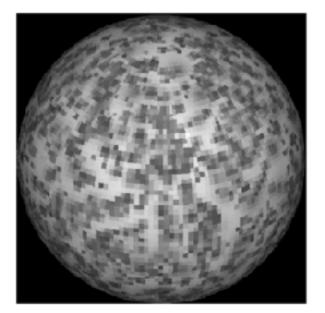


Reconstructed lightcurve



0-0

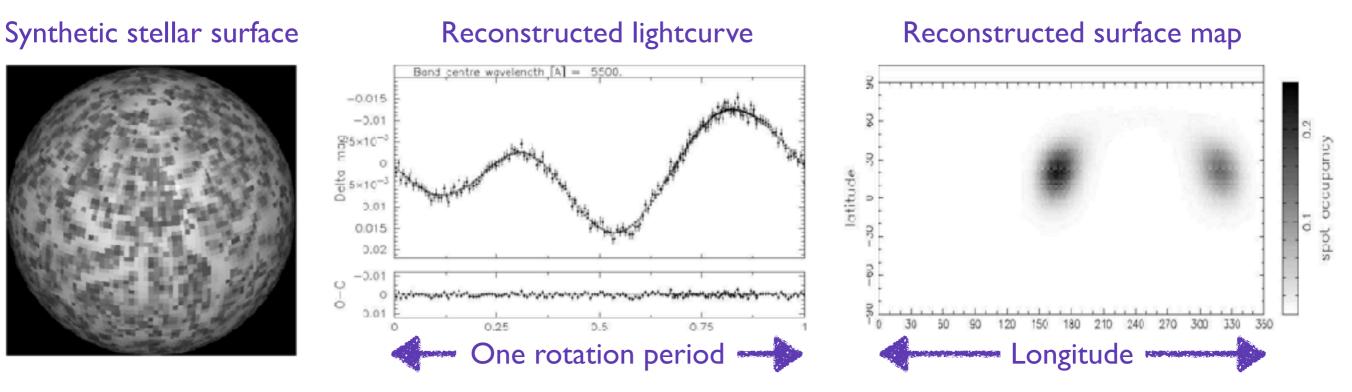
Synthetic stellar surface



Reconstructed lightcurve

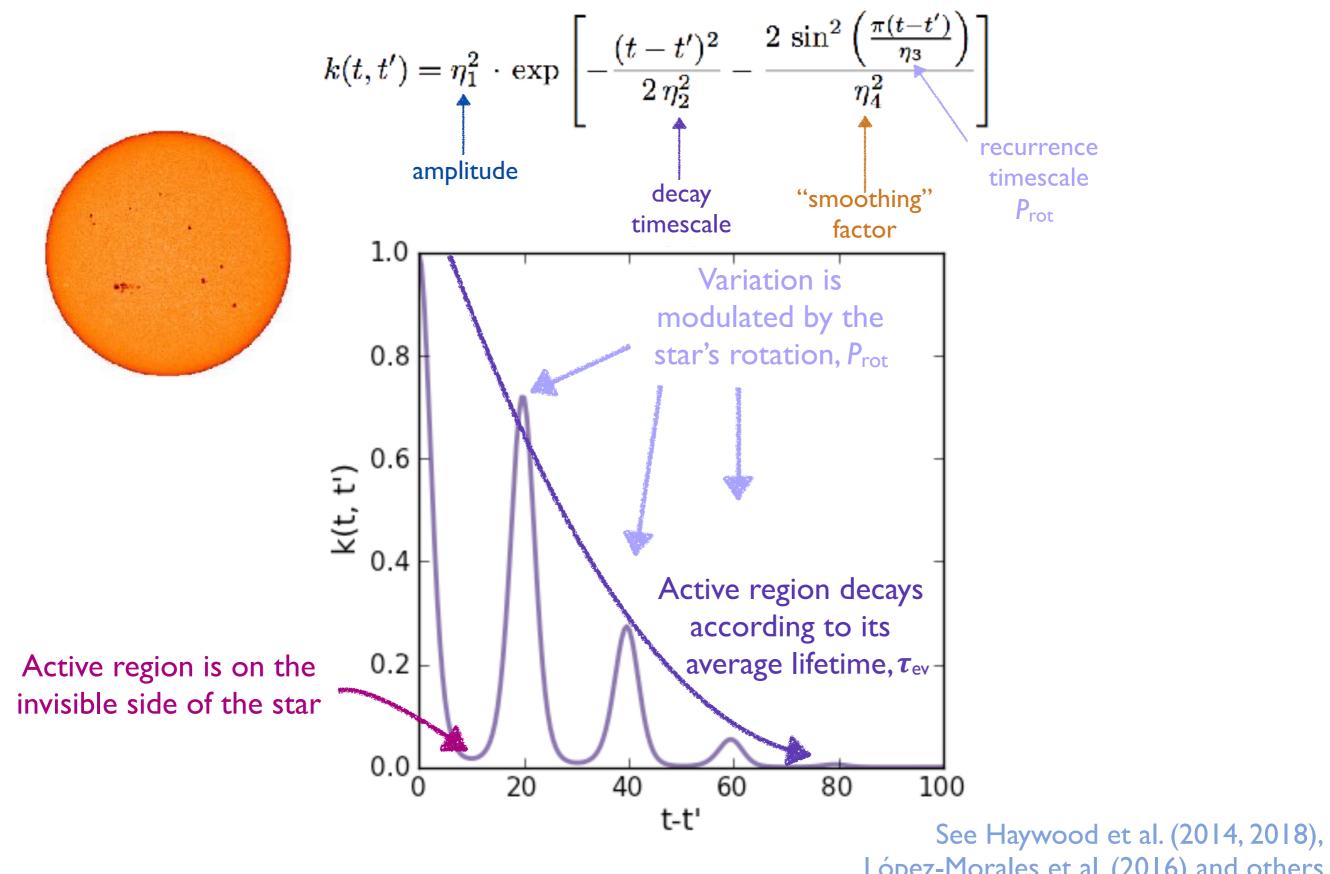
Band centre wavelencth [A] = 5500 -0.015 -0.0185×107 E spot accupancy Delto Delto atitud 0.01 d 0.015 0.02 -0.010 0.01 180 210 240 270 300 330 350 0.75 150 0.25 0.5 One rotation period -🗝 Longitude 🚥

