Classical Methods for Determining Stellar Masses, Temperatures, and Radii

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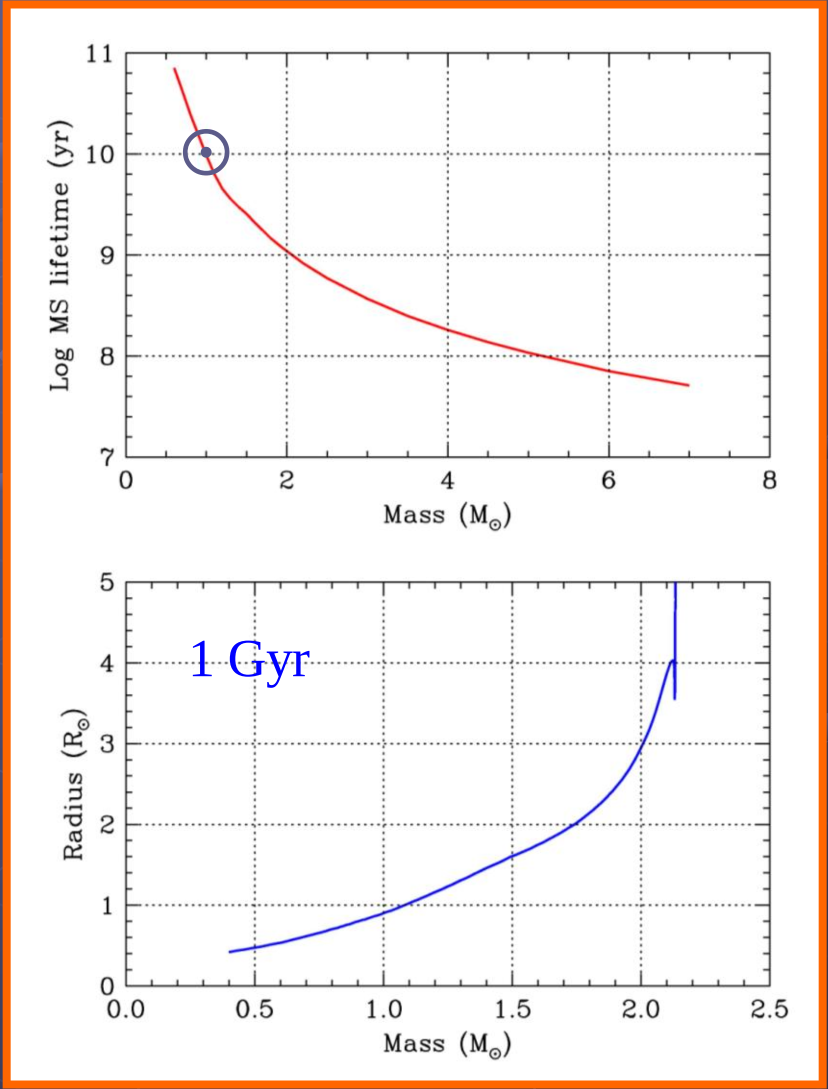
Outline

- Basic properties of stars
- Importance in the context of extrasolar planets (*"Stars as homes for habitable planetary systems"*)
- Other applications of stellar properties
- How are the fundamental properties of stars determined?
- Precision and accuracy: status
- Practical determination of M, R, and T_{eff} for exoplanet hosts

Fundamental Properties of Stars

- Stellar mass (drives evolution): usually available only in binaries
- Stellar radius
- Effective temperature
- Chemical composition
- Age: only from models; important for theory of planetary evolution

Luminosity: $L \propto R^2\, T_{\rm eff}^4$



Exoplanets and Their Host Stars

- Why do we care about the host star properties?
  - They influence habitability
  - They are needed to determine the planetary properties
- Basic data involved in determining the planet's properties (mass and radius):
  - Radial velocity measurements of host star's reflex motion
  - Photometric observations of transit
- Information that can be extracted about the planet's properties:

$$M_p \sin i = 4.919\ 10^{-3}\ P^{1/3}\ (1{-}e^2)^{1/2}\ K_*\ [(M_*{+}M_p)/M_\odot]^{2/3}$$

R_p/R_* (transit depth), i (or b, impact parameter), a/R_*

Other Applications of Stellar Properties

- Improve our understanding of the structure and evolution of single stars: tests of models (with cosmological implications: globular cluster ages, etc.)
- Use of general mappings such as the mass-luminosity relation to estimate the total mass in star clusters
- Use of binaries with well-determined properties as distance indicators in the Milky Way and beyond

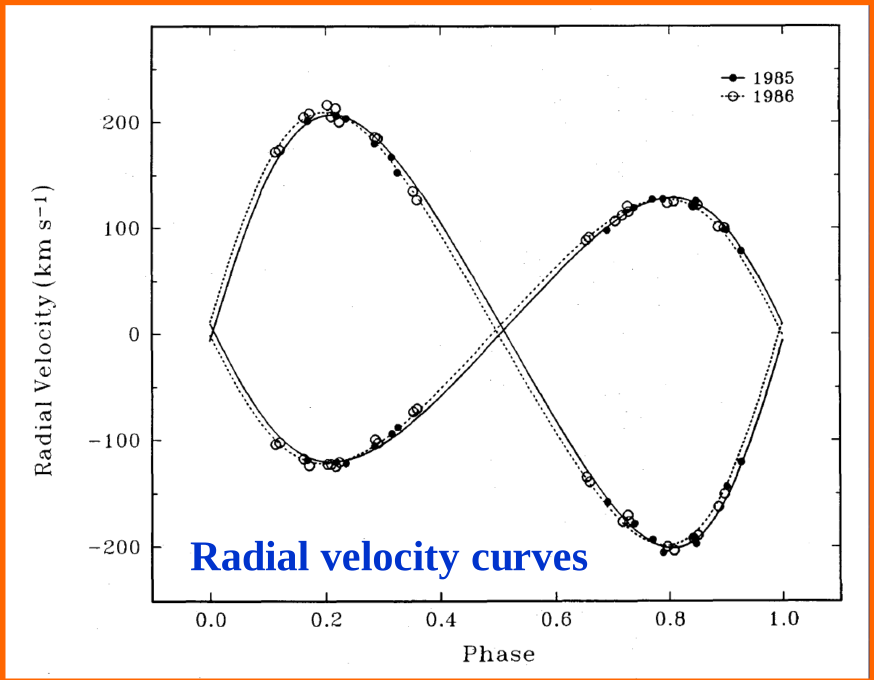
Stellar Mass Determinations

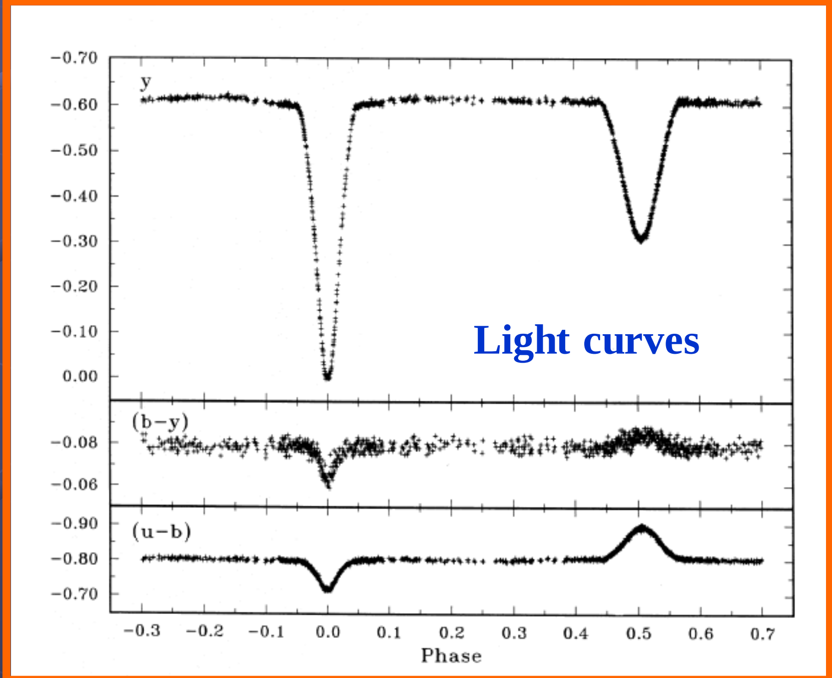
- Eclipsing double-lined binaries $\rightarrow P, K_1, K_2, e, i$

$$M_1 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_2$$
$$M_2 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_1$$

