

Astrophysical and Instrumental Noise Sources: Direct Imaging

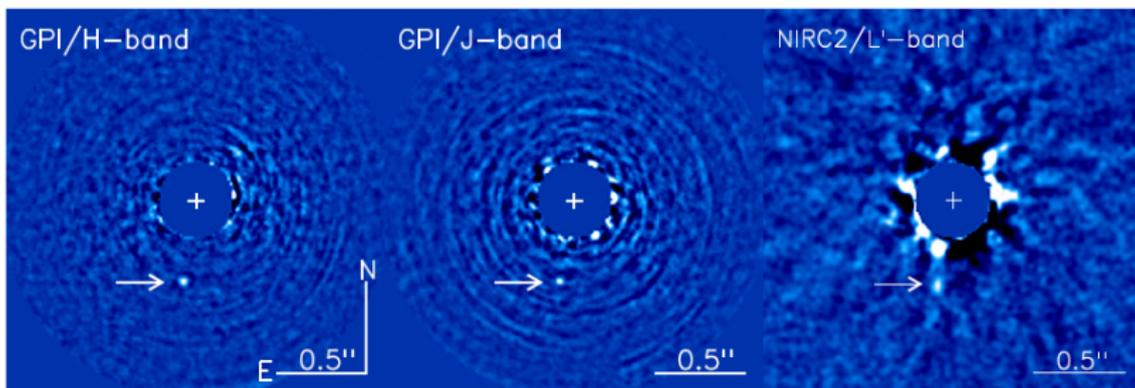
Laurent Pueyo, Space Telescope Science Institute

Sagan Summer Workshop, 2016

July 21, 2016

The example of 51 Eri b with the Gemini Planet Imager

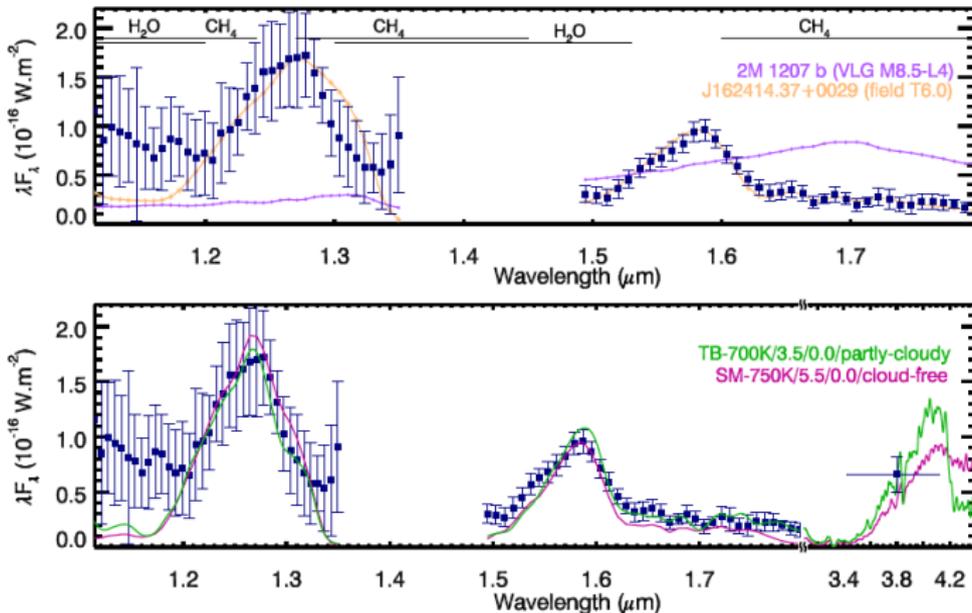
Images in multiple bands, Macintosh et al, 2015



How do we make blobs appear? How do we decide a blob might be a planet?

The example of 51 Eri b with the Gemini Planet Imager

Spectrum, Macintosh et al, 2015

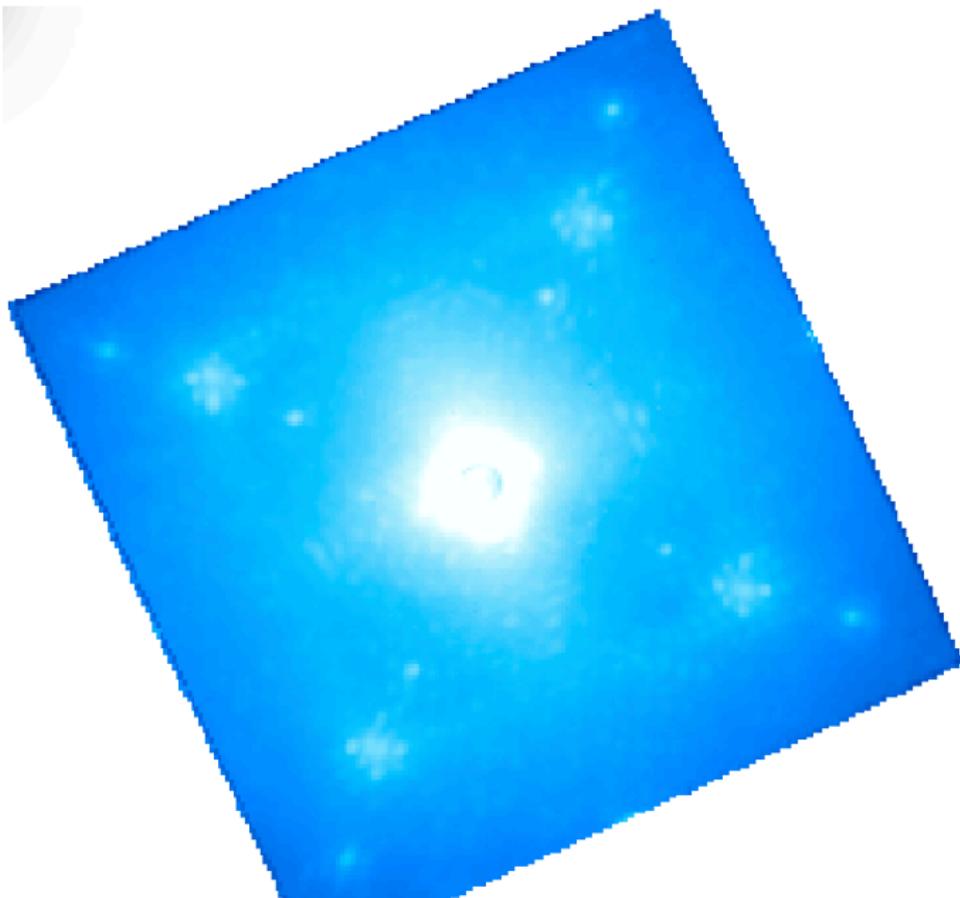


How do we get a spectrum?

This talk

- 1 High-contrast image formation theory.
- 2 High-contrast data analysis.
- 3 Handling astrophysical noise.

Fourier Transforms



Fourier Transforms

E. WOLF, PROGRESS IN OPTICS XIX © NORTH-HOLLAND 1981

V

THE EFFECTS OF ATMOSPHERIC TURBULENCE IN OPTICAL ASTRONOMY

BY

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Fourier Transforms

Let us denote $O(\alpha)$ the irradiance distribution from the object as a function of the direction α on the sky. $I(\alpha)$ will be the observed irradiance distribution, in the instantaneous image, as a function of the same variable α . A long exposure image will be considered as the ensemble average $\langle I(\alpha) \rangle$. Since astronomical objects are entirely incoherent, the relation between $\langle I(\alpha) \rangle$ and $O(\alpha)$ is linear. We shall moreover assume that it is shift invariant, i.e. the telescope is isoplanatic and the average effect of turbulence is the same all over the telescope field of view. In such a case, $\langle I(\alpha) \rangle$ is related to $O(\alpha)$ by a convolution relation

$$\langle I(\alpha) \rangle = O(\alpha) * \langle S(\alpha) \rangle \quad (4.1)$$

the point spread function $\langle S(\alpha) \rangle$ being the average image of a point source.

We shall define the two-dimensional complex Fourier transform $\tilde{I}(\mathcal{f})$ of $I(\alpha)$ as

$$\tilde{I}(\mathcal{f}) = \int d\alpha \cdot I(\alpha) \cdot \exp(-2i\pi\alpha \cdot \mathcal{f}) \quad (4.2)$$

with similar relations for the Fourier transform \tilde{O} and \tilde{S} of O and S . In these expressions the spatial frequency vector \mathcal{f} has the dimension of the inverse of the angle α and must therefore be expressed in radian^{-1} . With such a definition, (4.1) becomes, in the Fourier space

$$\langle \tilde{I}(\mathcal{f}) \rangle = \tilde{O}(\mathcal{f}) \cdot \langle \tilde{S}(\mathcal{f}) \rangle \quad (4.3)$$

where $\langle \tilde{S}(\mathcal{f}) \rangle$ is the optical transfer function of the whole system, telescope and atmosphere.

Fourier Transforms

chromatic point source, of wavelength λ . Again, we shall denote $\Psi_0(\mathbf{x})$ as the complex amplitude at the telescope aperture. The complex amplitude $\mathcal{A}(\boldsymbol{\alpha})$ diffracted at an angle $\boldsymbol{\alpha}$ in the telescope focal plane is proportional to

$$\mathcal{A}(\boldsymbol{\alpha}) \propto \int d\mathbf{x} \cdot \Psi_0(\mathbf{x}) P_0(\mathbf{x}) \exp(-2i\pi \boldsymbol{\alpha} \cdot \mathbf{x}/\lambda) \quad (4.4)$$

where $P_0(\mathbf{x})$ is the transmission function of the telescope aperture. For an ideal diffraction-limited telescope,

$$P_0(\mathbf{x}) = \begin{cases} 1 & \text{inside the aperture} \\ 0 & \text{outside the aperture.} \end{cases} \quad (4.5)$$

In the case of aberrated optics, wavefront errors are introduced as an argument of the complex transmission $P_0(\mathbf{x})$.

In the following, we shall make extensive use of the non-dimensional reduced variable

$$\mathbf{u} = \mathbf{x}/\lambda. \quad (4.6)$$

Let us call

$$\Psi(\mathbf{u}) = \Psi_0(\lambda \mathbf{u}) \quad \text{and} \quad P(\mathbf{u}) = P_0(\lambda \mathbf{u}). \quad (4.7)$$

With such notation (4.4) becomes

$$\mathcal{A}(\boldsymbol{\alpha}) \propto \mathcal{F}[\Psi(\mathbf{u}) \cdot P(\mathbf{u})] \quad (4.8)$$

Fourier Transforms

where \mathcal{F} is the complex Fourier transform defined by (4.2). The point spread function is the irradiance diffracted in the direction α

$$S(\alpha) = |\mathcal{A}(\alpha)|^2 \propto |\mathcal{F}[\Psi(\mathbf{u})P(\mathbf{u})]|^2. \quad (4.9)$$

Its Fourier transform is given by the autocorrelation function of $\Psi(\mathbf{u})P(\mathbf{u})$

$$\tilde{S}(\mathcal{J}) \propto \int d\mathbf{u} \cdot \Psi(\mathbf{u})\Psi^*(\mathbf{u}+\mathcal{J})P(\mathbf{u})P^*(\mathbf{u}+\mathcal{J}). \quad (4.10)$$

In the absence of any turbulence, we assume that $\Psi(\mathbf{u}) = 1$ (§ 3) so that, normalising $\tilde{S}(\mathcal{J})$ to unity at the origin,

$$\tilde{S}(\mathcal{J}) = \mathcal{S}^{-1} \int d\mathbf{u} \cdot P(\mathbf{u})P^*(\mathbf{u}+\mathcal{J}) = T(\mathcal{J}) \quad (4.11)$$

where \mathcal{S} is the pupil area (in wavelength squared units). Eq. (4.11) is the classical expression for the optical transfer function $T(\mathcal{J})$ of a telescope.

Fourier Transforms

In the presence of turbulence (4.11) becomes

$$\bar{S}(\mathcal{J}) = \mathcal{S}^{-1} \int d\mathbf{u} \cdot \Psi(\mathbf{u}) \Psi^*(\mathbf{u} + \mathcal{J}) P(\mathbf{u}) P^*(\mathbf{u} + \mathcal{J}) \quad (4.12)$$

and the optical transfer function for long exposures is

$$\langle \bar{S}(\mathcal{J}) \rangle = \mathcal{S}^{-1} \int d\mathbf{u} \langle \Psi(\mathbf{u}) \cdot \Psi^*(\mathbf{u} + \mathcal{J}) \rangle P(\mathbf{u}) P^*(\mathbf{u} + \mathcal{J}). \quad (4.13)$$

In (4.13) appears the second order moment

$$B(\mathcal{J}) = \langle \Psi(\mathbf{u}) \cdot \Psi^*(\mathbf{u} + \mathcal{J}) \rangle = B_0(\lambda \mathcal{J}) \quad (4.14)$$

the properties of which have been studied in § 3. Since $B(\mathcal{J})$ depends only upon \mathcal{J} , (4.13) can be written, taking (4.11) into account,

$$\langle \bar{S}(\mathcal{J}) \rangle = B(\mathcal{J}) \cdot T(\mathcal{J}) \quad (4.15)$$

showing the fundamental result that, for long exposures, the optical transfer function of the whole system, telescope and atmosphere, is the product of the transfer function of the telescope with an atmospheric transfer function equal to the coherence function $B(\mathcal{J})$.

Fourier Transforms

Let us denote $O(x)$ the irradiance distribution from the object as a function of the direction x on the sky. $I(x)$ will be the observed irradiance distribution, in the instrument image, as a function of the same variable x . A long exposure image will be considered as the ensemble average $\langle I(x) \rangle$. These astronomical objects are entirely incoherent, the relation between $I(x)$ and $O(x)$ is linear. We shall assume hence that it is shift invariant, i.e. the telescope is aperiodic and the average effect of turbulence is the same all over the telescope field of view. In such a case, $\langle I(x) \rangle$ is related to $O(x)$ by a convolution relation

$$\langle I(x) \rangle = O(x) * \mathcal{D}(x) \quad (4.1)$$

the point spread function $\mathcal{D}(x)$ being the average image of a point source. We shall define the two-dimensional complex Fourier transform $I(f)$ of $I(x)$ as

$$I(f) = \int dx \cdot I(x) \cdot \exp(-2\pi i x \cdot f) \quad (4.2)$$

with similar relations for the Fourier transforms O and \mathcal{D} of O and \mathcal{D} . In these expressions the spatial frequency vector f has the dimension of the inverse of the angle x and must therefore be expressed in rad^{-1} . With such a definition, (4.1) becomes, in the Fourier space

$$\langle I(f) \rangle = O(f) \cdot \mathcal{D}(f) \quad (4.3)$$

where $\mathcal{D}(f)$ is the optical transfer function of the whole system, telescope and atmosphere.

chromatic point source, of wavelength λ . Again, we shall denote $P_0(x)$ as the complex amplitude at the telescope aperture. The complex amplitude $a(f)$ diffracted at an angle x in the telescope focal plane is proportional to

$$a(x) = \int dx' \cdot P_0(x') \exp(-2\pi i x' \cdot x) \quad (4.4)$$

where $P_0(x)$ is the transmission function of the telescope aperture. For a sharp diffraction-limited telescope,

$$P_0(x) = \begin{cases} 1 & \text{inside the aperture} \\ 0 & \text{outside the aperture.} \end{cases} \quad (4.5)$$

In the case of aberrated optics, wavefront errors are introduced as an argument of the complex transmission $P_0(x)$.

In the following, we shall make extensive use of the non-dimensional reduced variable

$$u = x/\lambda. \quad (4.6)$$

Let us call

$$\Psi(u) = P_0(x) \quad \text{and} \quad \mathcal{P}(u) = P_0(u\lambda). \quad (4.7)$$

With such notation (4.4) becomes

$$a(x) = \mathcal{P}[\Psi(u) \cdot P(u)] \quad (4.8)$$

where \mathcal{P} is the complex Fourier transform defined by (4.2). The point spread function is the modulus squared of the direction x

$$I(x) = |a(x)|^2 = |\mathcal{P}[\Psi(u) P(u)]|^2. \quad (4.9)$$

In Fourier transform is given by the autocorrelation function of $\Psi(u) P(u)$

$$\mathcal{S}(f) = \int dx \cdot \Psi(u) \Psi^*(u+x) \cdot P(u) P^*(u+x). \quad (4.10)$$

In the absence of any turbulence, we assume that $\Psi(u) = 1$ (3.3) so that, normalizing $\mathcal{S}(f)$ to unity at the origin,

$$\mathcal{S}(f) = \mathcal{P}^2 \int dx \cdot P(u) P^*(u+x) \cdot T(f). \quad (4.11)$$

where \mathcal{P} is the pupil area (in wavelength squared units). Eq. (4.11) is the classical expression for the optical transfer function $\mathcal{T}(f)$ of a telescope.

In the presence of turbulence (4.11) becomes

$$\mathcal{S}(f) = \mathcal{P}^2 \int dx \cdot \Psi(u) \Psi^*(u+x) \cdot P(u) P^*(u+x). \quad (4.12)$$

and the optical transfer function for long exposures is

$$\langle \mathcal{S}(f) \rangle = \mathcal{P}^2 \int dx \cdot \Psi(u) \cdot \Psi^*(u+x) \cdot P(u) P^*(u+x). \quad (4.13)$$

In (4.13) appears the second order moment

$$\mathcal{M}(f) = \langle \Psi(u) \cdot \Psi^*(u+x) \cdot P(u) P^*(u+x) \rangle \quad (4.14)$$

the properties of which have been studied in 3.3. Since $\mathcal{M}(f)$ depends only upon f , (4.13) can be written, taking (4.11) into account,

$$\langle \mathcal{S}(f) \rangle = \mathcal{M}(f) \cdot \mathcal{T}(f) \quad (4.15)$$

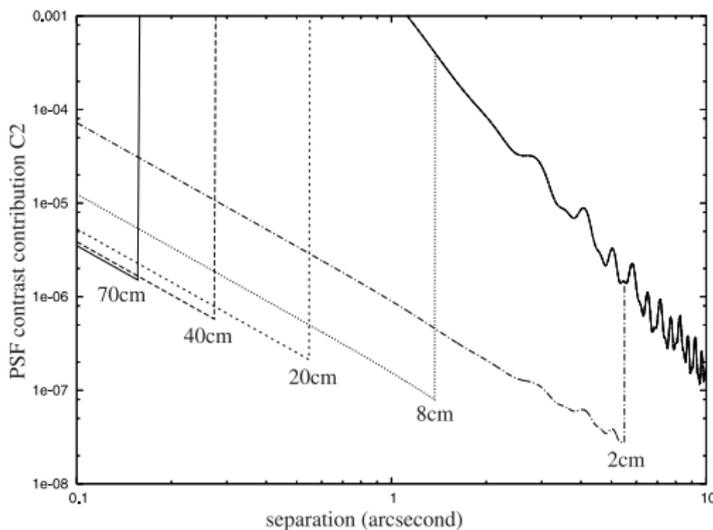
showing the fundamental result that, for long exposures, the optical transfer function of the whole system, telescope and atmosphere, is the product of the transfer function of the telescope with an atmospheric transfer function equal to the coherence function $\mathcal{M}(f)$.

