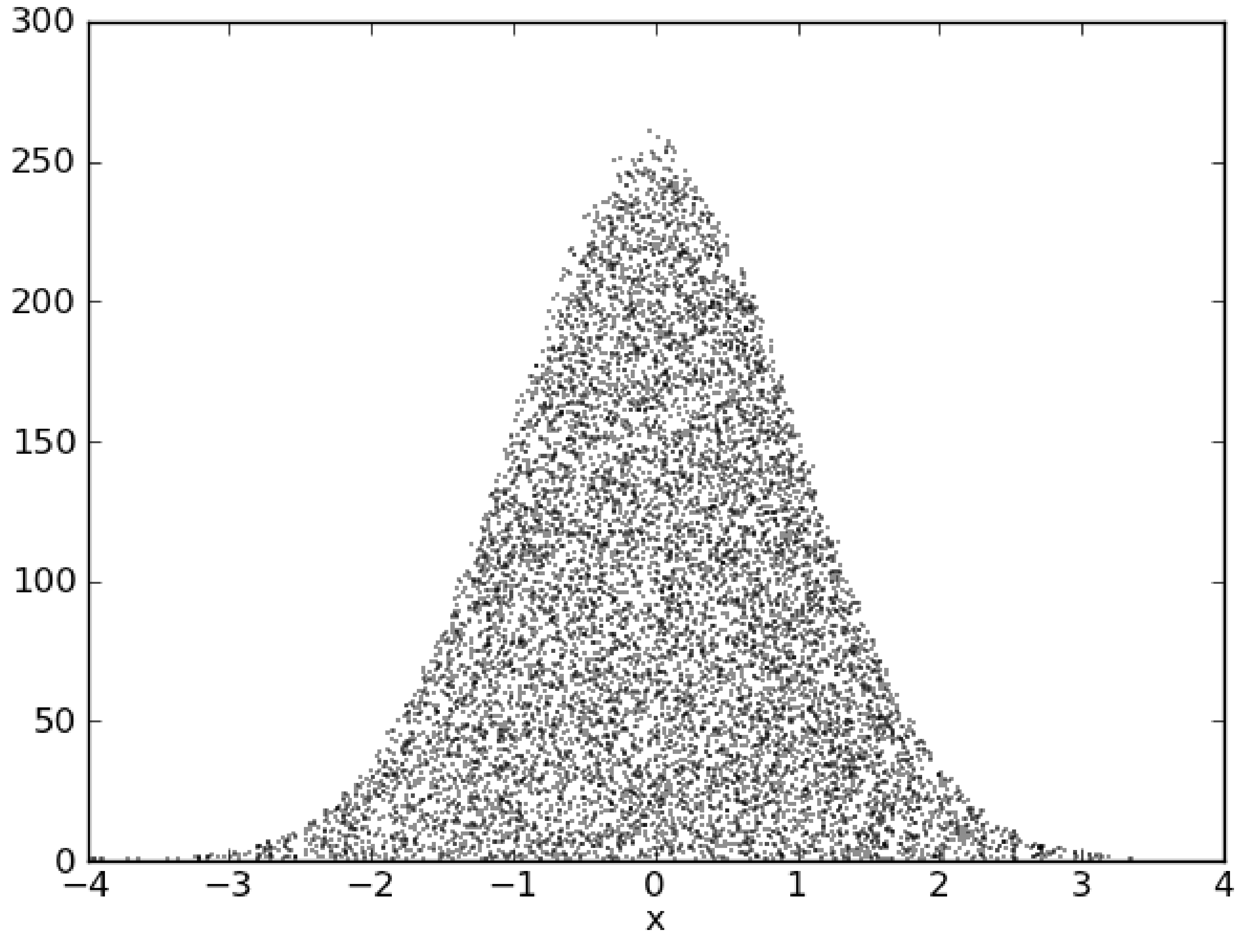
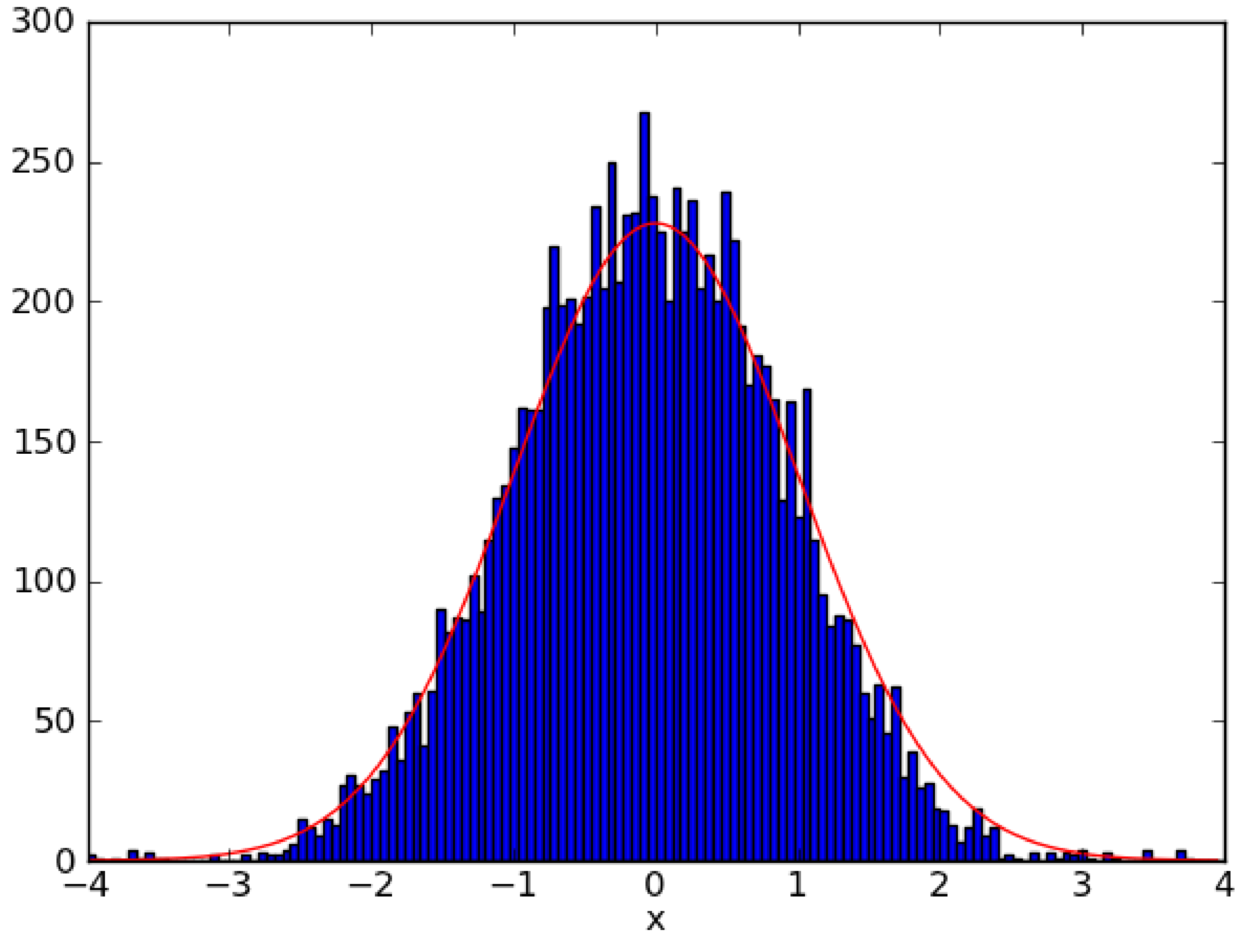
MCMC and Parameter Estimation

- Sampling
- MCMC
- Parameter estimation
- Transitional probability
- Jacobian and priors
- How many points we need

Sampling



Sampling



Monte Carlo Markov chain

(Metropolis-Hastings algorithm)

