



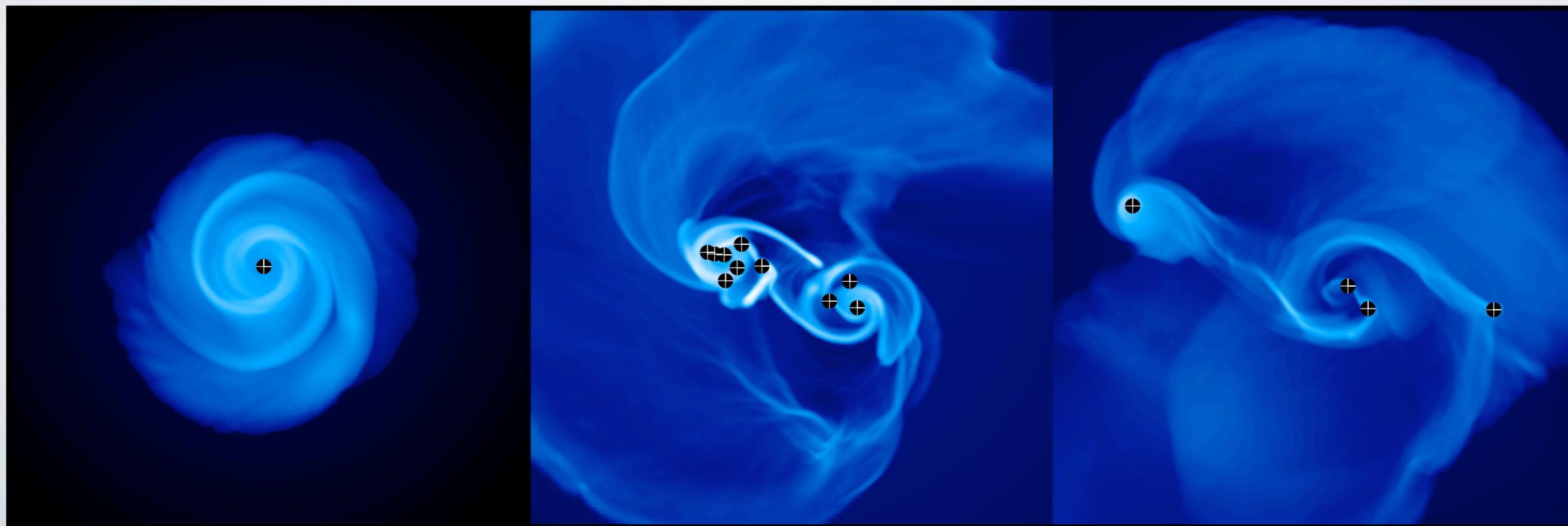
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COLLEGE OF SCIENCE

Astronomy
& Steward Observatory

PLANET FORMATION BY GRAVITATIONAL INSTABILITY ?

Kaitlin Kratter
University of Arizona

Sagan Summer Workshop, July 2015





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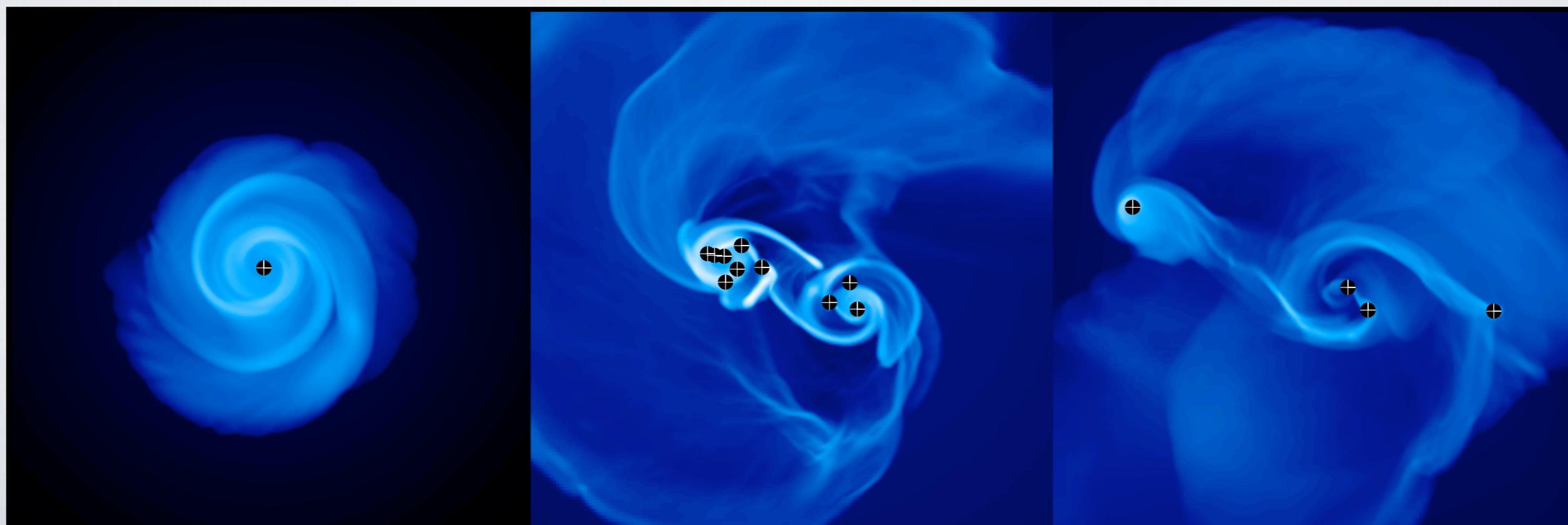
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PLANET FORMATION BY GRAVITATIONAL INSTABILITY ?

in GAS!

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THE BOTTOM LINE

- **Few** disks appear to be **massive enough to fragment** (except perhaps at very early times)
- Those that do are more likely to produce more massive objects like **brown dwarfs or m-stars**
- Inward migration followed by tidal disruption most often leads to **complete disruption**, rather than mass reduction. But might assist in solid core formation

THE BOTTOM LINE

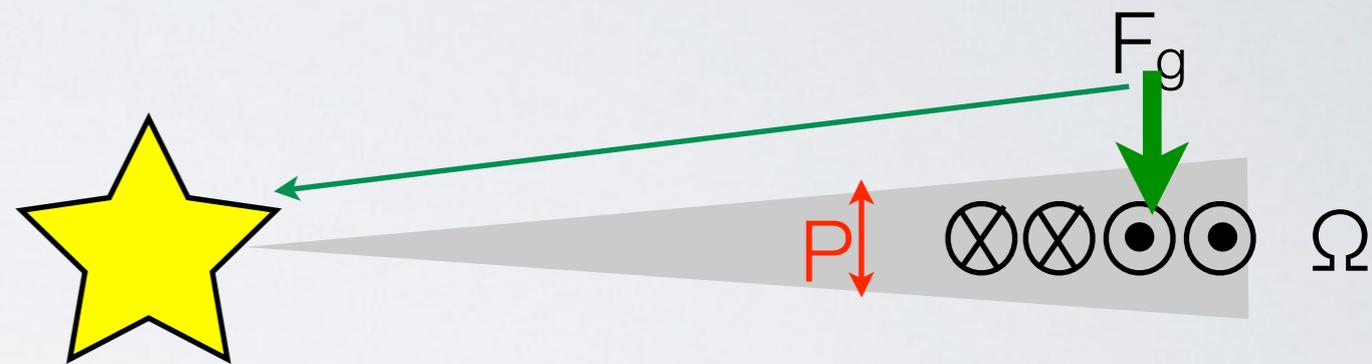
- **Few** disks appear to be able to fragment (except perhaps accretion disk studies were born in galactic / x-ray binary contexts, some of the standard assumptions made in these contexts are not well suited to protostellar / protoplanetary disks.
- Those that do fragment are more massive objects than stars.
- Inward migration of material is most often leads to mass loss rather than mass accretion.
 1. Thermodynamics are dominated by stellar irradiation
 2. H/R is not $\ll 1$

GLOSSARY

- **Self-Gravity:** the gravitational attraction of gas to itself is competitive with the central body
- **Gravitational Instability: (GI)** a linear, hydrodynamic instability that can arise in self-gravitating disks of gas, particles or both
- **Fragment:** a marginally bound gas clump that forms as the non-linear outcome of GI
- **Planet:** depends on whom you ask...

WHAT IS GRAVITATIONAL INSTABILITY?

- A hydrodynamic instability that arises in rotationally supported disks when **self-gravity** wins out over **pressure** support on small scales, and stabilization due to **shear** on large scales

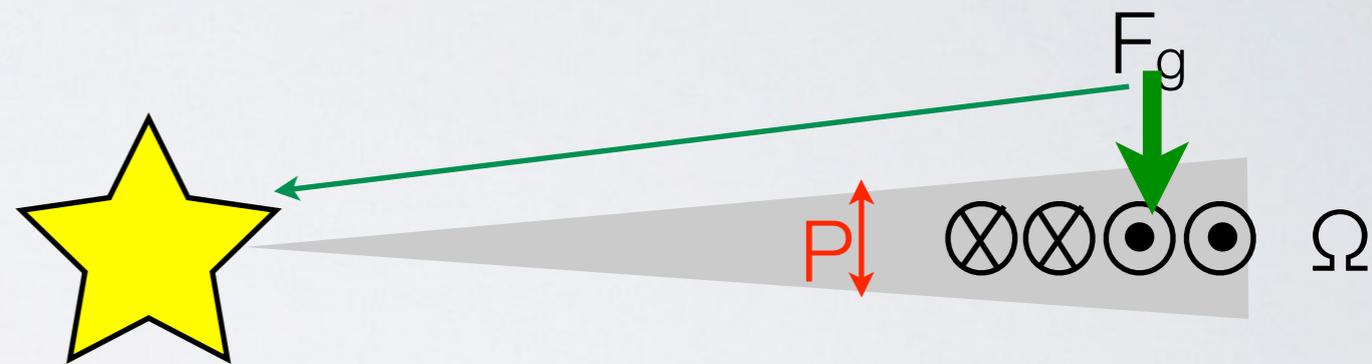


$$Q = \frac{c_s \Omega}{\pi G \Sigma} = f \frac{M_*}{M_D} \frac{H}{r}$$

Toomre's Q , rewritten this way can point us toward important, order of magnitude arguments

WHAT IS GRAVITATIONAL INSTABILITY? [^]WHEN $H \ll R$

- A hydrodynamic instability that arises in rotationally supported disks when **self-gravity** wins out over **pressure** support on small scales, and stabilization due to **shear** on large scales



$$(\omega - m\Omega(R))^2 = c_s^2 k^2 - 2\pi G \Sigma |k| + \kappa^2,$$

$$Q = \frac{c_s \Omega}{\pi G \Sigma} = f \frac{M_*}{M_D} \frac{H}{r}$$

Growth begins at $Q=1$ in the **local** approximation, for axisymmetric modes. since $H \ll R$, $M_D \ll M_*$

WHAT IS GRAVITATIONAL INSTABILITY? ^{WHEN $H < R$}

from Lau & Bertin 1978

$$(\omega - m\Omega)^2 = \kappa^2 + \left(k^2 + \frac{m^2}{r^2}\right)a^2(1 + \chi) - 2\pi G\sigma_0 \left(k^2 + \frac{m^2}{r^2}\right)^{1/2} (1 + \chi), \quad (12)$$

where

$$\chi = \mathcal{J}^2(\lambda/\lambda_c)^2 \quad (13)$$

and \mathcal{J} is defined in equation (6). In equation (13),

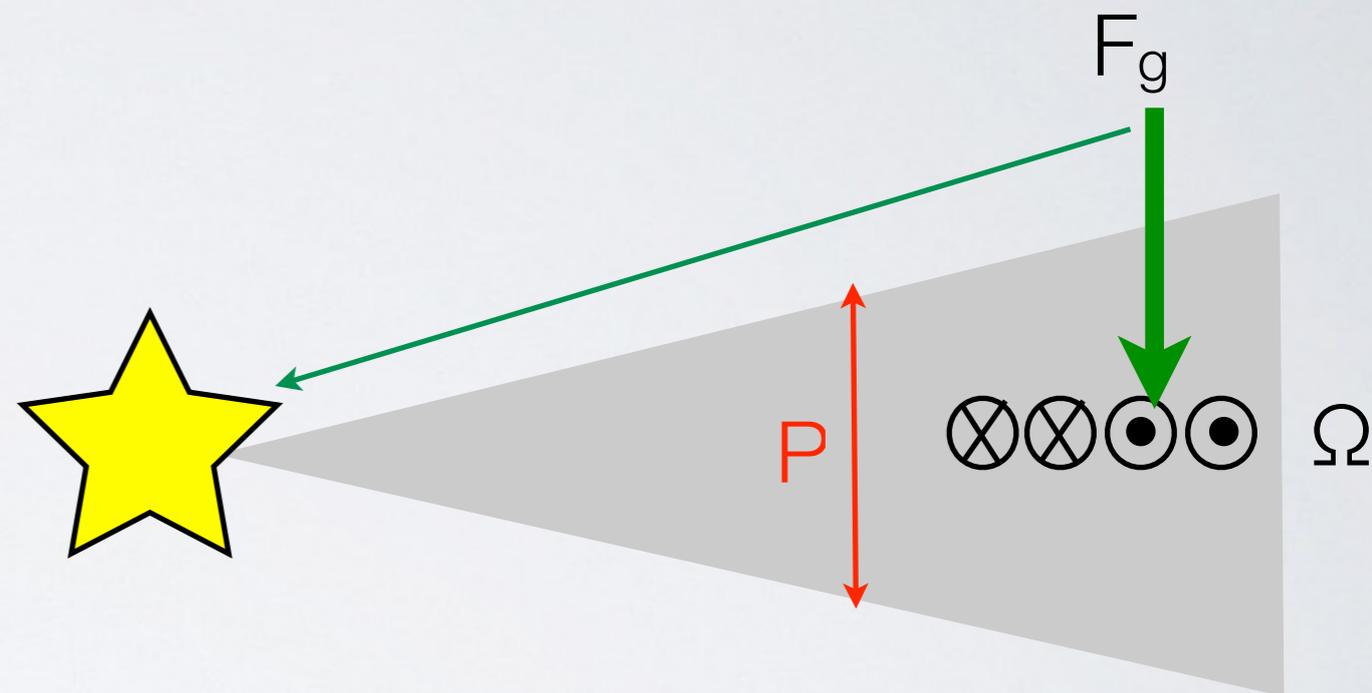
$$\lambda = 2\pi / (k^2 + m^2/r^2)^{1/2} \quad (14a)$$