Bottom Line

- Main sources of noise = whatever is at the telescope entrance, e.g. atmospheric turbulence and imperfections on the optics.
- In direct imaging data their **Fourier Transform** is the relevant quantity for noise estimation. For long exposures we care about the Fourier Transform of the auto-correlation of the errors at the telescope entrance averaged over time.

Fourier Transforms

Guyon (2005).



Bottom Line

- Main sources of noise = whatever is at the telescope entrance, e.g. atmospheric turbulence and imperfections on the optics.
- If we “broadly” know what they look like, we can predict what the images will look like.

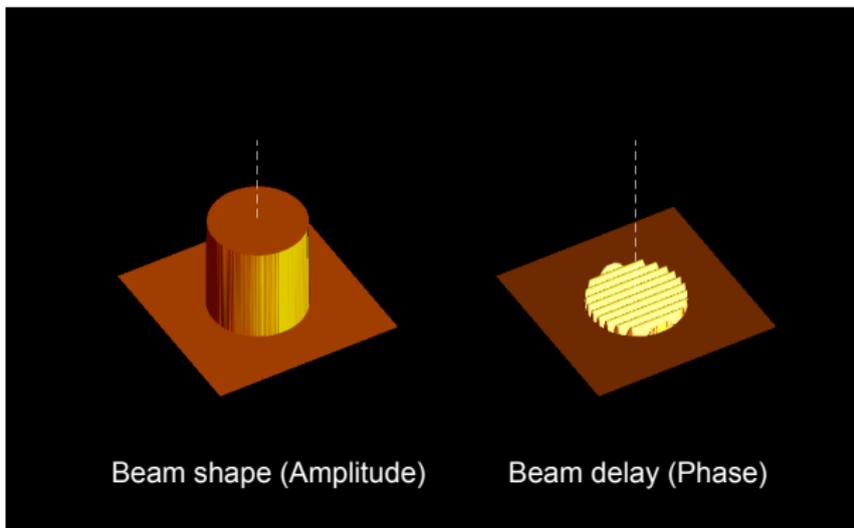
Speckles: symmetries

Complex amplitude at the entrance of the coronagraph, assuming no time variations:

$$\psi_0(\mathbf{x}) = \text{Beam}_{\text{Amplitude}}(\mathbf{x}) \exp[i\text{Beam}_{\text{Delay}}(\mathbf{x})]$$

$$\psi_0(\mathbf{x}) = [1 + \varepsilon_A(\mathbf{x})] \exp[i\varepsilon_{OPD}(\mathbf{x})/\lambda]$$

$$\psi_0(\mathbf{x}) \sim 1 + \varepsilon_A(\mathbf{x}) + i\varepsilon_{OPD}(\mathbf{x})/\lambda \sim \varepsilon_A(\mathbf{x}) + i\varepsilon_{OPD}(\mathbf{x})/\lambda$$



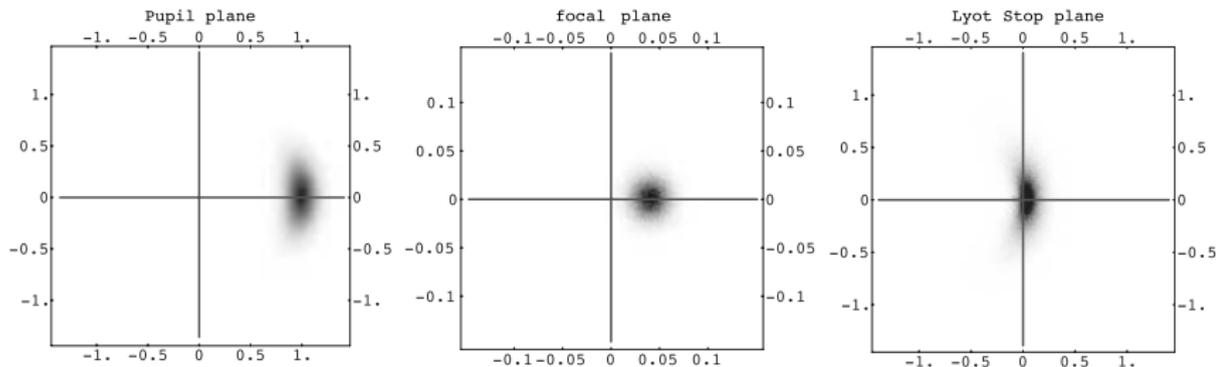
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Soummer et al. (2008).



Speckles: symmetries

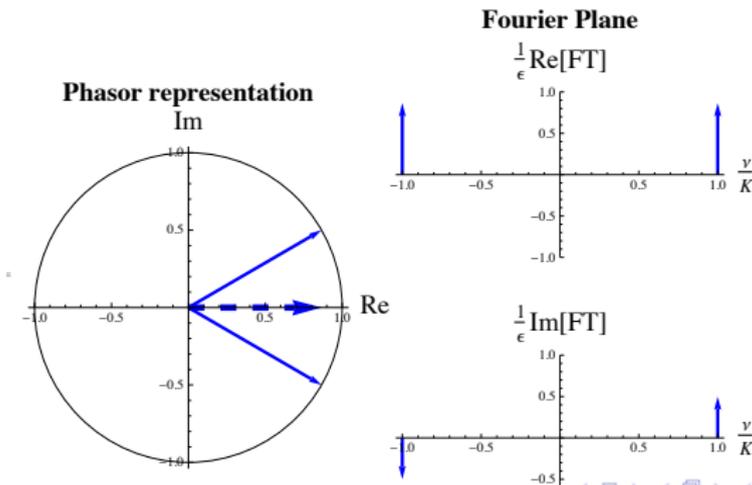
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$$\psi_0(\mathbf{x}) \sim \varepsilon \cos\left(\frac{2\pi}{D} n\mathbf{x} + \phi\right) \text{ and } \int d\mathbf{u} \psi_0(\mathbf{u}) \psi_0(\mathbf{u} + \mathbf{f})^* \sim \varepsilon \cos\left(\frac{2\pi}{D} n\mathbf{f} + \phi\right)$$

Pueyo et al. (2009).



Speckles: symmetries

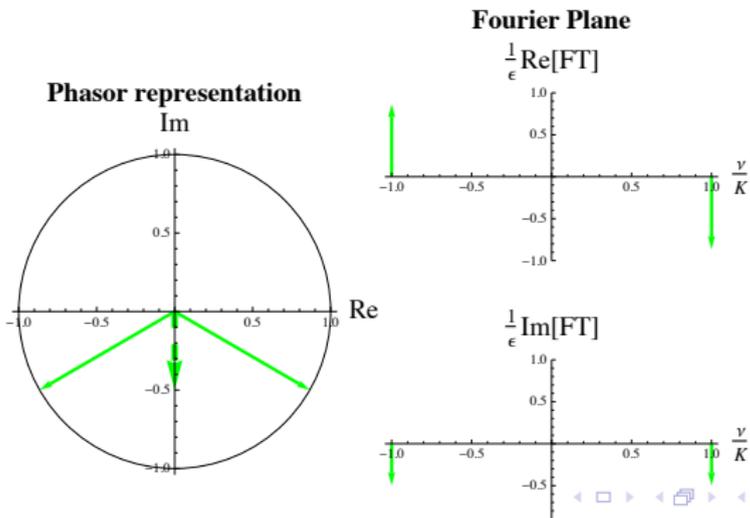
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Pueyo et al. (2009).

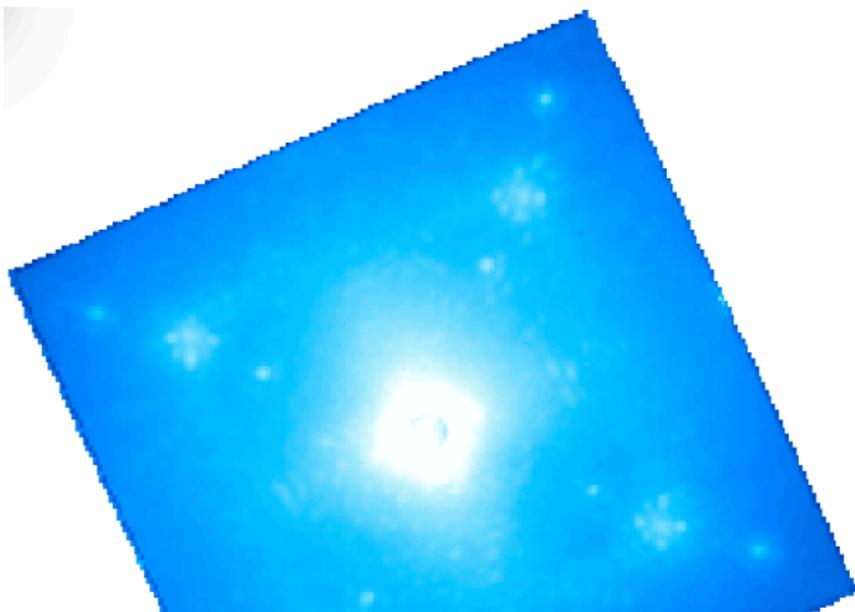


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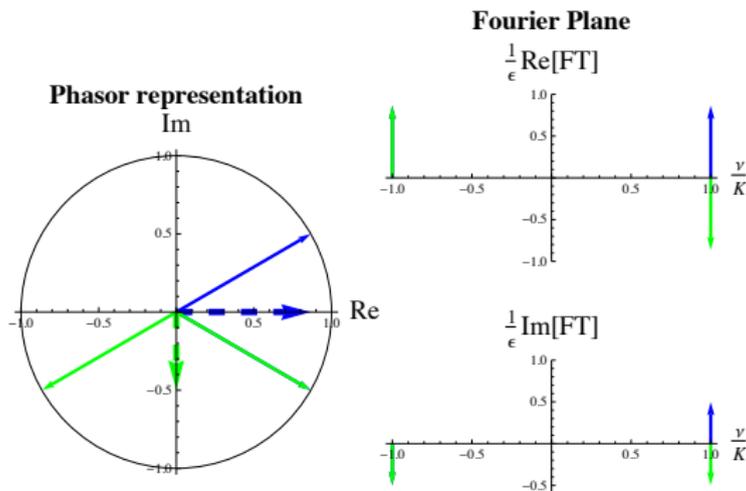
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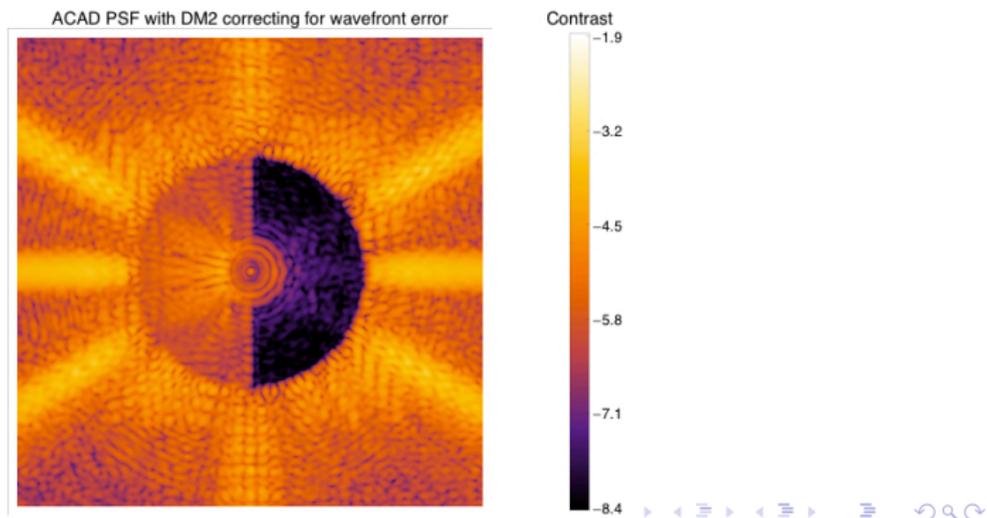
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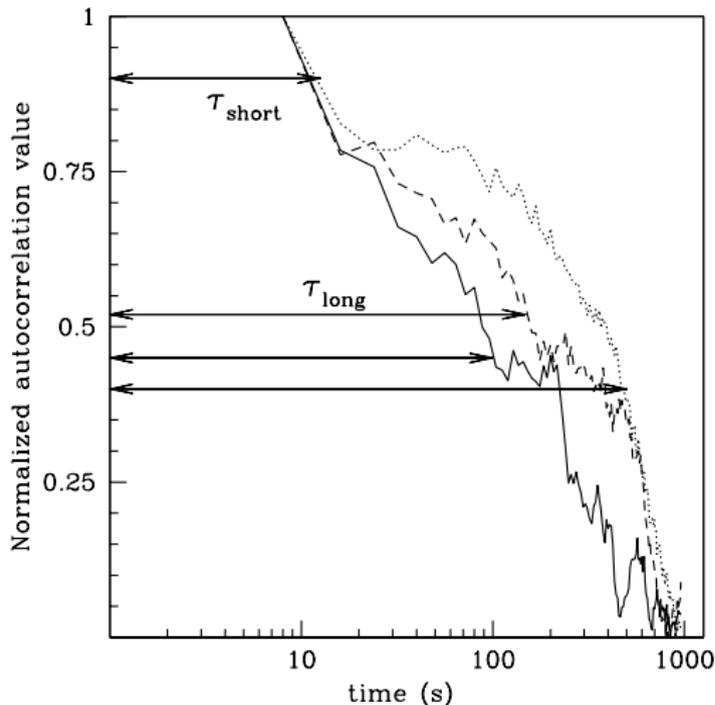
$$\psi_0(\mathbf{x}) \sim 1 + \varepsilon_A(\mathbf{x}) + i\varepsilon_{OPD}(\mathbf{x})/\lambda \sim \varepsilon_A(\mathbf{x}) + i\varepsilon_{OPD}(\mathbf{x})/\lambda$$

Pueyo and Norman (2013).



Speckles: Temporal evolution

Hinkley et al. (2007).



Quick derivation of the respective influence of atmospheric and “quasi-static” (e.g. from telescope/instrument optics) speckles.

$$\psi_0(\mathbf{x}) = [\varepsilon_{Atm}(t) + \varepsilon_{Tel}(t)] \cos\left(\frac{2\pi}{D} n\mathbf{x} + \phi\right)$$

$$S(\mathbf{f}) = \int d\mathbf{u} \langle \psi_0(\mathbf{u}) \psi_0(\mathbf{u} + \mathbf{f})^* \rangle_{T_{exp}}$$

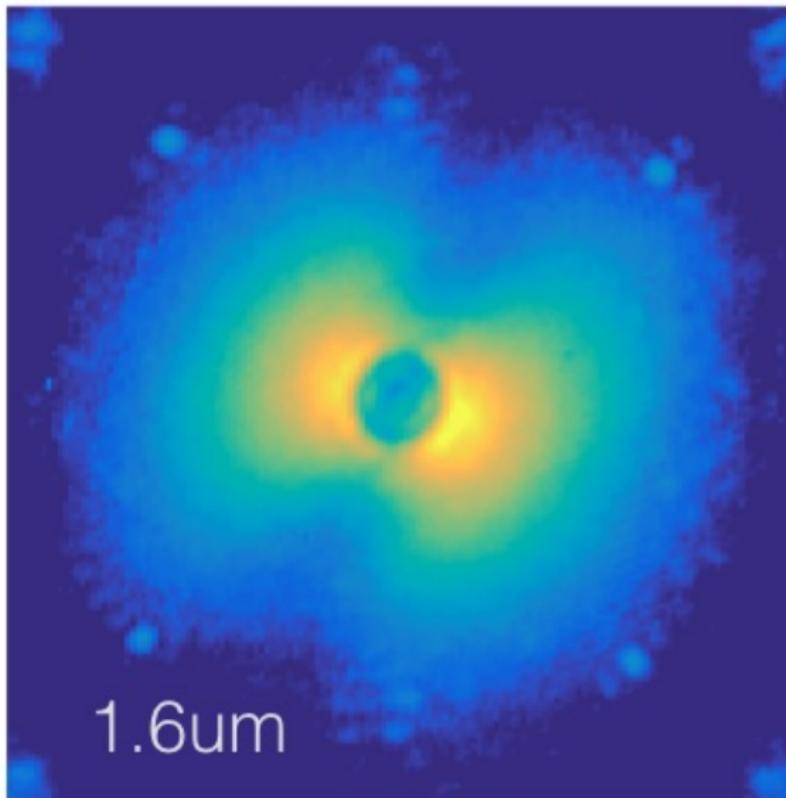
$$\sim [\sigma_{Atm}^2 + 2 \langle \varepsilon_{Atm}, \varepsilon_{Tel} \rangle_{T_{exp}} + \dots]$$

$$\dots + \langle \varepsilon_{Tel}, \varepsilon_{Tel} \rangle_{T_{exp}}] \cos\left(\frac{2\pi}{D} n\mathbf{f} + \phi\right)$$

Rigorous derivation in Perrin et al. (2005).

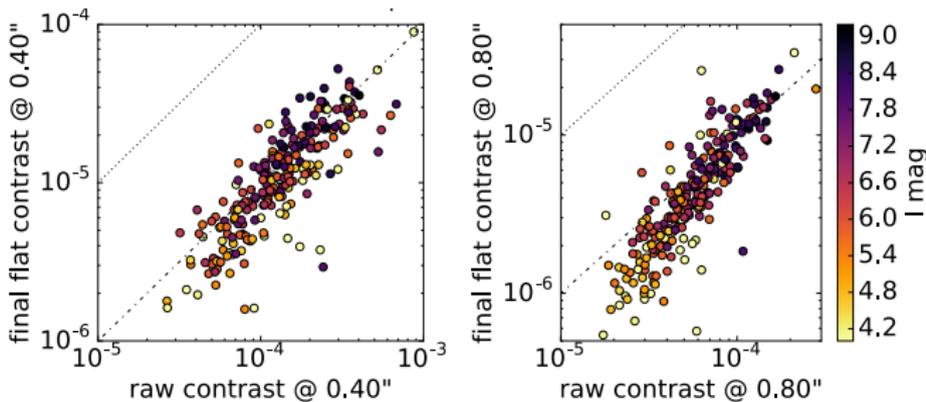
Speckles: Temporal evolution

Bailey et al. (2016).