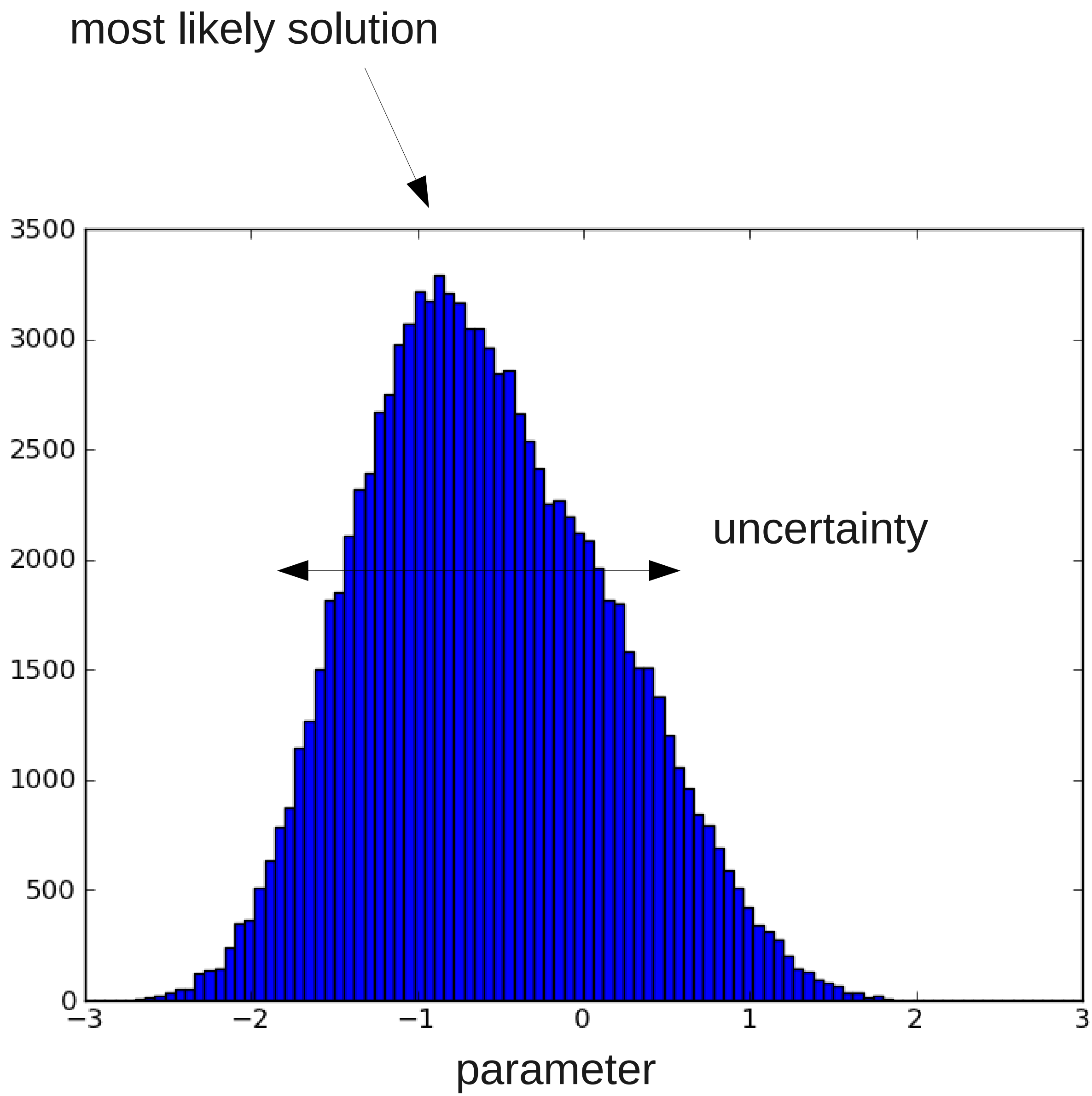
An algorithm to sample from the probability distribution when you do not know it!

(you only know how to compare two points in the space)

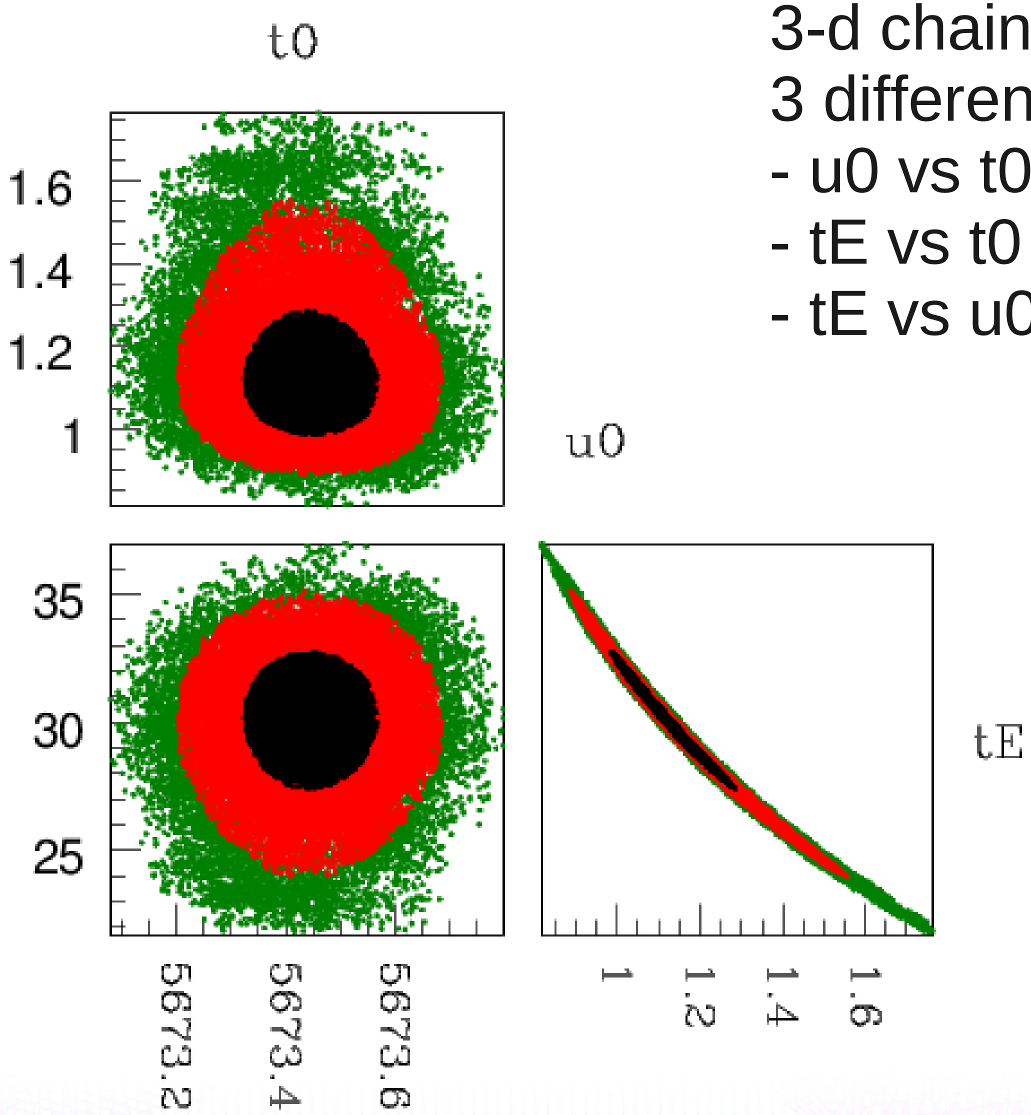
Algorithm

```
point=initial guess
loop
   trial_point = point + some_random_jump

   ratio =  p(trial_point)/p(point)

   if ratio > a_random_number(0 to 1) then:
      point=trial_point (accept trial point)
   else:
      (reject trial point, use the old one)

   chain.append(point)
end loop
(now you have a chain)
```

