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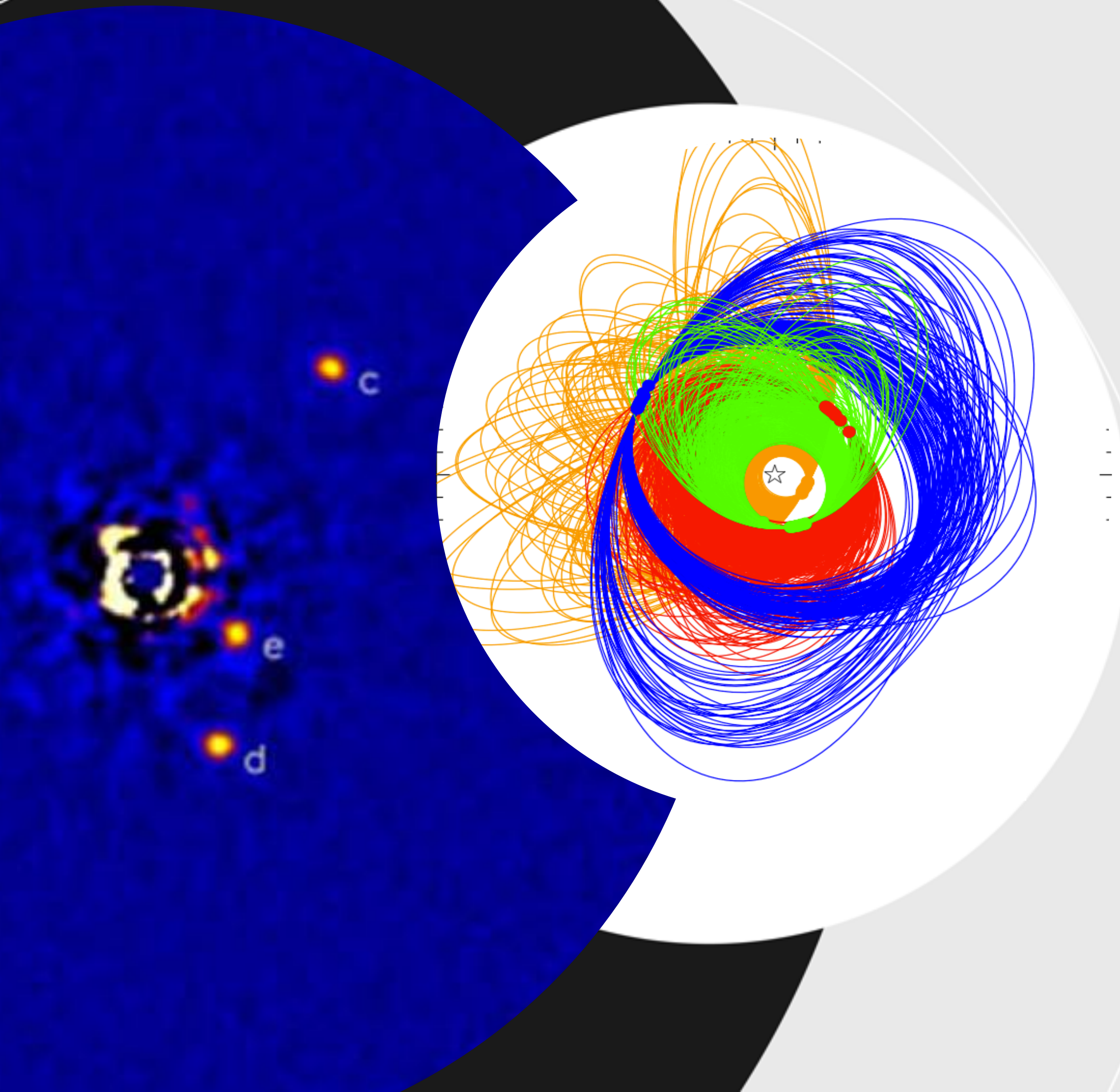
GOING FROM MEASUREMENTS TO ORBITS



Quinn Konopacky

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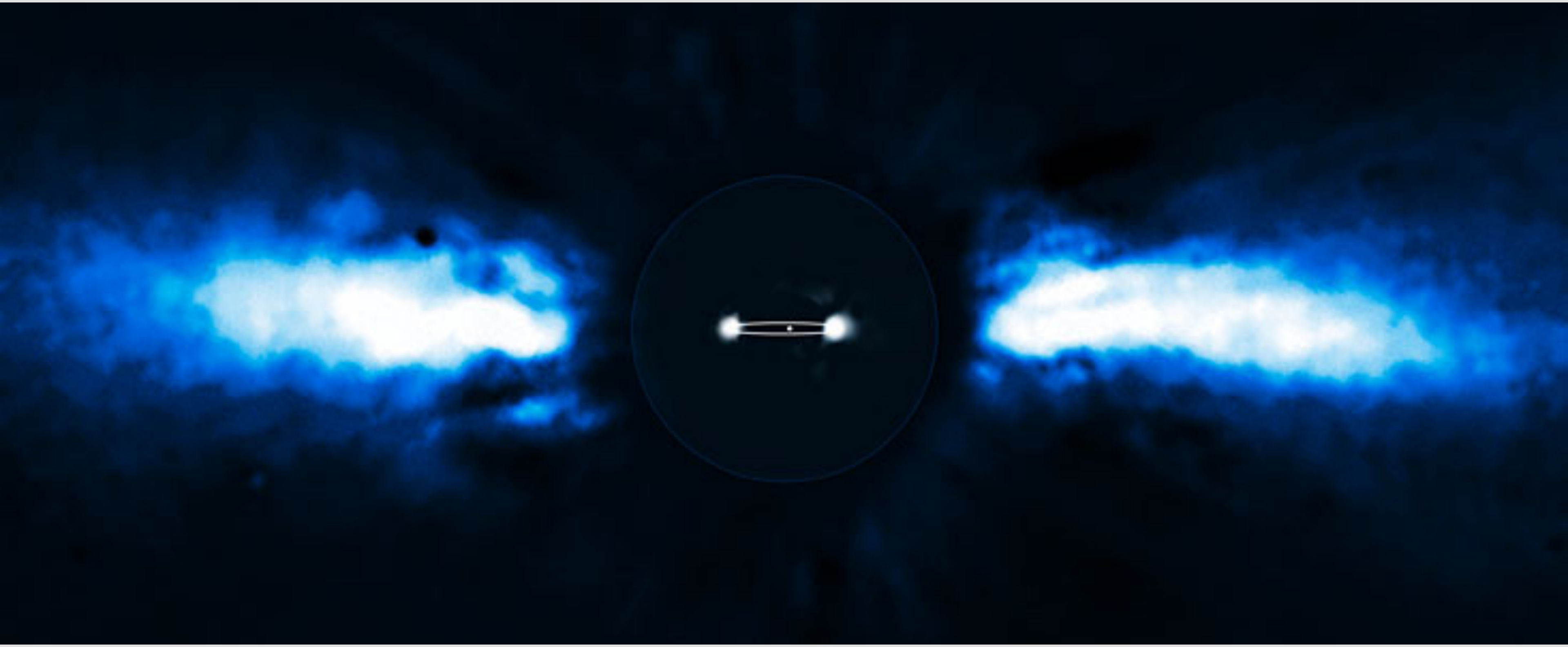
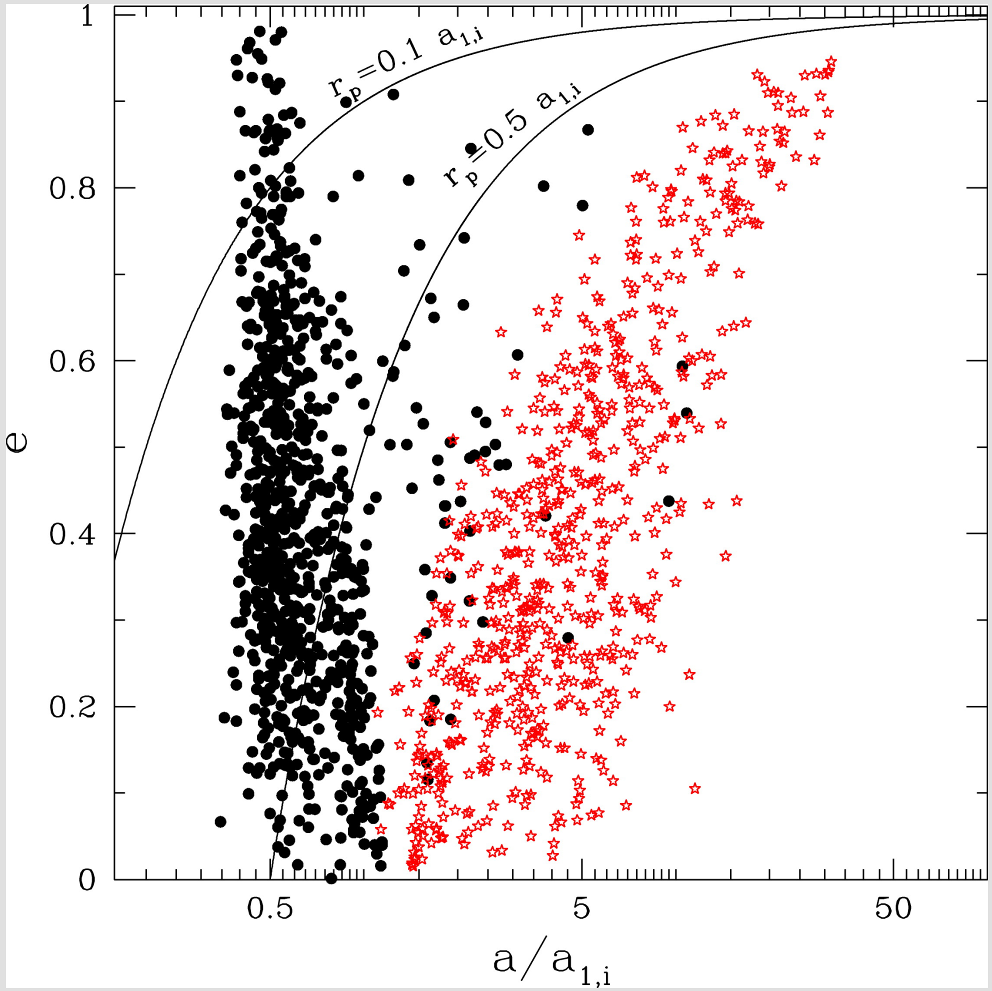
Understanding the orbital properties of directly imaged planets can provide powerful constraints on their evolution.



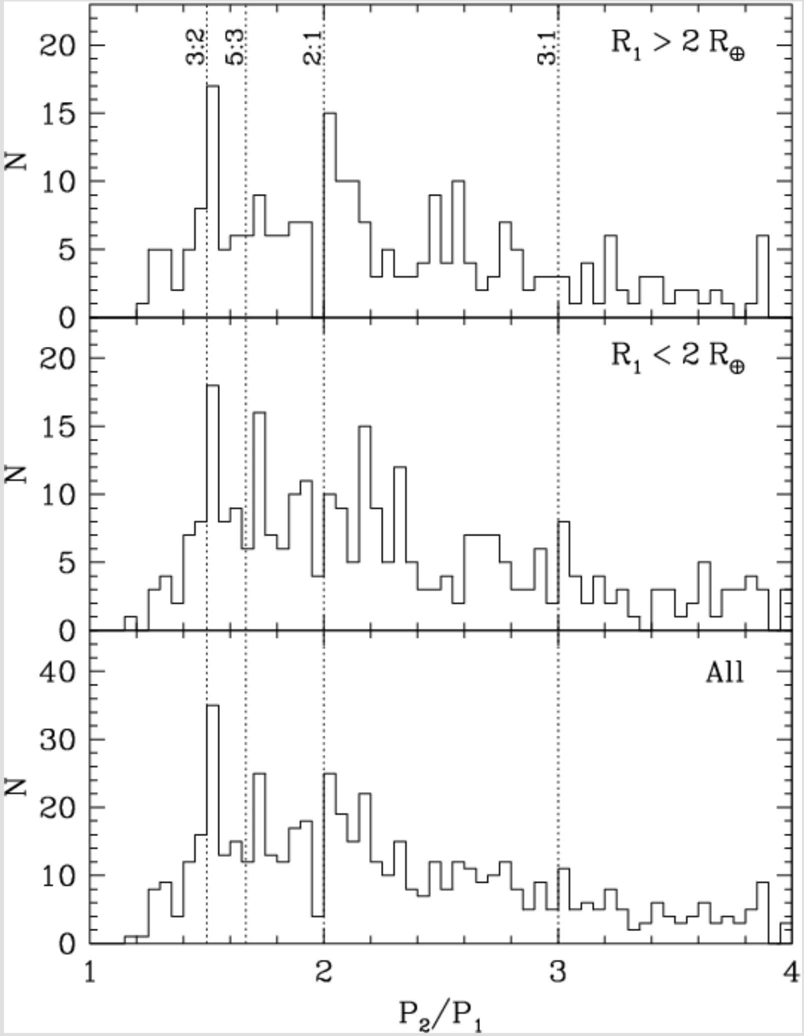
ESO

Orbital eccentricities can be used as a tracer of formation or subsequent dynamical processes.



Chatterjee et al. 2008

Identification of interesting dynamical features such as resonances could trace the early migration history of widely separated exoplanets.



Lee et al. 2013

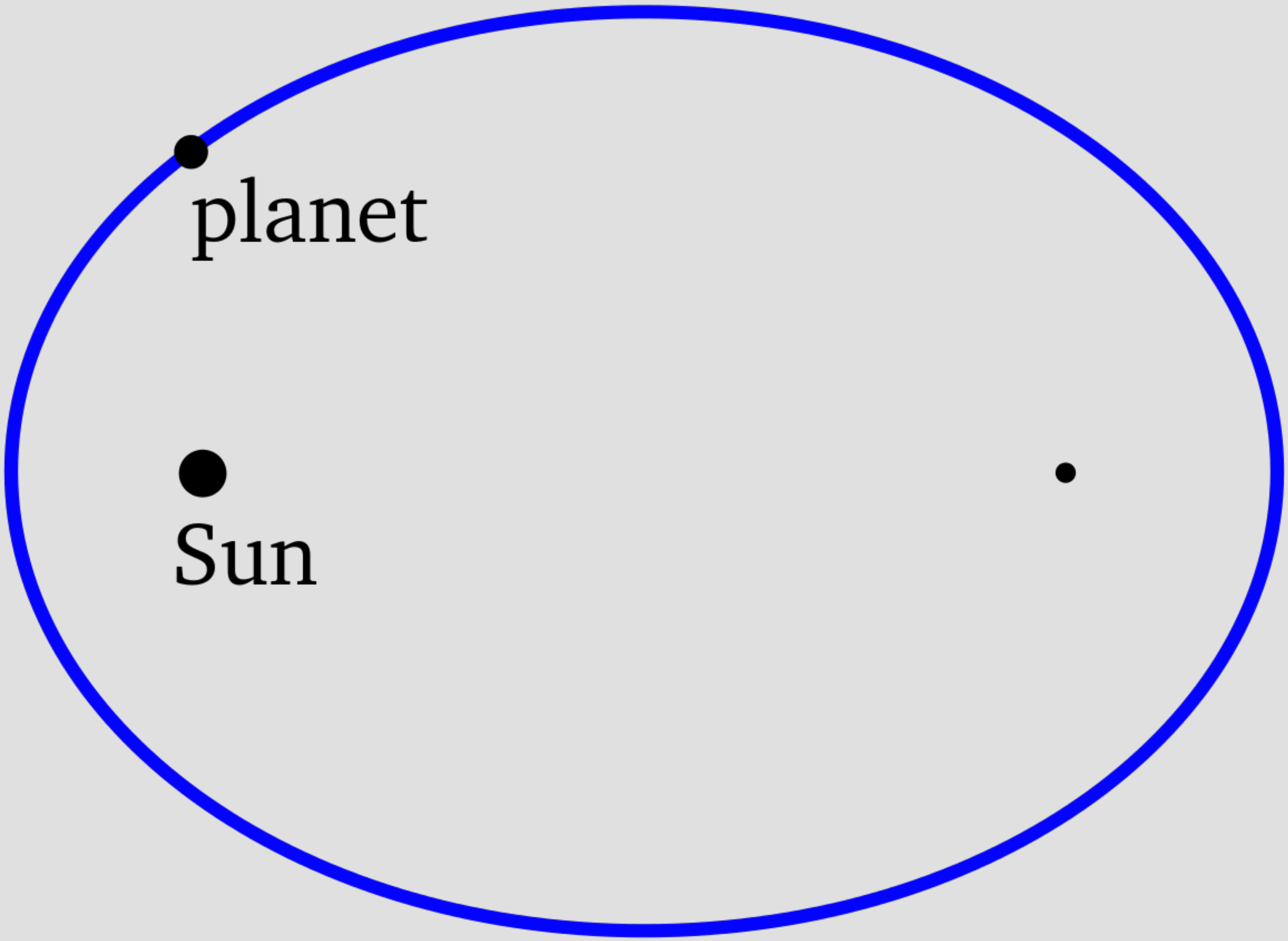
Orbit equations and derivations

References: "An Introduction to Close Binary Stars", R.W. Hilditch
"Spherical Astronomy", Robin M. Green

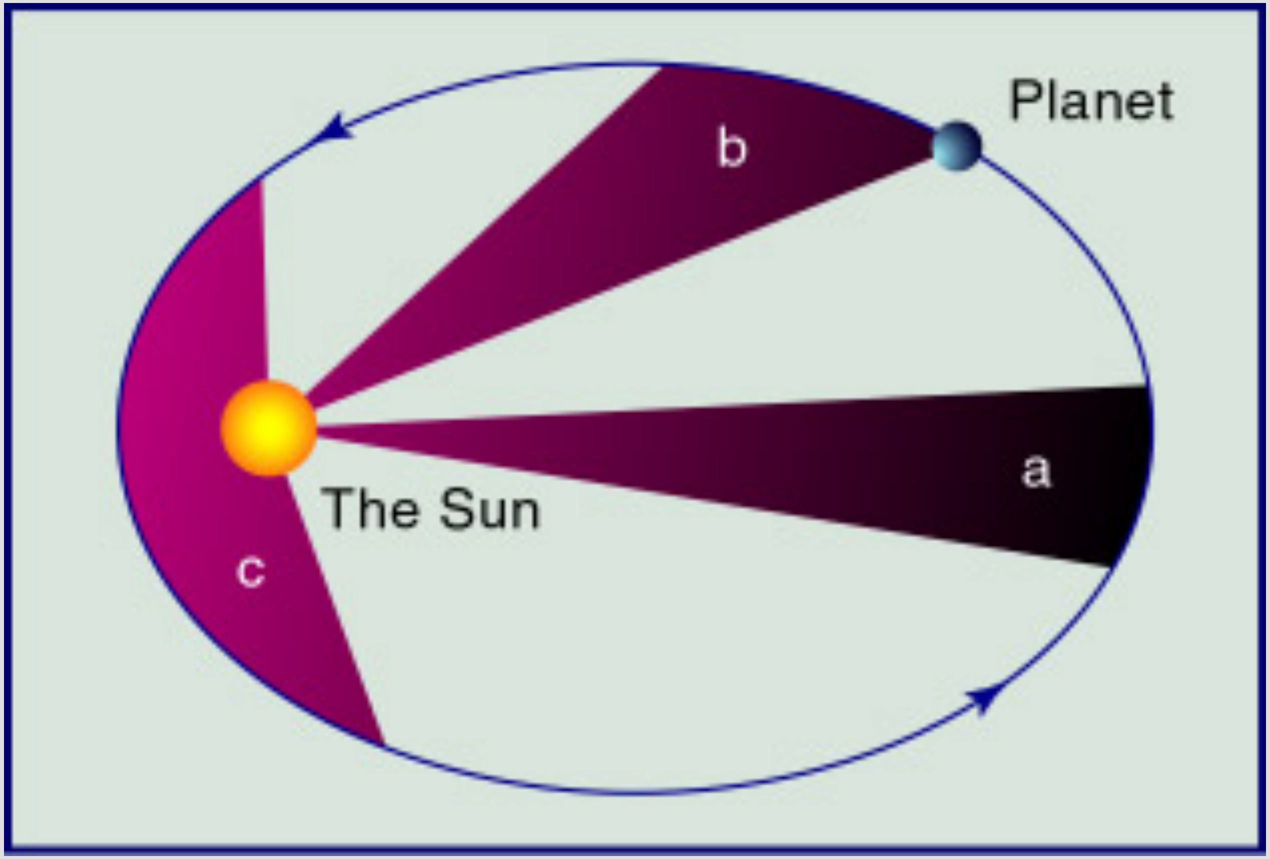
Some of the earliest analytic descriptions of orbits were derived by Johannes Kepler.