Speckles: Temporal evolution

Bailey et al. (2016).



Key temporal properties of speckles

- The atmosphere creates speckles, but they average out into a broad halo.
- Adaptive Optics performances dictates the shape of this “average halo”.
- The telescope+instrument speckles are pinned to the AO response.
- The telescope+instrument speckles have timescales ranging from exposure time to observing sequence.

Speckles: wavelength dependence

Speckles: wavelength dependence

chromatic point source, of wavelength λ . Again, we shall denote $\Psi_0(\mathbf{x})$ as the complex amplitude at the telescope aperture. The complex amplitude $\mathcal{A}(\boldsymbol{\alpha})$ diffracted at an angle $\boldsymbol{\alpha}$ in the telescope focal plane is proportional to

$$\mathcal{A}(\boldsymbol{\alpha}) \propto \int d\mathbf{x} \cdot \Psi_0(\mathbf{x}) P_0(\mathbf{x}) \exp(-2i\pi \boldsymbol{\alpha} \cdot \mathbf{x}/\lambda) \quad (4.4)$$

where $P_0(\mathbf{x})$ is the transmission function of the telescope aperture. For an ideal diffraction-limited telescope,

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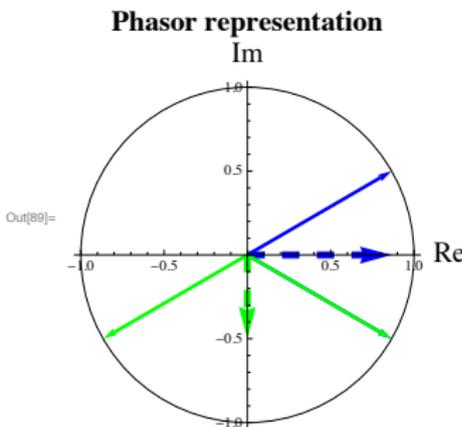
Let us call

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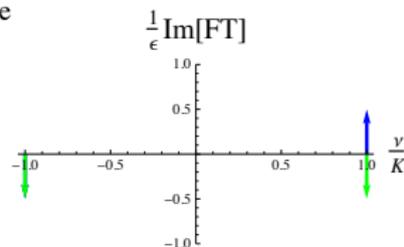
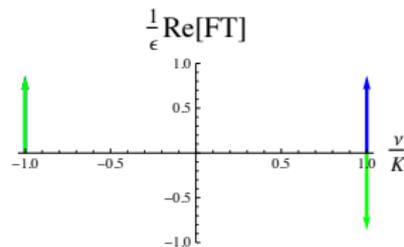
With such notation (4.4) becomes

$$\mathcal{A}(\boldsymbol{\alpha}) \propto \mathcal{F}[\Psi(\mathbf{u}) \cdot P(\mathbf{u})] \quad (4.8)$$

Speckles: wavelength dependence



Fourier Plane

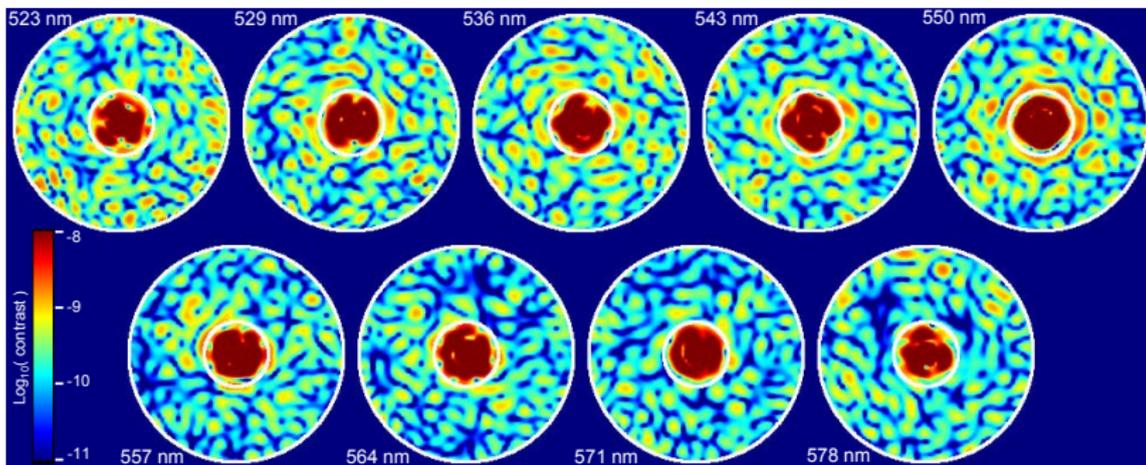


Key morphological properties of speckles

- Speckles look like planets.
- Speckles are symmetric (except when they are not).
- Speckles stretch with wavelength (except when they are not).

Speckles: wavelength dependence

Krist et al. (2016)

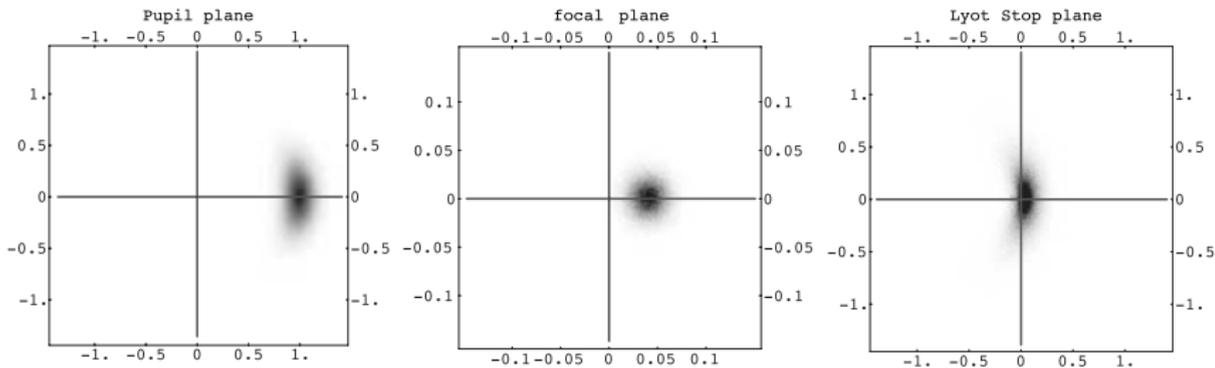


Key morphological properties of speckles

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Speckles: statistics

Soummer et al. (2008)



Speckles: statistics

Soummer et al. (2008)

$S(\mathbf{r}) \sim \mathcal{N}_c(0, I_c)$. The instantaneous intensity corresponding to the complex amplitude of equation (12) is simply

$$I = |S(\mathbf{r}) + \tilde{C}(\mathbf{r})|^2 \\ = \{\text{Re}[\tilde{C}(\mathbf{r}) + S(\mathbf{r})]\}^2 + \{\text{Im}[\tilde{C}(\mathbf{r}) + S(\mathbf{r})]\}^2, \quad (15)$$

where Re and Im denote the real and imaginary parts. Using the properties of circular Gaussian distributions, $\text{Re}[\tilde{C}(\mathbf{r}) + S(\mathbf{r})]$ and $\text{Im}[\tilde{C}(\mathbf{r}) + S(\mathbf{r})]$ are independent Gaussian random variables of the same variance $I_c/2$. We can rewrite the intensity with real and imaginary terms of variance unity,

$$I = \frac{I_c}{2} \left(\left\{ \text{Re} \left[\sqrt{2I_c^{-1}} \tilde{C}(\mathbf{r}) + S(\mathbf{r}) \right] \right\}^2 + \left\{ \text{Im} \left[\sqrt{2I_c^{-1}} \tilde{C}(\mathbf{r}) + S(\mathbf{r}) \right] \right\}^2 \right) = \frac{I_c}{2} \tilde{I}, \quad (16)$$

where $\text{Var}[\text{Re}[\sqrt{2I_c^{-1}} \tilde{C}(\mathbf{r}) + S(\mathbf{r})]] = \text{Var}[\text{Im}[\sqrt{2I_c^{-1}} \tilde{C}(\mathbf{r}) + S(\mathbf{r})]] = 1$.

The random variable \tilde{I} follows a decentered χ^2 with two degrees of freedom, $\chi^2_2(m)$, with a decentering parameter $m = 2I_c^{-1}I_c$ (Johnson et al. 1995, p. 433). The PDF for \tilde{I} is therefore

$$P(v) = 2^{-1} e^{-(m+v)/2} f_1\left(\frac{1}{4}mv\right), \quad v > 0, \quad (17)$$

where $f_0(z)$ is the regularized confluent hypergeometric function and ${}_0F_1(; q; z)$ is the confluent hypergeometric function defined as

$$f_0(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(q+n)!} z^n = \frac{{}_0F_1(; q; z)}{\Gamma(q)}. \quad (18)$$

Finally, the PDF of the intensity $I = I_c/2\tilde{I}$ is

$$p_I(I) = \frac{e^{-(I+I_c)/I_c}}{I_c} {}_0F_1\left(; 1; \frac{I_c I}{I_c^2}\right). \quad (19)$$

This expression is equivalent² to the “modified Rician distribution” derived by Goodman (1975) and used by Cagigal & Canales (1998, 2000) and Canales & Cagigal (1999, 2001):

$$p_I(I) = \frac{1}{I_c} \exp\left(-\frac{I+I_c}{I_c}\right) I_0\left(\frac{2\sqrt{I}\sqrt{I_c}}{I_c}\right), \quad (20)$$

This PDF corresponds to the well-known negative exponential density for a fully developed speckle pattern (e.g., laser speckle pattern; Goodman 2000). Finally, the distribution at photon counting levels can be obtained by performing a Poisson-Mandel transformation of the high-flux PDF in equation (20). An analytical expression of this PDF has been given in Aime & Soummer (2004b).

The mean and variance of the intensity can be obtained by several ways. A first method (Goodman 1975, 2000) is to express the mean intensity $E(I)$ and the second-order moment of the intensity $E(I^2)$ as functions of $C(\mathbf{r})$ and $S(\mathbf{r})$. The second-order moment for the intensity is the fourth-order moment for the complex amplitude, $E(I^2) = E((C+S)(C^*+S^*))^2$ (omitting the variables \mathbf{r} for clarity), which can be simplified using the properties of Gaussian distributions. With $E(SS^*SS^*) = 2E(SS^*)E(SS^*) = 2I_c^2$ we obtain $E(I^2) = I_c^2 + 4I_c I_c + 2I_c^2$. A second method is to derive a general analytical expression for the moments of the Rician distribution. This can be obtained either from the definition of the moments of equation (20) (Goodman 1975) or by computing the derivatives of the moment-generating function (Aime & Soummer 2004b). The instantaneous intensity in the focal plane (eq. [15]) can be written as

$$I = |C(\mathbf{r})|^2 + |S(\mathbf{r})|^2 + 2\text{Re}[C^*(\mathbf{r})S(\mathbf{r})]. \quad (22)$$

Since $E(S(\mathbf{r})^*) = E(S(\mathbf{r}))^* = 0$ (circular Gaussian distribution), the mean intensity is simply the sum of the deterministic diffraction pattern with a halo produced by the average of the speckles, $I_c + I_c$ or $I_c + I_c$, respectively, for direct and coronagraphic images. The variance also finds a simple analytical expression, and we have

$$E(I) = I_c + I_c, \\ \sigma_I^2 = I_c^2 + 2I_c I_c. \quad (23)$$

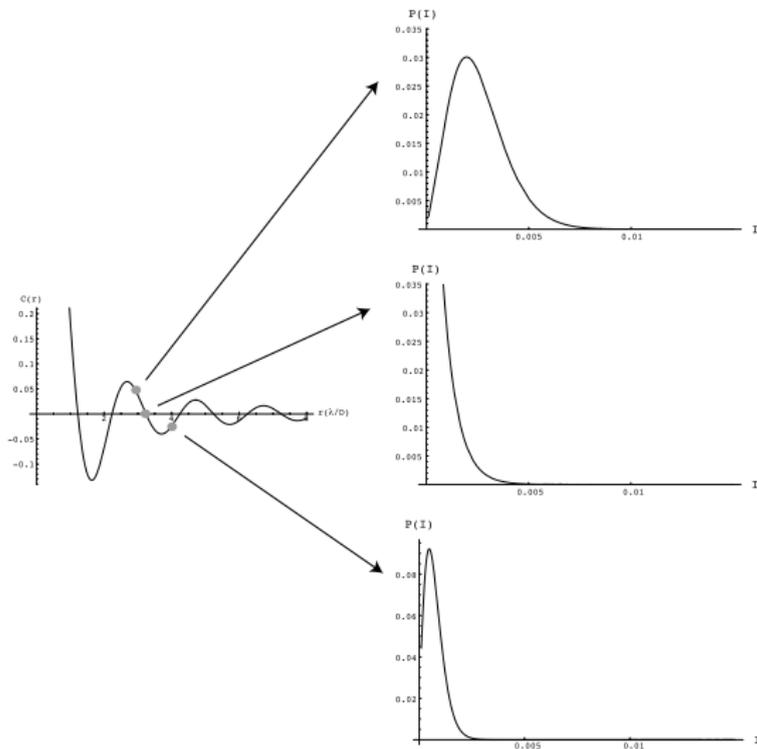
The variance associated with photodetection can be added to this expression to obtain the total variance $\sigma^2 = \sigma_I^2 + \sigma_P^2$, where σ_P^2 is the variance associated with the Poisson statistics, $\sigma_P^2 = I_c + I_c$. The total variance is therefore

$$\sigma^2 = I_c^2 + 2I_c I_c + I_c + I_c. \quad (24)$$

In the case of direct images, the term I_c corresponds to the perfect PSF scaled to the SR. In the case of coronagraphic images, the focal plane intensity is not invariant by translation, and therefore, it is technically not a true PSF. However, we use the

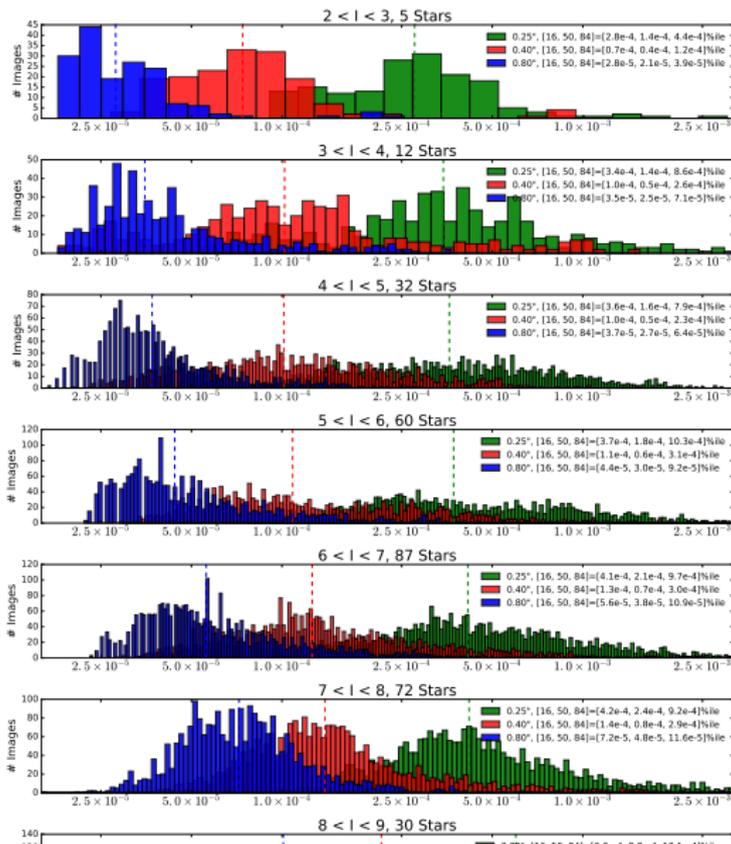
Speckles: statistics

Soummer et al. (2008)

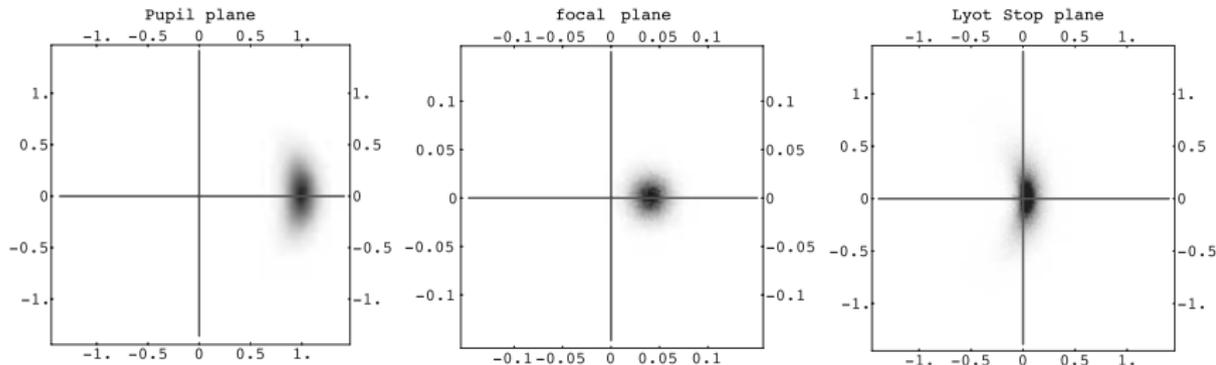


Speckles: statistics

Courtesy of A. Rajan and the GPI team.



Speckles: statistics



Key statistical properties of speckles

- Speckles follow a Modified Rician distribution (long positive tail).
- Second order moment depends on angular separation and on how well the coronagraph works.

Airing of grievances

Key annoying properties of speckles

- Speckles look like planets.
- Speckles follow a Modified Rician distribution (long positive tail).
- Second order moment depends on angular separation, on how well the coronagraph works and how well the atmosphere averages out.
- The telescope+instrument speckles have timescales ranging from exposure time to length of an observing sequence.

The most successful method to analyze direct imaging data so far has been to build an empirical model of the noise based on the data itself.

The problem(s)

Assume you have an image in which you are looking for a planet.

$$T(n) = I_{\psi_0}(n) + \varepsilon A(n).$$

We call ψ the random state of the telescope+instrument at the exposure.

The problem we want to solve is to figure out what are the relative contributions of the light diffracted within the instrument and of an hypothetical astrophysical signal.

Solutions

- We can have a really good model of our instrument.
- We “construct” a really good model of our instrument based on its data history (science frames+telemetry).
- We get more realizations of I_{ψ} for which we are sure that there is no astrophysical signal. We subtract them from T .

