Pupil Planes versus Image Planes
Comparison of beam combining concepts

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Outline

• Aims of this presentation
• Beam combiner functions
• Image plane vs Pupil plane
• Multiplexing multiple baselines
  – Cross-talk
• Field of View Issues
• Summary
Aims of Presentation

I am aiming to get across the following:
• Common features of image plane and pupil plane combination
• Differences
• Trades-off in combiner design
• Some instrument-related issues in interpreting visibility data
Beam Combiner Functions

• Generate fringe pattern(s) suitable for recording with detectors
• Want fringes on many interferometer baselines
  – Amplitude and phase of fringes on each baseline encode amplitude and
    phase of one Fourier component of source brightness distribution
• Want high-signal-to-noise fringes
  – Small collectors, low throughput; hence few photons
  – Atmosphere usually forces short integration times
• Since we cannot coherently amplify our signals, the previous two
  requirements usually conflict
Beam Combination

• The essential principle here is:
• Add the $E$ fields, $E_1 + E_2$, and then detect the time averaged intensity:

\[
\langle (E_1+E_2)(E_1+E_2)^* \rangle = \langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle + \langle E_1E_2^* \rangle + \langle E_2E_1^* \rangle \\
= \langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle + \langle 2|E_1||E_2| \cos \phi \rangle
\]

where $\phi$ is the phase difference between $E_1$ and $E_2$

• In practice there are two straightforward ways of doing this:
  – Image plane combination:
    • e.g. AMBER (VLTI), MIRC (CHARA), aperture masking experiments
  – Pupil plane combination:
    • e.g. NPOI, IOTA
Image Plane (Multi-Axial) Combination

- Mix the signals in a focal plane as in a Young’s slit experiment:

- In the focused image the transverse co-ordinate measures the delay

- Fringes encoded by use of a non-redundant “input” pupil

- Possible to use dispersion prior to detection in the direction perpendicular to the fringes
Pupil Plane (Co-Axial) Combination

- Mix the signals by superposing *afocal* beams:
- Focus superposed beams onto a single element detector
- Fringes encoded by use of a non-redundant modulation of delay of each beam
- Fringes are recorded by measuring intensity versus time
- Spectral dispersion can be used prior to detection
Integrated Optics Combiners/Fibre Couplers

- Co-axial combination in a waveguide
- Single-mode waveguide performs spatial filtering “for free”
- Everything I will say about “pupil plane” combination (usually refers to free-space co-axial combination) applies equally to IO unless otherwise stated
  - IO facilitates using static delays to encode fringes, rather than active modulation
Multiplexing – Image Plane

- Use non-redundant “input” pupil
- 1d pupil allows spectral dispersion perpendicular to fringes
- 2d pupil requires fewer detector pixels per baseline
- Possible to use optical fibres to remap pupil
Multiplexing – Pupil Plane

- Mix beams at successive beam-splitters (couplers)
- Modulate delays of input beams so that each baseline has a unique net velocity

- Fringe signal then appears at unique frequency for each baseline
Multiplexing – Pupil Plane

- Can use symmetry to decrease number of optical components
- The combiners below are functionally equivalent
  - Apart from angles of incidence
Signal-to-Noise Comparison

- Buscher (1988) showed that (all-one-one) pupil-plane and image-plane implementations give identical signal-to-noise, provided:
  - Noise-free detector
  - Fringe scanned in $\ll t_0$
  - Can coherently combine signals from all outputs of pupil-plane combiner
- Choice driven by practical considerations
  - Detector format & performance
  - Cost of detector(s)
  - Cross-talk/calibration
  - Alignment/stability
  - Spectral bandwidth
Crosstalk – Pupil Plane

• Delays of input beams are also being changed by atmosphere
  – Perhaps just residual from external fringe tracking
• This perturbs delay velocities
  – Smears fringe signal in frequency space
  – Peaks in power spectrum are broadened – can overlap unless fringe frequencies are well-separated ⇒ fast modulators and detectors
• Non-linear modulation also causes cross-talk
  – Mitigate with novel demodulation algorithms – see Thorsteinsson & Buscher (2004)
Crosstalk – Pupil Plane

