

# IS THERE A PLANET IN MY RADIAL-VELOCITY DATA?

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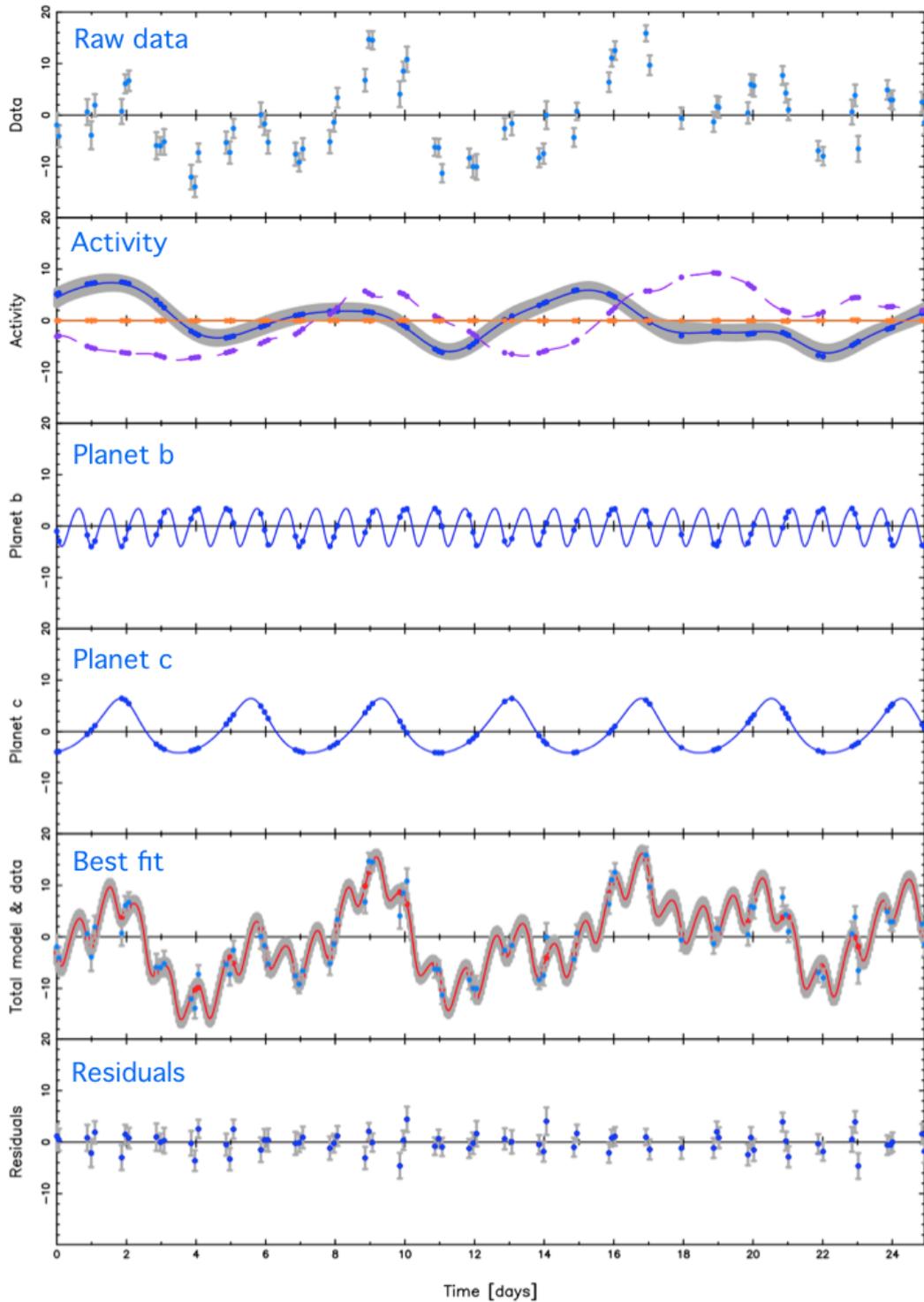


Figure 1: Gaussian Process regression to distinguish planetary and activity signals in the Corot-7 system (Haywood et al. 2014).