and

$$\lambda_c = 4\pi^2 G\sigma_0 / \kappa^2 \quad (14b)$$

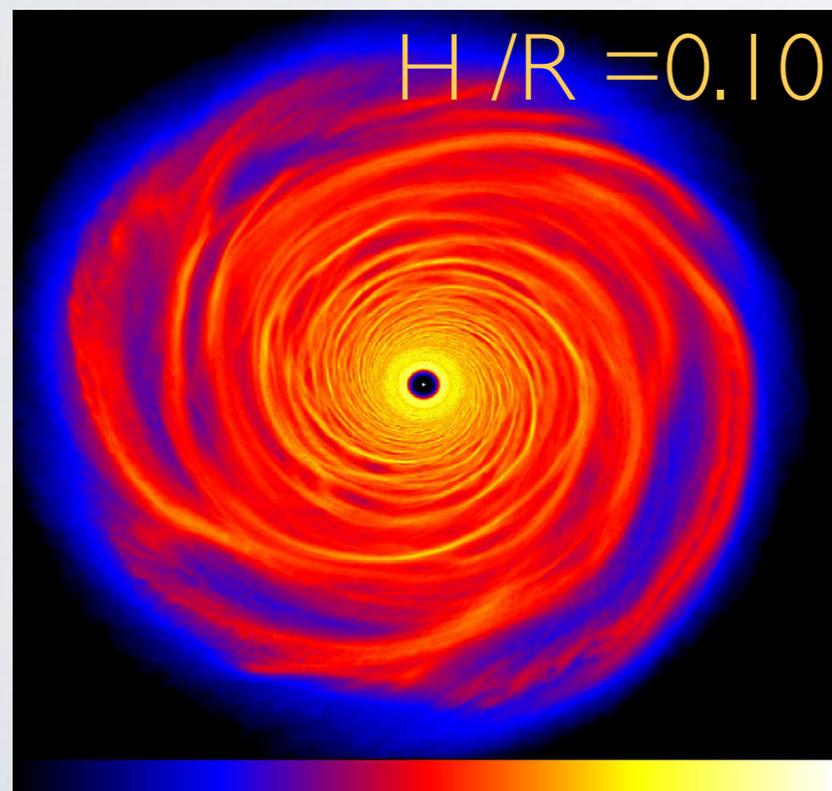
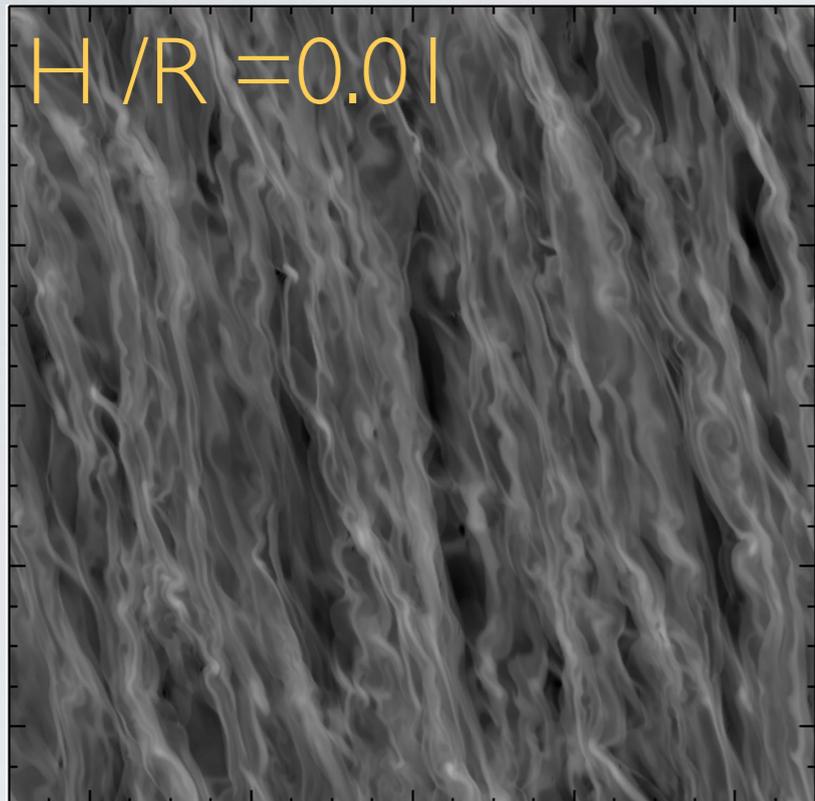
are respectively the local wavelength and the critical wavelengths defined by Toomre (1964). For stability, we obtain the condition

$$Q^2 \geq 4 \left[\frac{\lambda}{\lambda_c} - \frac{(\lambda/\lambda_c)^2}{1 + \mathcal{J}^2(\lambda/\lambda_c)^2} \right] \quad (15)$$

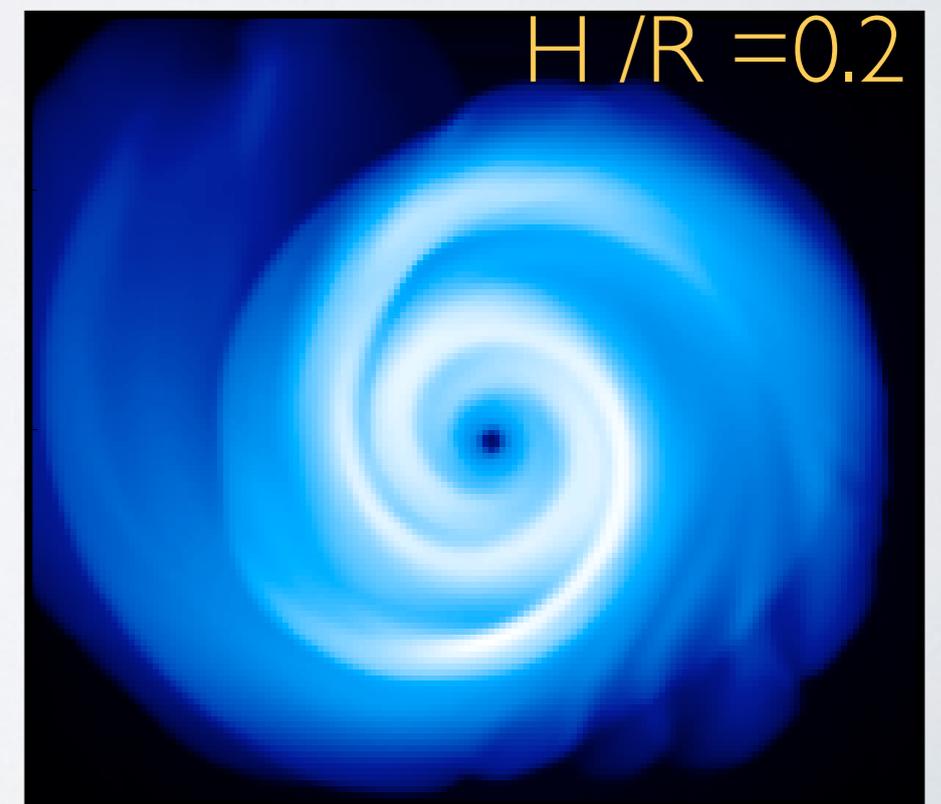
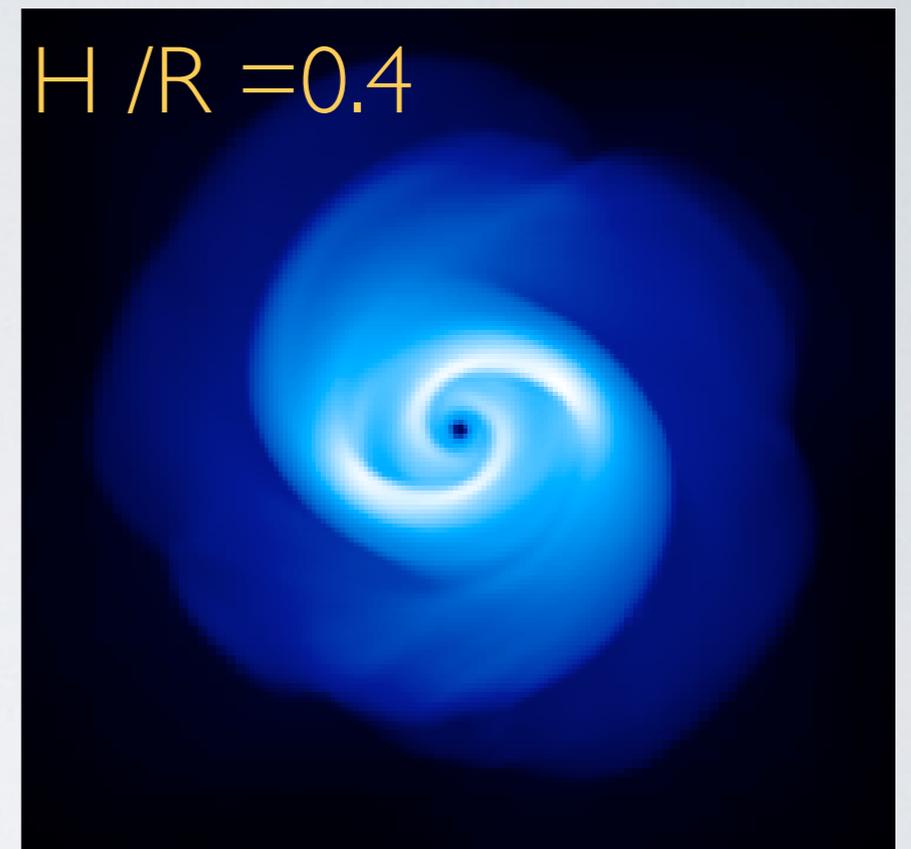


Growth begins at $Q > 1$ in the “global” approximation

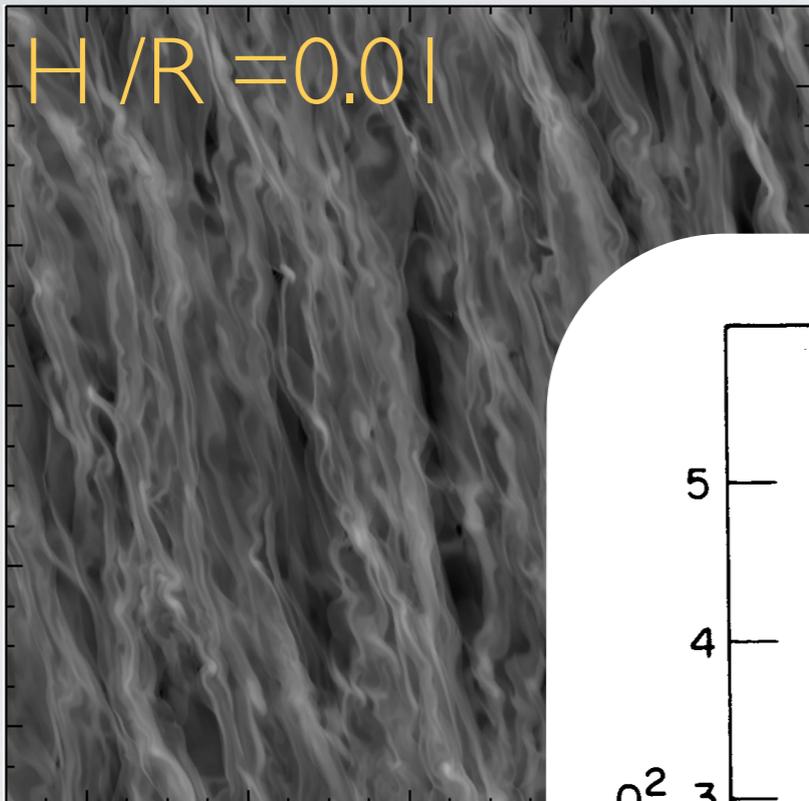
LOCAL VS GLOBAL GI



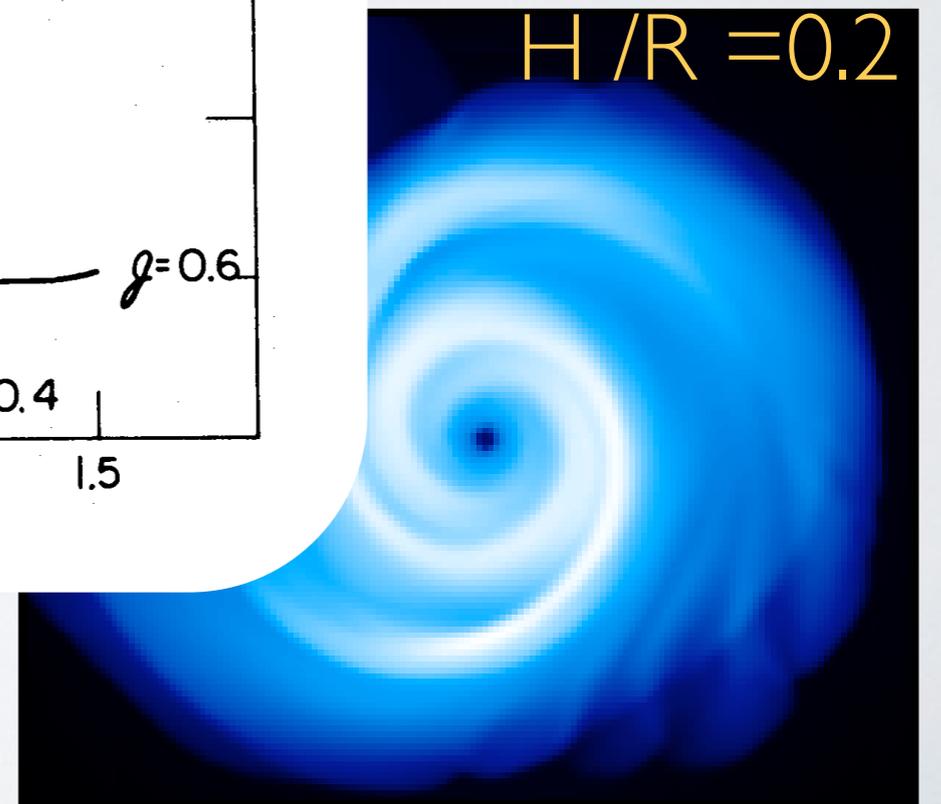
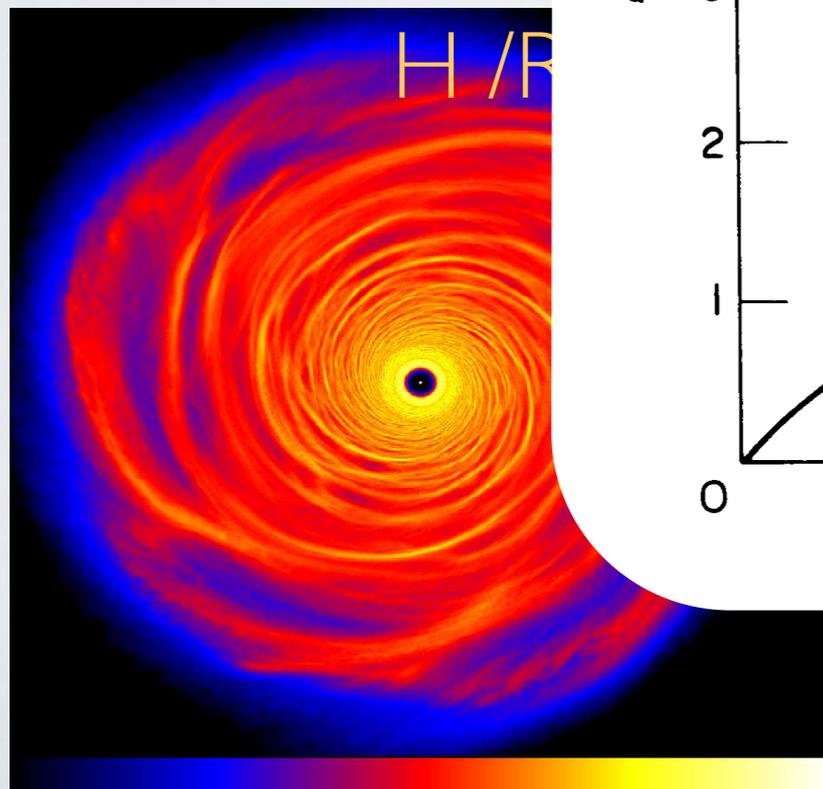
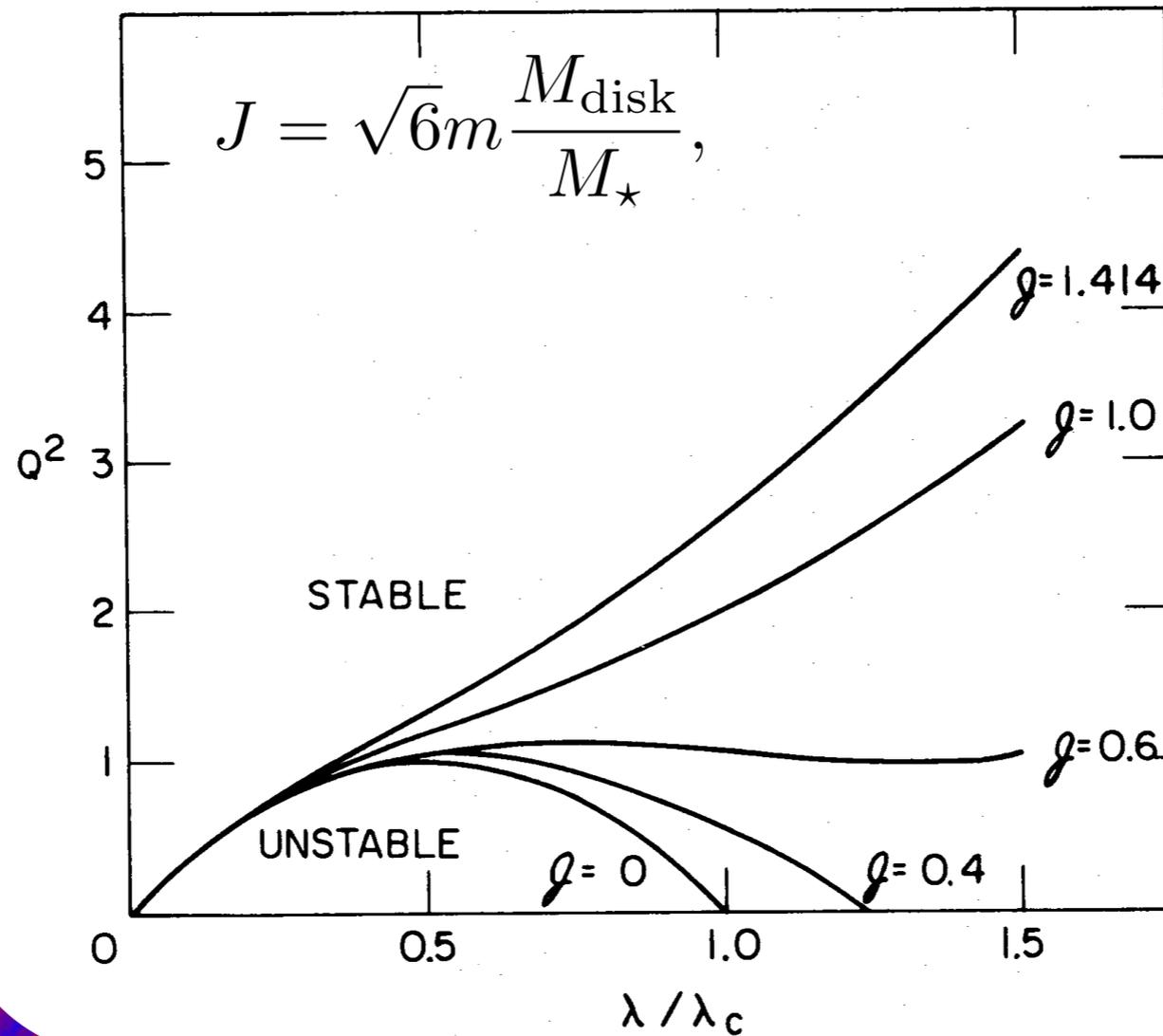
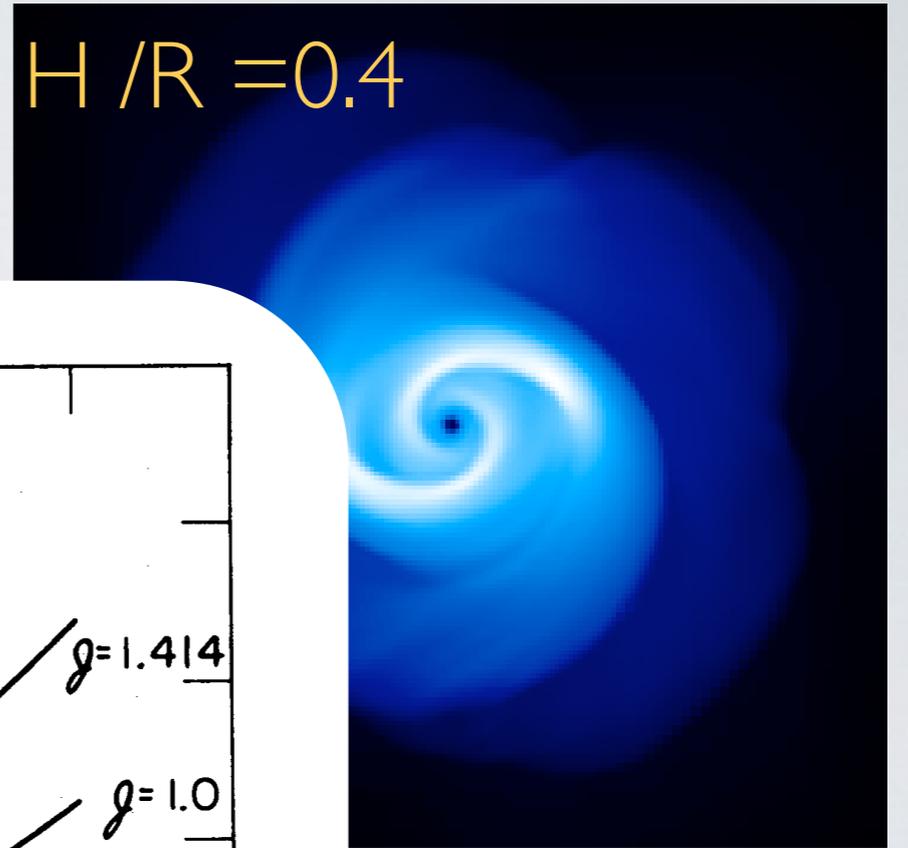
These two scales are governed by a different dispersion relations, thus different modes grow at different values of Q