3 Laws of Planetary Motion: 1609 and 1619

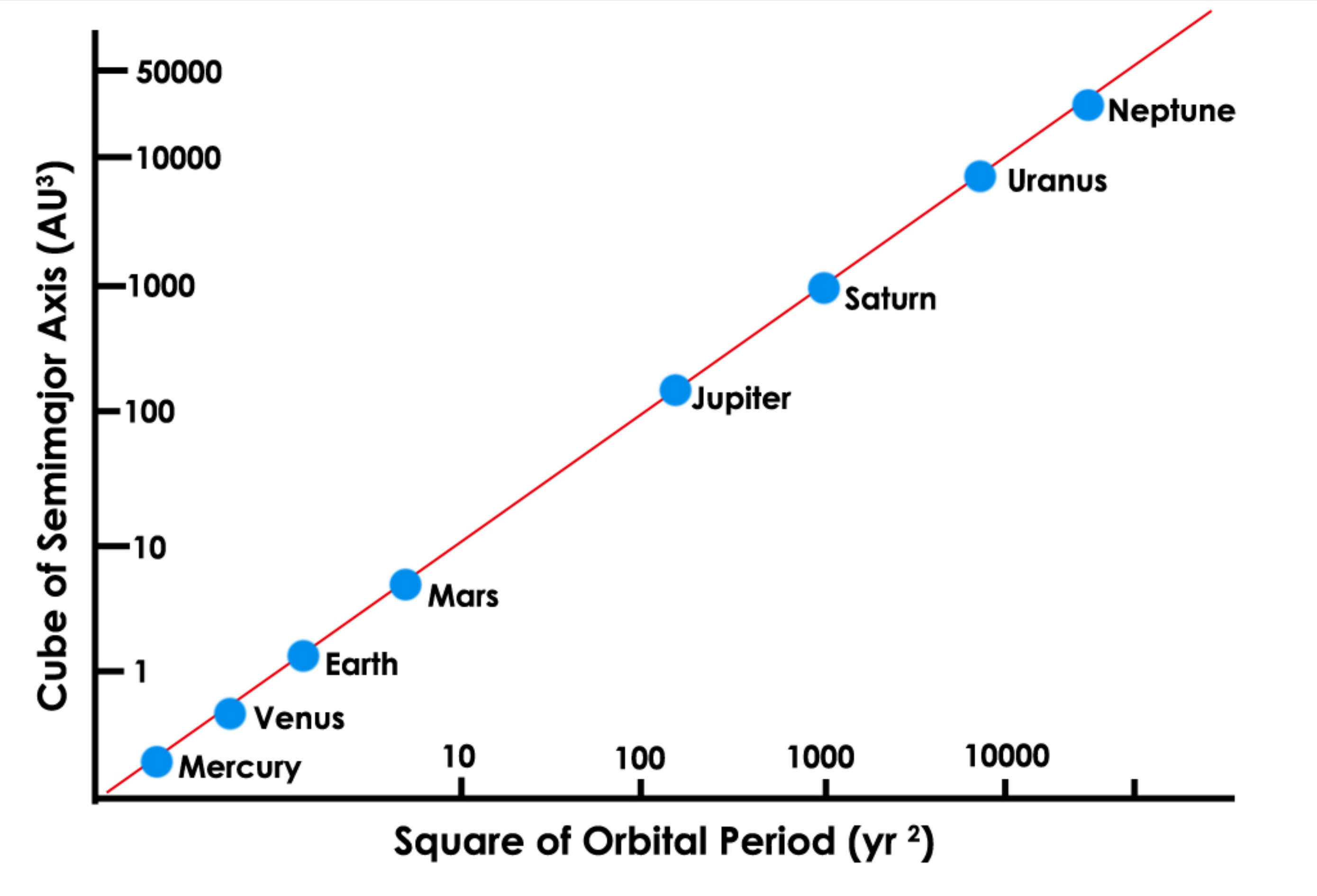
First Law: The orbit of a planet is an ellipse with the Sun at one focus.



Second Law: A line from the planet to the sun sweeps out equal areas in equal amounts of time.



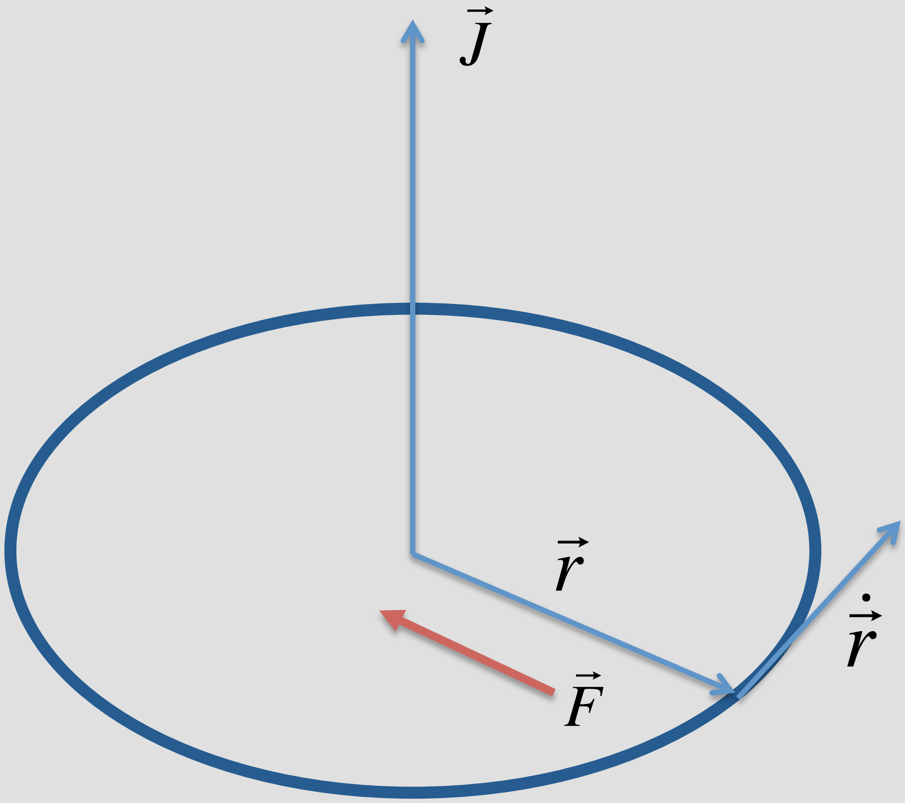
Third law: The period of a planet's orbit squared is proportional to its average distance from the Sun cubed.



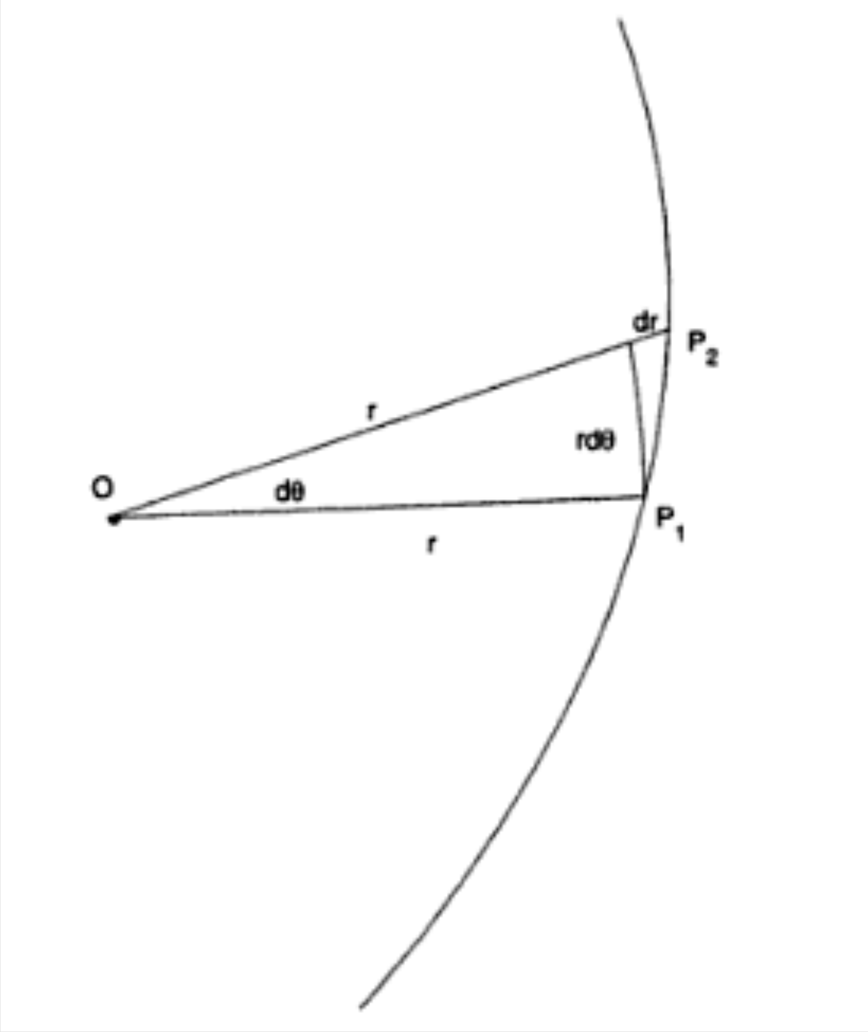
The "modern" formulation of orbital motion can be described by Newtonian gravitation.

$$\vec{J} = m\vec{r} \times \dot{\vec{r}} = const.$$

Angular Momentum



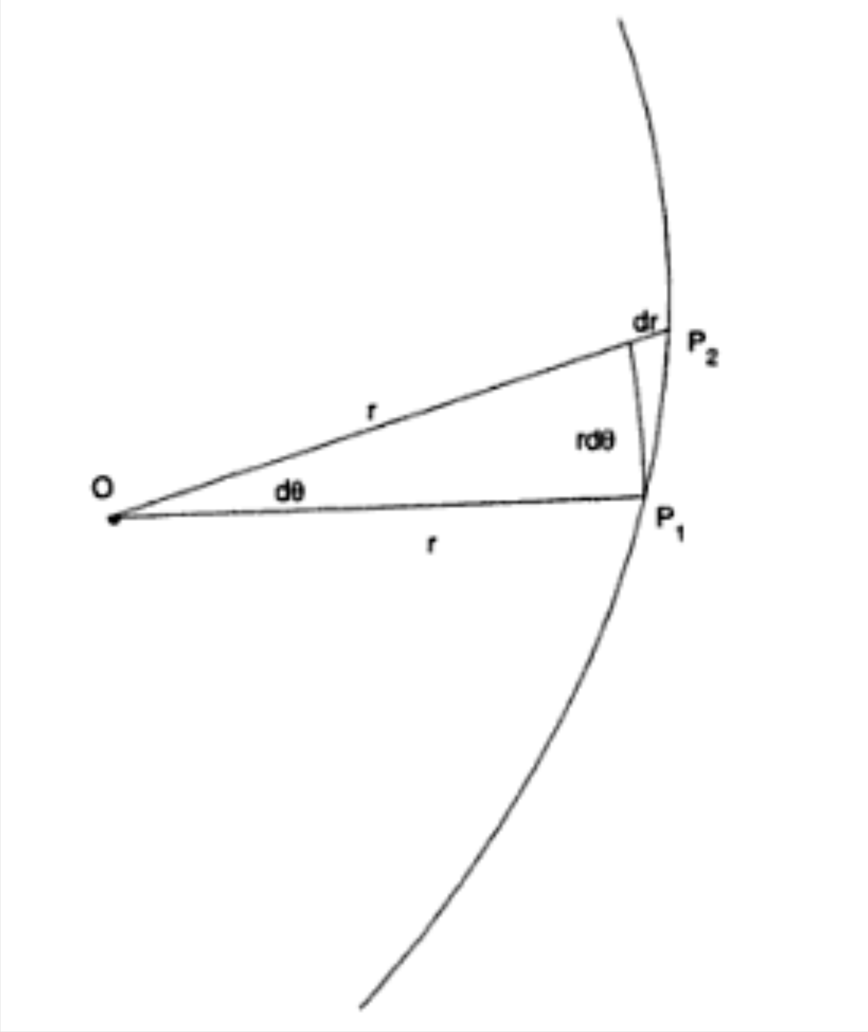
The motion of a body under the action of a central force.



$$r \rightarrow r + dr$$

$$\theta \rightarrow \theta + d\theta$$

Velocities can be separated into radial and transverse components.

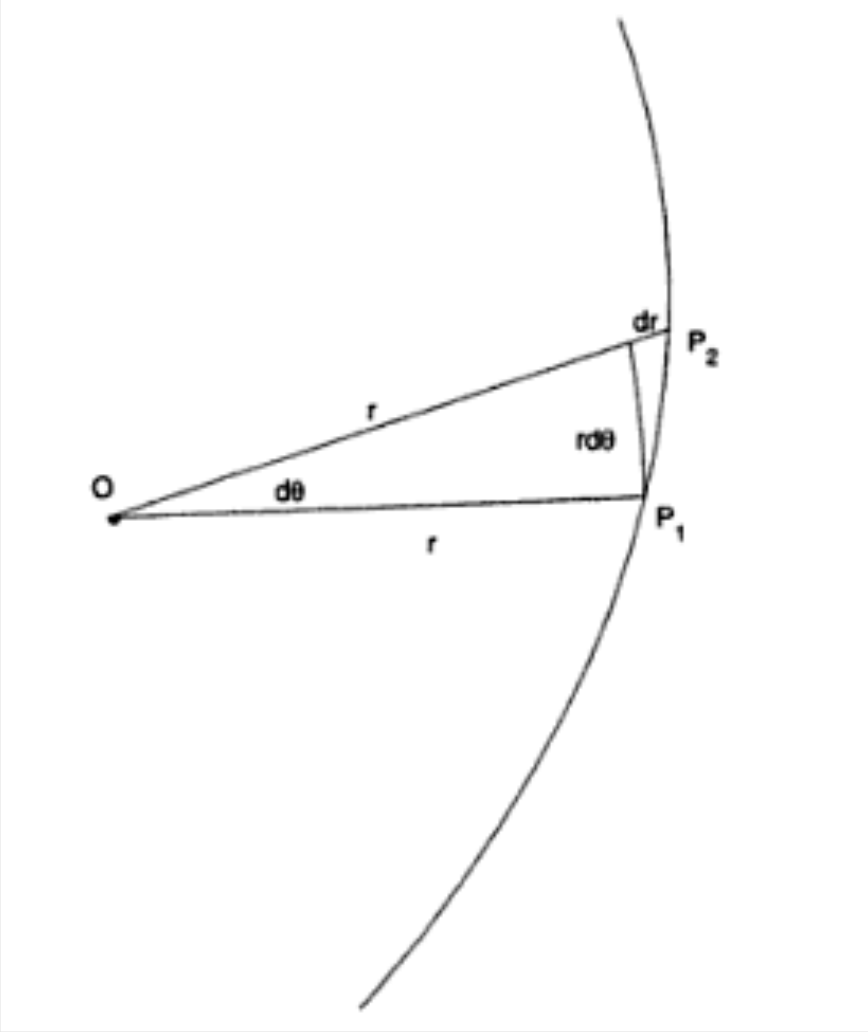


$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Orbital Speed

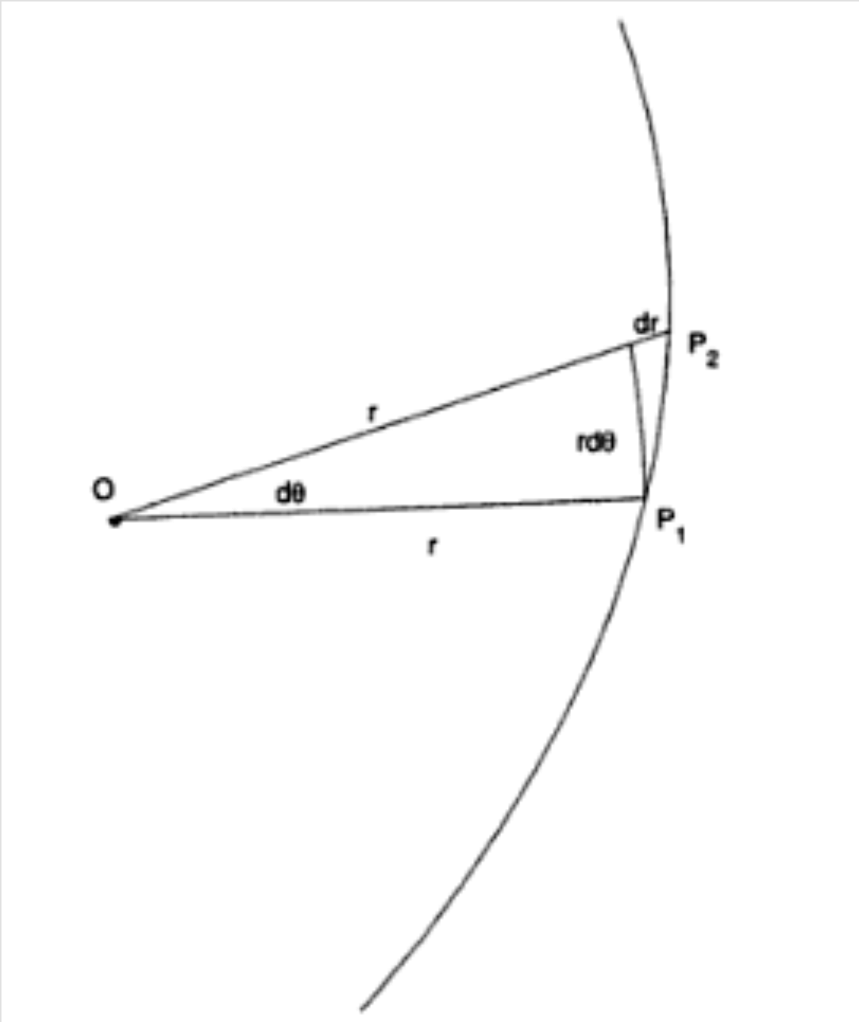
$$V^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

The components of acceleration and Kepler's second law.



$$\ddot{\vec{r}} = \underbrace{(\ddot{r} - r\dot{\theta}^2)\hat{r}}_{\text{Radial Component}} + \underbrace{(2r\dot{\theta} + r\ddot{\theta})\hat{\theta}}_{\text{Transverse Component}}$$

The components of acceleration and Kepler's second law.