# INTRODUCTION

**HISTORICAL CONTEXT** In 1995, Michel Mayor and Didier Queloz discovered the first planet orbiting a star other than our Sun, the exoplanet 51 Peg b (Mayor & Queloz 1995). This detection was made using the radial velocity (RV) technique, a method that uses high resolution spectrographs to measure the Doppler shift of spectral lines induced by the radial velocity variation of a star. In the presence of a planet, a star will show a periodic radial velocity variation induced by the gravitational pull of the planet. The detection of such a signal convinced Michel Mayor and Didier Queloz that they had discovered the first exoplanet. 51 Peg b was not the type of exoplanet people were looking for, because none of the planetary formation scenarios at the time could explain the presence of a *Hot Jupiter*, a Jupiter mass planet orbiting its host star in only three days. Therefore, a few scientists were not convinced at the time that the signal detected was induced by a planet, but instead thought it was induced by stellar signals. Indeed, the presence of spots on the stellar surface can mimic the presence of a planet (Saar & Donahue 1997b; Queloz et al. 2001), and acoustic waves propagating in the convective zone can induce perturbing signals in radial velocity (Hatzes 1996). Although these phenomena could not explain the signal detected by Mayor and Queloz, the detection of the *Hot Jupiter* HD209458 with the radial velocity technique in 1999 (Mazeh et al. 2000) followed a few months later by the observation of the planet transiting its host star (Charbonneau et al. 2000), i.e. passing between us and the star, definitively stopped the controversy. With the radial velocity measurement giving information on the mass and the transit giving information on the radius, it was possible for the first time to obtain the density of a *Hot Jupiter* and confirm that those type of objects are indeed planets.

**THE PROBLEM OF STELLAR SIGNALS** Before the first planet was discovered, scientists were trying to figure out how such a signal could be due to a star. From our knowledge today, a star like 51 Peg b cannot induce signals at the level of  $\sim 100 \text{ m s}^{-1}$ , the signal created by 51 Peg b. However, now that the  $\text{m s}^{-1}$  precision level has been reached by the best spectrographs, it is obvious that stars introduce signals at a similar level. Those stellar signals prevent the RV technique from detecting and measuring the mass of Earth-twins orbiting solar-type stars, i.e., Earth analogues orbiting in the habitable zone of GK dwarfs.

Stellar signals affecting RV measurements can be differentiated into four categories given our current state of knowledge:

- **Stellar Oscillations:** on a timescale of a few minutes for solar-like stars, produced by pressure waves propagating in stellar interiors (Dumusque et al. 2011c; Arentoft et al. 2008; Kjeldsen et al. 2005);
- **Stellar Granulation and Supergranulation:** on a timescale from a few minutes to 48 hours, resulting from convectively driven out-flowing material reaching the stellar surface (Dumusque et al. 2011c; Del Moro et al. 2004; Del Moro 2004; Lindegren & Dravins 2003; Dravins 1982);
- **Short-term Stellar Activity:** on the timescale of the stellar rotation period, induced by rotation in the presence of evolving surface inhomogeneities including spots and plages (Borgniet et al. 2015; Robertson et al. 2015, 2014; Dumusque et al. 2014; Boisse et al. 2012; Saar 2009; Meunier et al. 2010; Saar & 1997); and
- **Long-Term Stellar Activity:** on a timescale of several years, induced by stellar magnetic cycles and/or surface flows (Meunier & Lagrange 2013; Lovis et al. 2011; Dumusque et al. 2011a; Makarov 2010).

We know that stellar signals complicate the detection of small-mass exoplanets. There are several examples in the literature that show this: Gl 581 has been found to have between 3 and 6 planets (Hatzes 2016; Anglada–Escude & Tuomi 2015; Robertson et al. 2014; Baluev 2013; Vogt et al. 2012; Gregory 2011; Vogt et al. 2010; Mayor et al. 2009), HD 40307 between 4 and 6 planets (Diaz et al. 2011; Tuomi et al. 2013), and GJ 667C between 3 and 7 planets (Feroz & Hobson 2014; Anglada–Escude et al. 2012; Gregory 2012). In all of these systems, the number of planets found depends strongly on the model used to fit the data, and this is a sign that stellar signals are not properly modeled.

# HANDS-ON SESSION

**THE GOAL OF THIS HANDS-ON SESSION** The goal of this hands-on session is to give you some tools that you will be able to use later on when dealing with RV or similar data. The overall goal, however, is not for you to simply run the different codes, but to understand their strengths and limitations. Throughout the exercises, we encourage you to think about what the code is doing, what it is saying about the model, and what modifications you might want to make in order to improve the general analysis or to address a specific science question.