LOCAL VS GLOBAL GI



These two scales are



From GI to Fragmentation

- Fragmentation occurs when the instability does not saturate in the linear phase. Saturation typically occurs in one of two ways:
 - mode-mode coupling
 - thermal feedback

MODE-MODE COUPLING

- Interaction of multiple growing modes saturates the amplitude of density perturbations
- surface density may never get high enough *locally* for collapse, because global modes are triggered at *higher* Q .
- global modes provide very efficient angular momentum transport — may be important where MRI / disk winds fail

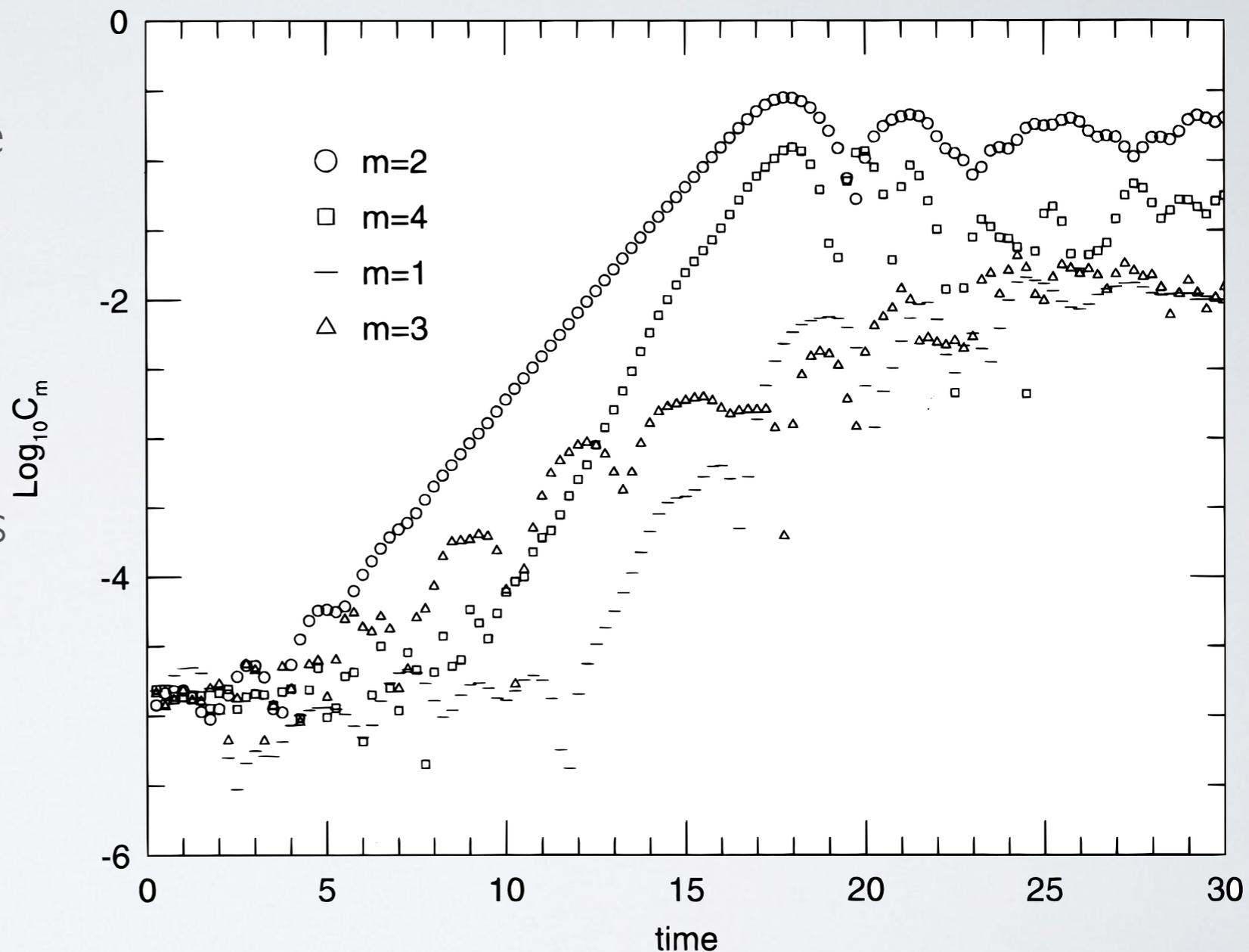


FIG. 6.—Global Fourier amplitudes, C_1 – C_4 , plotted as a function of time

THERMAL FEEDBACK: THE COOLING TIME CRITERION

- waves/spiral arms generate subsonic shocks, which heat the gas.
- too much heating, $Q > I$, self-regulated GI is possible
- too little heating, $Q < I$, fragmentation

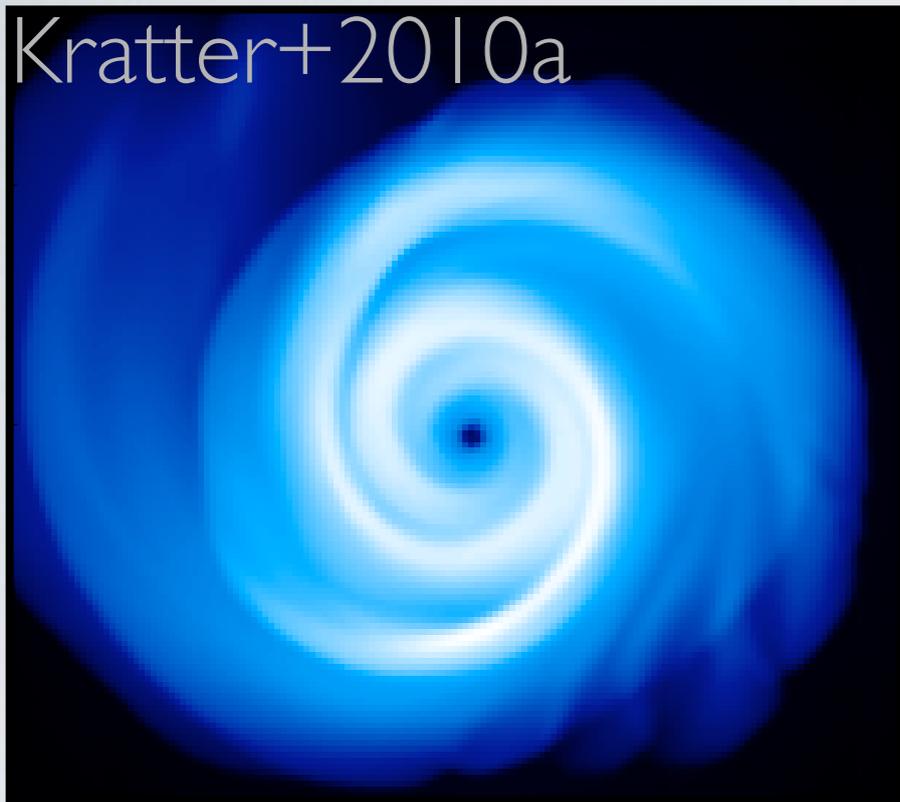
$$\tau_c = \beta \Omega^{-1} \approx \frac{\Sigma c_s^2}{\sigma T^4} f(\tau)$$

since the instability grows on a dynamical time, cooling on a similar timescale is too fast to stave off fragmentation.

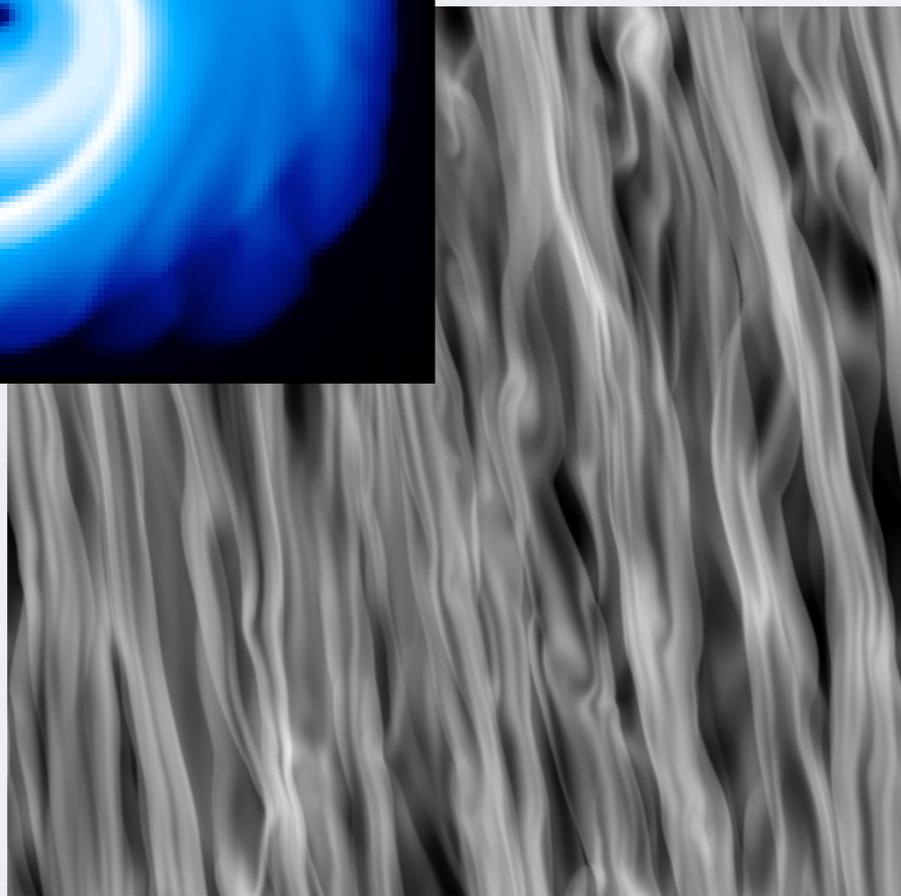
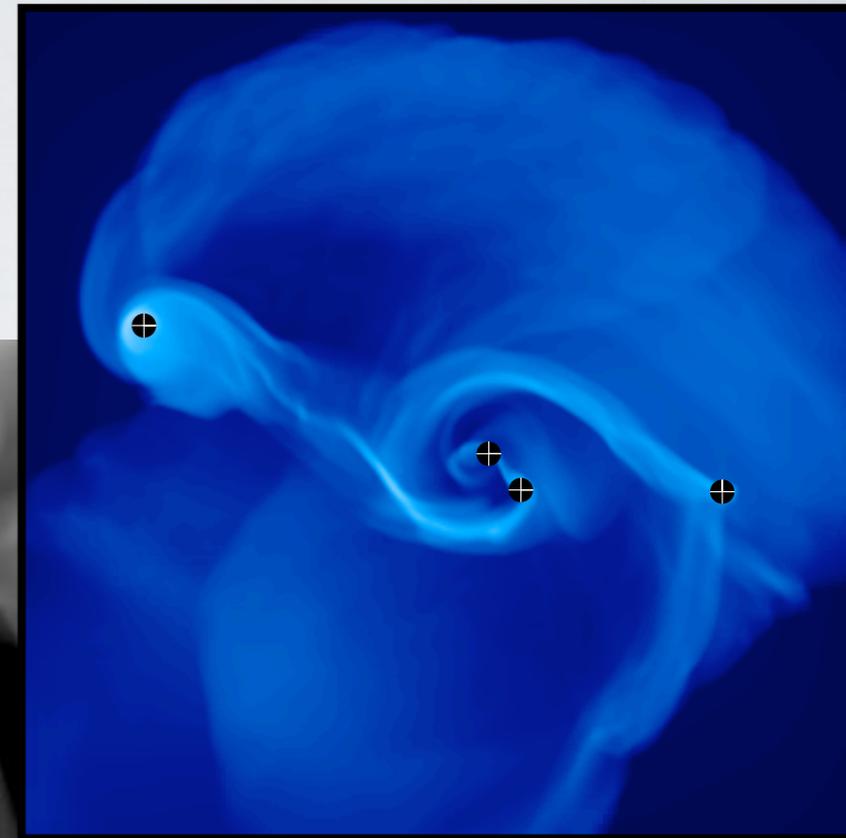
HOW FAST? WHAT IS β