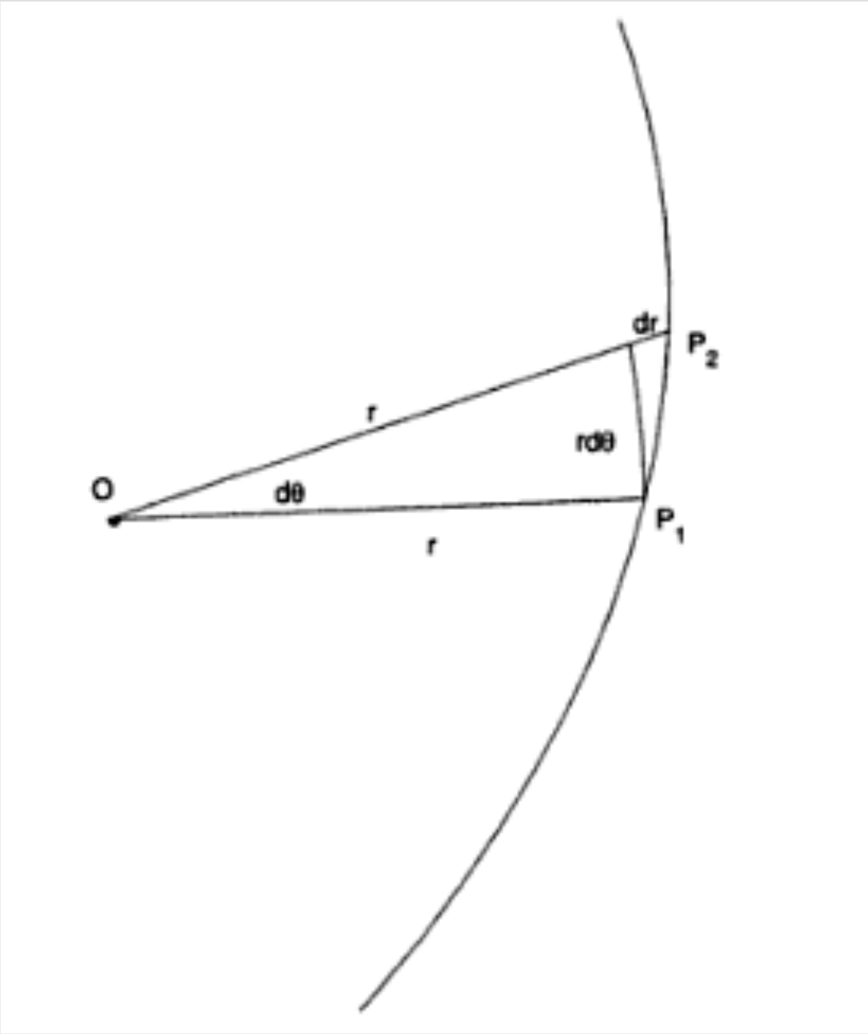
$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + \overset{0}{(2r\dot{\theta} + r\ddot{\theta})}\hat{\theta}$$

$$J = mr^2\dot{\theta} = const.$$

so

$$\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt} = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$$

The components of acceleration and Kepler's second law.



$$J = mr^2\dot{\theta} = const.$$

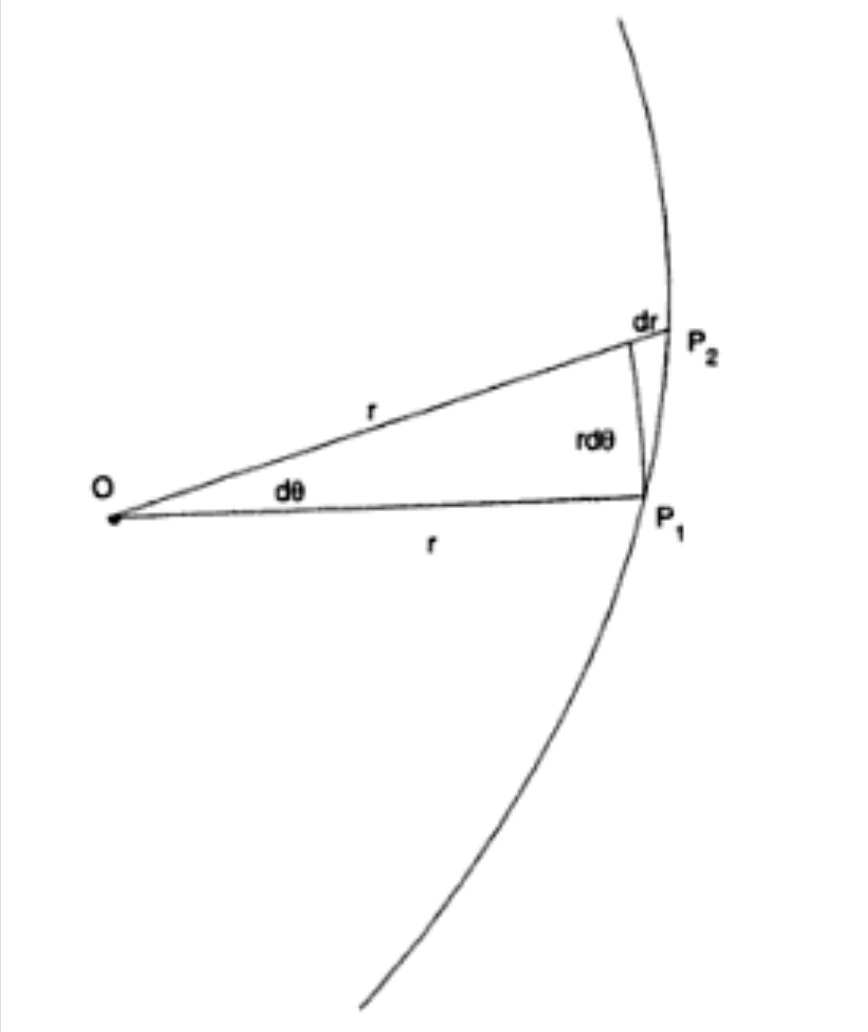
Area swept out by radius vector:

$$dA = \frac{1}{2} r^2 d\theta$$

so

$$\frac{dA}{dt} = \frac{1}{2} r^2 \dot{\theta} = \frac{J}{2m} = const.$$

Conservation of energy and the form of the orbit (Kepler's first law).



$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - G\frac{m_1 m_2}{r} = const.$$

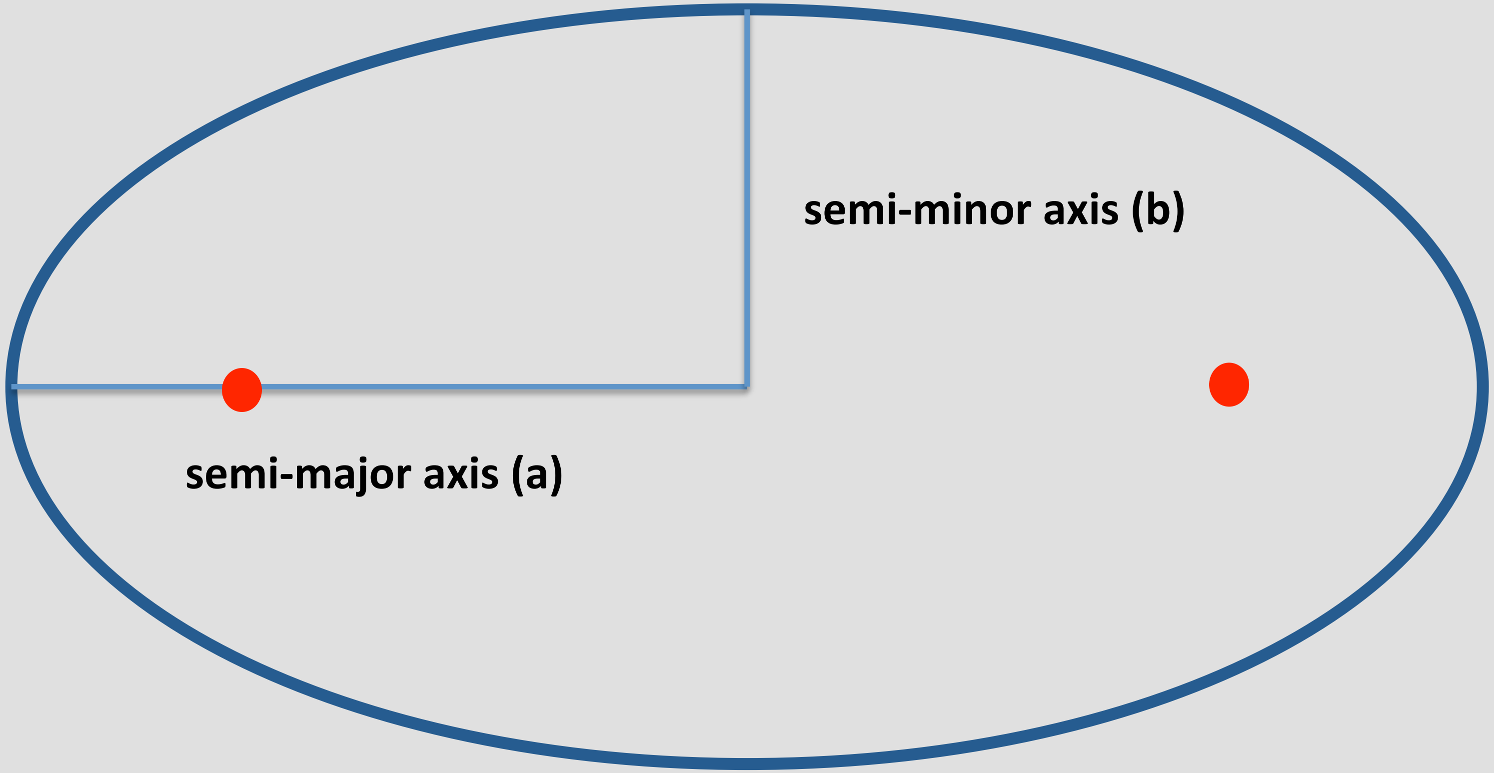
it can be shown that

$$r = \frac{l}{(1 + e\cos\theta)}$$

$e =$ **eccentricity (< 1)**

$l = \dfrac{J^2}{Gm_1 m_2} =$ **semi-lactus rectum**

Ellipses and Kepler's third law.

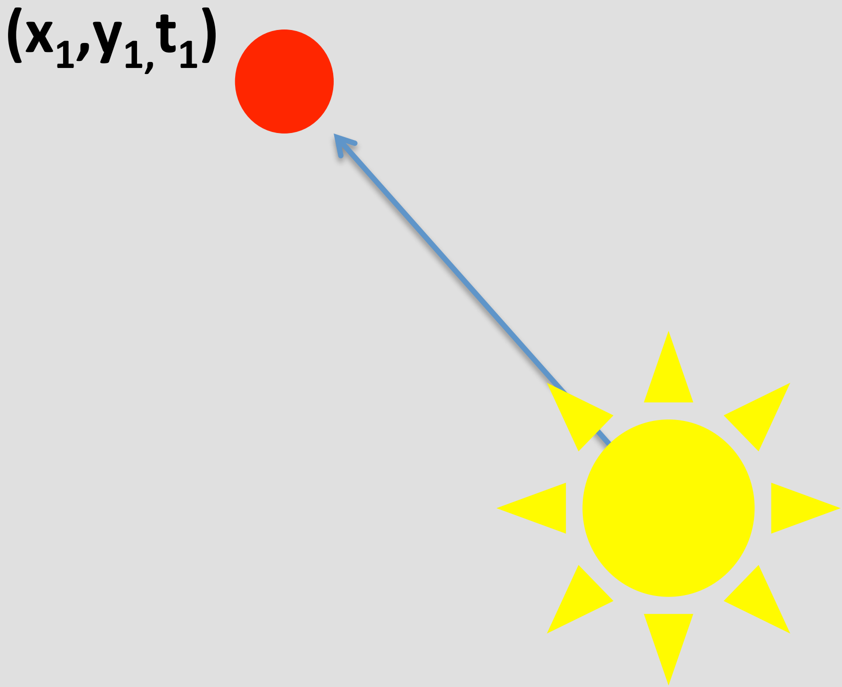


From the previous equations, it can be shown that

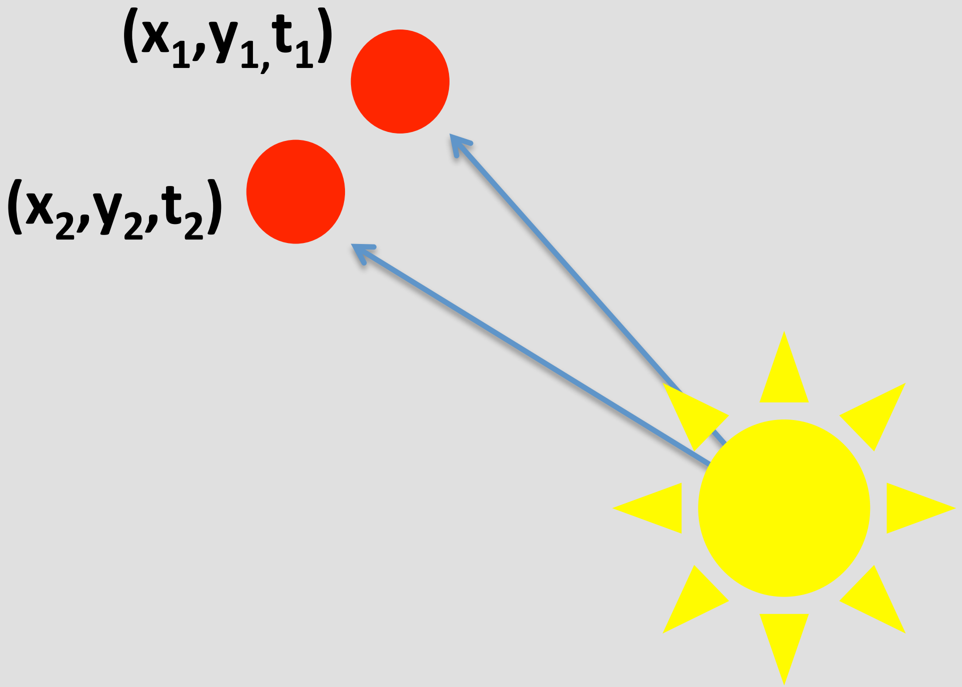
$$M = \frac{4\pi^2 a^3}{P^2}$$

Period

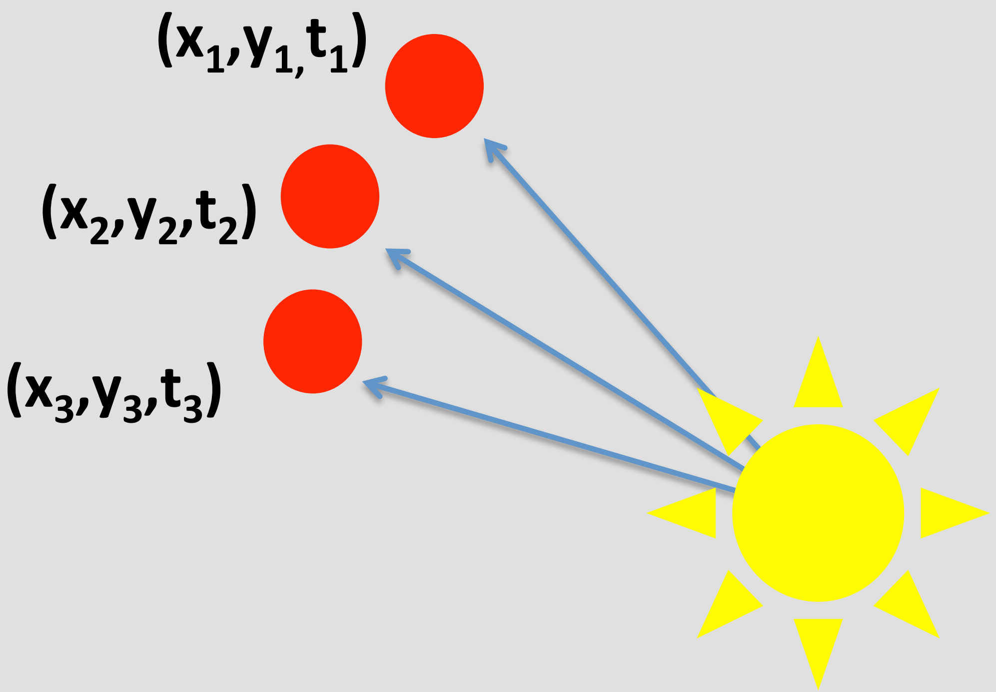
With direct imaging, what do we actually measure?