In this hands-on session, we will try to understand how stars can create an RV signal, and how this signal can be differentiated from a planetary signal. We will focus on the short-term stellar activity signal, which is the strongest and the most difficult stellar signal to deal with. We will first model the RV effect of a spot and a plage using the SOAP 2.0 code that I published two years ago (Dumusque et al. 2014), and then simulate the RV effect induced by a star like the Sun. We will inject a planetary signal into the RVs obtained from our solar simulation and see how stellar activity can complicate the detection of small-mass planets. Finally, we will use a Gaussian Process regression, an interesting approach to model short-term stellar activity signal, and thus extract planetary signals despite the stellar signal.

**RV EFFECT OF SPOTS AND PLAGES** RV measurements are affected by signals created by the presence of active regions (spots, plages) present at the surface of solar-type stars. These inhomogeneities are rotating with the stellar surface, and thus the period of the observed signal is linked to the rotation period of the star. What is normally observed, as many active regions are present at the same time (200 sunspots for the Sun), is a semi-periodic signal at the rotation period of the star, and the corresponding harmonics ( $P_{rot}/2$ ,  $P_{rot}/3$ ,  $P_{rot}/4$ , ...)

## Flux Effect

A spot will induce an RV variation due to its small flux compared to the average stellar surface. Since sunspots are  $\sim 700\text{ K}$  cooler than the effective temperature of the Sun, they have a much lower flux than the quiet solar photosphere regions. A spot will therefore break the flux balance between the blueshifted approaching limb and the redshifted receding limb of a rotating star, and will induce an RV variation as it passes through the visible stellar disc. A plage at the disc center is only slightly hotter than the average effective temperature and will induce a small flux effect. A plage on the limb will be brighter due to a center-to-limb brightness dependence, however at this location, the star emits less light due to limb darkening. Independent of its location, a plage will therefore induce a small flux effect compared to a spot, even if plages tend to be an order of magnitude more extended than spots.

## Convection effect

Plages and spots are regions affected by strong local magnetic fields. These magnetic fields will locally inhibit the convection, which will suppress the convective blueshift effect inside the active regions ( $\sim 300\text{ m s}^{-1}$  for the Sun). These regions therefore appear redshifted in comparison to the quiet photosphere, which induces an RV variation as active regions appear and disappear from the visible part of the stellar disk due to rotation. The convective blueshift effect for spots and plages of the same size will be similar. However plages tend to be an order of magnitude more extended than spots, therefore they are the main contributors to the convective blueshift effect.

The idea behind SOAP 2.0, schematised in Figure 2, is to divide a star into several cells, and in each of them put an average line profile. Limb-darkening is included, as well as stellar rotation and flux emerging from the given cells. The line profiles are obtained from solar observations taken in the quiet photosphere, and in a dark spot. The line profiles that we use will be seen in the end of the jupyter notebook.