- Best to coherently integrate forward and reverse scan together
  - Cancels slowly-varying part of leaked signal
- In this case $T_{\text{scan}} = 0.4t_0$ gives 1% fringe power leakage
Field of View: Co-Axial Combination

Condition for the off-axis object to contribute to the main fringe pattern:

Hence the field of view:

$$\alpha_{\text{max}} = \frac{\lambda}{B} \times \frac{\lambda}{\Delta \lambda}$$

*FOV is product of the spatial and spectral resolutions*
Golden Rule (Traub)

- FOV of an image-plane interferometer maximised when exit pupil is \textit{scaled version} of entrance pupil
  - Entrance pupil: array of collector pupils as seen from target
  - Exit pupil: input pupil of beam combiner
- Instruments that implement this are called \textit{homothetic mappers}
- If golden rule violated, FOV limited because white-light fringe for off-axis object doesn’t coincide with centre of its light
Homothetic Mapping: How To

• Easy way
  – Collectors on common mount
  – e.g. aperture masking, LBT

• Hard way
  – Collectors on independent mounts
  – Active relay optics to continuously adjust pupil mapping as Earth rotates
  – e.g. ……
Densified Pupils: “Hypertelescopes”

- Violate golden rule to concentrate light in fewer pixels
- Reduced field of view
- Aimed at direct imaging i.e. not via visibility measurement
  - Fringe pattern approximates target field convolved with compact PSF
FOV Limits

• Need to consider which of following give rise to FOV lower limit for each baseline of each observation:
  – FOV of collectors
  – Isoplanatic patch
  – FOV of interferometer optical train
  – Beam Combiner configuration – OPD effects
  – Spatial Filters
• For a dilute-aperture array, the above list is *usually* in order of decreasing FOV
  – Exchange the last two for lower spectral resolutions
• Remember that only the Fourier components corresponding to your projected baselines are sampled
  – Cannot image fields with many filled pixels unless many collectors
Interferometric (coherent) versus incoherent FOV

- In general, FOV over which target will contribute to measured fringe power (correlated flux) ≠ FOV for detected incoherent flux.
- Visibility amplitude est. is ratio of coherent to incoherent flux:
  \[ \text{ratio of energies} = \frac{|V|^2}{4} \]

- Incoherent field ≥ coherent (interferometric) field
  - Each part of field can contribute just DC signal, or both DC and fringe power, or not at all
- Centres of coherent and incoherent fields may not coincide precisely e.g. if target has non-uniform colour
  - Centre of coherent field related to fringe-tracking centre
  - Centre of incoherent field related to guiding centre
FOV Limits (again)

• Need to consider which of following give rise to FOV lower limit for each baseline of each observation:
  – FOV of collectors – limits incoherent field
  – Isoplanatic patch – limits coherent field
  – FOV of interferometer optical train – limits incoherent field
  – Beam Combiner configuration (OPD effects) – limits coherent field
  – Spatial Filters – limits incoherent field

• For a dilute-aperture array, the above list is usually in order of decreasing FOV
  – Exchange the last two for lower spectral resolutions
Restricted FOV effects

- Some examples:
  - coherent FOV = incoherent FOV < target size
    Interferometer “sees” smaller target => overestimates visibility

  - coherent FOV < target size < incoherent FOV
    Extra incoherent flux reduces visibility => net under- or overestimate

- Remember effects will have different magnitude on different baselines, so data need careful interpretation
Summary: Pupil planes versus Image planes

Image Plane Pros:
- Allows homothetic configuration to access larger fields

Image Plane Cons:
- Need large format detectors
- Usually need highly anamorphic optics to realise spectral resolution

Pupil Plane Pros:
- Fewer detector pixels needed

Pupil Plane Cons:
- Need fast modulators, fast detectors
- Cross-talk
- Potentially many optical components (not with IO or contacted optics)