Equatorial Superrotation on Tidally Locked Exoplanets

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Synopsis

- Most 3D atmospheric circulation models of tidally locked exoplanets—both gas giant and terrestrial—exhibit a circulation dominated by a fast eastward jet at the equator ("equatorial superrotation").

- This explains the fact that, according to Spitzer light curves, the hottest photospheric regions on HD 189733b are displaced eastward by tens of degrees longitude.

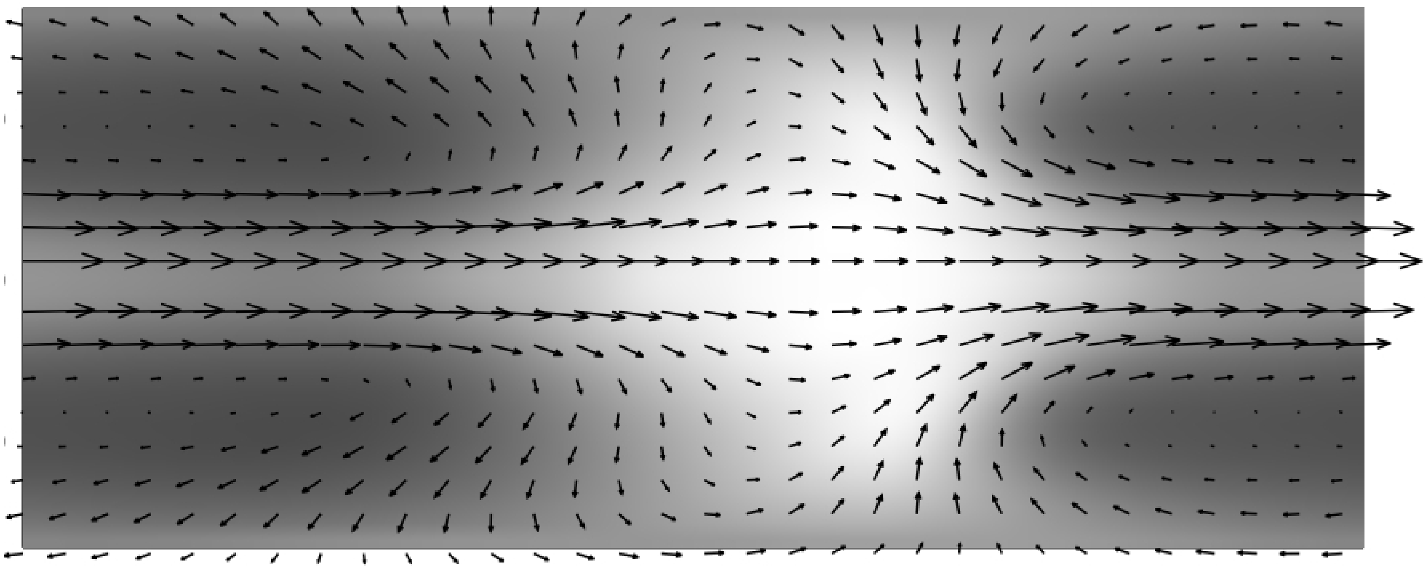
- However, the dynamical mechanisms that generate equatorial superrotation in these models have not previously been identified. Here, we show that the superrotation in these 3D models results from interaction of standing, planetary-scale Rossby and Kelvin waves with the mean flow.

- We demonstrate, analytically and numerically, that these waves are a direct response to the planetary-scale day-night heating contrast, and that they develop phase tilts that pump eastward momentum to the equator, thereby generating equatorial superrotation.

- Implications:
  - Jet results from direct, weakly nonlinear interaction of waves with mean flow. Turbulent eddy-eddy interactions are not necessary.
  - Jet width is set by the Rossby deformation radius ($\sim$planetary radius on typical hot Jupiters).
  - The theory provides a 0th-order understanding of the circulation regime relevant to a wide class of hot Jupiters and super Earths