- Physically: set by optical depth due to **dust**.
- It depends on the (effective) EOS (γ)
- Probably between 8-15 for protoplanetary disks with $\gamma = 7/5$
- *Numerical modelling required*

Saturation vs fragmentation

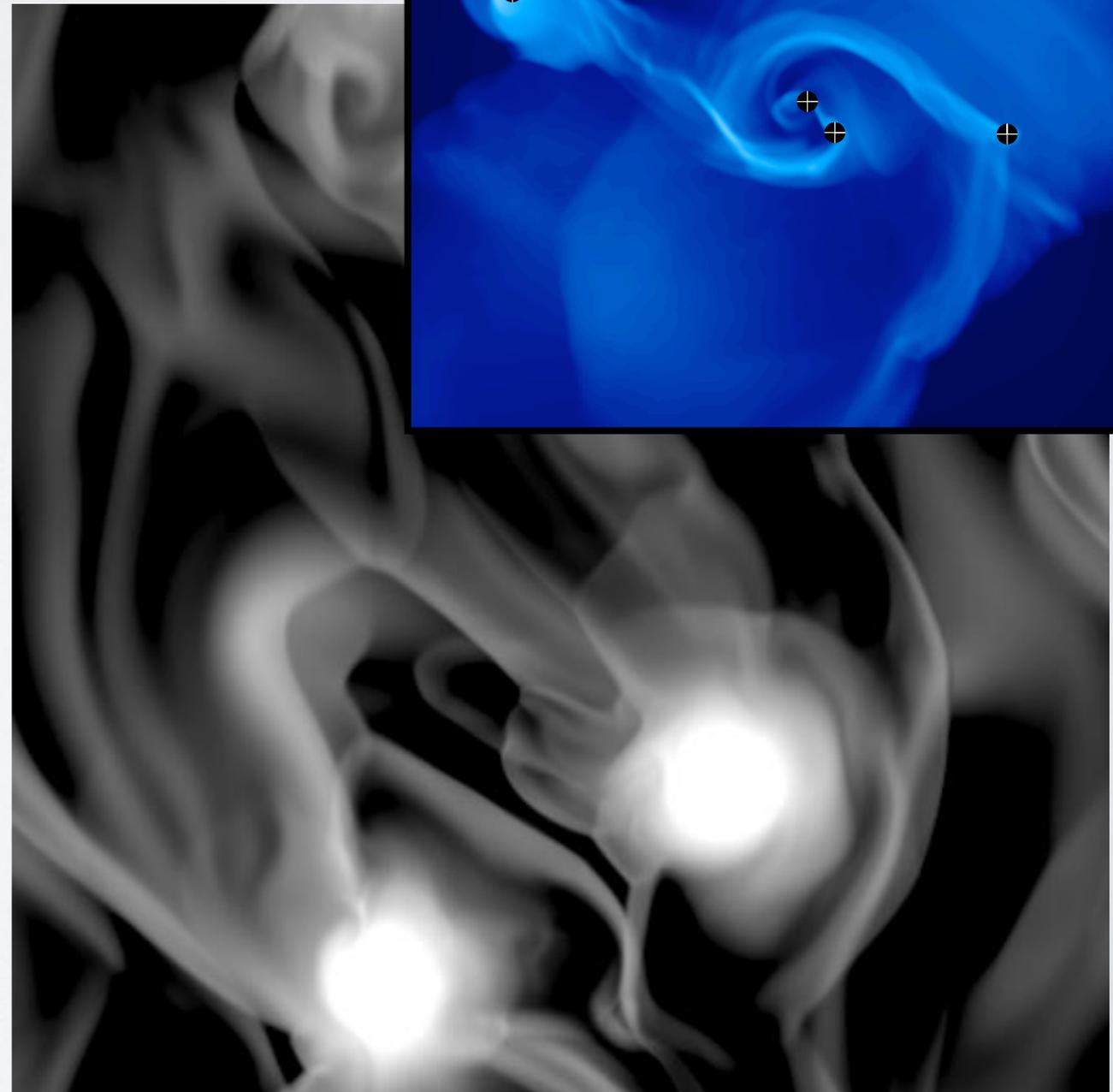


Fragments also occur on scales $\sim H$



Most power in “gravito-turbulence” occurs on scales $1 < H < 10$

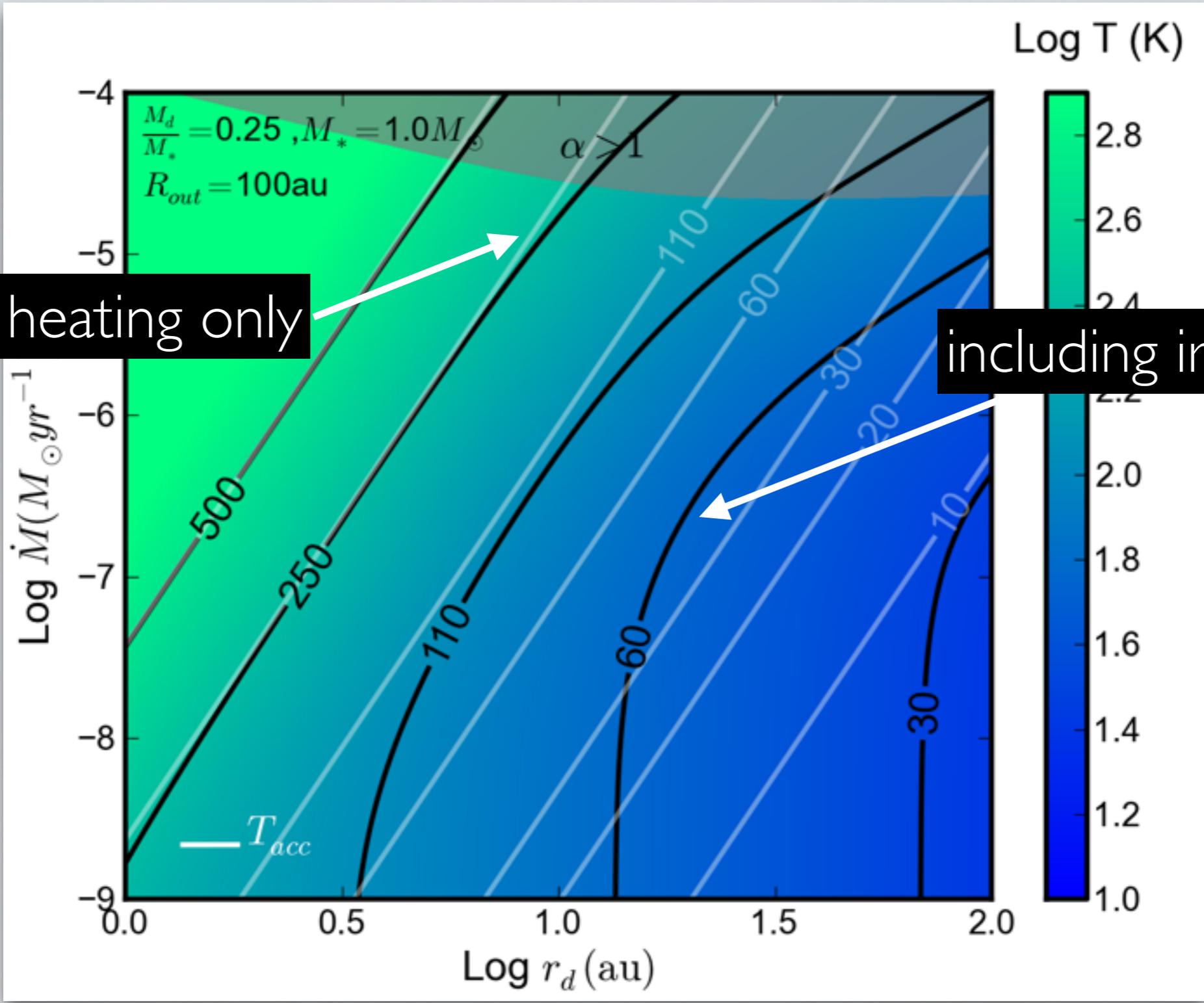
Gammie, 2001



WHAT ARE THE RIGHT
PARAMETERS FOR PROTOSTELLAR
AND PROTOPLANETARY DISKS?

Kratter & Lodato, in prep

$H < R$

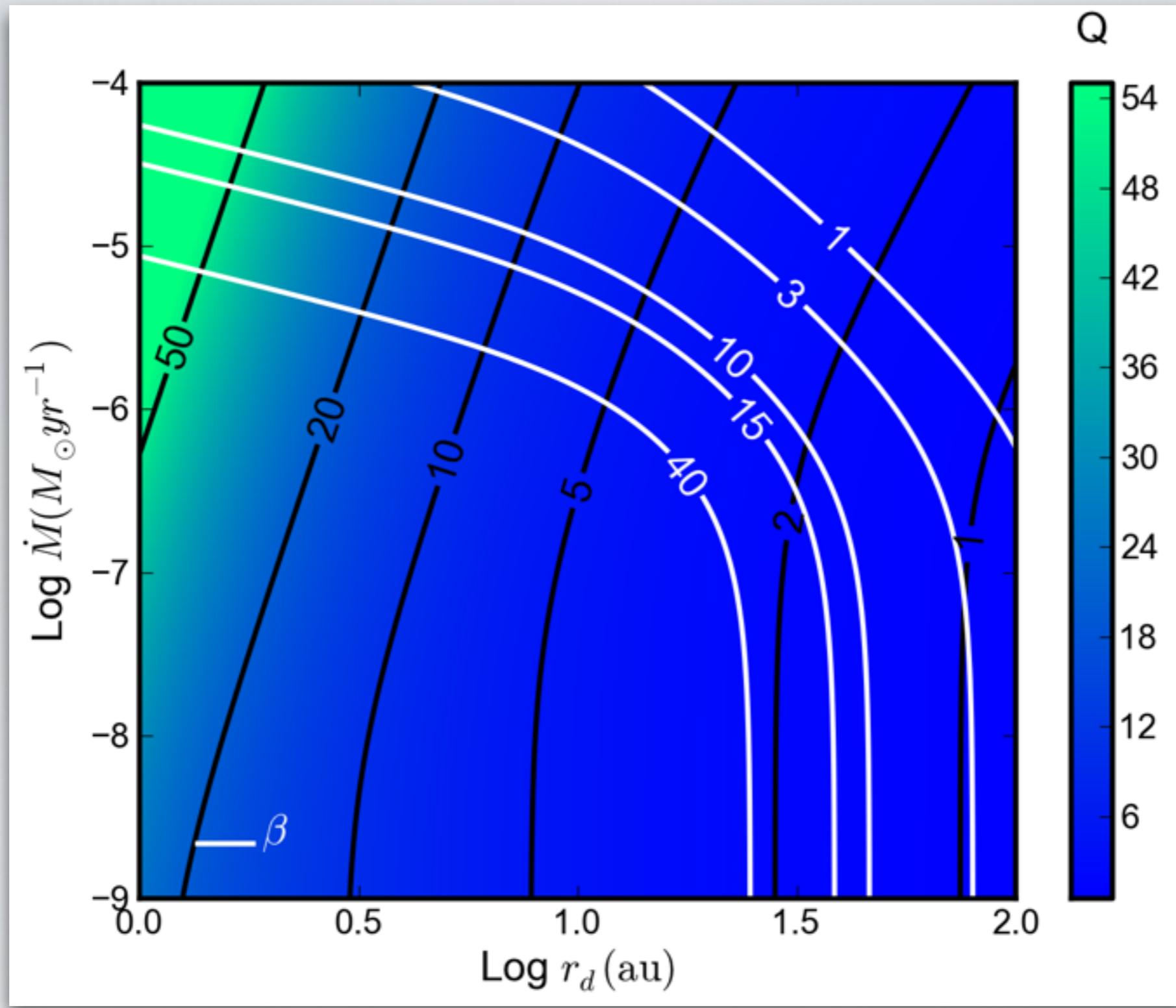


Conditions in a **massive, protostellar** disk around a sun-like star

$$\sigma T_{\text{mid}}^4 = \frac{3}{8} f(\tau_R) F_{\text{acc}} + \sigma T_{\text{h},*}^4 + \sigma T_{\text{ex}}^4 \quad F_{\text{acc}} = \frac{3}{8\pi} \dot{M} \Omega^2$$

