Grid and Reference Frames
Global Astrometry with SIM

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Grid

- SIM Grid solution is a global, one step LS adjustment of ~200K unknowns in ~300K equations
- Purpose of the Grid:
  - Determine instrument calibration and baseline orientation parameters for subsequent use in Narrow Angle and Wide Angle data processing
  - Establish SIM Reference Frame (SIMRF) better than 1 μas
  - Additional science (e.g., gravitational deflection) – you are encouraged to invent your own!
- Grid objects:
  - 1304 basic grid stars, RV vetted
  - 25—50 optically bright quasars
  - optionally, all NA targets and all reference stars
Regularized Delay Equation

\[ \delta d \equiv (B \cdot \delta s) + (\delta B \cdot s) + C + \delta F + \varepsilon \]

- Unknown baseline orientation (\(\delta B\), 2-vector), apparent position of star (\(\delta s\), 2-vector), path delay offset \(C\), calibration parameters (\(\delta F\), up to \(\sim 40\))
- Condition equation is severely underdetermined and can not be solved from a single measurement or a single tile, hence all sky, global solution needed

\[ \sigma_{\text{mission average per star}} = M \cdot \sigma_0 \]

where \(\sigma_0\) is single delay measurement precision (\(\sim 14\) \(\mu\)as), \(M\) is grid multiplier
- If the condition equations were perfectly conditioned, \(M \sim 1/\sqrt{N} \sim 0.07\), but in reality \(M \approx 0.26\) – the loss of condition comes from a coupling of \(\delta B\) and \(\delta s\) unknowns in the finite FOR
Correlated astrometric parameters

- The power spectrum of random errors defined by scalar or vector spherical harmonics is “red”, i.e., most error comes in large-scale perturbations
- Astrometric errors are *positively* correlated across the sky
- Differential Wide Angle mode can give a factor of 2 improvement in precision wrt global accuracy
Parallax zero-point error

- In a typical realization of grid, almost all parallax errors have the same sign because of a dominating zero-point error.
- A relatively small number of quasars (25—50) constrain low-order spherical harmonics and lead to dramatically better grid accuracy in parallax.
Quasars in the Grid

- USNO selected ~110 optically bright, low-variable quasars
- Simulations and covariance analysis of grid solutions with only 23 quasars reveal the benefits of quasar constraints:
  - Overall parallax accuracy improves ~17%
  - Parallax zero-point error improves ~60%
  - Greatly improved confidence intervals of parallax mission performance, e.g., the 0.99 confidence level on parallax error drops from 8.05 μas without quasars to 3.97 μas with only 23 grid quasars
  - The SIM Reference Frame (SIMRF) will be inertial to ~1 μas/yr in residual spin and ~1.7 μas in residual rotation
  - Some harmful systematic errors reduced by a factor of 5, e.g., certain systematic navigation errors and stellar aberration corrections
Why Grid performance is important?

• Correlated zonal errors propagate 100% into Wide Angle astrometry

• Accurate and inertial SIMRF entails fundamental (and free for you) science, for example
  – Galactic rotation and Galactocentric acceleration of the Sun
  – Gravitational bending of light
  – Constraints on relic gravitational waves
  – Speculatively, rotation of the Universe

\[ g = h \cos \omega t (xx - yy) \]
Wide Angle astrometric performance

Q: Can WA accuracy be better than the Grid?
A: Yes, but only slightly, only for bright stars and at a cost

Q: Can single visit integration time be traded for number of observations?
A: Yes, as this plot shows

Q: Does the measurement precision depend on the position within a tile?
A: Yes, see backup slide

Mission-average WA astrometric accuracy of proper motions as function of number of observations and single measurement precision
Optimized schedules

Q: Can accuracy be gained by clever scheduling of a given number of observations?
A: Absolutely, see the plot for a SVD-based optimization on parallax accuracy
Backup Slides
Grid multipliers

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Propagation of FDE

- Field-dependent instrument parameters will be determined in the grid and applied to WA delays
- Field-dependent errors sharply increase at the edge of the FOR (tile)