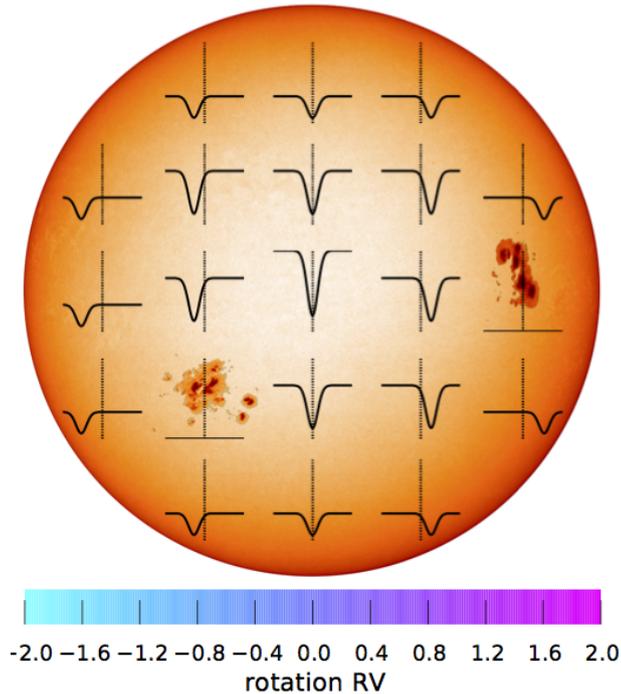


Figure 2: Figure explaining how SOAP 2.0 simulates the effect of active regions. The star is divided into a grid of  $NV$  cells, each of them having its own average line profile. Depending on the cell position, the average line profile is Doppler shifted to account for rotation. The rotational speed  $v \sin i$  in the case of the Sun is given as the horizontal bar at the bottom of the figure. The intensity of each cell is weighted by a limb darkening law, and the intensity is reduced (or increased) in presence of a dark spot (or a bright plage). Here dark spots with no emitting flux are considered. The integrated average line profile over the stellar disk is obtained by summing up all of the cells. Note that here all of the average line profiles have the same depth, however, the average line profiles on the limb appear shallower because they are weighted by limb darkening.

##### ACTION #####

In this exercise, we will simulate the effect of a spot and of a plage using the SOAP 2.0 code that I created 2 years ago. To start, open the jupyter notebook corresponding to SOAP 2.0: [open a Terminal](#), go to the folder SOAP2 (type "cd RVs/SOAP2") and type "jupyter notebook soap2.ipynb". Then we will follow the rest of the instructions using the notebook. To follow with me just click on the title and then move forward, from box to box using **Shift + Enter**. We will calculate the effect of a spot given the stellar and spot properties defined in the file "config.cfg" (in the "SOAP2" folder).

Once this is done, I ask you to calculate the RV effect of a spot and a plage with the following properties: longitude = 0, latitude = 15, size = 0.14. Open the "config.cfg" file (open a new Terminal and type "gedit RVs/SOAP2/config.cfg"), modify the corresponding parameters and save the file ("Ctrl + s" or top menu). Re-run the notebook from the very top, clicking on the title, and then moving forward with **Shift + Enter**

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**RV EFFECT INDUCED BY SPOTS AND PLAGES ON THE SUN** In the preceding section, we showed that we are able to estimate the effect of active regions on the different observables obtained from HARPS-like spectrographs using SOAP 2.0, including the RVs. It is not possible, however, to consider the appearance and size evolution of active regions with SOAP 2.0, which is necessary to model the activity of Sun-like stars. To do so, we used a model developed in Dumusque et al. (2011b). In this previous work, the authors used empirical laws from solar observations to simulate the appearance of spot groups on the solar surface because spots tend to appear in groups. Here, we improve this model by also considering plages, which are generally found to surround spots. In the new model, a plage

group will surround a spot group to form what we refer to as an active region group (ARG).

Considering an ARG is possible because the amplitude of the variations observed in RV, BIS SPAN, FWHM, and  $\log(R'_{HK})$  are proportional to the active region surface area (Dumusque et al. 2014; Dumusque et al. 2016). This ensures that considering the individual effect of ten active regions covering an ARG and then summing their contribution, or simply considering the effect of the ARG, will give the same results.

We present here a summary of the empirical laws derived from solar observations that we used to simulate the appearance of ARGs on the surface of a star. For more details we refer to Dumusque et al. (2011b). We note that the goal here is not to simulate the activity of the Sun, as this can be extremely complex and requires sophisticated magnetohydrodynamic models like CO5BOLD (Freytag et al. 2012), but to model in a simple way the observed phenomena that lead to RV variations.

We followed Dumusque et al. (2011b) to simulate the appearance and evolution of spot groups and started with an empirical law from solar observations that describes their lifetime distribution (Howard 2000):

- 50% of the spot groups have lifetimes of between 1 h and 2 days,
- 40% of the spot groups have lifetimes of between 2 days and 11 days, and
- 10% of the spot groups have lifetimes of between 11 days and 2 months.

For simplicity, we assumed that spot groups and the plage regions surrounding them, forming what we call ARGs, are tightly related. Plage groups appear and disappear at the same time as spot groups and evolve in size in a similar way. Plage groups cover a surface area that is ten times larger than the area covered by spot groups (Chapman-2001).

The lifetime of an ARG is correlated to its maximum surface area (Howard 2000). For a surface area measured in filling factors, that is, the surface area ratio between the active region and the visible hemisphere, the largest size of an ARG is given by

$$f_{ARG,max} = 10^{-4}T, \tag{1}$$

where the maximum filling factor of the ARG  $f_{ARG,max}$  is expressed in hemisphere, and the lifetime  $T$  in days. As in (Dumusque et al. 2011b), we did not consider small ARGs that last less than one day and affect the RV signal only negligibly.