Kratter &
Lodato, in
prep

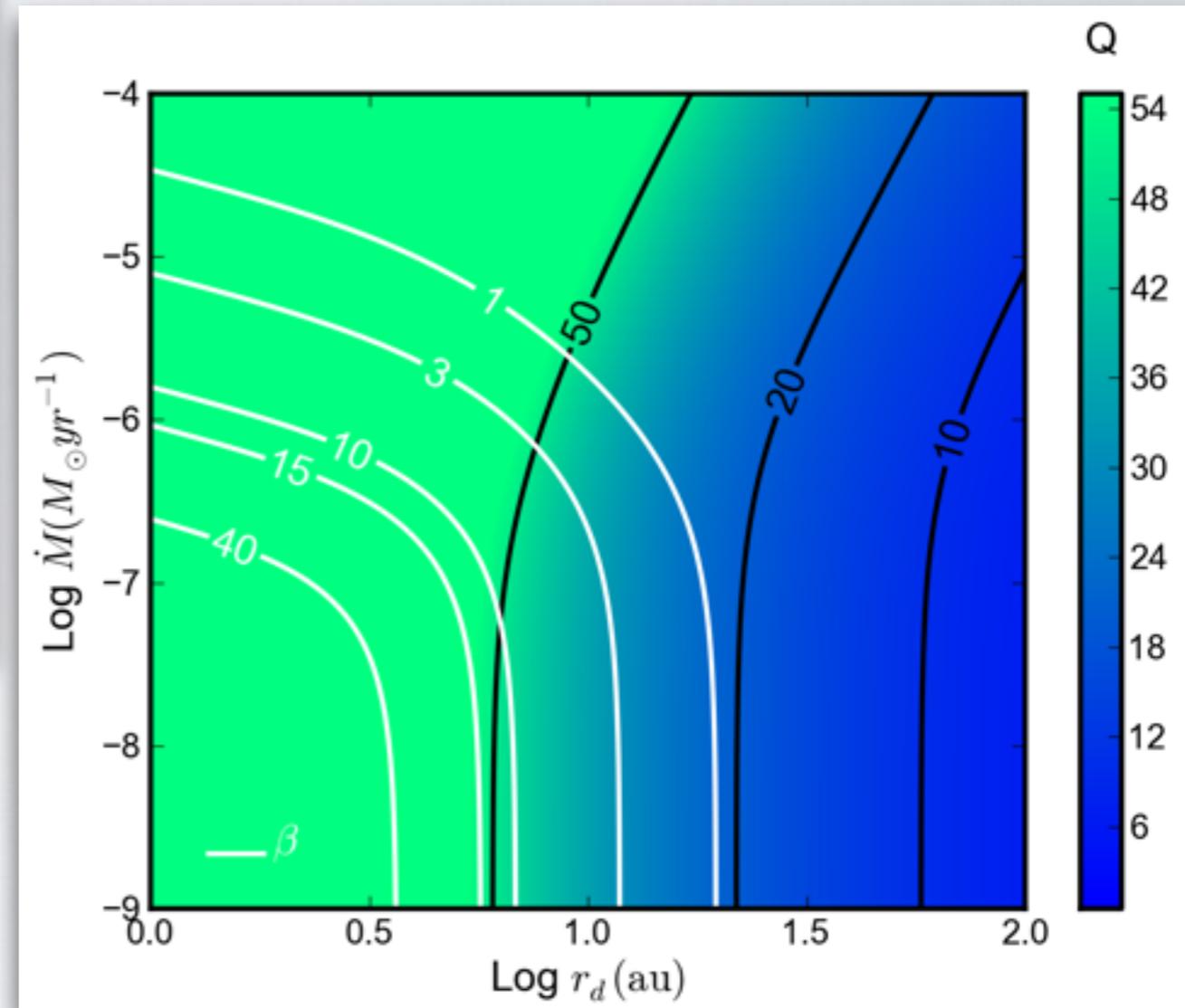
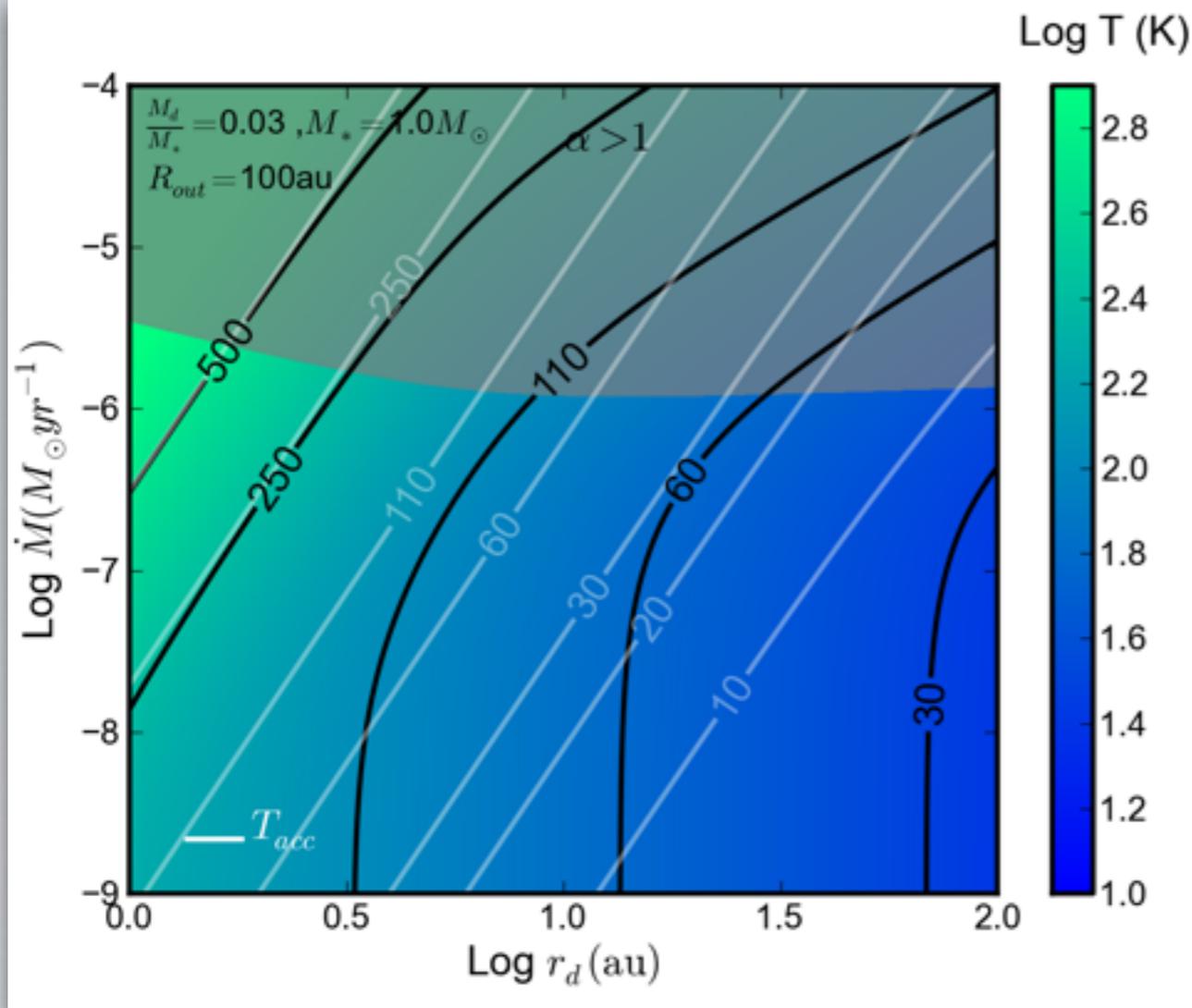
$H < R$



Conditions in a **massive** disk around a sun-like star

corresponding values of Q and cooling time, between
70-100 AU it is close enough to give rise to GI

$H \ll R$



Kratter & Lodato, in prep

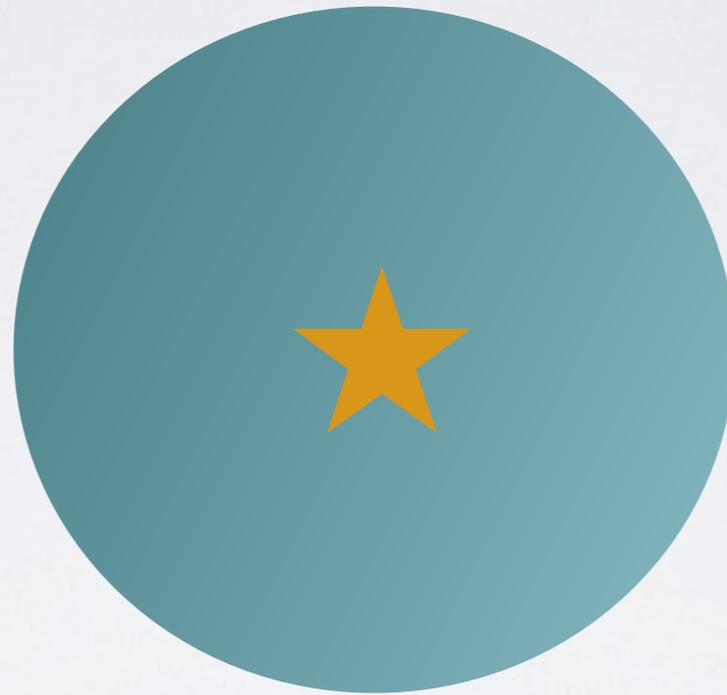
Conditions for measured Class I disks around sun-like stars

Disks that are low enough in mass to operate in the local regime are typically too hot to suffer from GI.

HOW DID WE GET HERE?

Disk with $Q > 1$

$$Q = \frac{c_s \Omega}{\pi G \Sigma} = f \frac{M_*}{M_D} \frac{H}{r}$$

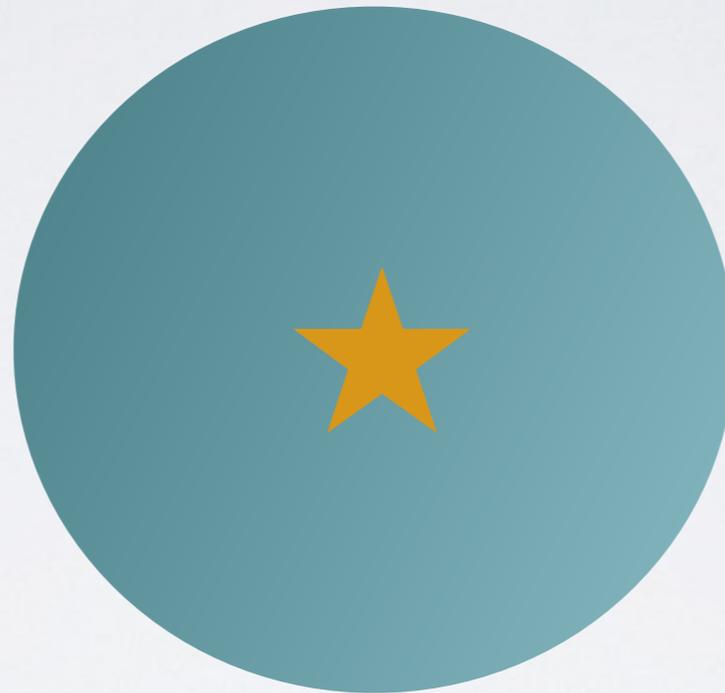


- The outer regions of protostellar disks are dominated by stellar irradiation, which fixes the temperature.

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add mass, raise
surface density

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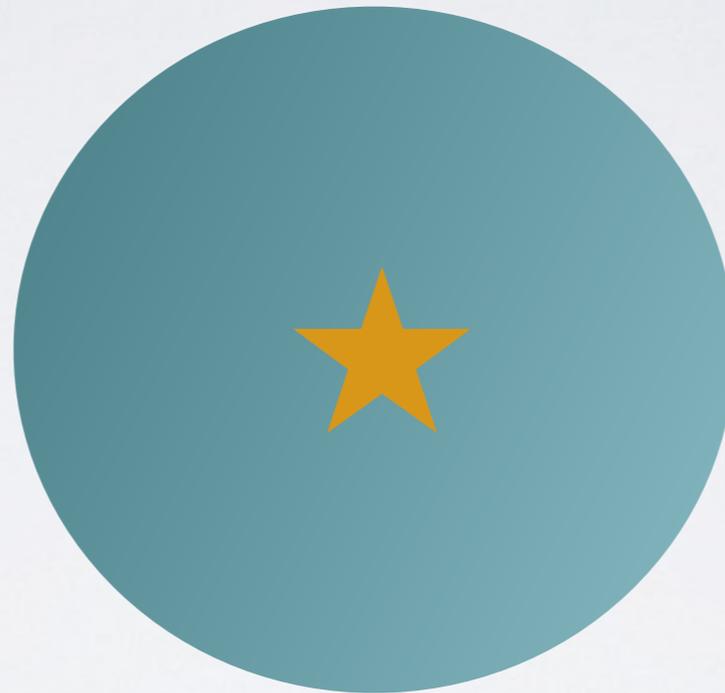
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cool the disk down



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add mass, raise surface density

- The outer regions of protostellar disks are dominated by stellar irradiation, which fixes the temperature.

only under very special circumstances can the disk reach instability by getting colder, rather than by adding mass