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Observing strategies

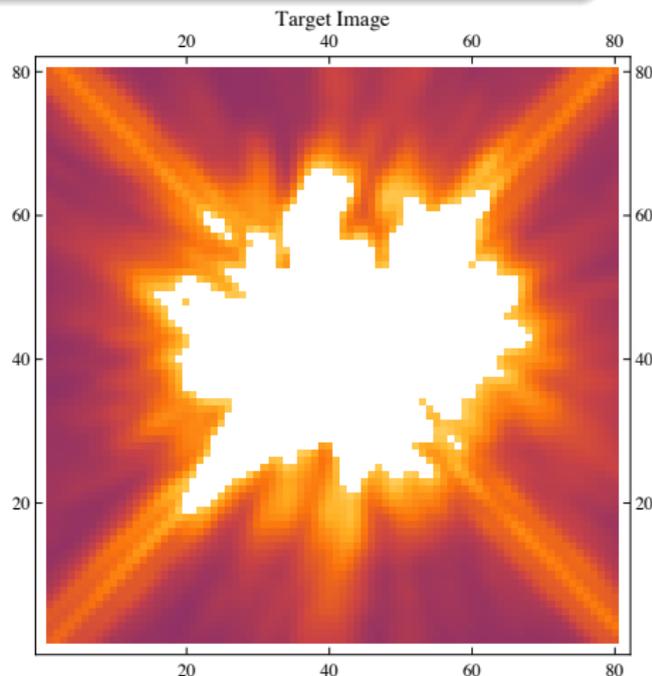
How to get more realizations of the instrument response?

- Take images of other sources.

$$\varepsilon A(n)? = I_{\psi_0}(n) - I_{\psi_1}(n)$$

What to watch for:

- The telescope + instrument must be very stable.
- The alignment of the images needs to be very precise (the star needs to be on the same fraction of a pixel).



Observing strategies

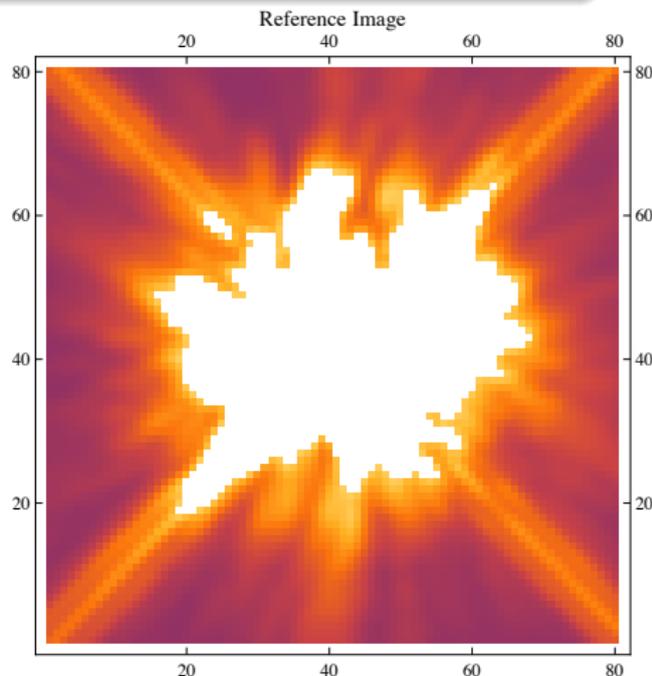
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- The alignment of the images needs to be very precise (the star needs to be on the same fraction of a pixel).



Observing strategies

How to get more realizations of the instrument response?

- Take images of other sources.
- Take images at other wavelengths/telescope orientations.

$$R(n) = I_{\psi_1}(n) + \varepsilon A(n - \delta n_{r,\theta}) \text{ or } R(n) = I_{\psi_1}(n - \delta n_{r,\theta}) + \varepsilon A(n)$$

LOCI - KLIP

Solving the least squares problem:

$$\min_{\{c_k\}} \left\{ \sum_n \left(T(n) - \sum_{k=1}^K c_k R_k(n) \right)^2 \right\}.$$

Equivalent to:

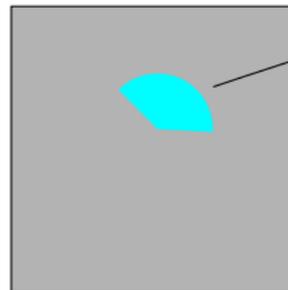
$$E[RR]C = T$$

where $E[RR]$ is the correlation matrix of the ensemble of references over the zone of the image we chose.

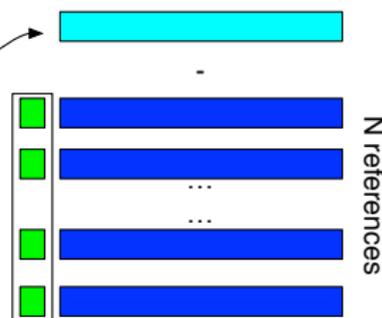
Several routes to invert this

- Tweak set up of the inverse problem (geometry, selection of references)
- Regularize of the inverse problem (SVD truncation, PCA)

Image, or part of image



K pixels in zone



LOCI - KLIP

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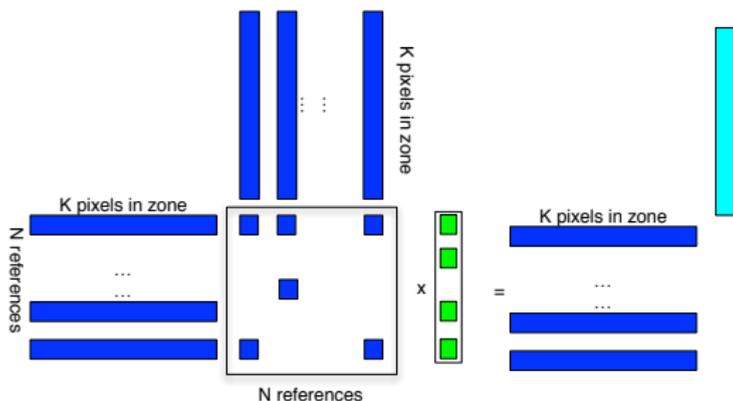
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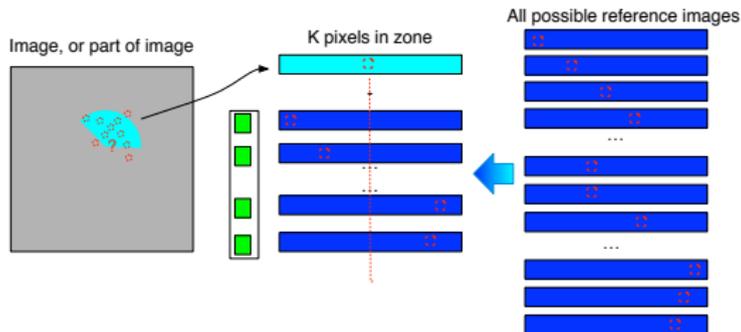
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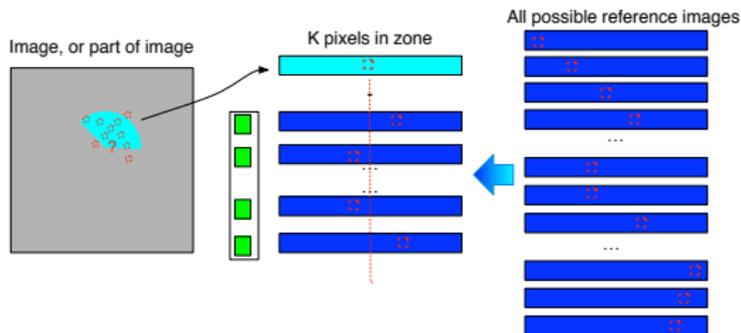
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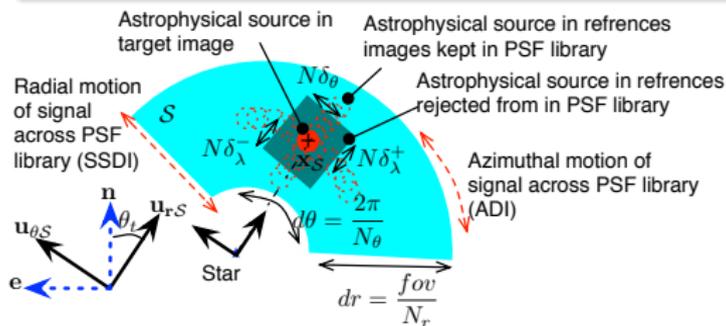
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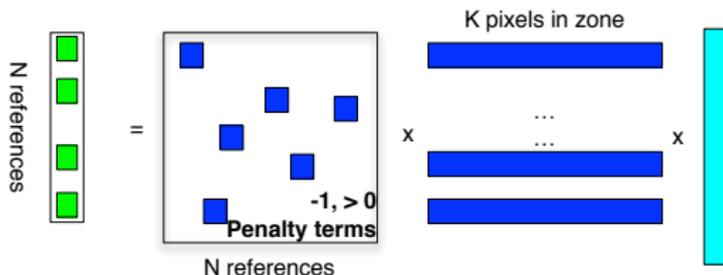
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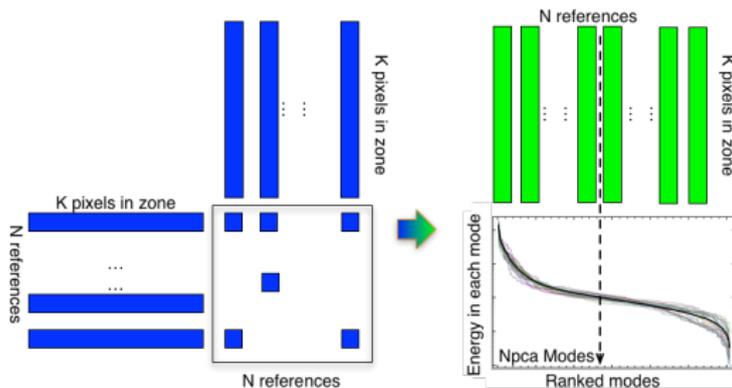
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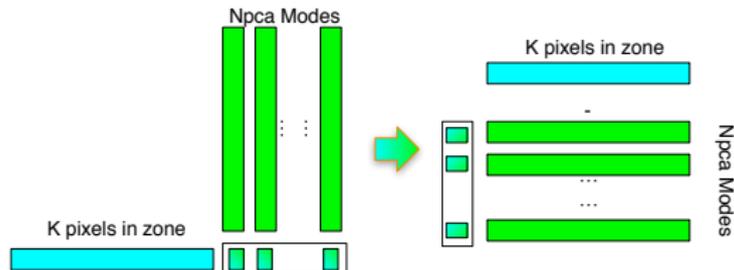
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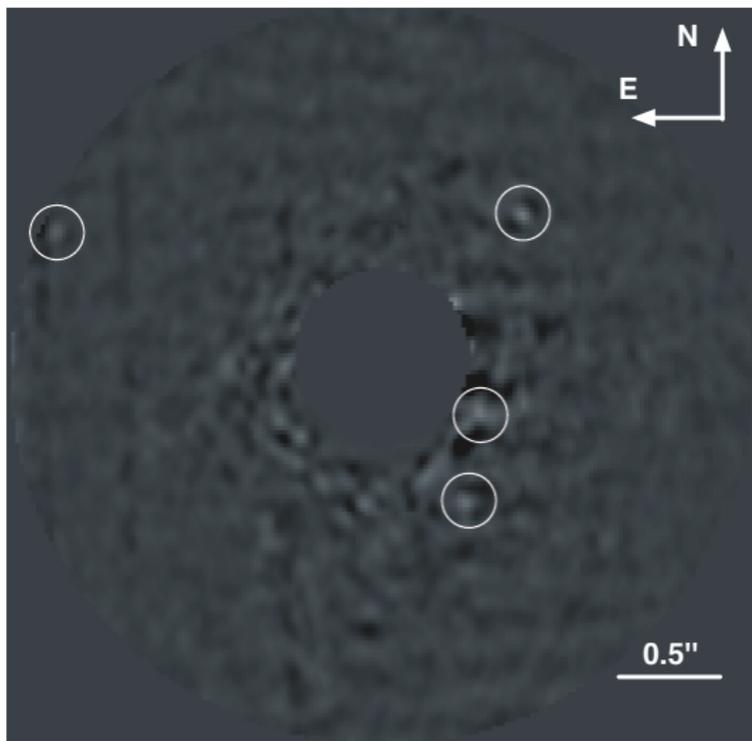
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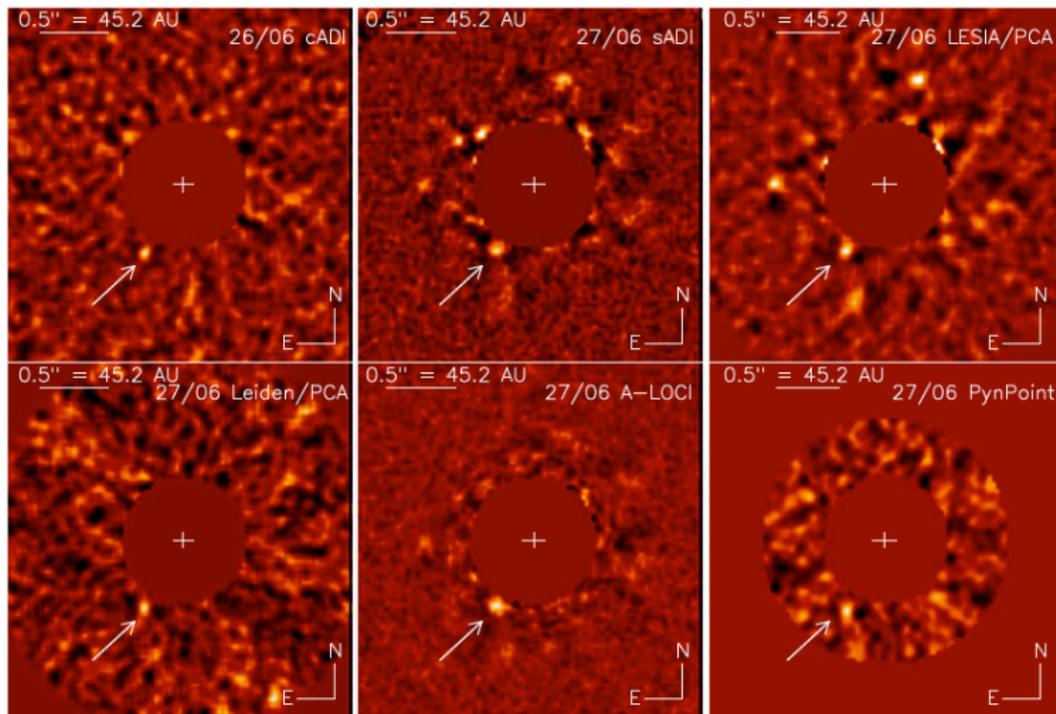
This is where the magic happens

Oppenheimer et al. (2013), Pueyo et al. (2015)



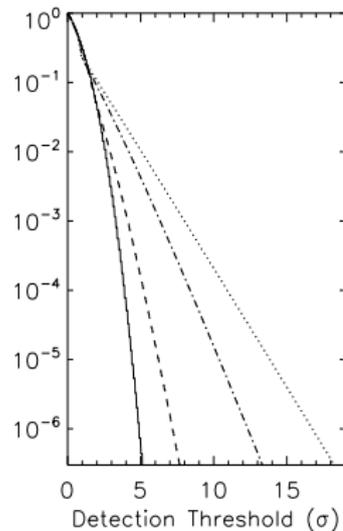
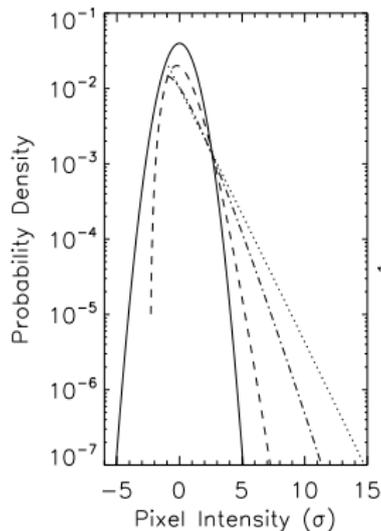
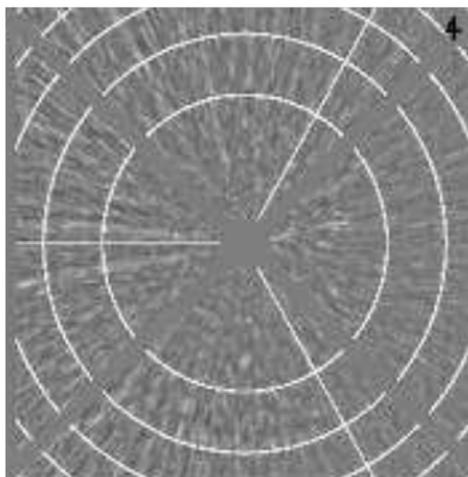
This is where the magic happens

Rameau et al. (2012)



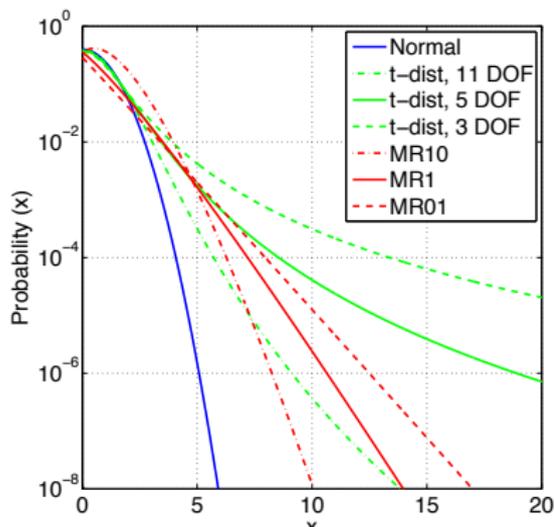
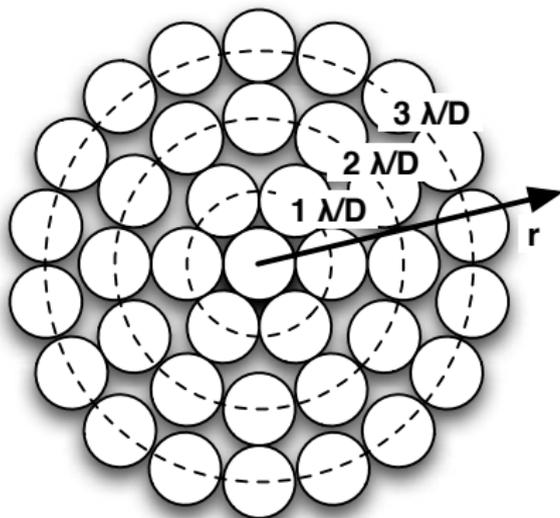
False positives and false negatives

The initial speckles follow Rice statistics, (hopefully) the steps above make them “more” Gaussian, Marois et al. (2007).



