What’s the Matter with Vega?:
The Next Generation of Model Atmospheres for Rapidly Rotating Stars

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What's the Matter with Vega?

Solar and Heliospheric Observatory (SOHO) 
Michelson Doppler Imager (MDI)

http://sohowww.nascom.nasa.gov/bestofsoho/images/large/mdi20031028.jpg

Very narrow band
Ni I 6768 Å

2003/10/28 06:24 UT

http://sohowww.nascom.nasa.gov/bestofsoho/images/large/mdi20031028.jpg

What's the Matter with Vega?
Limb Darkening Basics

(a) Deeper, hotter layers are visible near the disk center

(b) Shallower, cooler layers are visible near the disk limb

 Isothermal atmospheres do not exhibit limb darkening

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Oblate Saturn Viewed from the Hubble Space Telescope
Resolved Rapid Rotating Stars from Interferometry

*Disk of Regulus (B7 V) resolved as ellipsoid by CHARA (McAlister et al. 2005). Axial ratio: 1.32±0.02

*Disk of Achernar (B3 Vpe) resolved as ellipsoid by VLTI (A. Domiciano de Souza et al. 2003). Axial ratio: 1.56±0.05

*Disk of Altair (A7 V) resolved by CHARA (J. Monnier et al. 2007).

*Disk of Rasalhague (A5 III) resolved by CHARA (M. Zhao et al. 2009).

*Disk of Alderamin (A7 V) resolved by CHARA (M. Zhao et al. 2009).
Gravity Darkening vs. Limb Darkening

Gravity darkening: *Intrinsic to the star, a pole-to-equator effective temperature gradient resulting from rapid rotation. Local Teff on surface correlates with local gravity (e.g., $T_{\text{eff}} \propto g^{\frac{1}{4}}$)*

Limb darkening: An observer-dependent effect in which the intensity across a stellar surface varies due to a radial or depth dependent temperature gradient.

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A Pole-on Rapid Rotator: Vega (Aufdenberg et al. 2006)

Vega

Pole-on view (Earth view)

equator-on view

Vega

equatorial effective temperature: 7950±350 K

Vega

polar effective temperature: 10150±100 K

rotation period: 12.4±0.6 hours

inclusion: 4.7±0.3 degrees

equatorial diameter: 2.78±0.02 D☉

polar diameter: 2.26±0.07 D☉

The Sun

10150±100 K

7950±350 K

2.26±0.07 D☉

2.78±0.02 D☉
Vega is 47% more luminous than expected for its spectral type (A0 V)

AN EMPIRICAL H\(_\gamma\) LUMINOSITY CALIBRATION FOR CLASS V–III STARS

Christopher G. Millward and Gordon A. H. Walker
Geophysics and Astronomy Department, University of British Columbia

Vega brighter than mean W(H\(_\gamma\)) - \(M_V\) relation

Fig. 6.—Mean \(W(H_\gamma) - M_v\) relation
Vega is a Pole-on Rapid Rotator (R. Gray 1985, 1988)

Pole-on rapid rotators appear more luminous, have same color as slow rotators

From Tassoul (1978) “Theory of Rotating Stars”
originally from Maeder & Peytremann (1970)
What’s the Matter with Vega?

K-band
\( \lambda = 2.2 \, \text{\textmu m} \)

Fiber Linked Unit for Optical Recombination

beam combiner

http://www.chara.gsu.edu/CHARA/Slides/CHARAoverview.pdf
A Rotating Model Fit to the Visibility Data

Best Fit Parameters

- $T_{\text{eff}}$ (pole) = 10250 K, B9 spectral type
- $T_{\text{eff}}$ (equator) = 7900 K, A8 spectral type
- Log(g) (pole) = 4.10
- $\omega$ (angular break-up fraction) = 0.91±0.03
- $\theta_{\text{equator}}$ = 3.33±0.03 milliarcsecs

Best Fit $\chi^2 = 1.36$
A Comparison of Parameters for Vega from Aufdenberg et al. and Peterson et al.

