On the migration rate of giant planets protocores

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Abstract:

We present a large set of numerical simulations aimed at investigating a behavior previously observed in earlier work, namely a significant discrepancy between the linear migration time estimate and its value as obtained from simulations, for planet masses in the range 5-15 earth masses, characteristic of the solid cores of giant planets. For such masses, numerical simulations yield a much longer migration time, and can even display, for some sets of parameters, an outward migration. Our simulations show that this offset scales with the gradient of the specific vorticity, increases with the disk viscosity, and has a maximum for a planet mass proportional to $H^{2.6}$ (H being the disk thickness). These findings are compatible with non-linear effects associated to the corotation torque acting upon the planet.

Numerical simulations versus analytical estimates

The most recent analytical estimate of the torque acting upon a small mass planet embedded in a disk with power law surface density and temperature profiles is given by Tanaka et al. (2002), and yields the following migration timescale in a disk with uniform aspect ratio $h = H/r$:

$$\tau = (2.7 + 1.1\alpha)^{-1} q^{-1} \mu^{-1} h^2 \Omega_p^{-1}, \tag{1}$$

where q is the planet to star mass ratio, μ the disk to star mass ratio (namely $\Sigma a^2/M_*$, where Σ is the disk surface density at the planet orbit, which has radius a), and Ω_p is the planet angular frequency.

This expression has been obtained for a three dimensional isothermal disk, taking into account:

- The differential Lindblad torque acting on the planet (i.e. the torque that stems from the planet wake).
- The corotation torque (i.e. the torque that comes from the horseshoe region drag), assumed to be unsaturated.

D'Angelo et al. (2003) have tried to verify the analytical expression of Tanaka et al. (2002) by means of 3D numerical simulations. They used the code NIRVANA with a hierarchy of nested grids at the planet location to achieve a high resolution in the planet vicinity.

Their results are displayed on Fig. 1. They show that:

- at low mass ($M_p < 4\ M_\oplus$) one recovers the analytical estimate by Tanaka et al.
- at large mass ($M_p > 100\ M_\oplus$) the migration rate is no longer described by this linear analytical estimate. The disk response for that regime is strongly non-linear, a gap is emptied around the planet orbit, and the planet undergoes a so-called type II migration.
- In between these two extreme regimes, for $4\ M_\oplus < M_p < 20\ M_\oplus$, the migration rate drops significantly with respect to the analytic prediction. In particular, for $M_p \sim 10 M_\oplus$, the migration rate is one order of magnitude below the analytical estimate. It is this large discrepancy that motivated our work. For brief we shall refer hereafter to this discrepancy as the "offset".

2D versus 3D numerical simulations

We investigated this behavior by means of 2D and 3D simulations. The former cannot give a reliable value for the torque, since the torque value depends on the potential smoothing length. However, 2D simulations are CPU inexpensive and allow to investigate the existence and behavior of the offset for a large set of disk parameters.

3D simulations are not affected by smoothening issues and therefore give reliable values for the torque. However, as they are much more computationally expensive, they do not allow a parameter space exploration with the same level of detail as 2D simulations.