GG Lup

Andersen, Clausen & Giménez (1993), A&A, 277, 439

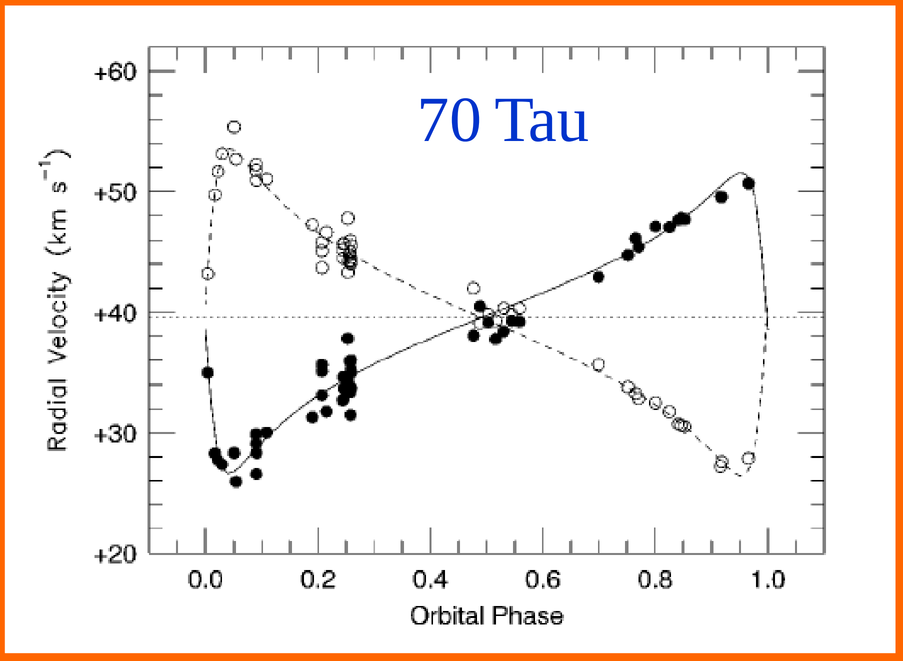


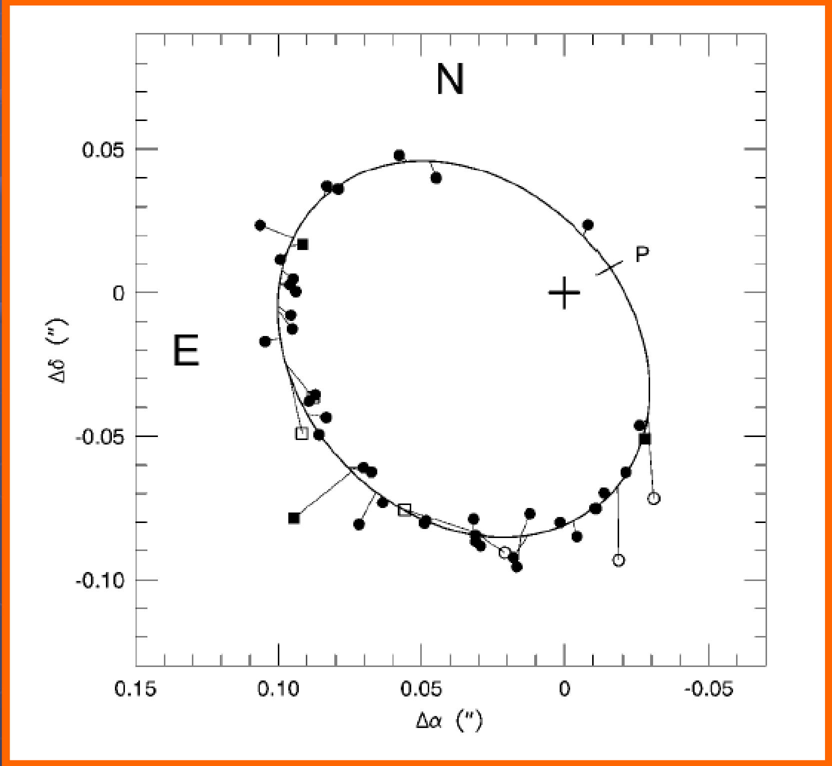


Double-lined astrometric-spectroscopic binaries

$$M_1 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_2$$
$$M_2 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_1$$

Torres, Stefanik & Latham (1997), ApJ, 479, 268





Radial velocity curves:

$\rightarrow P, \gamma, K_1, K_2, e, \omega, T$

Relative astrometric orbit:

$\rightarrow P, a'', e, i, \omega, \Omega, T$

Absolute astrometry (orbits measured for both stars)

(No spectroscopy)