Parameter estimation: Histogram



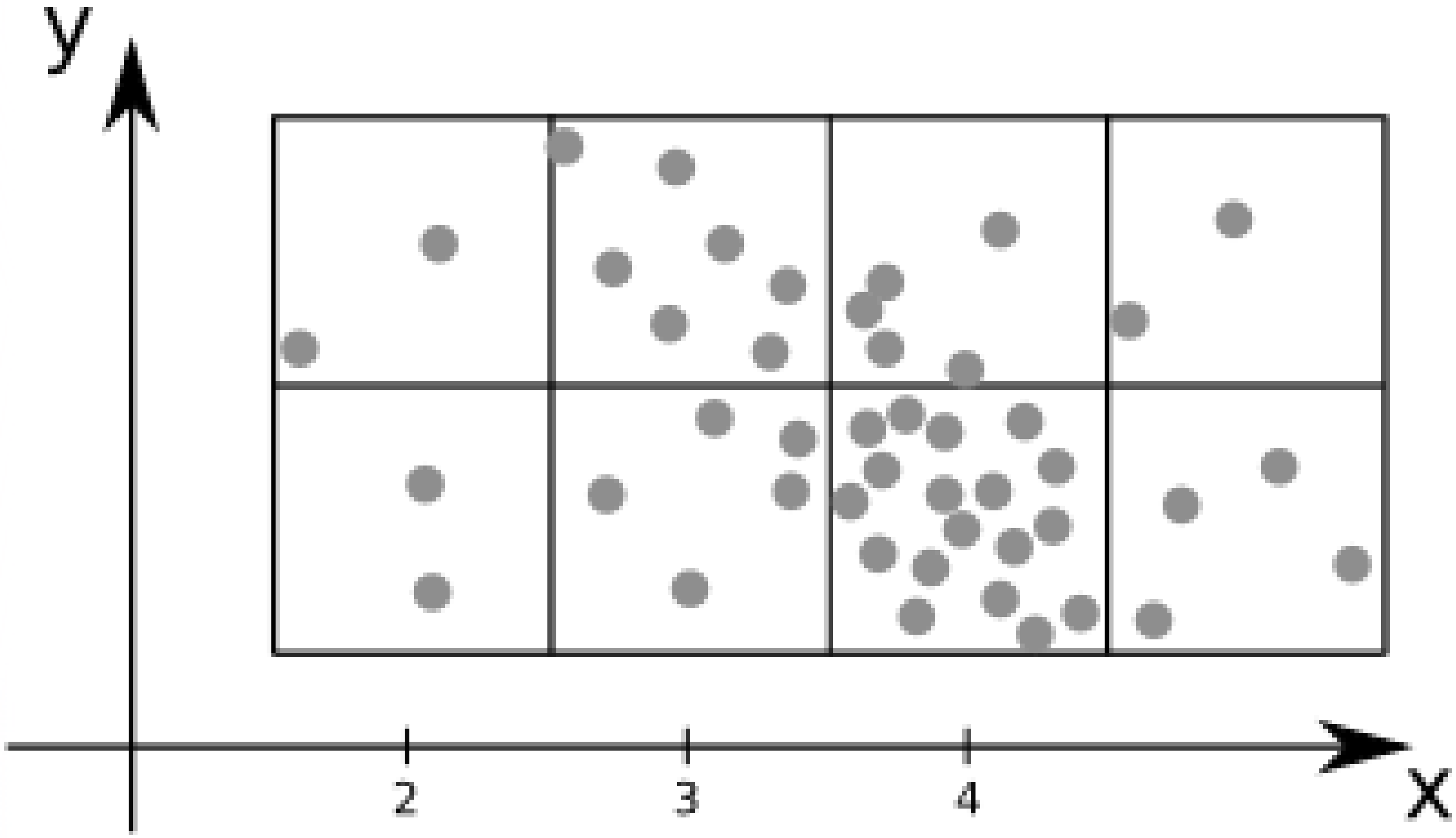
Correlations



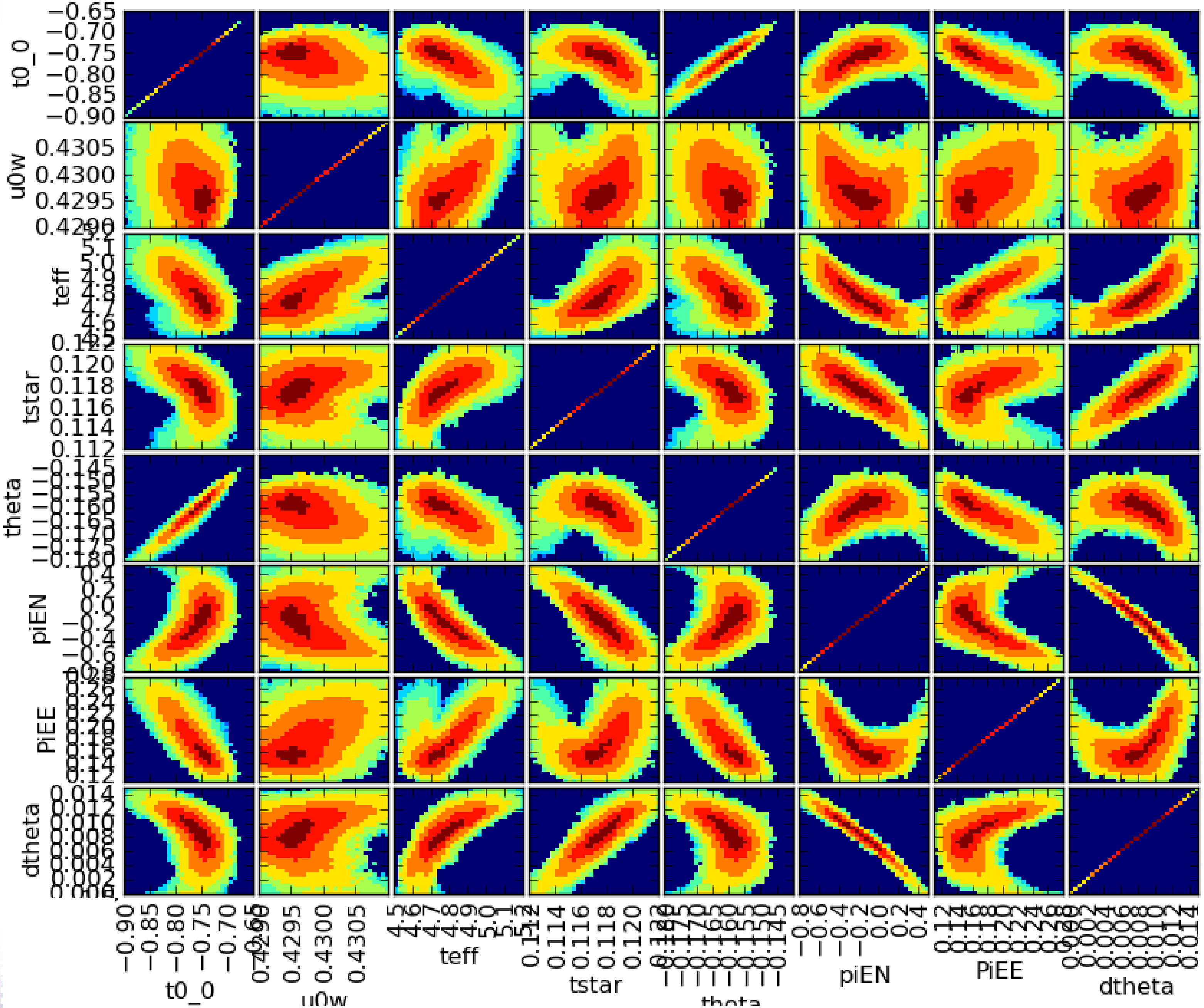
3-d chain projected onto
3 different planes:

- u0 vs t0
- tE vs t0
- tE vs u0

Density of points



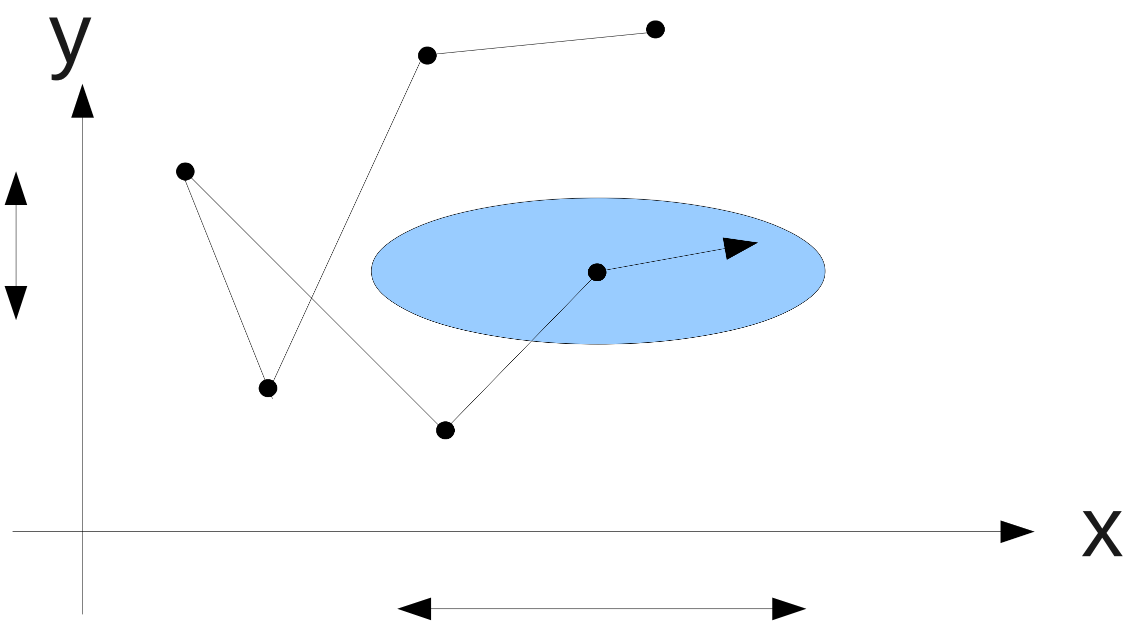
$$\sigma = \sqrt{2 \log \frac{L_{max}}{L}}$$



t0_0
u0w
teff
tstar
theta
piEN
PiEE
dtheta

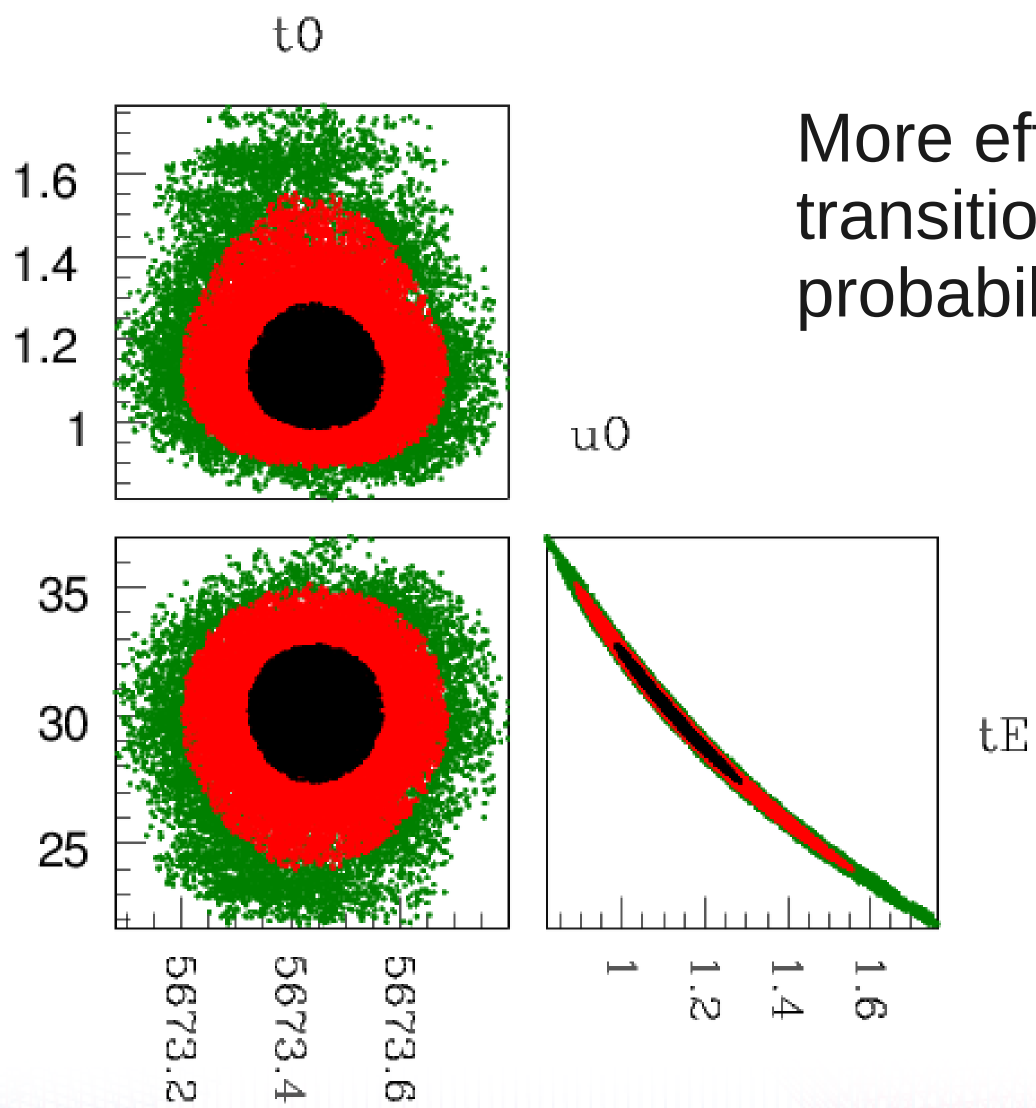
Transitional probability

- This is the part: `some_random_jump` in the code
- Usually we use multidimensional gaussian