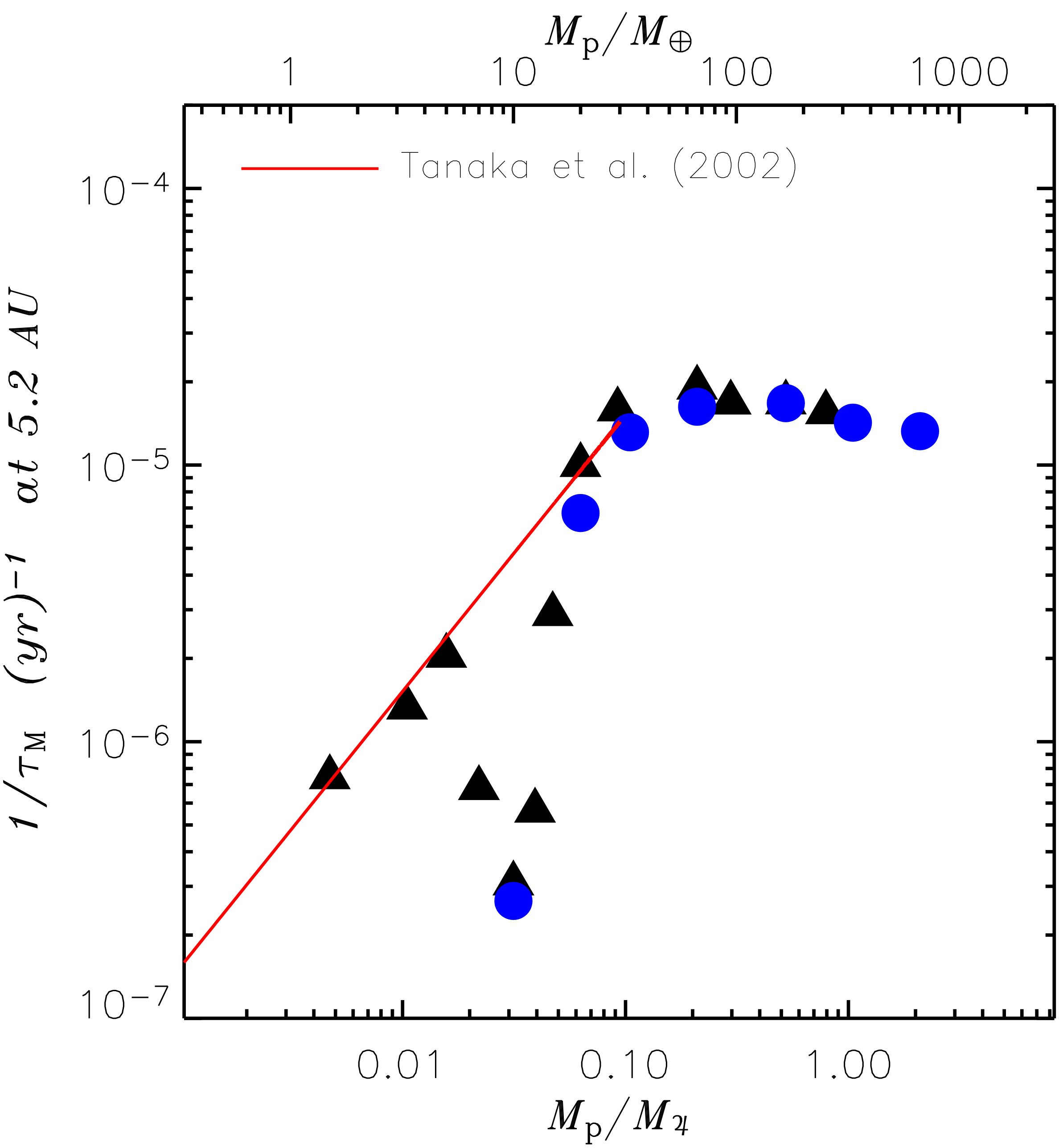

Figure 1: D'Angelo et al. (2003) numerical calculations vs. Tanaka et al. (2002) analytical predictions. The two different symbols correspond to different potential prescriptions, which both yield the same behavior.

Template disk parameters

In our 2D as well as 3D simulations, we consider a "template" disk with the following parameters:

- Fixed aspect ratio $h = H/r = 0.05$
- Power law surface density profile $\Sigma \propto r^{-\alpha}$, with $\alpha = 1/2$.
- Uniform kinematic viscosity ν, corresponding to an α parameter at the planet orbit $\alpha = 4 \cdot 10^{-3}$.

Any exploration of the parameter space is relative to this template. For instance, if we vary the disk thickness, we keep for the viscosity and surface density the prescriptions above.
This template corresponds to the disk parameters adopted by D'Angelo et al. (2003).