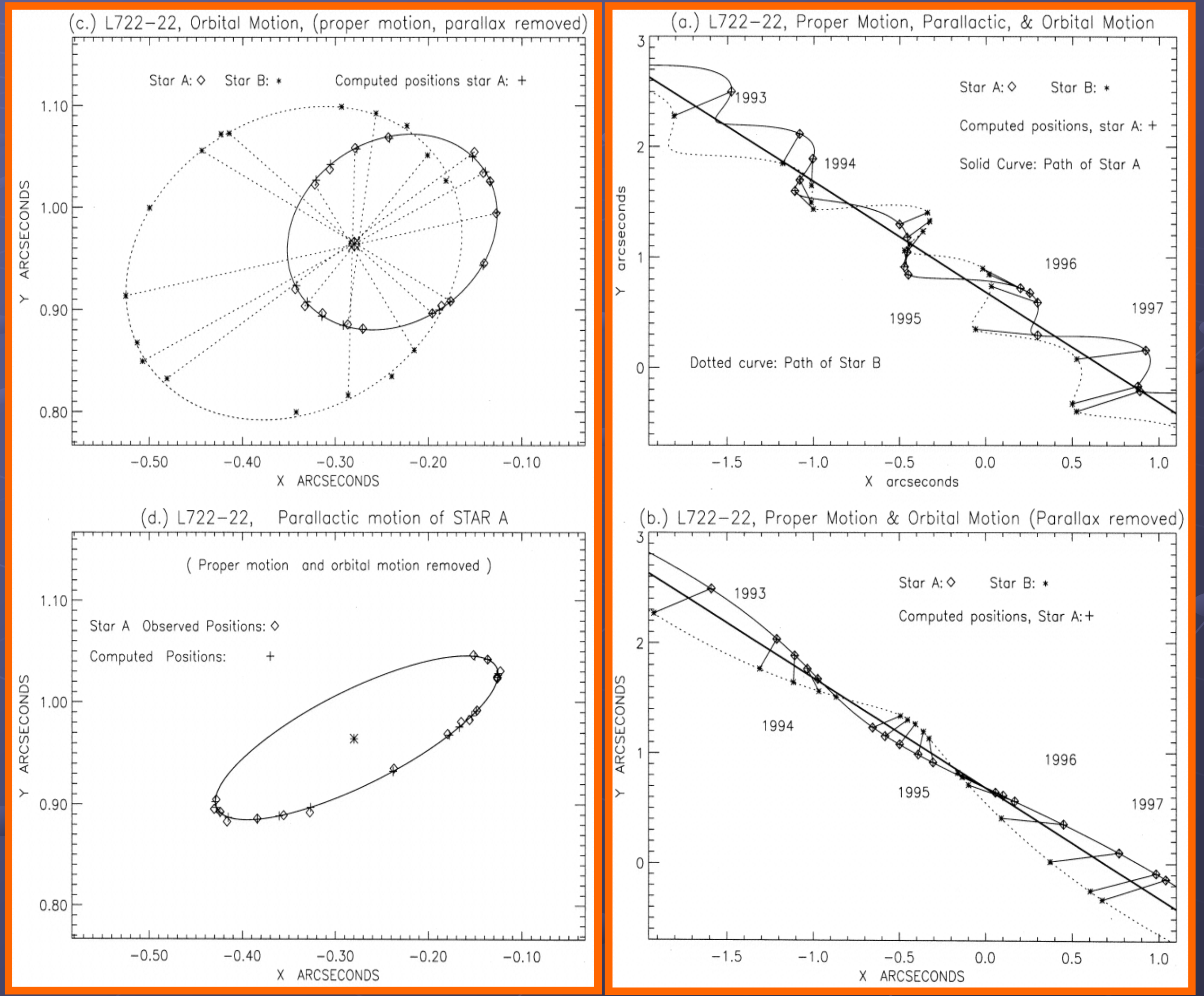
GJ 1005

Hershey & Taff (1998), AJ, 116, 1440

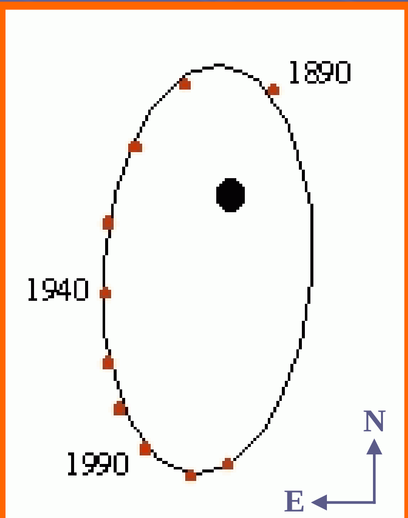
$$M_1 + M_2 \propto \frac{(a_1'' + a_2'')^3}{\pi^3 P^2}$$

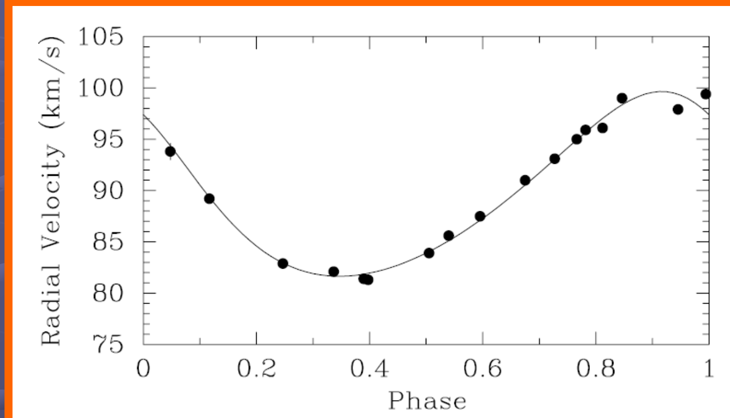
$$M_2/M_1 = a_1''/a_2''$$

- Period = 4.566 yr
- Distance = 6 pc
- *V* mag = 11.5 and 14.4
- Spectral type = dM4.5
- M_1 = 0.179   0.003 $M_\odot$
- M_2 = 0.112   0.002 $M_\odot$



- Single-lined spectroscopic binary + relative orbit + parallax





$$M_2 \sin i \;\propto\; K_1 P^{1/3} \sqrt{1-e^2} (M_1 + M_2)^{2/3}$$

$$M_1 + M_2 \;\propto\; (a''/\pi)^3 / P^2$$

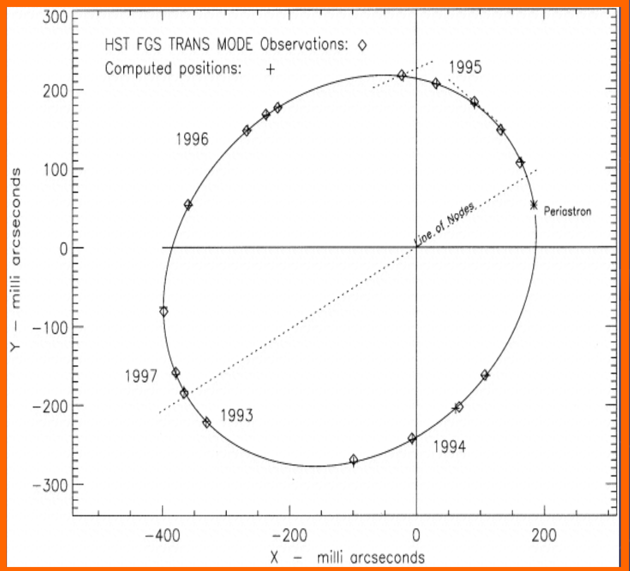
$$M_2 \;\propto\; \left(\frac{a''}{\pi}\right)^2 \frac{\sqrt{1-e^2} K_1}{P \sin i}$$

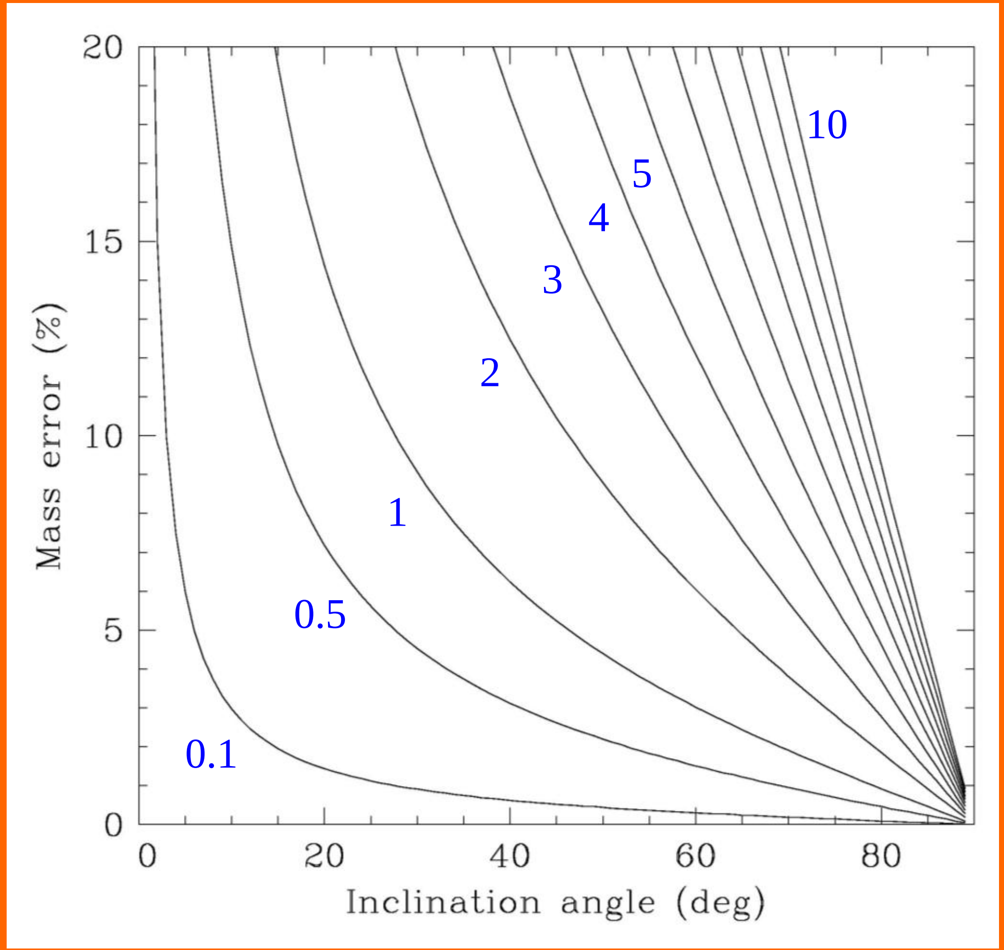
- Other configurations

Astrometric Requirements on Precision

Precision needed in the inclination angle of astrometric binaries, in order for the mass determinations **not** to be limited by the astrometry:

$$M_1 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_2$$
$$M_2 \sin^3 i \;\propto\; P(1-e^2)^{3/2}(K_1+K_2)^2 K_1$$





