Planetesimal Aggregation Driven by Impulse Magnetic Fields in Protoplanetary Nebulae

I. Túnyi[1], P. Guba[1,2], L. E. Roth[3] and M. Timko[4]

[1]Geophysical Institute, Slovak Academy of Sciences, Bratislava, Slovakia
[2]Faculty of Mathematicss, Physics and Informatics, Comenius University, Bratislava, Slovakia
[3]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, USA
[4]Institute of Experimental Physics, Slovak Academy of Sciences, Kosice, Slovakia

1. Introduction

- Effects of electrostatic lightnings and magnetic flares, namely fast melting, fast cooling and magnetization of chondrules, were investigated previously by a number of authors [see, e.g., 1-3]
- Recently [4], we put forward yet another effect associated with an impulse magnetic field: a coalescence of magnetized planetesimals due to forces of attraction connected with their remanent magnetization

Basic ideas underlying this mechanism:

- An impulse magnetic field can be viewed as being induced by a temporally isolated surge in the flow of charged particles, similar to the atmospheric lightning or electrostatic discharge
- Supposing such a strong impulse electric current passes along the lines of magnetic force in the protoplanetary nebula, a powerful magnetic field is generated
- Planetesimals containing ferromagnetic components gain hard remanent magnetization, and are attracted to the axis of instantaneous virtual electric conductor, stick together and coalesce into larger units

2. Aims

- To examine some quantitative aspects relevant to the nebular lightnings as a plausible candidate mechanism for producing space-localized, short-term magnetic fields
- To give quantitative support for our accretion model put forward previously [4, 5], in which a presence of impulse magnetic field was of a key necessity to form ferromagnetic accretional kernels

To accomplish these, two independent calculations based on

- **A classical model of electric conductor**
- **Lienard–Wiechert electromagnetic potentials**

have been performed

3. A classical model of electric conductor

Biot–Savart–Laplace law:

$$B(r) = \frac{\mu}{2\pi}\frac{1}{r} nevS$$

μ – magnetic permeability of ambient dust

r – perpendicular distance from the axis of conductor

n – dust density

e – electron charge

v – average particle speed

S – cross-section area of the conductor

The highest intensity occurs on the conductor surface:

$$B_c = B(r = r_c) = 2\pi \times 10^{-7} J r_c$$

J – current density

r_c – channel radius

Problem: Estimation for J

- Data on electrons accelerated to relativistic energies at travelling shocks: a value of $2.05 \times 10^{-8}\,\mathrm{A\,m^{-2}}$ for the fair-weather nebula (inferred from Saturn's inner magnetosphere [6])
- Scaling by a factor of 10^{17} derived from lightning events in the terrestrial atmosphere [7]

Altogether then, we get

$$r_c = \frac{10^7}{2\pi}\frac{B_c}{J} = 7.76 \times 10^{-4} B_c$$

for the corresponding radius of the lightning channel

- The greatest uncertainty along the way to obtaining this result is, of course, the scaling relation for J
- Nevertheless, it clearly suggests the possibility of strong, localized magnetic fields in the protoplanetary nebula: magnetic pulses ranging in intensity as high as 10 T require that the large discharges have thickness about 1 cm

4. Lienard–Wiechert electromagnetic potentials

In the Lorenz calibration, the scalar and vector electromagnetic potentials associated with spatially isolated system of electric charges $\rho(\mathbf{x},\mathbf{t})$ and currents $\mathbf{j}(\mathbf{x},\mathbf{t})$ are given by

$$\phi(\mathbf{x},\mathbf{t}) = \frac{\mu}{4\pi}\int\frac{\mathbf{j}(\mathbf{x}',\mathbf{t}')}{|\mathbf{x}-\mathbf{x}'|}\mathrm{d}^3\mathbf{x}',$$

$$\mathbf{A}(\mathbf{x},\mathbf{t}) = \frac{1}{4\pi\epsilon}\int\frac{\rho(\mathbf{x}',\mathbf{t}')}{|\mathbf{x}-\mathbf{x}'|}\mathrm{d}^3\mathbf{x}'$$

where

$t' \equiv t - |\mathbf{x}-\mathbf{x}'|/\mathbf{v}$
v – speed of electromagnetic waves in a given medium
μ – magnetic permeability of a given medium
ϵ – electric permitivity of a given medium

The corresponding electric and magnetic fields are then given by

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial\mathbf{t}}$$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

Simple case: electric charge Q localized to a point

- a point parametrized through $\mathbf{z}(t)$
- then $\rho = Q\delta[\mathbf{x} - \mathbf{z}(t)]$ and $\mathbf{j} = \rho\mathbf{u}(t) = Q\mathbf{u}(t)\delta[\mathbf{x} - \mathbf{z}(t)]$, where $\mathbf{u} = \mathbf{z}(t)$ and δ is the Dirac delta function

In a particular case of zero charge acceleration, the electric field can be shown to take the form

$$\mathbf{E}(\mathbf{x}, t) = \frac{Q}{4\pi\epsilon} \frac{\mathbf{r}}{[q(\mathbf{r})]^3} \left(1 - \frac{u^2}{v^2}\right)$$

where $\mathbf{r} = \mathbf{R} - \mathbf{u}R/v$, $\mathbf{R} = \mathbf{x} - \mathbf{z}(t')$ and $q(\mathbf{r}) = [r^2 - (\mathbf{r} \times \mathbf{u})^2/v^2]^{1/2}$.

Further, constraining ourselves to the situation when electric charge moves with constant speed, the strength of magnetic field in a distance r measured perpendicularly relative to $\mathbf{z}(t)$ at certain instant in time t can be expressed as

$$B = \frac{Q}{4\pi\epsilon} \frac{1}{r^2} \frac{u}{v^2} \left(1 - \frac{u^2}{v^2}\right).$$

For the relativistic charge velocity of, say, $u = v/2$, this finally reduces to

$$r = 4.159 \times (Q/B)^{1/2}.$$

Thus, for a magnetic field of 10 T to be present in a distance of 1 cm from a moving charge, this requires the electric charge of about $4 \times 10^4\ e$.

5. Conclusion

- We have performed two independent studies towards the assessment of the magnetic field strength during nebular lightnings, making use of the well-known results of classical electromagnetic theory
- We have found that magnetic fields of several Tesla in magnitude can accompany the events of nebular lightnings
- Magnetic fields of such intensity are capable of magnetizing the ferromagnetic materials to the saturation levels, thus enabling the formation of dust assemblages

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