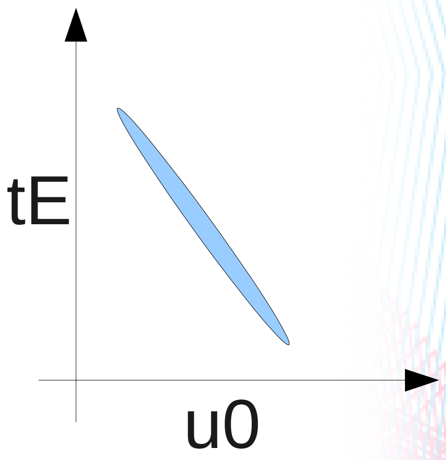


characteristic step size in x

Correlations

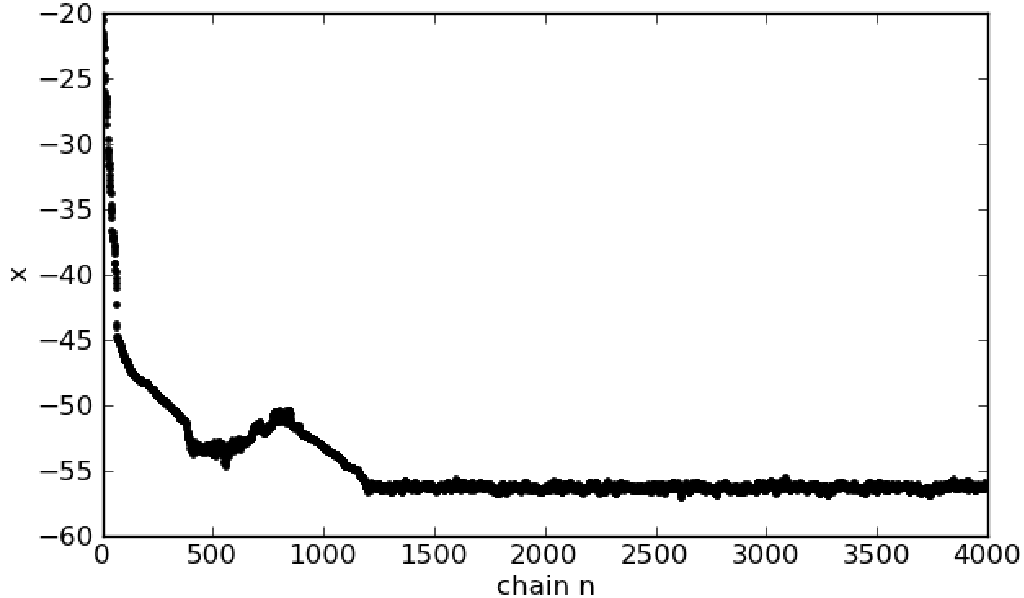


More efficient transitional probability:

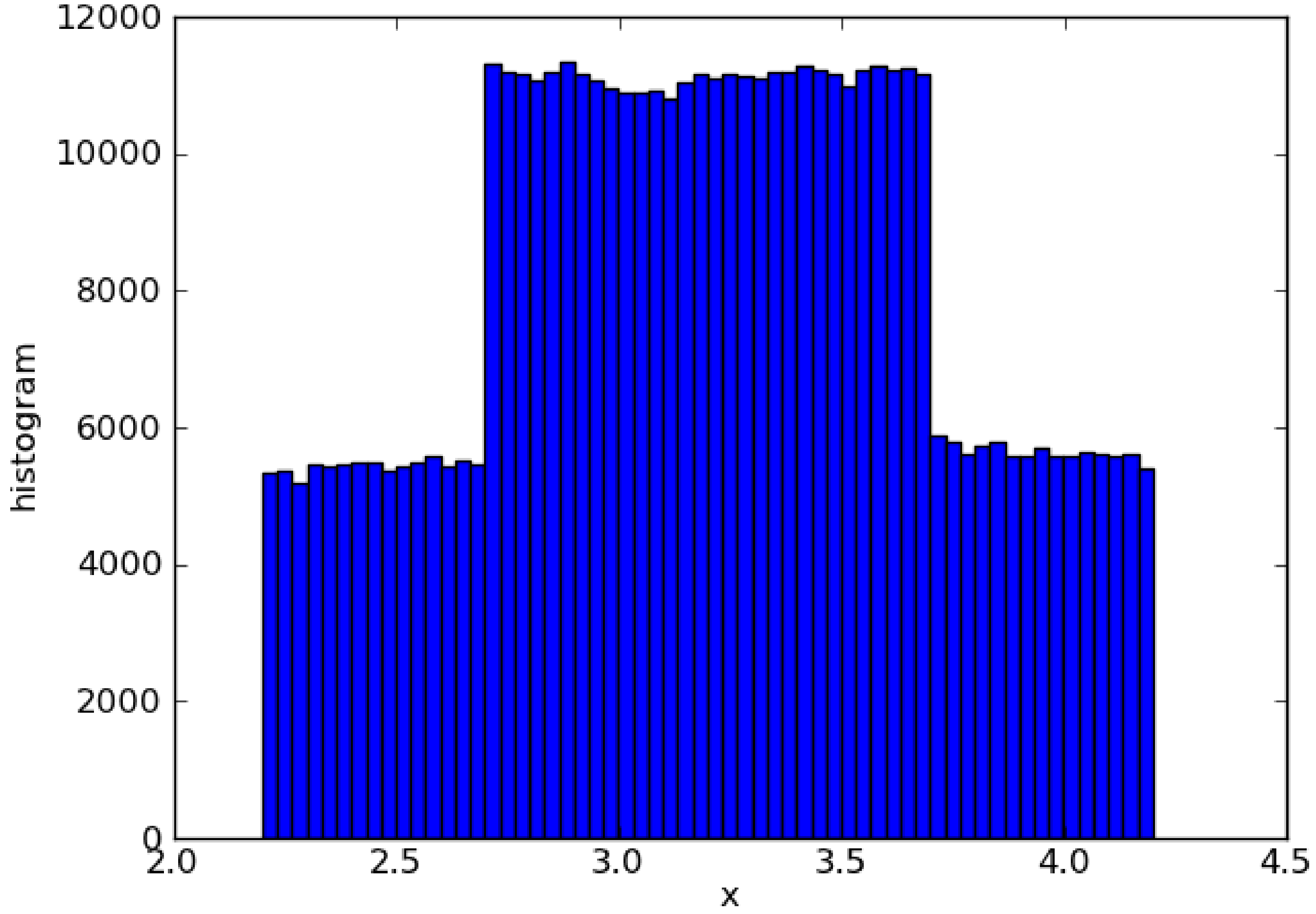


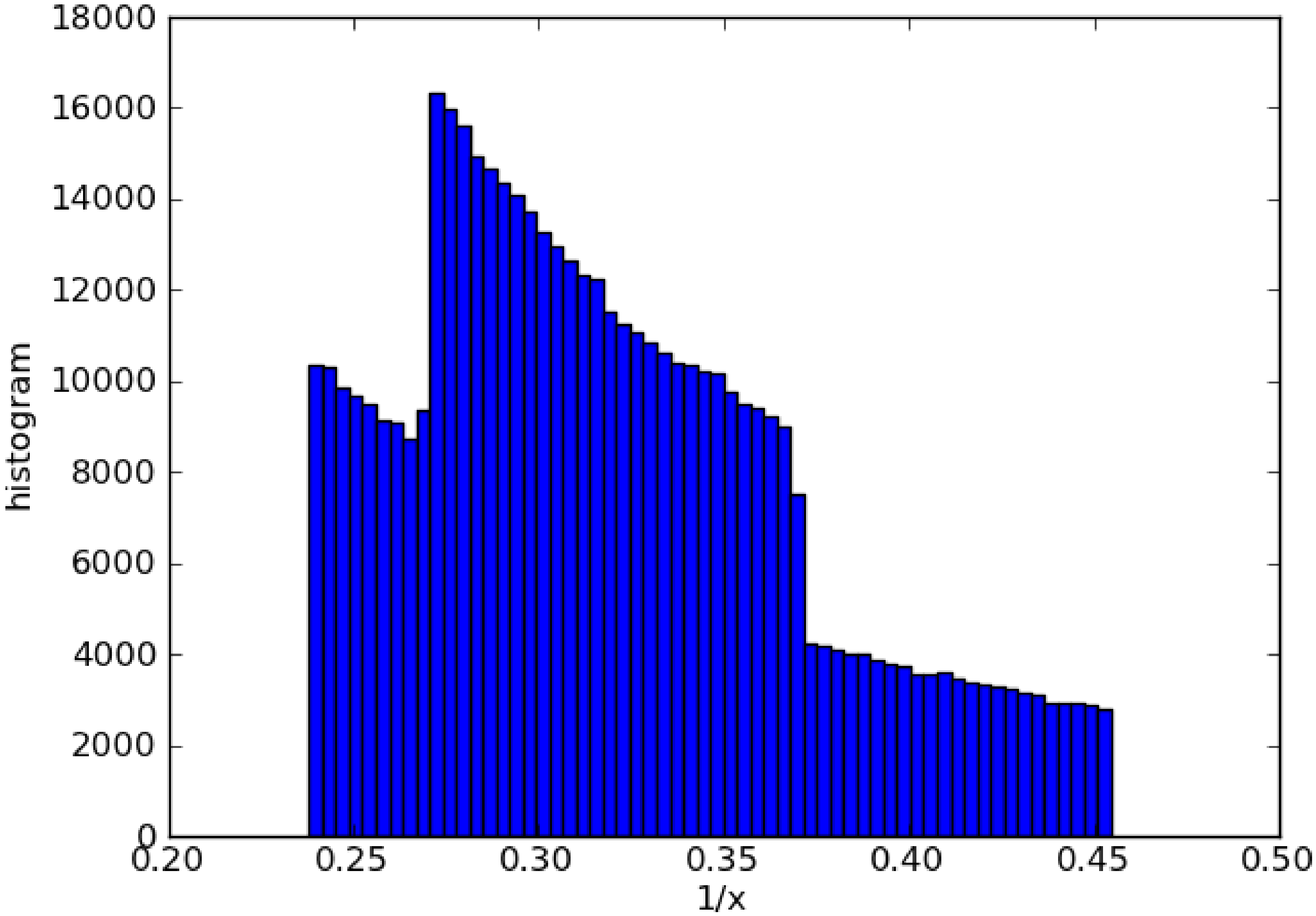
Burning

- In the initial part of MCMC we change transitional probability (trying to find a good correlation matrix and step sizes)
- (After a while) We fix it and start to collect chain links



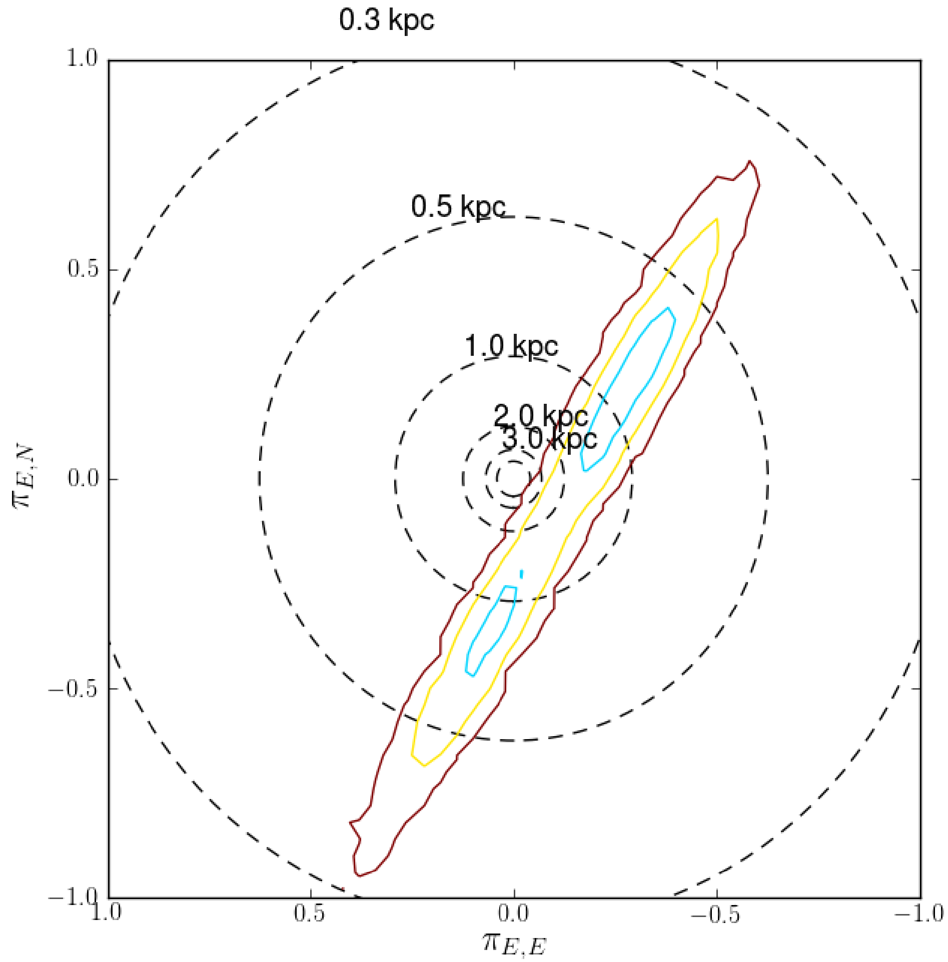
Jacobians and priors



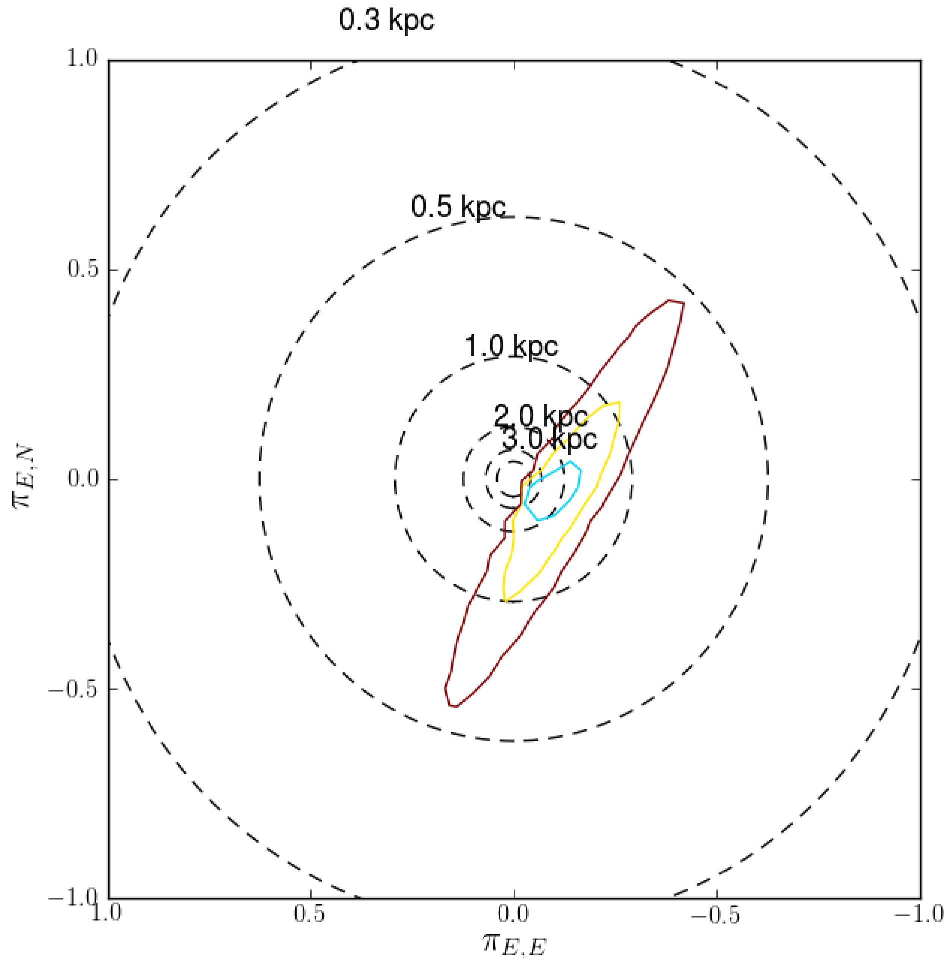


histogram
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0.20
0.25
0.30
0.35
0.40
0.45
0.50
1/x