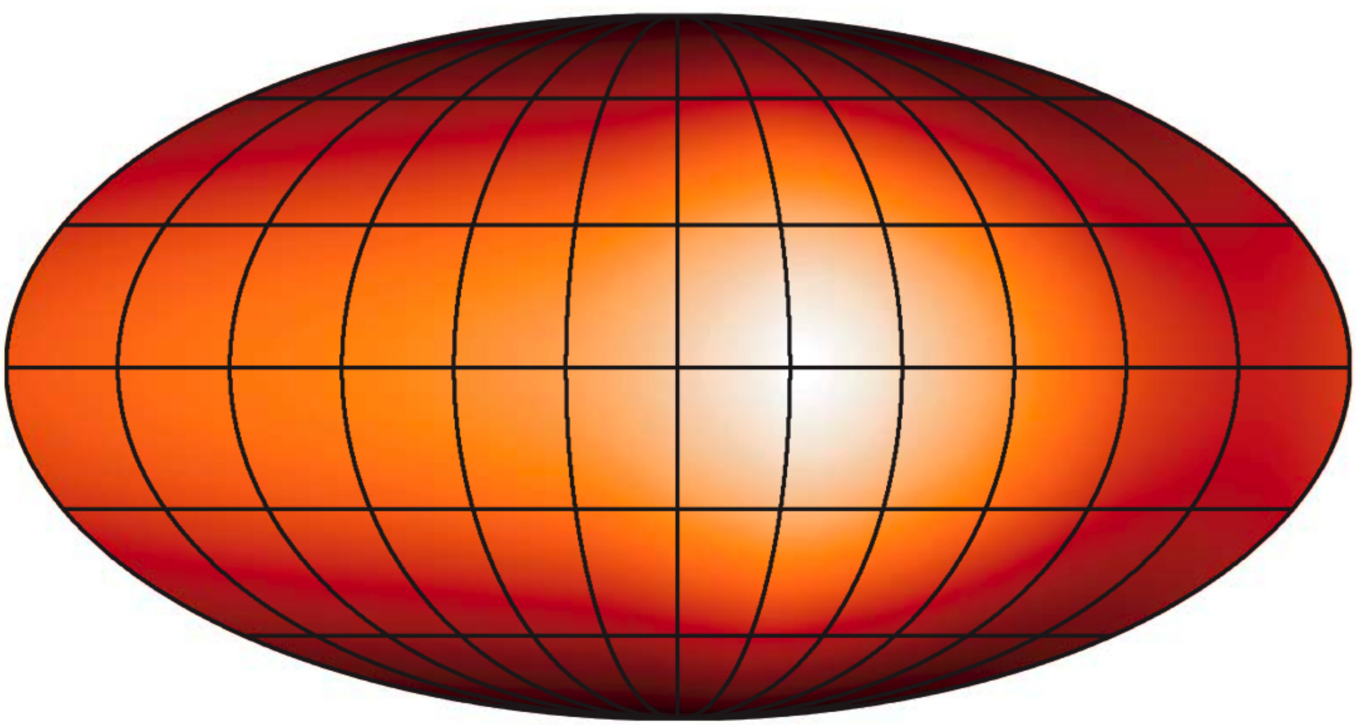
Backround

3D circulation models of tidally locked exoplanets typically predict a strong eastward equatorial jet, for example:



Showman & Guillot (2002)

which can explain the eastward offset of the hot spot subsequently inferred from Spitzer light curves of HD 189733b:



Knutson et al. (2007)