With direct imaging, what do we actually measure?

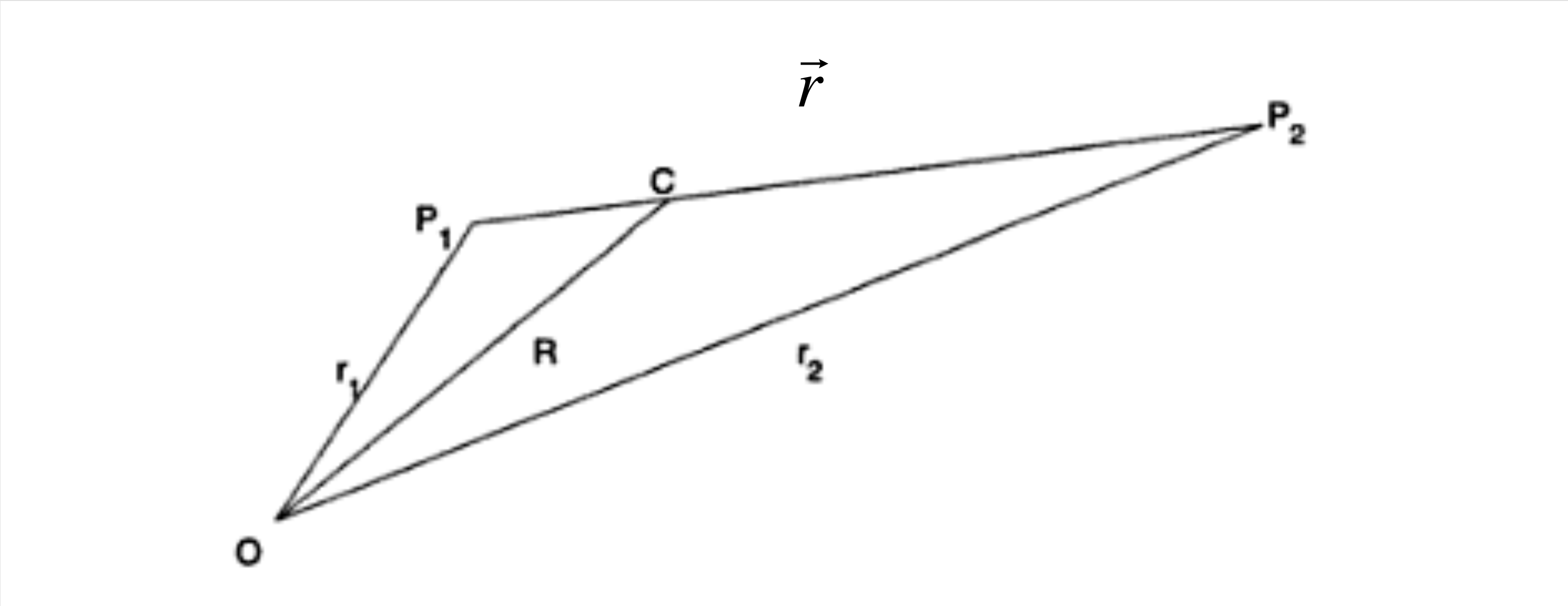


With direct imaging, what do we actually measure?



How do we convert from these measured quantities to orbits?

The two body problem and visual doubles.

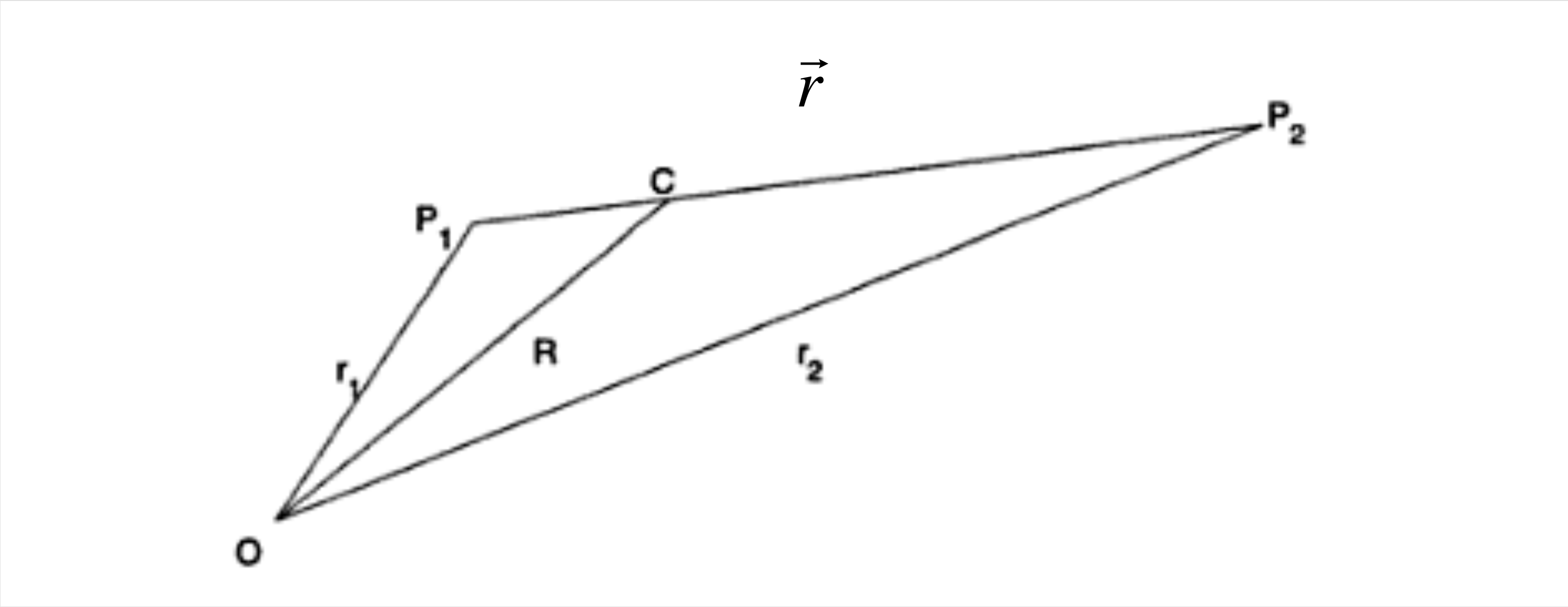


$$m_1\ddot{\vec{r}}_1 = -\frac{Gm_1m_2}{r^2}\hat{r}$$

and

$$m_2\ddot{\vec{r}}_2 = -\frac{Gm_1m_2}{r^2}(-\hat{r})$$

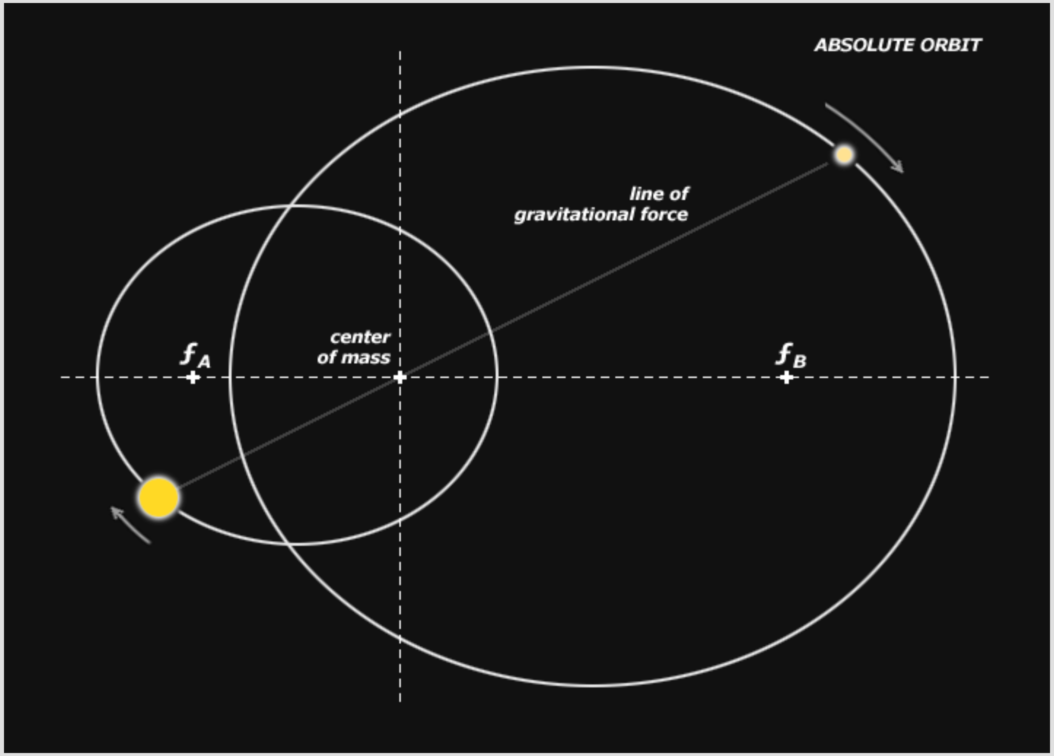
The two body problem and visual doubles.



$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = -\frac{G(m_1 + m_2)}{r^2}\hat{r} = \ddot{\vec{r}}$$

Relative orbits provide only the total system mass

How do barycentric and relative orbits relate to each other?



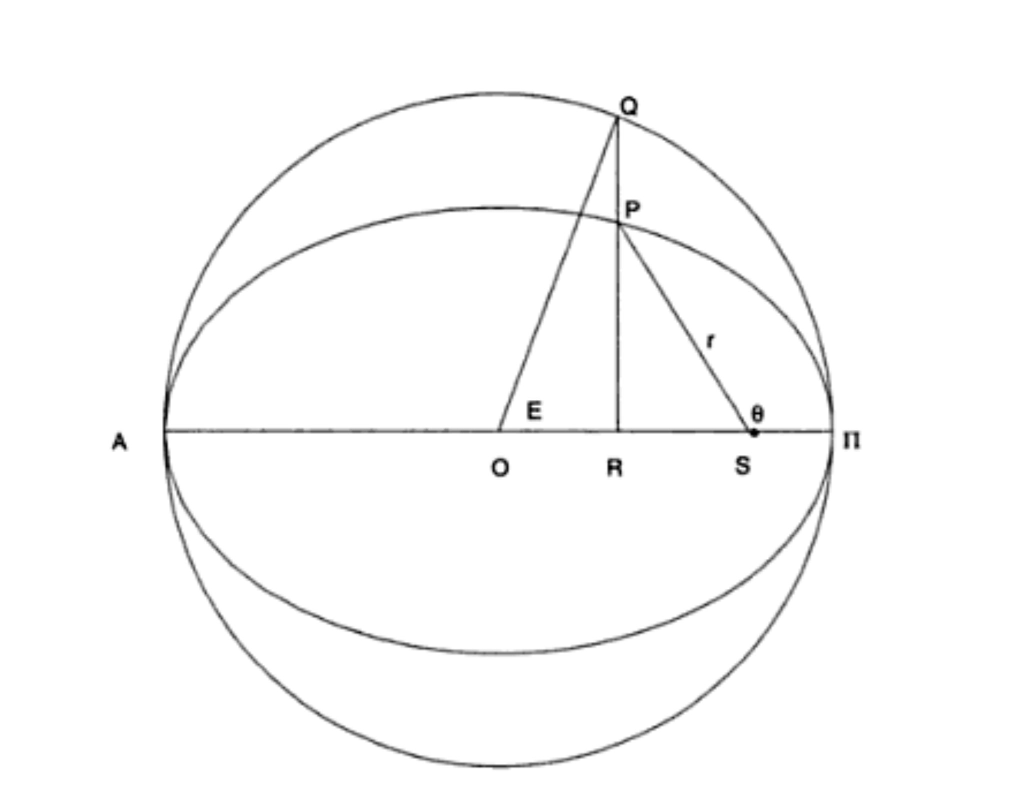
Bruce MacEvoy

$$a_{rel} = a_1 + a_2$$

$$e_{rel} = e_1 = e_2$$

$$a_1 : a_2 : a_{rel} =$$
$$m_2 : m_1 : (m_1 + m_2)$$

Important terms and angles for relative orbits.



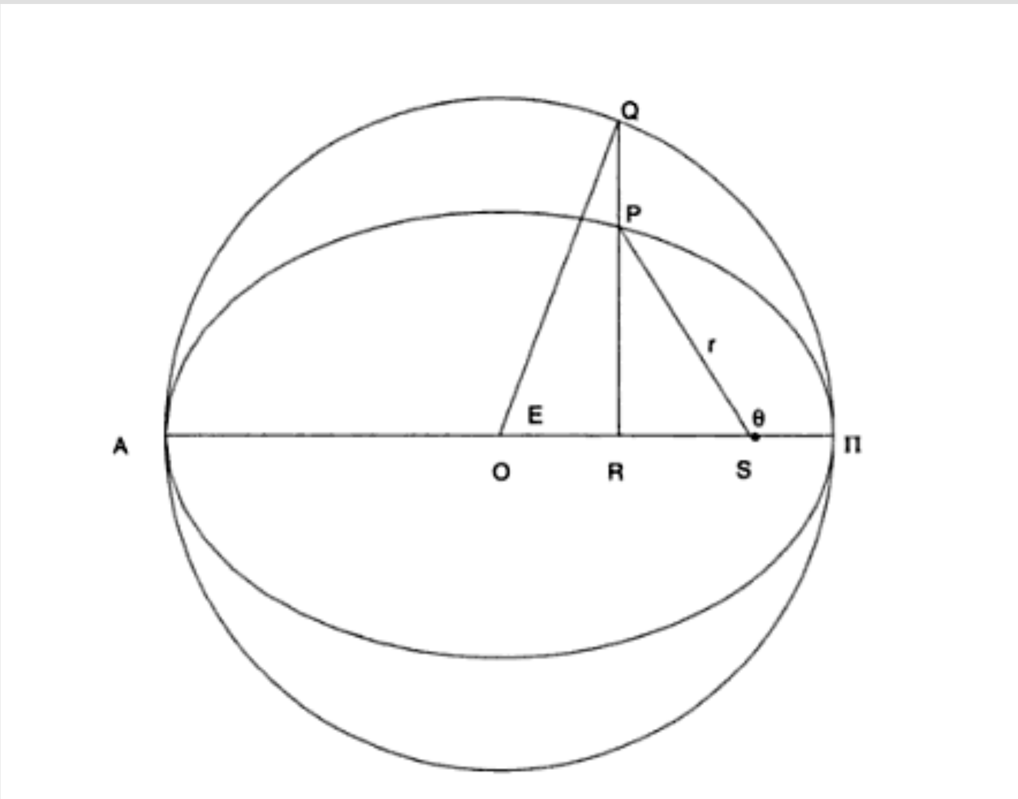
Θ = true anomaly

E = eccentric anomaly

periastron = closest distance between orbiting pair

apoastron = furthest distance between orbiting pair

Important terms and angles for relative orbits.



$$\tan\frac{\theta}{2} = \left[\frac{(1+e)}{(1-e)}\right]^{\frac{1}{2}} \tan\frac{E}{2}$$

$$r = a(1 - e\cos E)$$

so

$$r_p = a(1-e)$$

$$r_a = a(1+e)$$

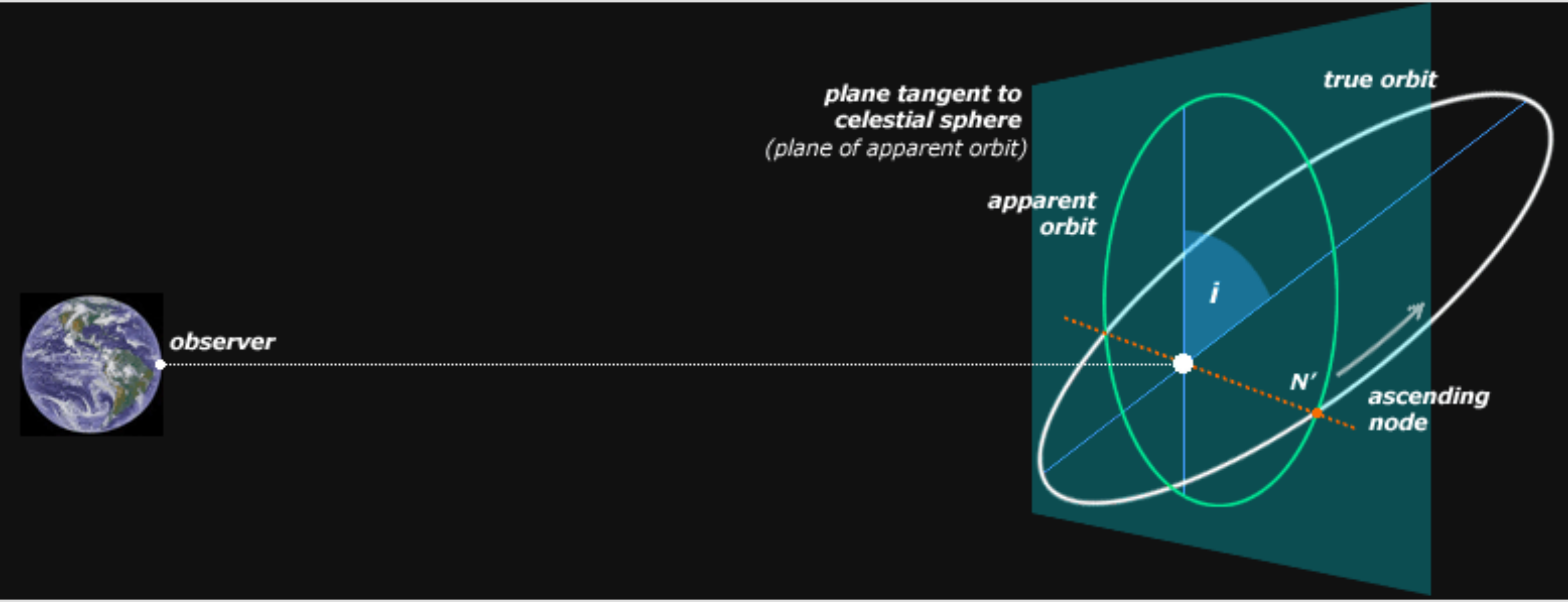
The eccentric anomaly can be solved for using Kepler's equation.