Stellar Radius Determinations

- Eclipsing double-lined spectroscopic binaries are the primary source of precise radius determinations: errors can be ≤1%, and the measurements are independent of distance

$$i\,,\quad R_1/a\,,\ R_2/a\,,\quad a\sin i\ \propto\ P\sqrt{1-e^2}(K_1+K_2)$$

- Angular diameters (θ) + parallaxes
  - Relatively few angular diameter measurements for dwarfs (especially for late spectral types)
  - Difficult in binaries

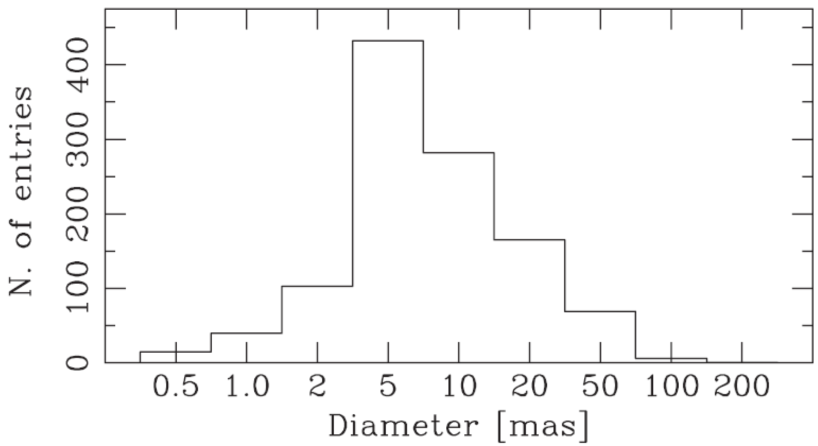


Fig. 4. Distribution of diameters. Note that the scale of the horizontal axis is arbitrary.

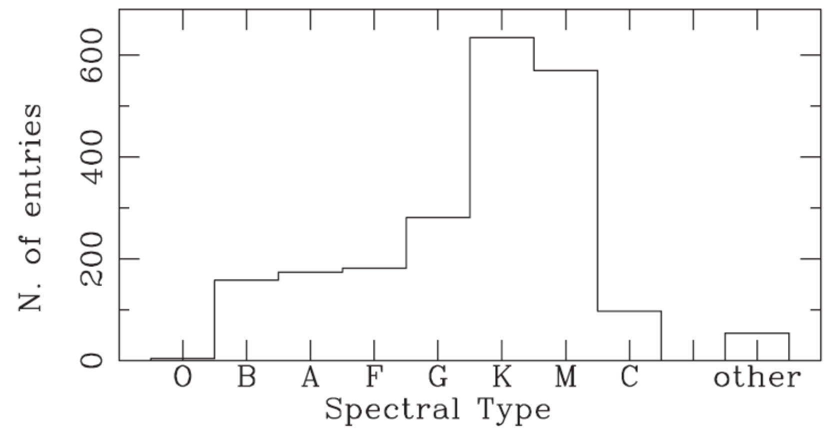


Fig. 6. Distribution of spectral type.

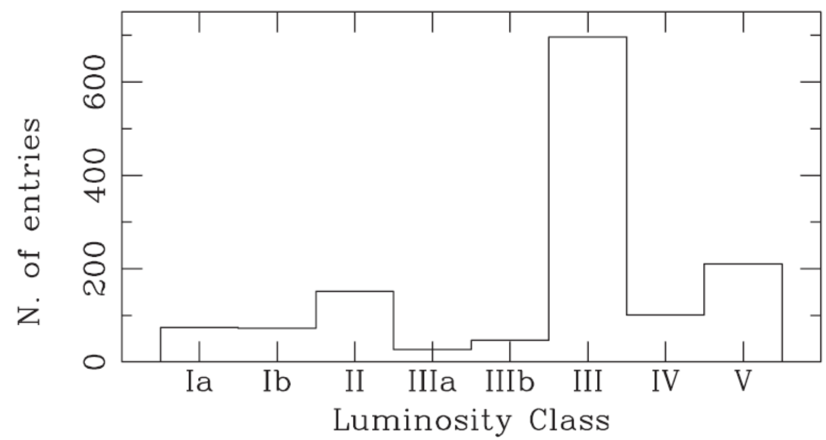


Fig. 7. Distribution of luminosity class.

Catalog of High Angular Resolution Measurements (CHARM)

Richichi & Percheron (2005), A&A, 431, 773

Effective Temperature Determinations

- Fundamental relation based on angular diameters and bolometric fluxes:

$$4\pi d^2 \int_0^\infty f_\lambda^\oplus d\lambda = 4\pi R^2 \sigma T_{\rm eff}^4 \quad \rightarrow \quad f_{\rm bol}^\oplus = \frac{\theta^2}{4}\,\sigma T_{\rm eff}^4 \;\;, \;\sigma = 5.67\times 10^{-5}\ {\rm erg\ s^{-1}\ cm^{-2}\ K^{-4}}$$

  - Relatively insensitive to errors in θ and $f_{\rm bol}^\oplus$
  - Very minor dependence on models (limb darkening)
  - Not applicable to every star (relatively few have accurate θ and $f_{\rm bol}^\oplus$)

- Infrared Flux Method (Blackwell et al. 1980, 1990): combines $f_{bol}^{\oplus}$ and NIR $f_{\lambda}^{\oplus}$ with model atmospheres to obtain T_{eff} (and θ)

$$f_{\rm bol}^{\oplus} = \frac{\theta^2}{4}\,\sigma T_{\rm eff}^4 \quad , \quad f_{\lambda}^{\oplus} = \frac{\theta^2}{4}\,\phi_\lambda(T_{\rm eff}, g, A)$$