In 2D runs, the resolution is $N_\phi = 600$ and $N_r = 153$, each calculation is run for 100 orbits and for each disk parameter set, a set of 35 planet masses are run (ranging from $q = 10^{-6}$ to $q = 10^{-3.5}$).
The 3D runs have the same nested grids as described in D'Angelo et al. (2003)

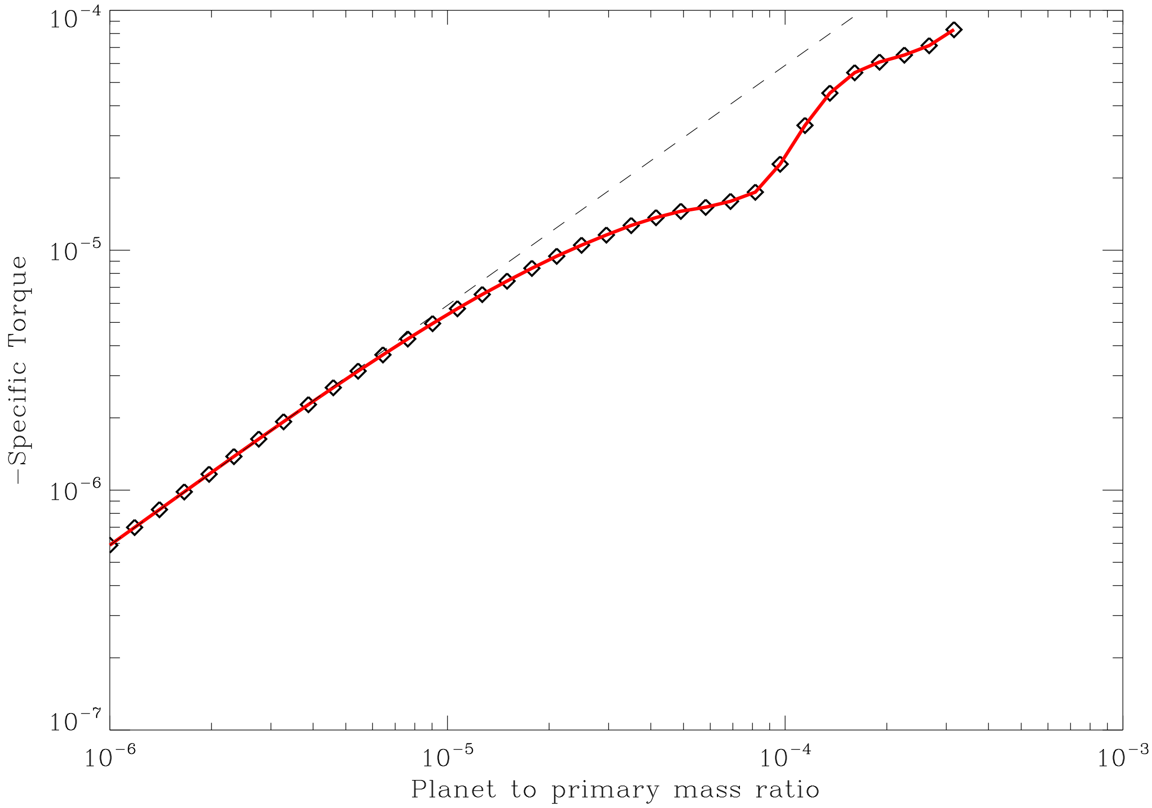

Figure 2: 2D Numerical simulations results for the template disk.

Template disk 2D results

The figure 2 shows the results obtained for the 2D runs with the 35 planet masses. The dashed line shows the linear extrapolation of the smallest mass result. The offset is present (although its depth and position is different from 3D simulations).

