Thanks to...

• Thomas Beatty
• Chris Burke
• Andrew Gould
• Gabriella Mallen-Ornelas
• Josh Pepper
• Frederic Pont
• Sara Seager
• Andrzej Udalski
Why Transit Surveys?

1. Find Planets!

2. Unusual/Extreme Populations

Statistics of Transit Surveys

- Statistics ⇔ Predicting planet yields

- Planning surveys ⇔ Planet yields
Predicting Planet Yields

Two approaches:

- **Backward**

\[
\langle N \rangle = \sum_k P_{\text{planet},k}(M, [\text{Fe/H}], P, r, ...) P_{\text{transit},k}(R, P, i) P_{\text{detect},k}(R, F, r, P, ...)
\]

- **Forward**

\[
\frac{d^n \langle N \rangle}{dM dR dP dr dld\Omega d...} = \frac{dn(l)}{dM dR dL} \frac{df}{dP dr} P_{\text{transit}}(R, P, i) P_{\text{detect}}(R, F, r, P, ...) l^2
\]

- **Hybrid: Forward constrained by # of stars**
Backward Approach

\[
\langle N \rangle = \sum_{k} P_{\text{planet}, k} (M, [\text{Fe/H}], P, r, ...) P_{\text{transit}, k} (R, P, i) P_{\text{detect}, k} (R, F, r, P...)
\]

- **Most Exact**
- **Difficult to implement**
  - Requires knowledge of stellar properties
  - Not widely applicable
- **Only robust for stellar systems**
Yields for Stellar Systems

- **General theory**
  Janes (1996), Gaudi (2000),
  von Braun et al. (2005),
  Pepper & Gaudi (2005, 2006)

- **One Parameter - Mass**

(Hartman et al., in prep)
Yields for Field Surveys

- General theory
  Pepper, Gould & DePoy (2003)

- Unknown Properties
  (distance, reddening)
  (mass, radius)

- Backward modeling hard
  (Hartman et al., in prep)
Forward Modeling

\[
\frac{d^n \langle N \rangle}{dM dR dP d r d l d \Omega d \ldots} = \frac{dn(l)}{dM dR dL} \frac{df}{dP dr} P_{\text{transit}}(R, P, i) P_{\text{detect}}(R, L, r, P \ldots) l^2
\]

- Distribution of stellar properties
- Probability planet will transit
- Probability planet will be detected
- Distribution of planet properties
- Volume element

\(l\)
Ingredients

- Distribution of stellar properties
  - Mass, radius, luminosity, metallicity dist.
  - Variation along the line of sight

- Distribution of planet properties
  - Period, radius distribution, metallicity, correlations
  - Dependence on mass?

- Transit probability

- Detection probability
  - Probability that a planet passes all cuts
  - Confirmed by high-precision RV follow-up
Detection Probability

Must account for all cuts consistently

• Algorithmic cuts (BLS: $\alpha$, SDE)
  – Limb darkening?
  – Ingress/Egress?
• Number of transits
• By-eye selection
• Magnitude limit/RMS cut
• Color cut
• RV follow-up
• Saturation
Lessons I: Simple Estimates

Fail

- Naïve estimate:

\[
\langle N \rangle \approx fP_{\text{transit}} N <_{\sigma} = 1\% \times 10\% \times 10^3 = 1
\]

Actual rates... one in $10^4$ or $10^5$
Why does the naïve estimate fail?

- Typical planet size
- Large star contamination
- S/N requirements
  - 1% not necessarily sufficient
  - Correlated errors & systematics
- Transit probability varies
- Multiple transits & aliasing
- Precision RV follow-up

(Bakos et al. 2006)
Why does the naïve estimate fail?

- Typical planet size
- **Large star contamination**
- S/N requirements
  - 1% not necessarily sufficient
  - Correlated errors & systematics
- Transit probability varies
- Multiple transits & aliasing
- Precision RV follow-up

(Gould & Morgan 2003)
Why does the naïve estimate fail?

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- Large star contamination
- **S/N requirements**
  - 1% not necessarily sufficient
  - Correlated errors & systematics (Pont et al 2006)
- Transit probability varies
- Multiple transits & aliasing
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(Pont et al 2006)
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\[ P_{\text{transit}} \approx \frac{R}{a} \propto M^{-1/3} R P^{-2/3} \]
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(Gaudi et al 2003)
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(O’Donovan et al 2006)
Lessons II: Selection Effects

\[ \langle N \rangle \approx \frac{\Omega}{3} nP_{\text{transit}} l_{\text{max}}^3 \]
Lessons II: Selection Effects

\[ \frac{S}{N} \approx N^{1/2} \frac{\delta}{\sigma} \] (white noise)

\[ N \approx \frac{R}{\pi a} N_{\text{total}} \]

\[ \sigma \approx N^{-1/2}_{\text{photons}} \propto L^{-1/2} l \]

\[ \delta \approx \left( \frac{r}{R} \right)^2 \]

\[ \frac{S}{N} \propto R^{-3/2} M^{-1/6} L^{1/2} r^2 P^{-1/3} l^{-1} \]
Lessons II: Selection Effects

\[
\frac{S}{N} \propto R^{-3/2} M^{-1/6} L^{1/2} r^2 P^{-1/3} l^{-1}
\]

\[
R \propto M, \quad L \propto M^{7/2}
\]

\[
\frac{S}{N} \propto M^{1/12} r^2 P^{-1/3} l^{-1}
\]