$$E - e\sin E = \frac{2\pi}{P}(t - T)$$

T = time of periastron passage

Kepler's equation is a transcendental equation (no analytic solution) that must be solved numerically (i.e., with a Newton-Raphson iterative solution)

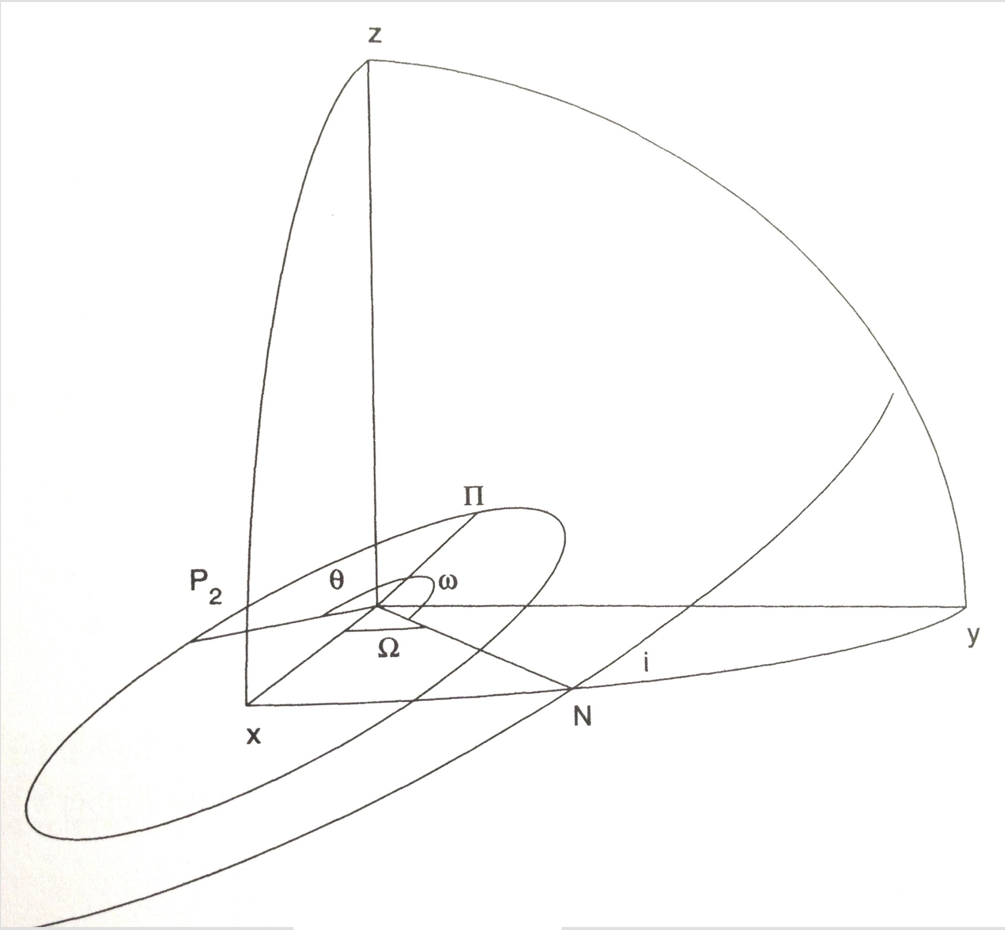
Equations for projected visual binary orbits.

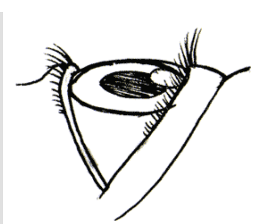


i = inclination

N = ascending node = where orbit plane intersects plane of the sky

Equations for projected visual binary orbits.





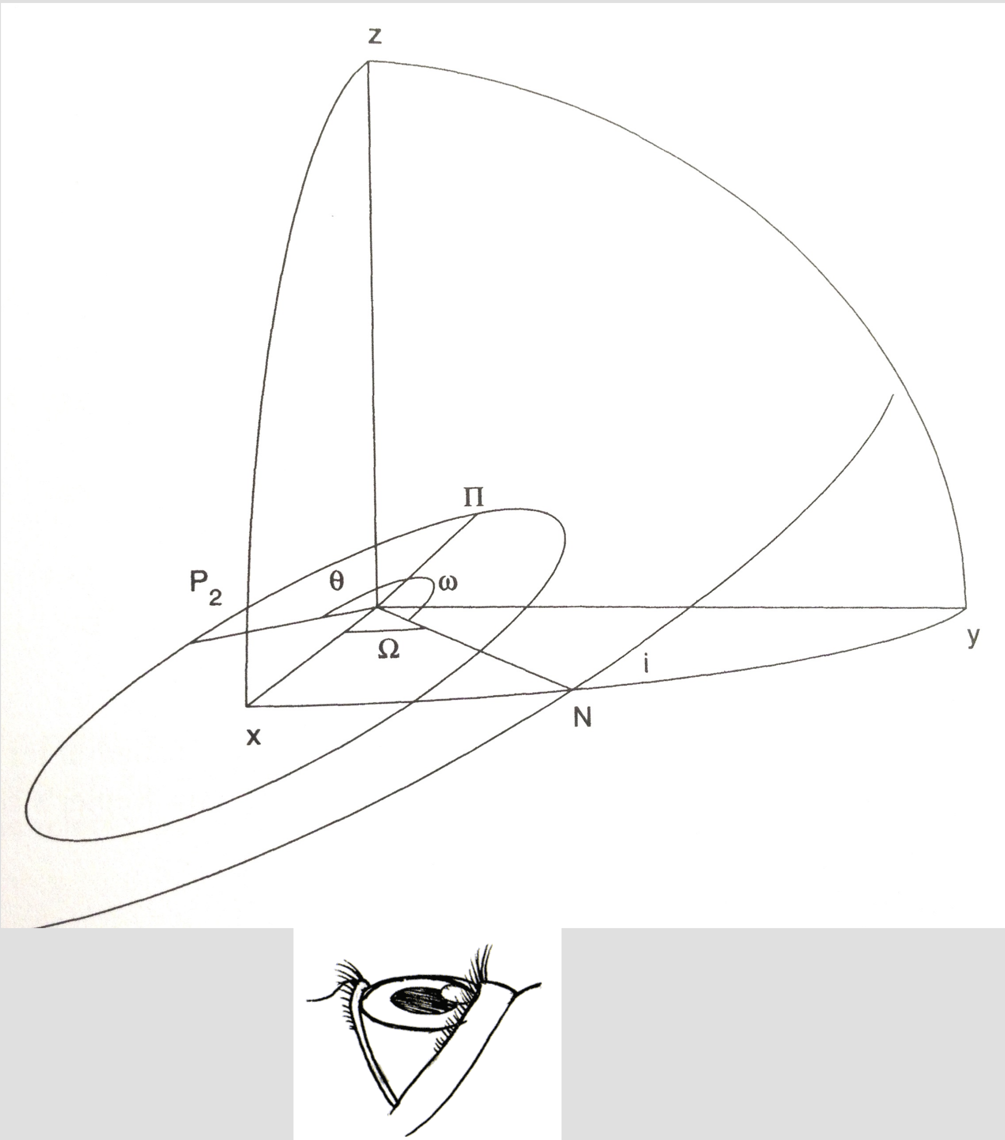
i = inclination

Ω = longitude of the ascending node

ω = longitude of periastron passage

With semi-major axis (a), these orbital elements are termed the "geometrical elements"

Equations for projected visual binary orbits.



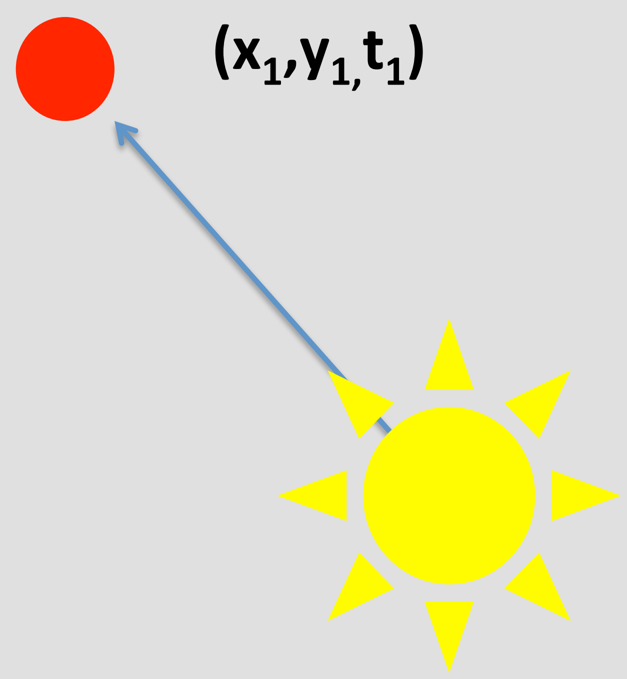
Dynamical Orbit Elements

P = period

T = time of periastron passage

e = eccentricity

How do we relate the measured positions as a function of time to the orbital elements?



Projecting the orbit onto the three coordinate axes.

$$x = r\left[\cos\Omega\cos(\theta+\omega) - \sin\Omega\sin(\theta+\omega)\cos i\right]$$

$$y = r\left[\sin\Omega\cos(\theta+\omega) + \cos\Omega\sin(\theta+\omega)\cos i\right]$$

$$z = r\left[\sin(\theta+\omega)\sin i\right]$$

The Thiele-Innes constants and elliptical rectangular coordinates.

Geometrical Elements

$$A = a\left[\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i\right]$$

$$B = a\left[\sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i\right]$$

$$F = a\left[-\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i\right]$$

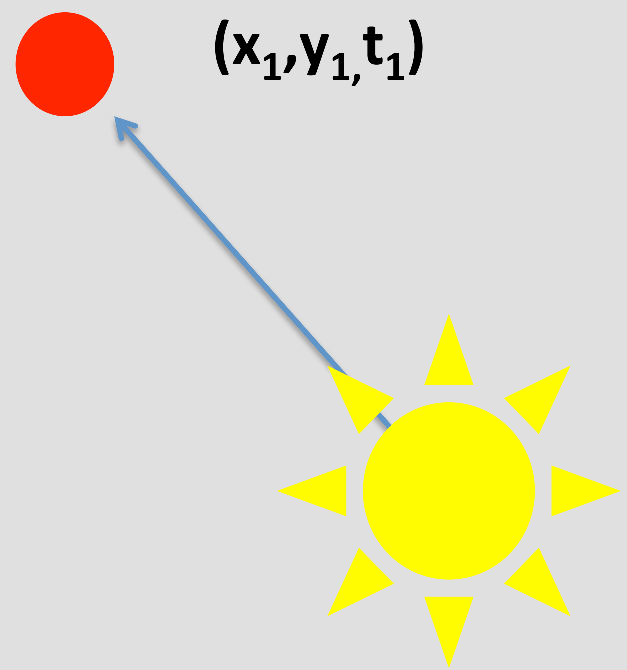
$$G = a\left[-\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i\right]$$

Dynamical Elements

$$X = \cos E - e$$

$$Y = (1 - e^2)^{\frac{1}{2}} \sin E$$

Now x and y positions are a simple function these constants and coordinates.



$$x = BX + GY$$

$$y = AX + FY$$

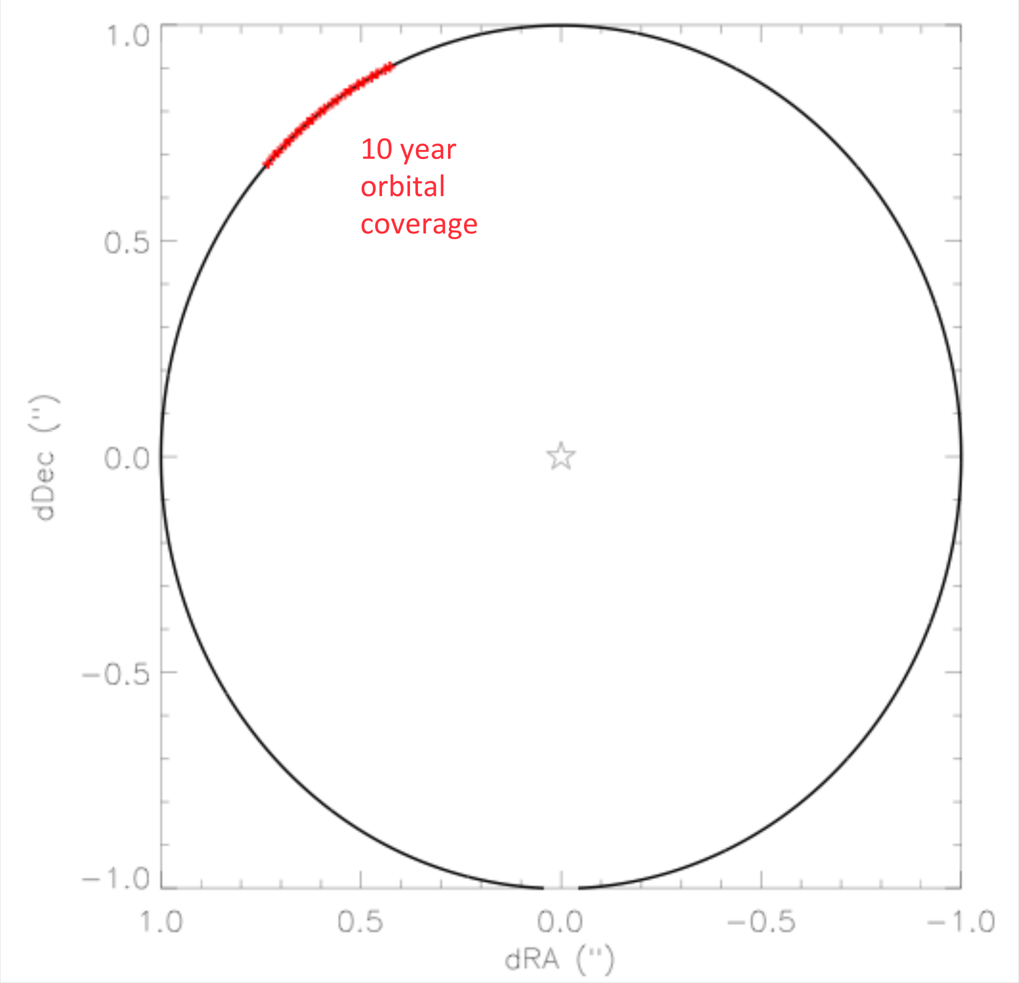
There are several methods of solving for orbits from astrometry.

- X^2 minimization
- X^2 minimization + Monte Carlo
- Markov Chain Monte Carlo
- MultiNest
- Others

Orbits in Action!

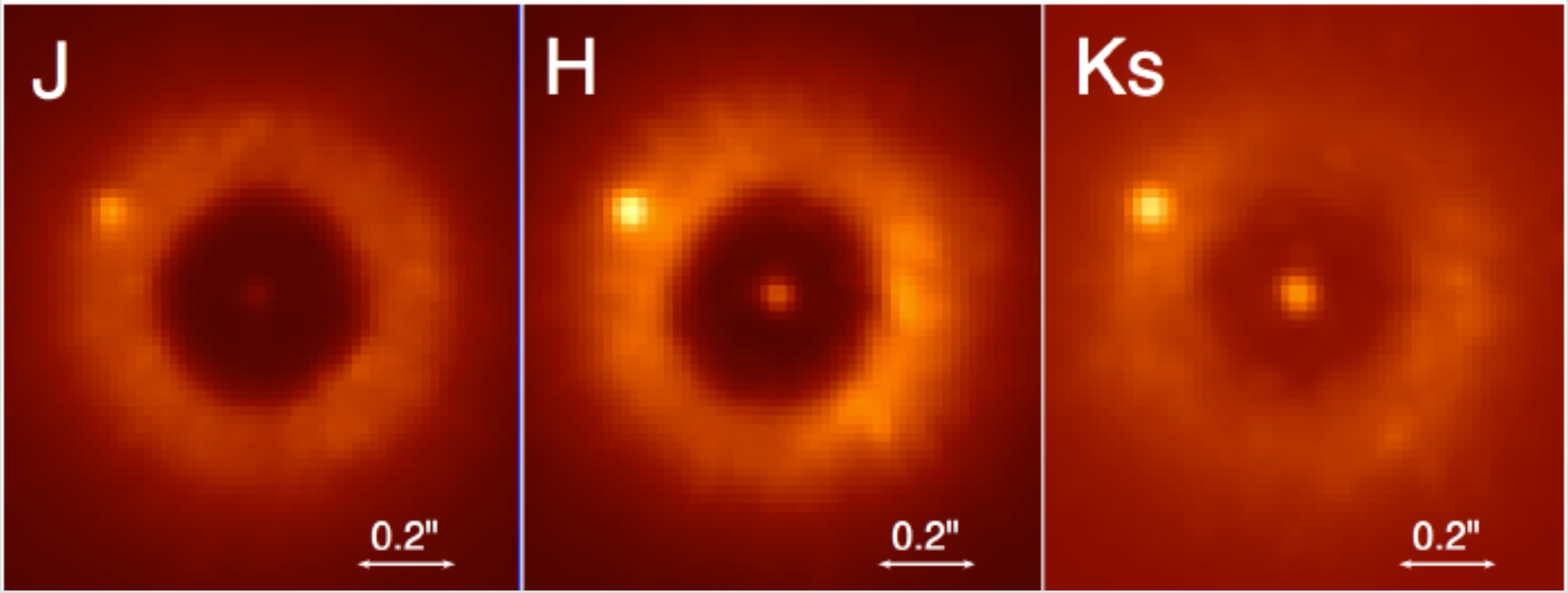
For directly imaged companions, in general the orbital phase coverage is tiny.