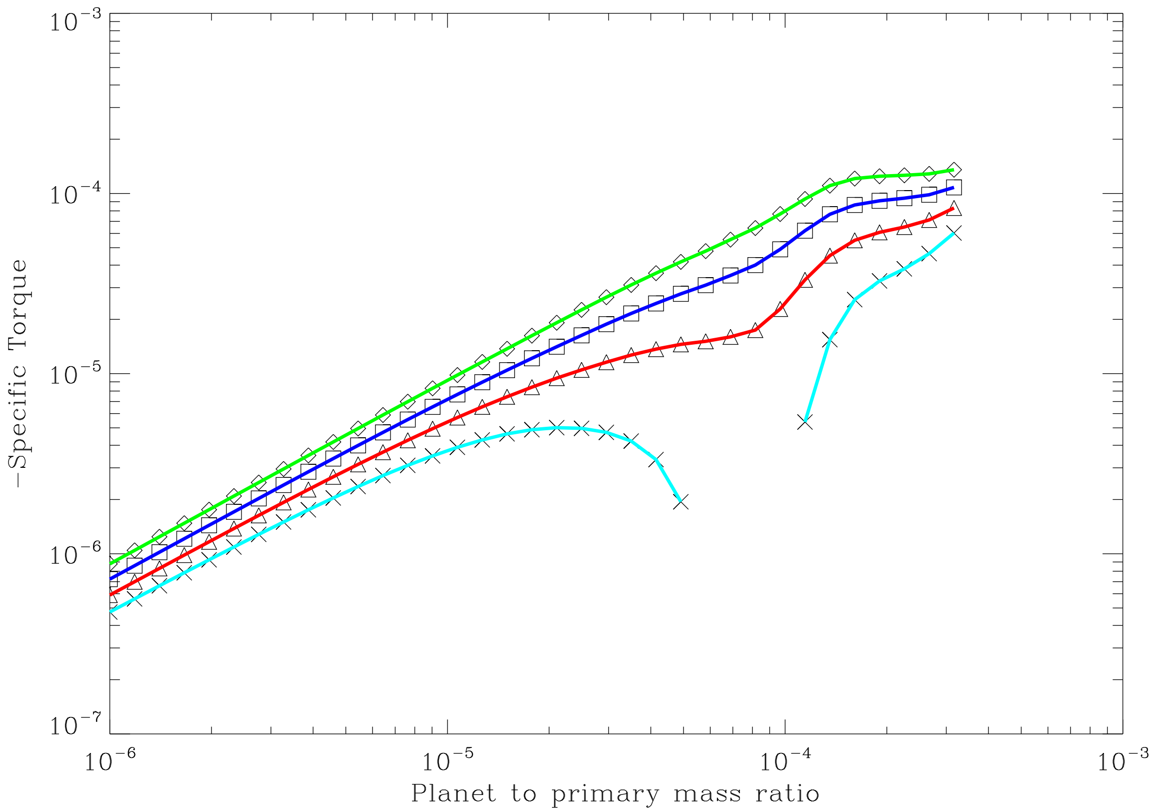

Figure 3: Results for different slopes: $\alpha = 3/2$ (green), $\alpha = 1$ (blue), $\alpha = 1/2$ (red, template set) and $\alpha = 0$ (cyan).

Dependency upon specific vorticity gradient

We have investigated what is the impact of the surface density slope α. We have repeated the template calculations with different slopes: $\alpha = 3/2$, $\alpha = 1$ and $\alpha = 0$. The results are reported on Fig. 3.

Fig. 4 shows the following quantity, for each slope:

$$y\text{-axis} = [T(q,\alpha) - T(q,\alpha = 3/2)]/q,$$

where $T(q,\alpha)$ is the torque acting on a planet with planet to primary mass ratio q, in a disk with surface density slope α. In addition, the dotted and dashed line show the $\alpha = 0$ curve scaled respectively by $2/3$ and $1/3$. Their good coincidence with the $\alpha = 1/2$ and $\alpha = 1$ curves respectively shows that the offset amplitude scales with $3/2 - \alpha$, i.e. the offset amplitude scales with the gradient of the disk specific vorticity.