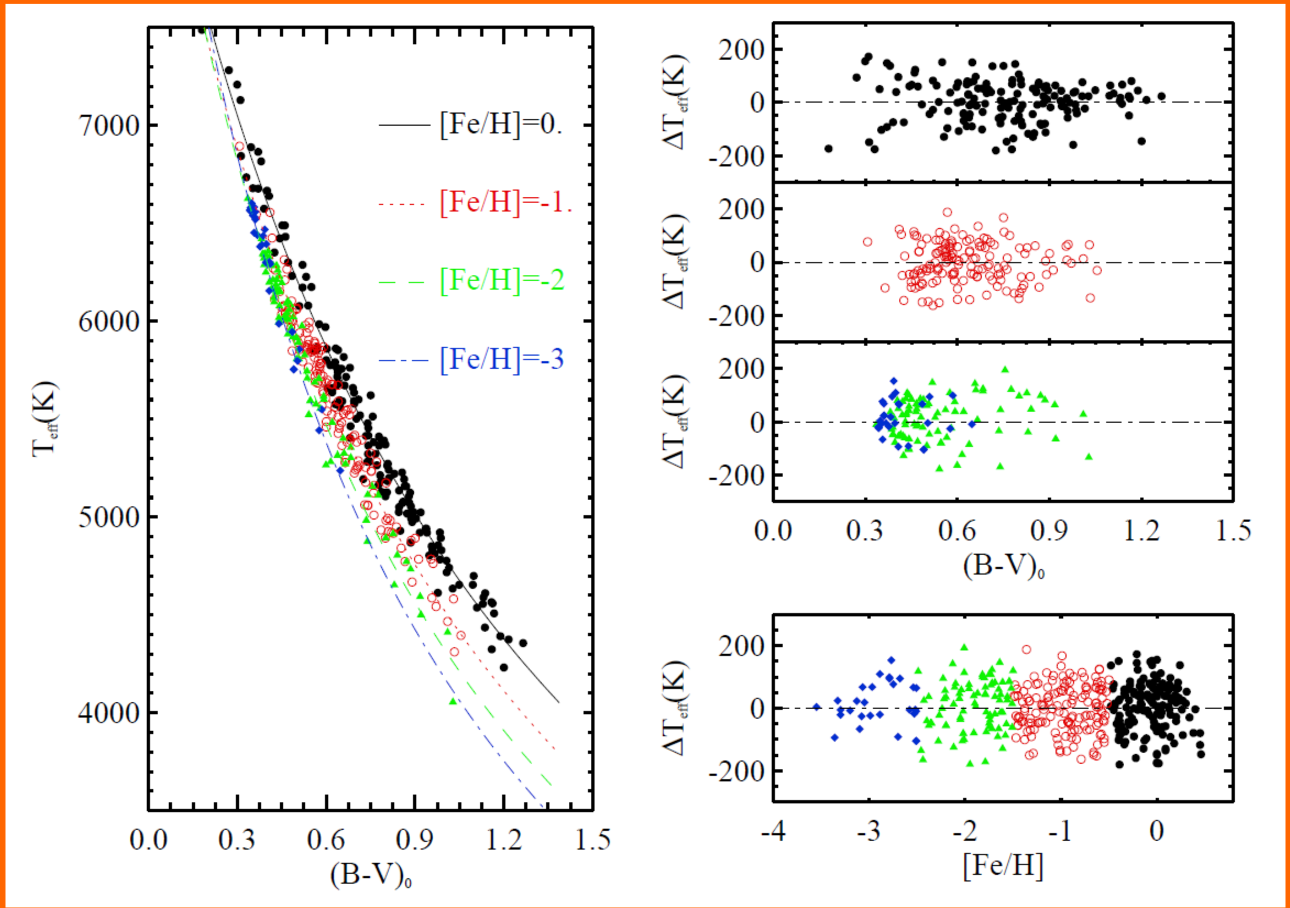
$$\frac{f_{\rm bol}^{\oplus}}{f_{\lambda}^{\oplus}} = \frac{\sigma T_{\rm eff}^4}{\phi_\lambda(T_{\rm eff}, g, A)} \equiv R_\lambda$$

Measured ← $\frac{f_{\rm bol}^{\oplus}}{f_{\lambda}^{\oplus}}$; $\frac{\sigma T_{\rm eff}^4}{\phi_\lambda(T_{\rm eff}, g, A)}$ → Can be calculated from models

  - $R_\lambda \propto T_{eff}^3$, and is relatively insensitive to extinction
  - Line blocking in the infrared is small, so result is insensitive to abundance; gravity effect is small

Practical determination of effective temperatures through color-temperature calibrations

González-Hernández & Bonifacio (2009), A&A, 497, 497



Caution:

Colors may be affected by interstellar reddening!

Luminosity Determinations

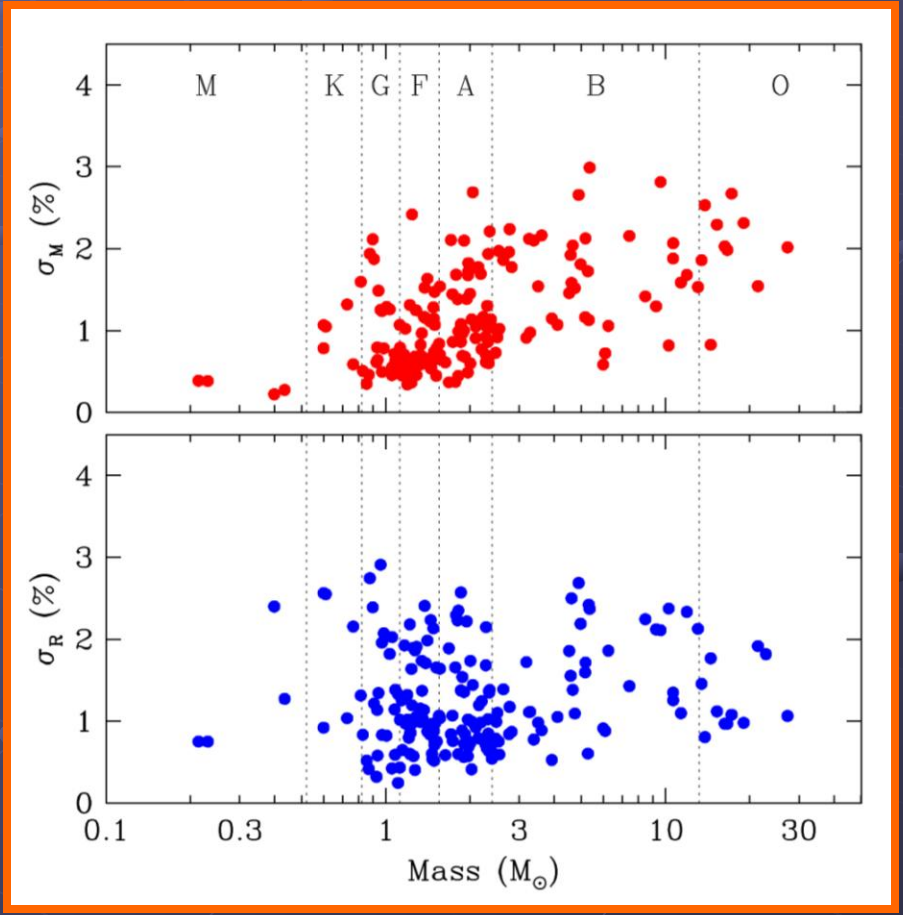
- From R and T_{eff} for eclipsing binaries (distance-independent, except for residual reddening effects on T_{eff}): $L \propto R^2\, T_{\mathrm{eff}}^4$
- From parallaxes for nearby stars
  - Trigonometric parallaxes (e.g., *HIPPARCOS*)
  - Orbital parallaxes in binaries:

$$\pi_{\mathrm{orb}} \propto \frac{a'' \sin i}{P\sqrt{1-e^2}(K_1+K_2)}$$

Progress in Knowledge of Accurate Stellar Masses and Radii Over the Years

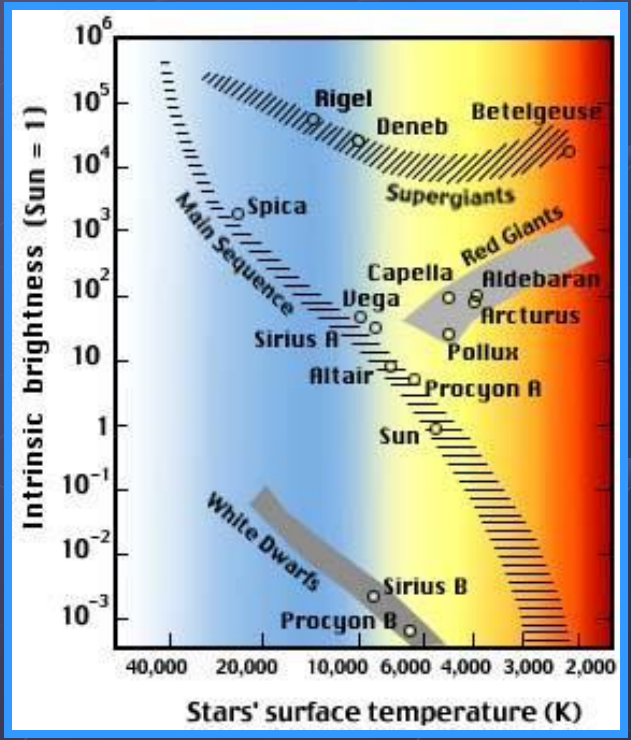
Binary Star Review	**σ_M, $\sigma_R \leq$ 3%**
Popper (1967)	2*
Popper (1980)	7
Andersen (1991)	45
Torres et al. (2010)	95