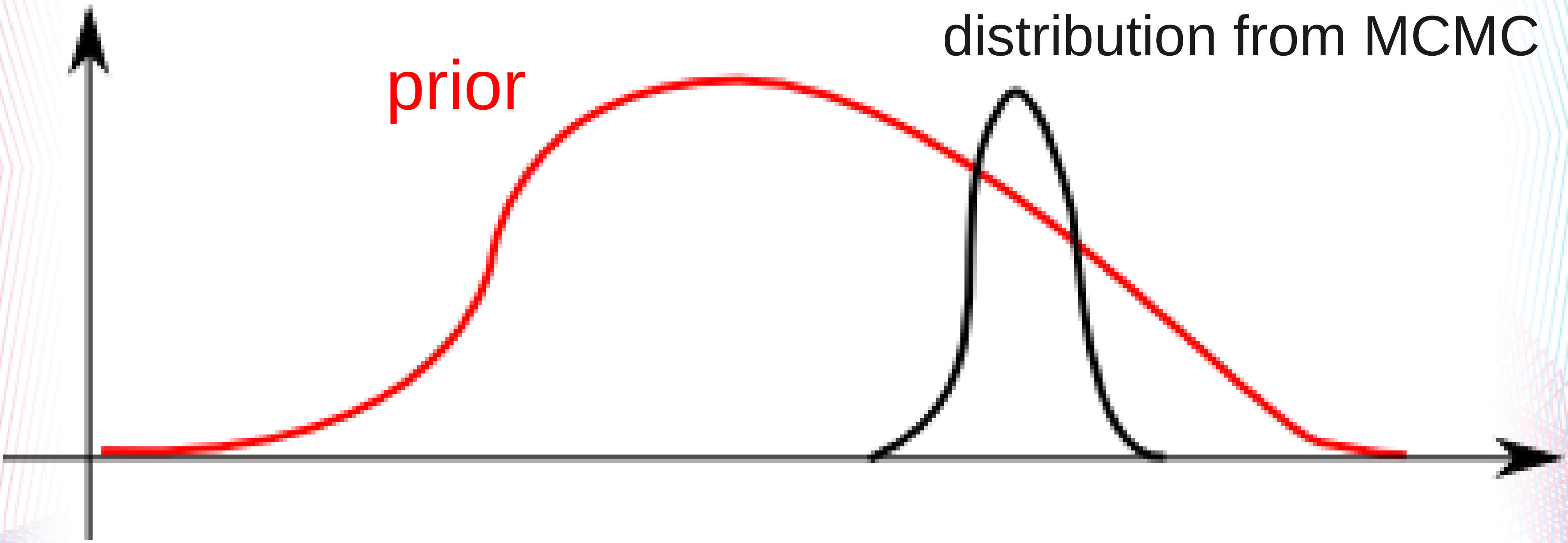
Distance to the lens



convenient parameterization for fitting, but...


0.3 kpc
0.5 kpc
1.0 kpc
2.0 kpc
3.0 kpc
1.0
0.5
0.0
−0.5
−1.0
$\pi_{E,N}$
$\pi_{E,E}$