The equatorial radii derived from CHARA (IR) and NPOI (Visible) data are inconsistent.

What's the Matter with Vega?
Vega’s Spectrum from Different Points of View

\[
T_{\text{eff}}(\text{pole}) = 10250 \text{ K} \quad \text{B9 spectral type}
\]
\[
T_{\text{eff}}(\text{equator}) = 7900 \text{ K} \quad \text{A8 spectral type}
\]


Bohlin and Gilliland (2004)
What's the Matter with Vega?

We then mosaicked the resulting images to produce the final 24 μm image of Vega's disk, shown in Figure 2. The mosaic does not show any saturation artifact at the center because of the subpixel dithers used in the observation: a pixel that was saturated in one image is partially overlapped by unsaturated neighboring pixels in an image from a different dither position. There were enough dither positions to provide partial coverage even at Vega’s position in the final mosaic. Because of this infilling of the saturated area, the effective exposure time in the central 600 is about half of that elsewhere. Photospheric removal at 70 μm was more straightforward. We registered the scaled reference PSF mosaic (HD 48915) to the Vega mosaic by centroiding, and subtracted. The results are shown in Figures 3a (coarse scale) and 3b (fine scale).

The MIPS 160 μm array suffers from a spectral leak caused by an internal reflection in the optical train allowing leakage from very blue and bright Rayleigh-Jeans sources to contaminate the signals at 160 μm. However, the spectral leak image is offset to one side of the true 160 μm image, and the brightness of the leak is proportional to the photospheric flux. Comparison of 160 μm images of stellar (blue) sources with images of asteroids shows that the near-infrared leak contributes very little to the 160 μm images on the opposite side of the source location. The predicted flux for Vega’s photosphere is 162 mJy, which is much fainter than the expected disk brightness at 160 μm. We took advantage of this situation by using only the half of the 160 μm image where the leak contribution is negligible. We also subtracted a scaled (red) 160 μm reference PSF (asteroid Harmonia) from the Vega mosaic, using the pointing information (accurate to <100) to register. The result is shown in Figure 4.

3. DISK MORPHOLOGIES AT 24, 70, AND 160 μm

We define our observed sensitivity based on the 1 μ background noise per pixel using the blank-sky area in the image. The 1 μ background noise in the PSF-subtracted image is 11 μ Jy arcsec−2 at 24 μm. The disk at 24 μm is symmetric and centered at the star position; no obvious asymmetry is seen in the image. At the 1 μ level, the 24 μm disk extends to r ∼ 4300 (330 AU) in radius. The total flux density (within the 1 μ contour) is ∼1.5 Jy (10%). This flux density value is in agreement with the IRAS 25 μm measurement. The quoted IRAS 25 μm flux density for Vega is ∼10.5 Jy (combining IRAS Point Source Catalog and Faint Source Catalog). Based on Kurucz models, Vega’s photosphere is ∼6.63 Jy at 25 μm. The relation between the IRAS quoted flux and Vega’s disk brightness is as follows:

- Aumann et al. (1984)
- Su et al. (2005)
A Debris Disk Around a Young Star (artist conception)
Amorphous Silicate Illuminated by Vega (Equatorial) Model Spectrum

Amorphous Silicate Illuminated by Vega (Equatorial) Model Spectrum

$\Delta T \simeq 100K$

M. Köhler (Univ. of Missouri)

Clues from Rotational Broadening of Spectral Lines

What's the Matter with Vega?

From Collins (1989)

Rotational Line Broadening

rotation speed km/s

Note: The image contains a diagram illustrating the effect of rotational broadening on spectral lines. The diagram shows how the shift in wavelength, Δλ, affects the peak intensity of the line, represented by the curves labeled with rotation speeds (0, 20, 40, 80 km/s). The wavelength is measured in Angstroms (Å). The diagram also includes a graph with the wavelength on the x-axis and the normalized peak intensity on the y-axis.
Takeda et al. (2008) find 175 km/s Equatorial Velocity


http://www.astro.uu.se/~oleg/structures.html
What's the Matter with Vega?