* Masses only



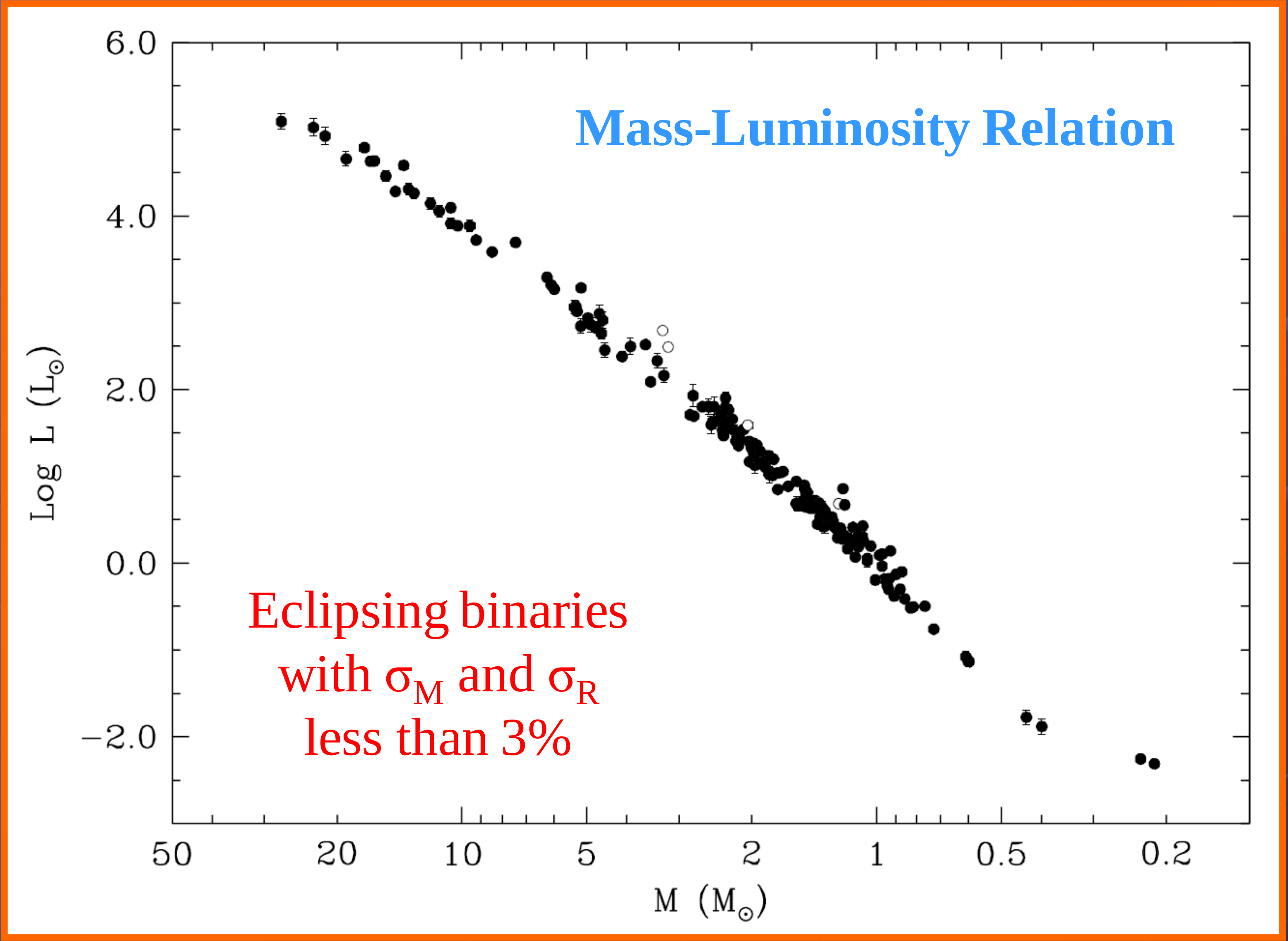
Status of Mass, Radius, and Temperature Determinations in Binaries

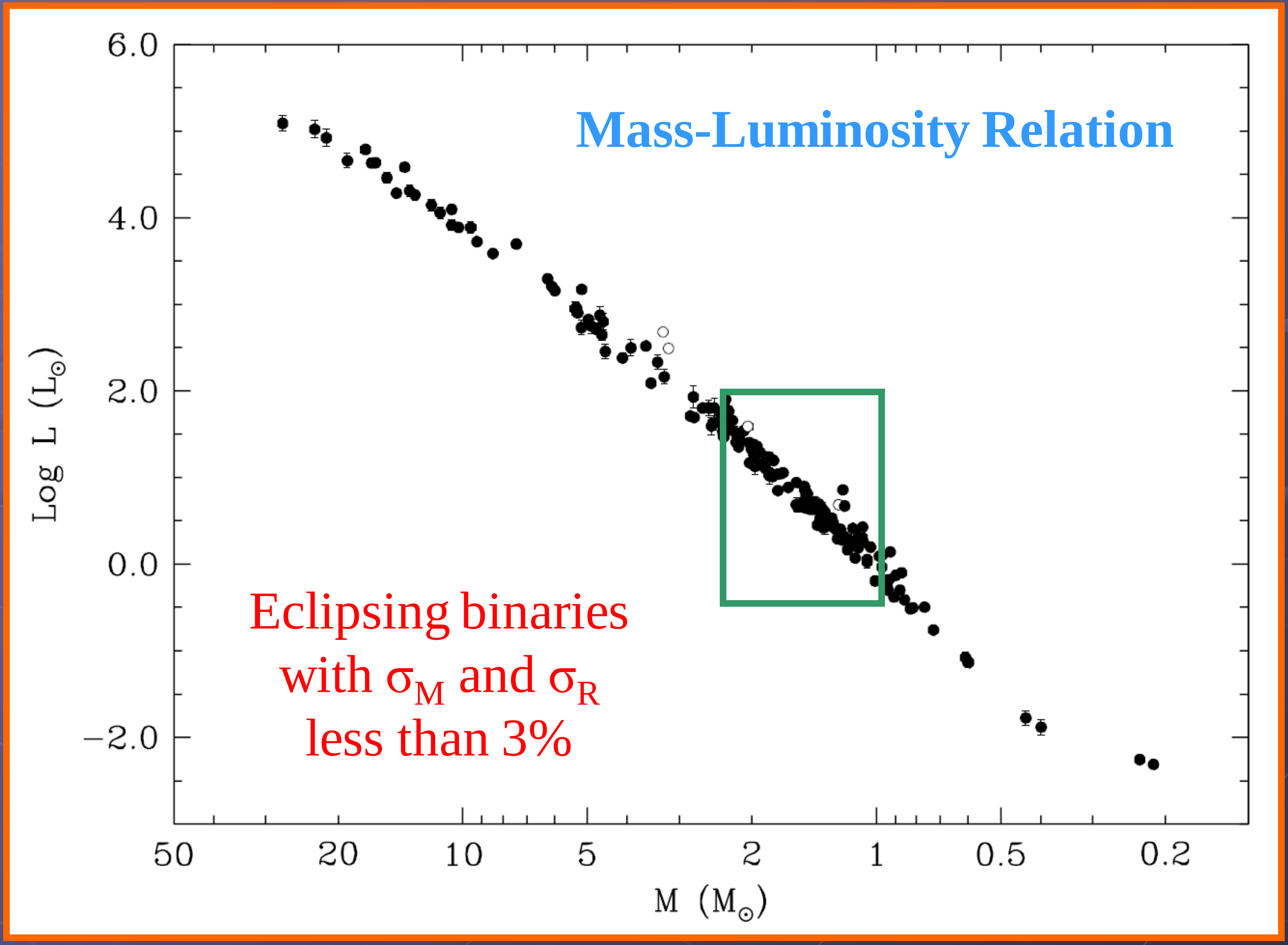
- Stars with the best determined properties (detached eclipsing binaries): less than 100 binary systems
- Areas of the H-R diagram that are well covered: ~1–10 $M_{\odot}$ main sequence stars
- Areas that need more work:
  - Low-mass stars
  - High-mass stars
  - Evolved stars (giants)
  - Pre-main sequence stars
  - Metal-poor stars