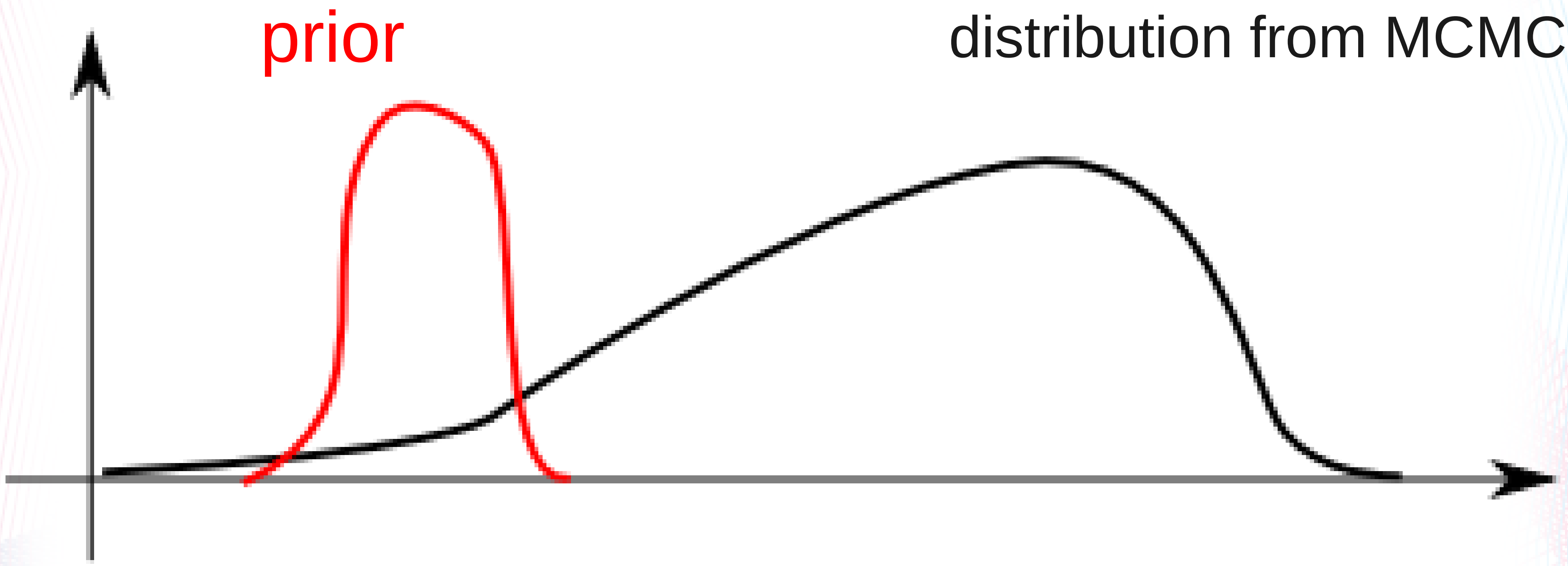
Good prior



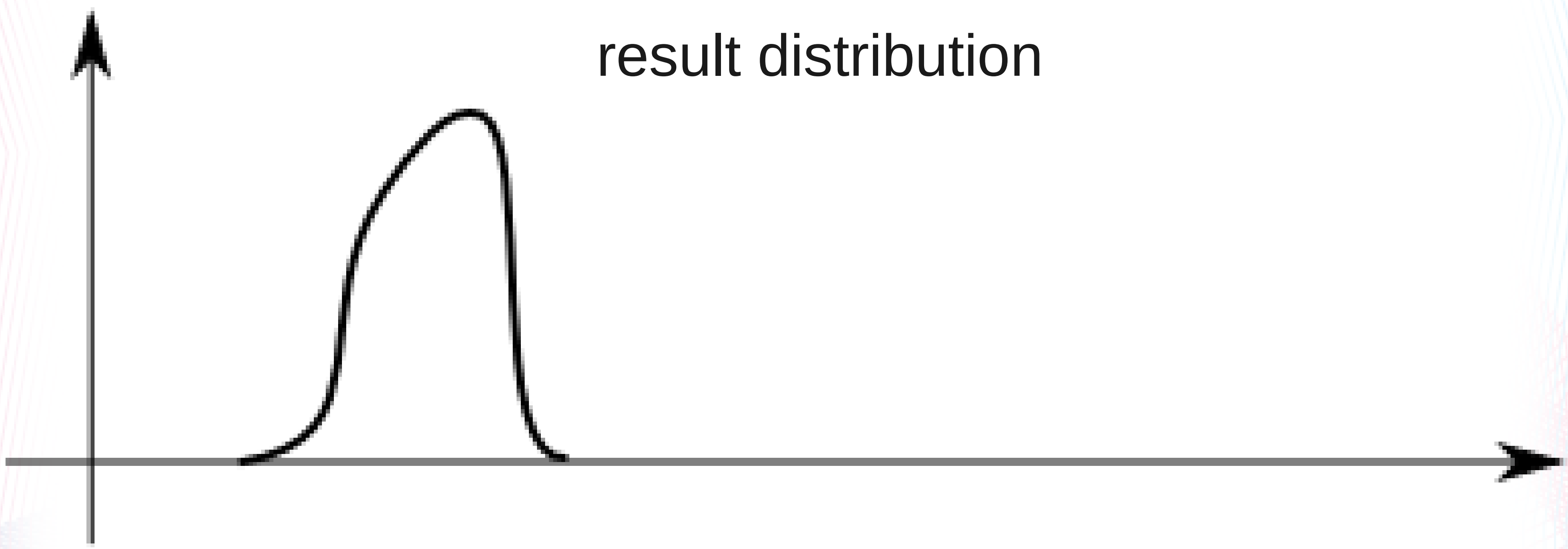
Good prior

prior modifies solution, but not overwhelms it

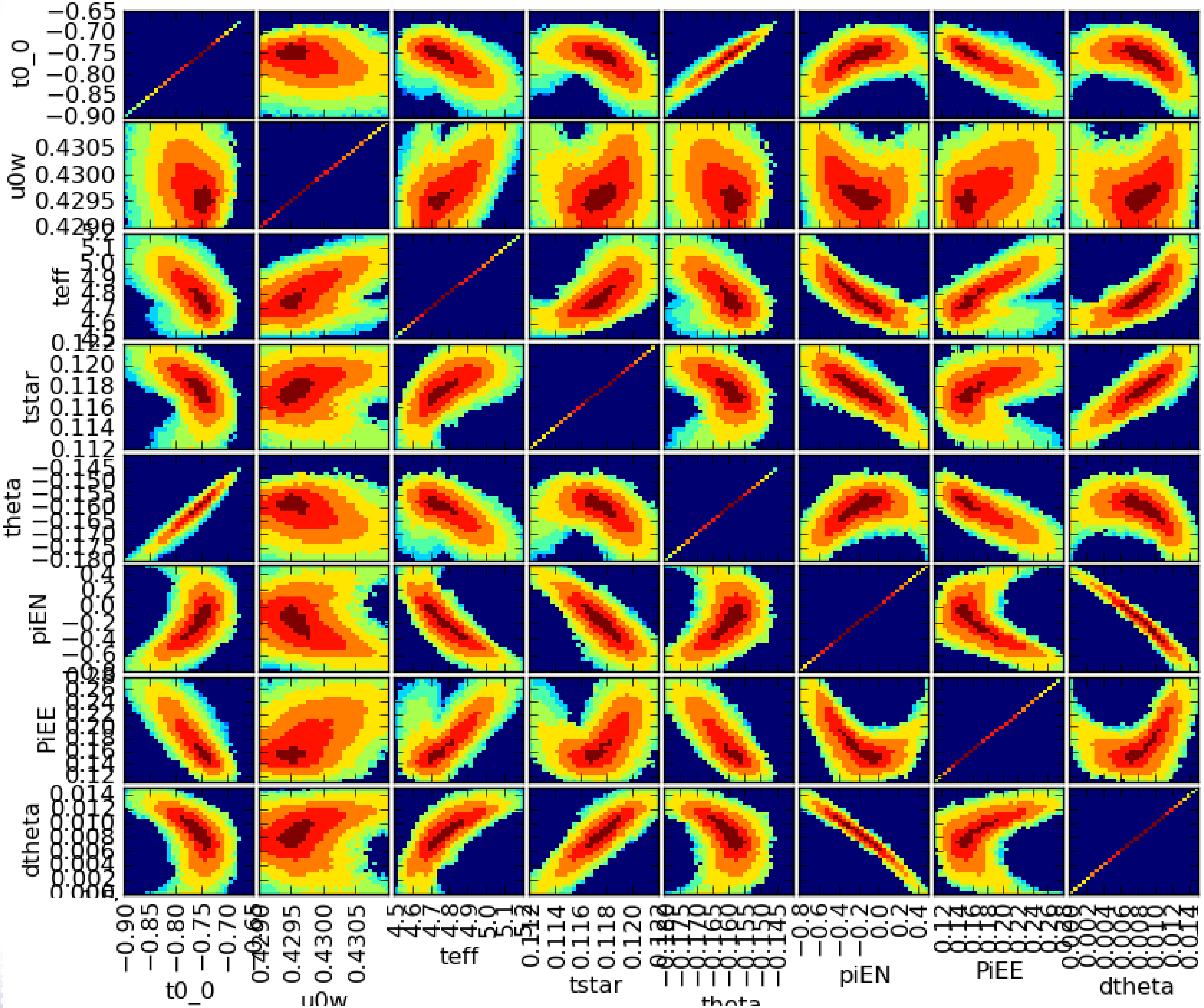
Over-constraining prior



Over-constraining prior

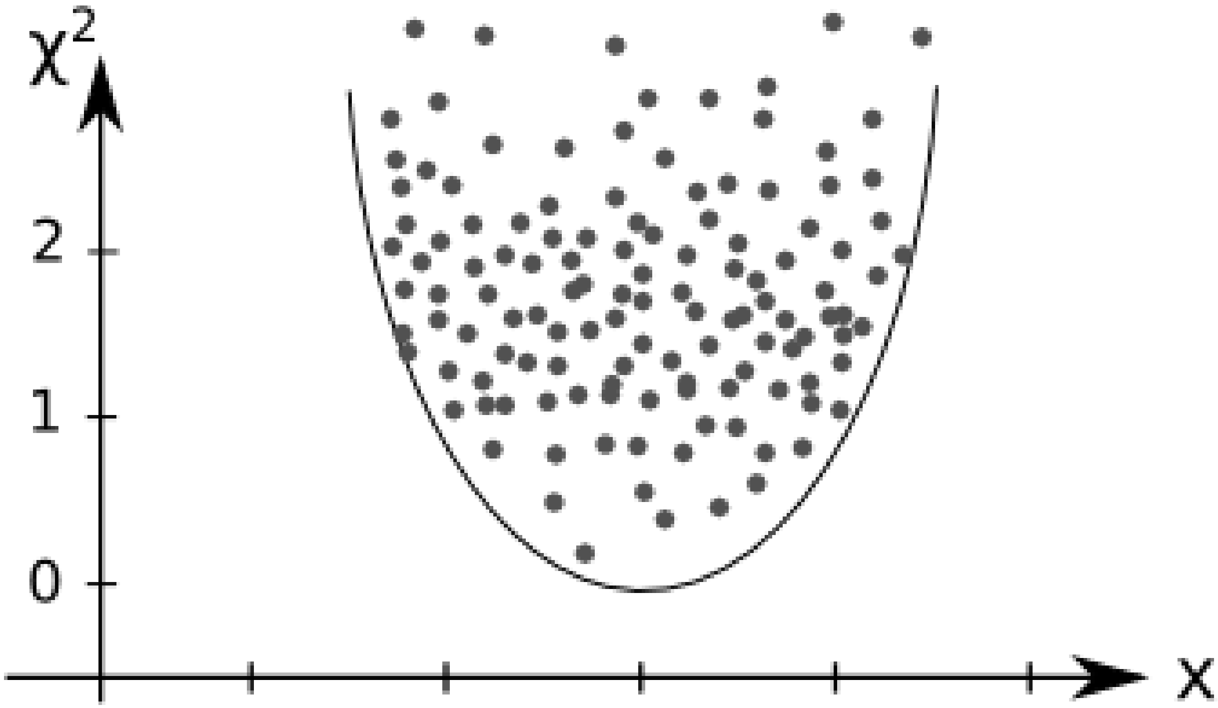


How many points we need



t0_0
u0w
teff
tstar
theta
piEN
PiEE
dtheta

How many points we need



Dong et al. 2007, 664, 862 gives this approximate formula:

$$N = 2^{k/2}\Gamma(\frac{k}{2})e^{1/2}$$

The End

- end for now, but workshop is going on