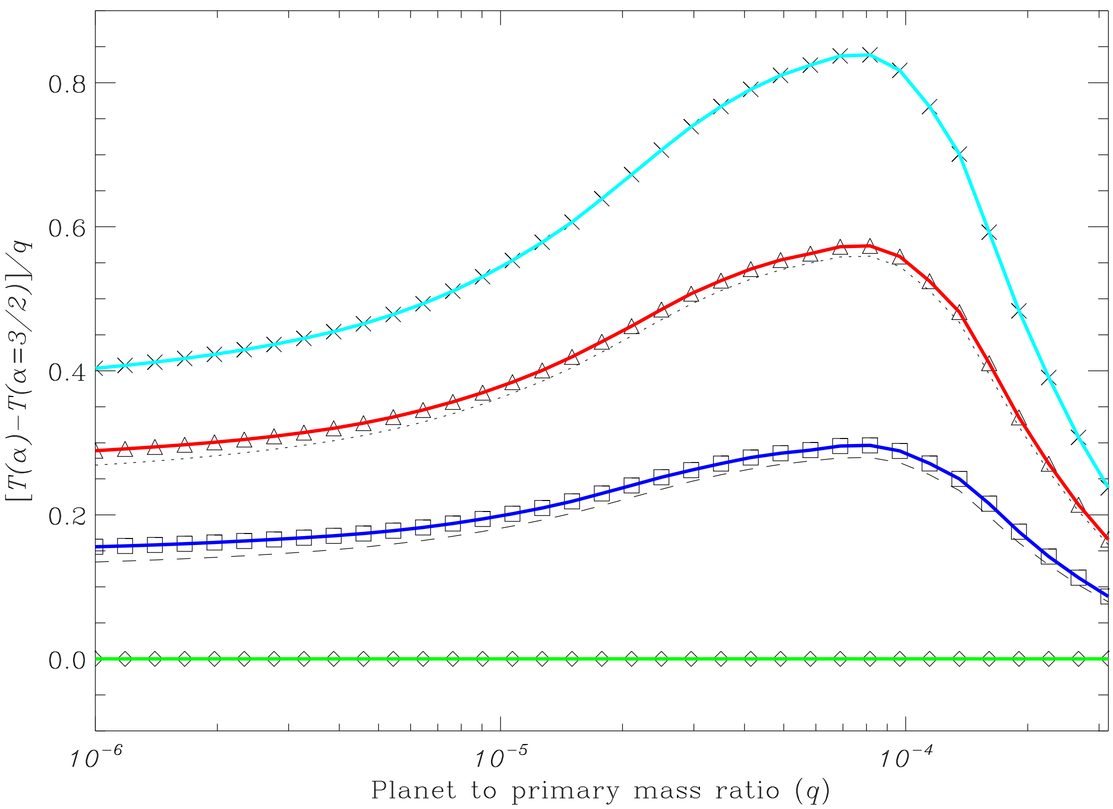

Figure 4: Scaled offset amplitude, w.r.t $\alpha = 3/2$ calculations. The color code is the same as for Fig. 3.