Generally, "best fit" orbits themselves are not interesting, but rather the *distribution* of orbital parameters for long period binaries



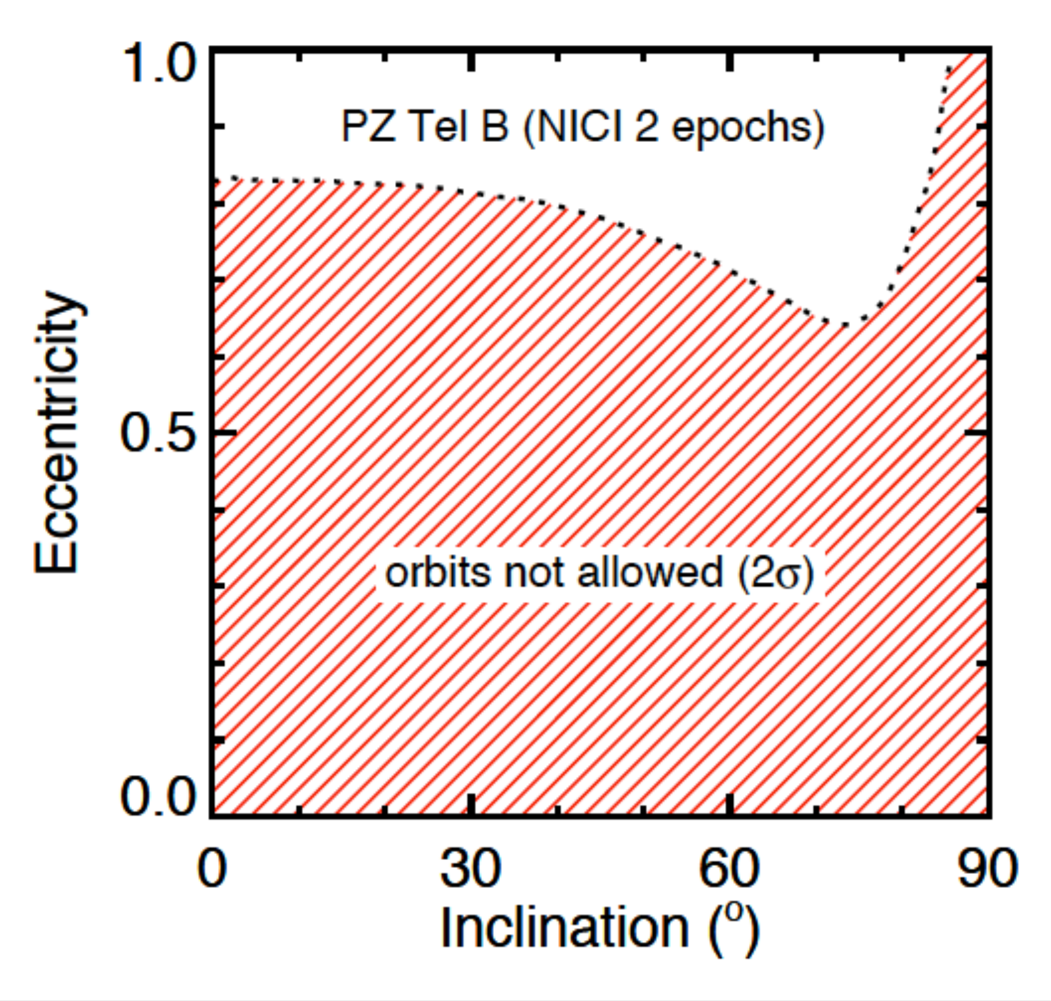
Circular, face on orbit with a=30 AU and distance=30 pc

Example – PZ Tel



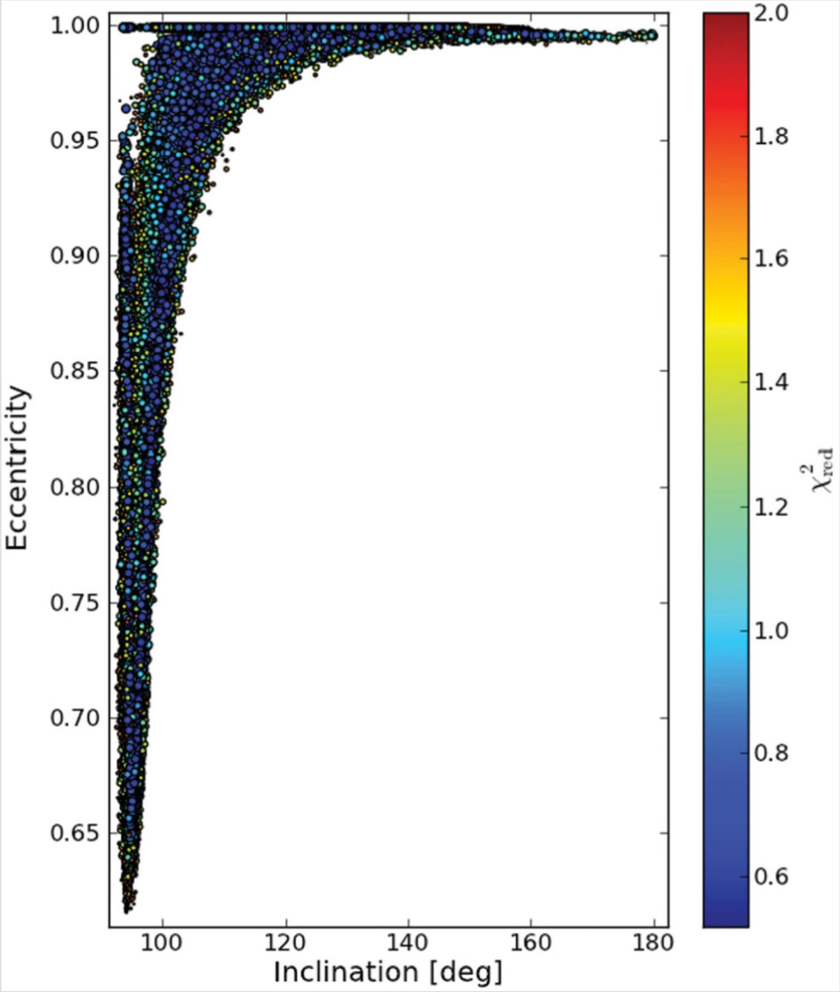
NICI; Biller et al. 2010

The orbital eccentricity of PZ Tel was constrained from only two measurements.



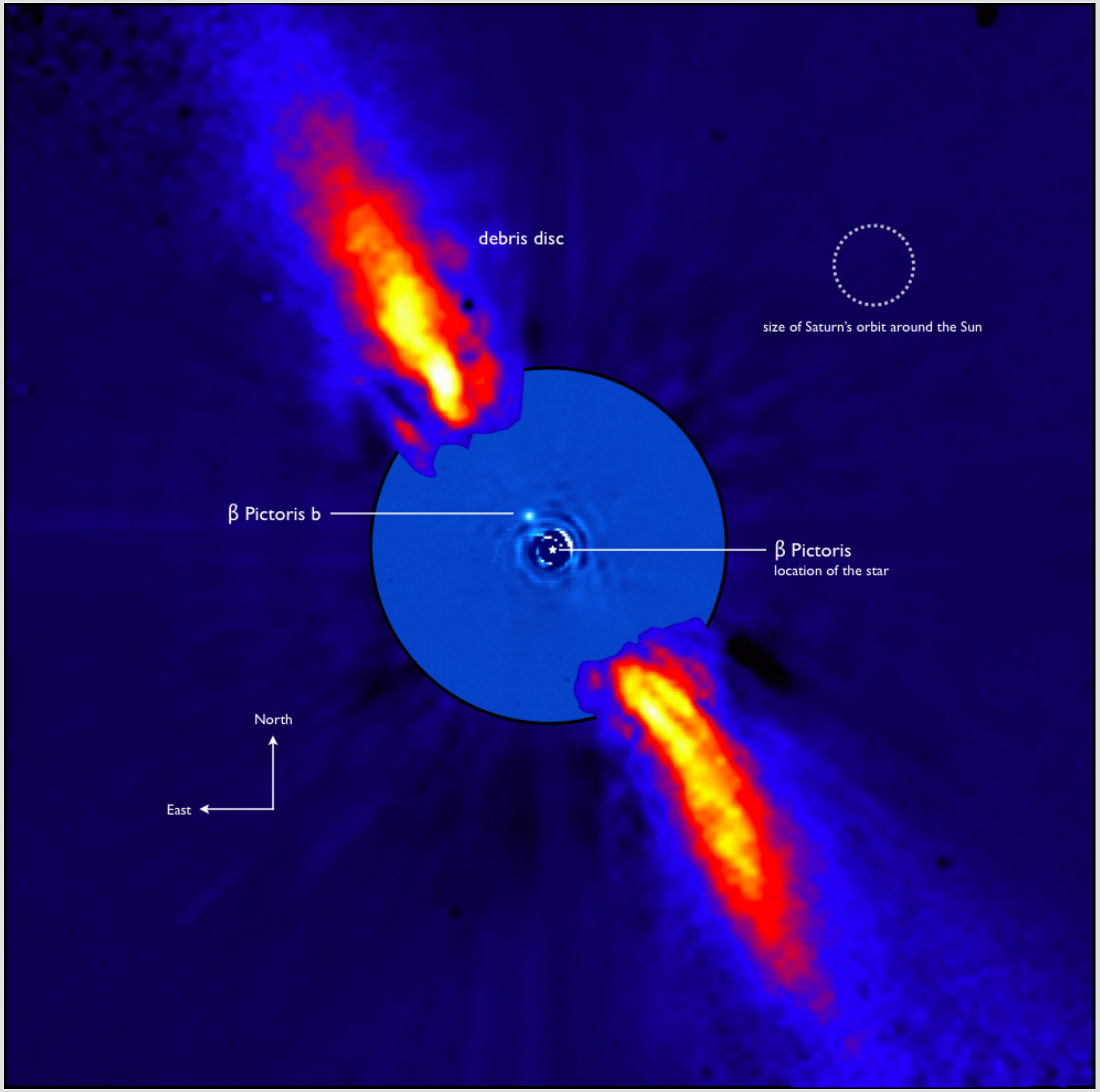
Biller et al. 2010

Additional observations with NACO have tightened eccentricity constraints.



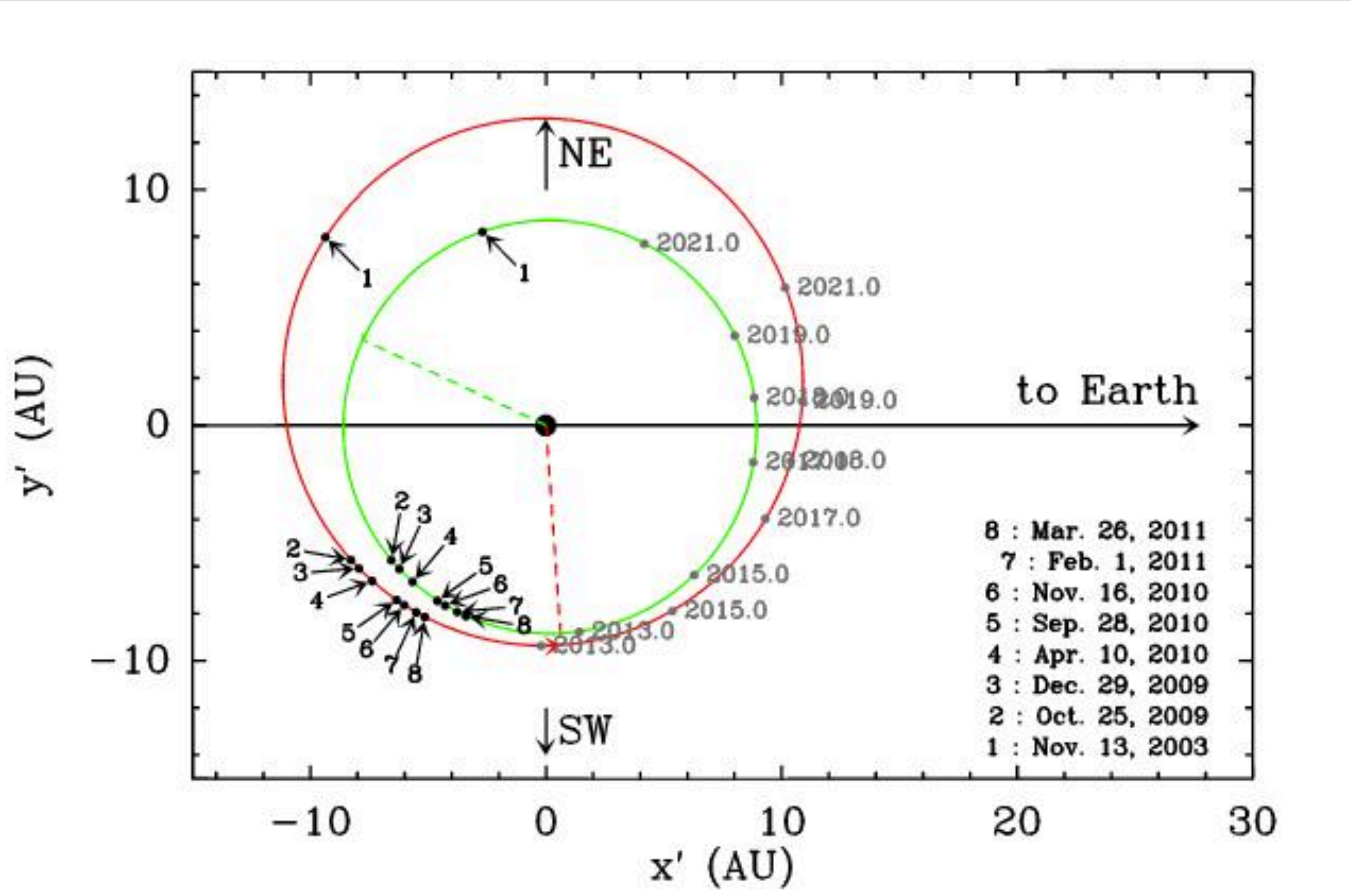
Mugrauer et al. 2012

Example - Beta Pic



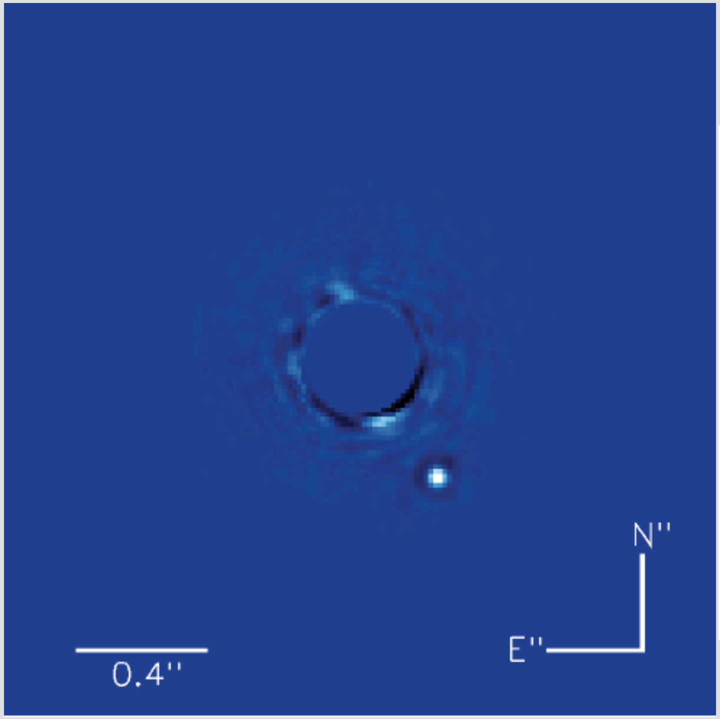
NACO; Lagrange et al. 2009

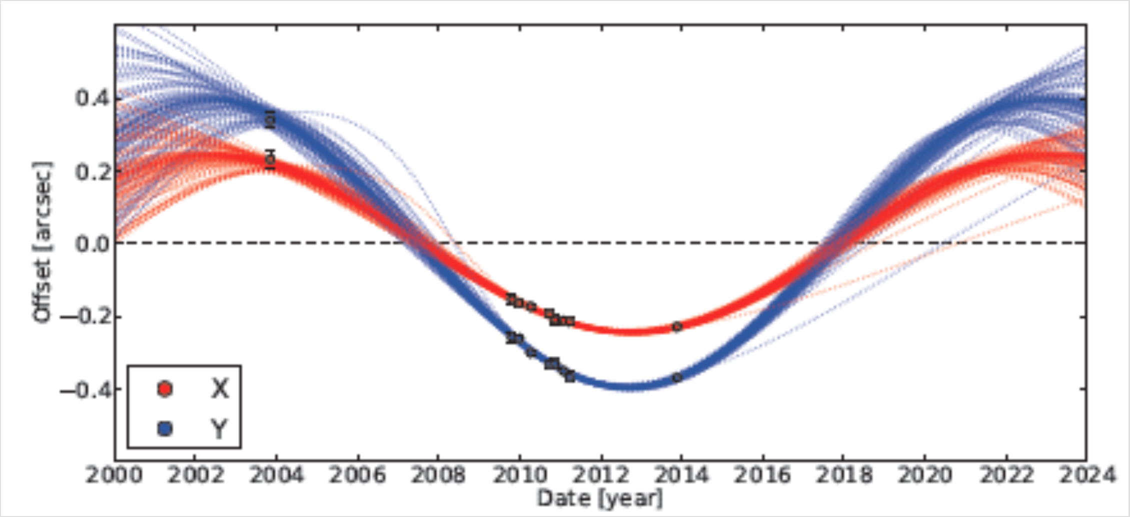
Beta Pic has a near-edge on orientation.



NACO; Chauvin et al. 2012

New observations show Beta Pic b has a 4% probability of transit in 2017.