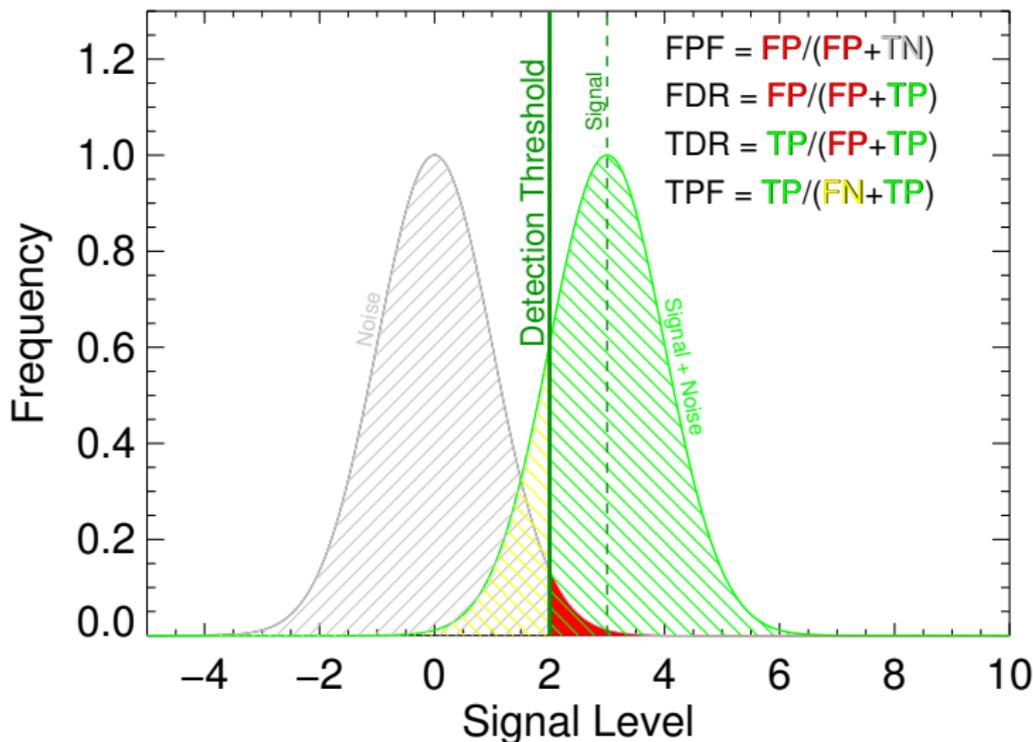
False positives and false negatives

When working at small separations a penalty term needs to be taken into account to include uncertainties associated with small number statistics when estimating the empirical variant of the noise, Mawet et al. (2014).



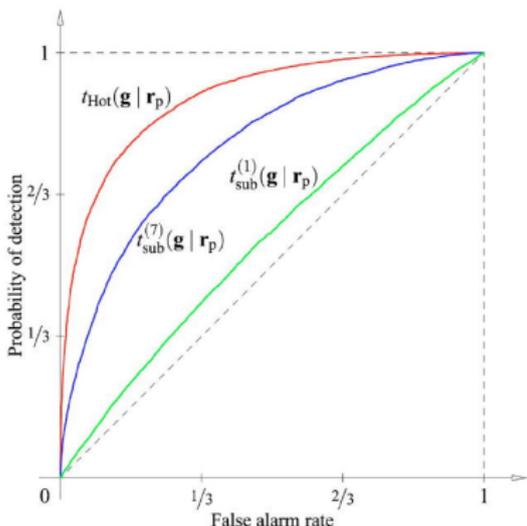
False positives and false negatives

In the case of a detection we care about the False Positive Fraction. In the case of upper limits we care about the True Positive Fraction, Wahhaj et al. (2015)



Receiver Operating Characteristic

An “observer” convert pixel maps into one scalar number that measures how the confidence in the detection of signal. The Receiver Operating Characteristic of a given observer illustrates how the FPF and TPF varies when the decision making threshold changes. Caucci et al. (2012).



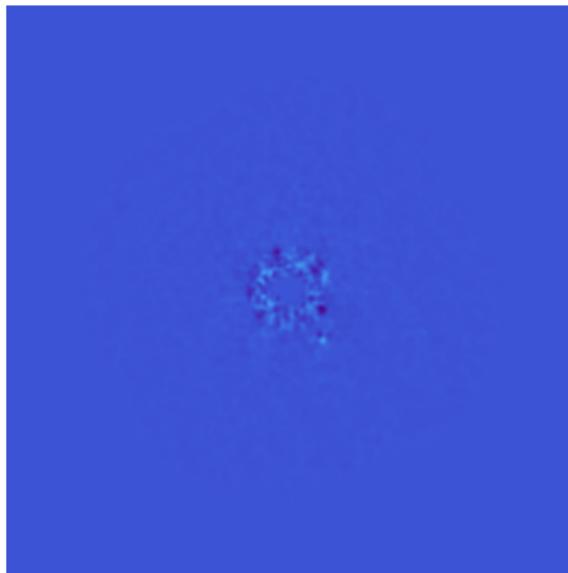
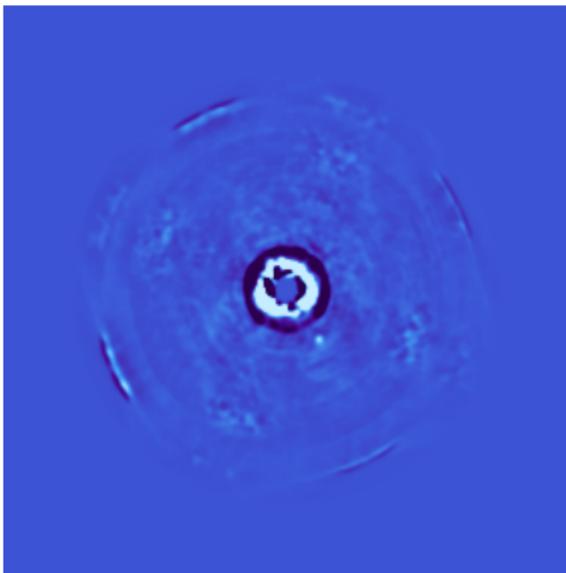
Decision making process

- Pick an algorithm to subtract noise **and** observer.
- Based on the noise properties and the observer calculate ROC.
- Figure out optimal threshold on the ROC to classify data under the assumption of a given utility function.

A utility function assigns costs:

- False Positives: cost is the non detections of a planet that is actually there.
- False Negative: cost is using telescope resources to follow up a “speckle” while those could be allocated to the detection a planet that is actually there, around another star.

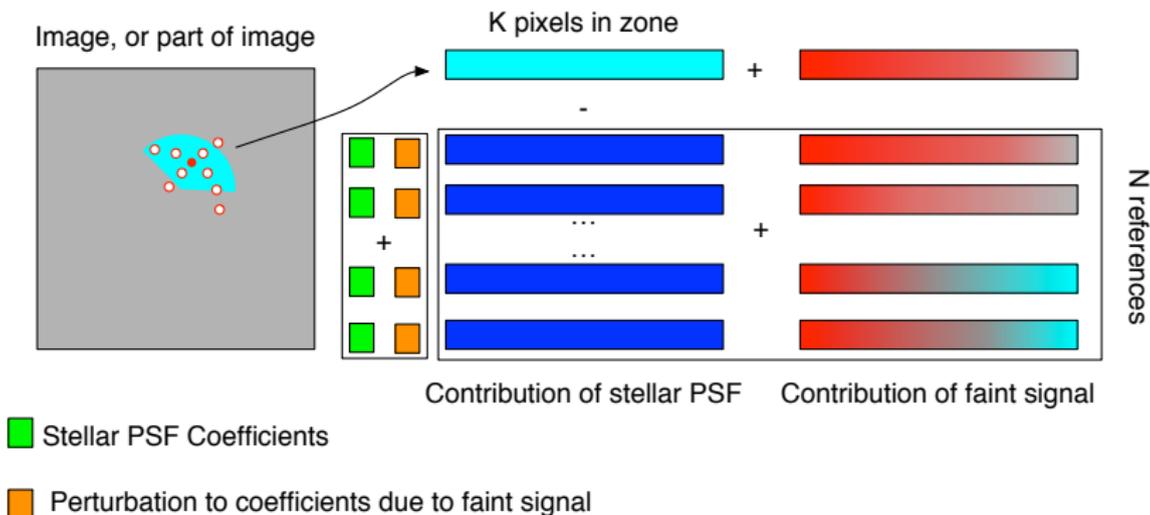
Problem....PSF subtraction algorithms also subtract the signal



Problem....PSF subtraction algorithms also subtract the signal

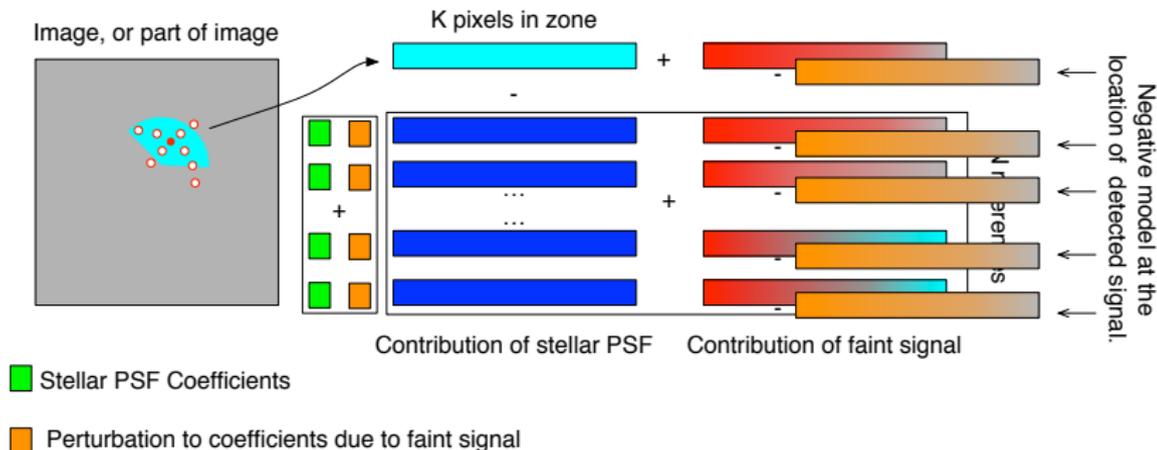
The least squares speckles fitting in the presence of signal can be written as:

$$\min_{\{c_k\}} \left\{ \sum_n \left([I_{\psi_0}(n) + A_0(n)] - \sum_{k=1}^K (c_k + \delta c_k) [I_{\psi_k}(n) + A_k(n)] \right)^2 \right\}.$$



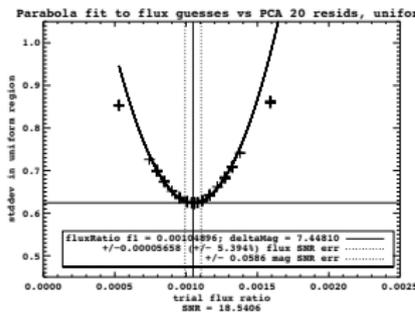
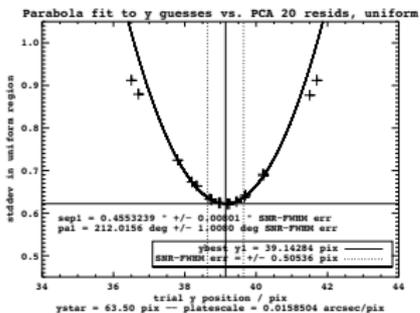
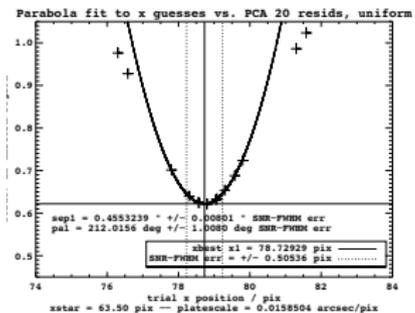
Problem....PSF subtraction algorithms also subtract the signal

Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables. This can be done in conjunction with any of the algorithms described before. Marois et al. (2010), Lagrange et al. (2012).



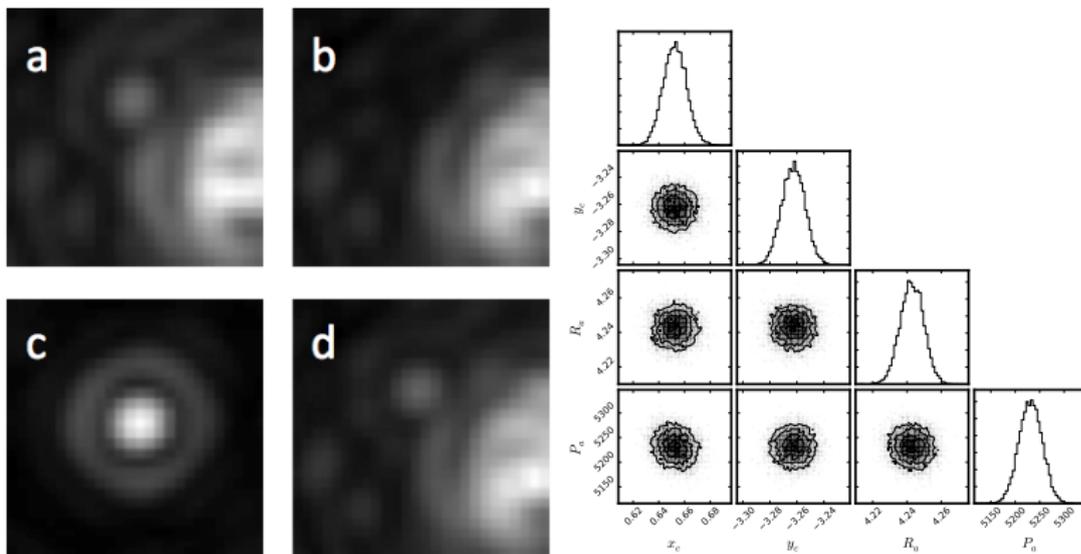
Problem....PSF subtraction algorithms also subtract the signal

Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables. Example of a grid search for astrometry and photometry, Morzinski et al. (2015).



Problem....PSF subtraction algorithms also subtract the signal

Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables. Example of an MCMC for astrometry and photometry, Bottom et al. (2014).



Problem....PSF subtraction algorithms also subtract the signal

Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables.

Main drawbacks

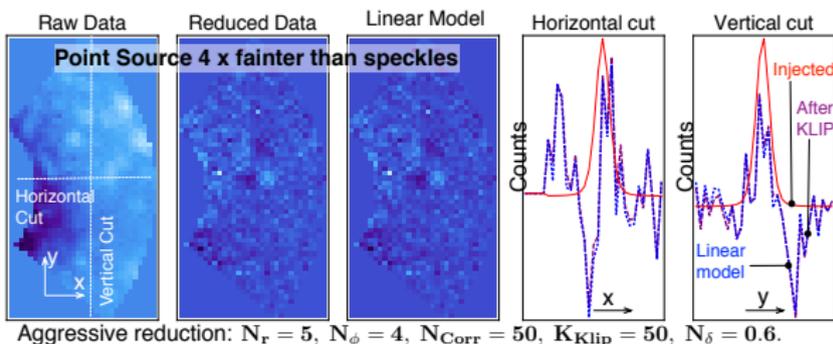
- The speckle subtraction algorithm has to be used each time around (involves a matrix inversion).
- There is no guarantee that the cost-function minimized/likelihood explored does not feature local minima. One might get stuck in them.
- In general these are not limiting factors in "small dimensional configurations" (astrometry and photometry = 3 dimensions).
- This becomes a severe limiting factor when trying to get spectrum (astrometry and spectrum = 39 dimensions with GPI).

Fortunately, we can actually predict what will happen

There is a way to write the influence of the astrophysical signal as:

$$PCA(\text{Speckles} + \text{Signal}) = PCA(\text{Speckles}) + \text{Signal} \delta PCA(\text{Speckles})$$

...and this applies to any algorithm relying on covariances. Pueyo (2016).



The linear model works:

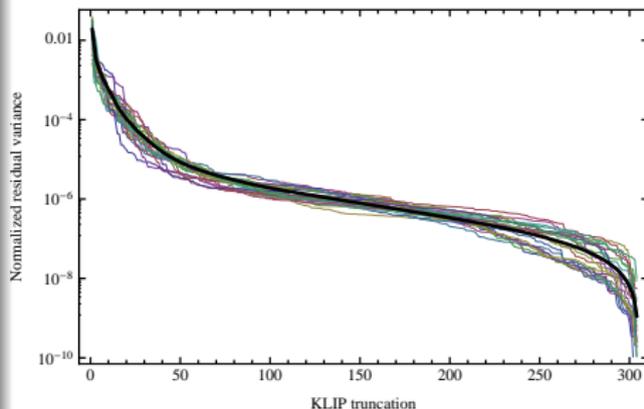
- If the astrophysical source is faint when compared to the speckles.
- If the astrophysical source is as bright as the speckles/brighter, **and** the algorithm parameters are chosen accordingly (not too aggressive).

What does it mean?

$Y_k(\mathbf{x}) = Z_k(\mathbf{x}) + \varepsilon \Delta Z_k(\mathbf{x})$. We can rank them in order of $\|\varepsilon \Delta Z_k(\mathbf{x}) / Z_k(\mathbf{x})\|$.