$$\Gamma = \frac{\dot{M}_{in}}{M_{*d}\Omega}$$

Low Mass Stars

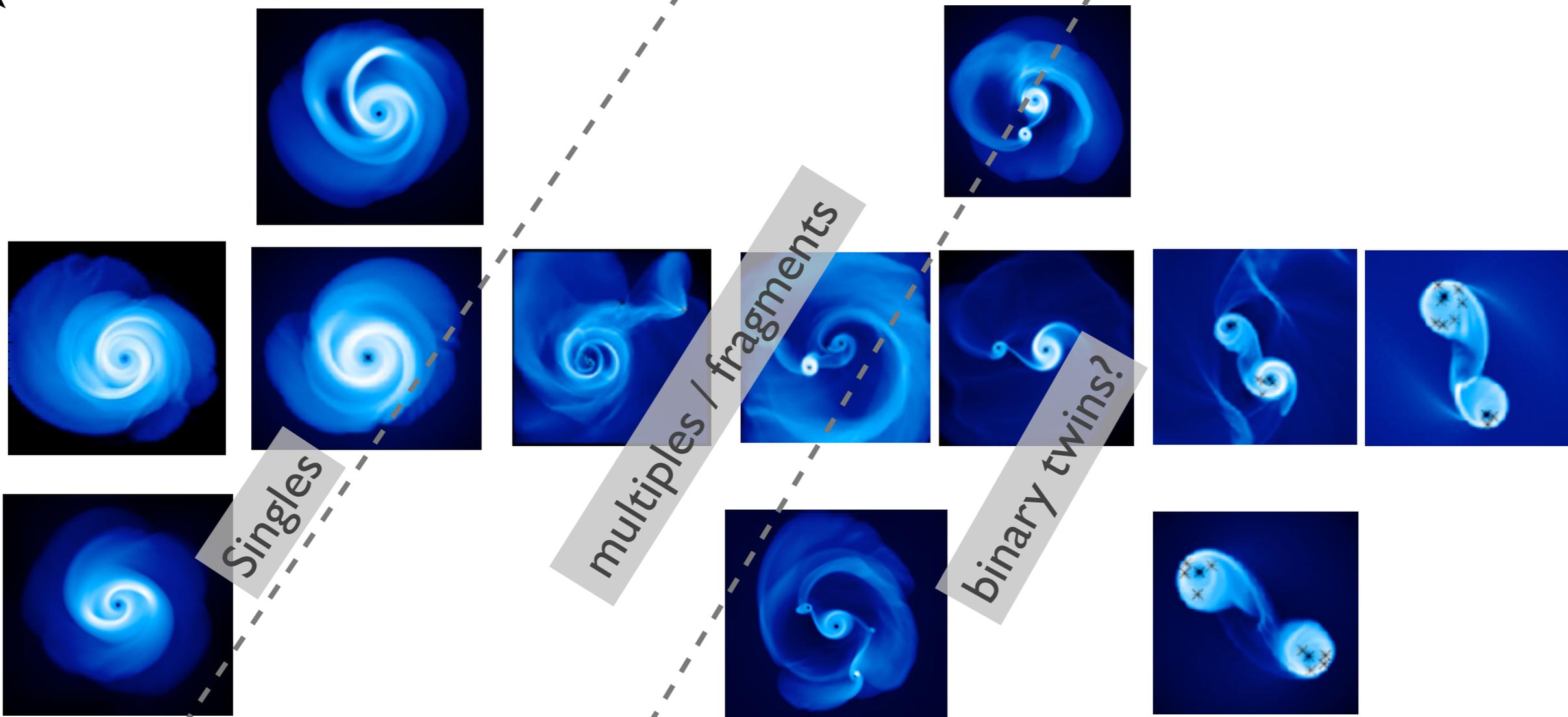
High Mass Stars

normalized accretion radius

0.1

0.01

0.005



1.4

2

3

5

5

normalized accretion rate

$$\xi = \frac{\dot{M}G}{c_s^3}$$

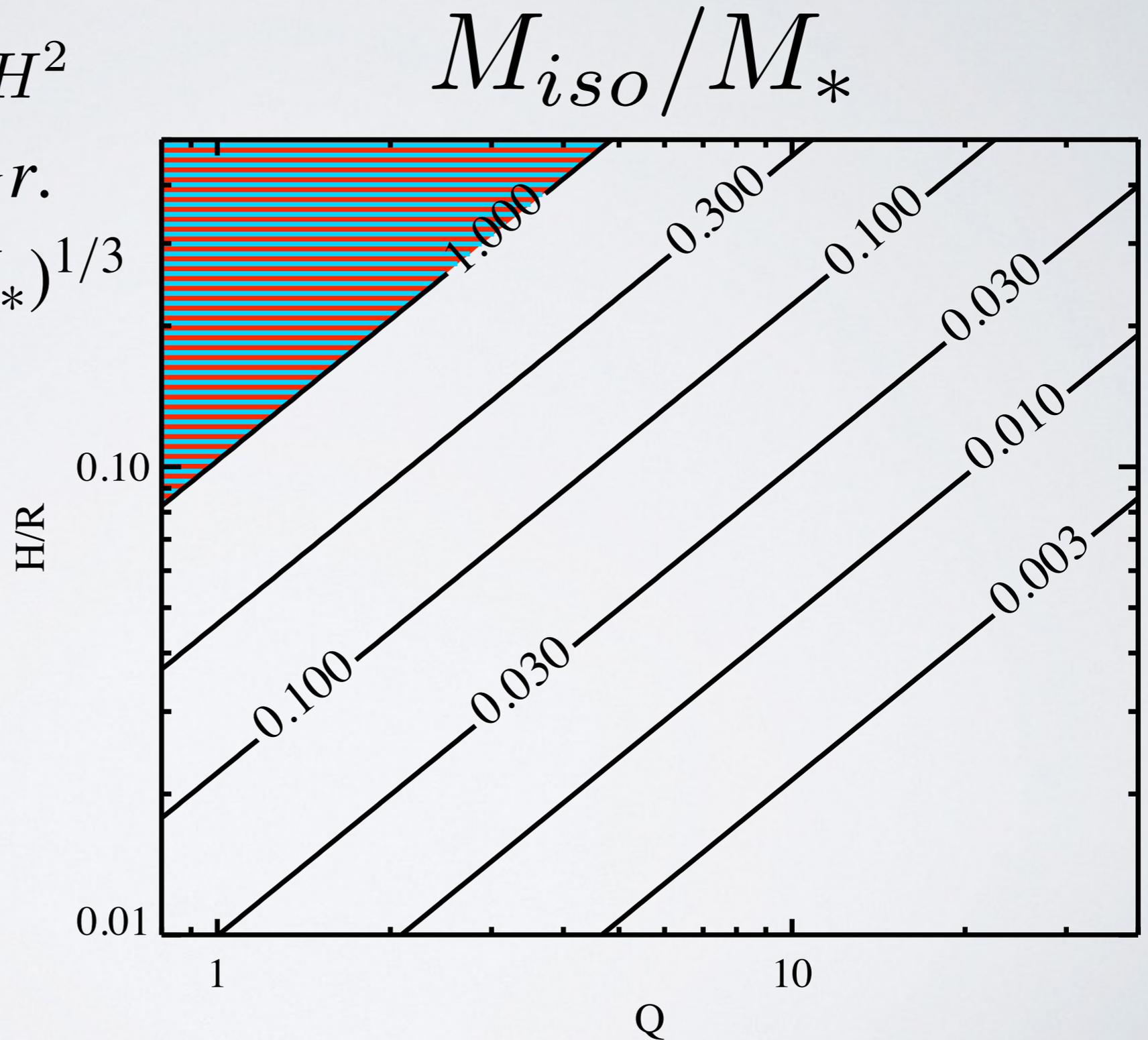
Fragment mass, zeroth order

$$M_{frag} = \Sigma \lambda^2 \sim \Sigma H^2$$

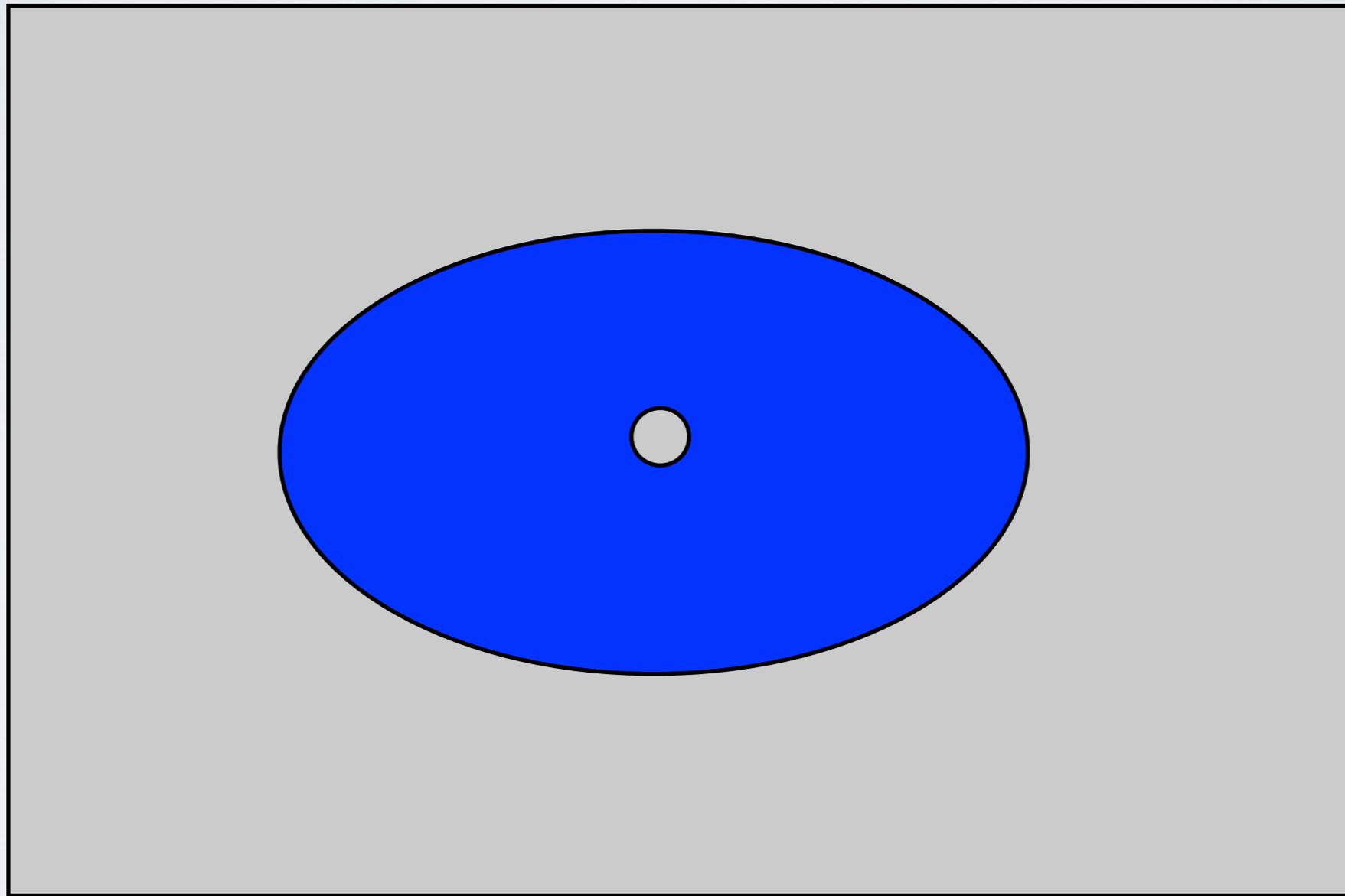
$$M_{iso} \approx 4\pi f_H \Sigma R_H r.$$

$$R_H = r (M_{iso}/3M_*)^{1/3}$$

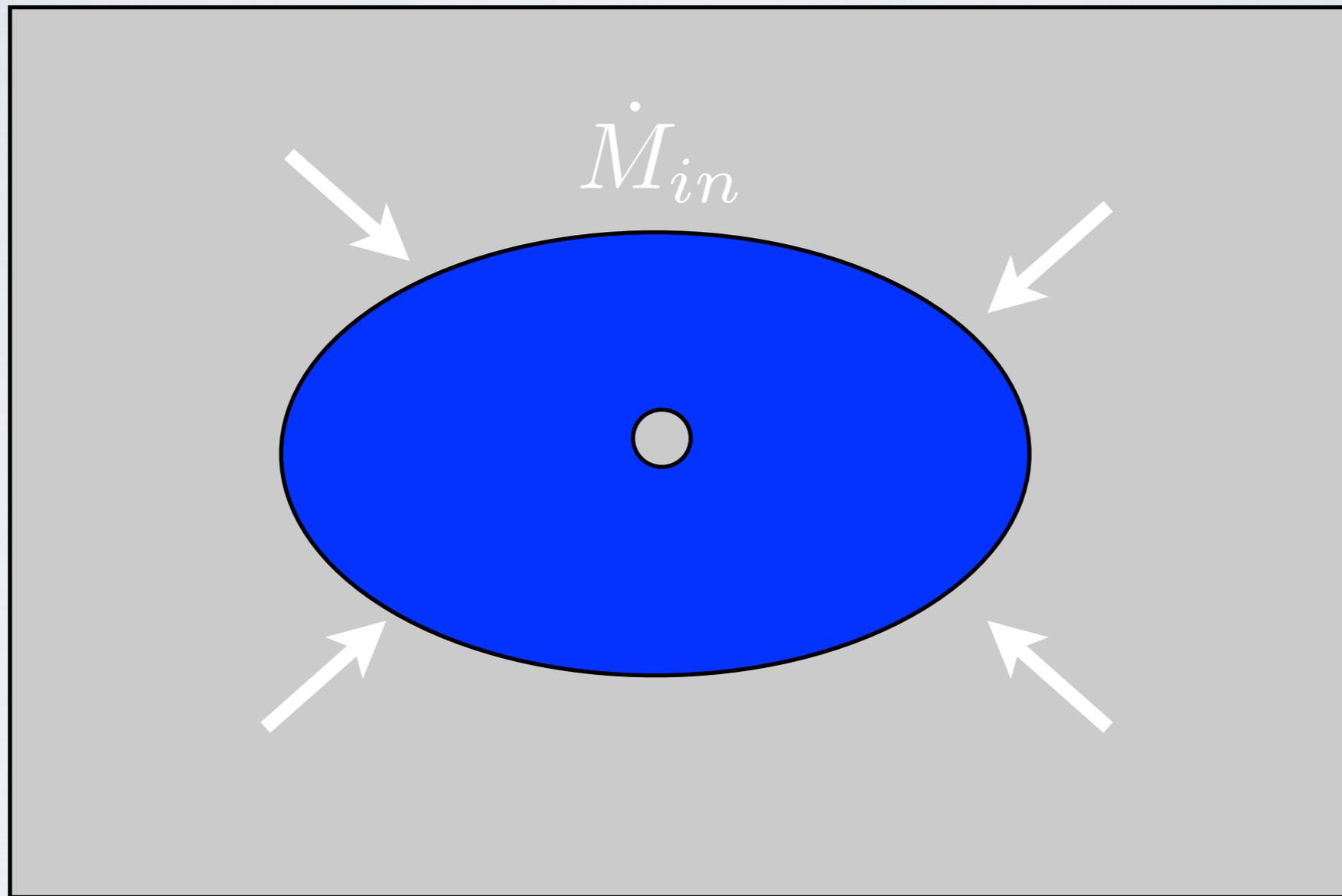
If disks are driven unstable by infall, it can be challenging to avoid isolation mass



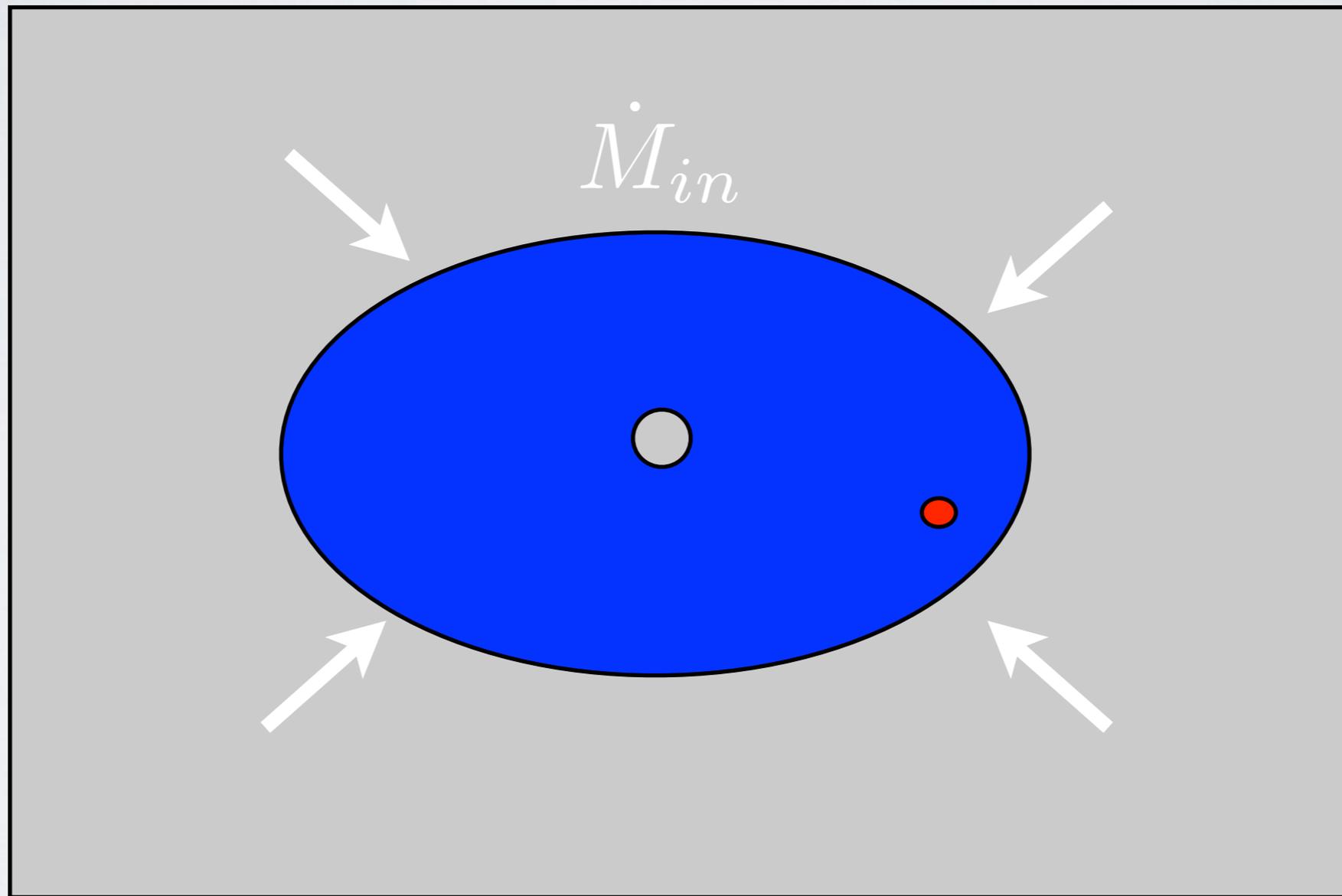
WHY DO FRAGMENTS GROW
IN DISKS WITH INFALL?