Reconstructed surface map



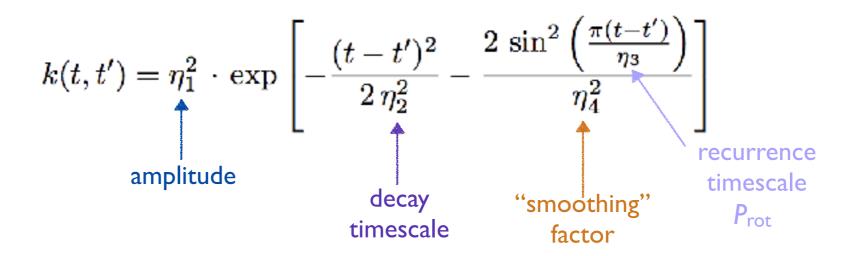
A lightcurve, or an RV curve, will only ever show 2-3 peaks per stellar rotation. This is equivalent to $\eta_4 \approx 0.5$.

$$k(t,t') = \eta_1^2 \cdot \exp\left[-\frac{(t-t')^2}{2\eta_2^2} - \frac{2\sin^2\left(\frac{\pi(t-t')}{\eta_3}\right)}{\eta_4^2}\right]$$

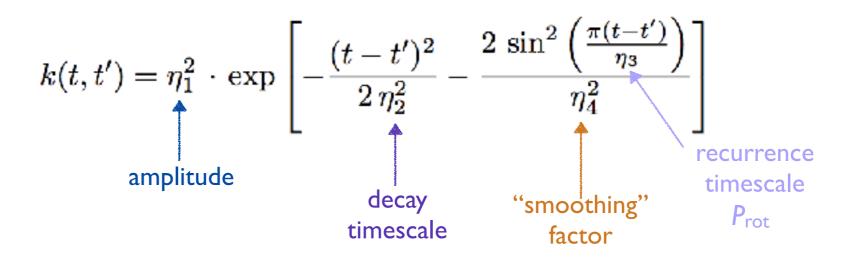


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Determining the hyperparameters