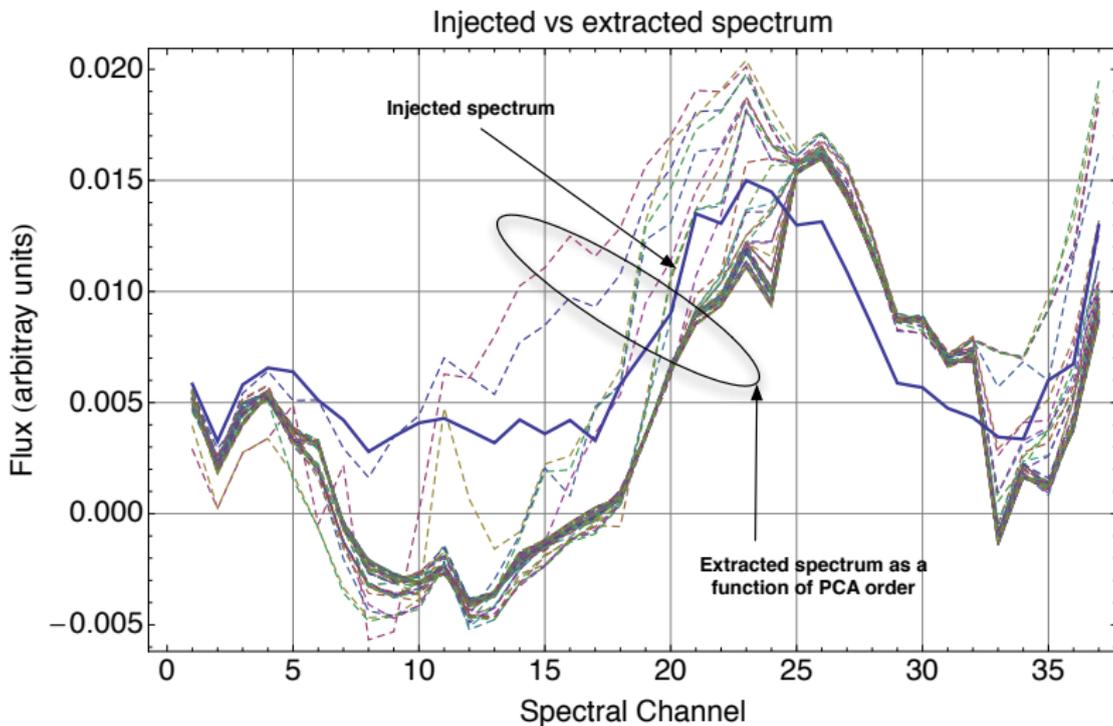
Three main terms:

- *over-subtraction*: unperturbed Principal Components $Z_k(\mathbf{x})$. Scales as $\|Z_k(\mathbf{x})\| = 1$.
- *direct self-subtraction*: presence of an astrophysical source at various parallactic angles and wavelengths in the observing sequence multiplied by LOCI coefficient. Scales as $\varepsilon / \sqrt{\Lambda_k}$.
- *indirect self-subtraction*: perturbation in the LOCI coefficient. Scales as ε / Λ_k .

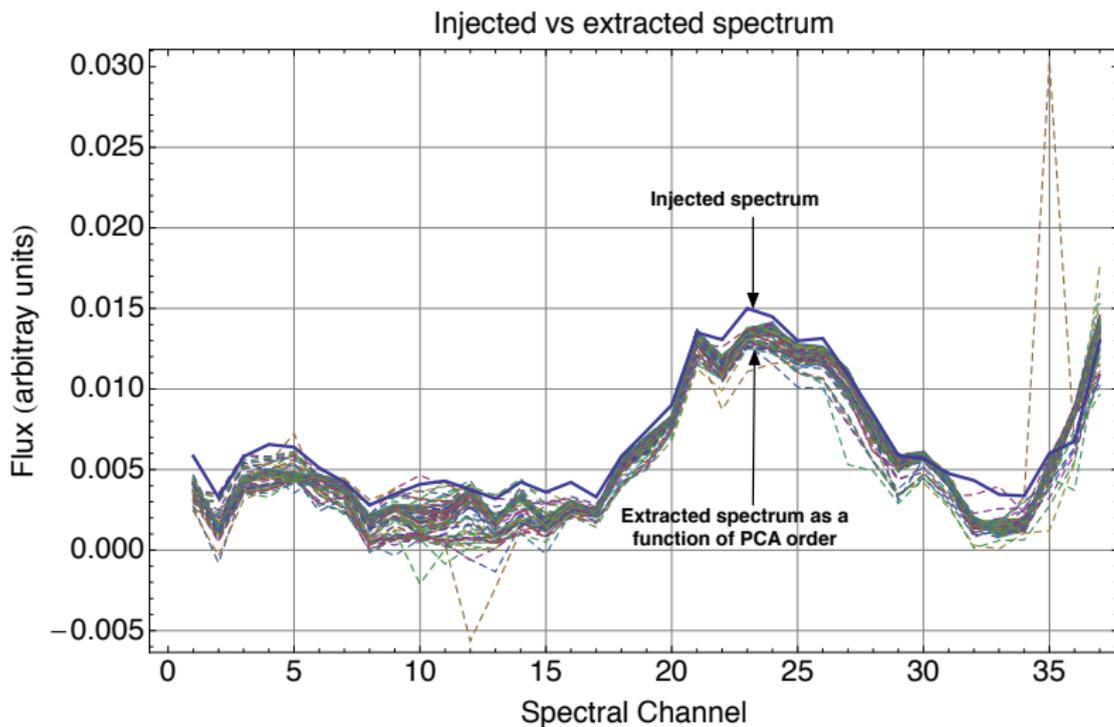


As K_{Klip} (e.g. Λ_k decreases) then self-subtraction becomes more and more dominant... estimation of astrophysical observables becomes increasingly complicated.

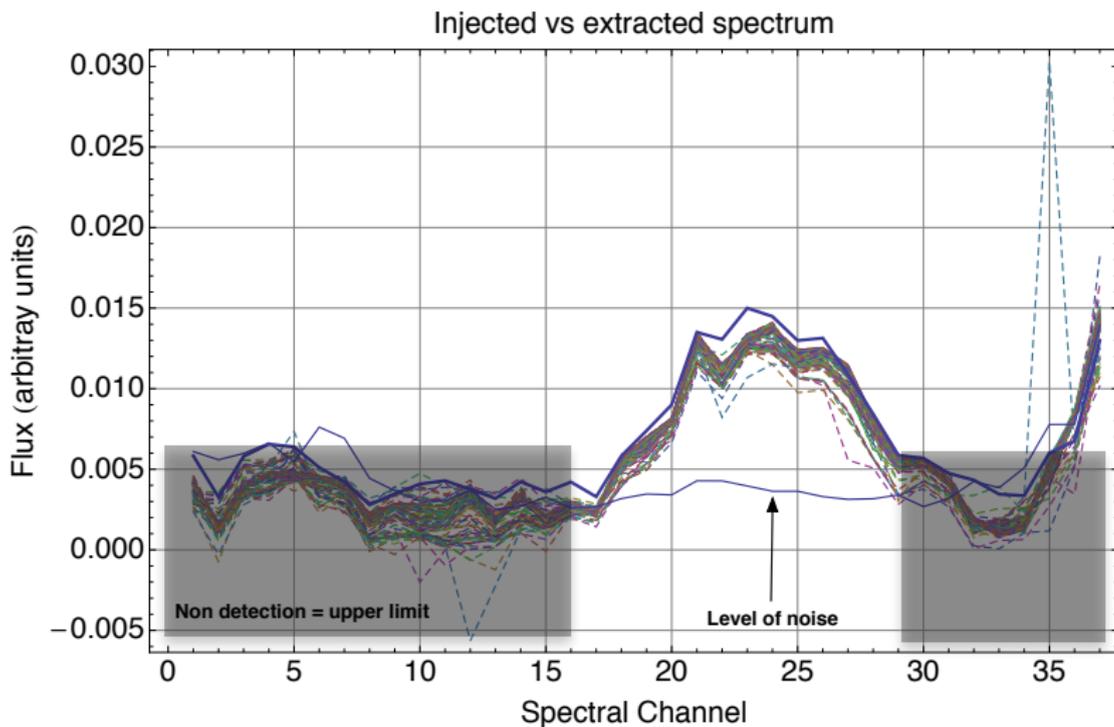
Application to spectral extraction



Application to spectral extraction

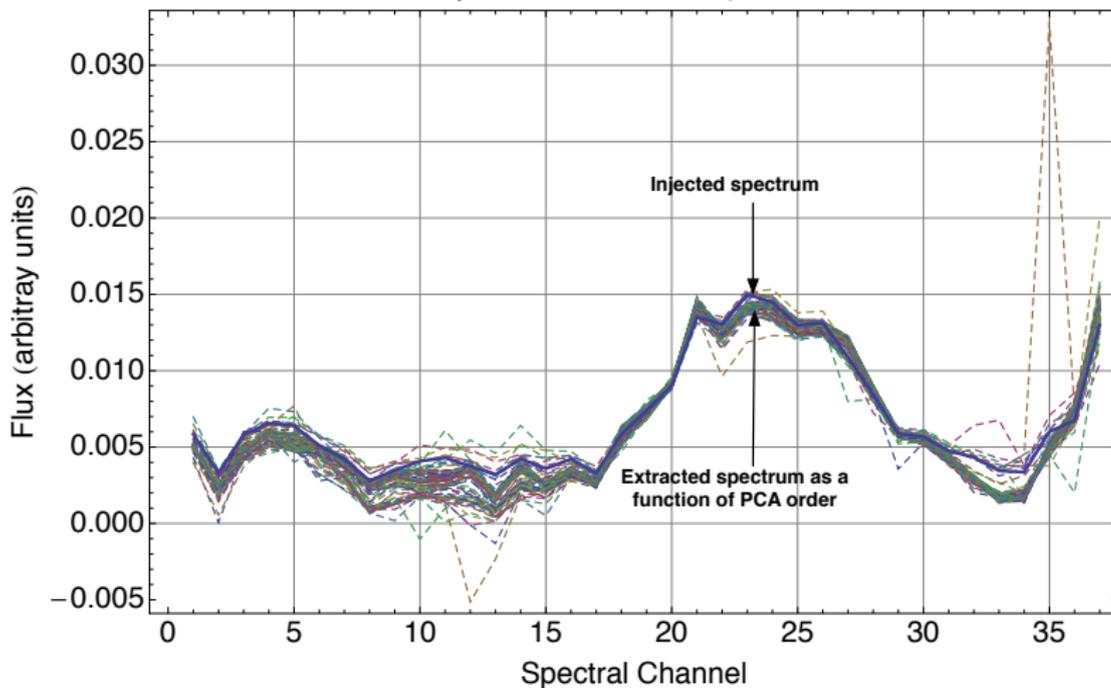


Application to spectral extraction



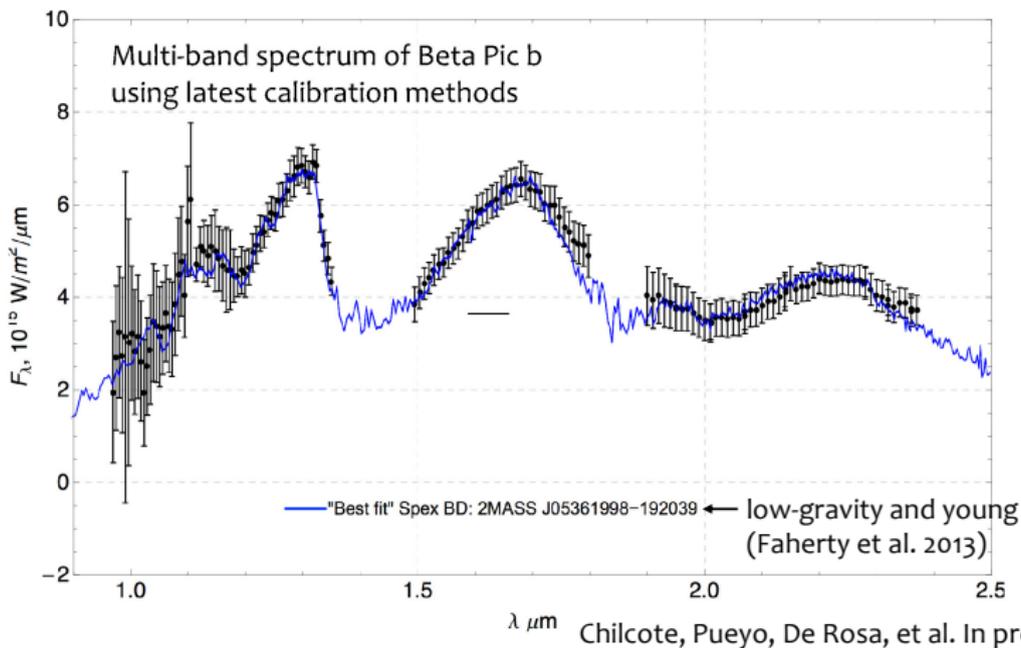
Application to spectral extraction

Injected vs extracted spectrum



Application to spectral extraction

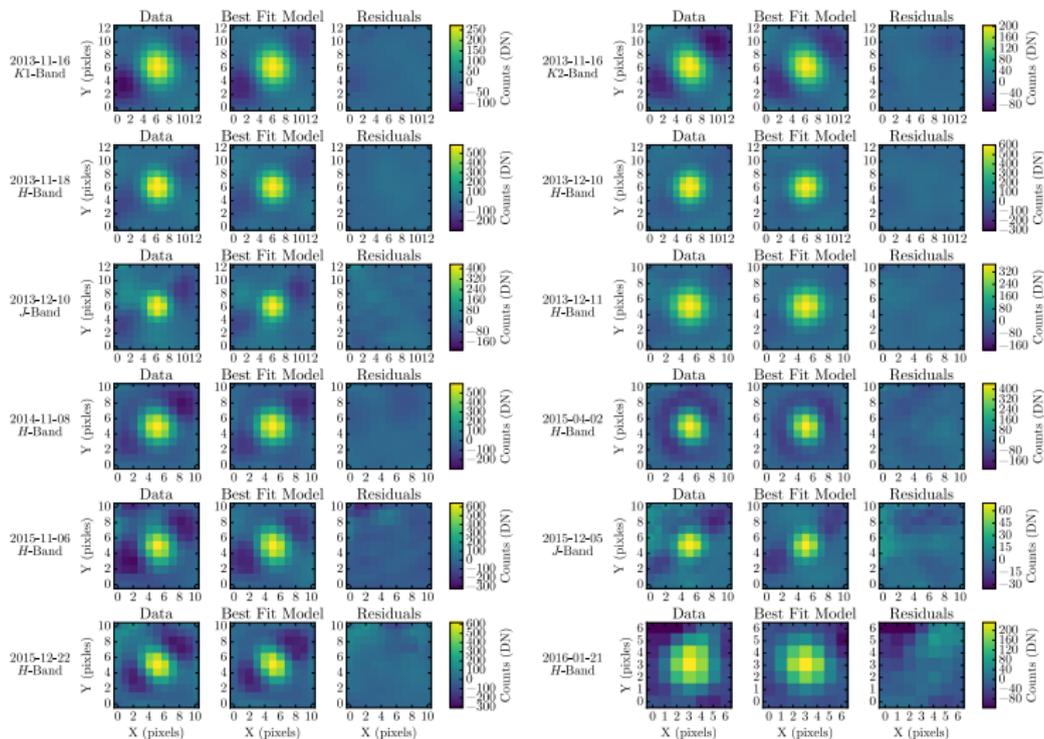
Application: YJHK Spectrum of β Pic b





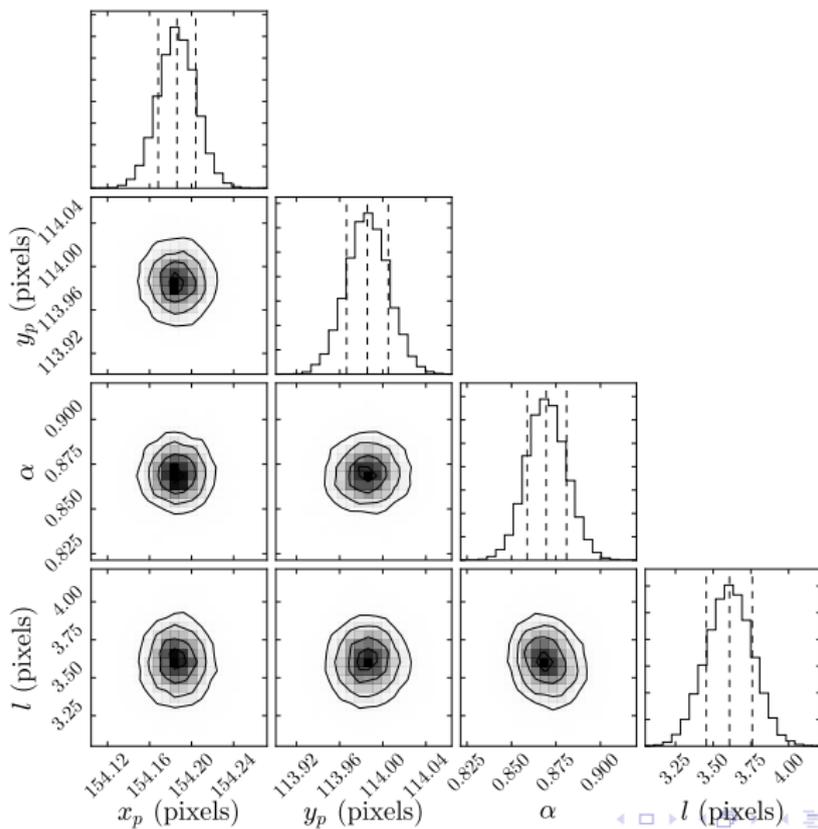
Application to astrometry

Wang et al. (2016).



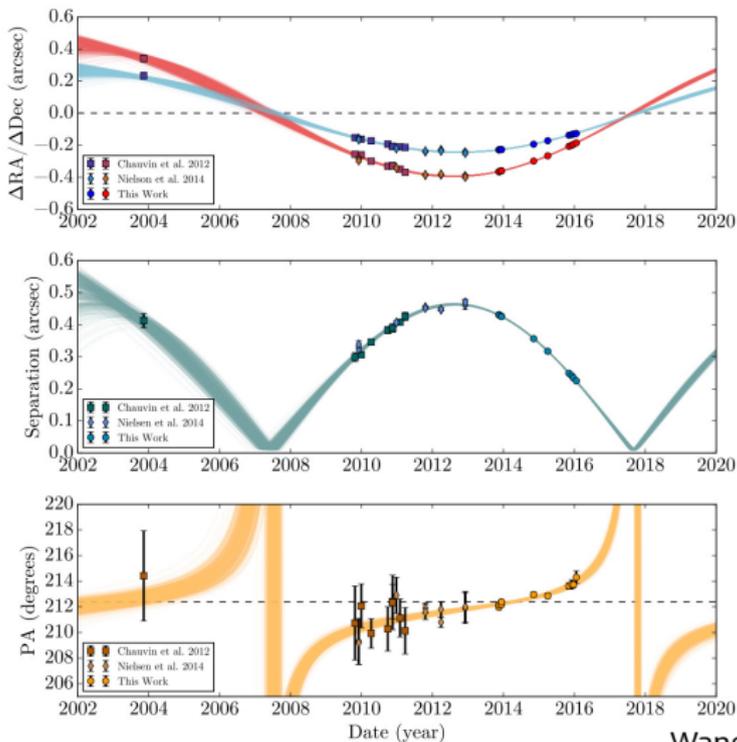
Application to astrometry

Wang et al. (2016).