At fixed distance, S/N nearly independent of mass!
Lessons II: Selection Effects

\[ l_{\text{max}} \propto M^{1/12} r^2 P^{-1/3} \left( \frac{S}{N} \right)^{-1} \]

\[ P_{\text{transit}} \approx \frac{R}{a} \propto M^{-1/3} R P^{-2/3} \]

\[ \langle N \rangle \approx \frac{\Omega}{3} n P_{\text{transit}} l_{\text{max}}^3 \]

\[ \langle N \rangle \propto P^{-5/3} r^6 \left( \frac{S}{N} \right)^{-3} \left( \frac{S}{N} \right)_{\text{min}} \]

(Gaudi et al. 2003, Gaudi 2005)
Lessons II: Selection Effects

• S/N-limited transit surveys have strong biases
  – Favor short periods
  – # of detections strong function of S/N
  – Overwhelmingly favor large planets

(Gaudi et al. 2003, Gaudi 2005, Pont et al. 2006)

\[ \langle N \rangle \propto P^{-5/3} r^6 \left( \frac{S}{N} \right)^{-3}_{\text{min}} \]
Lessons III: Correlated Noise

Most ground-based transit surveys are subject to correlated (i.e. red) noise

- Measurements correlated on transit timescales
- Fundamental limit to noise
- Changes the statistics

(Pont et al. 2006)
Lessons III: Correlated Noise

Relation between red and white noise

\[
\left( \frac{S}{N} \right)_r \approx N_{tr}^{1/2} \frac{\delta}{\sigma_{red}}
\]

\[
\left( \frac{S}{N} \right)_w \approx N^{1/2} \frac{\delta}{\sigma}
\]

\[
\left( \frac{S}{N} \right)_r = \left( \frac{S}{N} \right)_w \left( \frac{\sigma}{\sigma_r} \right) \left( \frac{N}{N_{tr}} \right)^{-1/2} \approx \left( \frac{S}{N} \right)_w n_k^{-1/2}
\]

Effective S/N considerably lower with correlated noise

(Pont et al. 2006)
Lessons III: Correlated Noise

\[
\frac{S}{N} \approx N_{tr}^{1/2} \frac{\delta}{\sigma_{red}}
\]

(\textit{red noise})

\[
N_{tr} \approx \frac{T}{P} f
\]

\[
\sigma_{red} \approx \text{constant}
\]

\[
\delta \approx \left(\frac{r}{R}\right)^2
\]

\[
\frac{S}{N} \propto R^{-2} r^2 P^{-1/2}
\]
Lessons III: Correlated Noise

When correlated noise dominates:

• Effective S/N considerably lower

• Detectability doesn’t depend on stellar brightness!

• Strong (inverse) dependence on stellar mass

• No volume effect - Threshold statistics
  • Stronger aliasing effects

• Changes the optimal observing strategy
Lessons IV: All Regimes

- Correlated noise
- Source noise
- Sky noise
  - often provides the fundamental limit
- Other noise sources
  - scintillation

(Hartman et al. 2006)
Future Considerations: Eccentricity

- Work by Chris Burke
- Eccentric orbits change detectability
  - Changes transit duration
    - Shorter near periastron, longer near apastron
  - Changes transit probability
    - Higher transit probability near periastron
Results

Upper Limits:
• 47 Tuc
• Open Clusters
• Field
• OGLE

Detailed analysis
– HJ Frequency = 1/310
– VHJ Frequency = 1/690

Comparison with RV
– metallicity bias

Gilliland et al. 2000
Weldrake et al. 2005
Results

Upper Limits:
• 47 Tuc
• **Open Clusters**
• Field
• OGLE

• Detailed analysis
  – HJ Frequency = 1/310
  – VHJ Frequency = 1/690

• Comparison with RV
  – metallicity bias

PISCES - Mochejska et al. (2005, 2006)
STEPSS - Burke et al. (2006)
Results

Upper Limits:
• 47 Tuc
• Open Clusters
• Field

OGLE
• Detailed analysis
  – HJ Frequency = 1/310
  – VHJ Frequency = 1/690
• Comparison with RV
  – metallicity bias

Hood et al. (2006)
Bramich & Horne (2006)
Results

Upper Limits:
- 47 Tuc
- Open Clusters
- Field

OGLE (seasons 1+2)
- Detailed analysis
  - Detailed forward model
  - Selection effects
- Comparison with RV
  - metallicity bias

Gould et al. (2006), Fressin et al. (2007)
OGLE vs RV

OGLE & RV consistent
Metallicity bias
⇒ generically expect fewer transiting planets

Gould et al. (2006)
Planet radii from OGLE

Bloated planets are rare.
Weak constraints on sub-Jupiter sized planets.