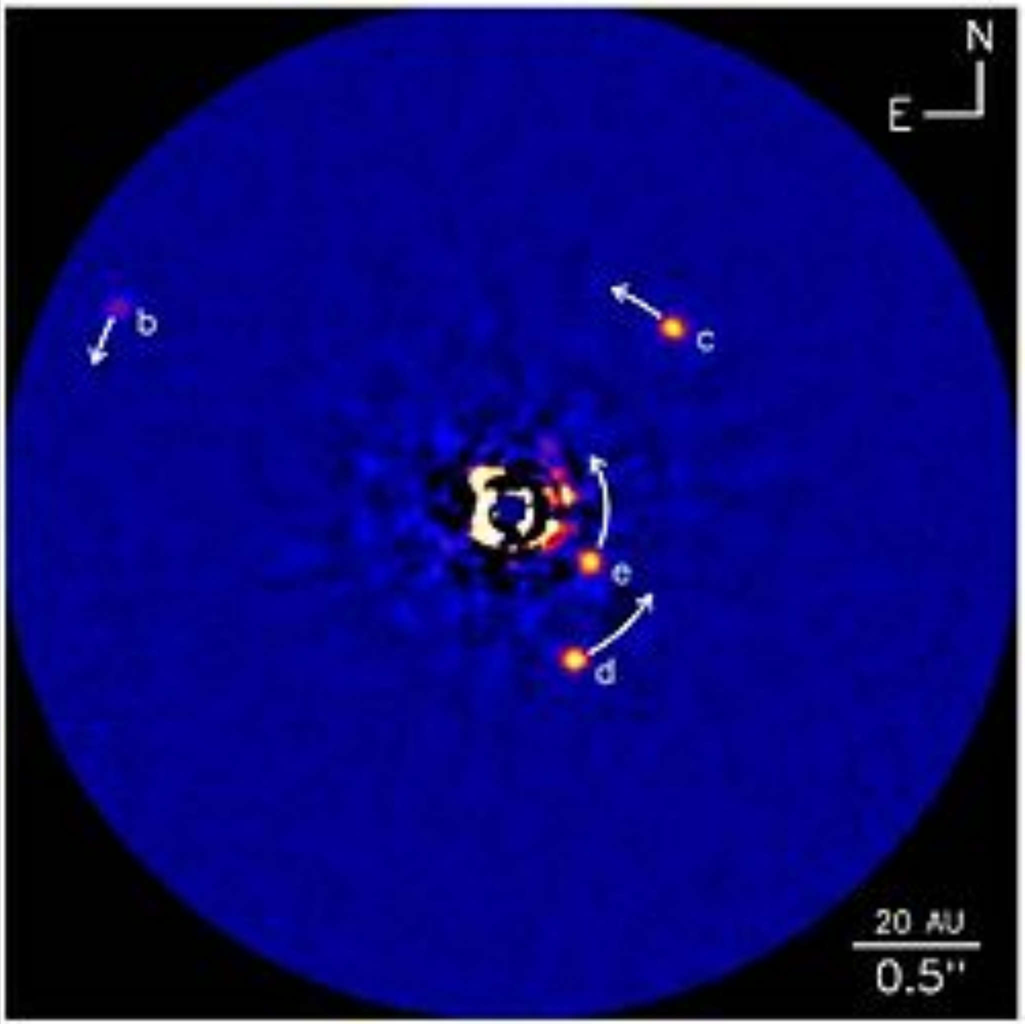


GPI; Macintosh et al. 2014

Example – HR 8799

Age ~ 30 Myr

Masses:
b ~ 5 M_{jup}
c ~ 7 M_{jup}
d ~ 7 M_{jup}
e ~ 7 M_{jup}

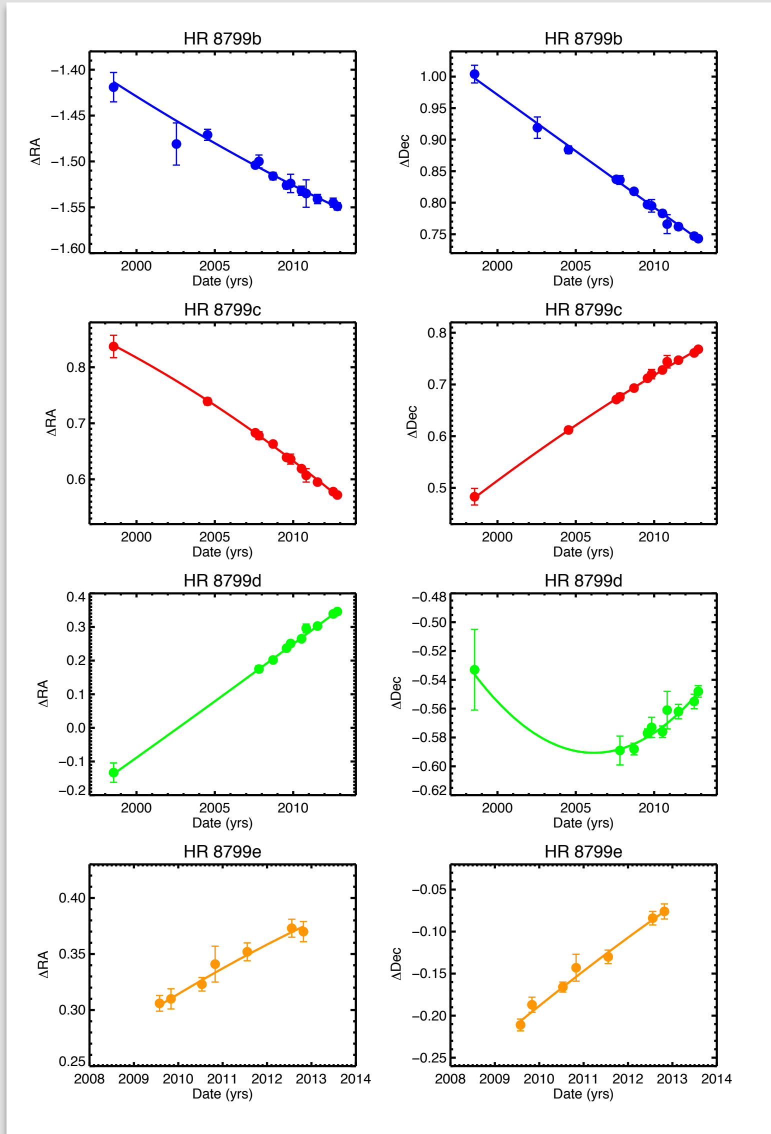


Keck NIRC2; Marois et al. 2010

Projected Separations:
b ~ 68 AU
c ~ 38 AU
d ~ 24 AU
e ~ 14.5 AU

The astrometric precision is sufficient for the first detection of acceleration in HR 8799c and d.

"Detection" defined by 3 sigma measurement of acceleration in the radial direction and a tangential acceleration consistent with zero



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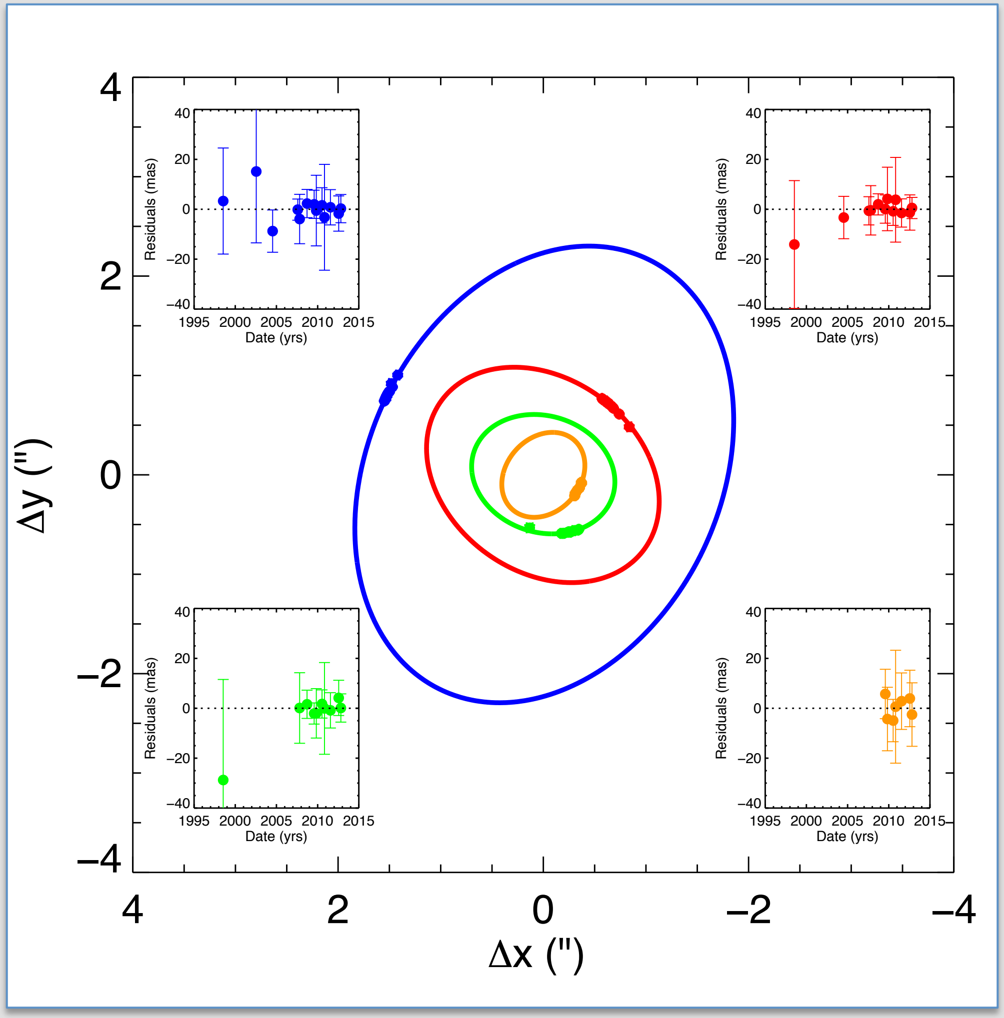
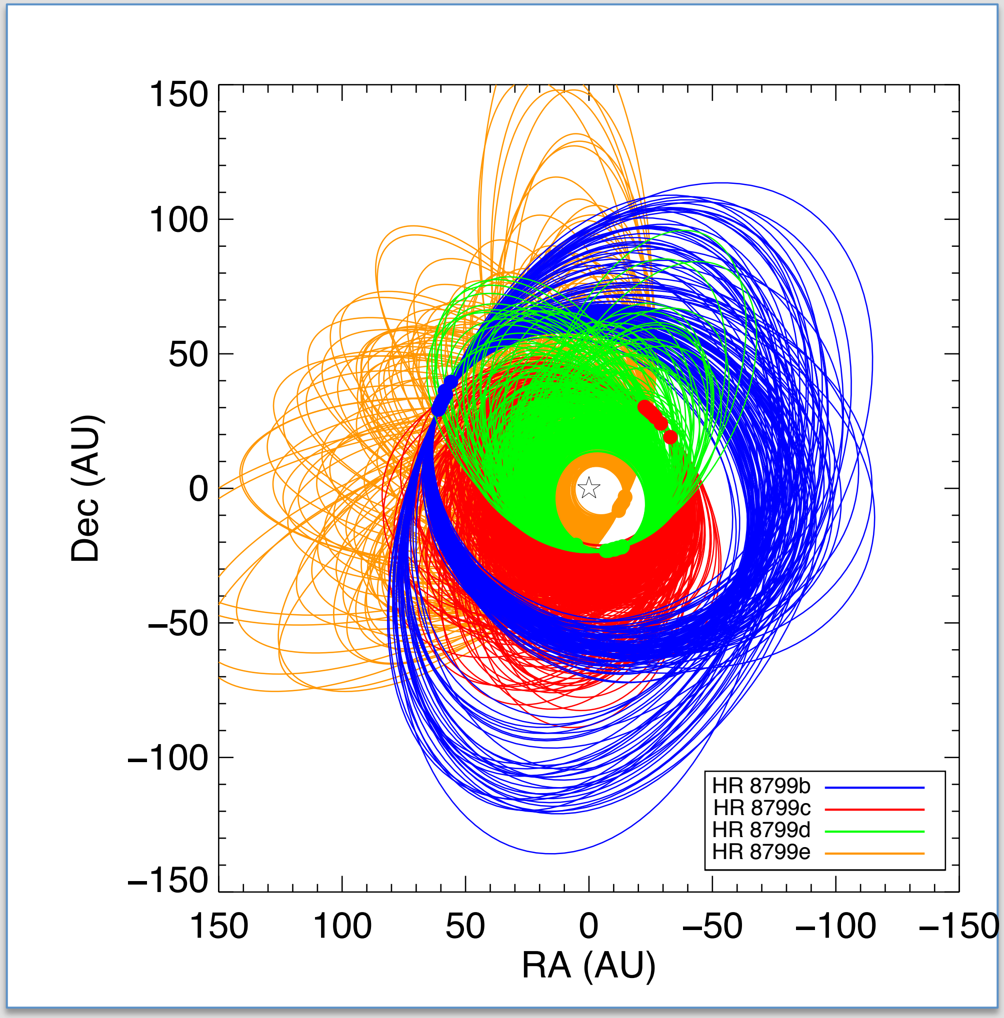
Including newest astrometry for HR 8799 provides ~15 years of orbit coverage.



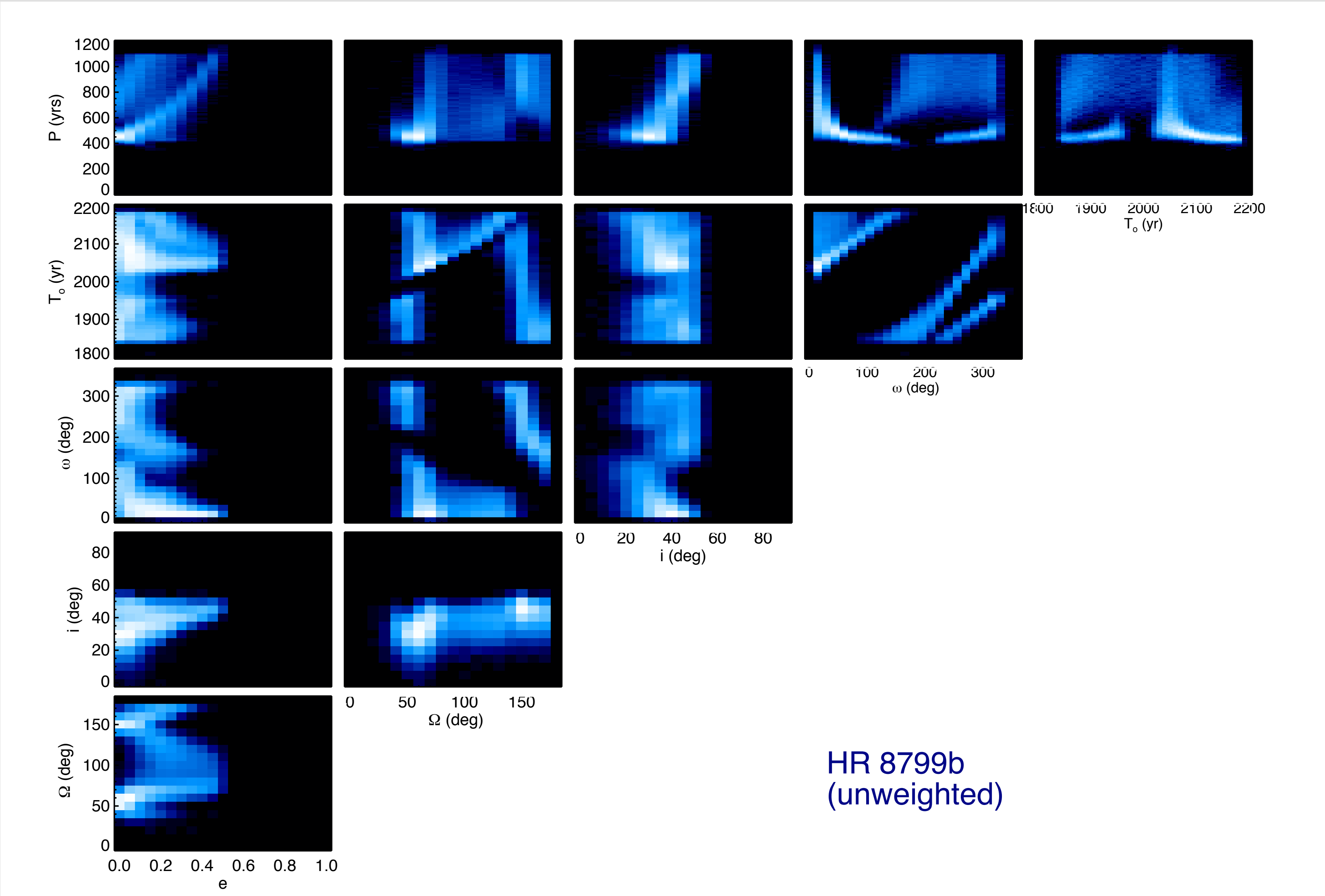
Konopacky et al. 2014

χ^2 minimization+Monte Carlo used to derive families of allowed orbits.



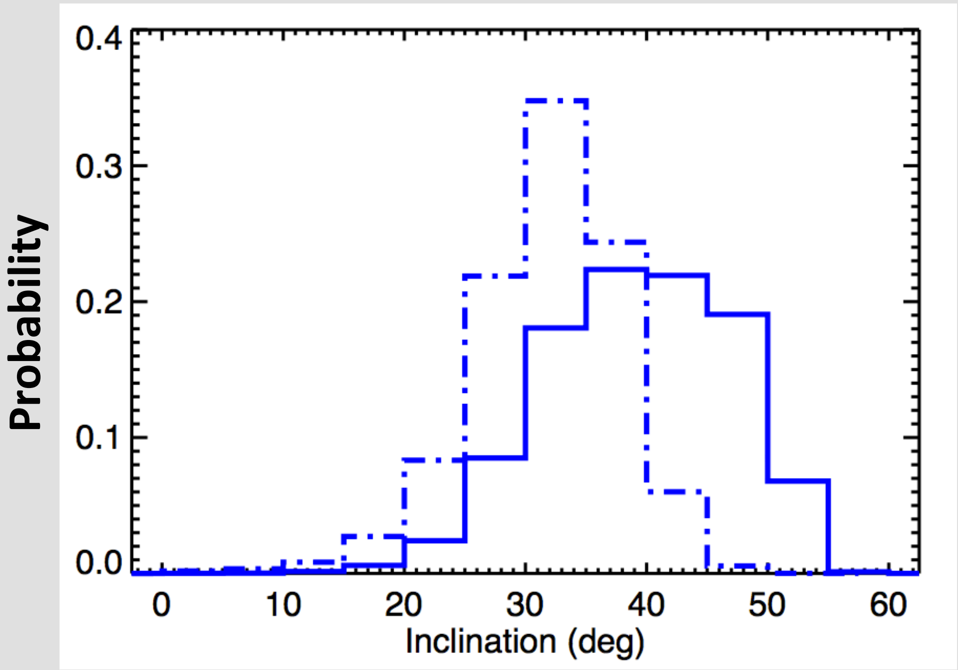
Konopacky et al. 2014

Orbital parameters for HR 8799b



Konopacky et al. 2014

Inclination constraints come more "easily" due to the orbital plane constraint from angular momentum.