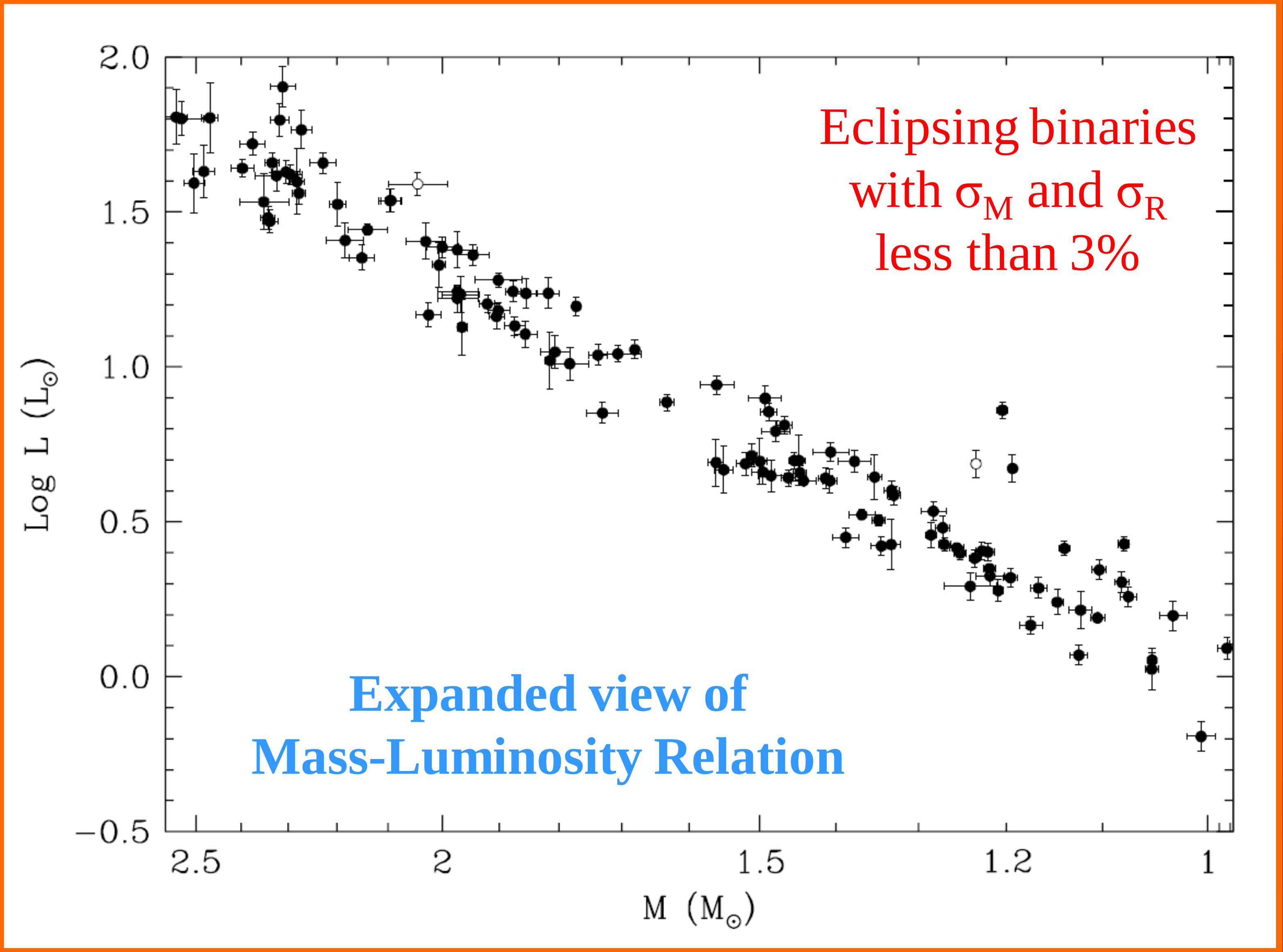


Determining Masses for Exoplanet Hosts

- Obtaining very precise (and accurate) absolute masses in binaries is not always easy, and requires high-quality observations
- Dynamical (fundamental = hypothesis-free) masses cannot be determined for single stars, so are not directly measurable for planet hosts (except HD 209458!)
- Asteroseismology
- Can we infer masses accurately from some other more easily measurable property such as the luminosity (if the parallax is known)? → The empirical *M*-*L* relation

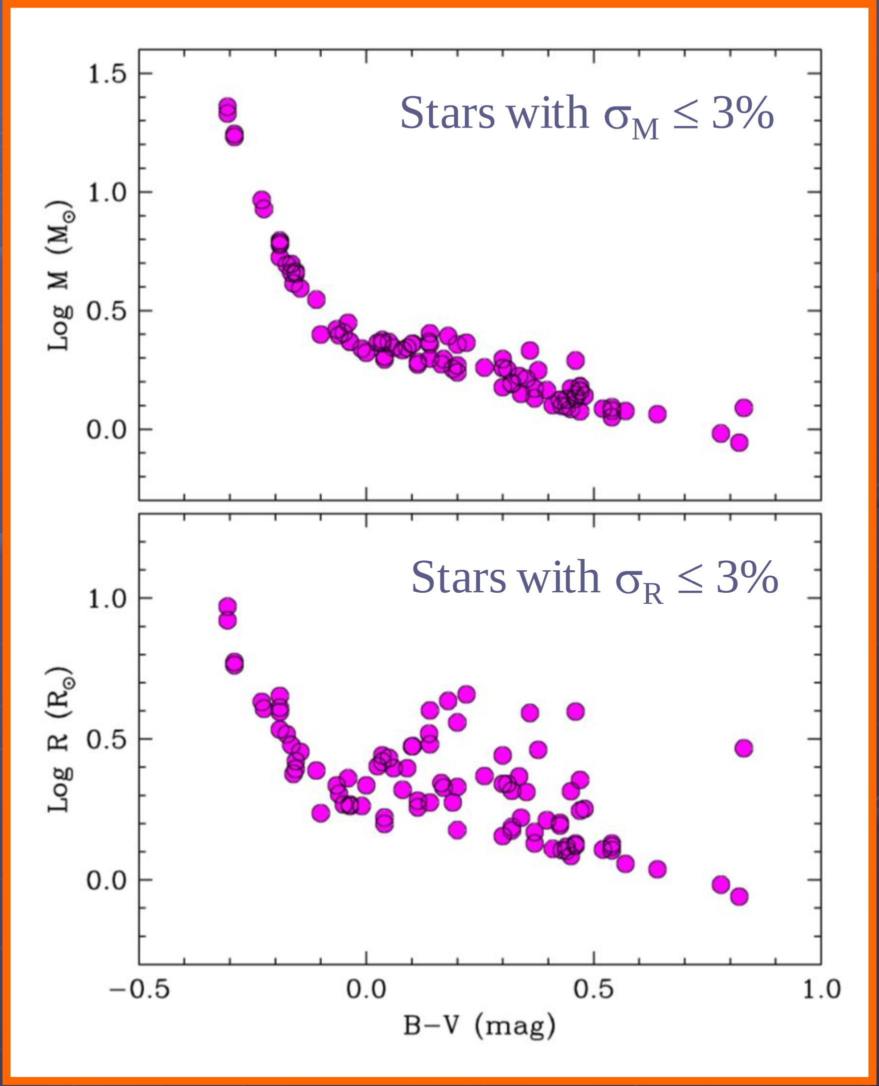

Mass-Luminosity Relation
Eclipsing binaries
with σ_M and σ_R
less than 3%
6.0
4.0
2.0
0.0
−2.0
Log L (L⊙)
50
20
10
5
2
1
0.5
0.2
M (M⊙)