Gould et al. (2006)
Increasing sophistication

For OGLE; consistent with Gould et al.
General agreement with RV + planet models
Predictions

- Extended model (w/ Thomas Beatty, CfA)
  - Galactic structure
  - Signal-to-noise ratio detection criteria
  - Noise sources (source, sky, scintillation, saturation, red & white noise)
  - Magnitude limit(s)
  - Mass, radius, effective temperature distribution
  - Arbitrary bandpasses
  - Visibility/transit requirements
  × False Positives
  × Blending
  × Binaries
Predictions: XO

Predicted XO Detections for V<12, S/N > 20, r=R_j

<table>
<thead>
<tr>
<th>00 hrs</th>
<th>04 hrs</th>
<th>08 hrs</th>
<th>12 hrs</th>
<th>16 hrs</th>
<th>20 hrs</th>
<th>Total</th>
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<tr>
<td>0.45</td>
<td>0.47</td>
<td>0.45</td>
<td>0.37</td>
<td>0.42</td>
<td>0.51</td>
<td>2.67</td>
</tr>
</tbody>
</table>

(Beatty & Gaudi, In prep)
Predictions: TrES

Predicted TrES Detections for R<13, S/N > 20, r=R_j

<table>
<thead>
<tr>
<th></th>
<th>And0</th>
<th>Cyg1</th>
<th>Cas0</th>
<th>Per1</th>
<th>UMa0</th>
<th>CrB0</th>
<th>Lyr1</th>
<th>And1</th>
<th>And2</th>
<th>Tau0</th>
<th>UMa1</th>
<th>Total</th>
<th>Total Red (S/N) &gt; 8</th>
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</thead>
<tbody>
<tr>
<td>VHJs</td>
<td>0.30</td>
<td>0.33</td>
<td>0.29</td>
<td>0.27</td>
<td>0.14</td>
<td>0.15</td>
<td>0.28</td>
<td>0.22</td>
<td>0.18</td>
<td>0.21</td>
<td>0.13</td>
<td>2.53</td>
<td>1.46</td>
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<tr>
<td>HJs</td>
<td>0.31</td>
<td>0.31</td>
<td>0.29</td>
<td>0.28</td>
<td>0.14</td>
<td>0.17</td>
<td>0.27</td>
<td>0.19</td>
<td>0.11</td>
<td>0.16</td>
<td>0.14</td>
<td>2.35</td>
<td>1.00</td>
</tr>
<tr>
<td>Both</td>
<td>0.61</td>
<td>0.64</td>
<td>0.58</td>
<td>0.55</td>
<td>0.28</td>
<td>0.32</td>
<td>0.55</td>
<td>0.41</td>
<td>0.29</td>
<td>0.37</td>
<td>0.27</td>
<td>4.86</td>
<td>2.46</td>
</tr>
</tbody>
</table>

(Beatty & Gaudi, in prep)
## Predictions: S/N limit

<table>
<thead>
<tr>
<th></th>
<th>VHJ</th>
<th>HJ</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/N &gt; 10</td>
<td>3.96</td>
<td>4.47</td>
<td>8.43</td>
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<tr>
<td>S/N &gt; 15</td>
<td>3.23</td>
<td>3.30</td>
<td>6.53</td>
</tr>
<tr>
<td>S/N &gt; 20</td>
<td>2.53</td>
<td>2.35</td>
<td>4.88</td>
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<tr>
<td>S/N &gt; 25</td>
<td>1.94</td>
<td>1.66</td>
<td>3.60</td>
</tr>
<tr>
<td>S/N &gt; 30</td>
<td>1.49</td>
<td>1.19</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Predicted TrES Detections for R < 13, r = R_j
Predictions: Space Surveys

- **Kepler**
  - $HJ+VHJ \sim 54$
  - Total # $J \sim 80$
  - Most fairly faint
  - TrES-2 likely only one with $V<12$
  - 50 Habitable Earths

- **CoRoT**
  - May find as many or more Jupiters

(Beatty & Gaudi, in prep)
Summary

• Interpretation and predictions require accurate simulations
• Naïve estimates fail.
• Strong selection effects.
• Correlated noise important
• OGLE surveys consistent with RV
• Current surveys require careful modelling
• Must include all selection cuts consistently
• XO & TrES predicted to have handfuls of detections