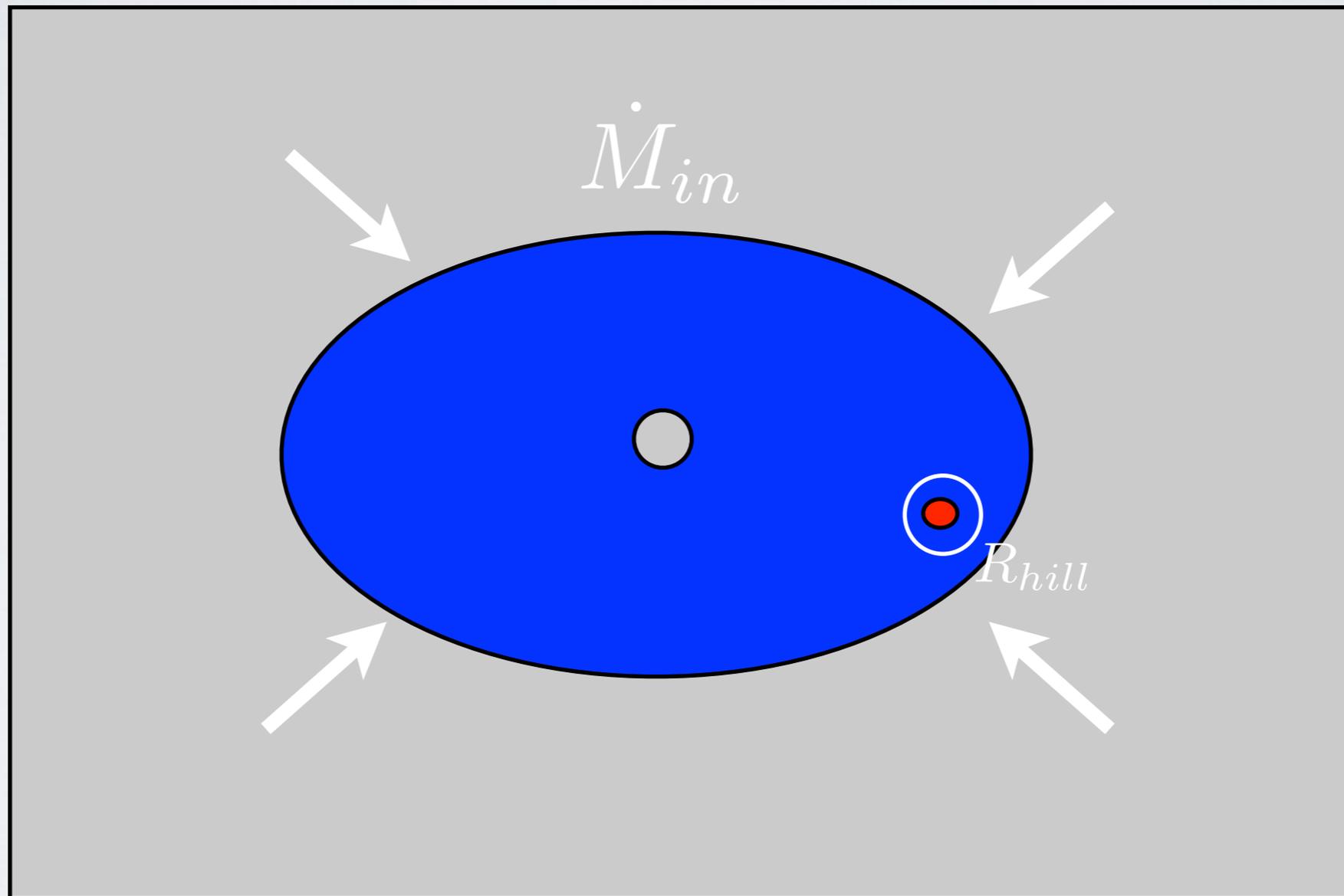
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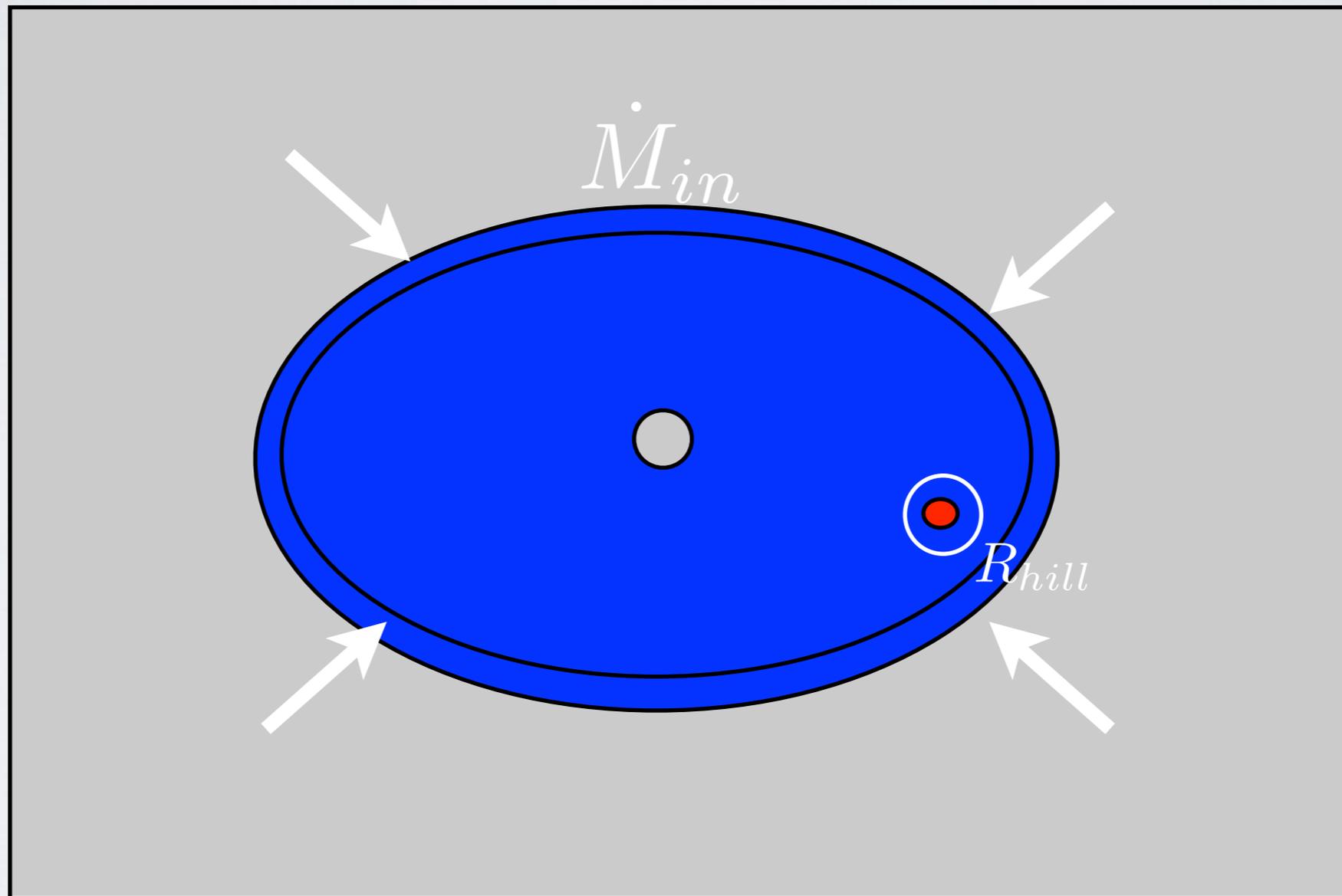
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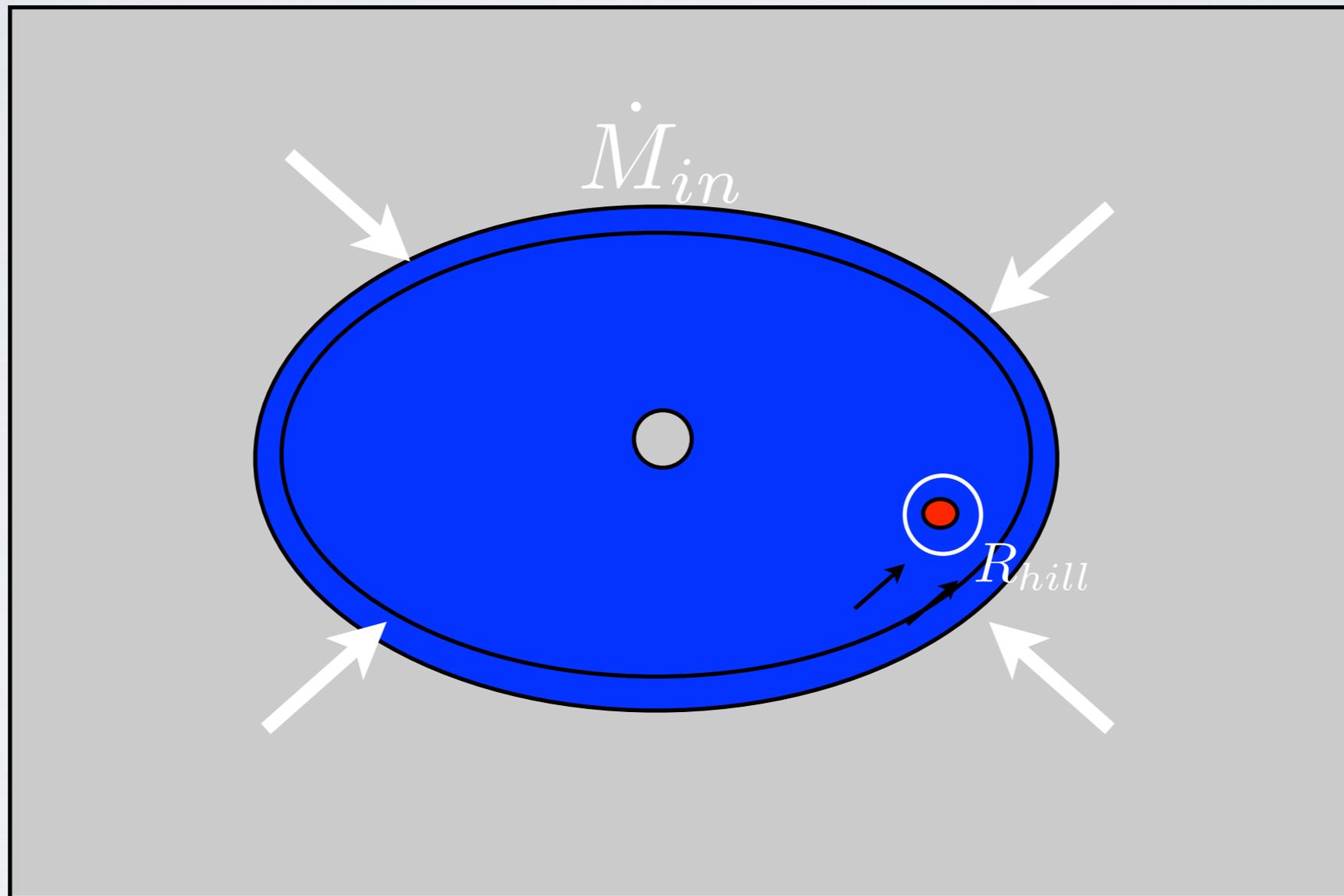
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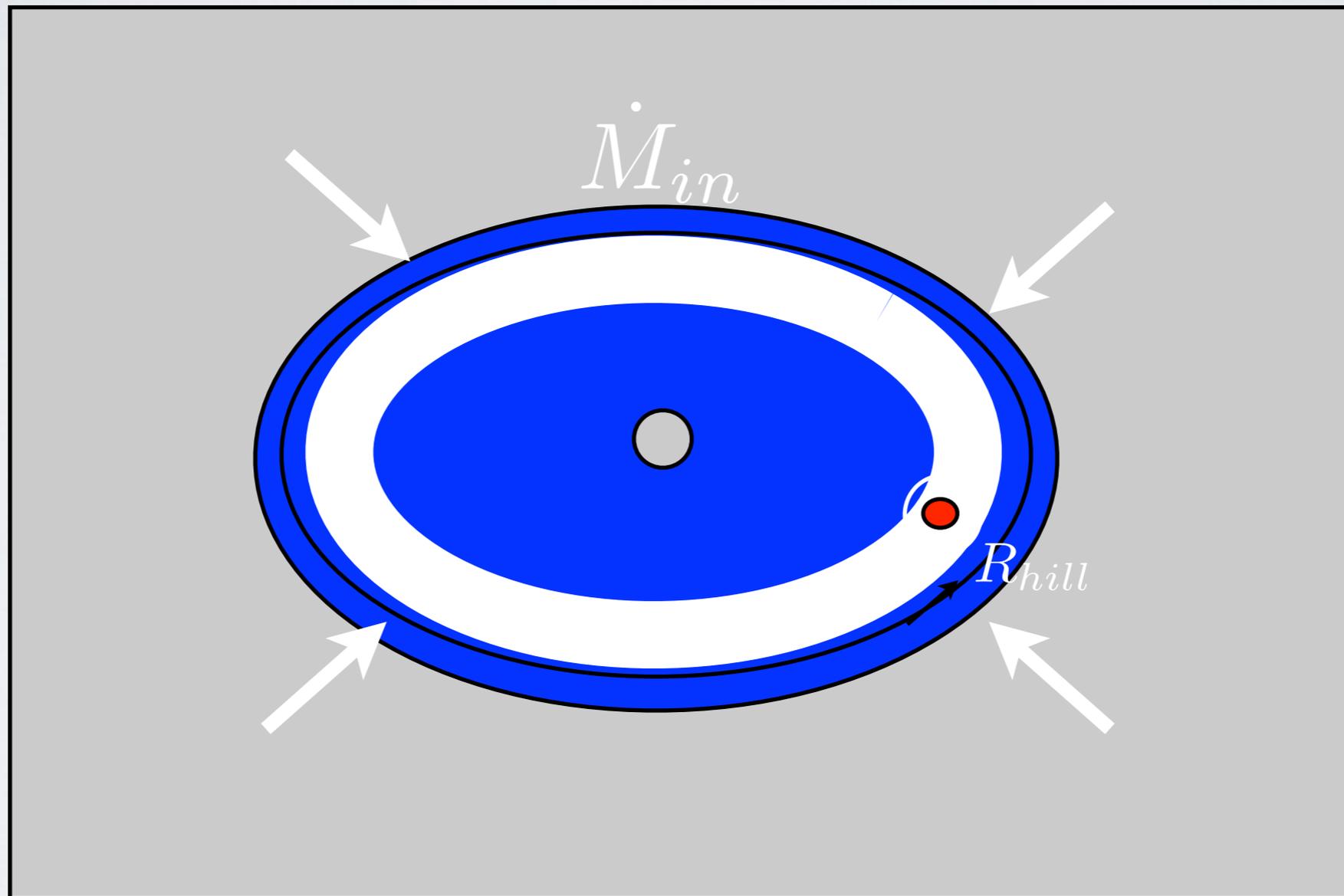
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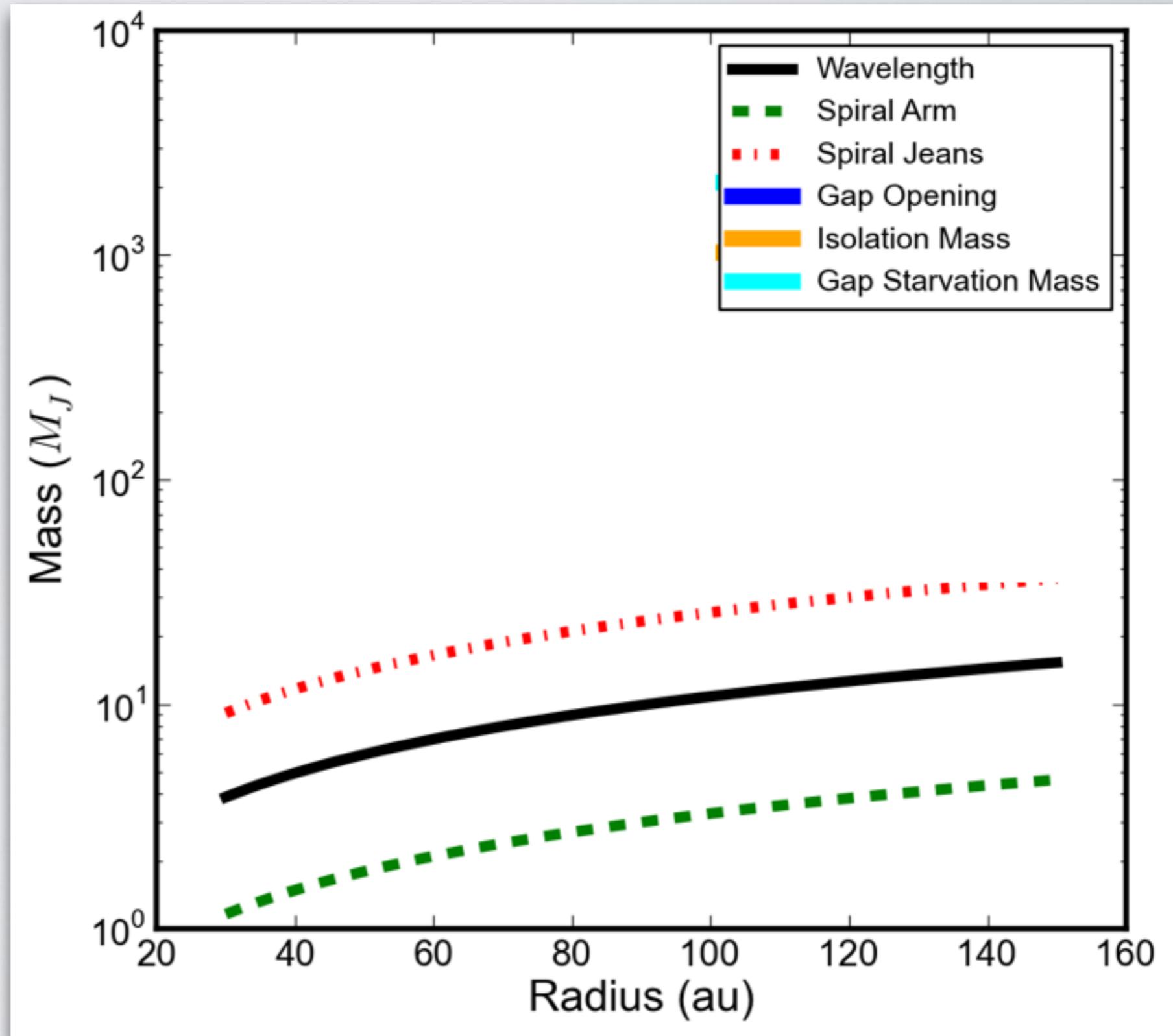
First order: how massive are fragments?