Application to astrometry

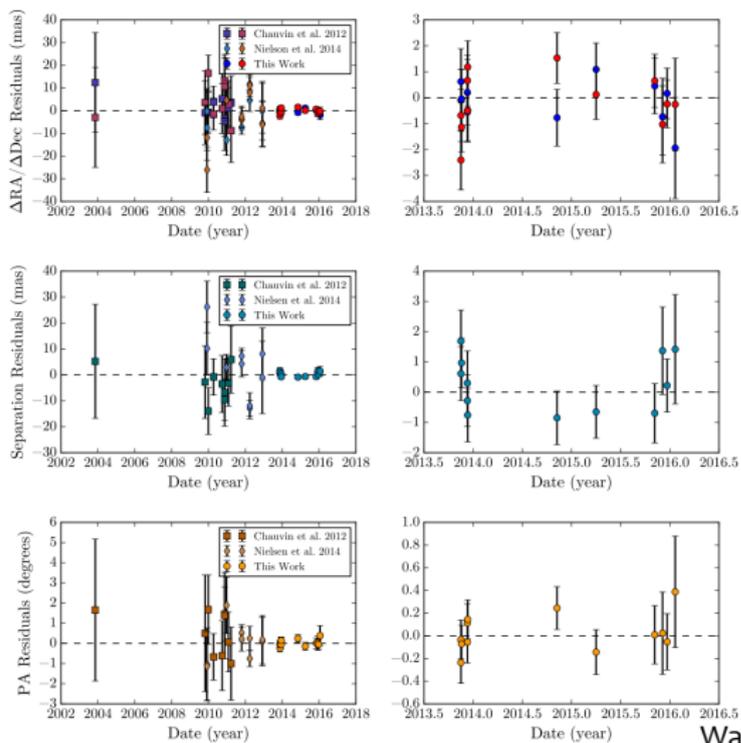
Wang et al. (2016).



Wang et al.

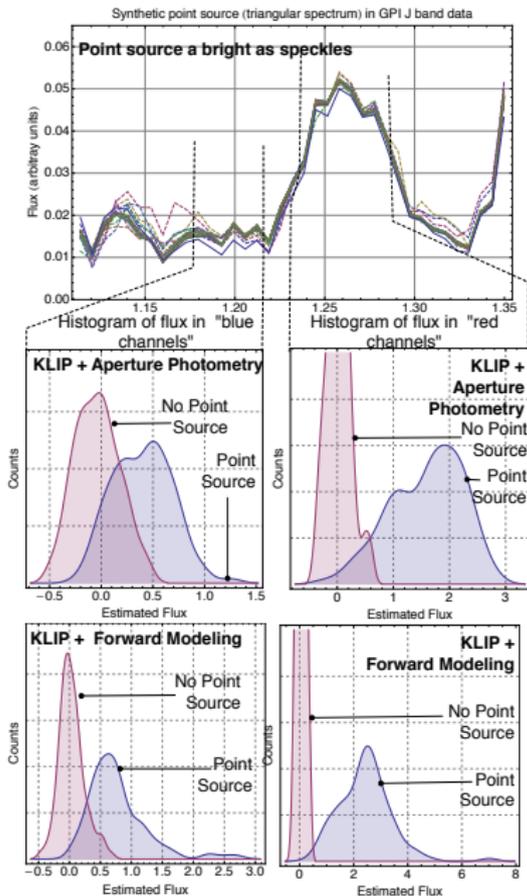
Application to astrometry

Wang et al. (2016).



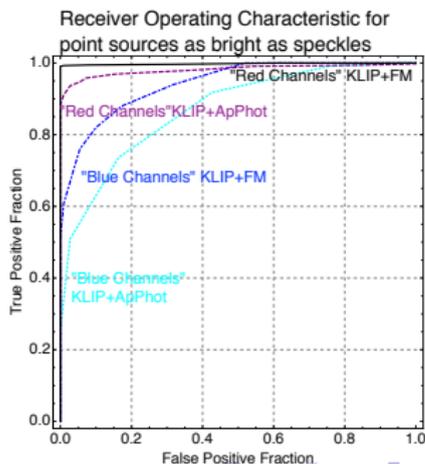
Wang et al.

Application to planet detection



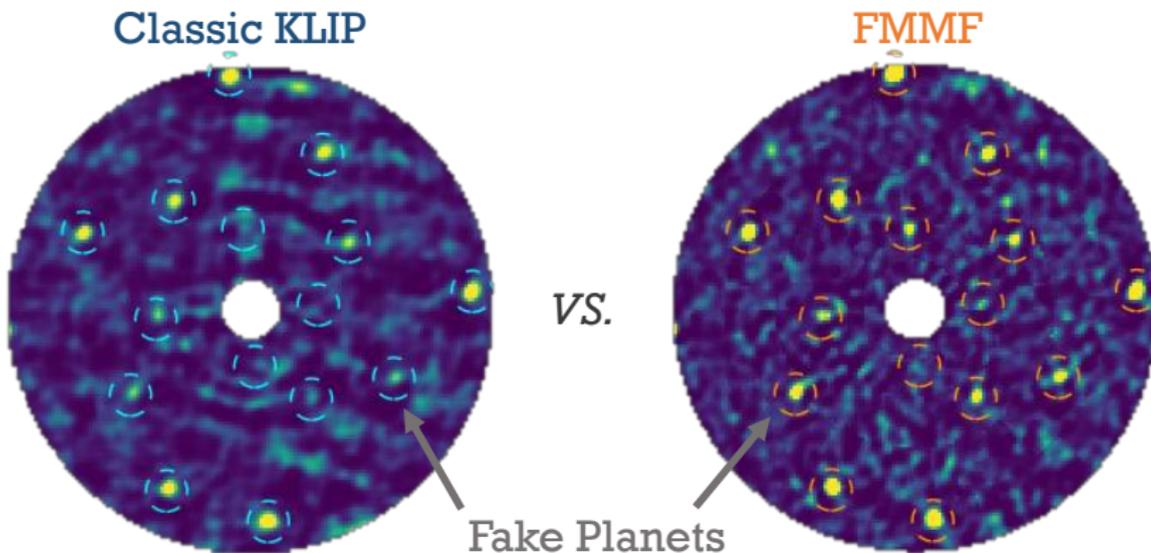
Forward Modeling for the detection problem

- Forward Modeling does not change the False Positive Fraction (= does not change the post KLIP speckles statistics).
- Forward Modeling changes the True Positive Fraction (= does change the post KLIP astrophysical flux).



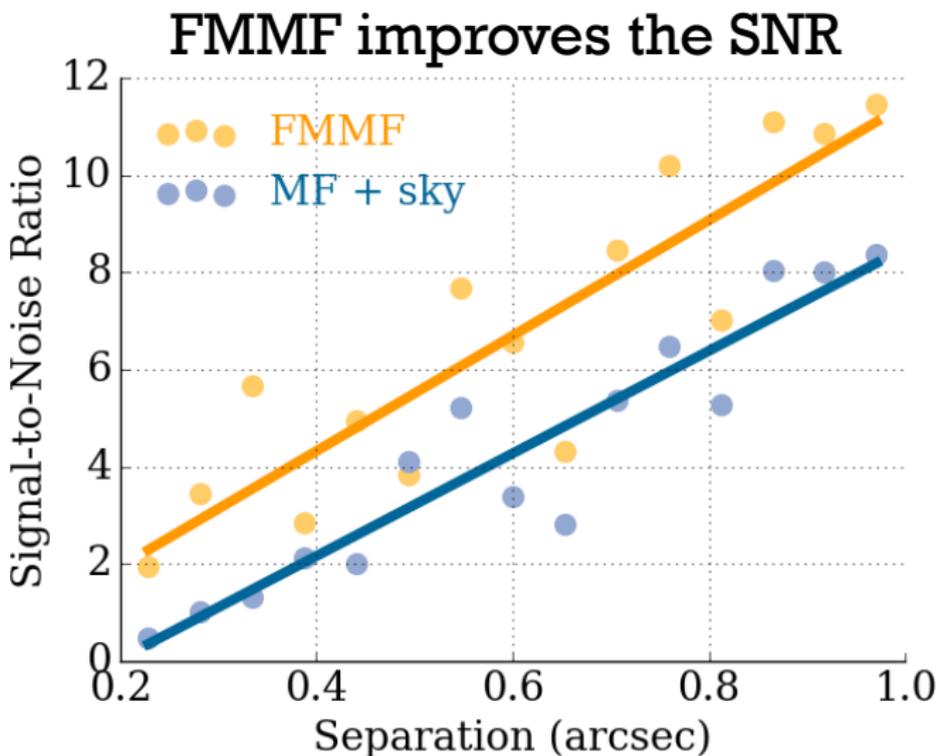
Application to planet detection

Ruffio et al., in prep.



Application to planet detection

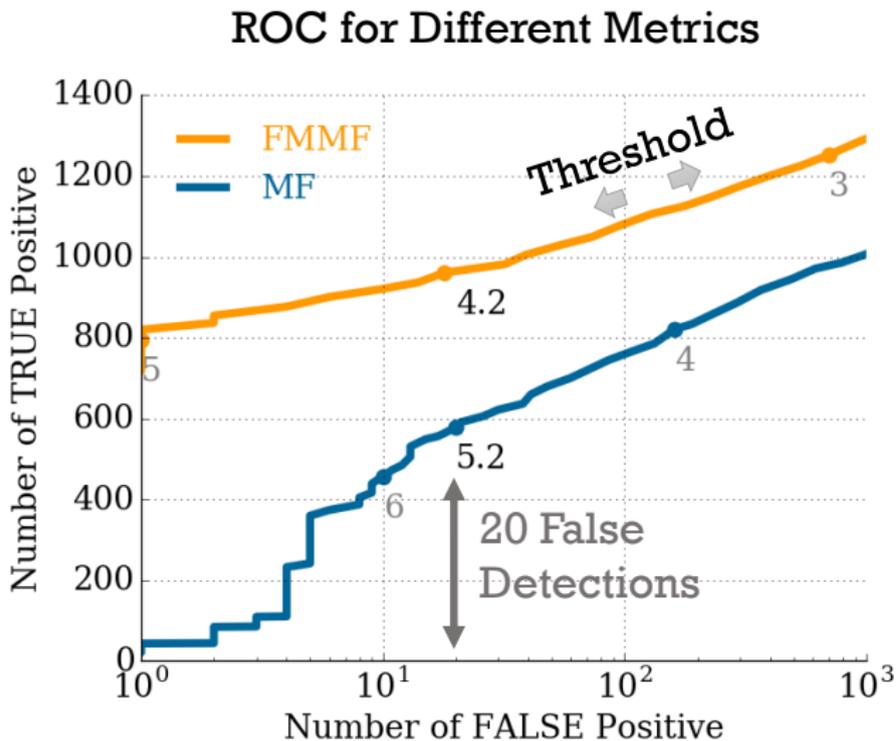
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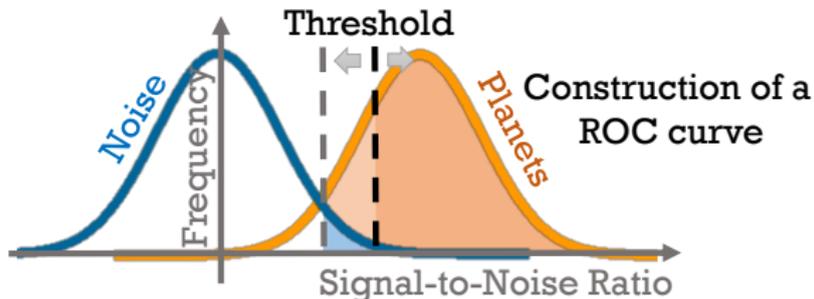
Fake planets injected in GPIES data



Noise

Application to planet detection

Ruffio et al., in prep.

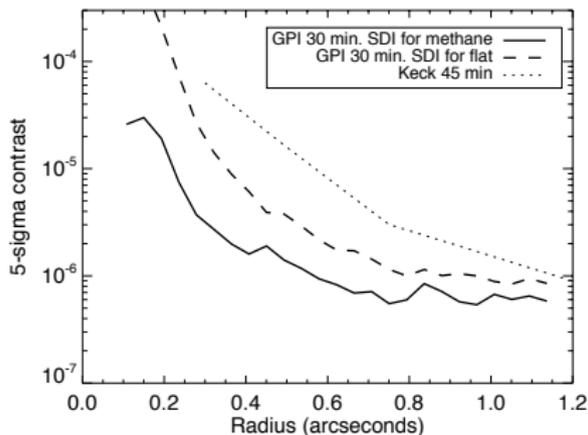


The **Receiver Operating Characteristic** (ROC) indicates the cost of a true detection in term of false positives. It is the right tool to compare detection metrics.

Contrast curves from different metrics should be drawn at the same false positive rate, which is not necessarily 5σ .

Contrast curves and completeness

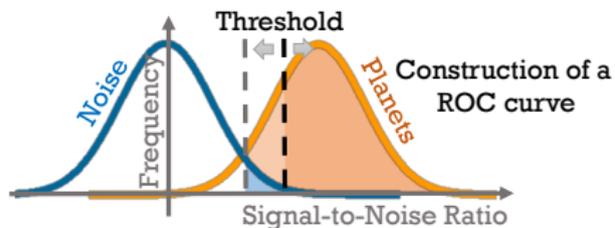
Macintosh et al. (2015)



How are survey results presented

- Pick the “right” contrast curve for each star. Delta mag vs separation.
- Convert into Mass vs SMA using your favorite model for mass-luminosity and Monte Carlo simulations to explore all possible orbits.
- Convert into Mass vs SMA using your favorite model for mass-luminosity and analytical propagation of priors.
- Sum over all stars in survey.

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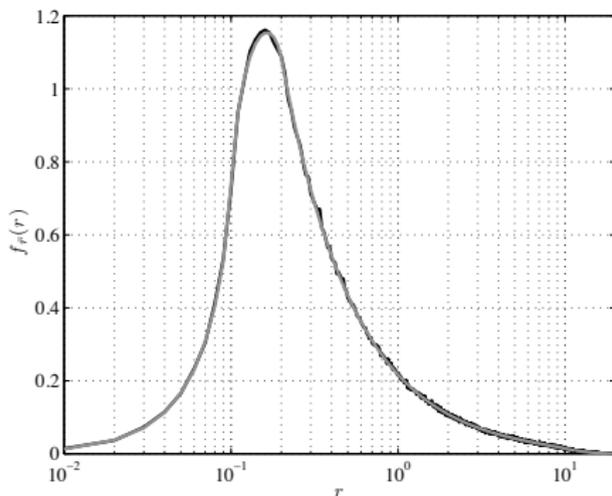
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Contrast curves and completeness

Savransky et al. (2010)



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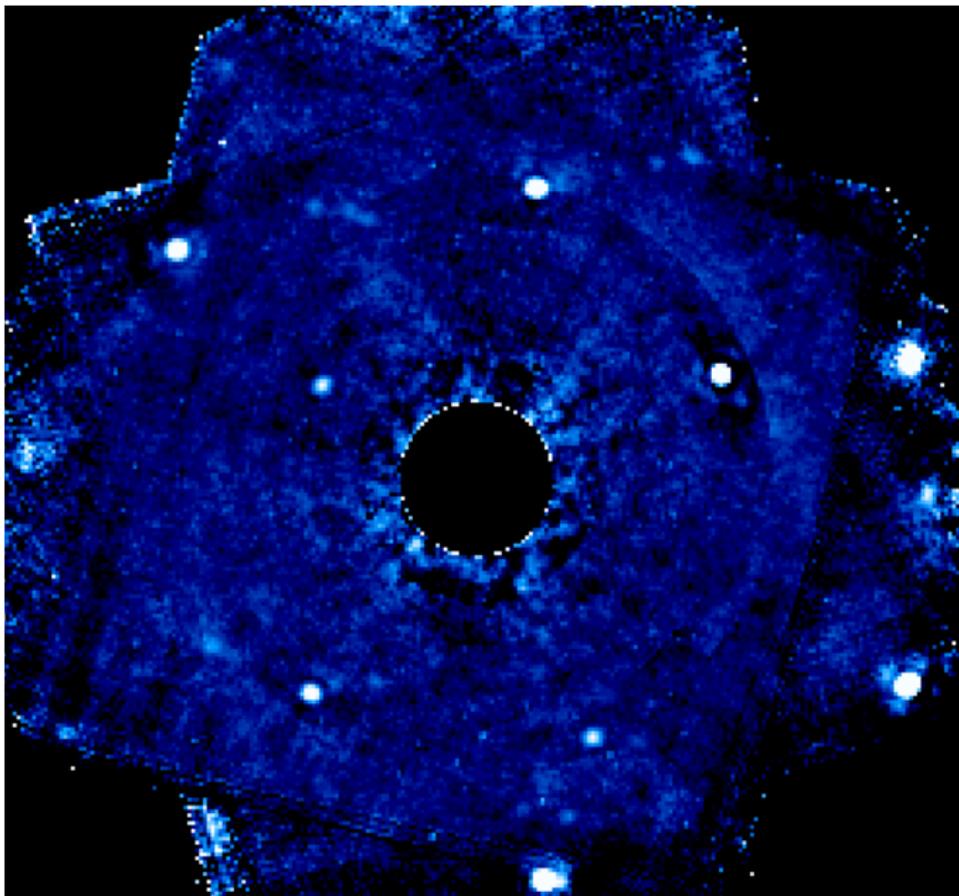
Other methods

Moving forward with data analysis

By and large most of the community is using “blind” Principal Component Analysis to analyze high-contrast imaging data. This is an ancient method! There is room to do better:

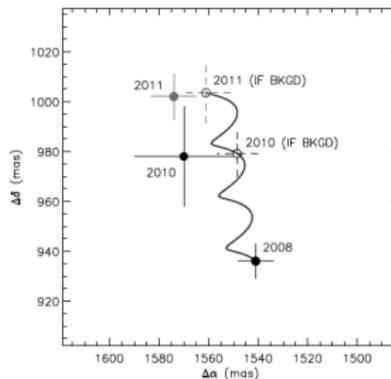
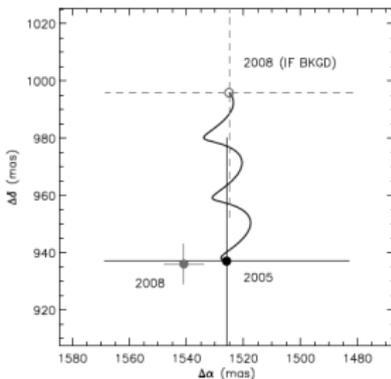
- Use correlation between telemetry and images (Vogt et al., 2010).
- Use the images (and maybe telemetry) a physical model of the complex field at the telescope entrance (Ygouf et al., 2012).
- Give up on the L2 norm (L1 norm?).
- Use only positive modes and positive coefficients (Non Negative Matrix Factorization).
- “Track” the motion of the planet in the data (low rank sparse decomposition, LLSG, Gomez et al., 2016).