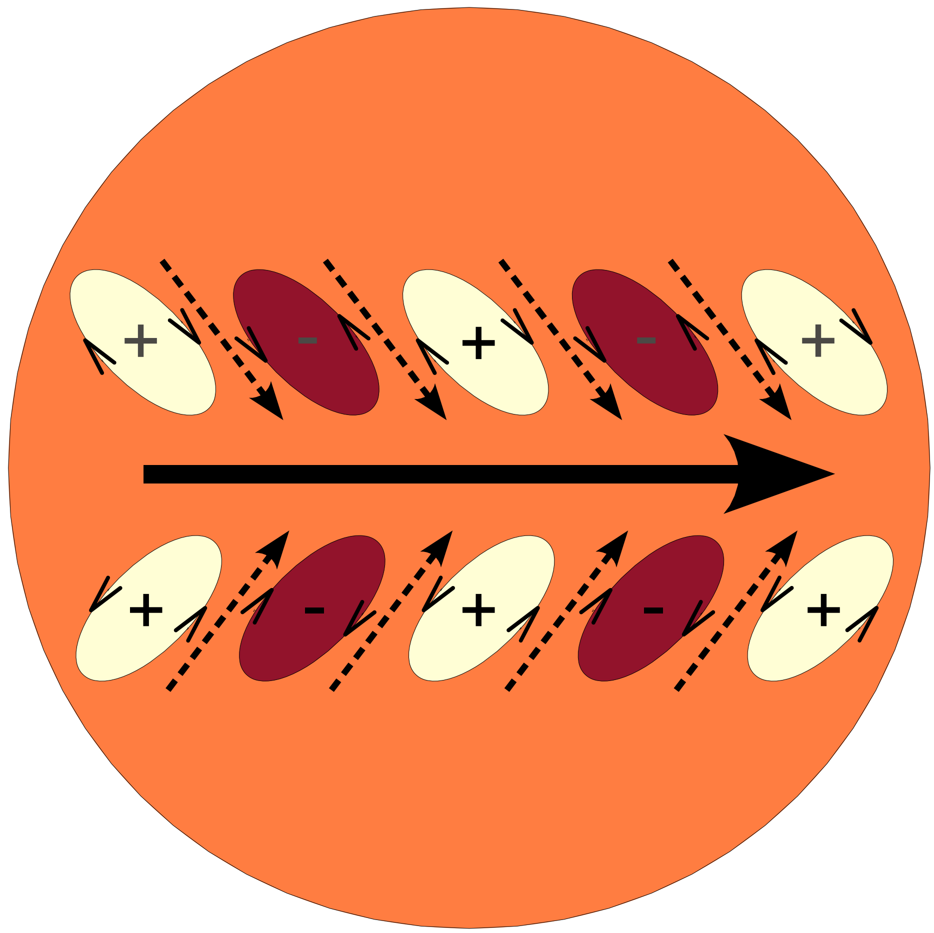
But, what is the mechanism for generating the equatorial eastward jet in these models? That has been a puzzle up to now.

Qualitative mechanism

To obtain superrotation, one wants eddies tilted like this:

Northern hemisphere: northwest-southeast

Southern hemisphere: southwest-northeast



These velocity tilts imply that equatorward-moving air has eastward eddy velocity while poleward-moving air has westward eddy velocity. Together, they induce a Reynolds stress[1] that pumps momentum equatorward.

But what could cause such tilted eddy structures? As we now show, they result naturally from standing, planetary-scale Rossby and Kelvin waves induced by the day-night heating/cooling pattern on a tidally locked planet.

[1]That is, $\overline{u'v'} < 0$ in northern hemisphere and $\overline{u'v'} > 0$ in southern hemisphere, where u' and v' are deviation of east-west (zonal) and north-south (meridional) wind components from their longitudinal means, respectively, and the overbar denotes a longitudinal average.

A simple model

We investigate the response to day-night thermal forcing in the simplest possible context. Imagine a two-layer model, with constant densities in each layer, where the upper layer represents the stratified atmosphere and the lower layer represents the deeper atmosphere and interior. In the limit where the lower layer becomes infinitely deep and the lower layer winds and pressure gradients remain steady in time, this system reduces to the shallow-water equations for the velocity $\mathbf{v}(\lambda, \phi, \mathbf{t})$ and thickness $h(\lambda, \phi, t)$ of the upper layer:

$$\frac{d\mathbf{v}}{dt} + g\nabla h + f\mathbf{k} \times \mathbf{v} = \mathbf{R} - \frac{\mathbf{v}}{\tau_{\text{drag}}} \tag{1}$$

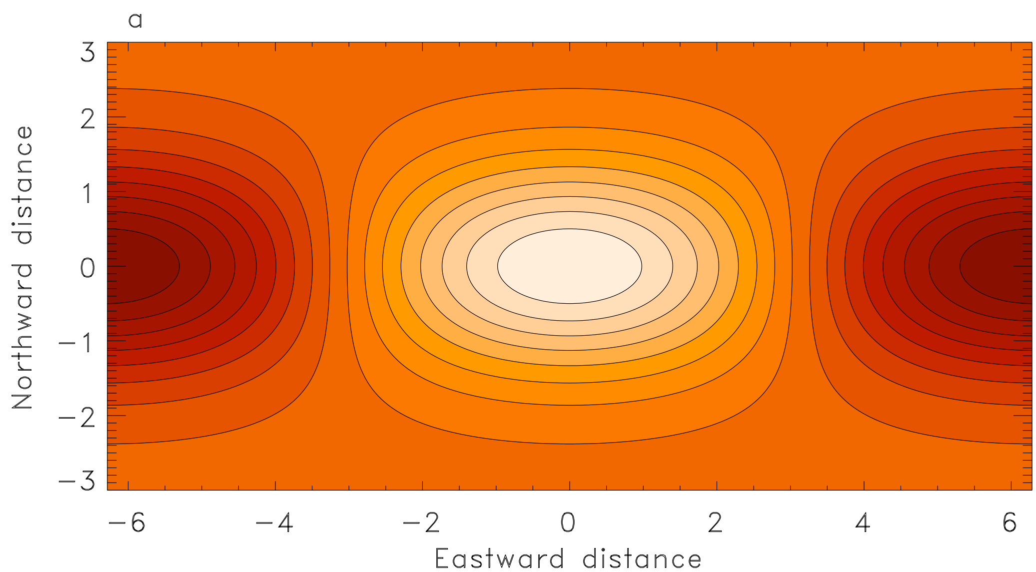
$$\frac{\partial h}{\partial t} + \nabla \cdot (\mathbf{v}h) = \frac{h_{\text{eq}}(\lambda, \phi) - h}{\tau_{\text{rad}}} \tag{2}$$