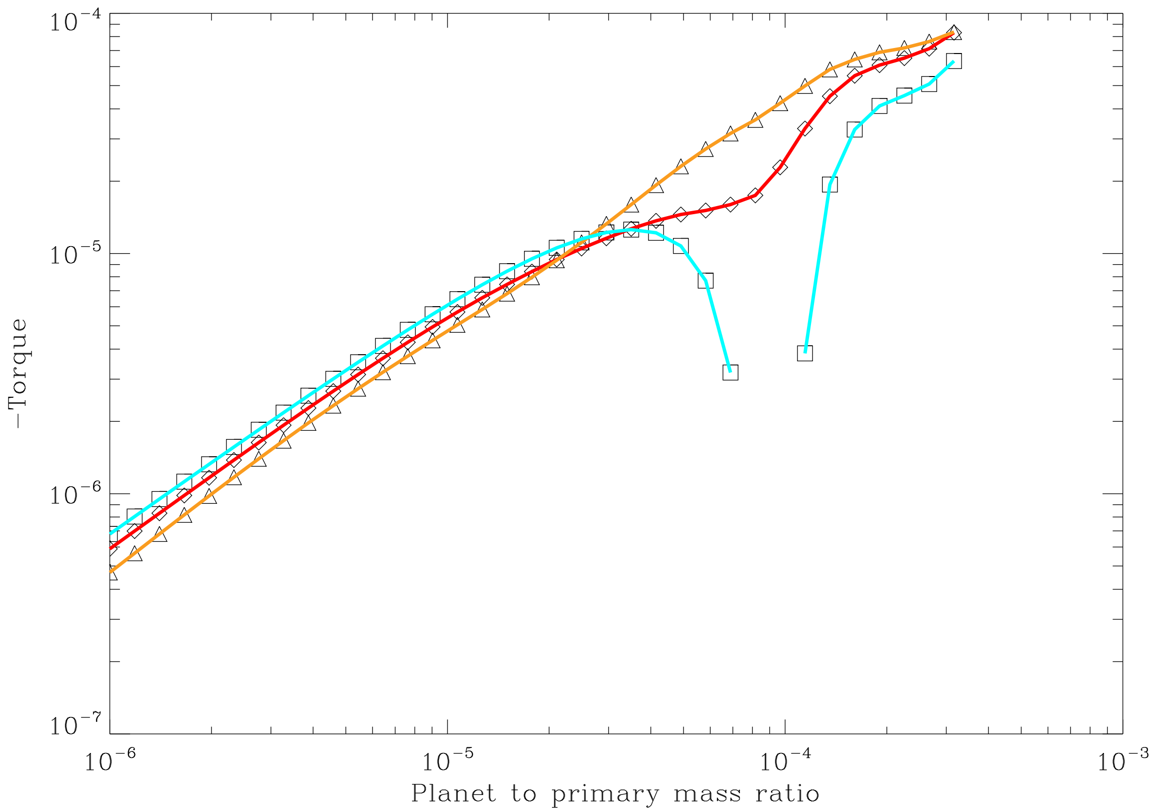

Figure 5: Low viscosity disk (orange), intermediate viscosity disk (i.e. template, in red), larger viscosity disk (cyan).

Dependency on viscosity

We have repeated the template runs with different values of the viscosity ($\nu = 5 \cdot 10^{-6}$ and $\nu = 2 \cdot 10^{-5}$). The results are shown on Fig. 5. The offset amplitude exhibits a strong dependence upon viscosity.

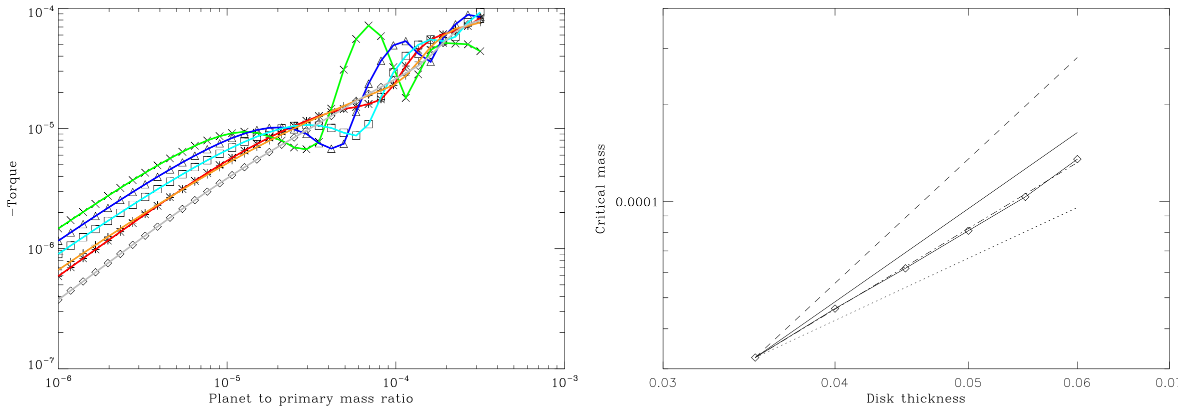

Figure 6: Left: Opposite of specific torque as a function of mass, for different disk thicknesses: $h = 0.035$ (green), $h = 0.04$ (blue), $h = 0.045$ (cyan), $h = 0.05$ (template, red), $h = 0.055$ (orange), $h = 0.06$ (gray). Right: position of minimum of $T_{\rm min}/q_{\rm min} - T/q$ as a function of disk thickness ($T_{\rm min}$ and $q_{\rm min}$ being respectively the torque and planet to primary mass ratio of the smallest mass planet. The dotted, dot-dashed, solid and dashed lines correspond to h^2, $h^{2.6}$, h^3 and h^4 dependencies.