Astrophysical false positives



Common proper motion for physical association

Combine proper motion and parallactic motion to establish physical association. Rameau et al. (2013), Mawet et al. (2012)

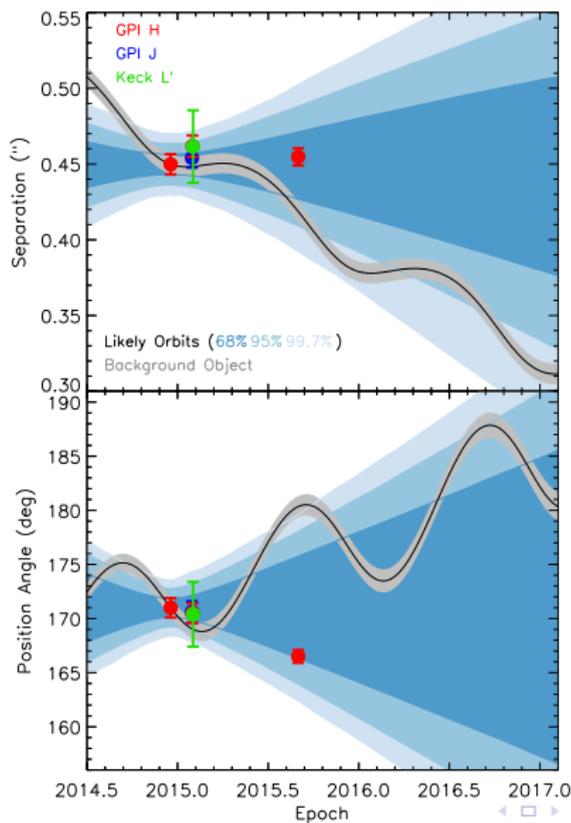


Speeding the process up

- This used to be a waiting game: proper and parallactic motion need to be larger than uncertainty in astrometry.
- Smaller error bars for astrometry do certainly help.
- How to use MCMC to speed things up?

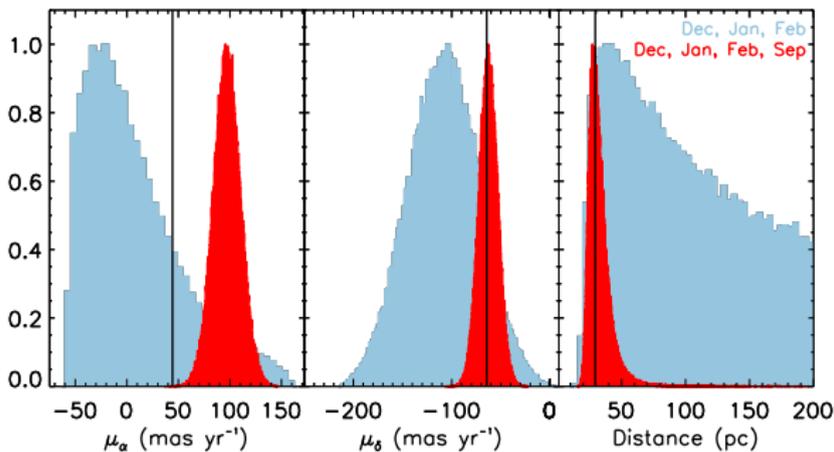
Common proper motion for physical association

De Rosa et al. (2015)



Common proper motion for physical association

De Rosa et al. (2015)

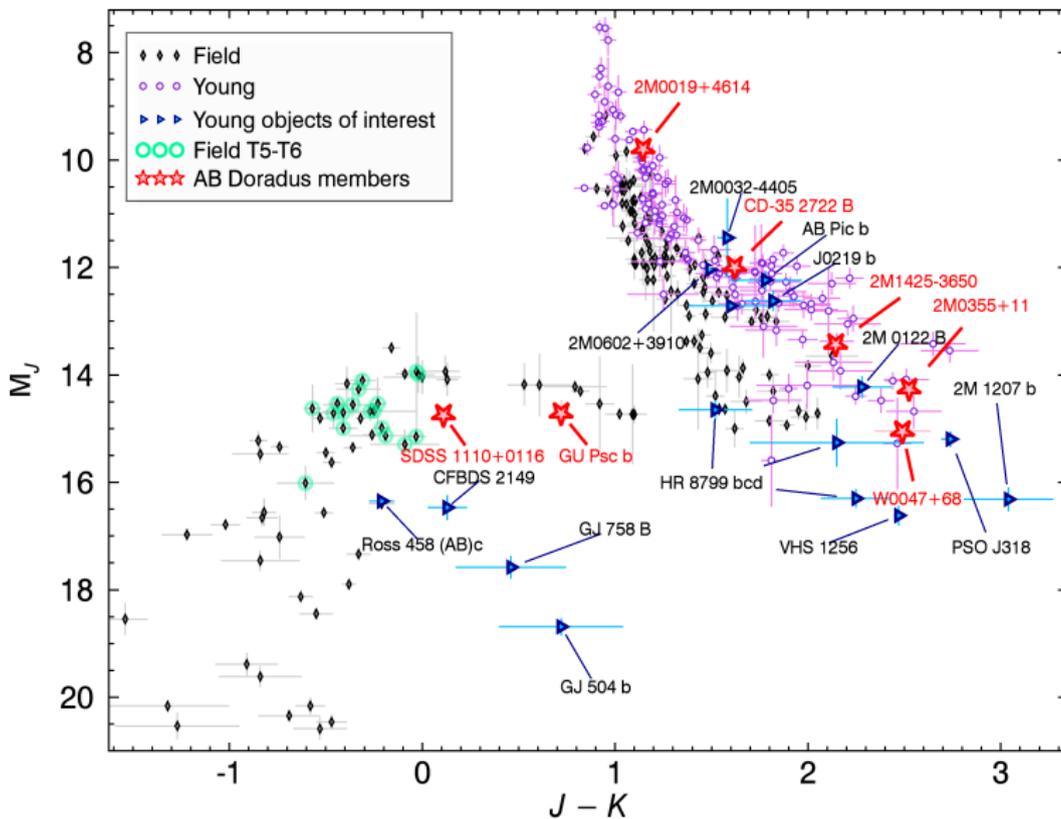


Speeding the process up

- This used to be a waiting game: proper and parallactic motion need to be larger than uncertainty in astrometry.
- Smaller error bars for astrometry do certainly help.
- The “astrophysical noise” hypothesis can also be fitted for.

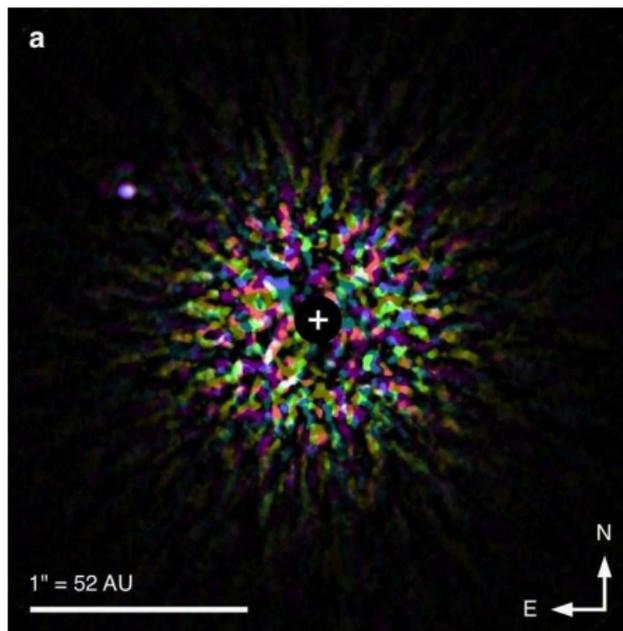
More information always helps

Gagne et al. (2015)



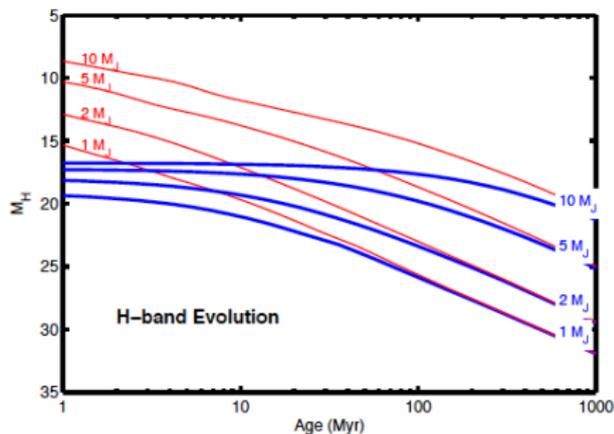
Age of stars: an oral story

Carson et al. (2009)



The mass of Kappa Andromeda

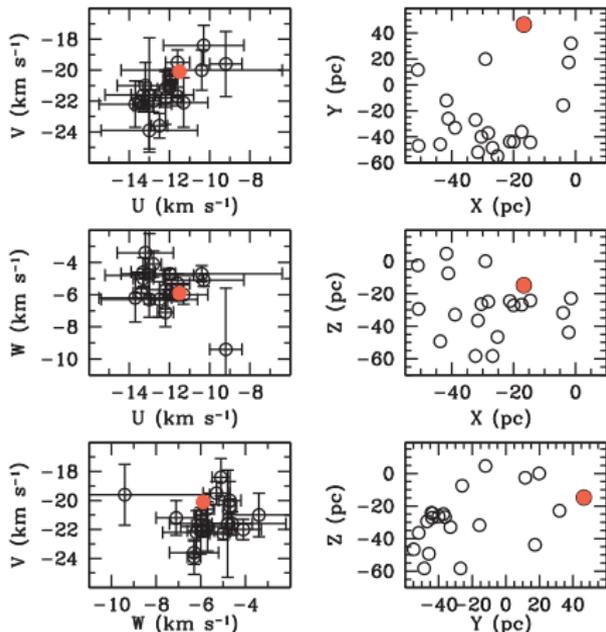
Spiegel and Burrows (2010)



We need the age of the system to tie the luminosity of the companion to its mass using evolutionary tracks

Age of stars: an oral story

Hinkley et al. (2013)

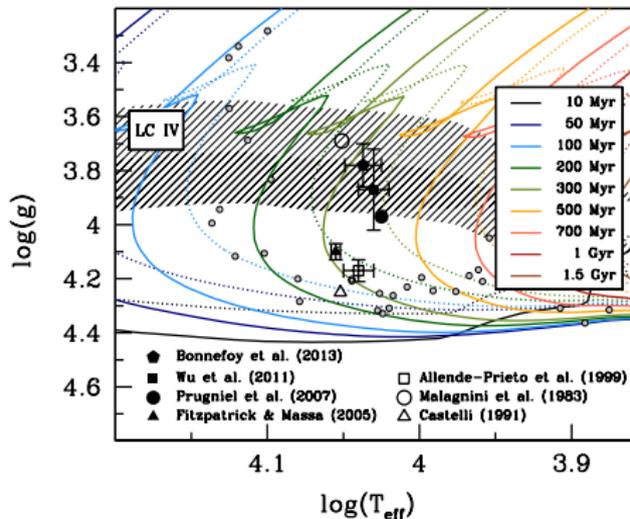


The mass of Kappa Andromeda

- Discovery paper, young (~ 50 Myrs) moving group, mass $\sim 12 M_{Jup}$.
- Second look: moving group membership not so convincing, star too bright to be young. Revised age ~ 200 Myrs, mass $\sim 30 M_{Jup}$.

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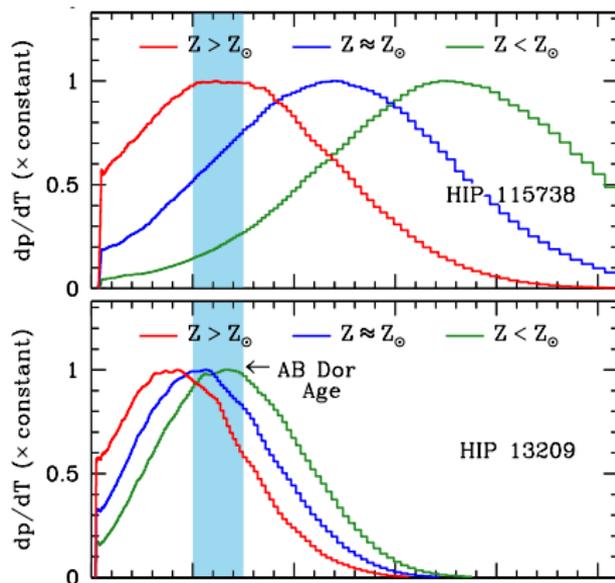


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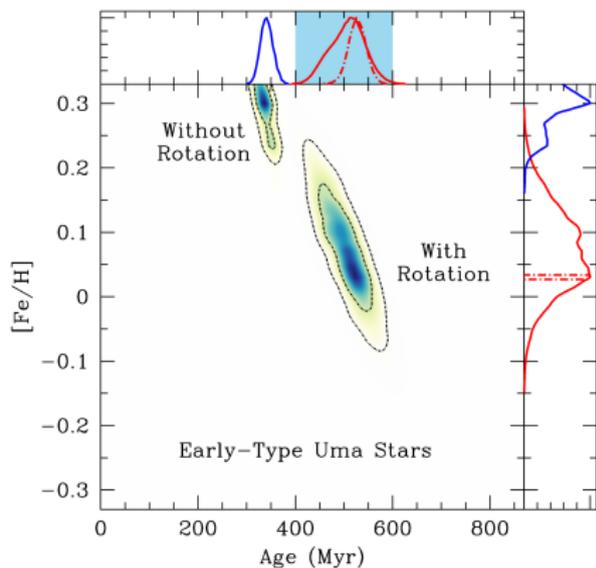
Age of stars with bayesian inference

Bayesian ages. Brandt (2015)



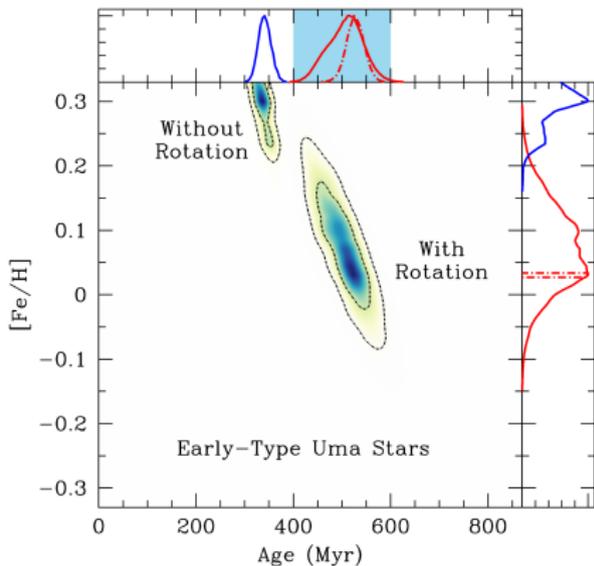
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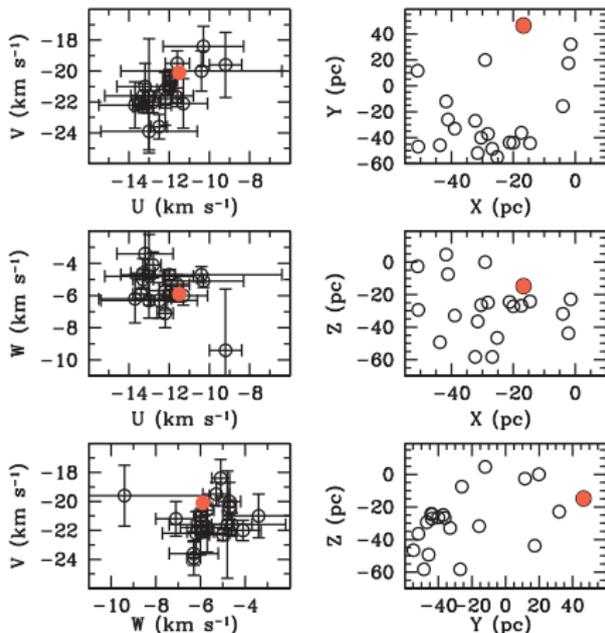


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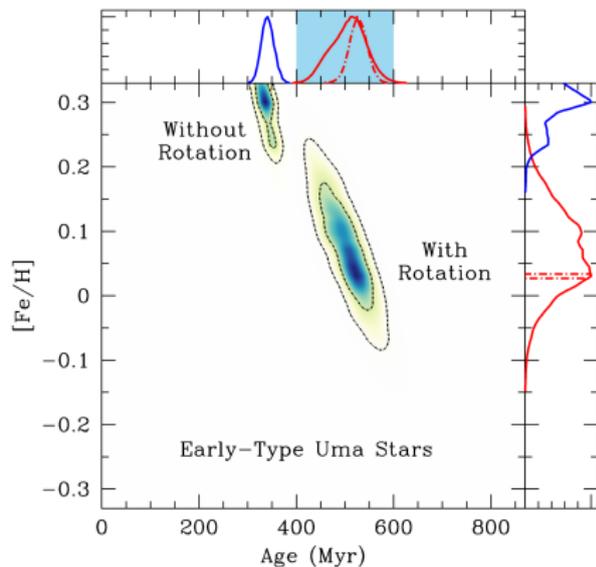


Bayesian moving group membership. Gagne et al. (2015)

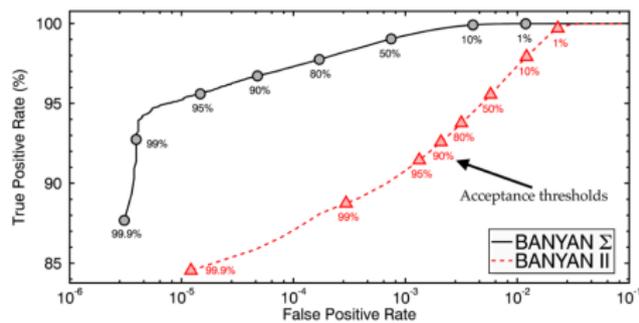


Age of stars with bayesian inference

Bayesian ages. Brandt (2015)



Bayesian moving group membership. Gagne et al. (2015)



Recap

Know your noise!

- Methods to mitigate astrophysical noise are somewhat more “modern” than for instrument noise.
- This is because we know more about the universe than about speckles.
- There is a lot of room for growth in the data analysis domain.

Key things to watch out for the future

- GPI and SPHERE (as instruments) are just starting. They are beautiful planet characterization machines.
- Solve the million dollar problem: reconcile RV and direct imaging Jupiter analog occurrence rates? Do we need deeper contrast? Do we need better angular resolution (... and wait for ELTs)?
- The possibility of obtaining short exposures times might completely change this story.
- JWST data might completely challenge the way we think about the instrument noise.
- Properly handling astrophysical noise will be critical for WFIRST.

Thank you