where λ, ϕ, t are longitude, latitude, time; g is gravity; $f \equiv 2\Omega \sin\phi$ is the Coriolis parameter (Ω is planetary rotation rate), and k is the vertical (upward) unit vector.

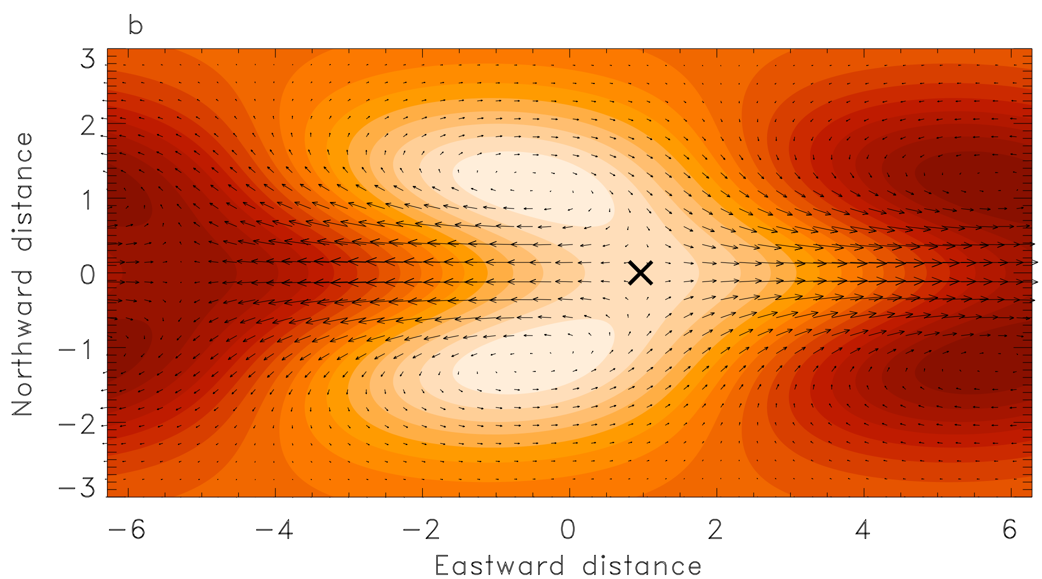
The interface between the layers represents an isentrope, across which mass flows in the presence of heating or cooling. Thus, heating/cooling is represented as a mass source/sink. We parameterize this as a relaxation to a radiative-equilibrium height field $h_{\text{eq}}(\lambda, \phi)$ over a characteristic radiative time constant τ_{rad}. h_{eq} is thick on the dayside and thin on the nightside (implying hot and cold, respectively). We also add drag, represented as a linear friction that relaxes the wind toward zero over a characteristic drag time constant τ_{drag}. R represents vertical momentum advection, and is $-\frac{Q\mathbf{v}}{h}$ in regions of heating and 0 in regions of cooling.

Analytic theory

We first linearize the equations and obtain steady, analytic solutions. Here is the assumed radiative-equilibrium height field (hot on dayside, cold on nightside):



This show an analytic solution, for parameter values appropriate to a hot Jupiter (orangescale is layer thickness, arrows are flow velocity):



This solution represents standing, planetary-scale Rossby and Kelvin waves.

Implications of Analytic theory for superrotation

Notice that the analytic solution has exactly the velocity tilts that are needed to generate equatorial superrotation!