Dependency on disk thickness

We have repeated the template runs with different disk thickness. The results are displayed on the Fig. 6. The offset minimum occurs at smaller masses in thinner disks. The right panel of Fig. 6 shows the mass q_m at the offset minimum as a function of the disk thickness. The non-linearity of the flow in the planet vicinity is controlled by the dimensionless parameter q/h^3 (Korycansky & Papaloizou, 1996), so if the offset corresponds to the onset of non-linear effects, we should expect a dependency $q_m \propto h^3$. This is close to the dependency that we measure from our calculations.

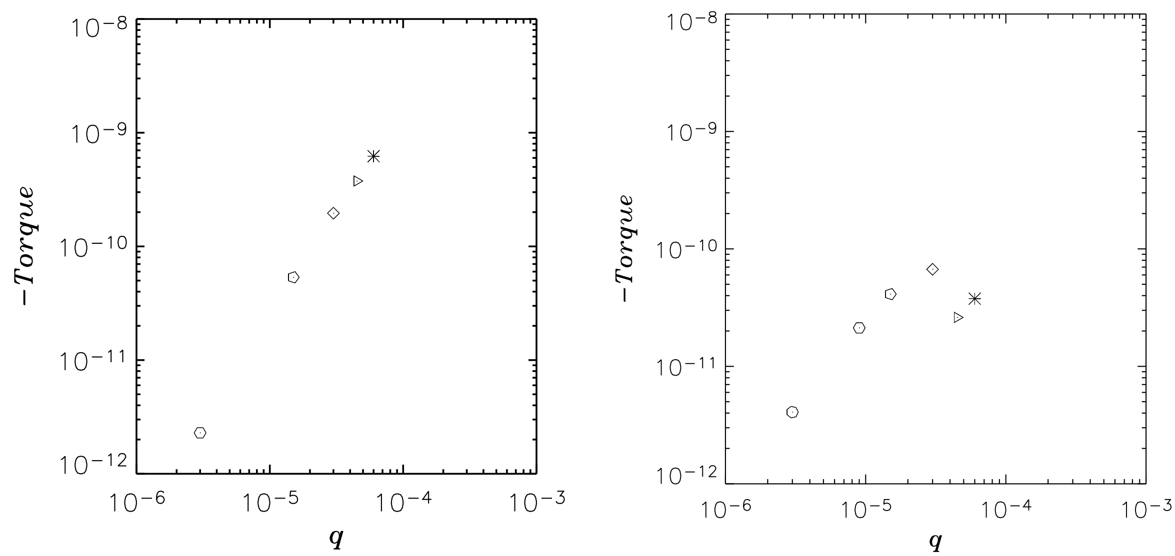

Figure 7: Left panel: $\alpha = 3/2$ slope; right panel: $\alpha = 0$ slope. The setup is the same as in D'Angelo et al. (2003), except that gas accretion onto the planets is not allowed.

3D results

We have repeated D'Angelo et al. (2003) calculations with a different slope ($\alpha = 3/2$). This corresponds to a vanishing gradient of specific vorticity. The results, which are preliminary as all calculations have not been completed yet, are compatible with the 2D results, and a scaling of the offset with the specific vorticity gradient.

Conclusions

Our findings indicate that the offset is due to an increase of the corotation torque above its linearly predicted value. Indeed, it scales with the surface density gradient, which is one characteristic of the corotation torque, and it depends on the viscosity (which is also the case of the corotation torque). This increase of the corotation torque corresponds to the onset of non-linear effects. This could have important consequences for planets with masses in the range $5 - 15\ M_{\oplus}$, which could have they migration halted or reversed in sufficiently shallow disks.

References

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Tanaka, H., Takeuchi, T. and Ward, W. R. 2002, ApJ, 565, 125