- Initial mass estimates all scale with

$$\Sigma H^2$$

- Fragments that are not disrupted can also easily grow from the parent disk

Kratter & Lodato, in prep, with data from Kratter+2010, Boley +2010, Forgan & Rice 2013, Young & Clarke 2015



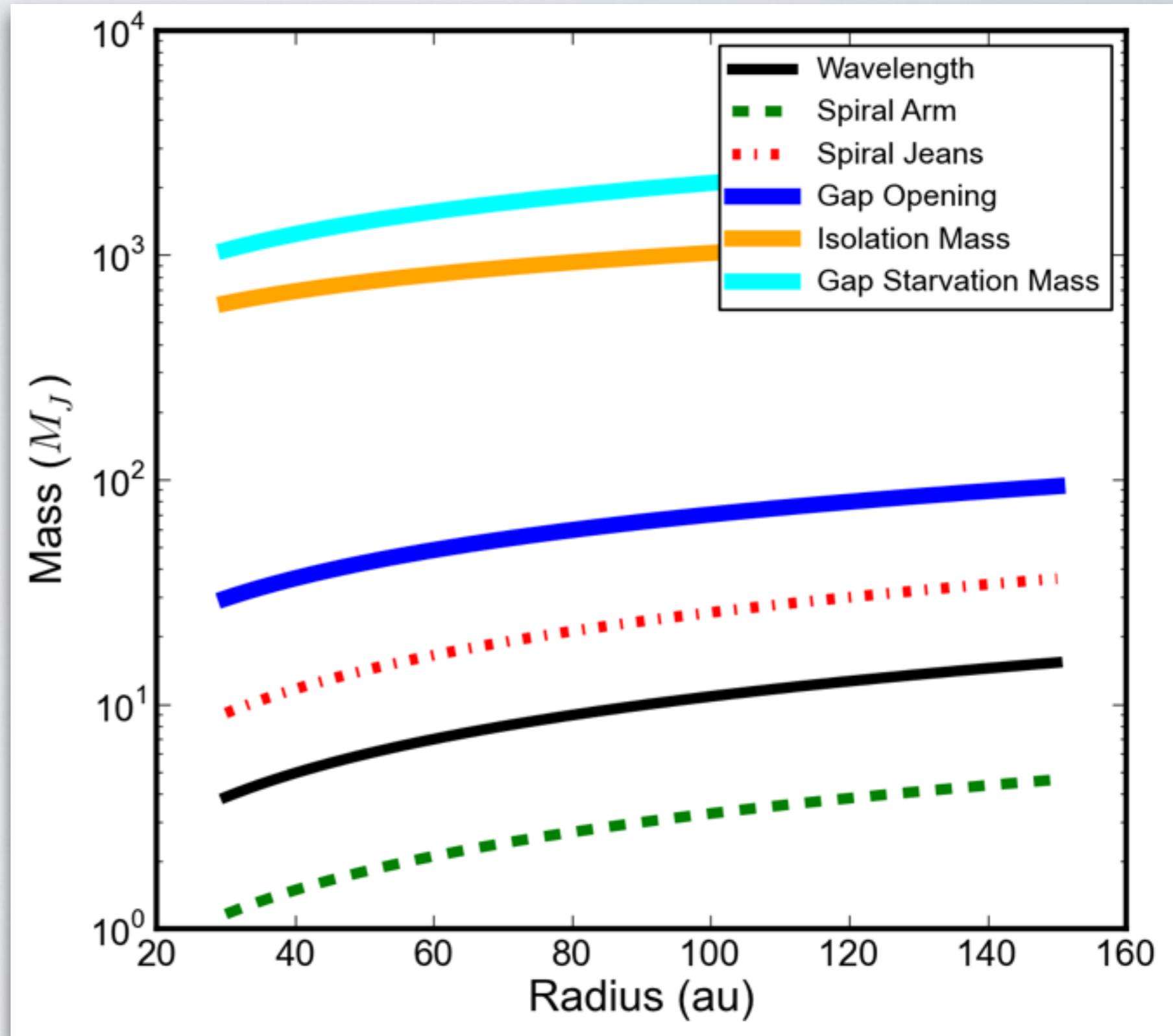
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Choose your own GI adventure!

local (cold, low mass) or global (hotter, higher mass)?

local

slow or fast cooling?

fast

slow

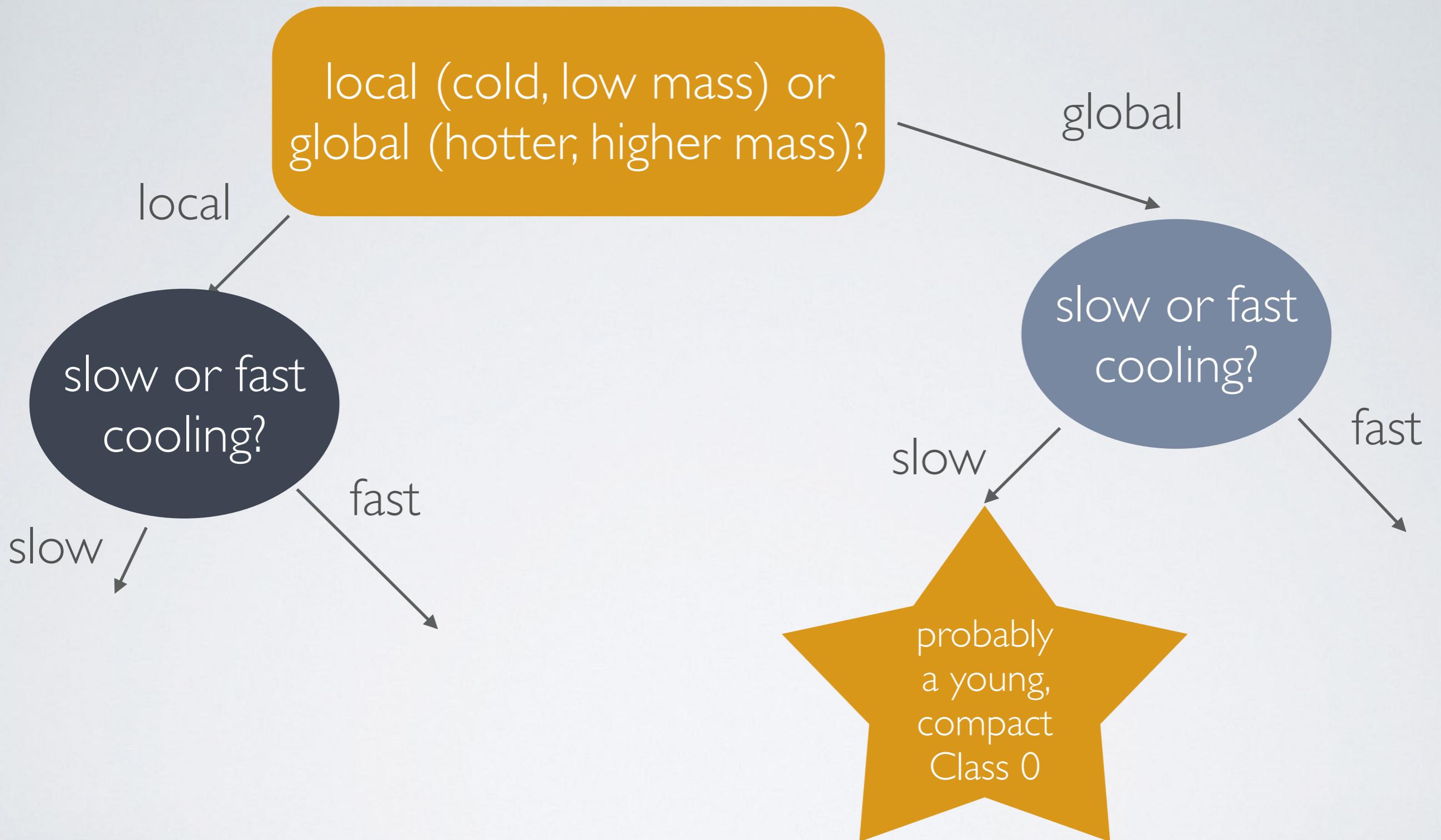
global

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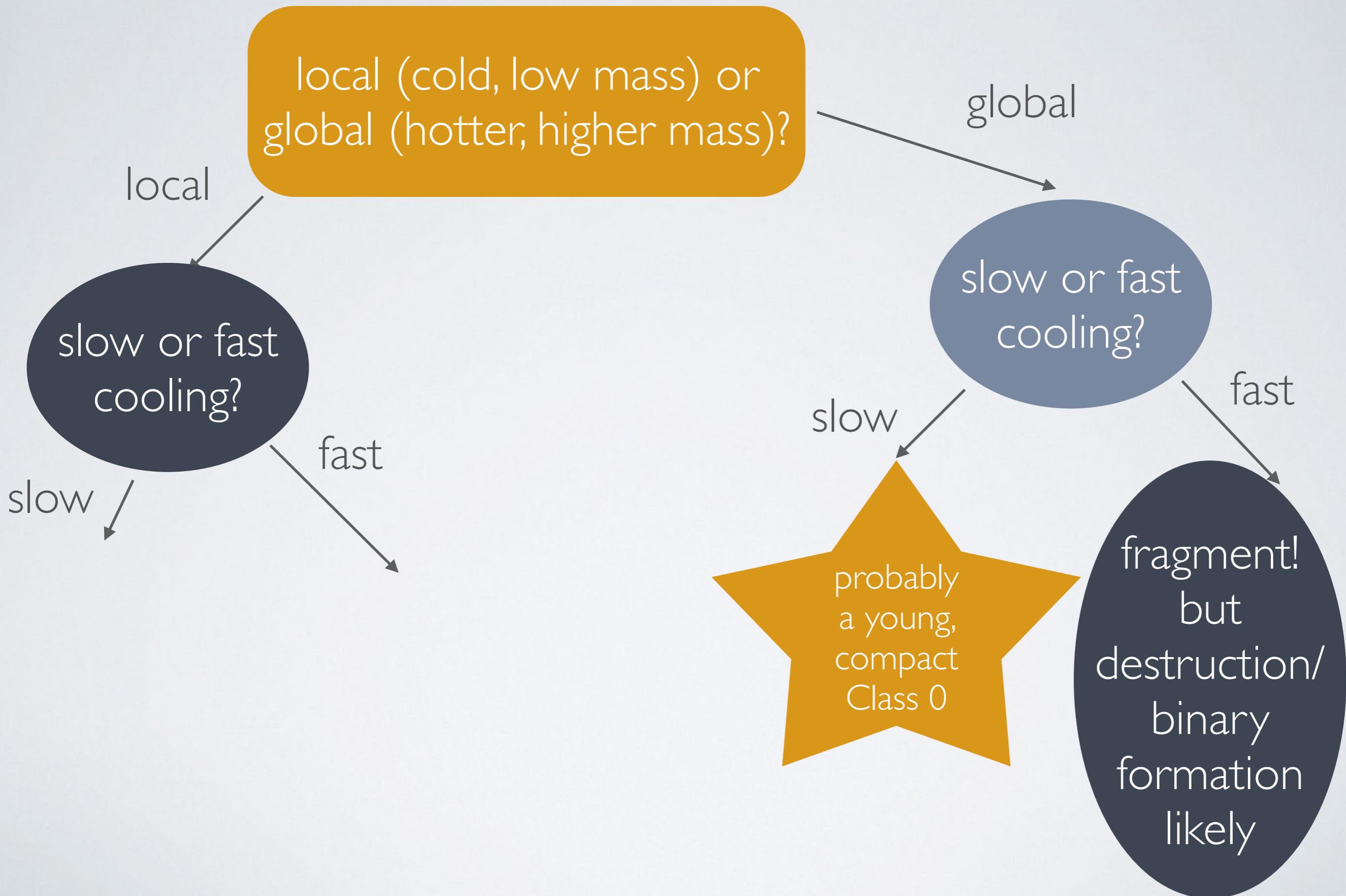
slow

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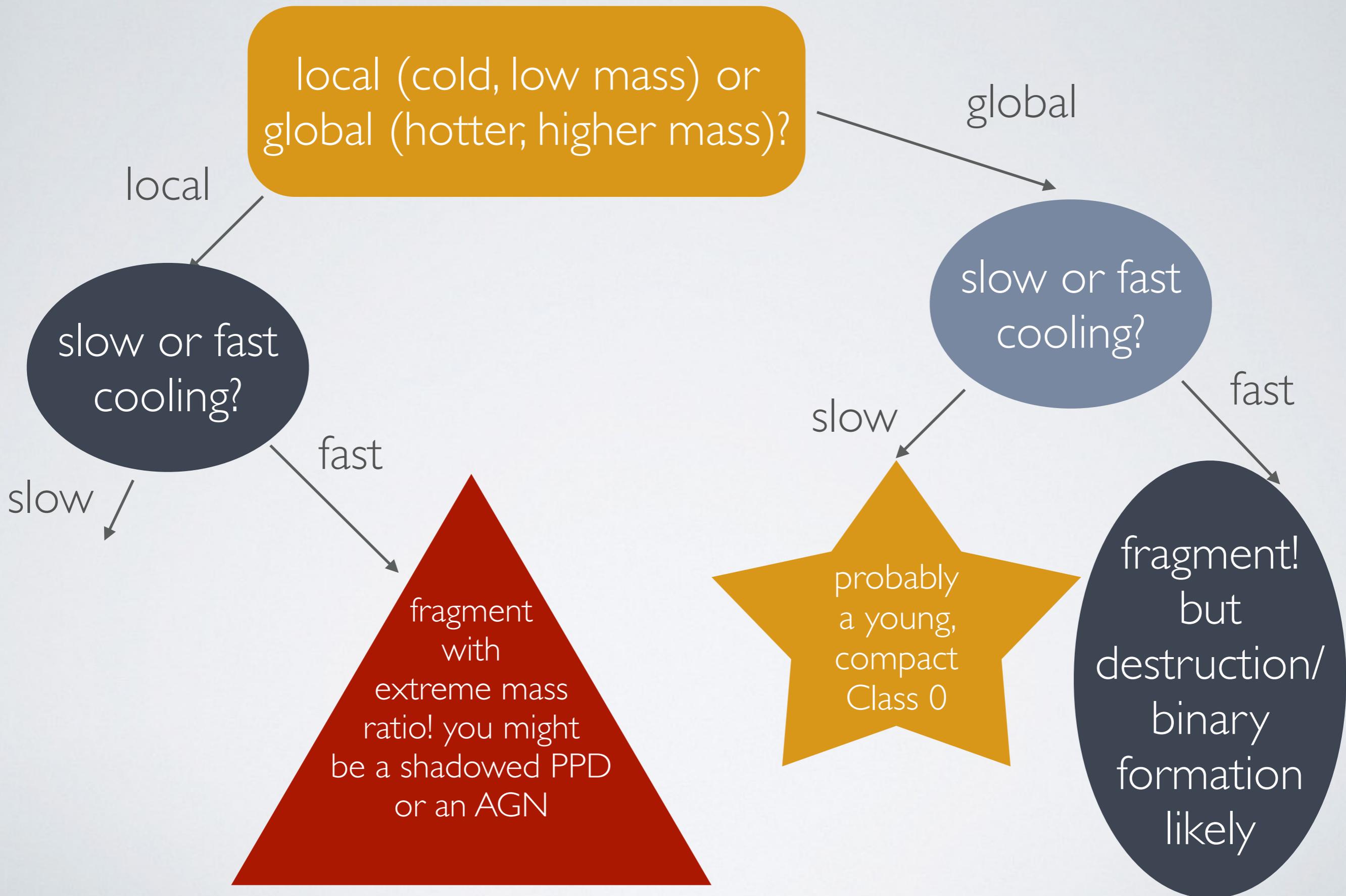
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slow or fast cooling?

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slow

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self-regulated GI
— you might be an AGN!

fragment with extreme mass ratio! you might be a shadowed PPD or an AGN

probably a young, compact Class 0

fragment! but destruction/binary formation likely

Choose your own GI adventure!

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infall

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ON THE NEXT INSTALLMENT....FATE OF FRAGMENTS

- Disruption on few dynamical times
- Partial or complete disruption due to inward migration
- Direct collapse / continued growth
- GI population synthesis, and other ways to make wide orbit planets