Mass-Luminosity Relation
Eclipsing binaries
with σ_M and σ_R
less than 3%
Log L ($L_\odot$)
6.0
4.0
2.0
0.0
−2.0
M ($M_\odot$)
50
20
10
5
2
1
0.5
0.2


Eclipsing binaries with σ_M and σ_R less than 3%
Expanded view of Mass-Luminosity Relation
Log L ($L_\odot$)
M ($M_\odot$)
2.0
1.5
1.0
0.5
0.0
−0.5
2.5
2
1.5
1.2
1

Predicting properties of single stars

- For a given spectral type or color index, errors can be σ_M ~ 15% and σ_R ~ 50%, or more
- Abundance and age effects in *M* and *R* are significant
- Bi-parametric fits including a luminosity indicator do somewhat better
- Averaging any number of accurate masses and radii in these empirical relations will not help



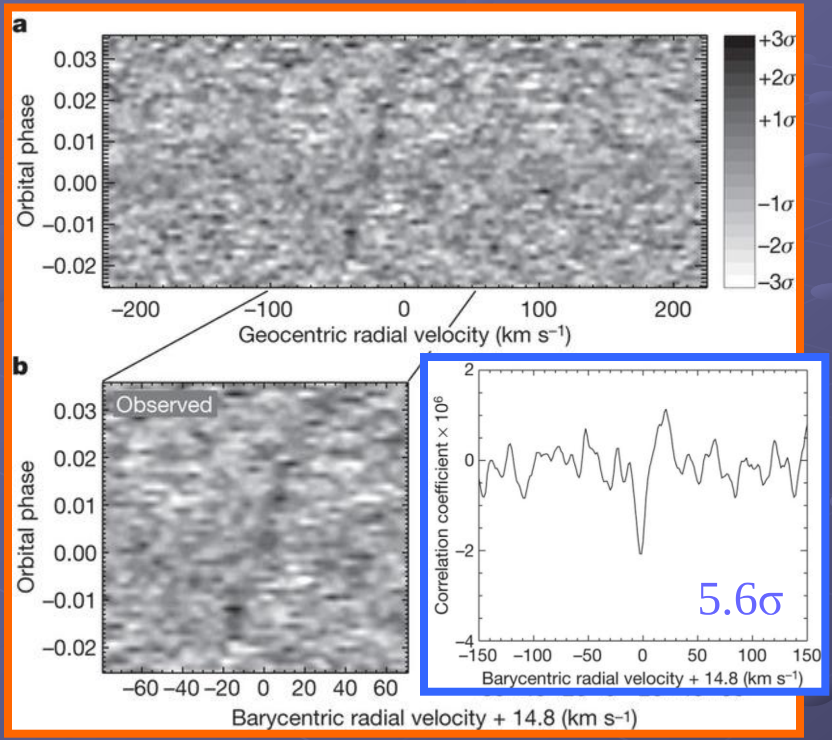
Summary:
M, R, and T_{eff} for Exoplanet Hosts

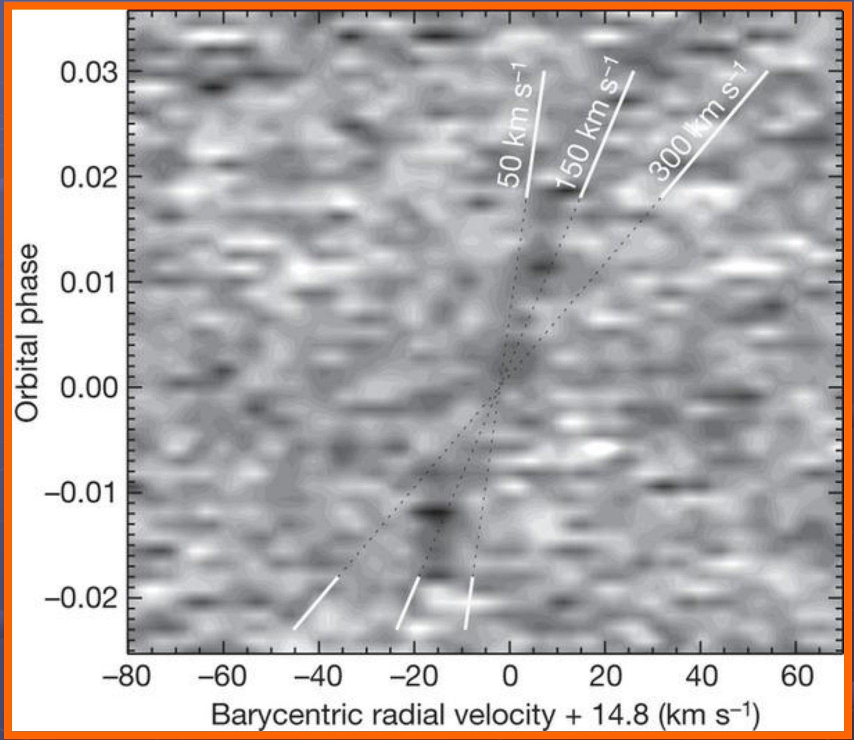
- Masses are usually inferred from stellar evolution models, with constraints from observations
  - Spectroscopic: T_{eff}, [Fe/H], log g (weak constraint)
  - Photometric: Transit light curves → mean stellar density: $\rho_* \approx \frac{3\pi}{GP^2}\left(\frac{a}{R_*}\right)^3$
  - Astrometric: Parallax
- Radii
  - Directly from angular diameters, if parallax is known
  - Inferred from transit light curves, with knowledge of the mass
- Effective temperatures
  - From the fundamental relation, or the Infrared Flux Method
  - From spectroscopy (relies on stellar atmosphere models)
  - Indirectly from color indices, and empirical calibrations
- Asteroseismology can provide independent constraints on M and R in some cases

Suggested Reading

- Alonso, A. et al. 1996, *The empirical scale of temperatures of the low main sequence (F0V-K5V)*, A&A, 313, 873
- Andersen, J. 1991, *Accurate masses and radii of normal stars*, A&ARv, 3, 91
- Andersen, J. 1997, *Precise stellar mass and mass-luminosity data,* IAU Symp. 189, 99
- Binnendijk, L. 1960, *Properties of double stars; a survey of parallaxes and orbits*, Univ. of Philadelphia Press
- Blackwell, D. E. et al. 1980, *Use of the infra-red flux method for determining stellar effective temperatures and angular diameters – The stellar temperature scale*, A&A, 82, 249
- Blackwell, D. E. et al. 1990, *Determination of temperatures and angular diameters of 114 F-M stars using the infrared flux method (IRFM)*, A&A, 232, 396
- Girardi, L. et al. 2000, *Evolutionary tracks and isochrones for low- and intermediate-mass stars: from 0.15 to 7* $M_{\odot}$*, and from Z=0.0004 to 0.03*, A&AS, 141, 371
- Heintz, W. D. 1973, *Double stars*, Reidel Pub. Co.
- Henry, T. J. & McCarthy, D. W. 1993, *The mass-luminosity relation for stars of mass 1.0 to 0.08 solar mass*, AJ, 106, 773
- Henry, T. J. et al. 1999, *The Optical Mass-Luminosity Relation at the End of the Main Sequence (0.08–0.20* $M_{\odot}$*)*, ApJ, 512, 864
- Popper, D. M. 1980, *Stellar masses*, ARA&A, 18, 115
- Torres, G. et al. 2010, *Accurate masses and radii of normal stars: modern results and applications*, A&ARv, 18, 67
- Yi, S. et al. 2003, *The* Y^2 *Stellar Evolutionary Tracks*, ApJS, 144, 259

Orbital motion of HD 209458b measured directly (Snellen et al. 2010, Nature, 465, 1049)





51 VLT/CRIRES spectra with *R* ≈ 100,000 taken during transit, cross-correlated against a CO template with 56 lines

$M_* = 1.00 \quad 0.22\ M_\odot$
$M_p = 0.64 \quad 0.09\ M_{Jup}$