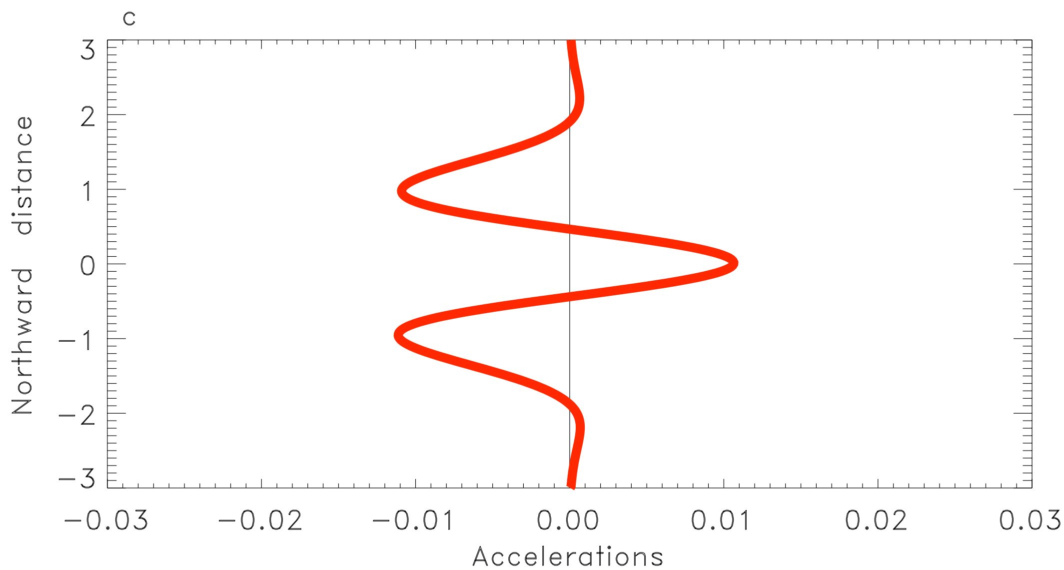
What is the mechanism for generating the phase tilts? There is more than one thing going on, but the dominant mechanism is simple:

- **The Kelvin waves straddle the equator; they manifest visually as the strip of eastward and westward wind along the equator, diverging from a point near the sub-stellar point and converging to a point near the anti-stellar point. The Rossby waves lie on their poleward flanks; they manifest as off-equatorial cyclones and anticyclones in the midlatitudes.**
- **Kelvin waves propagate to the east, whereas long Rossby wave propagate to the west.**
- **This differential east-west propagation causes an eastward phase shift of the height field at the equator and a westward phase shift of the height field at the midlatitudes.**
- **The result is a chevron pattern where the height contours, and velocities, tilt northwest-southeast in the northern hemisphere and southwest-northeast in the southern hemisphere.**

This mechanism is robust across a wide range of parameters, explaining why superrotation is so common in models of tidally locked exoplanets.

Implications of Analytic theory for superrotation, continued

We can further quantify the implications of the theory for superrotation by calculating the momentum flux from these analytical solutions. The divergence of the momentum flux gives the wave-induced accelerations of the mean flow that would result. This shows the answer, for typical parameter values:



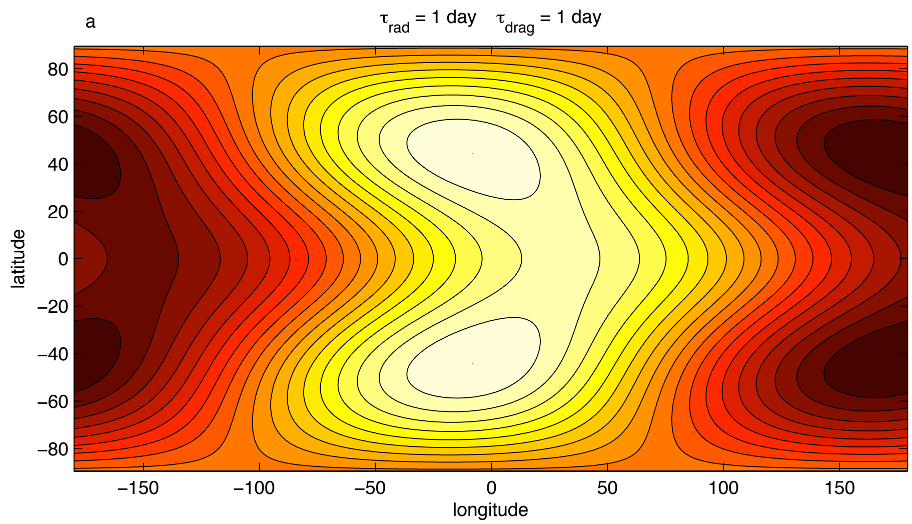
Thus, the analytic calculation predicts that the standing, planetary-scale Rossby and Kelvin waves will cause an eastward acceleration at the equator, typically along with westward acceleration at high latitudes. Equatorial superrotation would result.