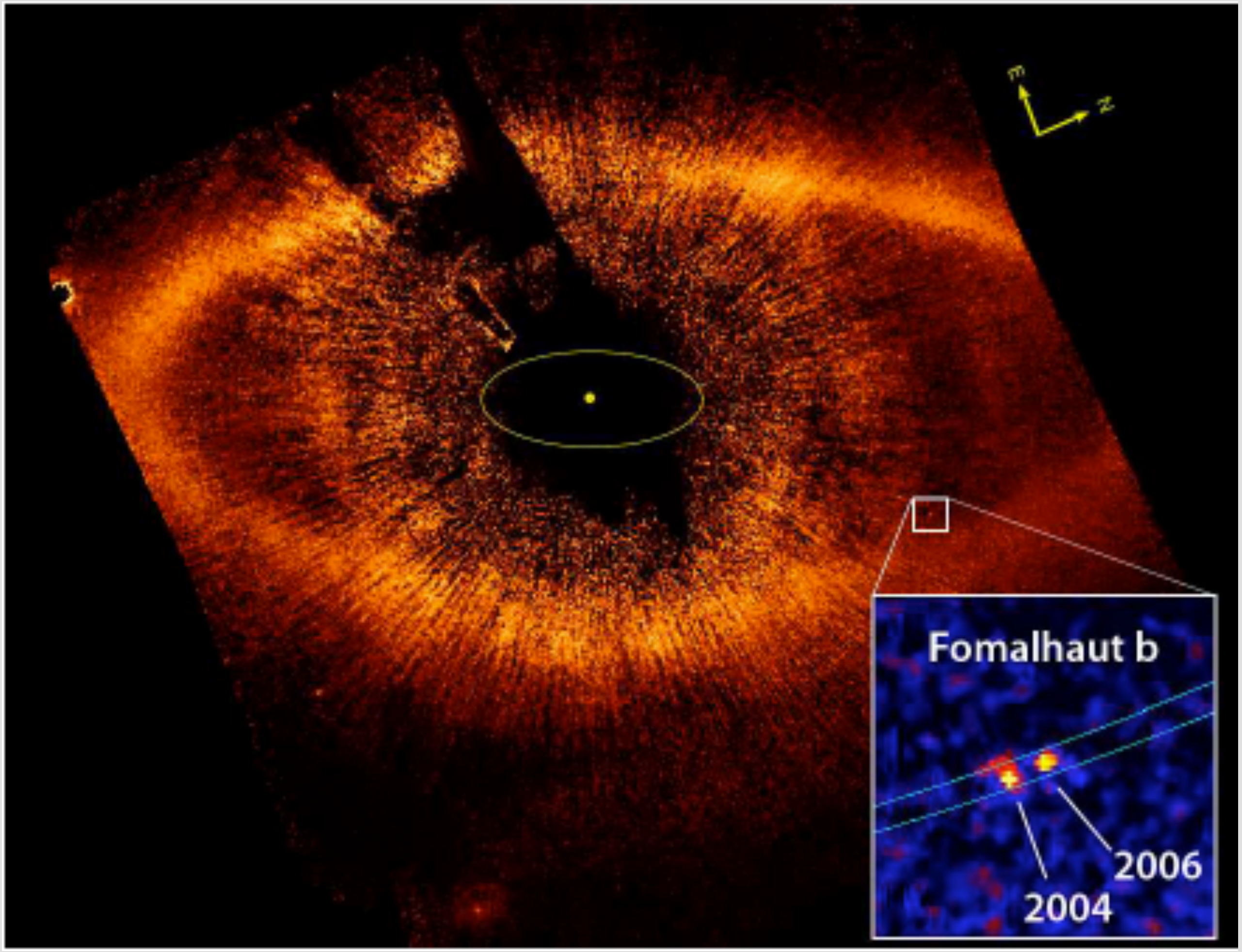
Konopacky et al. 2014

$$J_x = J \sin i \sin \Omega$$

$$J_y = -J \sin i \sin \Omega$$

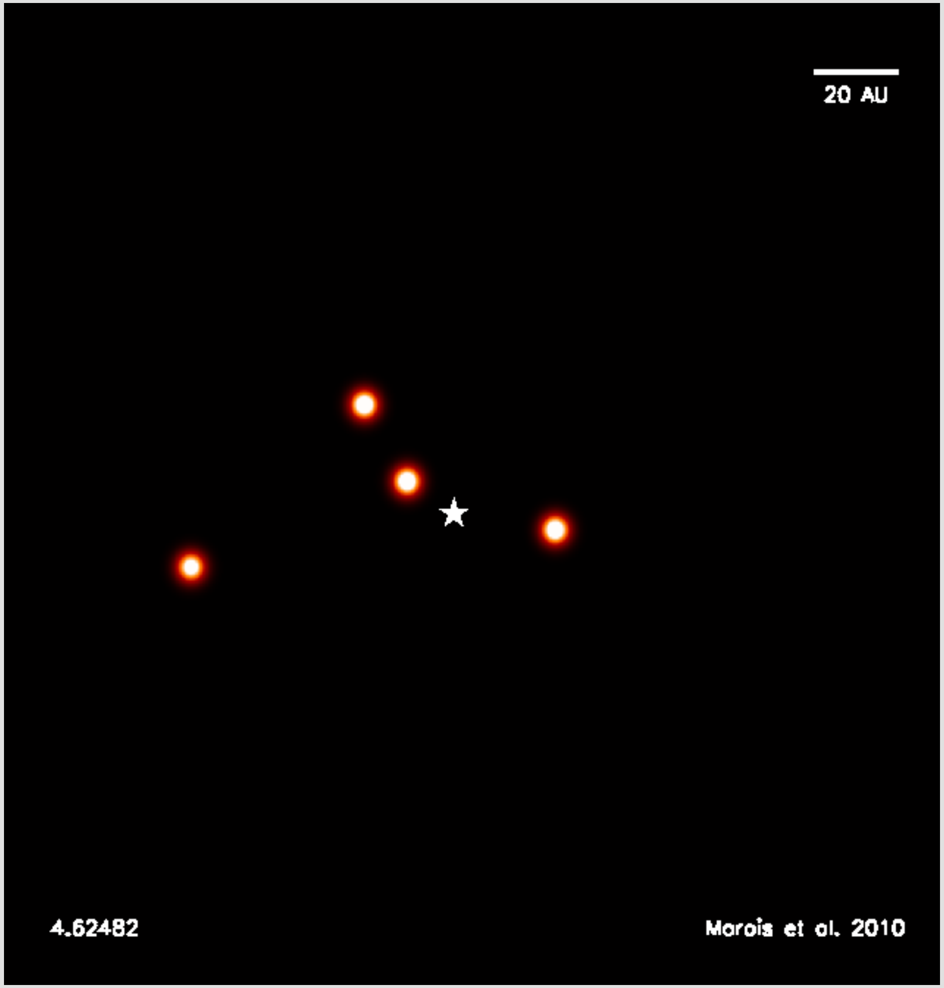
$$J_z = J \cos i$$

In some cases, orbital parameters can be additionally constrained by other aspects of the system, such as multiple planets or debris disks.

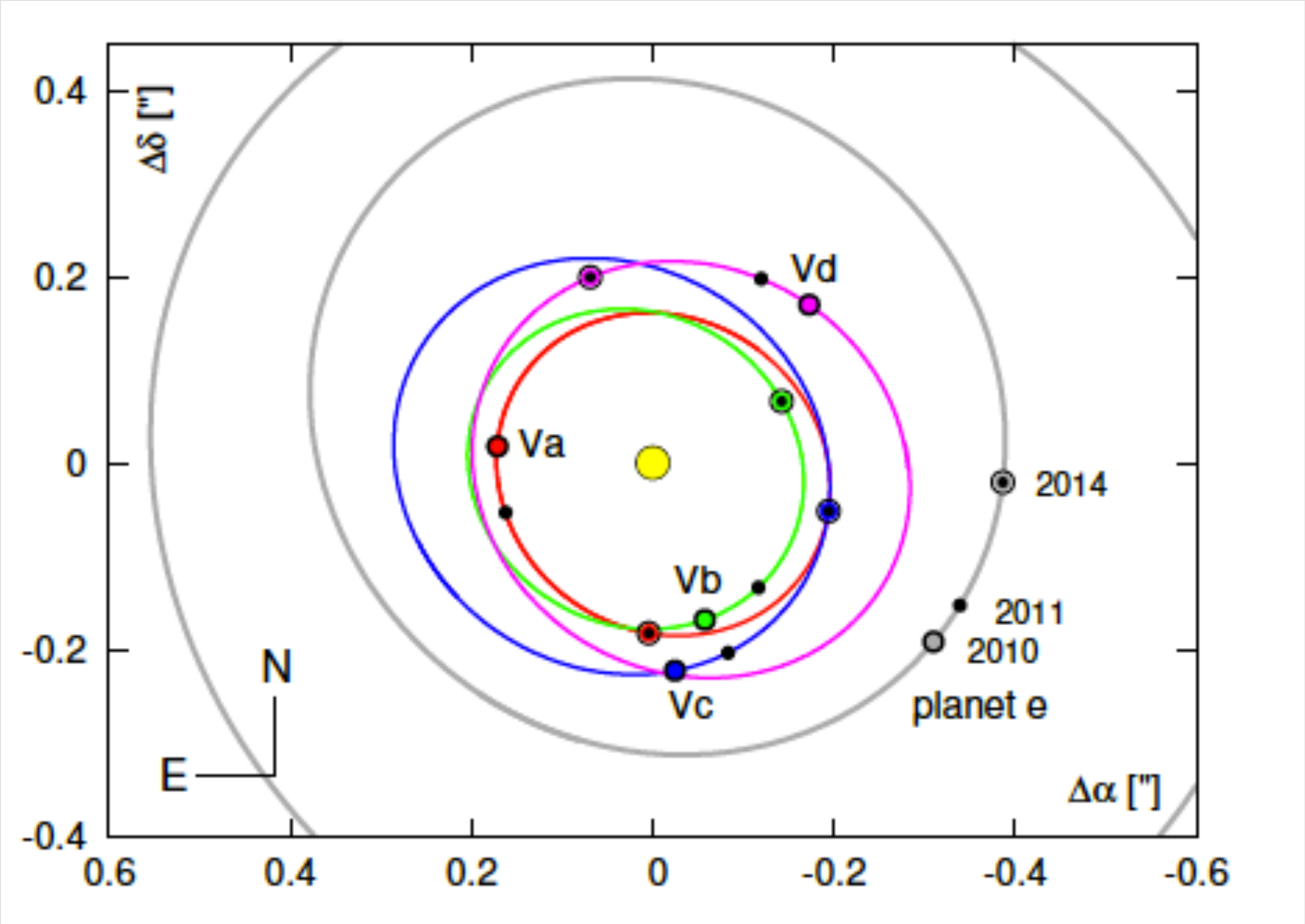


Fomalhaut b; Kalas et al. 2008

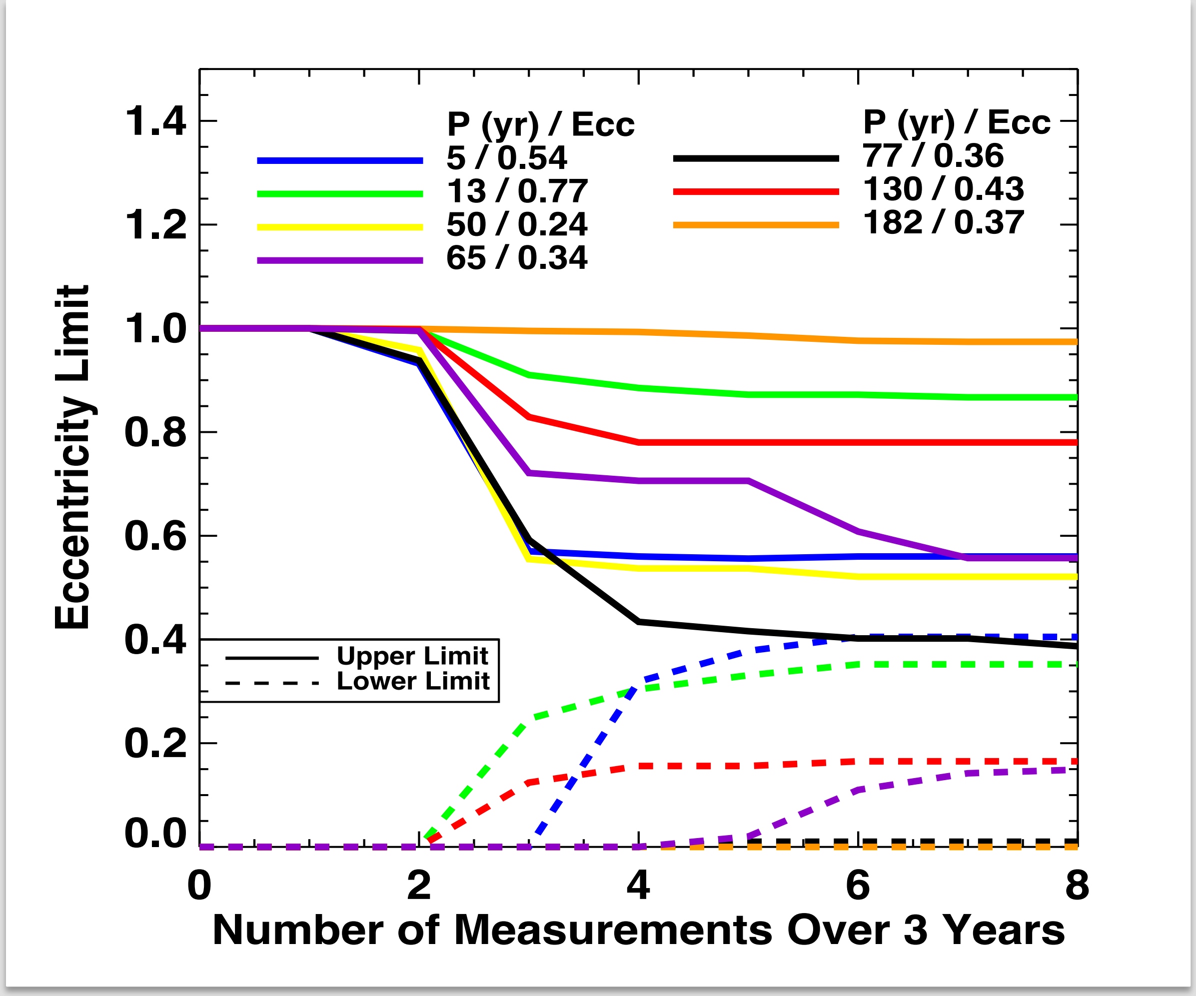
Stability analysis helps constrain masses and possible orbits for HR 8799.



New work by Gozdziewksi & Migaszewski (2013) suggest that HR 8799bcde must be in a 1:2:4:8 resonance.

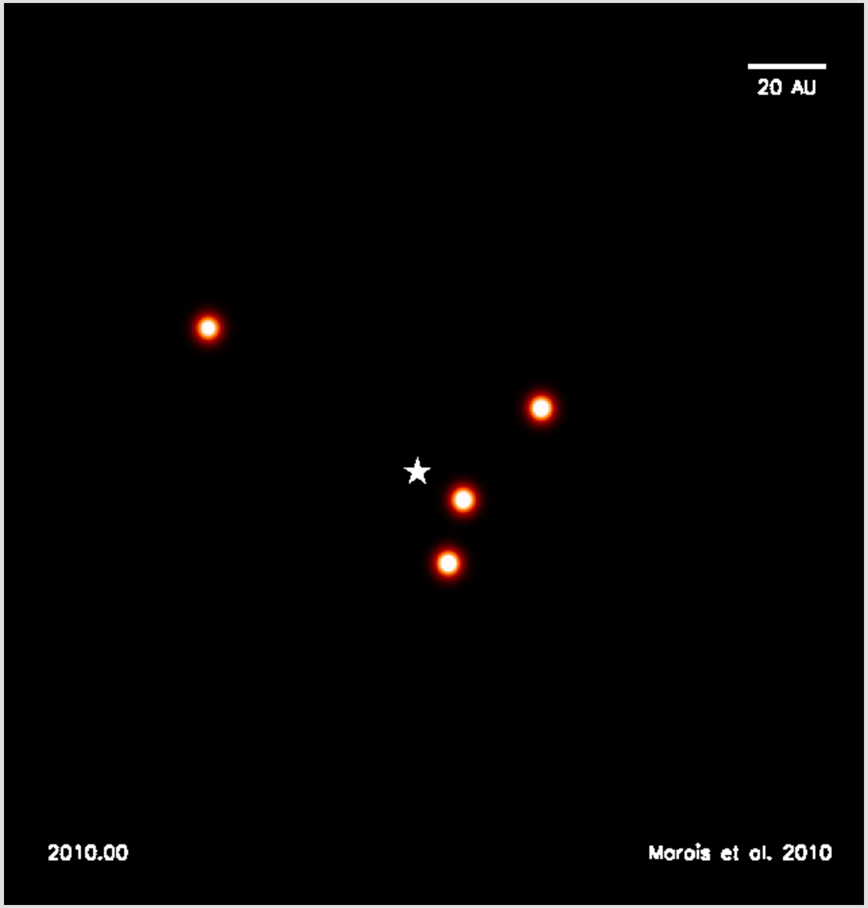


New instruments like GPI will be able to place orbital parameter constraints through precise astrometric measurements.



Conclusions

- Orbits are a powerful tool for directly imaged planet constrains
- Precise astrometry can be directly mapped into both geometrical and dynamical orbital parameters
- Many exciting new results have been coming out on the orbital parameters of directly imaged planets



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