Angular Momentum Transport in Protoplanetary Disks

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Abstract

Magnetic fields may regulate the evolution of protoplanetary disks by providing mechanisms that remove angular momentum from the gas, enabling most of it to be accreted. These mechanisms include MHD turbulence involving a disordered, small-scale field and hydromagnetic outflows driven by an ordered, large-scale field that threads the disk. Because of the low ionization fraction of the fluid, the magnetic coupling is weak and strongly dependent on the conductivity and its spatial variation.

We present two examples of magnetic activity in protoplanetary disks:
(1) The vertical structure and linear growth of the magnetorotational instability
(2) The launching of magnetocentrifugally driven winds.

The method incorporates a realistic ionization profile –produced by cosmic rays, X-rays and radioactive decay- as well as the effect of dust grains. The contributions of ambipolar diffusion, the Hall conductivity and Ohmic resistivity are all taken into account.

Parameters

- a Ratio of the Alfven to the sound speed at the midplane
- χ_0 Magnetic coupling at the midplane
- σ_H Hall conductivity
- σ_P Pedersen conductivity

The following parameters are also relevant for the wind solutions:

- ε Normalized midplane radial speed
- $\sigma_\parallel$ Field-aligned conductivity

Methodology

System of Equations

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{V}) = 0$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} - \left\{2\Omega V_\phi \hat{\mathbf{r}} + \tfrac{1}{2}\Omega V_r \hat{\boldsymbol{\phi}}\right\} - \left\{\frac{V_K^2}{r}\hat{\mathbf{r}}\right\} + \frac{C_s^2}{\rho}\nabla\rho + \nabla\Phi - \frac{\mathbf{J}\times\mathbf{B}}{c\rho} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B}) - c\nabla\times\mathbf{E}' - \left\{\tfrac{3}{2}\Omega B_r \hat{\boldsymbol{\phi}}\right\}$$

$$\mathbf{J} = \sigma_\parallel \mathbf{E}'_\parallel + \sigma_H \hat{\mathbf{B}}\times\mathbf{E}'_\perp + \sigma_P \mathbf{E}'_\perp$$

which leads to

$$\mathbf{E}' = \frac{\mathbf{J}}{\sigma_\parallel} + \frac{\sigma_H}{\sigma_\perp^2}\frac{\mathbf{J}\times\mathbf{B}}{B} - \left(\frac{\sigma_P}{\sigma_\perp^2} - \frac{1}{\sigma_\parallel}\right)\frac{(\mathbf{J}\times\mathbf{B})\times\mathbf{B}}{B^2}$$

$$\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

MRI: This system is linearized about an initial state where the fluid is in Keplerian rotation and B is vertical. The terms in {} apply only in this case. The equations are integrated vertically from the midplane to the surface.

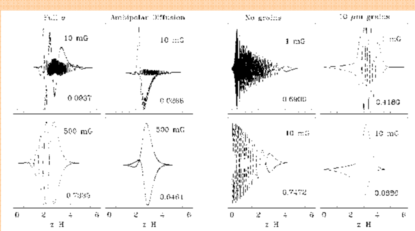
Jets: The system is integrated vertically from the midplane until the position of the sonic point and values of the variables there can be estimated. Then the solution is integrated backwards to a fitting point. The process is iterated until the full solution converges.

Magnetorotational instability

(1) Structure of the perturbations

(a) Conductivity regime

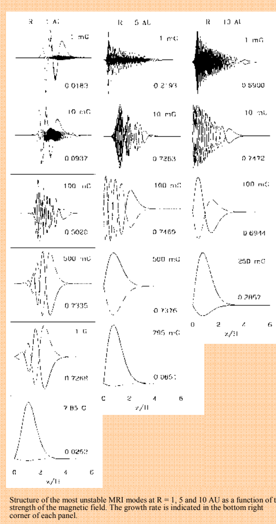
Left panels: Full conductivity modes have higher wavenumber, and grow closer to the midplane, than modes in the ambipolar diffusion limit.
Right panels: When dust grains are present, both the wavenumber and growth rate of the perturbations are reduced and the dead zone is more extended.



Structure of the most unstable modes of the MRI for the minimum-mass solar nebula model as a function of the magnetic field strength (top right corner of each panel). The maximum growth rate (ν_{max}) is shown in bottom right corners. Solid lines display δB_r and dashed ones correspond to δB_ϕ. *Left panel*: At R = 1 AU for different configurations of the conductivity tensor. *Right panel*: R = 10 AU. The left column shows the case where dust grains have settled, while the right one displays results when 0.1μm grains are present.