The wave-induced acceleration at the equator predicted by this model is eastward—suggesting superrotation—across a wide range of $\tau_{\rm rad}$ and $\tau_{\rm drag}$ values.

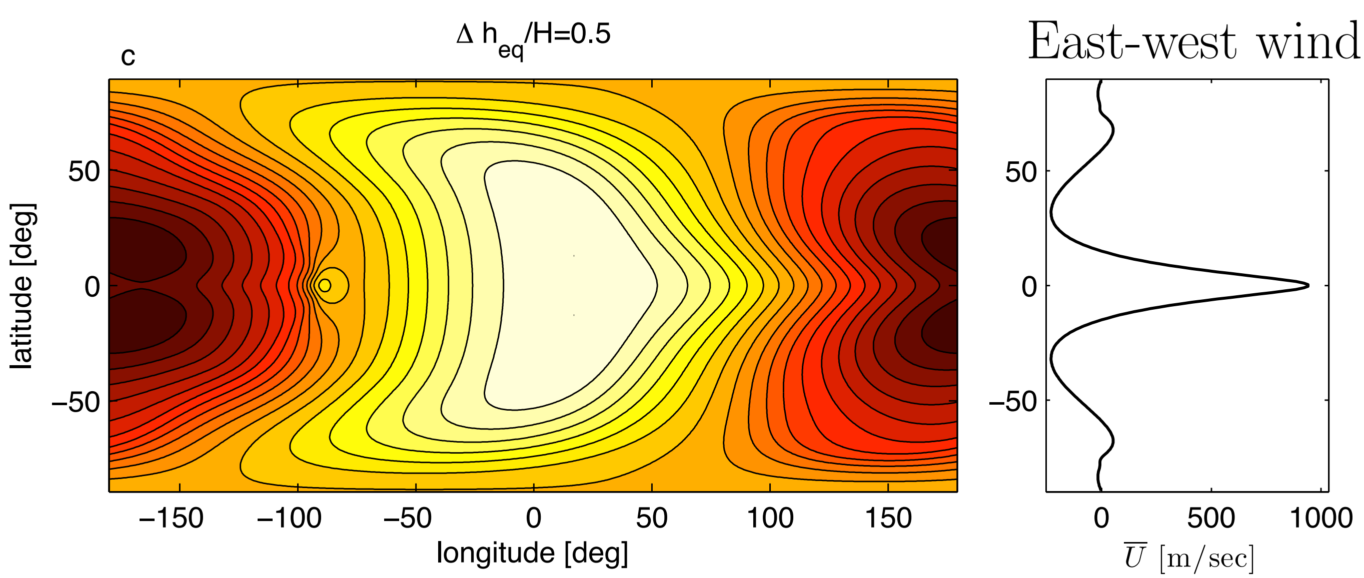
Numerical results

The analytic theory neglects nonlinearities. To increase the realism, we then solved the fully nonlinear form of the exact same system (the one-layer shallow-water equations with thermal forcing and drag) to see what happens.

When we make the amplitude very small, the results look very similar to the analytic theory. This is encouraging:



When we increase the amplitude so that day-night fractional thermal differences approach unity, appropriate to a strongly forced hot Jupiter, here is a typical result:

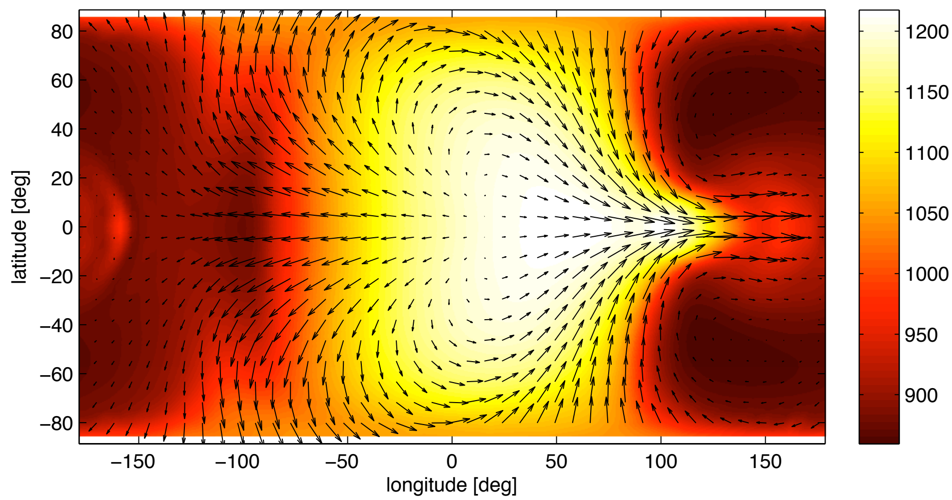


The flow is dominated by equatorial superrotation!

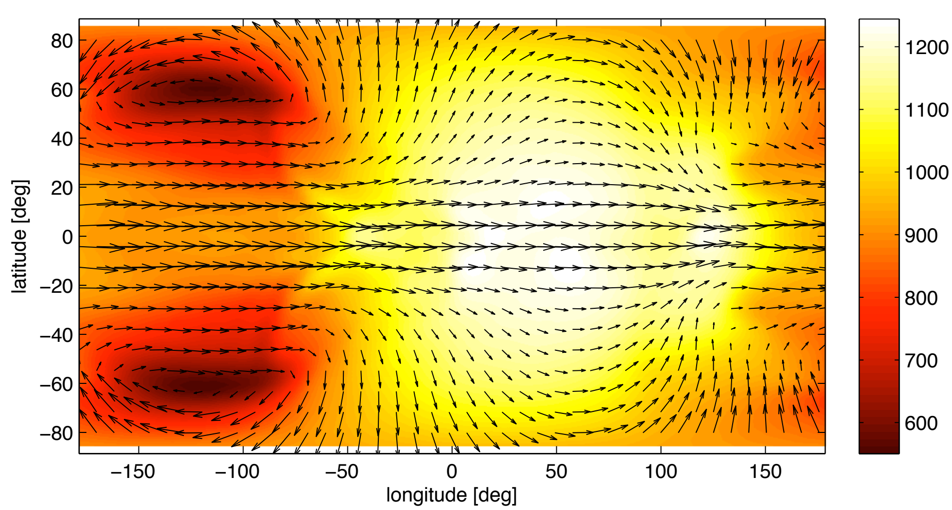
Three-dimensional model

But what is the connection between this theory and the results of full 3D general circulation models (GCMs)?

This shows the results from the 3D model of HD 189733b from Showman et al. (2009)—which couples the dynamics to a realistic treatment of non-grey radiative transfer—during the spin-up phase (this is shown at 30 mbar):



Notice the striking similarity between this flow pattern and the analytic theory. The Kelvin and Rossby wave structures are obvious, and they produce the necessary velocity tilts to generate superrotation. Once the model has spun up, superrotation indeed results:



Conclusions

- **Equatorial superrotation is a common outcome of 3D models of tidally locked exoplanets—both hot Jupiters and terrestrial planets—and explains the eastward offset of the hot spot in Spitzer light curves of HD 189733b.**
- **We have shown that equatorial superrotation results naturally from the interaction of thermally forced, planetary-scale standing Kelvin and Rossby waves with the mean flow. The mechanism is the differential east-west propagation of these waves—Kelvin waves to the east and Rossby waves to the west—which induces velocity tilts that then pump momentum from midlatitudes to the equator.**
- **We demonstrated this mechanism with an analytic theory and a suite of one-layer and 3D numerical simulations. In steady state, the eastward equatorial acceleration caused by the horizontal eddy fluxes is balanced by westward acceleration caused by drag and/or vertical eddy-momentum transport.**
- **The waves have a latitudinal half-width controlled by the Rossby deformation radius, which is comparable to a planetary radius on typical hot Jupiters. This explains why hot Jupiter circulation models tend to only exhibit a small number of wide jets. Since the waves, and the jet they produce, are planetary in scale, this also explains why typical hot Jupiter GCMs require only modest spatial resolution.**
- **Our theory provides, for the first time, a fundamental explanation for the basic circulation regime seen in most 3D models of tidally locked exoplanets. See Showman & Polvani (2011), arXiv 1103:3101 for the preprint.**