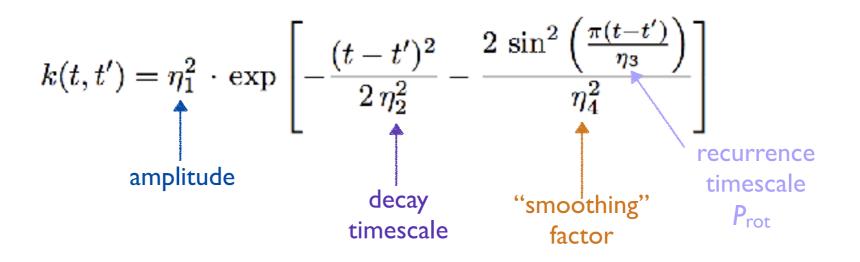


Determining the hyperparameters



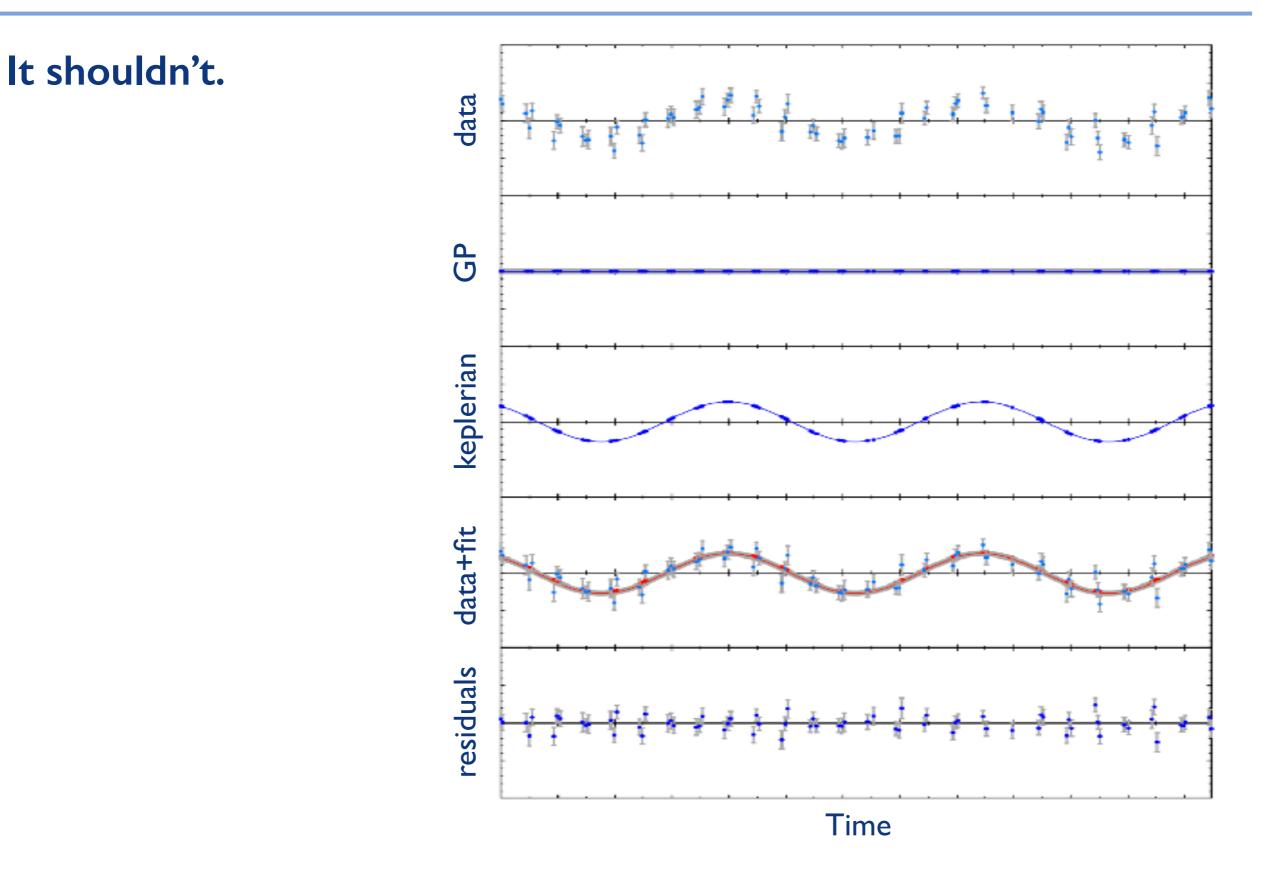
Can "train" the GP on an auxiliary dataset, eg. the lightcurve (Haywood et al. 2014, see also Grunblatt et al. 2015, Cloutier et al. 2017 and others), the spectroscopic indicators like FWHM, BIS (Rajpaul et al. 2015, Jones et al. 2017 and others), in some cases even the RVs themselves (Faria et al. 2016, Barros et al. 2017)