Takeda et al. (2008) Model Fits Many Neutral and Singly Ionized Lines

Our calculations indicate 270 km/s equatorial speed too high

![Graph showing atmospheric absorption in Vega's spectrum with labeled velocities.]

- $V_{equ} [\text{km/s}] = 270.354$
- $V_{equ} [\text{km/s}] = 115.682$
- Calculations indicate equatorial speed is too high.
... But Yoon et al. (2008) find 275 km/s Equatorial Velocity

The difference is additional macro-turbulence broadening of 10 km/s.
Meridional Circulation May Provide “Macro-turbulence”

Fig. 1.—Circulation in a uniformly rotating star.

Gridding 1-D models on to 3-D stars neglects velocity shear between latitudes.
Standard Roche-Von Zeipel Model Atmosphere

- $\omega$ Fraction of the angular break-up speed
- $\theta_{\text{equ}}$ Equatorial Angular Diameter
- $T_{\text{pole}}$ Effective Temperature at the pole
- $g_{\text{pole}}$ Surface gravity at the pole

$$R_{\text{equ}} = 107.48 \frac{\theta_{\text{equ}}}{\pi_{\text{hip}}}$$

$$R_{\text{pole}} = \frac{\omega R_{\text{equ}}}{3 \cos \left[ \frac{\pi + \cos^{-1}(\omega)}{3} \right]}$$

$$R(\vartheta) = 3 \frac{R_{\text{pole}}}{\omega \sin \vartheta} \cos \left[ \frac{\pi + \cos^{-1}(\omega \sin \vartheta)}{3} \right]$$

$$g(\vartheta) = \left[ g_r(\vartheta)^2 + g_\vartheta(\vartheta)^2 \right]^{1/2}$$

$$\frac{T_{\text{eff}}(\vartheta)}{T_{\text{pole}}} = \left( \frac{g(\vartheta)}{g_{\text{pole}}} \right)^\beta$$

- Standard Rotating Star Model
  1. Point-mass approximation for the potential
  2. Uniform (non-differential) rotation
  3. Stellar shape is an equipotential surface, the sum of gravitational and centrifugal potentials
  4. Von Zeipel gravity darkening law
  5. Interpolated 1-D atmosphere models

$$\Omega = \omega \Omega_{\text{crit}} = \omega \left[ \frac{8}{27} \frac{GM}{R_{\text{pole}}^3} \right]^{1/2}$$

$$V_{\text{equ}} = R_{\text{equ}} \Omega$$

$$i = \sin^{-1} \left[ \frac{v \sin i}{V_{\text{equ}}} \right]$$

$$\Sigma \approx 4\pi GM \left[ 1.0 - 0.19696 \omega^2 - 0.094292 \omega^4 + 0.33812 \omega^6 - 1.30660 \omega^8 + 1.8286 \omega^{10} - 0.92714 \omega^{12} \right]$$

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What’s the Matter with Vega?
Visualization of the results from the PHOENIX 3-D code (Hauschildt & Baron)

The voxel (volume element) grid has $65^3$ elements.

The intensity image is shown for $(\theta, \varphi) = (0°, 0°), (45°, 45°), (140°, 250°)$ and $(89°, 139°)$.

The intensities are mapped linearly to 255 shades of gray.

In spherical coordinates $(n_r, n_\theta, n_\varphi) = (65, 33, 65)$ voxels, an angular resolution on the stellar surface of about 5.5 degrees.