The appearance of ARGs is ruled by a Poisson law:

$$P[(N(t + \tau) - N(t)) = k] = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!} \quad k = 0, 1, \dots, \tag{2}$$

where  $P$  is the appearance probability of an ARG,  $N$  is the number of ARGs,  $t$  is the time,  $\tau$  is the time step, and  $\lambda$  is the average appearance rate of ARGs per unit of time. When an ARG appears on the stellar surface, it starts with a null size that increases during the first third of the ARG lifetime up to its maximum size as defined by Eq. 1. The size then decreases during the remaining two thirds of its lifetime.

Active region groups appear around preferred active longitudes, with a Gaussian distribution centered on 0 and a dispersion of 15 degrees. Compared to the work of Dumusque et al. (2011b), where 4, 5, or 6 active longitudes could exist, we here only considered the four case scenario.

### ##### ACTION #####

In this exercise, we will simulate the appearance of active regions on the surface of a star like the Sun, and calculate the resulting RV effect. To start, open the jupyter notebook corresponding to the solar activity model: [open a Terminal, go to the folder Solar\\_activity\\_simulation \(type "cd RVs/Solar\\_activity\\_simulation"\) and type "jupyter notebook Solar\\_activity\\_model.ipynb"](#). Then we will follow the rest of the instructions using the notebook. To follow

with me just click on the title and then move forward, from box to box using **Shift + Enter**. We will calculate the effect of a spot given the stellar and spot properties defined in the file "config.cfg" (in the "SOAP2" folder).

Once the notebook is finished, you can have a look at the animation created by the notebook, showing spots and plages evolving on the stellar surface while the star is rotating (open a new Terminal and type "vlc RVs/Solar\_activity\_simulation/animation/active\_region.mp4")

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**GAUSSIAN PROCESSES** A Gaussian process (GP) is a non-parametric way to model  $n$  data points. Its kernel is an  $n \times n$  covariance matrix  $K$  in which each element contains information about how much each pair of data are correlated with each other. The matrix is determined by a model covariance function  $k(t_i, t_j)$  whose form reflects correlation between the data points. Different types of kernels exist to consider different types of correlated data. One kernel widely used is the exponential-squared kernel defined as (Rasmussen 2006):

$$k(t_i, t_j) = a^2 \exp\left(-\frac{(t_j - t_i)^2}{2\tau^2}\right), \quad (3)$$

where  $a$  is the amplitude of the variability,  $t_i$  and  $t_j$  two different times, and  $\tau$  the exponential decay term. Such a kernel is used when points are correlated over a typical timescale  $\tau$ , and when no correlations are observed for longer timescales.

The parameters of a kernel are called *hyper-parameters*, and the best set of *hyper-parameters* can be estimated for a data set by maximising the following log-likelihood function:

$$\ln \mathcal{L} = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \mathbf{K}) - \frac{1}{2} \underline{r}^T \cdot \mathbf{K}^{-1} \cdot \underline{r}. \quad (4)$$

$\mathbf{K}$  is the covariance matrix built from the covariance function in Equation 4, and  $\underline{r}$  is the RVs. The maximum of the log-likelihood can be found either using a Maximum A Posteriori algorithm, or using a MCMC sampler.

##### ACTION #####

In this exercise, we will introduce GP regression in the simple case of fitting a straight line through correlated data. To start, open the jupyter notebook corresponding to the GP introduction: **open a Terminal, go to the folder Introduction\_to\_GPs** (type "cd RVs/Introduction\_to\_GPs") and type "jupyter notebook introduction\_to\_GP.ipynb". Then we will follow the rest of the instructions using the notebook. To follow with me just click on the title and then move forward, from box to box using **Shift + Enter**. During the process, you will be asked to write the log-likelihood of the GP.

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**EXTRACTING PLANETARY SIGNAL DESPITE STELLAR SIGNALS USING GP** We introduced Gaussian Processes (GPs) in the preceding section using a simple kernel. Now let's see how we can use GP in the case of RV data to model stellar activity. We will first load the data that we generated with our model of stellar activity presented two sections above and then inject a planetary signal. Then, we will fit those data using a GP regression model only, and using a GP regression plus planet model. We finally compare the two models to see which model is a better fit to the data, which will tell us if the planet is detected or not.