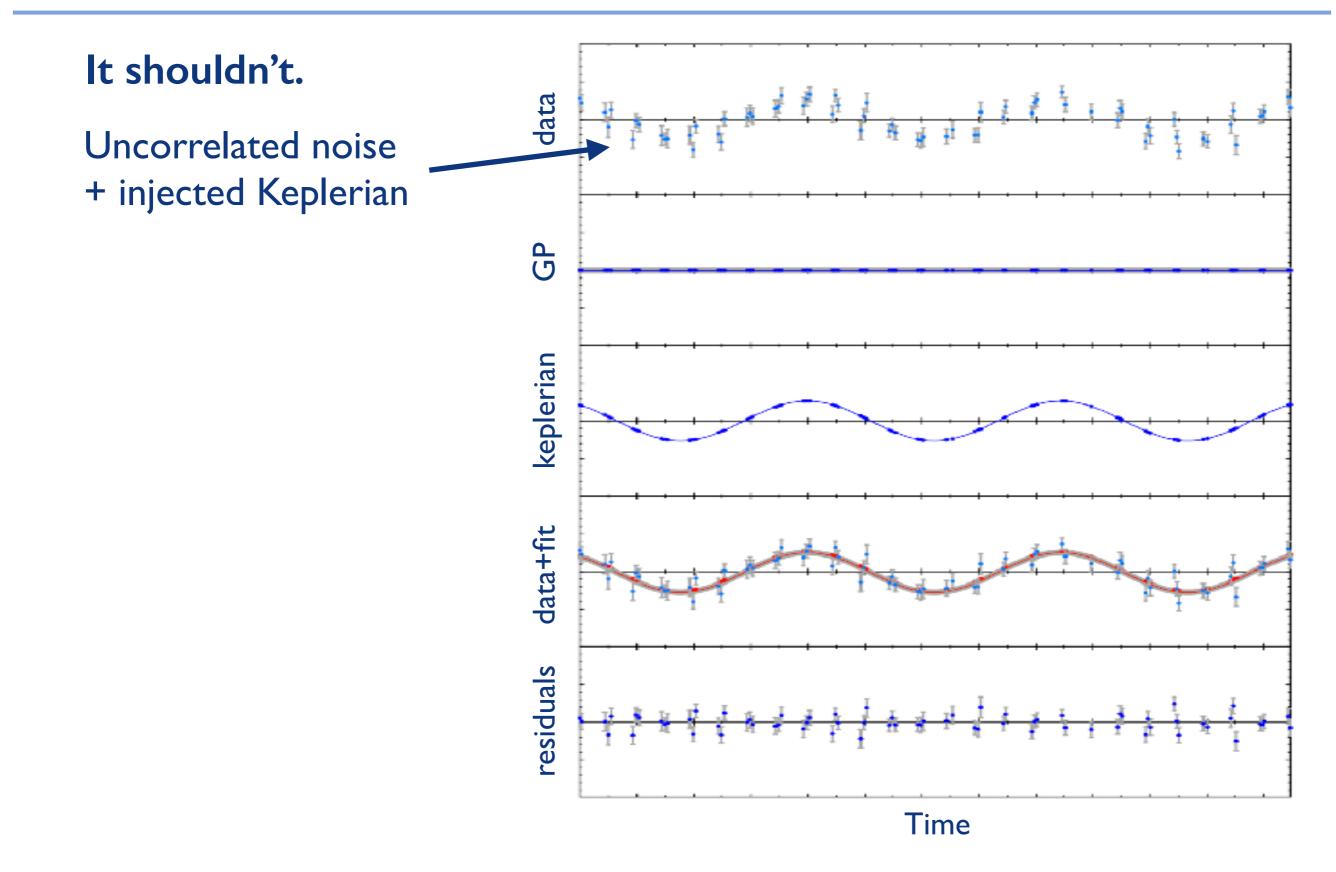
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- Can "fix" the hyperparameter values using Gaussian priors, based on prior knowledge/analysis (López-Morales et al. 2016, Dittmann et al. 2017, Haywood et al. 2018 and others)

It shouldn't.