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What’s the Matter with Vega?
Self-Consistent Field (SCF) Model for a Rotating Star
Mass Distribution is Fully Self-Consistent with Effective Potential

\[ g = \nabla \Psi \]  
\[ \Psi = \Phi + \Phi' \]  
\[ \nabla^2 \Phi = -4\pi G \rho \]  
\[ \Phi' = \int_0^\infty \Omega^2(\varpi) \varpi \, d\varpi \]  
\[ \eta = \Omega_0/\Omega_{cr} \]  
\[ \Omega_e/\Omega_0 = 1/(1 + \alpha^2) \]  
\[ \Omega(\varpi) = \frac{\Omega_0}{1 + (\alpha \varpi/R_e)^2} \]  
\[ \Omega(\varpi) = \Omega_0 \left[ 1 + (\alpha \varpi/R_e)^2 \right] \]

Effective gravitational acceleration
Effective potential
Gravitational potential (Poisson’s Equation)
Centrifugal potential
Ratio of the angular velocity at the pole to the critical angular velocity
Ratio of the angular velocity at the equator to the angular velocity at the pole (differential rotation)
“Anti-solar” rotation: pole spins faster than equator
“Solar” rotation: equator spins faster than pole
One to Two Solar Mass SCF Differentially Rotating Models

<table>
<thead>
<tr>
<th>$M$, $\alpha$, $\eta$</th>
<th>$1,M_\odot$, 1.5, 1.55</th>
<th>$1,M_\odot$, 3.75, 3.58</th>
<th>$1,M_\odot$, 5, 4.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $1,M_\odot$ 0.93$R_\odot$</td>
<td>(b) $1,M_\odot$ 0.74$R_\odot$</td>
<td>(c) $1,M_\odot$ 0.77$R_\odot$</td>
<td></td>
</tr>
<tr>
<td>(d) $1.2,M_\odot$ 0.90$R_\odot$</td>
<td>(e) $2,M_\odot$ 2.93$R_\odot$</td>
<td>(f) $2,M_\odot$ 1.51$R_\odot$</td>
<td></td>
</tr>
<tr>
<td>1.2 $M_\odot$, 4, 3.74</td>
<td>2 $M_\odot$, 3, 5.64</td>
<td>2 $M_\odot$, 4.75, 5.9</td>
<td></td>
</tr>
</tbody>
</table>

Convective

Radiative


What’s the Matter with Vega?
More Massive SCF Models for Alpha Eri (Jackson et al.)

(A) $M=6 \ M_\odot$, $R_{eq} = 12.19 \ R_\odot$

(B) $M=9 \ M_\odot$, $R_{eq} = 11.73 \ R_\odot$

(C) $M=12 \ M_\odot$, $R_{eq} = 11.57 \ R_\odot$

(D) $M=15 \ M_\odot$, $R_{eq} = 12.41 \ R_\odot$

Trial SCF Model for Alderamin (Alpha Cep)

M = 1.9\,M_\odot, \eta = 1.60, \alpha = 0.08
i=26.5^\circ \text{ at 1.6 microns}
What's the Matter with Vega?

Trial SCF Model at higher inclinations
Searching for a Vega Model
SCF Model Shapes and Pole-to-Equator Temperature Profiles

6: M = 2.55M\odot, \eta = 2.05, \alpha = 1.50, \beta = 0.18
7: M = 2.55M\odot, \eta = 1.37, \alpha = 0.80, \beta = 0.12
8a: M = 2.52M\odot, \eta = 1.40, \alpha = 0.80, \beta = 0.08
7: $M = 2.55 M_\odot$, $\eta = 1.37$, $\alpha = 0.80$, $\beta = 0.12$

Too much UV flux, Temperature profile too hot
What's the Matter with Vega?

$8a: M = 2.52M_\odot, \eta = 1.40, \alpha = 0.80, \beta = 0.08$

Better UV Match; line blanketing is too strong
Future Work

1. Identify and quantify which radiative features are sensitive to which model features.

2. Find a best fit match of the SCF models to the spectrophotometric, spectroscopic, and interferometric data sets.

3. Include main-sequence evolutionary effects in the SCF rotating models.