In the case of stellar activity, the lifetime of active regions is approximately two rotational periods, while the signal's periodicity is equal to the rotation period. Therefore we prefer a quasi-periodic kernel, that adds a periodicity to the simpler exponential-squared kernel (see Eq. 3) :

$$k(t_i, t_j) = a^2 \exp\left(-\frac{(t_j - t_i)^2}{2\tau^2} - \Gamma \sin^2\left(\frac{\pi(t_j - t_i)^2}{P_\star}\right)\right), \quad (5)$$

with  $P_*$  the rotation period of the star or of one of its harmonics, and  $\Gamma$  the amplitude of this periodic signal.

##### ACTION #####

In this exercise, we will see how we can use GP in the case of RV data to model stellar activity. To start, open the jupyter notebook corresponding to planet fitting: open a Terminal, go to the folder `Stellar_activity_and_planet` (type `cd RVs/Stellar_activity_and_planet`) and type `jupyter notebook Planet_stellar_activity_and_GP.ipynb`. Then we will follow the rest of the instructions using the notebook. To follow with me just click on the title and then move forward, from box to box using `Shift + Enter`.

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**QUESTIONS** Here is a list of questions for you to answer at the end of the hands-on session.

1. What happens if a planet with the same amplitude is moved to shorter periods? Try with a period of 2.2 days with the same amplitude. What do you think, is it easier or more difficult to detect? Is it possible to detect an Earth twin at very short periods? If yes, can you detect smaller signals? If no, what type of signal can you detect? You can first see if the signal appears in the periodogram to give a first guess, and then use the GP modeling.
2. What happens if a planet with the same amplitude is moved to longer periods? Try with a period of 100 days with the same amplitude. What do you think, is it easier or more difficult to detect? Is it possible to detect an Earth twin with a period of 100 days? If yes, can you detect smaller signals? If no, what type of signal can you detect at this period? You can first see if the signal appears in the periodogram to give a first guess, and then use the GP modelling.
3. What happens if you change the stellar inclination? Is the activity RV signal more or less important? You can first try with one active region using SOAP 2.0, and then model the activity for a star like the Sun inclined with an angle of 45 degrees. Is it more difficult or easier to detect planets with this inclination? Do you prefer to observe equator-on stars, or pole on stars for planet detection? If you have time, give an example using the GP regression and fitting a planetary signal.
4. What happens if you increase the rotation period of the star? Does the activity RV signal increase or decrease? You can first try with one active region using SOAP 2.0, and then model the activity for a star like the Sun with a rotational period of 40 days. Is the effect of spots bigger than the effect of plages? Do you know why? How do the correlations between RVs and the other parameters (BIS SPAN, FWHM, photometry) change? Do you prefer a star that is rotating faster or slower to detect exoplanets?
5. What happens if you decrease the rotation period of the star? Is the activity RV signal increasing or decreasing? You can first try with one active region using SOAP 2.0, and then model the activity for a star like the Sun with a rotational period of 5 days. Is the effect of spots bigger than the effect of plages? Do you know why? How do the correlations between RVs and the other parameters (BIS SPAN, FWHM, photometry) change? Do you prefer a star that is rotating faster or slower to detect exoplanets?
6. What happens if you keep the same period but you increase or decrease the amplitude of the planet. Is it more difficult to detect smaller amplitude planetary signals, or large amplitude ones? Using the RV data set used in the example and GP modeling, try to find at which amplitude you can confidently detect an exoplanet with a period of 10 days.
7. What happens if you change the number of RV observations (change the `half_data` parameter to `int(len(time)/4.)`, `int(len(time)/3.)` and `int(len(time))`). Is it more difficult or easier to detect the same exoplanet? Try for different

numbers of measurements to detect what is the smallest planetary signal you can detect and try to see how this evolves? Is this following a Gaussian distribution ( $\sqrt{N_{\text{meas.}}}$ ) or not? Do you know why? The computational time for *half\_data*= int(len(time)) can be long, so do it at the end if you have time.

8. What happens if you decrease the difference between the spot temperature and the quiet photosphere? Is the activity RV signal larger or smaller? You can first try with one spot using SOAP 2.0, and then model the activity for a star like the Sun with a difference in spot temperature of 100 K (fix the rotation period to 5 days). How sensitive is the effect of spots to this change? Can you detect smaller planetary signals than when comparing with an activity model of the Sun? Do you think it is therefore better to observe G dwarfs or M dwarfs to look for small planets? Why (think about the Planck energy distribution)?

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