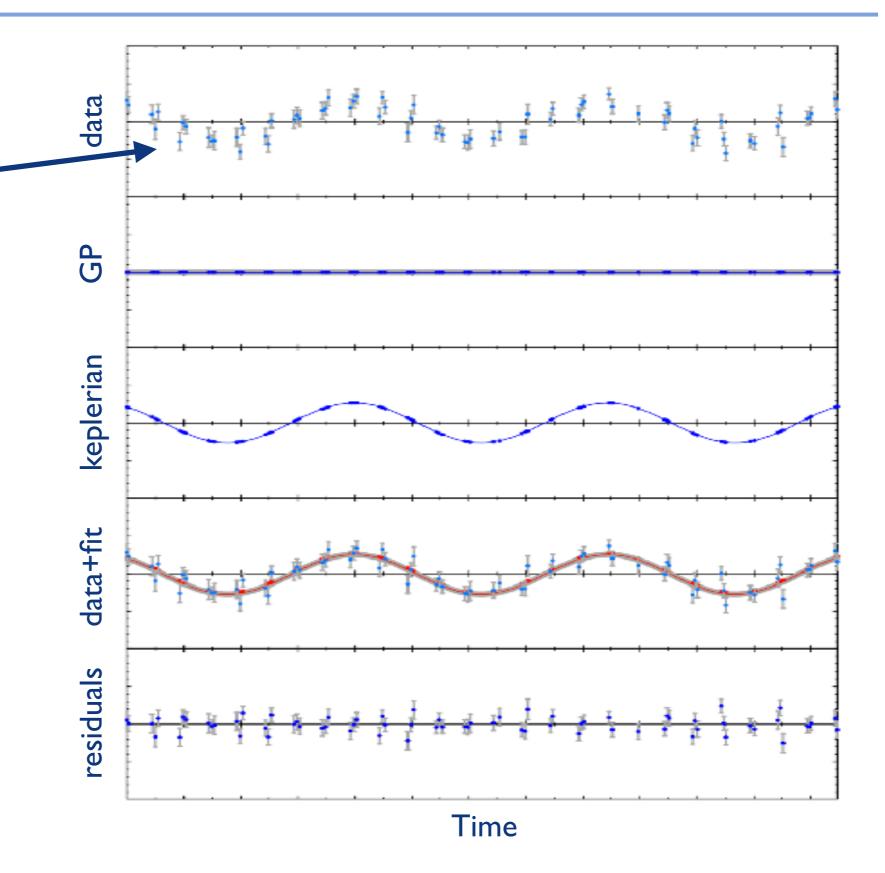


lt shouldn't.

Uncorrelated noise + injected Keplerian

Occam's razor:

The flexibility of the GP is balanced by a penalty term in the likelihood function. The Keplerian is a simple model (less flexible but perfectly adequate for a keplerian signal) so it is favoured by the likelihood function.



• Aim for an accurate (*i.e.* unbiased) measurement. Precision is a great advantage, *only if* the measurement is also accurate.

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Papers & textbooks:

- The classic textbook reference (in which you will find all the equations and statistical jargon): C. E. Rasmussen & C. K. I. Williams, Gaussian Processes for Machine Learning, the MIT Press, 2006, (online: www.GaussianProcess.org/gpml).
- A very clear introduction to GPs (not specific to exoplanets): Roberts et al. (2012)
- GPs to account for stellar activity in RV analyses: Baluev (2013), Haywood et al. (2014), Grunblatt et al. (2015), Rajpaul et al. (2015), Faria et al. (2016), Anglada-Escudé et al. (2016), López-Morales, Haywood et al. (2016), Barros et al. (2017), Jones et al. (2017), Cloutier et al. (2017), Haywood et al. (2018) and others.
- GPs applied to transmission spectroscopy for the study of planetary atmospheres: Gibson et al. (2011) and Czekala et al. (2014) and others.
- Machine learning to detrend Kepler and K2 lightcurves: Foreman-Mackey et al. (2015), Crossfield et al. (2015), Foreman-Mackey et al. (2014), Ambikasaran et al. (2014), Aigrain et al. (2015) and Barclay et al. (2015), Armstrong et al. (2016) and others.

Develop your GP intuition:

- This is a fantastic lecture on the nature of Gaussian processes by David MacKay. I thoroughly recommend watching it! <u>http://videolectures.net/gpip06_mackay_gpb/</u>
- Suzanne Aigrain and her group have given many talks and tutorials, all available here: <u>http://splox.net/tag/gps/</u>
- Read Chapter 2 of my PhD thesis (Haywood 2015): <u>https://research-repository.st-andrews.ac.uk/handle/10023/7798?</u> mode=full&submit_simple=Show+full+item+record
- Discussion on "astrophysically-motivated" GPs: Haywood et al. (2018), López-Morales et al. (2016)

Useful codes:

- Dan Foreman-Mackey's George, celerite (and emcee) codes are publicly available at: <u>http://dan.iel.fm/research/</u>.
- João Faria's kima, for exoplanet detection in RVs with DNest4 and GPs: <u>https://github.com/j-faria/kima</u>
- Radvel, a radial velocity modelling toolkit co-written by BJ Fulton, Erik Petigura, Sarah Blunt and Evan Sinukoff: <u>https://</u> <u>radvel.readthedocs.io/en/latest/</u>

The use of GPs in exoplanet science is growing fast; this is only a small selection of papers/codes/etc. and is in no way exhaustive.