(b) At different radii

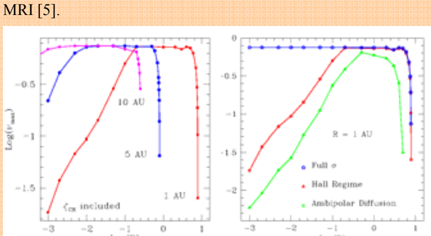
The wavelength of the perturbations increases with the strength of the field [1]. There is a central dead zone for B < 500 mG (1 AU) and B < 100 mG (5 AU). For R = 10 AU, ideal-MHD holds throughout the disk cross-section and modes grow at the midplane.



Structure of the most unstable MRI modes at R = 1, 5 and 10 AU as a function of the strength of the magnetic field. The growth rate is indicated in the bottom right corner of each panel.

(2) Growth Rates

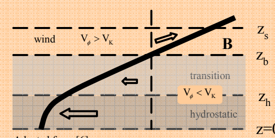
Left panel: Perturbations grow for B < 8 G (1 AU), B < 800 mG (5 AU) and B < 250 mG (10 AU). For a range of B, ν_{max} is of order the ideal-MHD rate (0.75Ω) [1, 5].
Right Panel:Hall conductivity modifies the growth rate of global unstable modes at 1 AU for all magnetic field strengths that support MRI [5].



Left panel: Growth rate of the most unstable modes of the MRI for R = 1, 5 and 10 AU as a function of the strength of the magnetic field for the minimum-solar nebula disk and including the ionization provided by cosmic rays, X-rays and radioactive decay.
Right panel: Growth rates of the fastest growing modes at 1 AU as a function of the strength of the magnetic field for different configurations of the conductivity tensor

Jets & Outflows

(1) Illustrative vertical disk structure



Adapted from [6]

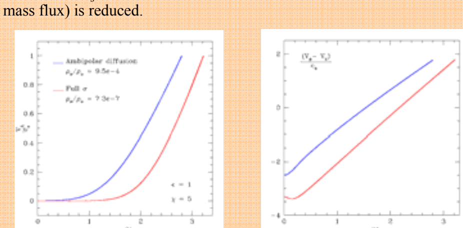
Representative field line and poloidal velocity of the fluid (arrows). Z_h denotes the disk scale height, z_b is the base of the wind and z_s is the sonic point.

In the hydrostatic layer, magnetic field lines are radially bent and sheared. The field takes angular momentum from the matter.

In the transition layer, field lines are nearly straight as magnetic energy dominates. They overtake the fluid at z_b and accelerate matter centrifugally.

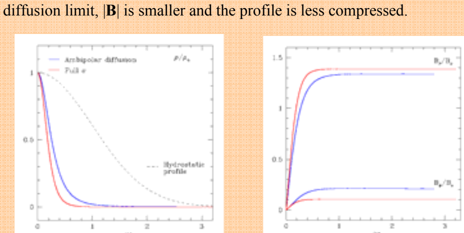
(2) Solutions and conductivity regime

(a) When Hall conductivity is included, the hydrostatic region is more extended and the sonic point lies higher above z=0 than in the ambipolar diffusion limit. The density ρ_s at the sonic point (a measure of the wind's mass flux) is reduced.



Normalized vertical (left panel) and azimuthal (right panel) velocity of representative wind solutions as a function of height above the midplane. The blue line shows the ambipolar diffusion limit while the red line displays a solution including Hall conductivity. The magnetic coupling (χ) and inward radial speed (ε) are the same in both cases.

(b) The density distribution of the solutions is steeper than the hydrostatic profile, evidence that the disk is magnetically confined. In the ambipolar diffusion limit, |**B**| is smaller and the profile is less compressed.



Similar to (a) above but showing the density (left panel) and components of the magnetic field strength (right panel). The hydrostatic density profile is also displayed.

Future Work

- Complete analysis of wind solutions using a realistic ionization profile.
- Match local wind solutions onto self similar global wind solutions [2, 6, 3].
- Model the fractions of angular momentum transported via the MRI and outflows, respectively, as a function of position in protoplanetary disks.
- Investigate the implications of the disk structure to planet formation and migration

References

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